

MATH 632: Galois Theory

MWF 9:15– 10:15

Gruening 308

<http://www.dms.uaf.edu/~eallman/classes/632-fall-09/632-2009.html>

Instructor: Elizabeth S. Allman

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Office Hours: T 9-10, W 10:15-11:15, F 10:15-11:15, and by appointment.

Prerequisites: MATH 631 with a grade of C or better.

Textbook: *Galois Theory*, 3rd ed., by I. Stewart, Chapman and Hall

Final Exam Date: December 16, 8:00 - 10:00 am

COURSE OVERVIEW AND GOALS:

Galois theory was invented by the French mathematician Evariste Galois (1811-1832) whose remarkable and short life was ended tragically after a duel. Galois theory is concerned with finding solutions to polynomial equations and, in particular, with Galois' discovery of deep and remarkable connections between group theory and roots of polynomials equations.

It is well known that a quadratic equation $ax^2 + bx + c = 0$ has a general solution given by the quadratic equation, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Less well known is that there are similar more complicated equations for solving the general cubic and quartic equations. While Abel proved that there is *no* such formula for the general quintic $a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$, one of Galois' magnificent contributions was the discovery of the intimate relationship between this lack of formula and the structure of the symmetric group S_5 .

In this course, we develop Galois theory by introducing notions of separable extensions, normal extensions, and finally Galois extensions of fields. This study culminates in the main theorem connecting the *Galois group* G of a polynomial to a Galois extension of fields. The Galois extension reflects many of the properties of its Galois group; for instance, the lack of a general formula for roots of quintics follows from the structure of S_5 , in particular, since S_5 is not *solvable*.

We also discuss solutions to classic problems of the Greeks, like those on ruler and compass constructions.

COURSE MECHANICS:

Class meetings will be run as interactive lectures, to the extent possible given the enrollment. That means that while I will be presenting material at the board, and you will be taking notes, I will also be asking for suggestions, ideas, and questions about the material as we go along. I don't expect 'correct' answers, but I do expect you to be actively following and participating — that makes the class more interesting for us all.

As some topics in MATH 632 require more background than you received in MATH 631, there will be some review of polynomials rings and other special rings, together with coverage of some additional topics, during the first weeks of this course before we begin Galois Theory.

Class attendance is expected, although I will not formally take roll. If you miss a class, you should get notes from another student. Homework assignments will be given in class, and also posted on the course web page in a timely fashion.

Homework will be assigned regularly and collected once a week on a day agreeable to all. There will rarely be time at the beginning of class for simple questions on homework, so you should expect to get your homework questions answered during office hours¹. There is an emphasis on proof-writing in algebra courses, and substantial time and effort must be expended to develop clear and lucid proof-writing techniques.

I encourage you to work with others on the homework, and to share ideas for solutions, but you must *write up solutions independently*. You will learn nothing from simply copying a solution. Even though you may find you can't do every problem without help or hints, as a group we should be able to give proofs for all problems. The entire homework assignment will be checked by the instructor, with most problems graded quite closely.

Homework will be accepted until 5pm on its due date, either at my office or in my mailbox in the math department office. I will not accept *any* late homework that has not been cleared ahead of time or is not due to a genuine emergency (e.g., a death in the family).

Missed examinations or homework papers that are not approved in advance will result in an 'F' on that work. No make-ups will be given except in extreme circumstances (e.g., family death, documented illness, etc.). Notifying me by email or a note that you will miss an exam is not sufficient for advance approval; you must speak with me to be excused.

Grades:

As should be clear from the above, the largest contribution to your final grade will be from your graded work on homework assignments. This is an elective class, and I expect students to work maturely and hard to learn advanced algebra. This includes filling in gaps in background independently, when necessary. Grades will be assigned using the following weights:

Homework	75 %
Class Participation	10 %
Final Exam	15 %

Grade Bands: A, A- (90 - 100%); B+, B, B- (80 - 89%); C+, C, C- (70 - 79%); D+, D, D- (60 - F%); 69 (0 - 59%). On rare occasion, I may lower the thresholds. Also, in an effort to reward the student who makes significant improvement over the course of the term, marked improvement over the semester may overcome earlier deficiencies and result in a better final grade.

Other Policies:

Course accommodations: If you need course adaptations or accommodations because of a disability, please inform your instructor during the first week of the semester, after consulting with the Office of Disability Services, 203 Whitaker (474-7403).

University and Department Policies: Your work in this course is governed by the UAF Honor Code. The Department of Mathematics and Statistics has specific policies on incompletes, late withdrawals, and early final exams, some of which are listed below. A complete listing can be found at

<http://www.dms.uaf.edu/dms/Policies.html>.

¹or possibly during our extra weekly session. Special arrangements will be discussed on the first day of class

Prerequisites: The prerequisite for MATH 632 is MATH 631 with a grade of C or better. Students not meeting this prerequisite are not eligible to take this course and will be dropped.

Late Withdrawal: This semester the last day for withdrawing with a 'W' appearing on your transcript is October 30. If, in my opinion, a student is not participating adequately in the class, I may elect to drop this student.

Graded Coursework: Please keep all graded work for MATH 632 until final grades have been assigned.

Academic Honesty: Academic dishonesty, including cheating and plagiarism, will not be tolerated. It is a violation of the Student Code of Conduct and will be punished according to UAF procedures.

Courtesies: As a courtesy to your instructor and fellow students, please arrive to class on time, turn your cell phones and iPods off during class, and pay attention in class.