

Instructions: Show all work for full credit. Poor notation or sloppy work will be penalized. Point values as indicated.

1. (10 pts. - 5 pts. each) Simplify. Your answer should have no negative exponents. You may assume all variables are positive.

(a) $\frac{\sqrt[3]{16x^{17}y^2}}{\sqrt[3]{2x^2y^{-4}}}$

$$= \sqrt[3]{\frac{16}{2} \cdot \frac{x^{17}}{x^2} \cdot \frac{y^2}{y^{-4}}}$$

$$= \sqrt[3]{8} \cdot \sqrt[3]{x^{15}} \cdot \sqrt[3]{y^6}$$

$$= \boxed{2x^5y^2}$$

(b) $\left(\frac{8x^{-4}y^2}{27x^2y^{-1}}\right)^{-\frac{2}{3}}$

$$= \left(\frac{27x^2y^{-1}}{8x^{-4}y^2}\right)^{\frac{2}{3}} = \left(\frac{27x^6}{8y^3}\right)^{\frac{2}{3}}$$

$$= \frac{27^{\frac{2}{3}} (x^6)^{\frac{2}{3}}}{8^{\frac{2}{3}} (y^3)^{\frac{2}{3}}} = \boxed{\frac{9x^4}{4y^2}}$$

2. (12 pts. - 6 pts. each) Multiply using a special product formula.

(a) $(\sqrt{y}-2)(\sqrt{y}+2)$

$$\boxed{y-4}$$

(b) $(x-2y)^3$

$$x^3 - 3(x)^2(2y) + 3(x)(2y)^2 - (2y)^3$$

$$= \boxed{x^3 - 6x^2y + 12xy^2 - 8y^3}$$

3. (10 pts. - 5 pts. each) Factor completely.

(a) $(x^2 - 3)^2 - 4(x^2 - 3) - 5$

$$([x^2 - 3] - 5)([x^2 - 3] + 1)$$

$$= \boxed{(x^2 - 8)(x^2 - 2)}$$

(b) $3x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}$

$$3x^{-1/2}(x^2 - 3x + 2)$$

$$\boxed{3x^{-1/2}(x-2)(x-1)}$$

4. (7 pts.) Rationalize the denominator. Then cancel any common terms.

$$\frac{h}{\sqrt{x+h} - \sqrt{x}} \cdot \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) = \frac{h(\sqrt{x+h} + \sqrt{x})}{(x+h) - x} = \frac{h(\sqrt{x+h} + \sqrt{x})}{\cancel{h}}$$

$$\boxed{\sqrt{x+h} + \sqrt{x}}$$

5. (10 pts.)

(a) (3 pts.) Give the formula for simple interest. Define all quantities in your answer.

$$I = Prt$$

\uparrow Interest \rightarrow Principal \rightarrow rate \rightarrow time

(b) (7 pts.) Cal invests \$10,000 in a Certificate of Deposit (CD) at an interest rate of 4%. After t years, \$1200 in simple interest has been earned. Find the number t of years that this money was invested.

$I = \$1200$ $P = \$10,000$ $r = .04$ Find t .

$$I = Prt$$

$$1200 = 10,000(.04)t$$

$$1200 = 400t$$

$$\frac{1200}{400} = t$$

$$\boxed{t = 3 \text{ years}}$$

6. (16 pts. - 4 pts. each) Find all real solutions. If there are no real solutions, write "No real solutions." Check your answers, if necessary.

(a) $2a^2 - a = 7$

$$2a^2 - a - 7 = 0$$

Quadratic Formula.

$$\begin{aligned} a &= \frac{1 \pm \sqrt{1 - 4(2)(-7)}}{4} \\ &= \frac{1 \pm \sqrt{1 + 56}}{4} \\ &= \frac{1 \pm \sqrt{57}}{4} \end{aligned}$$

(b) $\left[\frac{3}{x} + \frac{5}{x+2} = 2 \right] x(x+2)$

$$3(x+2) + 5x = 2x(x+2)$$

$$3x + 6 + 5x = 2x^2 + 4x$$

$$8x + 6 = 2x^2 + 4x$$

$$0 = 2x^2 - 4x - 6$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$x=3$

Check: $\frac{3}{3} + \frac{5}{3+2} \stackrel{?}{=} 2$
 $1 + 1 = 2 \quad \checkmark$

$x=-1$

Check: $\frac{3}{-1} + \frac{5}{-1+2} \stackrel{?}{=} 2$
 $-3 + 5 \stackrel{?}{=} 2$
 $2 = 2 \quad \checkmark$

$x=3$
 $x=-1$

(d) $x^2 + 4 = 0$

$$x^2 = -4$$

No Real Solutions

(e) $\sqrt{x+5} = x-1$

$$(x+5) = (x-1)^2$$

$$x+5 = x^2 - 2x + 1$$

$$0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

$$x=4 \quad x=-1$$

Check:

$x=4: \sqrt{4+5} \stackrel{?}{=} 4-1$
 $3 = 3 \quad \checkmark$

$x=-1: \sqrt{-1+5} \stackrel{?}{=} -1-1$
 $2 \stackrel{?}{=} -2 \quad \text{No}$

Only $x=4$

7. (15 pts. - 5 pts. each) Solve each inequality. Give your answer either in interval notation or using inequality notation.

(a) $3x + 2 \leq 7x - 1$

$$3 \leq 4x$$

$$\frac{3}{4} \leq x$$

$$\boxed{x \geq \frac{3}{4}}$$

(b) $\left| \frac{3x-1}{2} \right| > 7$

$$|3x-1| > \frac{7}{2}$$

$$\frac{3x-1}{2} < -7$$

$$3x-1 < -14$$

$$3x < -13$$

$$x < -13/3$$

$$\frac{3x-1}{2} > 7$$

$$3x-1 > 14$$

$$3x > 15$$

$$x > 5$$

$$\boxed{x < -\frac{13}{3} \text{ or } x > 5}$$

(c) Use a number line to solve this inequality. Include test points as part of your answer. Place your answer on the line provided.

$$\frac{2x-3}{x+1} \leq 1$$

$$\frac{2x-3}{x+1} - 1 \leq 0$$

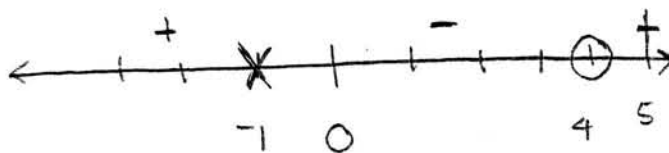
$$\frac{2x-3-(x+1)}{x+1} \leq 0$$

$$\frac{2x-3-x-1}{x+1} \leq 0$$

$$\frac{x-4}{x+1} \leq 0$$

Zero: $x = 4$

Undefined: $x = -1$



Test points for $\frac{x-4}{x+1}$

$$x = -2: \frac{-2-4}{-2+1} = \frac{-6}{-1} = 6 > 0$$

$$x = 0: \frac{0-4}{0+1} = -4 < 0$$

$$x = 5: \frac{5-4}{5+1} = \frac{1}{6} > 0$$

Answer: $\boxed{(-1, 4]} \text{ or } -1 < x \leq 4$

8. (10 pts. - 5 pts. each) Consider the two points $P(1, -2)$ and $Q(5, 1)$.

(a) Find the distance $d(P, Q)$ between the two points.

$$d = \sqrt{(1-5)^2 + (-2-1)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

(b) Find the equation of the circle with center P and containing the point Q .

$$(x-1)^2 + (y+2)^2 = 25$$

$$R = 5$$



9. (10 pts.) Amy buys an old car. The value V in dollars of this car after t years is given by the linear function

$$V = -100t + 1000 \text{ dollars}$$

for \$1000 which depreciates at a rate of ~~\$400~~¹⁰⁰ per year.

(a) (6 pts.) Find the t intercept of this line.

The t -intercept is where $V=0$

$$t = 10$$

(b) (4 pts.) Explain the meaning of the t intercept.

After $t = 10$ years, the car is worthless.