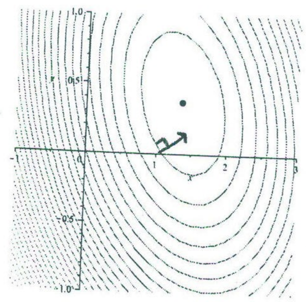
MATH 202
Quiz 8

Name : ____________________

April 1, 2008

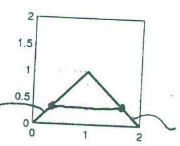
Instructions: This quiz is worth five points. You get one point for taking this quiz.

1. (1 pt.) Below is a contour plot of a function $f(x,y)$ with a unique maximum value at the point (x,y) indicated by a dot. At the point $(1,0)$ draw the gradient vector $\nabla f(1,0)$. (Your answer will only be a positive scalar multiple of the $\nabla f(1,0)$ since you do not know the magnitude.)



$\nabla f(1,0)$ is orthogonal to the contour line and points in the direction of maximal increase for f

2. (2 pts.) Find the volume of the solid that lies below the surface $z = xy$ and above the pictured triangle with vertices $(0,0)$, $(2,0)$, and $(1,1)$.



Easiest to integrate "dxdy"

$y = 2 - x \Rightarrow x = 2 - y$

$y = x \Rightarrow x = y$

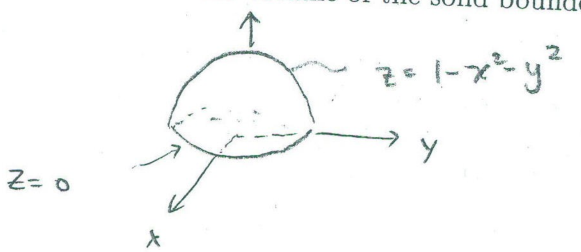
$$\text{Vol} = \int_0^1 \int_y^{2-y} xy \, dx\,dy = \int_0^1 \tfrac{1}{2}x^2 y \Big|_y^{2-y} dy$$

$$= \int_0^1 \tfrac{1}{2}(2-y)^2 y - \tfrac{1}{2}y^2 y \, dy = \int_0^1 \tfrac{1}{2}(4 - 4y + y^2)y - \tfrac{1}{2}y^3 \, dy$$

$$= \int_0^1 2y - 2y^2 + \tfrac{1}{2}y^3 - \tfrac{1}{2}y^3 \, dy = \int_0^1 2y - 2y^2 \, dy = y^2 - \tfrac{2}{3}y^3 \Big|_0^1$$

$$= (1 - 2/3) - 0 = \boxed{\tfrac{1}{3}}$$

3. (1 pt.) Set up, but do not evaluate, an iterated integral in polar coordinates that computes the volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$.



$$\iint_R 1 - x^2 - y^2 \, dA = \int_0^{2\pi} \int_0^1 (1 - r^2)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^1 r - r^3 \, dr\, d\theta$$