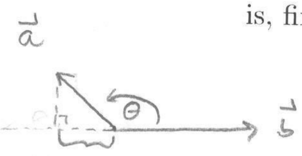
MATH 202
Quiz 2

Name : SOLUTION

February 8, 2008

Instructions: Each question is worth 2 points. You get one point for taking this quiz.

1. (a) Let $\mathbf{a} = (-1,\ 1,\ 0)$ and $\mathbf{b} = (2,\ 1,\ 1)$. Find the *scalar* component of $\mathbf{a}$ onto $\mathbf{b}$; that is, find $\text{comp}_{\mathbf{b}}\mathbf{a}$.



$$\text{Comp}_{\vec b}\vec a = \frac{\vec a \cdot \vec b}{|\vec b|} = \frac{(-1)(2) + (1)(1) + (0)(1)}{\sqrt{2^2+1^2+1^2}} = \boxed{\frac{-1}{\sqrt{6}}}$$

↳ this "shadow" is $\text{comp}_{\vec b}\vec a$

$= |\vec a| \cos\theta$

Note: since $\frac{-1}{\sqrt{6}} < 0$

$\frac{\pi}{2} < \theta < \pi$

(b) Use your answer to (a), to find the *vector* component of $\mathbf{a}$ onto $\mathbf{b}$; that is, find $\text{proj}_{\mathbf{b}}\mathbf{a}$.

$$\text{proj}_{\vec b}\vec a = \frac{-1}{\sqrt6}\left(\frac{\vec b}{|\vec b|}\right) = \frac{-1}{\sqrt6}\cdot\frac{1}{\sqrt6}(2,1,1) = \left(\frac{-2}{6}, \frac{-1}{6}, \frac{-1}{6}\right)$$

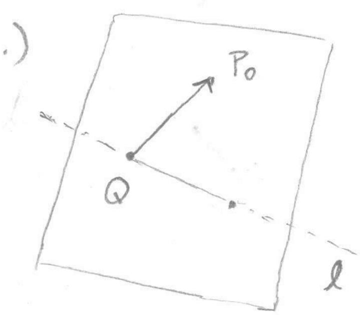
$$= \left(\frac{-1}{3}, \frac{-1}{6}, \frac{-1}{6}\right)$$

2. Give the equation of the plane that contains the point $P_0(3, -1, 1)$, and contains the line given by parametric equations $x = 1 - t$, $y = -1 + t$, $z = t$.

The plane must contain the direction vector of the line $\vec v = (-1, 1, 1)$.

Also, the point $Q(1, -1, 0)$ is on the line (Set $t = 0$.)

Therefore, $\overrightarrow{QP_0}$ is also in the plane, $\overrightarrow{QP_0} = (2, 0, 1)$



The normal vector is $\vec n = \overrightarrow{QP_0} \times \vec v = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 2 & 0 & 1 \\ -1 & 1 & 1 \end{vmatrix}$

$$= -\hat\imath - 3\hat\jmath + 2\hat k = (-1, -3, 2)$$

Using the point P_0 and $\vec n$, we find

$$\vec n \cdot \vec x = \vec n \cdot \vec{p_0}$$

$$-x - 3y + z = (-1, -3, 2) \cdot (3, -1, 1)$$

$$\boxed{-x - 3y + 2z = 2}$$