MATH 310: Numerical Analysis
Details for Homework #5
due Wednesday, Oct 14 at 9 am

Written problems:

1. Use Gaussian elimination to solve the linear system of equations.

$$\begin{pmatrix} 2 & 1 & 3 \\ 6 & 4 & 8 \\ -2 & 1 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}.$$

2. Section 3.1: 1
3. Section 3.1: 3

   Give *only* the number n of iterations you need to perform to attain the desired accuracy.
4. Section 3.2: 1 e
5. Section 3.2: 7
6. Section 3.2: 9
7. Section 3.5: 1
8. Section 3.5: 2
9. Section 3.6: 1

Programming assignment.

You will need to write three programs and include printouts of these programs with your homework assignments. As always, deposit copies of your code in the shared directory.

- A function that returns the trapezoid estimate $T_n(f)$ to the definite integral $\int_a^b f(x)dx$.
- A function that returns an approximation to a root of a function $f(x)$ using the bisection rule.
- A function that returns an approximation to a root of a function $f(x)$ using the Newton's method. For this Newton's method, make sure your function takes (among others) an argument M for the maximal number of iterations to perform, and implements a stopping criterion like that on the top of page 104 in your textbook.

Now do the following problems as part of your programming assignment. Please include written explanations of your work, where this is necessary.

1. Section 2.5, 11 a, b, c

2. Section 3.3: 3

   Use BOTH bisection and Newton's method on these functions. For your solution, I *only* want to see the values of the root you find.

3. Solve the equation $x = \tan x$ using Newton's method. Find the roots nearest 4.5 and 7.7.

4. Find the positive minimum point of the function $f(x) = \dfrac{\tan x}{x^2}$ by applying Newton's method to the derivative.

5. The equation $f(x) = 2x^4 + 24x^3 + 61x^2 - 16x + 1$ has two roots near 0.1. Find them using either Newton's method or bisection.

6. The polynomial $p(x) = x^3 + 94x^2 - 389x + 294$ has roots at $x = 1$, 3, and -98. The point $x_0 = 2$ should be a good starting point for computing the small zeroes. Either, by hand, or with your computer, do a few iterations of Newton's method with $x_0 = 2$ and see what happens. Explain.

7. The function $f(x) = \arctan(x)$ has what is known as a 2-cycle when applying Newton's method for a very specific initial value of x_0. That is, if you make the right choice, using Newton's method you find the pattern of roots $x_0, x_1, x_0, x_1, x_0, x_1, x_0, x_1, \ldots$. Either using algebra or your program, see if you can find this value of x_0. Explain your method.

8. Use your favorite numerical computing device to get an estimate for $r = \sqrt{2}$. Using your programs, or combining them to write another one, write a short program that runs the bisection method and Newton's method to find a root of $f(x) = x^2 - 2$. Use $[1, 2]$ for the starting interval $[a_0, b_0]$ in the bisection method and use $x_0 = 1$ for your initial guess in Newton's method. For each iteration you run, calculate the error $e_n = r - x_n$ for both methods. Output these errors e_n to the screen. Compare the errors *and comment.* (You will be graded on your comments.)