MATH 161
Exam 2

Name : SOLUTIONS
October 29, 2012

Instructions: Show all work for full credit. Poor notation or sloppy work will be penalized. Point values as indicated.

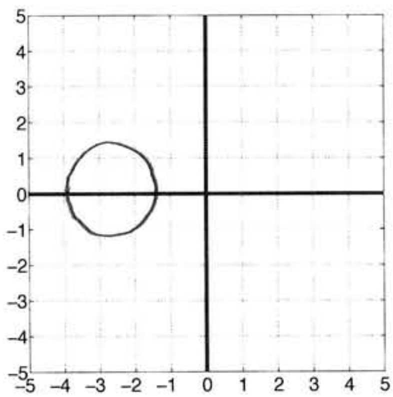
1. (9 pts. total – 3 pts. each)

   (a) Give the definition of a function.

   Variations on the following are correct:

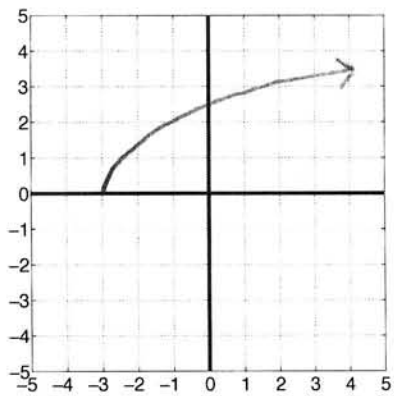
   A function is a rule of assignment

   (b) Sketch a graph of something which is NOT a function.

   fails the Vertical Line Test

   (c) Now sketch the graph of something which is a function.

   passes the vertical line test.

2. (10 pts.) Divide $P(x) = 2x^3 - 7x^2 + 5$ by $D(x) = x - 3$. Give both the quotient $Q(x)$ and remainder $R(x)$.

$$
\begin{array}{r}
2x^2 - x - 3 \\
x-3 \,\big)\, 2x^3 - 7x^2 \qquad + 5 \\
-\ (2x^3 - 6x^2) \\
\hline
-x^2 \\
-\ (-x^2 + 3x) \\
\hline
-3x + 5 \\
-\ (-3x + 9) \\
\hline
-4 \quad R
\end{array}
$$

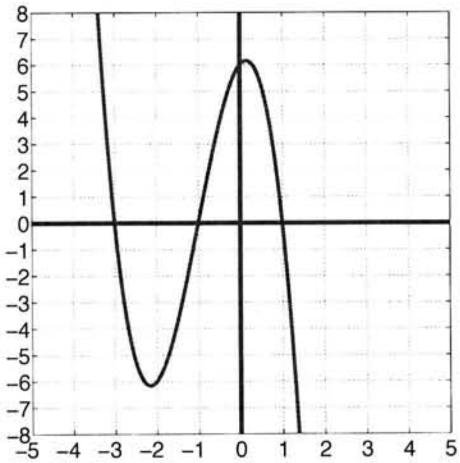
OR

$$
\begin{array}{r|rrrr}
3 & 2 & -7 & 0 & 5 \\
 & & 6 & -3 & -9 \\
\hline
 & 2 & -1 & -3 & -4 \quad R
\end{array}
$$

The quotient $Q(x)$ is $\boxed{2x^2 - x - 3}$. The remainder $R(x)$ is $\boxed{-4}$.

3. (20 pts. – No partial credit.)

Consider the graph of the following polynomial $P(x)$.



(a) (2 pts.) What is the degree of the polynomial? 3

(b) (2 pts.) Is the leading coefficient positive or negative? negative

(c) (2 pts.) Fill in the blanks.

As $x \to \infty$, $P(x) \to$ $-\infty$. As $x \to -\infty$, $P(x) \to$ $+\infty$.

(d) (2 pts.) Is $P(x)$ 1-1? Justify your answer.

No. It fails the horizontal line test.

(e) (2 pts.) Give the *definition* that a function $f(x)$ is 1-1. (You must be precise for credit.)

$f(x)$ is 1-1 if $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

(f) (3 pts.) What are the zeros of $P(x)$? $-3, -1, 1$

(g) (2 pts.) What is the y-intercept of $P(x)$? 6

(h) (5 pts.) Using your answers to the previous questions, give an equation for $P(x)$. (You may leave your answer in factored form.)

$$P(x) = a(x+3)(x+1)(x-1)$$

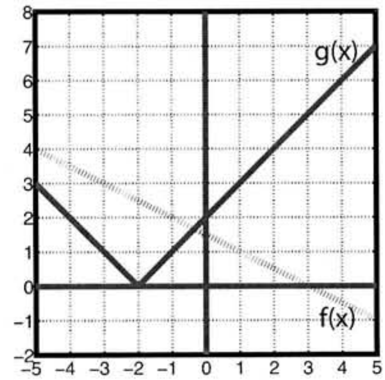
From (g), $P(0) = a(3)(1)(-1) = 6$

$$-3a = 6$$

$$a = -2$$

$$\boxed{P(x) = -2(x+3)(x+1)(x-1)}$$

4. (21 pts. – No partial credit. (a) - (c) are 2 pts. each. (d) - (h) are 3 pts. each.)
Consider the following graph with functions $f(x)$, $g(x)$ as labeled.



Compute, if possible, the following quantities. If there is not enough information to compute the quantity, write "IMPOSSIBLE" or "UNDEFINED" as appropriate.

(a) $(g \circ g)(1) = g(g(1)) = g(3) = \boxed{5}$

(b) $\left(\frac{f}{g}\right)(-2) = \frac{f(-2)}{g(-2)} = \frac{5/2}{0}$ $\boxed{\text{Undefined}}$

(c) $g(f(3)) = g(0) = \boxed{2}$

(d) $\left(\frac{g}{f}\right)(-1) = \frac{g(-1)}{f(-1)} = \boxed{\frac{1}{2}}$

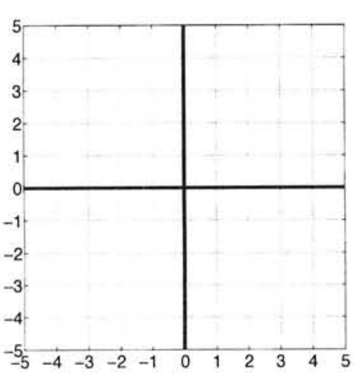
(e) $f^{-1}(2) = \boxed{-1}$

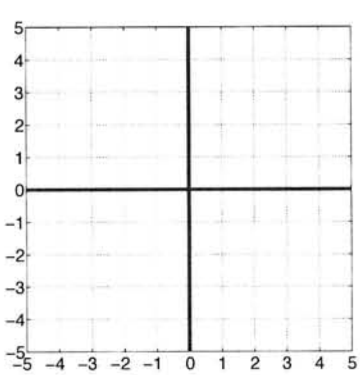
(f) $g^{-1}(1) =$ IMPOSSIBLE, g is NOT 1-1.

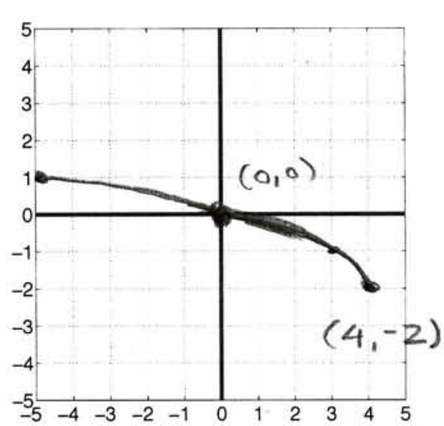
(g) $(f - g)(1) = f(1) - g(1) = 1 - 3 = \boxed{-2}$

(h) The average rate of change of $g(x)$ from $x = -3$ to $x = -1$.
$\frac{g(-3) - g(-1)}{-3 - (-1)} = \frac{1-1}{-2} = \boxed{0}$

5. (10 pts.) Starting with the function $y = \sqrt{x}$, sketch a graph of $y = \sqrt{-x+4} - 2$ on the axes below. You have been given three sets of axes here to use for a sequence of transformations. Please place your final answer on the axes on right. A complete answer has both x- and y-intercepts labeled.







Final Answer: $y = \sqrt{-x+4} - 2$

6. (20 pts.) If a ball is thrown directly upward with an initial velocity of 40 ft/s, its height in feet after t seconds is given by

$$h(t) = 40t - 16t^2 \text{ feet}$$

(a) (5 pts.) Compute $h(2)$ and explain its meaning. Include units in your answer.

$h(2) = 40(2) - 16(2)^2$

$= 80 - 64 = \boxed{16 \text{ ft}}$

After 2 seconds, the ball is 16 ft in the air.

(b) (10 pts.) What is the maximum height attained by the ball? and at what time t does the ball reach this height?

$h(t) = 40t - 16t^2$

$= -16t^2 + 40t$

$= -16(t^2 - \frac{5}{2}t)$

$= -16(t - \frac{5}{4})^2 + (16)(\frac{5}{4})^2$

$= -16(t - \frac{5}{4})^2 + 25$

Max height $= 25$ ft

a time $t = 1.25$ seconds

(c) (5 pts.) At what time does the ball hit the ground?

When $h(t) = 0$ so $40t - 16t^2 = 0$

$8t(5 - 2t) = 0$

$t = 0$ $\boxed{t = 5/2}$

$\boxed{\text{At } t = 2.5 \text{ seconds}}$

7. (10 pts.) It is possible to check that $g(x) = \dfrac{4x-1}{x+3}$ is a 1-1 function. (You can trust me on this.)

(a) (7 pts.) Compute the inverse function $g^{-1}(x)$.

$y = \frac{4x-1}{x+3}$. Switch x and y.

$x = \frac{4y-1}{y+3}$

$x(y+3) = 4y - 1$

$xy + 3x = 4y - 1$

$3x + 1 = 4y - xy$

$3x + 1 = y(4 - x)$

$y = \frac{3x+1}{4-x}$

$\boxed{g^{-1}(x) = \frac{3x+1}{4-x}}$

(b) (3 pts.) Without performing any calculations at all, give $g(g^{-1}(\sqrt{\pi + 2}))$. Explain briefly how you got your answer *without computation.*

Since $g(x)$ and $g^{-1}(x)$ are inverse functions, $g(g^{-1}(\sqrt{\pi+2})) = \boxed{\sqrt{\pi+2}}$