HW 3 PROBLEMS

1-7. Hassett, Chapter 2, #1, 2, 3 (Note: the c_β are non-zero elements of k.), 4a-c, 6 (Note: this shows actually that monomial ideals in $\mathbb{C}[x, y]$ are finitely generated, and by extension that monomial ideals in $\mathbb{C}[x_1, \ldots, x_n]$ are finitely generated.) 7, 8.

8. Hassett, Chapter 2, Application of number 11.

   Digest the definition of a reduced Groebner basis. Then consider the affine variety defined by the following system of linear equations, where the rows correspond to the equations $f_i = 0$, for $i = 1, 2, 3$.

$$\begin{array}{ccccccccc} x & + & y & + & z & & = & 0 \\ & & y & + & 2z & +\,w & = & 0 \\ & & y & + & z & +\,w & = & 0. \end{array}$$

   (a) Give a Groebner basis for the ideal $I = \langle f_1, f_2, f_3 \rangle$ with respect to $>_{\text{lex}}$.

   (b) Give a reduced Groebner basis for the ideal $I = \langle f_1, f_2, f_3 \rangle$ with respect to $>_{\text{lex}}$.

   (c) Give a geometric description of the variety defined by the vanishing of these three polynomials. Also give a parametric description of this variety.

9,10. Hassett, Chapter 2, #13, 14