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- Result : the line through two points $\mathbf{x}$ and $\mathbf{x}'$ is $\mathbf{l} = \mathbf{x} \times \mathbf{x}'$.

Some quick vector operations

$$\mathbf{x} \times \mathbf{y} = \mathbf{x}_\times \cdot \mathbf{y} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = \begin{pmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - y_1 x_2 \end{pmatrix}$$

$$\mathbf{x}_\times = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

Mixed product : $\mathbf{x}^T(\mathbf{y} \times \mathbf{z}) = |\mathbf{x} \;\; \mathbf{y} \;\; \mathbf{z}|$ (the volume of the parallelepiped defined by the three vectors)

Singular value decomposition

Theorem (SVD) :

Let $\mathbf{A}$ be an $m \times n$ matrix. $\mathbf{A}$ may be expressed as :

$$\mathbf{A} = \mathbf{U\Sigma V}^T = \sum_{i=1}^{\min(m,n)} \sigma_i U_i V_i^T$$

where $\mathbf{\Sigma}$ is a $m \times n$ diagonal matrix with $\sigma_i = \mathbf{\Sigma}_{ii} \geq 0$, and $\mathbf{U}$ $(m \times m)$ and $\mathbf{V}$ $(n \times n)$ are composed of orthornormal columns

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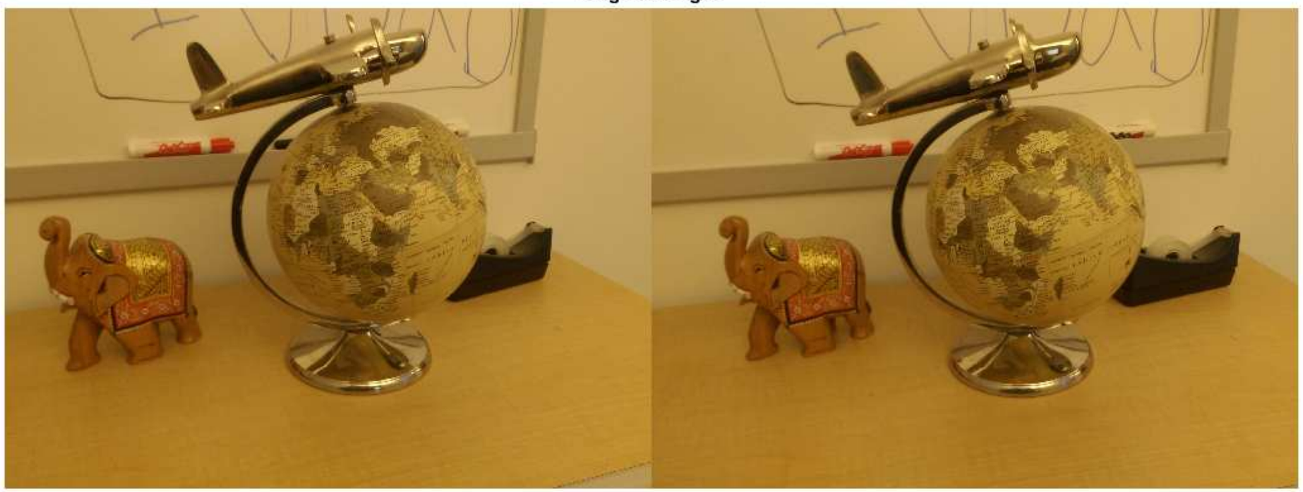
- By convention, the σ_i are aligned in descending order by the decomposition algorithms.

Outline

- The 3D representation of points
- The pinhole camera model
- Applying a coordinate transformation
- Homogeneous representations and algebraic operations
- The fundamental matrix
- The essential matrix
- Rectification

Why is this part "fundamental" ? (cheap joke)

What we can get from two views :



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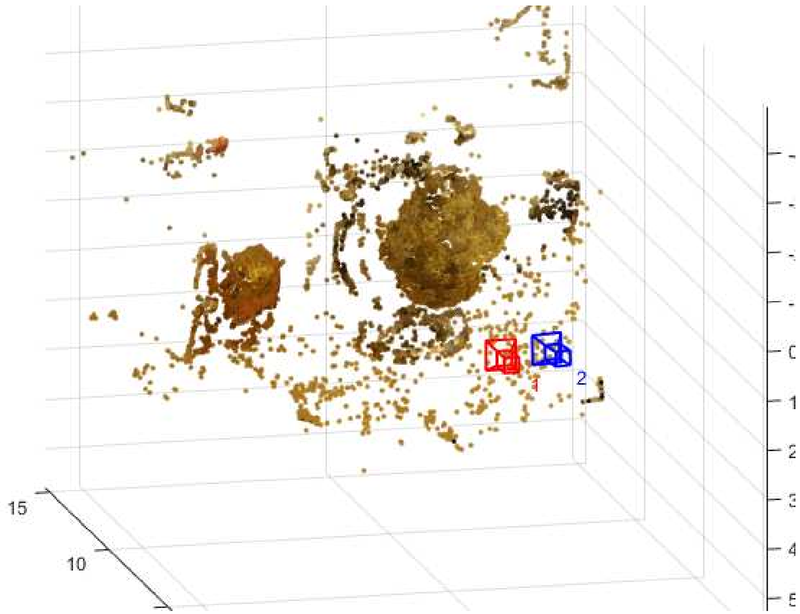

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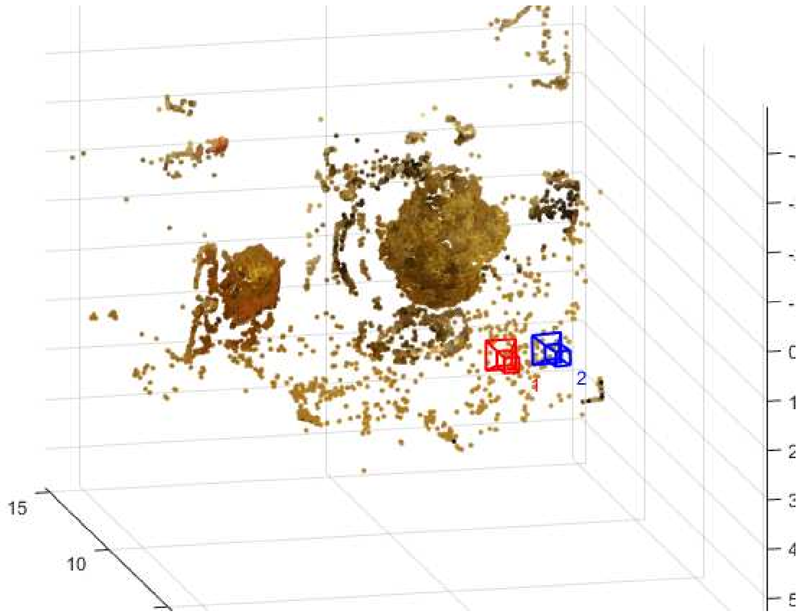

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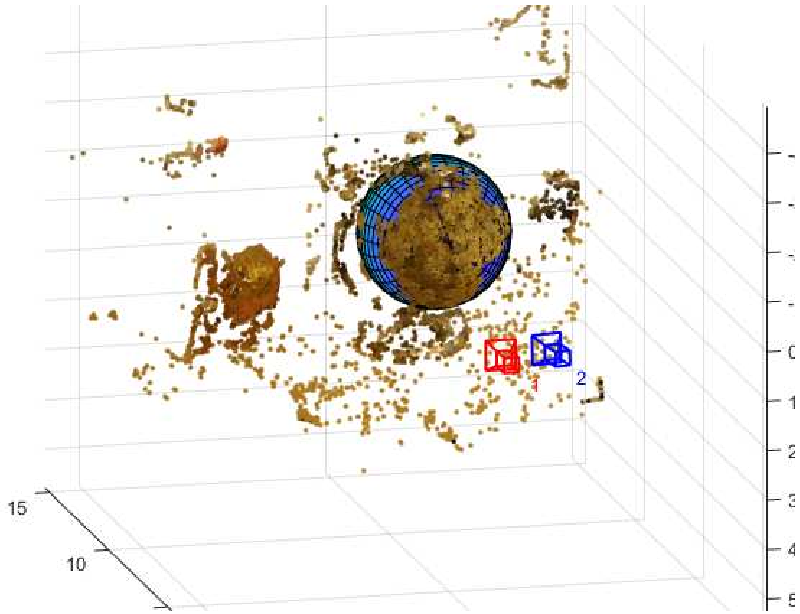

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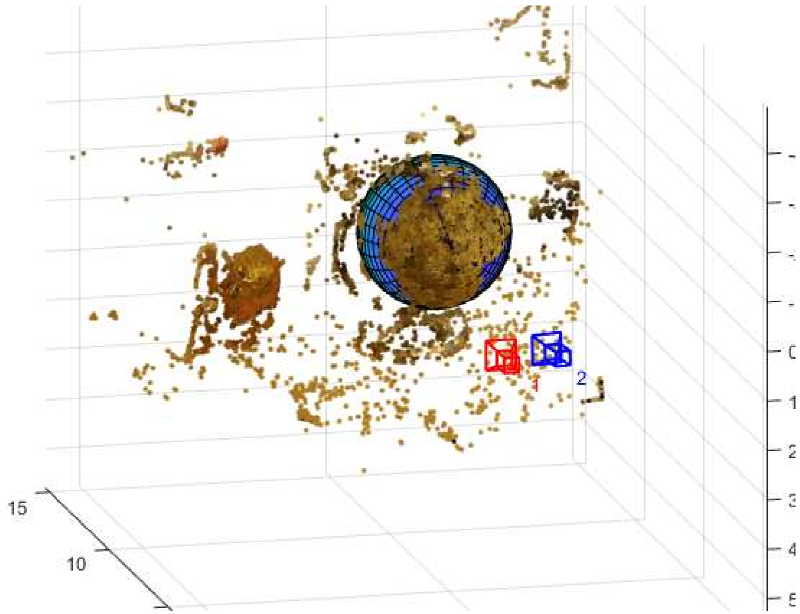

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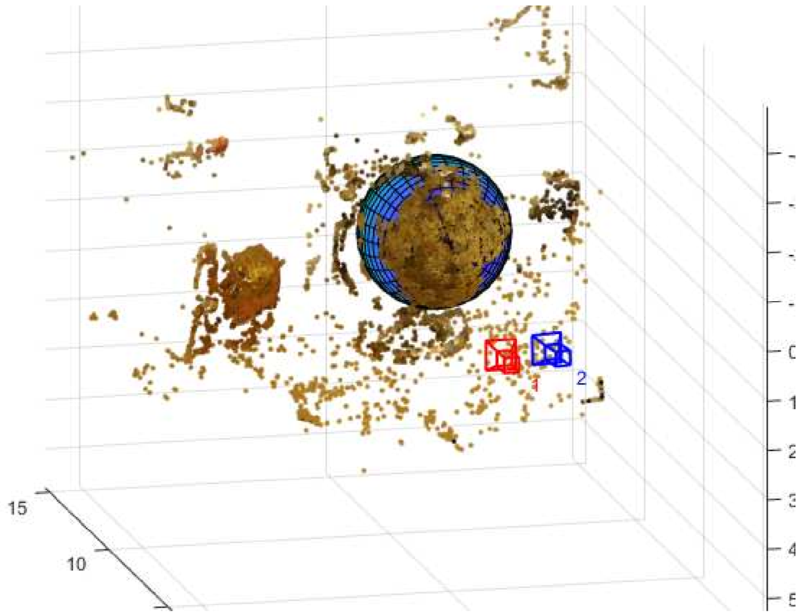

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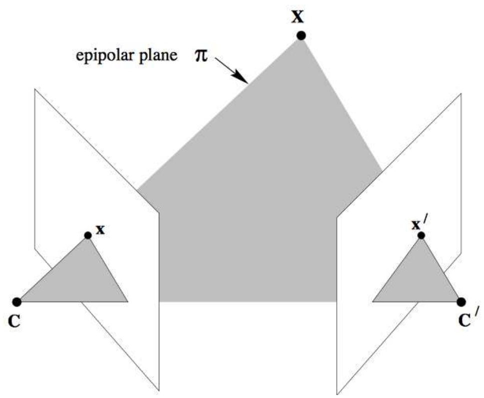
Some important observations :

- the pixel projection is along the ray defined by the 3D point and the camera center (i.e. as for **x**, **X** and **C**)



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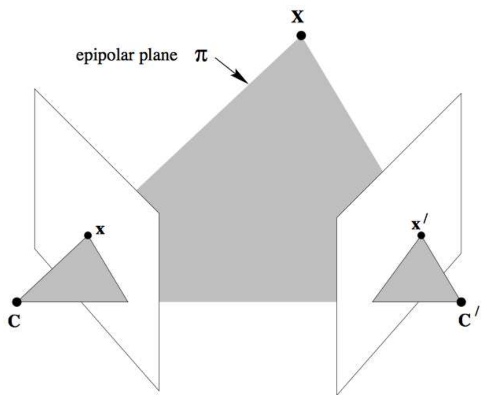

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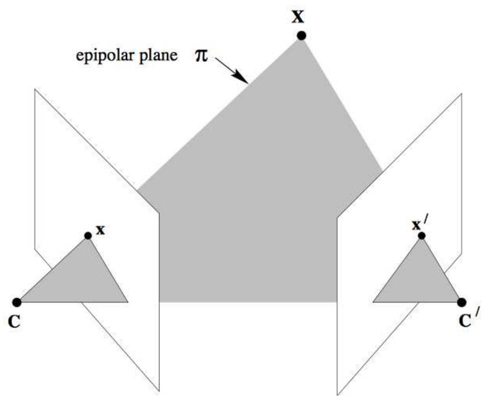

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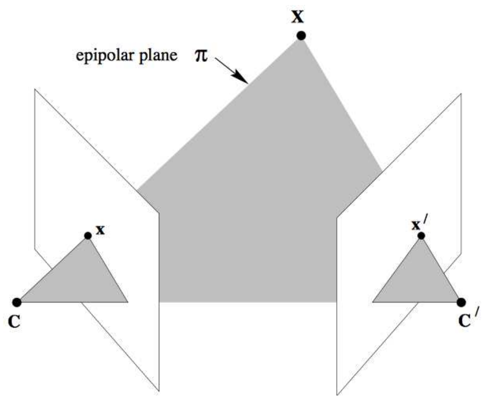

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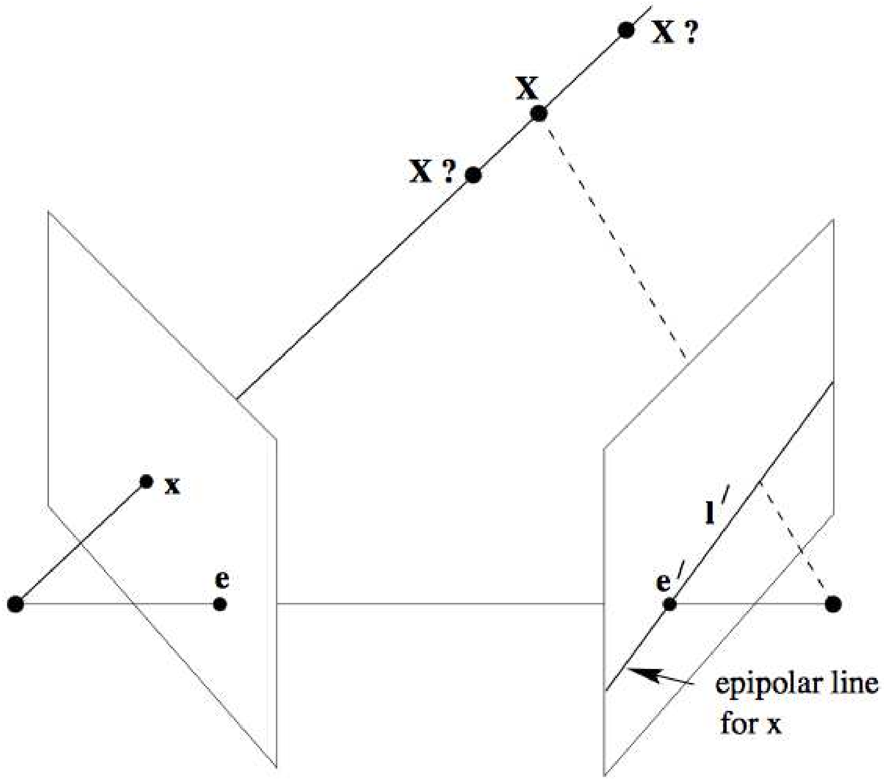

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Finally, by transposing $\mathbf{K}'^{-1}\mathbf{x}'$ and ignoring the scalar λ we get :

$$\mathbf{x}'^{T} \underbrace{\mathbf{K}'^{-T}\mathbf{t}_\times \mathbf{R}\mathbf{K}^{-1}}_{\mathbf{F}} \mathbf{x} = 0$$

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- ... but also $\mathbf{F} = \mathbf{K'}^{-T} \mathbf{t}_\times \mathbf{R} \mathbf{K}^{-1}$ encodes, along with the calibration matrices, *the rotation and translation* between views

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Theorem

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  $\left\|\mathbf{U}\mathbf{D}\mathbf{V}^T\mathbf{f}\right\| = \left\|\mathbf{D}\mathbf{V}^T\mathbf{f}\right\|$, and $\|\mathbf{f}\| = \left\|\mathbf{V}^T\mathbf{f}\right\|$. We have to minimize $\left\|\mathbf{D}\mathbf{V}^T\mathbf{f}\right\|$ subject to $\left\|\mathbf{V}^T\mathbf{f}\right\| = 1$. If $\mathbf{y} = \mathbf{V}^T\mathbf{f}$, then we minimize $\|\mathbf{D}\mathbf{y}\|$ subject to $\|\mathbf{y}\| = 1$. Since $\mathbf{D}$ is diagonal with values in descending order, it means that $\mathbf{y} = (0, 0 \dots, 1)$, and $\mathbf{f} = \mathbf{V}\mathbf{y}$ is the last column of $\mathbf{V}$. (*A5.3, Hartley and Zisserman*)

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- This algorithm is also preferred as fewer observations are needed

Outline

- The 3D representation of points
- The pinhole camera model
- Applying a coordinate transformation
- Homogeneous representations and algebraic operations
- The fundamental matrix
- The essential matrix
- Rectification

Using the camera calibration and the essential matrix

If the calibration matrices $\mathbf{K}$ and $\mathbf{K}'$ are known :

- we may recover the pose information from $\mathbf{F} = \mathbf{K}'^{-T}\mathbf{t}_\times\mathbf{R}\mathbf{K}^{-1}$:

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- Knowing $\mathbf{E}$: interesting for relative pose estimation
- Main disadvantage : $\mathbf{K}$ and $\mathbf{K}'$ are required to get to $\mathbf{E}$

Recovering R and t from E

It has been shown that the decomposition of **E** is possible and there are actually four valid solutions (*9.6.2, Hartley and Zisserman*) :

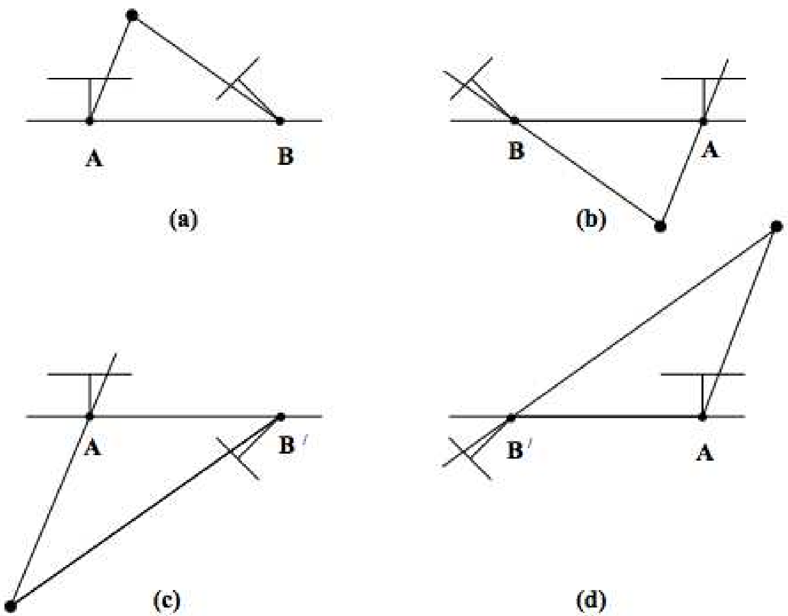

Fig. 9.12. **The four possible solutions for calibrated reconstruction from E.** *Between the left and right sides there is a baseline reversal. Between the top and bottom rows camera B rotates* 180° *about the baseline. Note, only in (a) is the reconstructed point in front of both cameras.*

- Identify the correct solution : cheirality check (the 3D points have to be in front of the camera) with an additional match from the two views

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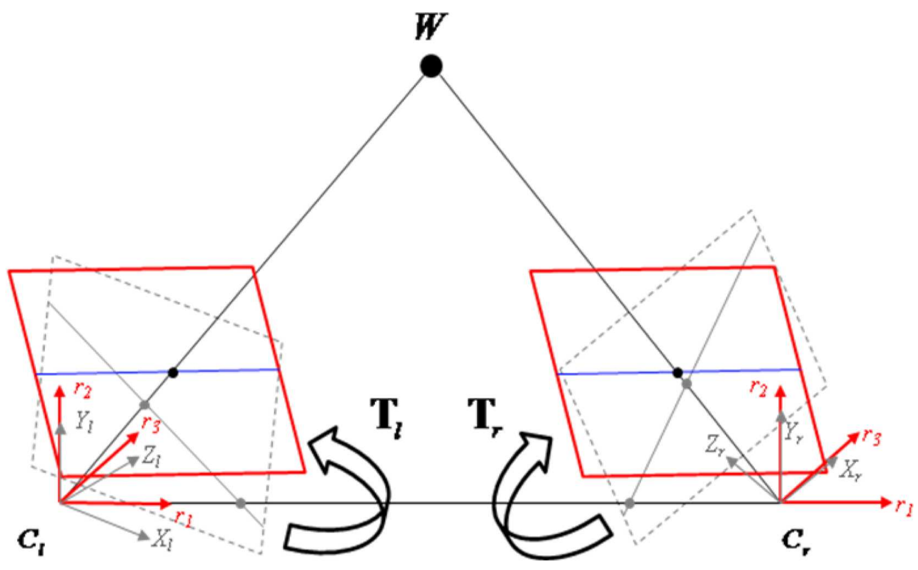
Rectification

Using $\mathbf{F}$, we restrict the search for the corresponding projection $\mathbf{x}'$ of a point $\mathbf{x}$ to a line (the epipolar line $\mathbf{l}' = \mathbf{F}\mathbf{x}$).

Stereo rectification

- Apply an adjustment to the images in order to get horizontal epipolar lines in both views



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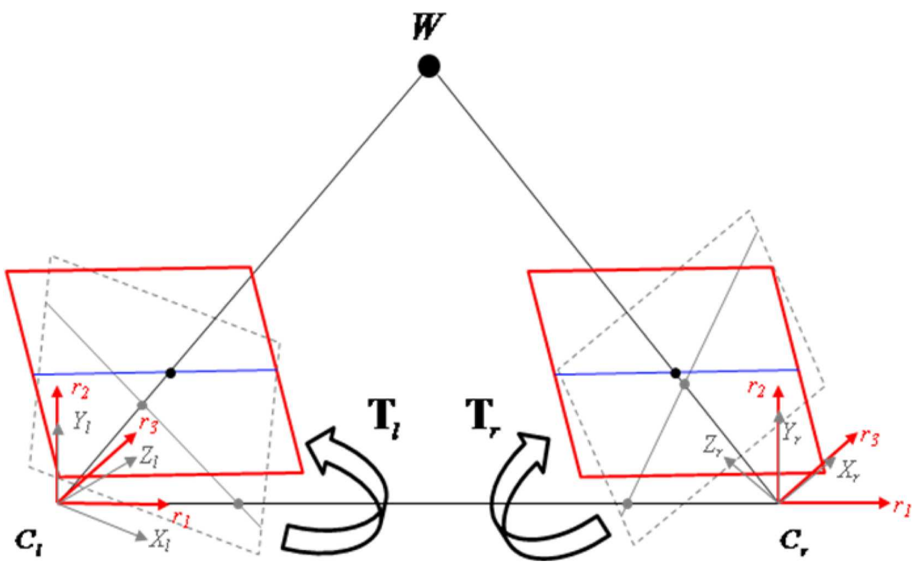

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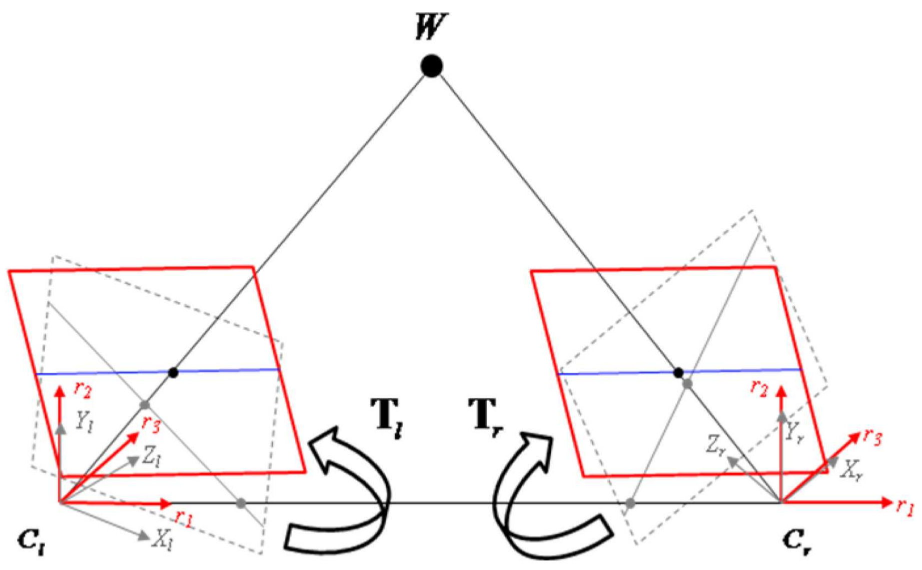

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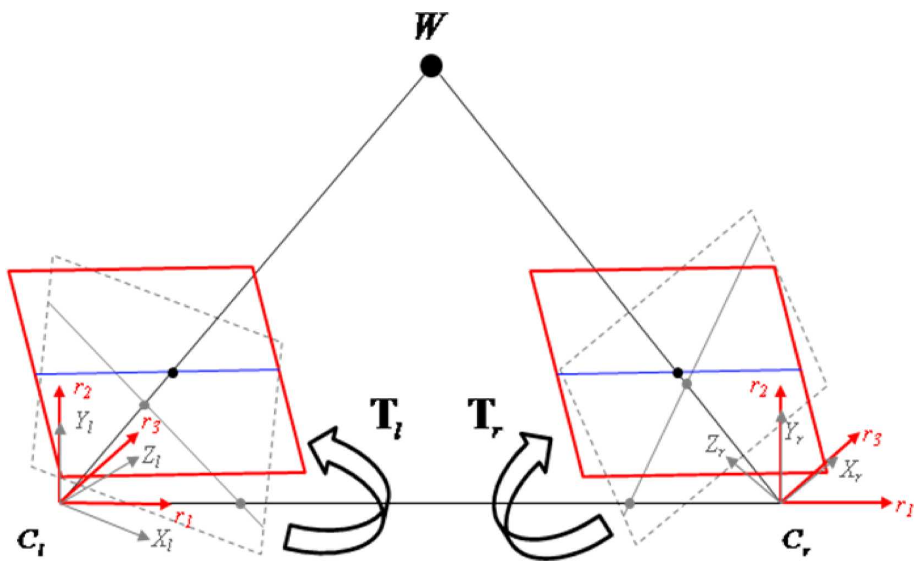

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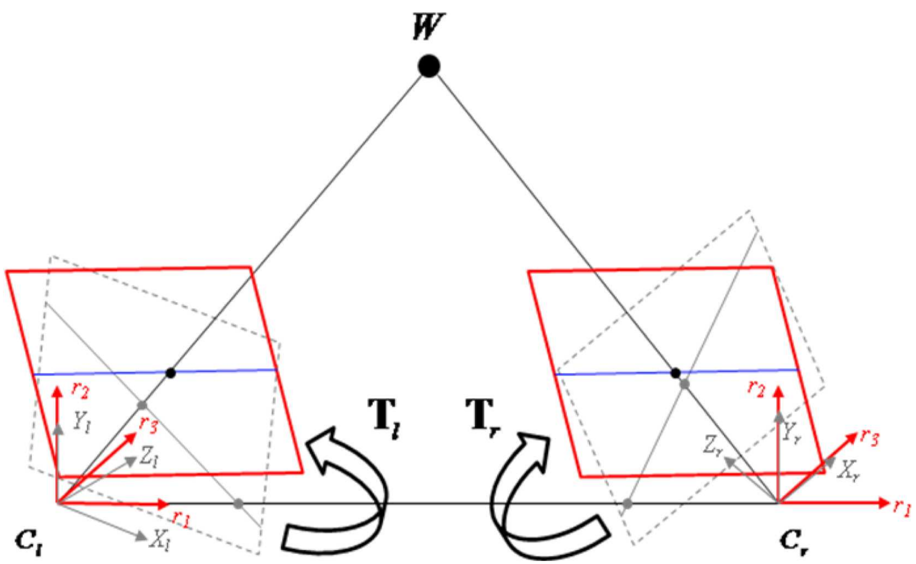
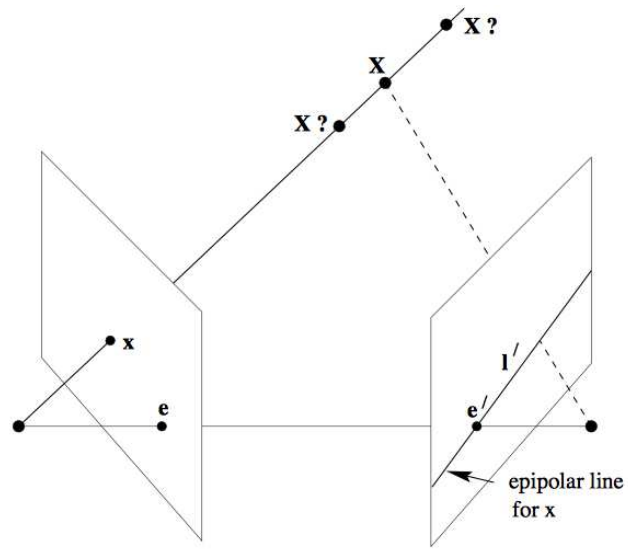
- An interpolation is required for creating the new images, but high computation gain overall



Triangulation - the building block of 3D reprojections

We have the pose $\mathbf{R}, \mathbf{t}'$ between cameras and the projection locations $\mathbf{x}, \mathbf{x}'$. What now ?



Get $\mathbf{X}$: triangulate the point in 3D

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- There are other algorithms which are more accurate, but costlier
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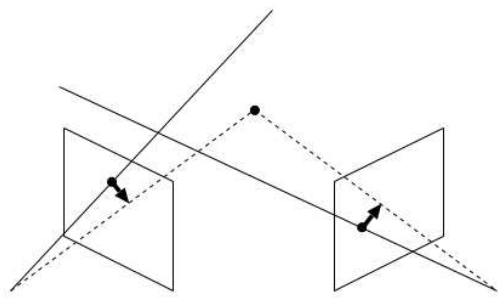
- The linear approach is reasonably good, and it is effective especially if used as an initialization for a nonlinear refinement (as we will see in the following slides)

Triangulation - how to use multiple views

If we have multiple views, the unknown $\mathbf{X}_j$ may be constrained by multiple observations $\mathbf{z}_{j,\tau}$ from cameras C_τ characterized by some pose parametrization $\mathbf{s}_\tau$. How to use them effectively together ?

Nonlinear optimization



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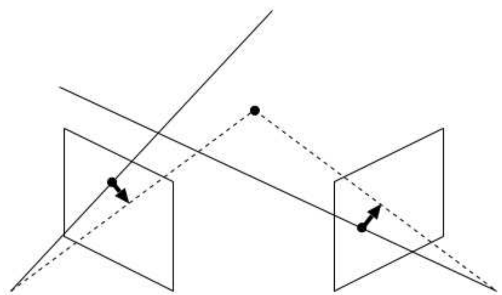

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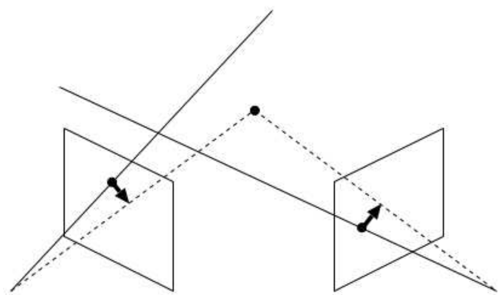
$$\hat{\mathbf{X}}_j = \arg\min_{\mathbf{X}_j} \sum_\tau e(\mathbf{s}_\tau, \mathbf{X}_j, \mathbf{z}_j)^T e(\mathbf{s}_\tau, \mathbf{X}_j, \mathbf{z}_j)$$

- Use Gauss-Newton or LM (usually the optimum is not far from a reasonable initialization)



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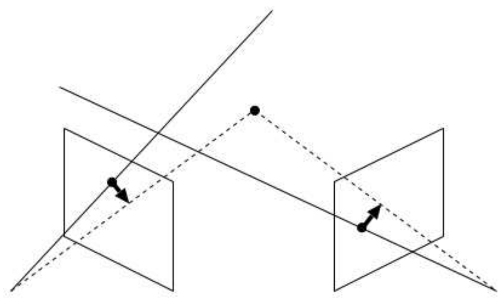
- More than one 3D point may be refined, but in this way the optimizations are decoupled



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Opposite problem : we have a set of 3D points $\mathbf{X_j}$ (computed previously) which are visible from camera C_τ. Based on current observations $\mathbf{z}_{j,\tau}$ from C_τ we would like to estimate its pose $\mathbf{s}_\tau$.

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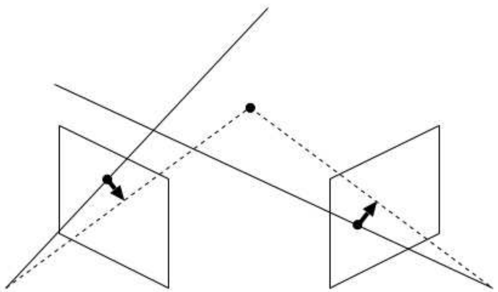

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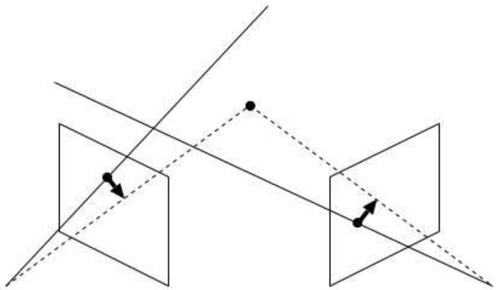

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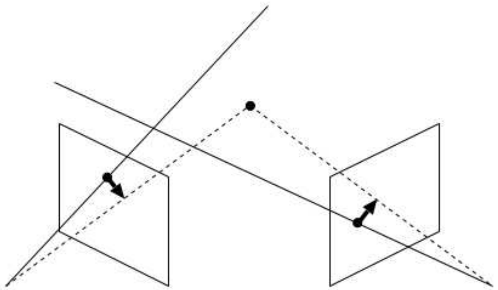

Pose estimation - how to use multiple views

Opposite problem : we have a set of 3D points $\mathbf{X_j}$ (computed previously) which are visible from camera C_τ. Based on current observations $\mathbf{z}_{j,\tau}$ from C_τ we would like to estimate its pose $\mathbf{s}_\tau$.

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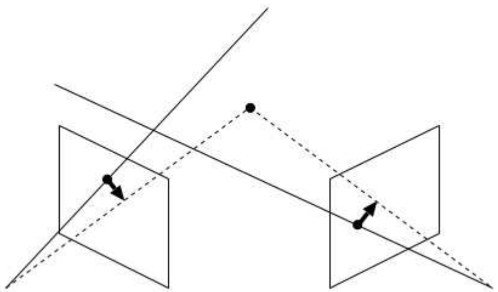

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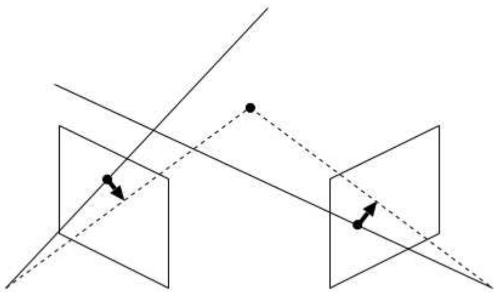
  - from the projection of three 3D points in space and their projections, one may compute the camera pose in a closed form (the P3P problem)



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- in reality all estimations we perform are noisy
- if we also apply the process iteratively (triangulation, pose estimation and repeat) the errors will be amplified (drift)

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- the interest of the initial step is just to provide a quality initialization for $\mathbf{s}_\tau$ as $\hat{\mathbf{s}}_t$

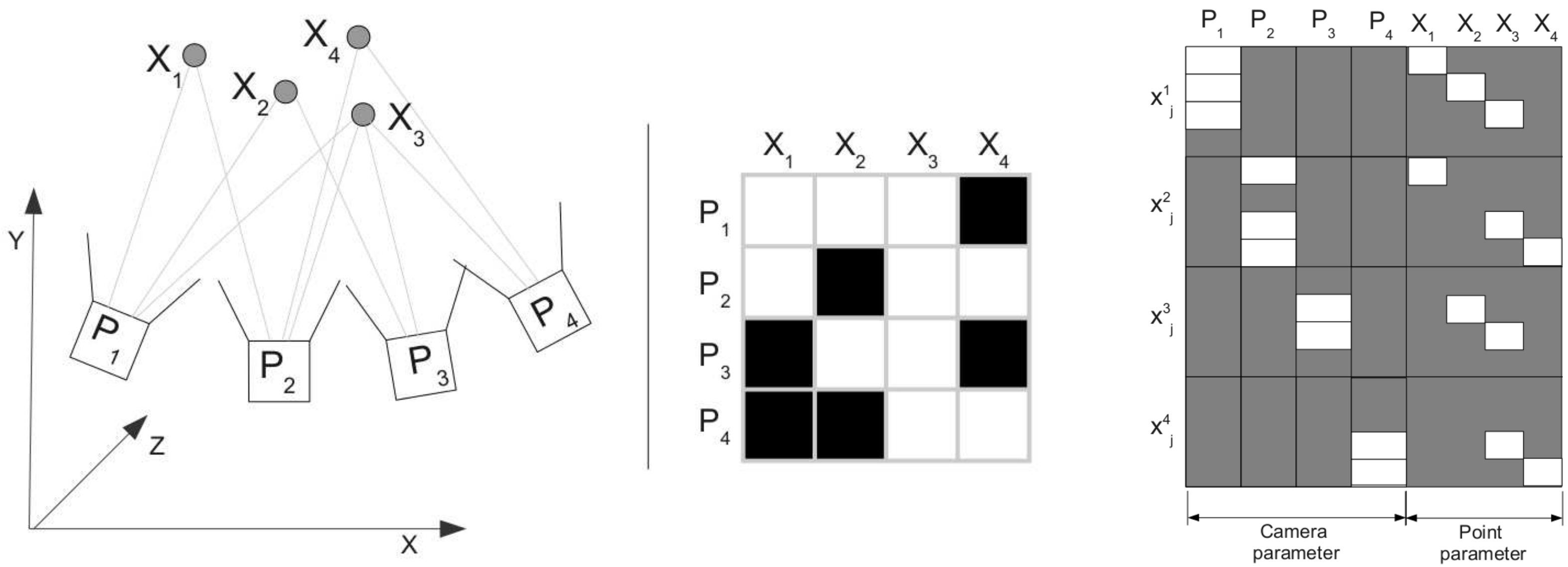
Global optimization - final step

We compute the MAP (Maximum A Posteriori) for the maximum amount of preliminary estimations and observations that we have at that moment (brutal, massive optimization). The solution we search this time is provided by :

$$\tilde{\mathbf{S}}_{0:t}, \tilde{\mathbf{X}} = \arg\min_{\mathbf{S}_{0:t}, \mathbf{X}} \sum_{\tau=0}^{T} \sum_{j=1}^{M} e(\mathbf{s}_\tau, \mathbf{X}_{j,\tau}, z_{j,\tau})^T \, e(\mathbf{s}_\tau, \mathbf{X}_{j,\tau}, z_{j,\tau})$$

The complexity of this algorithm, once we exploit the sparseness of its Jacobian : $O(T^3 + MT^2)$, which is very interesting since $M \gg T$.

Towards real time reconstruction



An example of configuration : 5207 3D points, 54 poses, 24609 projections, 15945 variables, 21 it., 7.99 sec.
Not fast enough !

- Selection of key-frames
- Parallel execution of tracking et BA (initial and final steps)
- Limit the number of iterations (when needed)
- Local Bundle Adjustment

Typical architecture for RT optimization