MATH 202
Quiz 7

Name : SOLUTIONS

October 27, 2009

Instructions: You get one point for taking this quiz. (Note: This quiz has two pages.)

1. Consider the function $f(x, y) = e^{4x-x^2-y^2}$.

   (a) (1 pts.) Find all critical points of $f(x, y)$.

Find points where $f_x, f_y = 0$ OR f_x or f_y fail to exist.

$f_x(x,y) = e^{4x-x^2-y^2}(4-2x) = (4-2x)e^{4x-x^2-y^2}$

$f_y(x,y) = -2ye^{4x-x^2-y^2}$

Require: $f_x = 0$ and $f_y = 0$ $\Rightarrow$ $(4-2x)e^{4x-x^2-y^2} = 0 \Rightarrow 4-2x = 0 \Rightarrow x=2$

and $-2ye^{4x-x^2-y^2} = 0 \Rightarrow y=0$

The only critical point is $(2,0)$

The critical points are $(2,0)$

   (b) (2 pts.) Use the Second Derivative Test to determine if the critical points are local maxima, local minima, or saddle points. Justify your answer.

1. Find second partial derivatives first:

$f_{xx} = (4-2x)e^{4x-x^2-y^2}(4-2x) + -2e^{4x-x^2-y^2} = e^{4x-x^2-y^2}\left[(4-2x)^2-2\right]$

$= ((4-2x)^2-2)e^{4x-x^2-y^2}$

$f_{yy} = -2y(e^{4x-x^2-y^2})(-2y) + (-2)e^{4x-x^2-y^2} = e^{4x-x^2-y^2}\left[(-2y)^2-2\right]$

$= (4y^2-2)e^{4x-x^2-y^2}$

$f_{xy} = (4-2x)e^{4x-x^2-y^2}(-2y) = -2y(4-2x)e^{4x-x^2-y^2}$

2. Evaluate these 2nd partials at $(2,0)$:

$f_{xx}(2,0) = ((4-2(2))^2-2)e^{4(2)-2^2-0^2} = -2e^4$

$f_{yy}(2,0) = (4(0)^2-2)e^4 = -2e^4$

$f_{xy}(2,0) = -2(0)(4-2(2))e^4 = 0$

3. Form $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} -2e^4 & 0 \\ 0 & -2e^4 \end{vmatrix} = 4e^8 > 0$

4. Since $D > 0$ and $f_{xx}(2,0) = -2e^4 < 0$, we have a $\boxed{\text{LOCAL MAX}}$ at $(2,0)$

2. (1 pt.) Suppose $g(x, y)$ has a critical point at $(3, 3)$, and the values of the second partial derivatives are given as follows:

$$\left.\frac{\partial^2 g}{\partial x^2}\right|_{(x,y)=(3,3)} = 2 \qquad \left.\frac{\partial^2 g}{\partial y \partial x}\right|_{(x,y)=(3,3)} = -1$$

$$\left.\frac{\partial^2 g}{\partial x \partial y}\right|_{(x,y)=(3,3)} = -1 \qquad \left.\frac{\partial^2 g}{\partial y^2}\right|_{(x,y)=(3,3)} = -3$$

Classify the critical point $(3, 3)$ (as a local max, local min, or saddle point), and give an informal explanation to justify this. Your answer should discuss the signs of $g_{xx}(3, 3)$ and $g_{yy}(3, 3)$.

$$D = \begin{vmatrix} 2 & -1 \\ -1 & -3 \end{vmatrix} = 2(-3) - (-1)(-1) = -6 - 1 = -7 < 0$$

The 2nd derivative Test says $(3,3)$ is a $\boxed{\text{SADDLE POINT}}$

Justification:

Since $g_{xx}(3,3) = 2 > 0$ and $g_{yy}(3,3) = -3 < 0$

we (informally) think of $g(x,y)$ as concave down at the curve given by the intersection of $g(x,y)$ with the plane $y = 3$ and concave up at the curve given by the intersection of $g(x,y)$ with the plane $x = 3$.