Exam 1 Name:_____

Math 202 F01 02/15/06

1. A particle is initially at the point $\mathbf{r}(0) = (-1, -2, 0)$. It moves so that its velocity is given by $\mathbf{v}(t) = (3\cos(5t), 4, 3\sin(5t))$.

   (a) (6 pts.) Find the acceleration of the object at $t = \pi/2$. (You should simplify your answer so that no trigonometric functions appear.)

   (b) (6 pts.) Find the position of the object at all times.

   (c) (8 pts.) Find the length of the path the object has followed between the times $t = \pi$ and $t = 3\pi$.

   (d) (3 pts.) At what time (if ever) does the object cross the xz-plane?

2. Consider the three points in space: $A = (1, 2, 1)$, $B = (0, 2, -1)$, and $C = (2, 1, 1)$.

   (a) (5 pts.) Which of B and C is the closest to A?

   (b) (7 pts.) Give a parameterization of a line through B that is parallel to the line between A and C.

   (c) (9 pts.) Give an equation for the plane in which the points A, B, and C lie.

3. (6 pts.) An object is acted on by a force of $\mathbf{F} = (2, 1, 1)N$. However, other constraints on the object allow it to move only in the direction given by $\mathbf{d} = (1, 1, -1)$. Calculate a vector representing the part of the force $\mathbf{F}$ that can actually affect the motion of the object.

4. Consider the two parameterized paths $\mathbf{r}(t) = (t^2, 2, t)$ and $\mathbf{s}(t) = (1 + \ln t, 2t, 3t - 2)$.

   (a) (5 pts.) Show that particles following these paths would collide.

   (b) (5 pts.) At what angle would the particles hit one another? Your answer may involve an inverse trigonometric function.

   (c) (3 pts.) Is the angle in part (b) acute (less than $\pi/2$), right, or obtuse (greater than $\pi/2$)? Explain how you know. (No points will be given for an answer without explanation.)

5. Suppose you are given three vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. What simple formulas could you use to calculate each of the following?

   (a) (4 pts.) The volume of the parallelepiped with edges $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

   (b) (4 pts.) The area of the parallelogram with edges $\mathbf{a}$ and $\mathbf{b}$.

6. The following equations can all be graphed relatively easily in $\mathbb{R}^3$. For each, with *one phrase or sentence* indicate what about the equation makes it possible to graph it without much effort, and then give the graph.

   (a) (7 pts.) $z = 9 - y^2$

   (b) (7 pts.) $z = 9 - x^2 - y^2$

   (c) (7 pts.) $x + 2y + z = 1$

7. Convert between coordinate systems, as indicated.

   (a) (4 pts.) $(1, 1, \sqrt{2})$ in rectangular coordinates = ? in spherical coordinates

   (b) (4 pts.) $(2, \pi, -3)$ in cylindrical coordinates = ? in rectangular coordinates