MATH 202 Name :_____

Midterm 1 February 17, 2012

Instructions: Show all work for full credit. Poor notation or sloppy work will be penalized.

1. (6 pts.) Let $P(-1, 2, 0)$ and $Q(1, 1, -3)$ be points in $\mathbb{R}^3$. Give parametric equations for the line containing P and Q.

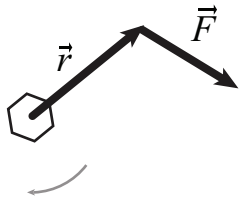
2. (20 pts. – 5 pts. each) Give brief clear explanations to the following questions.

   (a) Let $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ be three vectors in $\mathbb{R}^3$. What is the geometric meaning of $\left|\mathbf{v}_1 \cdot (\mathbf{v}_2 \times \mathbf{v}_3)\right|$?

   (b) Suppose $\mathbf{a}$ and $\mathbf{b}$ are unit vectors in $\mathbb{R}^3$ and $\mathbf{a} \cdot \mathbf{b} = -\dfrac{\sqrt{3}}{2}$, what can you say about the vectors?

   (c) Let $P(x_1, y_1, z_1)$ be a point in $\mathbb{R}^3$ at the tip of vector $\mathbf{p}$, and $\mathbf{n} = < n_1, n_2, n_3 >$, $\mathbf{x} = < x, y, z >$. Explain why the equation $(\mathbf{x} - \mathbf{p}) \cdot \mathbf{n} = 0$ gives the equation of a plane. (Drawing a picture might help.)

   (d) The magnitude of the torque vector $\tau = \|\boldsymbol{\tau}\| = \|\mathbf{r} \times \mathbf{F}\|$ measures the tendency of an object (a bolt, for example) to rotate. (See figure.) Using the formula for the magnitude of the cross product of vectors, explain the effect of lengthening the lever arm on this tendency to rotate. Does this agree with your experience?



3. (17 pts.) Let $\mathbf{a} = < 1, -1, 3 >$ and $\mathbf{b} = < 2, 0, 1 >$ be vectors in $\mathbb{R}^3$.

(a) (8 pts.) Find the equation of the plane that contains the vectors $\mathbf{a}$ and $\mathbf{b}$.

(b) (4 pts.) Find the area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$. (Your answer will contain an angle θ.)

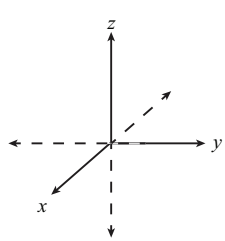
(c) (5 pts.) Find the *vector projection* of $\mathbf{a}$ onto $\mathbf{b}$; that is, find $\text{proj}_{\mathbf{b}}(\mathbf{a})$.

4. (12 pts. – 4 pts. each)

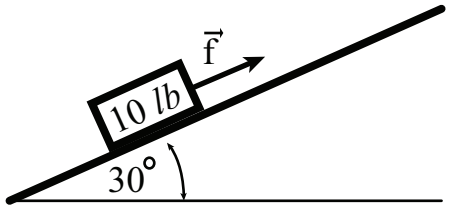
(a) Convert the point P with rectangular coordinates $P(-2, 2\sqrt{3}, 1)$ to cylindrical coordinates.

(b) If Q is the point with rectangular coordinates $Q(-3, 0, 0)$, what are the spherical coordinates of Q?

(c) Describe and sketch the surface in $\mathbb{R}^3$ whose equation in spherical coordinates is given by $\phi = \dfrac{3\pi}{4}$.



5. (5 pts.) A 10 lb weight is placed on an incline forming an angle of 30° with the horizontal as shown. Find the magnitude of the force **f** needed to keep the weight from sliding.



6. (15 pts.) Consider the planes in $\mathbb{R}^3$ given by the equations below:

$$\begin{aligned} \text{Plane 1:} \qquad & 2x - 3y + z = 1 \\ \text{Plane 2:} \qquad & -4x + 6y - 2z = 2 \end{aligned}$$

(a) (5 pts.) Are the planes parallel or do they intersect? Explain your answer briefly.

(b) (3 pts.) Give the equation of a plane parallel to Plane 1 that passed through the origin.

(c) (2 pts.) Find the point Q on the first plane with x-coordinate 1 and y-coordinate 1.

(d) (5 pts.) Using the point $Q(x, y, z)$ found in the previous problem, find the distance from Q to the second plane.

7. (8 pts. – 4 pts. each) The trajectory in the plane of two particles are given by the equations

$$\text{Particle 1:} \quad \mathbf{r}(u) = < e^{2u},\, 2\cos(u) - 1 >$$

$$\text{Particle 2:} \quad \mathbf{s}(v) = < \tan(v),\, 1 + \ln(\frac{4}{\pi}v) >$$

where u and v are both measured in seconds.

(a) Show that the graphs of $\mathbf{r}(u)$ and $\mathbf{s}(v)$ intersect at the point $(1, 1)$. (Justify your answer for full credit.)

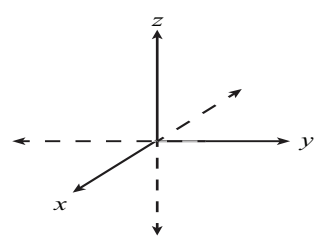
(b) Do the two particles collide at $(1, 1)$. Explain briefly.

8. (17 pts.) An particle moves in $\mathbb{R}^3$ with position given by

$$\mathbf{r}(t) = < \cos(t), t^2, 4\sin(t) > \text{ meters for } t \geq 0,$$

where t is measured in seconds.

(a) (5 pts.) On the axes below, sketch the trajectory of the particle. Indicate the orientation of the trajectory with an arrow. Include the rectangular coordinates of $\mathbf{r}(0)$, $\mathbf{r}(\frac{\pi}{2})$, and $\mathbf{r}(\frac{3\pi}{2})$ on your graph.



(b) (5 pts.) Find the velocity $\mathbf{v}(t)$ of the particle at time t. Include units in your answer.

(c) (4 pts.) Find the speed of the particle at $t = \frac{\pi}{6}$.

(d) (3 pts.) At time $t = e^{\pi}$, are the velocity and acceleration vectors perpendicular? Explain briefly.