Bayesian approaches and ML techniques in multisource fusion systems

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Outline

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Multisource fusion

Combine information from multiple sources and sensors in order to achieve analysis and decision-supporting inferences that cannot be achieved with a single sensor or source.

Main objectives

- Data fusion : combine multiple sources of information (e.g. sensor data) in order to provide an optimal estimation of the state of the system of interest
- Information fusion : take the optimal decision, given multiple sources of information (e.g. classifiers)

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Typical challenges:

- Varying levels of uncertainty of sources
- Missing or sparse data
- Non homogeneity of spatio-temporal scales
- Computational cost

Estimation

The first focus will be on estimation and we will focus on:

- Bayesian recursive filtering and how to apply it in a tractable manner
- Kalman filtering (and friends)
- Data fusion for estimation with a complete example

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The principles

A relatively simple and general framework:

- Data fusion objective : obtain an estimation $\hat{x}$ of an unknown vector state x given a noisy measurement vector z provided by a range of sensors:

$$z = h(x, v)$$

  where v represents the noise.

- The famous Bayes rule : $p(x, y) = p(x|y)p(y) = p(y|x)p(x)$
- $p(x)$ and $p(y)$: pdf (probability density function) of x and y
- $p(x, y)$: joint pdf of x and y
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Our estimation

Considering the Bayes rule, it is straightforward to obtain:

$$p(x|z) = \frac{p(z|x)p(x)}{p(z)}$$

- Likelihood $p(z|x)$: the link between the observation and the unknown state x
- Prior $p(x)$: the prior knowledge we have on x
- $p(z)$ is the knowledge we have on z irrespective of x; since it does not involve the state, it is rarely used in practice

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The likelihood

- Note that z is a random variable while x is known : measurement characterization knowing x
- Typically provided by $z = h(x, v)$

Estimators

Even if we know $p(x|z)$, we usually want a good estimation $\hat{x}$ of the variable x

Properties

- We characterize an estimator by its bias $B_{\hat{x}} = \mathbb{E}[\hat{x} - x] = \mathbb{E}[\hat{x}] - x$, which is ideally 0 (i.e. unbiased estimator)
- And by its variance $\mathbb{E}[\tilde{x}] = \mathbb{E}[(\hat{x} - x)^2] = \mathbb{E}[\hat{x}^2] - \mathbb{E}[x]^2$
- The variance should be as small as possible
- The minimum variance can be estimated theoretically: it is the CRLB

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Common estimators

- The MAP $\hat{x}_{MAP}$ is the value x such that $p(x|z)$ is maximum: $\hat{x}_{MAP} = \arg\max_x p(x|z)$
- The MMSE mimizes the squared error sum:

$$\hat{x}_{MMSE} = \arg\min_x \mathbb{E}[(\hat{x}] - x)^T(\hat{x}] - x)]$$

- We can demonstrate that $\hat{x}_{MMSE} = \mathbb{E}[x|z] = \int x p(x|z) dx$

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Estimators - in practice

- The behaviour is approximately the same for symmetrical distributions $p(x|z)$
- Multimodal posterior distributions : significant errors / unstable estimations
- The MAP is usually easier to compute
- The denominator in the Bayes rule $p(z)$ does not matter since it does not depend on x

Exercise

Let us consider a scalar measurement z such that $z = \theta + w$ with :

- w : noise such that $w \sim \mathcal{N}(0, \sigma_w^2)$
- the prior distribution for $\theta \sim \mathcal{N}(\theta_0, \sigma_\theta^2)$

Determine the Bayesian estimator $\hat{\theta}$ of θ.

Solution

We need to compute $p(z|\theta)p(\theta)$.
Since z $= \theta +$ w and w $\sim \mathcal{N}(0, \sigma_w^2)$, we have z $\sim \mathcal{N}(\theta, \sigma_w^2)$.
Therefore, $p(z|\theta) = \mathcal{N}(\theta, \sigma_w^2)$, and we also know that $p(\theta) = \mathcal{N}(\theta_0, \sigma_\theta^2)$. The product of two Gaussian functions $\mathcal{N}(\mu_1, C_1)$ and $\mathcal{N}(\mu_2, C_2)$ is also a Gaussian function given by $k\mathcal{N}(\mu, C)$, with:

$$C = [C_1{}^{-1} + C_2{}^{-1}]^{-1} \qquad \mu = [C_1{}^{-1} + C_2{}^{-1}]^{-1}[C_1{}^{-1}\mu_1 + C_2{}^{-1}\mu_2]$$

So, $p(z|\theta)p(\theta)$ will be an un-normalized Gaussian function

$$p(z|\theta)p(\theta) \propto \mathcal{N}(\mu, \sigma^2) \propto \mathcal{N}(z, \sigma_w^2)\mathcal{N}(\theta_0, \sigma_\theta^2)$$

With :

$$\sigma^2 = \frac{\sigma_w^2\sigma_\theta^2}{\sigma_w^2 + \sigma_\theta^2} \qquad \mu = \frac{\sigma_w^2\theta_0 + \sigma_\theta^2 z}{\sigma_w^2 + \sigma_\theta^2}$$

Since the result is a Gaussian function, we have for MAP/MMSE the same result $\hat{\theta} = \mu$

Dynamic Estimation

This problem is more challenging : take into account the temporal evolution of the system.

Notations

- x_k : real state vector at time k
- z_k : measurement performed at time k
- Z_k : set of measurements performed up to time k

The objective is to estimate x_k by exploiting all the available observations:

$$p(x_k|Z_k) = \frac{p(Z_k|x_k) \times p(x_k)}{p(Z_k)}$$

Definition

Under a Markovian assumption, we can formulate :

$$p(x_{0:t}|z_{1:t}) \propto p(z_t|x_t)p(x_t|x_{t-1})p(x_{0:t-1}|z_{1:t-1})$$

Proof

$$p(x_{0:t}|z_{1:t}) \quad = \quad p(x_{0:t}|z_t, z_{1:t-1})$$

Proof

$$\begin{aligned} p(x_{0:t}|z_{1:t}) &= p(x_{0:t}|z_t, z_{1:t-1}) \\ &= \frac{p(z_t, z_{1:t-1}|x_{0:t})p(x_{0:t})}{p(z_t, z_{1:t-1})} \end{aligned}$$

Bayes rule

$$p(A|B,C) = p(B,C|A)\frac{p(A)}{p(B,C)}$$

Proof

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\begin{aligned}
p(x_{0:t}|z_{1:t}) &= p(x_{0:t}|z_t, z_{1:t-1}) \\
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$$p(A, B) = p(A|B)p(B)$$

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Proof

Markovian hypotheses

- the current state depends only on the previous state:

$$p(x_t|x_{0:t-1}, z_{1:t-1}) = p(x_t|x_{t-1})$$

- the observations are conditionally independent with respect to the states, and depend only on the current state:

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Using recursion

$$p(x_{0:t}|z_{1:t}) \propto \underbrace{p(z_t|x_t)}_{\text{likelihood}} \underbrace{p(x_t|x_{t-1})}_{\text{transition}} p(x_{0:t-1}|z_{1:t-1})$$

Solutions

- Kalman filter : linear functions, Gaussian densities, analytical solution
- Extended Kalman filter : linearized functions
- Particle filter : tractable approximation for general functions

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Kalman Filter

State equations

$$x_t = F_t x_{t-1} (+B_t u_t) + w_t$$

- x_t : state vector at time t
- u_t : control vector
- F_t : state transition matrix between $t-1$ and t
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- w_t : state noise vector, $w_t \sim \mathcal{N}(0, W_t), \mathbb{E}[w_t w_t^T] = W_t$

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In the following, we will ignore the control term:

$$x_t = F_t x_{t-1} + w_t$$

Kalman Filter

Measurement equation

$$z_t = H_t x_t + v_t$$

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Kalman Filter

Two-step recursion

- Given $x_{t-1|t-1}$ the optimal non-biased estimator of x_{t-1}
- Given the observation z_t

1. Compute the non-biased estimator $x_{t|t-1}$ of x_t without using the observation z_t
2. Compute the non-biased estimator $x_{t|t}$ of x_t using the observation z_t

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Kalman Filter

Two-step recursion

- Given $x_{t-1|t-1}$ the optimal non-biased estimator of x_{t-1}
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Principle : feedback control

Objective : recursively minimize the error between estimation and state

1. Prediction (theoretical value) step : with the parameters from $t-1$
2. Correction step : using the current observation to correct the current prediction

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Densities

- Posterior density at $t-1$:

$$p(\mathbf{x}_{0:t-1}|\mathbf{z}_{1:t-1}) = \mathcal{N}(\mathbf{x}_{t-1|t-1}, \mathbf{P}_{t-1|t-1})$$

- Prior density at t :

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General algorithm

Initialization

- $x_0 = x_{0|0} = \mathbb{E}[x_0] = \mu_0, P_0 = \mathbb{E}[(x_0 - \mu_0)(x_0 - \mu_0)^T]$

Prediction of $x_{t|t-1}$

Using only the state equation :

$$x_{t|t-1} = \mathbb{E}[x_t | z_{1:t-1}]$$

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General algorithm

Prediction error

$$e_{t|t-1} = x_t - x_{t|t-1}$$

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General algorithm

Prediction of the state covariance error $P_{t|t-1}$

By definition :

$$P_{t|t-1} = \mathbb{E}[e_{t|t-1} e_{t|t-1}^T]$$

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General algorithm

Correction : exploiting the observation z_t

Update of the prediction based on the difference between the predicted observation $z_{t|t-1} = H_t x_{t|t-1}$ and the observation (innovation)

$$x_{t|t} = x_{t|t-1} + Q_t(z_t - H_t x_{t|t-1})$$

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Question : how to set the gain matrix Q_t so that it minimizes the MSE between the state and its estimation?

General algorithm

Prediction error at t

$$e_{t|t} = x_t - x_{t|t}$$

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Prediction error at t

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Discussion of the gain

The Kalmain gain must account for the uncertainties of the current estimation and of the observation :

- if the observation uncertainty V_t is small compared to $P_{t|t-1}$: strong gain (the observation is reliable)
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Finding the gain

- It minimizes the MSE of the estimation : trace $(P_{t|t})$
- We can show that the optimal gain is:

$$Q_t = P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + V^T)^{-1}$$

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Conclusion

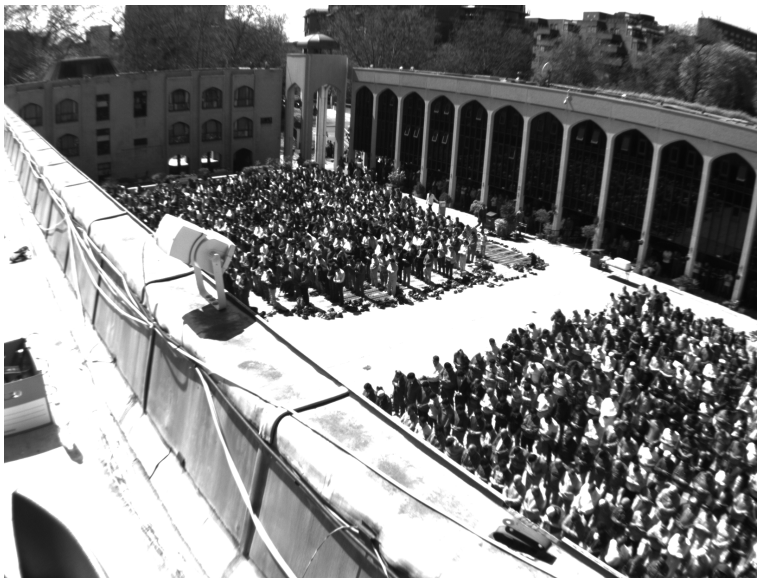
- Linear system and Gaussian noise assumptions : analytical, optimal solution
- Positive definite matrixes : matrix calculus
- Ideally suited for sensor fusion
- However, in reality it has its limitations (e.g. computer vision, tracking etc.): strongly nonlinear or unknown dynamics, non-parametric and multimodal densities
- More general approaches are needed (e.g., sequential Monte Carlo methods), but the cost is significant

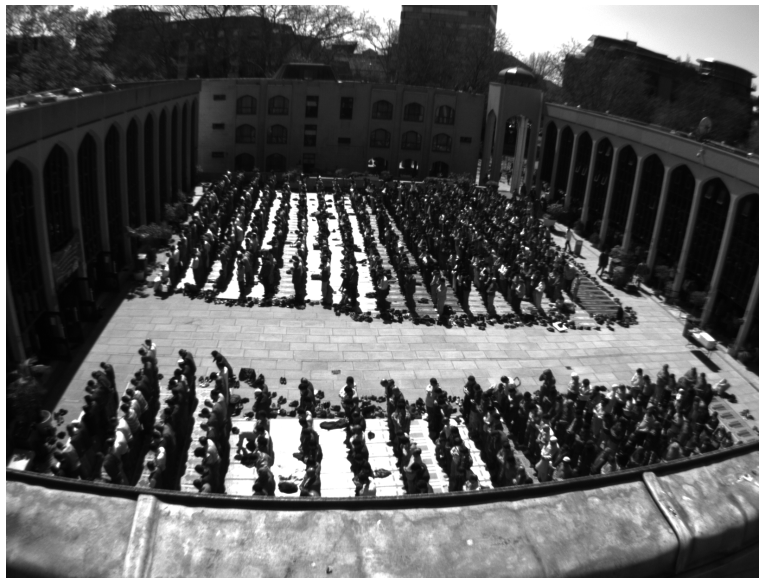
Outline

The context of this work

Head detection in crowded urban environments:

- Video surveillance
- Crowd modeling
- Action recognition





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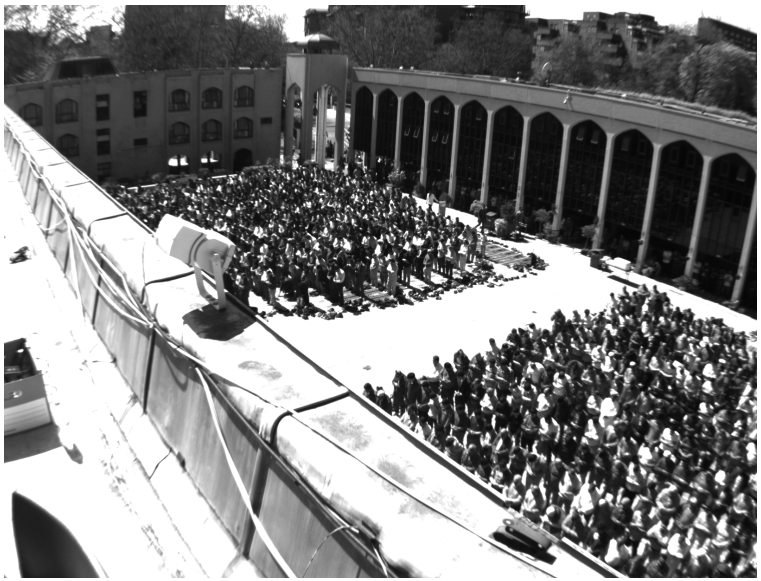

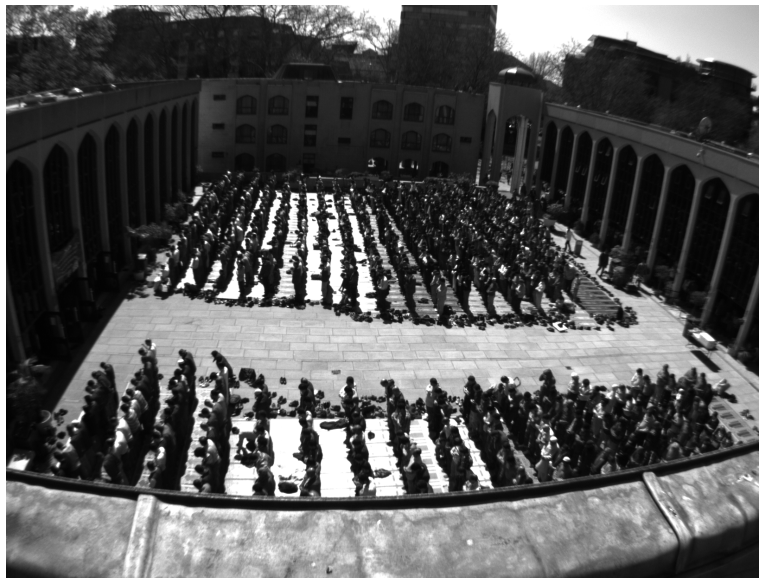

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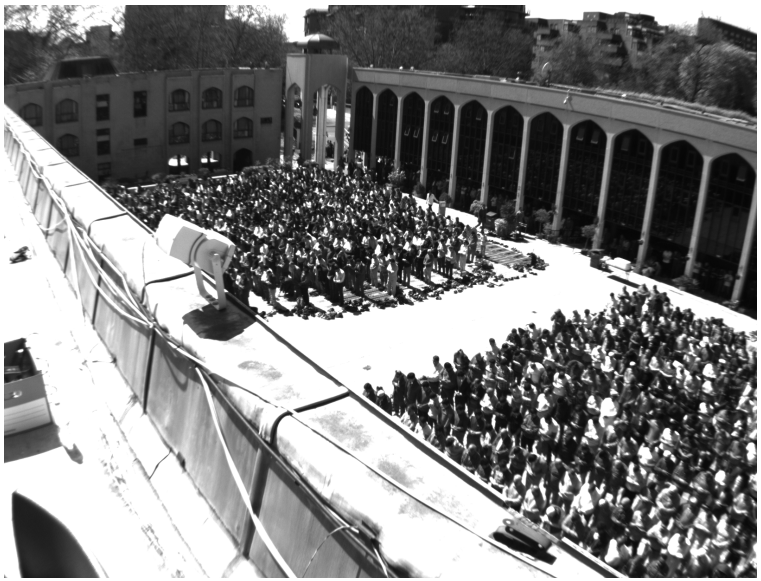

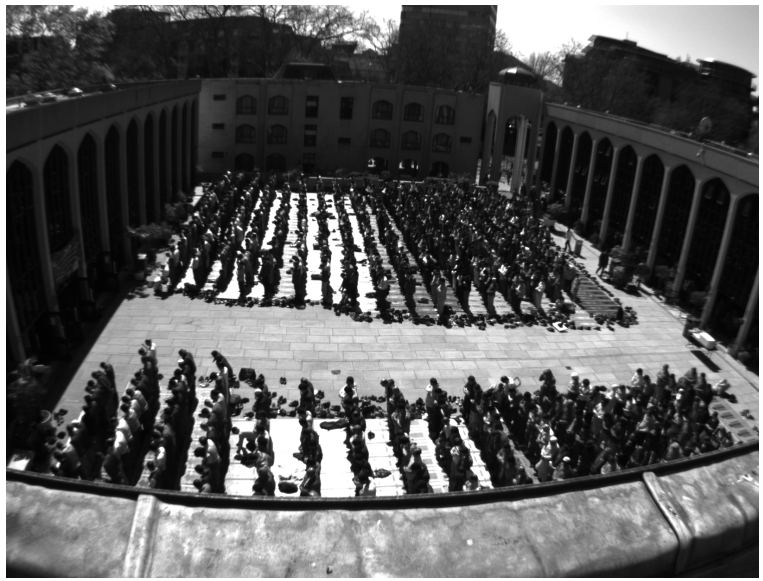

The context of this work

Dense crowds scenarios introduce novel challenges:

- Strong occlusion
- Background homogeneity
- Invisible ground plane and body parts
- Low detection performance in individual sensors

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How to overcome dense crowd challenges in an unsupervised manner?

Camera network with multiple overlapping views.

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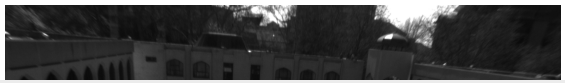
The setting

Images recorded in Regents Park Mosque, London.

- Camera registration is performed at low densities.



- Head detection is performed on the crowd leaving the mosque.

Pedestrian map computation

Pairwise MRF energy function

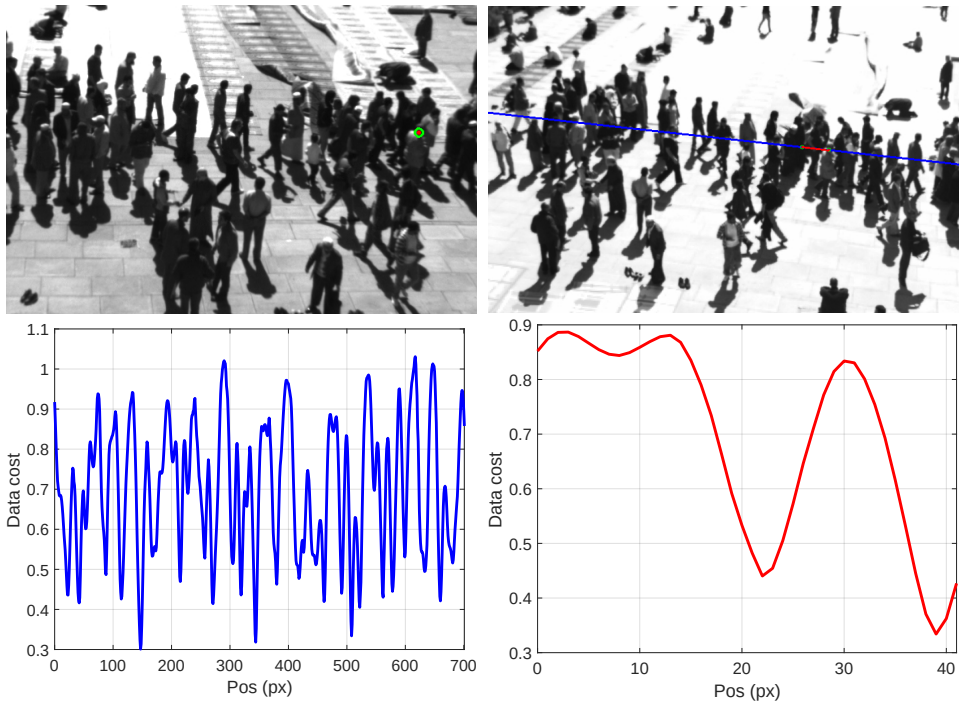
$$E(I) = \sum_{p \in \mathcal{I}} D_p(I_p) + \lambda \sum_{(p,q) \in \mathcal{N}} V_{p,q}(I_p, I_q)$$

- p pixel belonging to the image $\mathcal{I}$.
- $I \in \mathcal{L}$ labeling assigment.
- $\mathcal{N}$ set of edges of the image graph (4-connectivity).
- D_p **data** cost function.
- $V_{p,q}$ **discontinuity** cost function.
- λ regularization parameter.

Geometry of the problem

Inferring scene and camera geometry

- **Objective**: Restrict the search volume of a head in the 3D space.
- Volume contained between the planes at height 1.4 and 2 meters.
- In the image space it corresponds to a **segment** on the epipolar line.
- Extremes of the segments are found by variable-height homographies projections.



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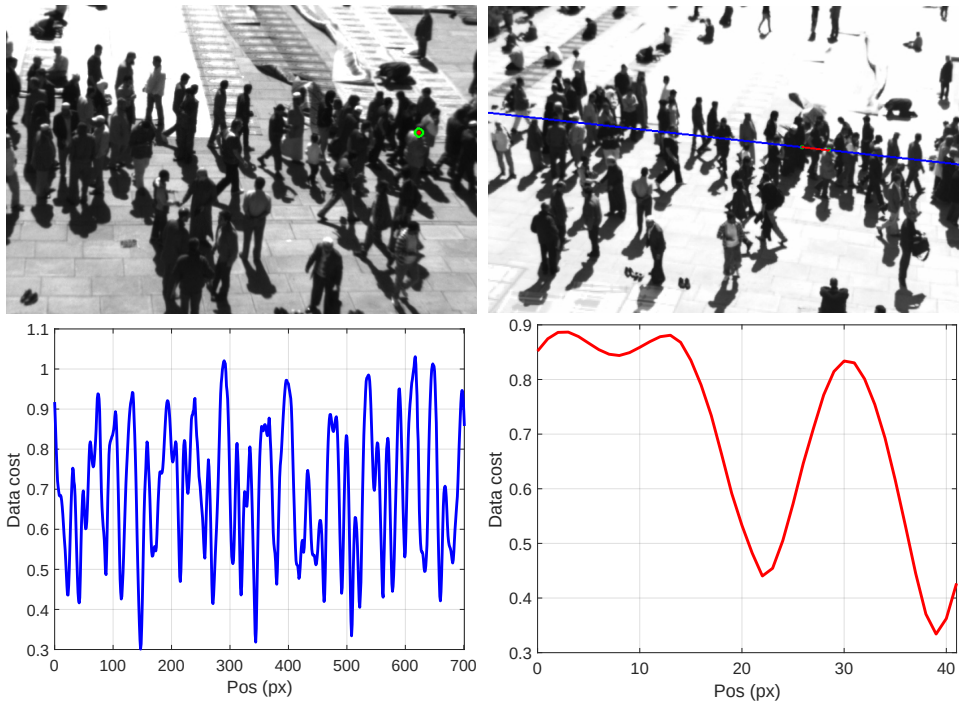

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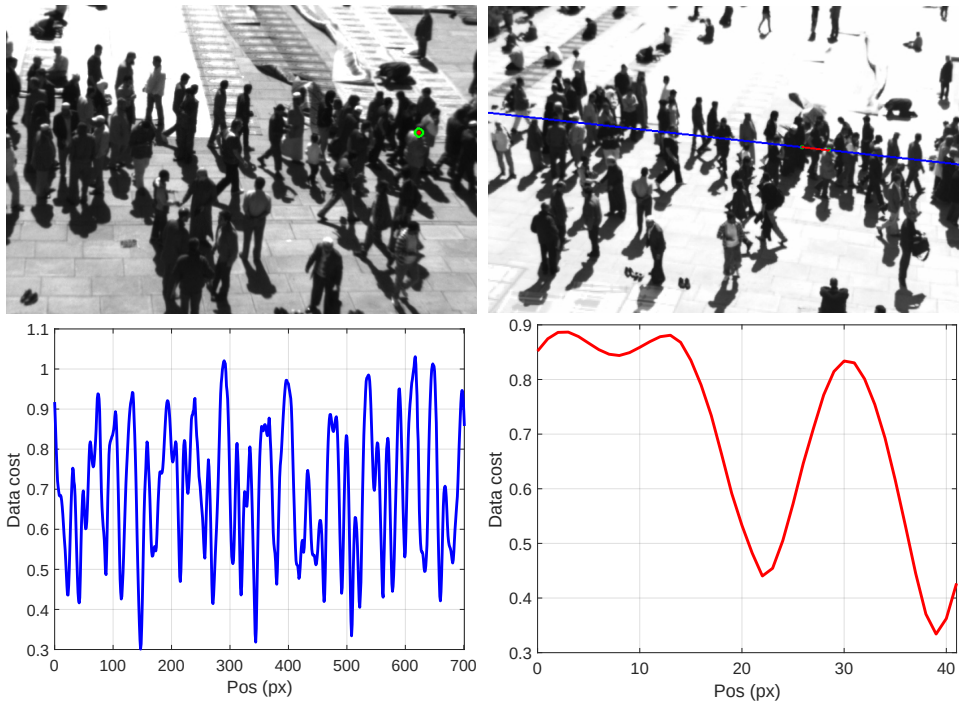

Geometry of the problem

Inferring scene and camera geometry

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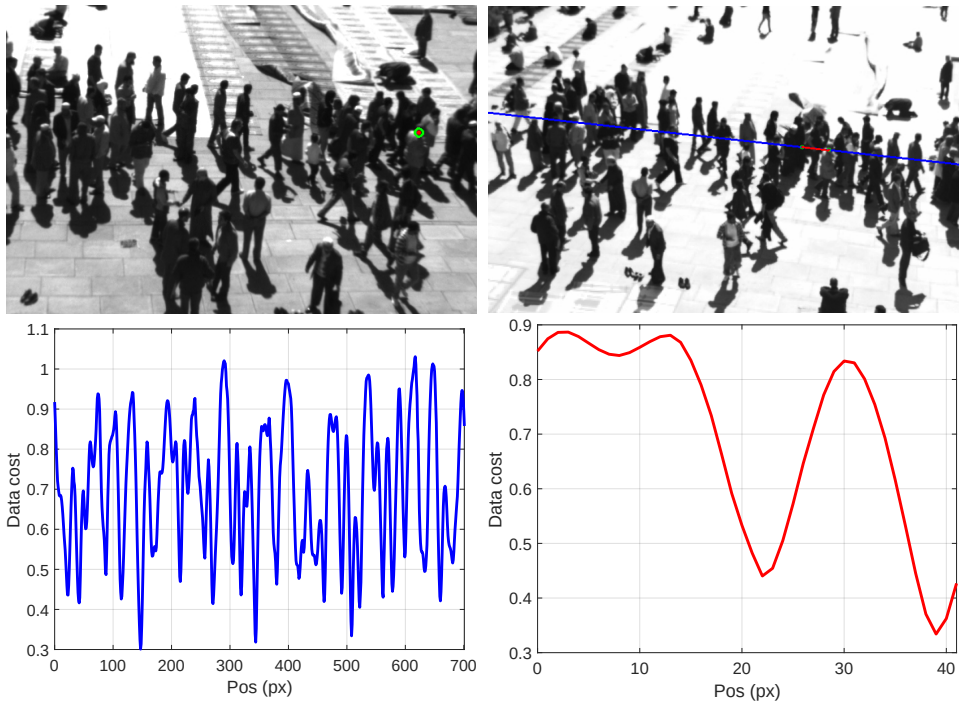

Label definition: height-based optimization

- State-of-the-art stereo matching: *depth* as label.
- This method performs a **height-based optimization**.

Advantages of height over depth

- Use of height comes naturally from the search space definition.
- Height is a ground plane related variable.
- Height allows more sophisticated discontinuity constraints than the classic constant-depth assumption.

- Discretized label set, ranging from 1.4 meters to 2 meters, with step $\Delta_h = 2.5$ centimeters.
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Relative pose estimation

Estimation of the **fundamental matrix** $F_{i,j}$ between any pair of cameras using [Pellicanò et al., 2016]:

- Iterative estimation of $F_{i,j}$ from a synchronized video stream.
- Moving pedestrians add new information at each frame.
- **Output**: $F_{i,j}$, $S_{i,j}$ (inliers matches set).

Bundle adjustment is used to enforce a global scale.

The only metric information needed is the **laser distance** between cameras

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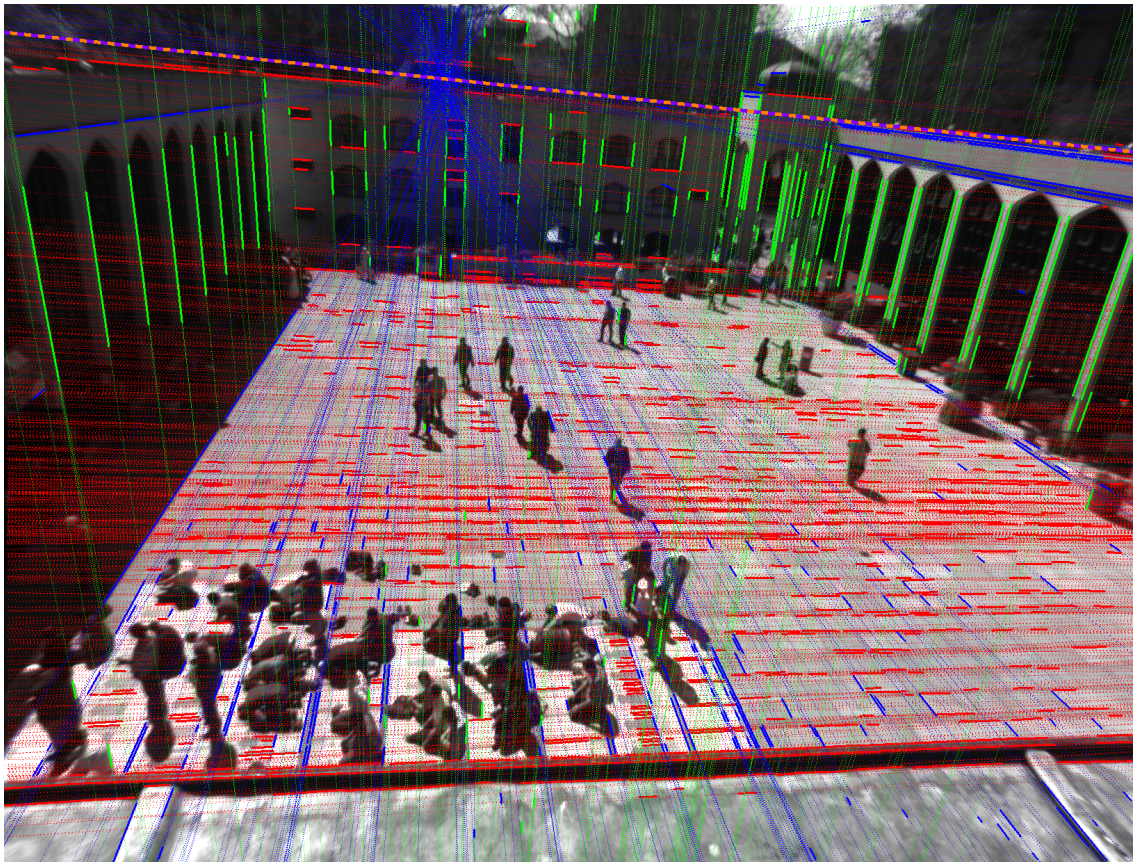
Vanishing points

Automatic vanishing points extraction from image segments [Lezama et al., 2014].



The position of the ground plane is also estimated automatically based on the Manhattan-world assumption.

Data cost function

- We need a local region descriptor suited for a wide-baseline scenario.
- **DAISY descriptor**:
  - Good computational efficiency.
  - Robust to perspective and illumination changes.
- Data cost expressed as a **dissimilarity function** between the DAISY descriptors.
- Given N cameras, $N-1$ dissimilarity functions will be produced.
- **Data cost fusion**:
  - If $N = 3$, simple cost averaging will be effective.
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Discontinuity cost function

- We model the head surface as a planar patch which is **perpendicular to the ground plane**.
- At any pixel location p, the direction of maximum height variation is on the r_p line connecting p to the vertical vanishing point.
- Assuming local planarity, the height varies with a constant gradient along r_p.
- The variation gradient $|\nabla_p|$ can be estimated **from the geometry** and is dependent on the pixel position.
  - Height variation per pixel gets larger in locations further from the reference camera.

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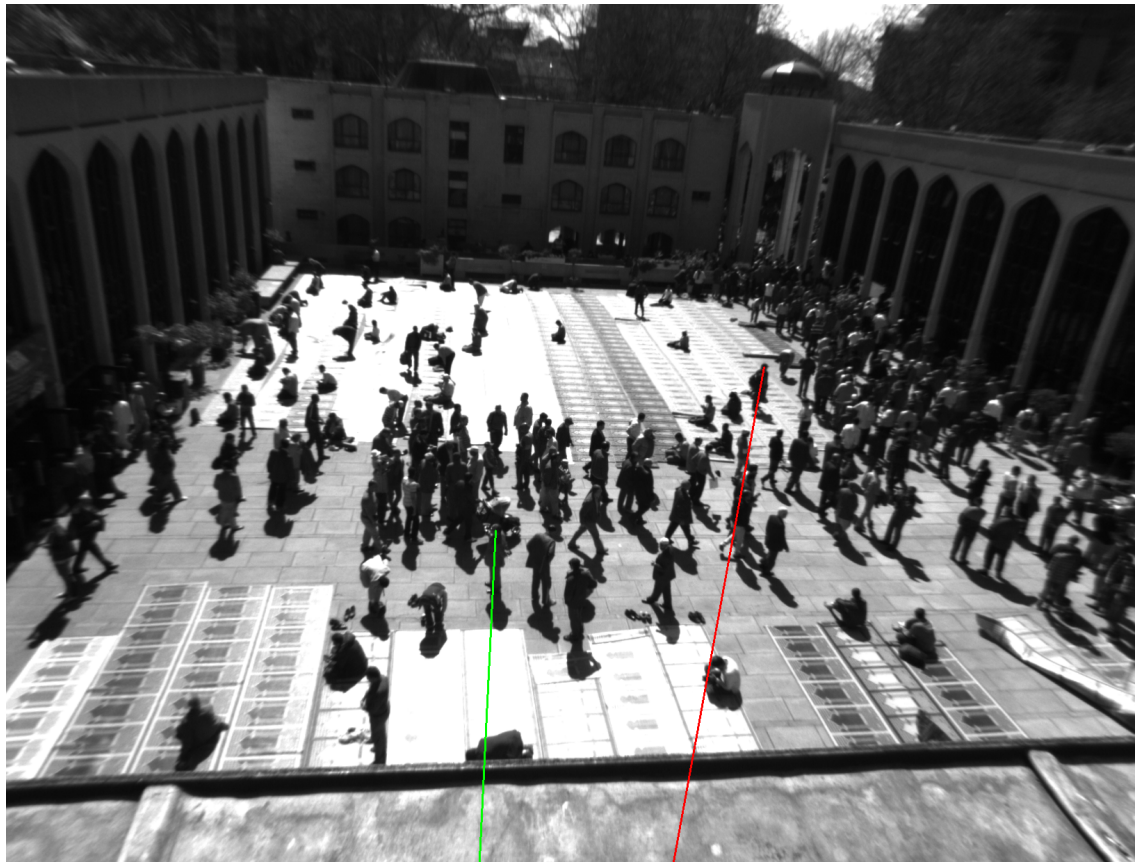

- Red: Head radius $\approx 4px$; $|\nabla_p| = 2.5cm$.
- Green: Head radius $\approx 6px$; $|\nabla_p| = 1.6cm$.

Discontinuity cost function

Given the point p and the neighbor point q:

- Compute the orthogonal projection $q^{\perp}$ of q on the line r_p
- Expected height variation ξ_h: $|\nabla_p| d(p, q^{\perp})$.
- Distance function:

$$D_{p,q}(I_p, I_q) = \left| I_p - I_q - s_p \xi_h \right|$$

where s_p is the expected sign of the variation.

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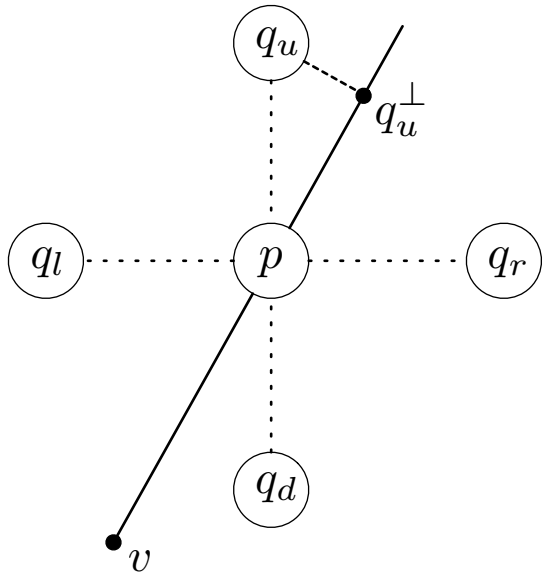
When r_p is almost completely vertical:

- $\xi_h(q_l) = \xi_h(q_r) \approx 0$
- $\xi_h(q_u) = \xi_h(q_d) \approx |\nabla_p|$



Discontinuity cost function

Truncated distance function

$$\hat{V}_{p,q}(l_p, l_q) = \min\left[\frac{\max\left(D_{p,q}(l_p, l_q), D_{q,p}(l_q, l_p)\right)}{\Delta_h}, K\right]$$

Final discontinuity cost function

$$V_{p,q}(l_p, l_q) = \begin{cases} \hat{V}_{p,q}(l_p, l_q) & l_p \neq unknown \wedge l_q \neq unknown \\ K & l_p = unknown \wedge l_q \neq unknown \\ K & l_p \neq unknown \wedge l_q = unknown \\ 0 & l_p = unknown \wedge l_q = unknown \end{cases}$$

Optimization algorithm

- $V_{p,q}(l_p, l_q)$ is not **submodular**: not suitable for Graph-Cuts.
- Results provided with Loopy Belief Propagation.

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Height map example

Color ranges from red (1.4 meters) to green (2 meters).

Temporal filtering

Objective: Output only motion consistent fragments of the height map.

Tracklets building stage

1. Project the height map on the reference plane (geometry+PCA)
2. Find height local maxima in the projected data.
3. Cluster points around closest maxima and compute a centroid per cluster.
4. Tracklets creation and extension. Associate centroid to an existing tracklet if:
   - It is located closer than $\theta_h = 20cm$ from the predicted location of the tracklet.
   - Its height closer than $\theta_h = 10cm$ to the tracklet average height.
5. Any tracklet which is not extended is terminated.

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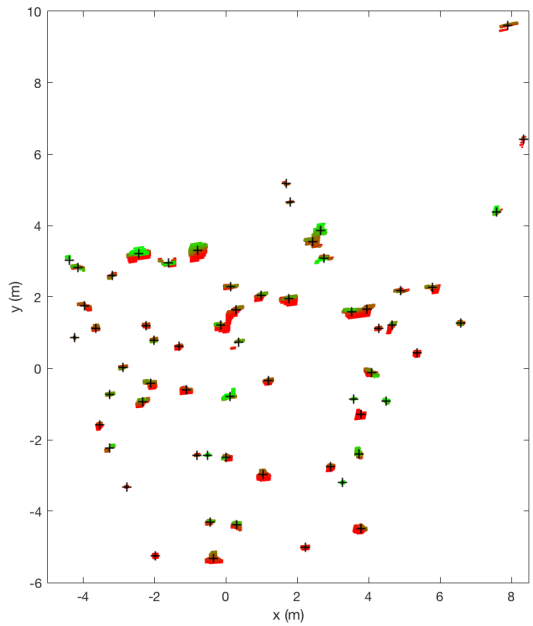
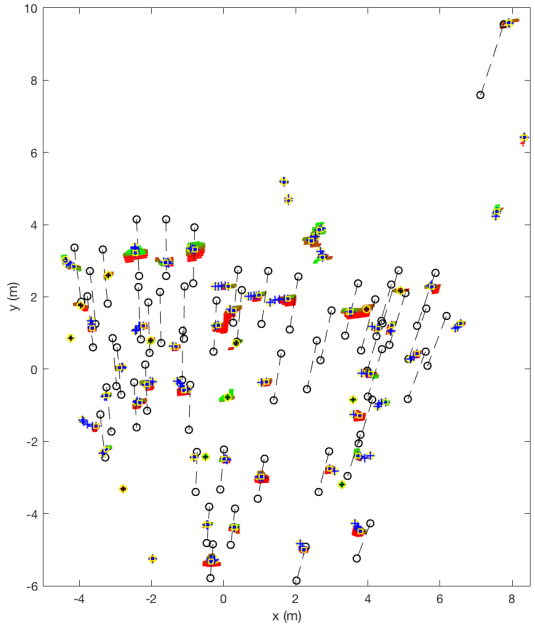
Filtering stage

- Select a tracklet thresholding paramater θ_l ($\theta_l = 0$, no filtering).
- Discard all the fragments associated to tracklets of length less or equal to θ_l.





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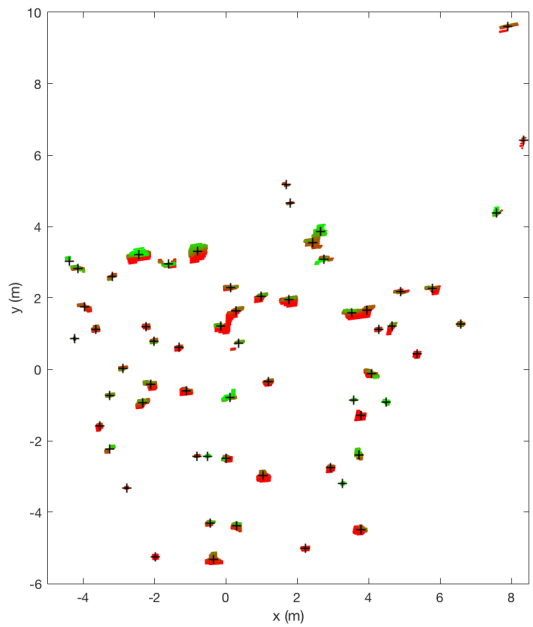

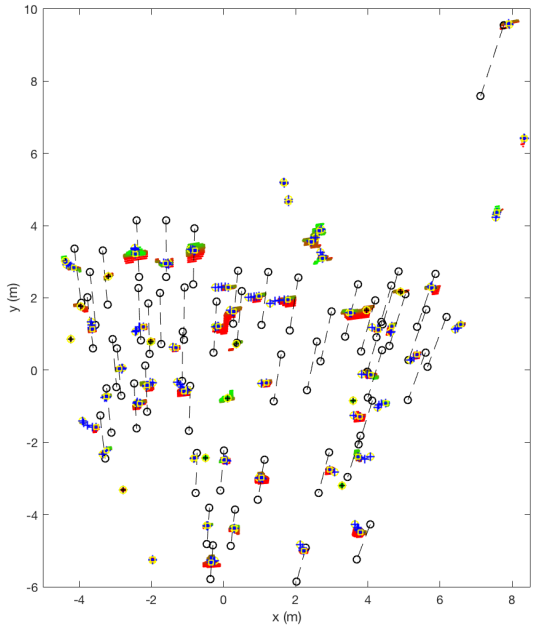

Dataset

- Three synchronized video streams per sequence.
- *Dense*: 18567 pedestrians annotations, clustered towards exit areas.
- *Sparse*: 2969 pedestrians annotations.

- Quantitative evaluation performed with respect to manually annotated tracks.

Dense sequence results

θ_l value	Recall	Precision
0	85.60%	65.67%
1	80.20%	72.35%
2	75.10%	77.18%
3	70.26%	79.81%

Table: Recall and precision on the *Dense* sequence depending on the tracklet threshold θ_l.

- $\lambda = 0.07$
- High recall, even with strong occlusion and no appearance related term.
- Temporal filtering gives a strong benefit on the precision metric.

Sparse sequence results

λ value	Recall	Precision
0.08	89.89%	42.65%
0.15	74.37%	66.78%
0.20	62.23%	82.82%

Table: Recall and precision on the *Sparse* sequence depending on the regularization parameter λ. Here $\theta_{\mathrm{l}} = 1$.

- λ has a strong impact on the precision-recall results.
- At low densities ground related **persistent phantoms** can appear, being detrimental for precision.
- Absence of an appearance term makes impossible to differentiate phantoms from pedestrians.

Concluding remarks

Conclusions

- A method for locating pedestrian heads in cluttered outdoor scenes.
- **Low level** data fusion.
- Modeling the problem is very important : the right space for the hidden variable, the right optimizer
- Cost may be a significant issue, but hardware can help : 1000x acceleration on GPU for this problem

References I

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