MATH 161 Name :______________________________

Math 161 Review Problems Decemb34 10, 2012

Section 1.1: Sets, Intervals, and Absolute Value

1. Find the union and the intersection of $A = \{1, 3, 8, 9, 10, 101\}$ and $B = \{-1, 2, 3, 8, 101, 120\}$.
2. In the set $\{-7, \frac{2}{5}, -0.7, \sqrt{7}, 0, \pi, -\sqrt{100}\}$, list all numbers that are A) integers, B) rational, and C) irrational.
3. Rewrite the expression in an equivalent form without absolute value bars.
   (a) $|\sqrt{3} - 5|$ (b) $|2x - 1|$
4. What are the possible values of $\dfrac{|2x+1|}{2x+1}$?
5. Using the absolute value symbol, express each statement:
   (a) the distance between x and 1
   (b) the distance between x and 1 is more than 5 units
6. Interpret $|x| \leq 2$ as distance from the origin. Draw this set on a number line. Express this set using interval notation.
7. Express the graphed set using interval notation.
8. Represent the solution graph using absolute value.

Section 1.2: Exponents and Radicals

1. Simplify. Write all answers using positive exponents only.
   (a) -6^0 (b) $\dfrac{5^{-3}}{5}$ (c) $3(2x^{-2}y^3)^5$ (d) $\dfrac{36x^4y^9}{64x^{-5}y^{12}}$ (e) $\dfrac{\sqrt[3]{54x^5y}}{\sqrt[3]{2x^2y^{-5}}}$ (f) $\left(\dfrac{27x^{-4}y^2}{8x^2y^{-1}}\right)^{\frac{-2}{3}}$
2. Simplify the following expressions. Assume all variables represent non-negative numbers.
   (a) $\sqrt{75x^4}$ (b) $\dfrac{\sqrt{80x^5}}{\sqrt{5x}}$ (c) $\sqrt{63x} - \sqrt{28x}$ (d) $\sqrt{(-11)^2} + \sqrt[3]{64}$
   (e) $\left(\sqrt[5]{x^2y}\right)^{\frac{5}{2}}$ (f) $\sqrt{18xy^3} \cdot \sqrt[3]{2^4x^4y^7}$ (g) $\left(8x^{-6}y^3\right)^{\frac{1}{3}}\left(x^{\frac{5}{6}}y^{\frac{-1}{3}}\right)^6$

Section 1.3: Algebra and Polynomials

1. Simplify
   (a) $-6^2 + 4$ (b) $3(10 - 2(1-4)^2)^2$ (c) $2(3xy)4(xy)$
2. Multiply the following and simplify.
   (a) $(2x-3)(x^2-4x+3)$ (b) $(3x-2)^2$ (c) $(2\sqrt{x}-1)^2$ (d) $(3x-4)^3$
   (e) $[8y+(7-3x)][8y-(7-3x)]$ (f) $(x-y-3)(x-y+3)$ (g) $(3x+1)(x^2+9)(3x-1)$

3. Factor completely.

   (a) $12x^4 - 18x^3 - 54x^2$ (b) $8x^2 + 33x + 4$ (c) $6x^4 + 6x^2 - 12$ (d) $16x^4 - 81$

   (e) $(5x + 2y)^2 - (5x - 2y)^2$ (f) $125x^6 - 27$ (g) $2(x + 3)^{\frac{1}{2}} - 10(x + 3)^{\frac{5}{2}}$

   (h) $3(x + 1)(2x + 3)^2 - 9(x + 1)^2(2x + 3)$ (i) $(x + 1)^{\frac{1}{3}} + x(x + 1)^{\frac{-2}{3}}$ (j) $2(x + 3)^{\frac{-1}{2}} - 5(x + 3)^{\frac{1}{2}}$

   (k) $(x^2 - 3)^2 - 4(x^2 - 3) + 3$

Section 1.4: Rational Expressions

1. Perform the indicated operations. Also state any values that should be excluded from the domain of each expressions.

   (a) $\dfrac{2x^2 + 8x}{8x}$ (b) $\dfrac{2}{5x + 1} + \dfrac{3}{5x}$ (c) $\dfrac{x^2 - 25}{x^2 + 3x - 10} \div \dfrac{x^2 + 7x + 10}{x^2 + 8x + 15}$ (d) $\dfrac{1 + x^{-1}}{1 - x^{-2}}$

   (e) $\dfrac{3}{5x + 2} + \dfrac{5x}{25x^2 - 4}$ (f) $\dfrac{2(x + 1)^{\frac{1}{2}} - x(x + 1)^{\frac{-1}{2}}}{x + 1}$ (g) $\dfrac{\frac{3}{h+1} - 3}{h}$ (h) $\dfrac{\frac{1}{2+x} - \frac{1}{2}}{x}$

   (i) $\dfrac{15x^4(x^2 - 1)^2 + 12x^2(x^2 - 1)^3}{x^4(x^2 - 1)(3x + 2)}$ (j) $\dfrac{8x(x + 2)^2 - 6x^2(x + 2)}{6x^3(x + 2)^6}$ (k) $\dfrac{x - 3}{x^2 - 4} - \dfrac{x + 2}{x^2 - 4x + 4} - \dfrac{2}{2 - x}$

2. (a) Rationalize the denominator $\dfrac{23}{5 + \sqrt{2}}$ (b) Rationalize the numerator $\dfrac{\sqrt{x + 3} - \sqrt{x}}{5}$

Section 1.5: Solving equations

1. Solve the following equations for the unknown.

   (a) $\dfrac{3}{x + 3} = \dfrac{5}{2x + 6} + \dfrac{1}{x - 2}$ (b) $\dfrac{1}{y - 3} - \dfrac{2}{y + 1} = \dfrac{8}{y^2 - 2y - 3}$ (c) $-\dfrac{9}{x^2} + 8 = -\dfrac{1}{x^4}$

   (d) $\sqrt{20 - 8a} = a$ (e) $\sqrt{x + 5} - \sqrt{x - 3} = 2$ (f) $(2x + 7)(x - 6) = -39$

   (g) $w^2 - 13w = -36$ (h) $3(y + 4)^2 - 5 = 22$ (i) $2c^2 + c - 5 = 0$ (j) $x^{-2} - 3x^{-1} - 4 = 0$

2. Solve by completing the square.

   (a) $6x^2 - 12x - 3 = 0$ (b) $3x^2 + x - 2 = 0$

Section 1.6: Modeling with Equations

See text # 25, 27, 29, 61, 63, 67, 69.

Section 1.7: Inequalities

1. Solve

   (a) $\dfrac{3x}{10} + 1 \geq \dfrac{1}{5} - \dfrac{x}{10}$ (b) $-5 \leq \dfrac{1}{2}x - 4 < -3$ (c) $3x^2 < 8x$ (d) $\dfrac{x^2 - 4}{x^2 - 2x - 3} \leq 0$

   (e) $\dfrac{4}{x} \leq x$ (f) $-\dfrac{1}{x} \leq x - 2$ (g) $x + 2 \leq \dfrac{3}{x}$

2. Find the domain of $\dfrac{\sqrt{x + 3}}{\sqrt{x - 1}}$.

Section 1.8 and 1.10: Symmetries, Lines and Circles

1. Determine any symmetry. Prove your answers by applying the appropriate symmetry test.

   (a) $y = x^4 + \dfrac{|x|}{x^4 + x^2 + 2}$ (b) $y = 2x^7 - 3x + 1$ (c) $x = y^4 - 5y^2$

2. Find the equation of the line

   (a) passing through the points $P(1, 2)$ and $Q(5, -2)$

   (b) passing through $P(1, 3)$ and parallel to $3x - 6y = 2$

   (c) passing through $P(1, 3)$ and perpendicular to $3x - 6y = 2$

3. Find x such that $(x, 4)$ is 5 units from $(3, 1)$.

4. Find the equation of the circle that

   (a) has center $(2, 1)$ and radius $r = 3$

   (b) has center $(2, -3)$ and passes through the point $P(1, 1)$

5. Find the center and radius of the circle with equation $2x^2 + 2y^2 + 20x - 36y - 30 = 0$

Section 2.1: Functions Main ideas: evaluation, domains

1. Evaluate the functions $f(x) = \begin{cases} x^2 - 1, & \text{for } x > 2 \\ \frac{1}{x^2+2}, & \text{for } x \leq 2 \end{cases}$, and $g(x) = \dfrac{x^2 - 4}{2x^2 - 9x - 5}$ at $x = -2, 0, \frac{1}{2}, 1, 5$.

2. Give the domain of the following functions:

   (a) $\dfrac{\sqrt{x-2}}{x^2 - 2x - 3}$ (b) $\sqrt[3]{t-1}$ (c) $\sqrt[4]{2y+5}$ (d) $x + \dfrac{1}{x}$

Section 2.2: Graphs of functions Main ideas: sketch by plotting points, domain and range from graphs, vertical line test

1. Sketch the following functions. Give the domain and range.

   (a) $f(x) = 2x^2 - 3$ (b) $g(x) = |x + 3|$ (c) $h(x) = \sqrt{2x + 1}$ (d) $h(x) = (x + 5)^3$

   (e) For each of the functions above, describe the function as a transformation. For example, '*the function is a vertical translation by 10 units up.*'

2. State the *Vertical Line Test* for a function. Now sketch a graph that a) IS a function and b) IS NOT a function.

Section 2.3: Local Minima and Maxima of graphs, Increasing and decreasing functions Main ideas: You will need to know and use the definitions of minima, maxima, and intervals of increase and decrease.

1. (a) Textbook 1-4 (b) Textbook 19, 21 (c) Textbook 31, 33 (d) Textbook 43

Section 2.4: Average rate of change of a function Main ideas: Understand how to compute average rates of change and to interpret them as slopes of secant lines.

1. If an object is dropped from a tall building, the distance it has traveled after t seconds is given by $d(t) = 16t^2$ ft.

   (a) Find its average speed between (i) $t = 1$ second and $t = 5$ seconds (ii) $t = 2$ s and $t = 6$ s

   (b) Sketch a graph of $d(t)$ for $t \geq 0$. Explain the meaning of your answer to (i) and (ii) above. Your answer should include words like 'slope' and include the sketch of some line segments.

Section 2.5: Transformations of Functions Main ideas: Understand horizontal and vertical translations of graphs, dilations/contractions, reflections.

1. A function $f(x)$ is given. Explain how to obtain the formula for the

   (a) y-axis reflection of $f(x)$ (b) the x-axis reflection of $f(x)$

   (c) a horizontal translation of $f(x)$ five units left (d) a vertical translation of $f(x)$ ten units down

2. Starting with the function $y = x^3$. Graph the following. Include all intercepts on your sketches.

   (a) $y = -(x+2)^3 + 1$ (b) $y = -(-x+2)^3 + 1$ (c) $y = -2(x+1)^3$

3. Starting with the function $y = \sqrt{x}$. Graph the following. Include all intercepts on your sketches.

   (a) $y = -\sqrt{-x+4} + 1$ (b) $y = \sqrt{3-x} - 1$

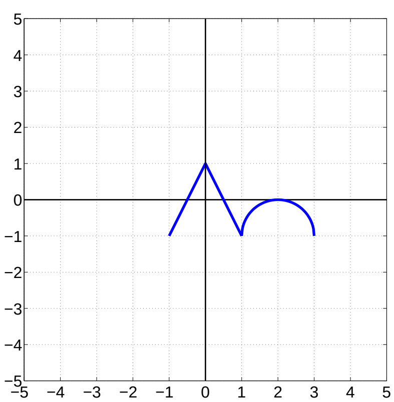
4. Starting with the function $g(x) = |x|$. Give the formula for the function obtained by the following sequence of transformations. Then sketch the graphs of these functions.

   (a) a horizontal translation 4 units right, then a y-axis reflection, then a vertical translation 3 units down

   (b) an x-axis reflection, followed by a vertical translation 5 units up.

5. Below is the graph of the function $y = f(x)$. Graph

   (a) $y = -f(-x)$ (b) $y = -f(x-3) - 1$ (c) $y = f(-x)$ (d) $y = -2f(x)$ (e) $y = |f(x)|$



Section 2.6: Combining Functions Main ideas: Adding, subtracting, multiplying, dividing functions. VERY important: composition of functions.

1. Let $f(x) = |x+1|$, $g(x) = x^3 - 1$, and $h(x) = \sqrt[3]{x+1}$. Compute (if possible)

   (a) $f(g(1))$, $f(g(-1))$, $g(f(1))$, $g(f(-1))$, $g(h(\pi))$, $h(g(\pi))$, $h(g(x))$, $f(g(h(7)))$, $h(g(f(2)))$

(b) $(f+g)(1)$, $(f-g)(2)$, $(gh)(-1)$, $(\frac{f}{g})(-4)$, $(\frac{f}{h})(26)$

(c) $(f \circ g)(x)$, $(g \circ g)(x)$, $(h \circ f)(x)$, $(g \circ h)(2)$, $(f \circ h)(x)$

(d) The domains of $(\frac{g}{f})(x)$, $(g \circ h)(x)$, $(\frac{g}{h})(x)$

Section 2.7: 1-1 functions, inverse functions Main ideas: Horizontal line test; Definition of 1-1 function; Finding inverses of 1-1 functions; Graphing inverses of functions

1. Give the definitions of 1-1 *function* and *inverse of a function* $f(x)$.

2. State the horizontal line test. Determine if the functions below are 1-1 or not.

3. Suppose $f(2) = 3$, $f(4) = 10$, and $f(5) = 2$ and that f^{-1} exists. Find, if possible,

   (a) $f^{-1}(2)$, (b) $f^{-1}(3)$, (c) $f^{-1}(4)$, (d) $f^{-1}(10)$, (e) $(f \circ f)(5)$ (f) $(f \circ f^{-1})(x)$

4. Find the inverse of

   (a) $g(x) = 2x + 1$, (b) $h(x) = \dfrac{4x-2}{3x+1}$, (c) $p(t) = 4 - \sqrt[3]{t}$.

5. Sketch a graph of $g(x) = 2x - 1$. Show that $g(x)$ is 1-1. Then sketch a graph of $g^{-1}(x)$. Finally, give a formula for $g^{-1}(x)$.

Section 3.1: Quadratic functions and models Main ideas: Plotting parabolas, putting a quadratic polynomial in 'standard form' by completing the square, find the coordinates of minima and maxima, find the maximum/minimum value

1. Consider the quadratic polynomial functions below. For each of these, (i) find the coordinates of the vertex; (ii) sketch the function; (iii) find the maximum or minimum value; (iv) state the domain and range.

   (a) $f(x) = x^2 + 4x + 1$ (b) $g(x) = -x^2 + 6x + 5$, (c) $h(x) = 3 - 4x - 4x^2$,

2. Find the maximum or minimum value of the function

   (a) $h(t) = 3 - x - \frac{1}{2}x^2$, (b) $h(t) = 100x(x - 150)$

3. 3.1 #65

Section 3.2: Polynomial functions and their graphs Main ideas: Sketching polynomials, determining end behavior, using zeros and sign charts to graph polynomials

1. Determine the 'end behavior' of the following polynomials. Find the zeroes of the polynomials. Finally, sketch the graph of the polynomial. Make sure your graph displays all intercepts and exhibits the correct end behavior.

   (a) $g(x) = -2x^4 - x^3 + 3x^2$ (b) $h(t) = t^3 - 2t^2 - 4t + 8$

Section 3.3: Dividing polynomials Main ideas: Long division of polynomials, synthetic division. Vocabulary: quotient, remainder, divisor, dividend

1. Use long division to perform the following divisions:

   (a) $\dfrac{4x^3 + 2x^2 - 2x - 3}{2x + 1}$ (b) $\dfrac{2x^5 - 7x^4 - 13}{4x^2 - 6x + 8}$ (c) $\dfrac{x^6 + x^4 + x^2 + 1}{x^2 + 1}$

2. Use synthetic division to find the quotient and the remainder.

   (a) $\dfrac{x^2-5x+4}{x-3}$ (b) $\dfrac{x^3+2x^2+2x+1}{x+2}$

Sectons 3.4, 3.7, Chapter 4, Sections 10.1, 10.2, 10.9

Section 3.4: Find real roots of polynomials Main ideas: Rational Root Theorem, Descartes Rule of Signs, Factoring after finding roots

1. Give a list of all possible rational roots of the function $f(x) = -12x^4 + 100x - 2$.

2. Find all rational zeros of

   (a) $f(x) = x^3 - 3x^2 - 4$ (b) $g(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$ (c) $h(x) = 6x^3 + 11x^2 - 3x - 2$

3. Find all real roots of $x^3 + 4x^2 + 3x - 2$. (Textbook 47)

4. Sketch a plot of $x^4 - 5x^3 + 6x^2 + 4x - 8$ (Textbook 61)

Section 3.7: Rational functions Main ideas: Plotting rational functions; finding vertical, horizontal and slant asymptotes; understanding 'end behavior'

Sketch the following. In your analysis, you should
1) Factor numerator (zeros) and denominator (vertical asymptotes).
2) Find $x-$ and y-intercepts.
3) Determine behavior of graph near vertical asymptotes (to ∞ or $-\infty$)
4) Determine the horizontal asymptotes and/or end behavior.
5) Sketch.

1. $R(x) = \dfrac{6x^3-2}{2x^3+5x^2+6x}$ (Textbook 29)

2. $f(x) = \dfrac{x^2+2}{x-1}$ (Textbook 31)

3. $W(x) = \dfrac{x^4+2}{x-1}$ (Variation on Textbook 31)

4. $g(x) = \dfrac{x^2-2x+1}{x^3-3x^2}$ (Textbook 63)

Section 4.1: Exponential functions Main ideas: Graphs of exponential functions; compound interest.

1. Sketch (a) $y = -2^{-x} + 5$ (b) $h(x) = -\left(\dfrac{1}{3}\right)^x + 2$.

2. Suppose you invest \$2000 at an interest rate of 5%.

   (i) Compute the amount of money in your account assuming the interest is

   (a) compounded annually (b) compounded semiannually (c) compounded quarterly

   (d) compounded monthly (e) compounded daily

   (ii) Now suppose at the end of 1 year you have \$2016.20 in your account, but that interest was computed using the **simple** interest formula. What the **simple** interest rate for this year?

Section 4.2: The natural exponential e^x Main ideas: Plotting and computing with $y = e^x$, continuously compounded interest.

1. Use a calculator to compute $e^{1.2}$, $e^{-.1}$, e^{-2}, e^3.
2. Sketch a plot of $y = -e^x + e$. Include intercepts and asymptotes.
3. Suppose you invest \$10,000 in an account with interest rate $r = 1.1\%$.
   (a) If the interest is compounded continuously, how much money is in the account after 1 year? 2 years? 3.5 years?
   (b) If the interest is compounded quarterly, how much money is in the account after 1 year? 2 years? 3.5 years?

Section 4.3: Logarithm functions and Section 4.4 Laws of logarithms Main ideas: Plotting and computing with $y = \log x$, $y = \ln x$, understanding logarithms and how to simplify them. Using the laws of logarithms. Change of base formula.

1. Without a calculator simplify
   (a) $\log(100)$ (b) $\log(\frac{2}{200})$ (c) $\log(4) + \log(25)$ (d) $\log(100^x)$ (e) $\log(10^{\ln e})$ (f) $\log(10^{\ln \pi})$
   (g) $\ln(\frac{1}{e})$ (h) $\frac{1}{2}\ln(e^2)$ (i) $\ln(e^{x^2}) - \ln(e)$ (j) $\ln(1)$ (k) $e^{\ln(100)}$ (l) $\ln(12e) - \ln(12)$
   (m) $\log 25 + 4\log 2 - 2\log 2$ (n) $\log_4 16^{100}$ (o) $\log(\log(10^{10{,}000}))$ (p) $\log_3(\sqrt{27})$ (q) $\log_9(\frac{1}{3})$
   (r) $\log_2(8^5)$ (s) $\log_\pi(\pi)$
2. Without a calculator, answer whether the following expressions are positive, negative, zero, or undefined.
   (a) $\log_2(-.1)$ (b) $\ln(3)$ (c) $\ln(\frac{1}{2})$ (d) $\log(.7)$ (e) $\ln e$
3. Textbook 4.4: 7-51 odd
4. Sketch $y = \ln(x + 1) - 2$. Include intercepts and asymptotes. Give the domain and range.
5. Without a calculator, find integers a, b so that
   (a) $a < \log(76) < b$ (b) $a < \ln(\frac{1}{2}) < b$ (c) $a < \log_4(70) < b$
6. Suppose you invest \$10,000 in an account where the interest is compounded continuously. After two years you have \$10,202.01 in the account. What was the interest rate r?
7. Use a calculator to estimate the following. Round your answer to three decimal places.
   (a) $\log_5(23.2)$ (b) $\log_2(\frac{1}{3})$
8. Write down the three laws of logarithms.
9. Sketch $y = |\ln(x + 3)|$.

Section 4.5: Exponential and Logarithmic Equations Main ideas: Solving equations with the variable in the argument of a logarithm or in the exponent position. Doubling time for investment, population growth.

1. Solve for x. Check your answers.

   (a) $\log(x+2) = -1$ (b) $\log(x^2-1) = 3$ (c) $\ln(x+1) = 2 - \ln(x)$ (d) $\log_2(3x-1) = 2$
   (e) $2\log_2 x = -3$ (f) $e^{2x} - e^x = 6$ (g) $3^{\frac{x}{14}} = .1$ (h) $2^{3x-1} = 3^{x+2}$ (i) $e^{\ln(x^2-3x)} = 4$
   (j) $e^x = -4$

2. Explain in your own words why

   (a) $\log\left(\frac{2}{3}\right) \neq \frac{\log(2)}{\log(3)}$ (b) $\log(2+x) \neq \log(2) + \log(x)$ (c) What are the rules of log's?

3. A student invests \$3000 in an account which pays 3% interest per year. When will there \$5000 in the account, if

   (a) The interest in compounded quarterly? (b) The interest in compounded monthly?
   (c) The interest in compounded continously?

Section 4.6: Modeling with exponential and logarithm functions Main ideas: Doubling/Tripling/Halving times and exponential growth; Relative growth rates.

1. A certain breed of rat was introduced on an island 8 months ago. The current rat population on the island is estimated to be 4100 and doubling every three months.
   (a) What was the initial size of the population?
   (b) Estimate the population one year after the rabbits were introduced to the island.
   (c) Sketch a graph of the rabbit population.

2. Population of California: (Textbook 15) The population of California was 29.76 million in 1990 and 33.87 million in 2000. Assume that the population grows exponentially.

   (a) Find a function that models the population t years after 1990.
   (b) Find the time required for the population to double.
   (c) Use the function from part (a) to predict the population in 2010.

Section 10.1: Systems of linear equations in two variables Main ideas: Solving such systems; counting the number of solutions. Understanding the solutions as intersections of lines.

1. Textbook 33, 35, 37

Section 10.2: Systems of linear equations in several variables Main ideas: Transforming to an equivalent upper triangular system. Unique solutions, No solutions $\equiv$ *inconsistent system*, Infinitely many solutions.

1. Textbook 23, 24, 27, 31

Section 10.9: Solving systems of inequalities Main ideas: Partitioning the plane into regions corresponding to '$>$', '$<$', and '$=$'.

1. Solve

$$x^2 + y^2 < 25$$
$$x + 2y \geq 5$$