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A MODEL OF THE HELIOSPHERIC MAGNETIC FIELD CONFIGURATION

S.-I. AKASOFU, P. C. GRAY and L. C. LEE
Geophysical Institute, University of Alaska, Fairbanks, AK 99701, U.S.A.

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Abstract—A three-dimensional model of the magnetic field configuration in the heliosphere is constructed by assuming that the interplanetary magnetic field consists of four components, (i) the solar dipole, (ii) a large number of small spherical dipoles located along an equatorial circle just inside the Sun (representing the magnetic field line arcade), (iii) the field of the poloidal current system generated by the solar unipolar induction and (iv) the field of an extensive current disc around the Sun lying in the ecliptic plane. The magnetic field intensity at a distance of 1 A.U. (about 20 $R_\odot$ above the ecliptic plane) is normalized to fit the observed spiral configuration.

1. INTRODUCTION

The Archimedean spiral configuration of the interplanetary magnetic field (IMF) lines was first suggested by Parker (1958) and was subsequently studied by a number of workers in understanding various interplanetary phenomena, such as the propagation of solar protons, cosmic ray particles and of the blast waves.

The spiral configuration of the IMF is intuitively understandable in the ecliptic plane, but a study of the magnetic field configuration in the entire heliosphere requires knowledge on how the electric currents responsible for the IMF are generated, how they close (or diffuse away from the heliosphere) and how the interstellar magnetic field interacts with the heliosphere. These problems were most recently discussed by Alfvén (1977).

In Parker's treatment and those which followed (Axford, 1972; Stenflo, 1972), it was assumed that the magnetic field configuration in the heliosphere is dipolar within a sphere of radius of $r = b = 5\times10^{11}$ cm ($\simeq 7\,R_\odot$) and that outside this sphere ($r > b$), the magnetic field $\mathbf{B}$ and the solar wind velocity $\mathbf{V}$ are related by $\mathbf{B} = \alpha\mathbf{V}$ and $\mathbf{B}_\theta = V_\theta = 0$. From Parker's equation for $\mathbf{B}$ one can easily see that r and ϕ components of the electric current in spherical coordinates are given by

$$J_r = \frac{B_b b^2}{4\pi r^2}[3\cos^2\theta - 1](\omega/V) \tag{1}$$

$$J_\phi = \frac{B_b b^2}{4\pi r^3}\sin\theta$$

and thus

$$\int_{r=b} J_r\,\mathrm{d}S = 0, \tag{2}$$

where the integral is over the entire surface of the sphere with radius b. This implies the electric currents generating $\mathbf{B}$ are distributed over the entire heliosphere such that J_r changes the direction at $\theta = 54.7°$ and the total net current across the sphere of radius $r = b$ is null. However, the generation mechanisms and the boundary condition is implicit in Parker's treatment. Further, the explicit assumptions of $B_\theta = V_\theta = 0$ at $r > b$ and $V = V_r$ have not been confirmed well above the ecliptic plane. In fact, the Pioneer 11 observations at heliographic latitudes up to 16° by Smith *et al.* (1978) suggest that the interplanetary magnetic field has a stretched dipolar structure along the equatorial plane ($B_\theta \neq 0$), extending well beyond the distance of the Earth. There are also some solar wind and magnetic field observations which indicate that the magnetic field lines from coronal holes in high heliographic latitudes reach the ecliptic plane at a distance of 1 A.U. For a review of this subject, see Hundhausen (1977). The magnetic field line tracing by using Type IV bursts also supports such a view (Fitzenreiter *et al.*, 1977).

In order to consider the origin of the B_θ component and for that matter of the other components, it is worthwhile to find out the distribution of the electric currents in the heliosphere and their dynamo processes. Alfvén (1977) has recently pointed out that the Archimedean spiral magnetic field line configuration should be associated with spiral electric current lines which are perpendicular to the magnetic field lines and suggested that the radial component of the spiral current arises from the EMF due to the sun acting as a unipolar inductor; see also Alfvén (1950, p. 160). He suggested also two possibilities about the radial

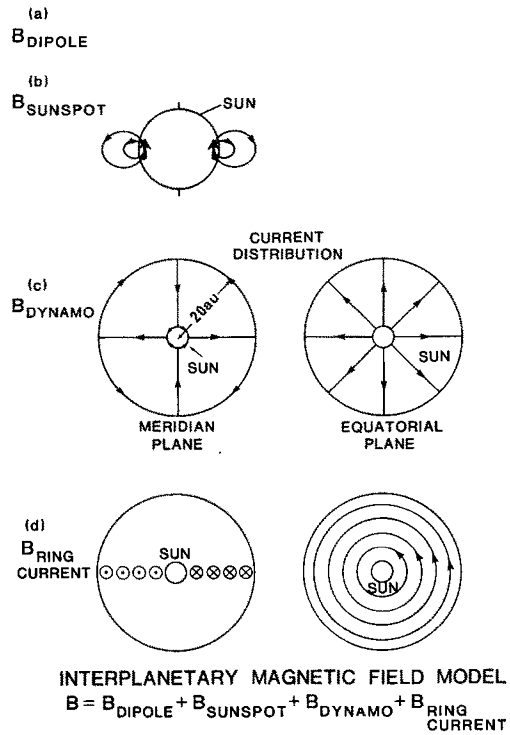

FIG. 1. SCHEMATIC ILLUSTRATION SHOWING THE FOUR COMPONENTS OF THE INTERPLANETARY MAGNETIC FIELD.

component of the current, either that it goes to infinity (becoming increasingly diffuse) or that it is connected to the current from the polar region of the sun along the outer surface of the heliosphere.

In this paper, we assume that the IMF consists of the following four components, and that they are confined in the heliosphere which is assumed to be a sphere of radius of 20 A.U. Figure 1 illustrates them schematically.

(i) The dipole component ($\mathbf{B}_{\text{dipole}}$).

(ii) The sunspot component ($\mathbf{B}_{\text{sunspot}}$). This component will be discussed in connection with the fourth component.

(iii) The dynamo component ($\mathbf{B}_{\text{dynamo}}$) arises from a poloidal current system illustrated in Fig. 1. In the ecliptic plane, this current has the radial component. Following Alfvén (1977), we assume that it is generated by a dynamo process as the result of the rotating sun in the dipole field. During an odd sunspot cycle (the north polar region has the S pole), the induced elective motive force is directed poleward on the photosphere, generating the radially outward current from the pole. It is assumed that this radial current from the pole reaches the north polar region of the heliosphere and then flows along the outer (spherical) surface of the heliosphere toward the ecliptic plane. This current is then connected to the inward-directed equatorial current. Alfvén (1977) estimated the total radial current in the ecliptic plane to be of the order of 3×10^9 A. Figure 1 refers to the even cycle situation.

(iv) The ring current component ($\mathbf{B}_{\text{ring current}}$) arises from a thin equatorial sheet current around the sun. It is assumed that this sheet current has a radial extent of 20 A.U. During an even sunspot cycle, this "ring current" is directed westward.

It should be noted that during an odd sunspot cycle, the polarity of the dipole field reverses, and the direction of all the currents reverses, resulting in the reversal of the magnetic field vector $\mathbf{B}$ as well.

The solar dipole moment $M_\odot$ is fairly well known ($\sim3.4\times10^{32}$ gauss cm^3). Its polar intensity on the photosphere is assumed to be 2 gauss, so that its equatorial intensity is 1 gauss. We have an extensive measurement of the IMF at 1 A.U. Its radial component is about 3–5 gammas on the average and decreases outward. It is reasonable to assume that the current sheet extends radially from the coronal region to the boundary of the heliosphere ($\sim$20 A.U.). If so, any reasonable current sheet which can produce 3–5 gammas at 1 A.U. has a magnetic moment ($\sim50\,M_\odot$) which is much greater than that of the solar dipole. Therefore, it is not possible to construct an IMF model in which all field lines of the equatorial sheet current can be connected to the solar surface, unless there is a strong magnetic field on the solar surface which does not, however, affect significantly the solar dipole field. We have solved this problem by placing 180 small spherical dipoles along a circle of radius of $0.8\,R_\odot$; the radius of each sphere is $0.1\,R_\odot$ and the magnetic field at its equator is 500 gauss. These small spherical dipoles constitute the so-called "magnetic field line arcade" just above the photosphere.

2. FORMULATION

In this section, we present an analytical expression for the four components of the interplanetary magnetic field: (i) the dipole component $\mathbf{B}_{\text{dipole}}$, (ii) the sunspot component $\mathbf{B}_{\text{sunspot}}$, (iii) the dynamo-driven component $\mathbf{B}_{\text{dynamo}}$, and (iv) the equatorial ring current component $\mathbf{B}_{\text{ring current}}$.

(i). *Dipole component* $\mathbf{B}_{\text{dipole}}$

The dipole component of the solar magnetic field can be written in cylindrical coordinates (z, ρ, ϕ) as

$$B_{z\ \mathrm{dipole}} = -\left(\frac{B_s r_1^3}{2}\right)(2z^2-\rho^2)(z^2+\rho^2)^{-5/2}$$

$$B_{\rho\ \mathrm{dipole}} = -\left(\frac{3B_s r_1^3}{2}\right)\rho z(z^2+\rho^2)^{-5/2}$$

$$B_{\phi\ \mathrm{dipole}} = 0 \tag{3}$$

where $(B_s r_1^3/2)$ is the moment of the dipole. If we choose r_1 to be the radius of the Sun, then, B_s is the dipole field at the north pole of the sun and is chosen to be 2 gauss in the following numerical calculation.

(ii). *Sunspot component* B_{sunspot}

The combined magnetic field of 180 spherical dipoles of radius of 0.1 $R_\odot$, distributed uniformly along a circle of radius of 0.8 $R_\odot$ in the equatorial plane, can be calculated in a similar way as (3).

(iii). *Uni-polar dynamo component* $\mathbf{B}_{\mathrm{dynamo}}$

Next we calculate the magnetic field $\mathbf{B}_{\mathrm{dynamo}}$ generated by the current system driven by the dynamo process on the photosphere of the sun as shown in Fig. 1. The radial surface current density i_ρ in the equatorial plane can be written as

$$i_\rho = i_{\rho_0}(\rho_0/\rho). \tag{4}$$

Note that $i_{\rho_0} = I_0/2\pi\rho_0$ is the current density at $\rho=\rho_0$. The total radial current I_0 is related to the azimuthal interplanetary magnetic field $B_{\phi 0}$ at $\rho = \rho_0$, $z=0^+$ (0^+ denotes a position right above the equatorial thin current sheet) by the following relation

$$I_0 = 2\pi\rho_0 i_{\rho_0} = B_{\phi 0}\rho_0 c. \tag{5}$$

Following the method used by Smythe (1950, Chapter 7) we find that the magnetic field due to the current system in Fig. 1 can be written as

$$\mathbf{B}_{\mathrm{dynamo}} = \nabla\times(\nabla\times\mathbf{W}_{\mathrm{dynamo}}) \tag{6}$$

where

$$\mathbf{W}_{\mathrm{dynamo}} = \begin{cases} B_{\phi 0}\rho_0|z|\phi\hat{\mathbf{z}}, & \text{for } r_1<(z^2+\rho^2)^{1/2}<r_2 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Here $\hat{\mathbf{z}}$ is a unit vector in the z direction. We obtain from equation (6) and (7)

$$\mathbf{B}_{\mathrm{dynamo}} = \begin{cases} \pm B_{\phi 0}(\rho_0/\rho)\hat{\boldsymbol{\phi}}, & \text{for } r_1<(z^2+\rho^2)^{1/2}<r_2 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where $\hat{\boldsymbol{\phi}}$ is a unit vector in the ϕ direction and the plus (minus) sign is for $z>0$ ($z<0$). In the following calculation, we choose $B_{\phi 0} = 3.5\ \gamma$. Note that the dynamo component $\mathbf{B}_{\mathrm{dynamo}}$ dominates the other two components at large distance from the Sun.

(iv). *The ring-current component* $\mathbf{B}_{\text{ring current}}$

Following Alfvén (1977), the ring current density in the equatorial plane can be written as

$$i_\phi(\rho) = \begin{cases} i_{\phi 0}(\rho_0/\rho)^2, & \text{for } \rho_1<\rho<\rho_2 \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

The ring current density $i_{\phi 0}$ is related to the radial interplanetary magnetic field $B_{\rho 0}$ at $\rho=\rho_0$, $z=0^+$ by $2\pi i_{\phi 0}/c = B_{\rho 0}$. We can use the observed value of $B_{\rho 0}$.

Following Smythe (1950, Chapter 7), we write

$$\mathbf{B}_{\text{ring current}} = \nabla\times(\nabla\times\mathbf{W}_{\text{ring current}}) \tag{10}$$

where $\mathbf{W}_{\text{ring current}} = W_{\text{ring current}}(\rho, z)\hat{z}$,

$$W_{\text{ring current}}(\rho, z) = \int_0^\infty dk f(k) J_0(k\rho)e^{-kz} \tag{11}$$

and $f(k)$ is a function to be determined by the boundary condition in the equatorial plane. Note that $J_0(k\rho)$ is the Bessel function of the first kind. At $z=0$, we have

$$B_{\rho\,\text{ring current}}(z=0^+, \rho) - B_{\rho\,\text{ring current}}(z=0^-, \rho) = \frac{4\pi}{c} i_\phi \tag{12}$$

where i_ϕ is given by (9). Note that $B_{\phi\,\text{ring current}}$ and $B_{z\,\text{ring current}}$ are continuous across the equatorial plane. From the boundary condition in (12), we find that

$$f(k) = \frac{2\pi}{kc}\int_0^\infty \rho\, d\rho i_\phi(\rho) J_1(k\rho). \tag{13}$$

In order to obtain an analytical expression of $\mathbf{B}_{\text{ring current}}$, we approximate the surface density $i_\phi(\rho)$ in (9) by the following equation

$$i_\phi(\rho) = i_{\phi 0}\left(\frac{\rho_0}{\rho}\right)^2(e^{-\rho/\rho_2} - e^{-\rho/\rho_1}). \tag{14}$$

With the surface current density given by (13), the ring-current magnetic field component $\mathbf{B}_{\text{ring current}}$ is obtained from (10)–(12),

$$B_{\rho\,\text{ring current}} = \pm B_{\rho_0}\rho_0^2\int_0^\infty dk k G(k) J_1(k\rho)e^{-k|z|}$$

$$B_{z\,\text{ring current}} = B_{\rho_0}\rho_0^2\int_0^\infty dk k G(k) J_0(k\rho)e^{-k|z|} \tag{15}$$

$$B_{\phi\,\text{ring current}} = 0$$

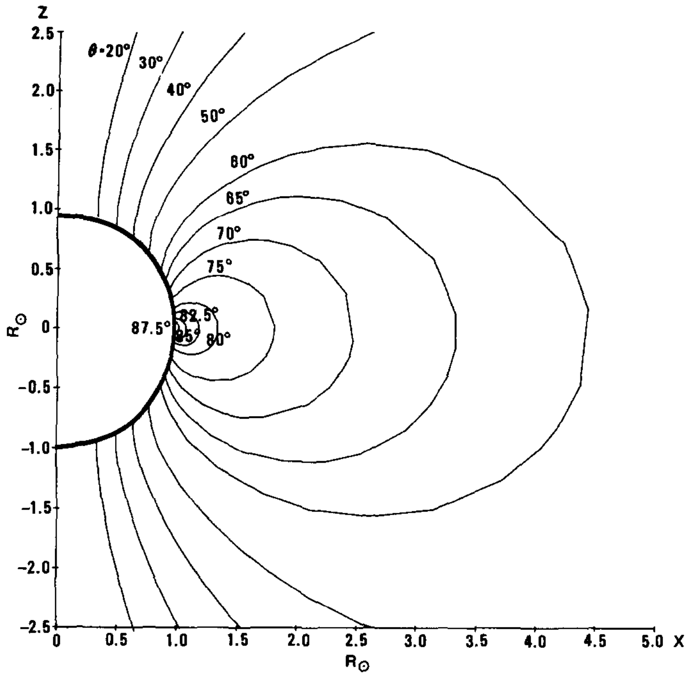

FIG. 2. MERIDIAN PROJECTION OF MAGNETIC FIELD LINES IN THE VICINITY OF THE SUN (EXCLUDING THE B_ϕ COMPONENT, NAMELY THE B_{dynamo} COMPONENT).

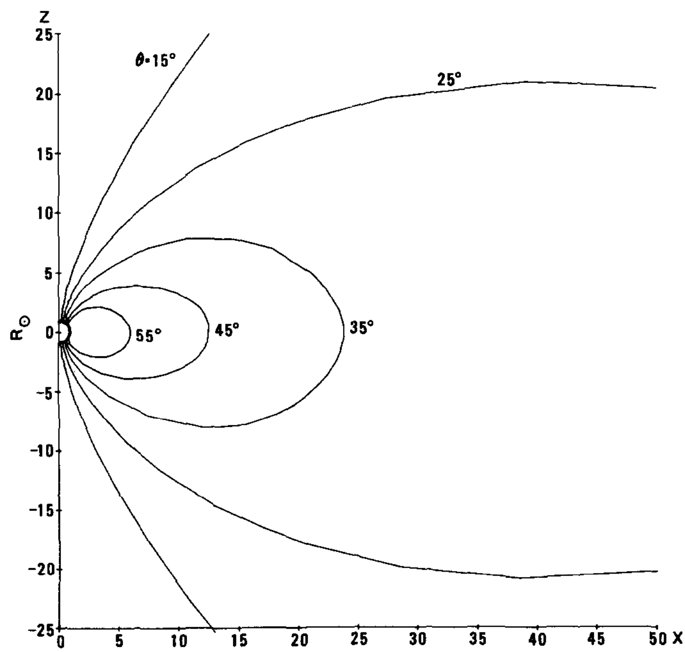

FIG. 3. MERIDIAN PROJECTION OF MAGNETIC FIELD LINES UP TO A DISTANCE OF 50 $R_\odot$ (EXCLUDING THE B_ϕ COMPONENT, NAMELY THE B_{dynamo} COMPONENT.)

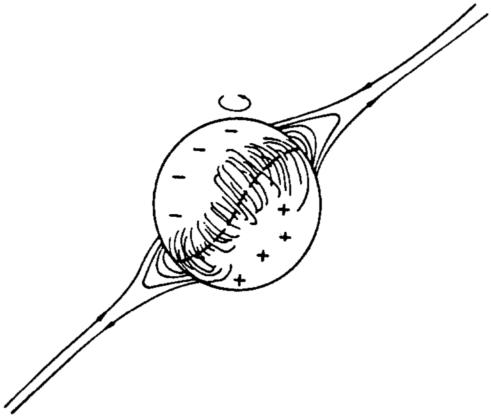

FIG. 4. GEOMETRICAL RELATIONSHIP BETWEEN THE MAGNETIC FIELD LINE ARCADE (INDICATING THE LOCATION OF THE MAGNETIC EQUATORIAL PLANE NEAR THE PHOTOSPHERE) AND THE HELMET STREAMER (INDICATING THE LOCATION OF THE CURRENT SHEET IN THE CORONAL LEVEL).
(After Svalgaard, L., Wilcox, J. M. and Duvall, T. L., (1974 *Solar Phys.* **37,** 157).

where

$$G(k)=\frac{1}{k}\left\{\left(k^2+\frac{1}{\rho_2{}^2}\right)^{1/2}-\left(k^2+\frac{1}{\rho_1{}^2}\right)^{1/2}+\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right)\right\}. \tag{16}$$

Unfortunately, the convergence of the integral in (15) is very slow. Therefore, in the present paper, we replaced the current sheet by a number of rings. The magnetic field of a circular loop current can be found in a standard book (*cf.* Stratton, 1941). The innermost ring is located at 10 $R_\odot$, and the rings are spaced by a distance of 10 $R_\odot$. The current intensity in the rings varies as $1/r^2$ and is adjusted to give $B_{\rho_0} = -3.5\gamma$ gammas at a distance of 1 A.U. from the Sun and 20 $R_\odot$ from the ecliptic plane.

3. RESULTS AND DISCUSSIONS

Figure 2 shows the meridian plane projection of the magnetic field configuration near the Sun ($<5\ R_\odot$), without including the B_Φ component. It can be seen that the magnetic field near the equatorial plane is dominated by the sunspot field, constituting the so-called magnetic field line arcade, but the polar field is dipolar. Figure 3 shows also the meridian plane projection of the magnetic field configuration to a distance of about 50 $R_\odot$, without including the B_Φ component which is not important up to this distance. One can see clearly the effects of the current sheet around the Sun, stretching the magnetic arcade field lines. For comparison, Fig. 4 shows the magnetic field configuration near the Sun, including the magnetic field line arcade and the coronal streamer. The latter indicates the

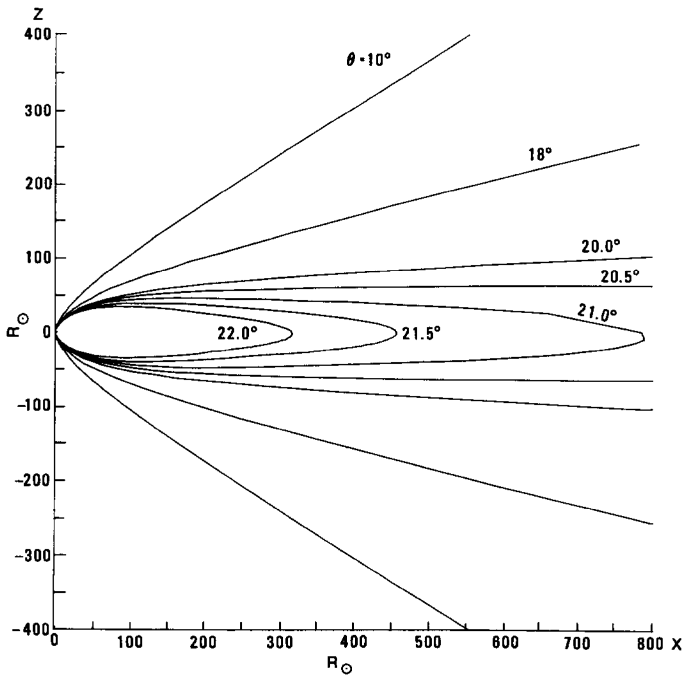

FIG. 5. MERIDIAN PROJECTION OF MAGNETIC FIELD LINES UP TO A DISTANCE OF 800 $R_\odot$ (EXCLUDING THE B_Φ COMPONENT, NAMELY THE $B_{\rm dynamo}$ COMPONENT).

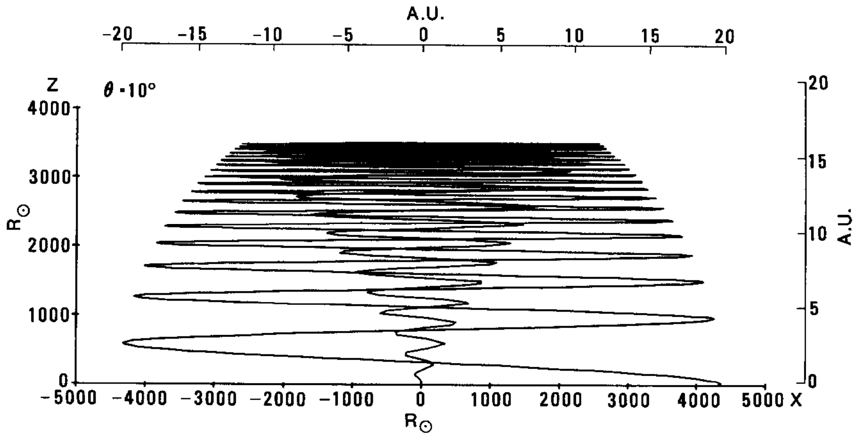

FIG. 6. MERIDIAN PROJECTION OF A MAGNETIC FIELD LINE "ORIGINATING FROM $\theta = 10°$ FROM THE POLE OF THE SUN.

location of the current disc and the solar magnetic equatorial plane. In our model, the magnetic equatorial plane coincides with the ecliptic plane.

Figure 5 shows also the meridian plane projection of the magnetic field configuration up to a distance of $800\, R_{\odot}$, without including the B_{Φ} component which causes spiraling of the meridian (making it difficult to see clearly the meridian component).

One can see more clearly a large-scale magnetic field structure in this figure than in Fig. 4.

In Figure 6 we show the meridian plane projection of a magnetic field line which "originates" at the polar angle $\theta = 10°$ on the photosphere. In this figure, we have included the B_{Φ} component which tends to produce a helical structure. In this model, the B_r and B_{Φ} components are normalized in such a way that the angle between the total **B** vector and a solar radial line is 135° at a radial distance of 1 A.U. and of a distance of $20\, R_{\odot}$ from the ecliptic plane; this is close to an average IMF configuration at the Earth's distance. Actually, the helical structure in high heliographic latitudes is also present in Parker's model. The main difference

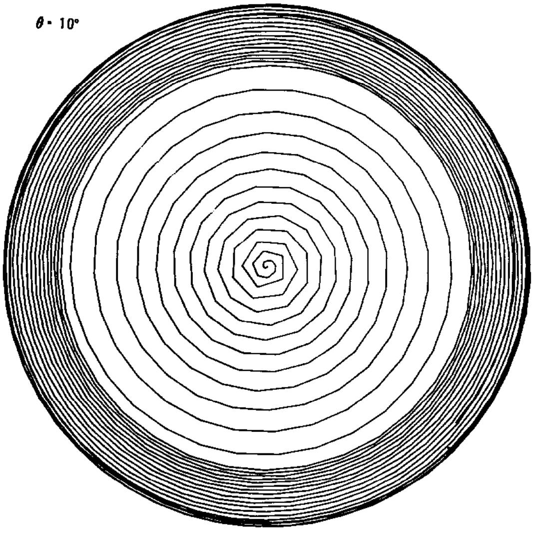

FIG. 7. EQUATORIAL PROJECTION OF A MAGNETIC FIELD LINE "ORIGINATING" FROM $\theta = 10°$ FROM THE POLE OF THE SUN

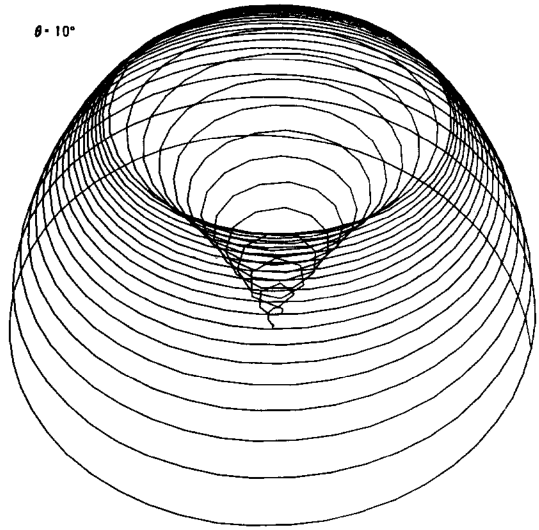

FIG. 8. VIEW OF THE MAGNETIC FIELD LINE IN FIGS. 6 AND 7, FROM A MIDDLE HELIOGRAPHIC LATITUDE.

between his and our models is that his current is a distributed current, while ours is a radial line current from the pole. Figure 7 shows the equatorial projection of this field line ($r = 10°$). In order to see this helical structure a little better, Fig. 8 shows a view of the field line $\theta = 10°$ from a middle heliographic latitude, rather than directly from above the pole.

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REFERENCES

Alfvén, H. (1950). *Cosmical Electrodynamics*, Oxford University Press, Oxford.

Alfvén, H. (1977). Electric currents in cosmic plasmas. *Rev. Geophys. Space Phys.* **15,** 271.

Axford, W. I. (1972). The interaction of the solar wind with the interstellar medium, in *Solar Wind*, (Ed. C. P. Sonett, P. J. Coleman, Jr. and J. M. Wilcox), p. 609. NASA SP-308, Washington, D.C.

Fitzenreiter, R. J., J. Fainberg, R. R. Weber, H. Alvarez, F. T. Haddock and Potter, W. H. (1977). Radio observations of interplanetary magnetic field structure, out of the ecliptic. *Solar Phys.* **52,** 477.

Hundhausen, A. J. (1977). An interplanetary view of coronal holes, in *Coronal Holes and High Speed Wind Streams*, (Ed. J. B. Zirker,) p. 225. Colorado Associated University Press, Boulder, Colorado.

Parker, E. N. (1958). Dynamics of the interplanetary gas and magnetic fields. *Ap. J.* **128,** 664.

Smith, E. J., Tsurutani, B. T. and Rosenberg, R. L. (1978). Observations of the interplanetary sector structure up to heliographic latitudes of 16°: *Pioneer* 11. *J. geophys. Res.* **83,** 717.

Smythe, W. R. (1968). *Static and Dynamic Electricity*, Chap. 7, McGraw-Hill, New York.

Stenflo, J. O. (1972). The electromagnetic structure of interplanetary space, in *Solar Wind*, (Ed. C. P. Sonett, P. J. Coleman, Jr. and J. M. Wilcox) p. 115. NASA SP-308, Washington, D.C.

Stratton, J. A. (1941). *Electromagnetic Theory*, p. 263, McGraw-Hill, New York.

Svalgaard, L., Wilcox, J. M. and Duvall, T. L. (1974). A model combining the polar and sector structured solar magnetic fields. *Solar Phys.* **37,** 157.

Note added in proof: Our model differs from others in one important respect, namely the presence of the "closed field" region along the ecliptic plane. Without such a closure, it is difficult to explain the fact that Forbush decreases recover much more slowly at greater heliocentric distances [Van Allen, J. A. (1979). Propagation of a Forbush decrease in cosmic ray intensity to 15.9 A.U., *Geophys. Res. Lett.*, **6,** 566. March 31, 1980].