MATH 631: Theory of Modern Algebra I
http://www.dms.uaf.edu/~eallman/classes/631-fall_08/631.html
MWF 9:15 - 10:15
Gruening 308

Instructor: Elizabeth S. Allman, Chapman 303B, e.allman@uaf.edu and 474-2479.

Office Hours: M TBA, W 1:00 – 2:00, F 10:30 – 11:30 and by appointment. These office hours may change at a later date, depending on student demand and scheduling concerns. Please note that the best way to reach me is by e-mail.

Textbook: *Abstract Algebra* by D. Dummit and R. Foote. 3rd edition, Wiley, 2004.

Grading: There will be one midterm exam and a cumulative final exam in MATH 631. In addition, there will be regular homework assignments and formal solution write-ups as described below. Grades will be assigned using the following weights:

Homework	20 %
Solutions	20 %
Midterm	30 %
Final Exam	30 %

Content: Broadly speaking, the field of Abstract Algebra studies sets together with operations $+$, $\cdot$ on the elements of these sets. The main objects of study are *groups*, *rings*, and *fields*. A group $< G, \cdot >$ with a single binary operation that satisfies a few axioms is the most general algebraic object familiar to you from undergraduate study. Groups arise naturally in many fields of mathematics. Ones familiar to you include those arising from a geometric setting (dihedral groups D_{2n}), permutation groups (S_n and A_n), ones of importance in number theory ($< \mathbb{Z}, + >$, $\mathbb{Z}/n\mathbb{Z}$), and matrix groups ($GL_n(\mathbb{R})$, $SL_n(\mathbb{R})$, etc.)

A ring $< R, +, \cdot >$ is a non-empty set endowed with two binary operations that satisfies the ring axioms. The model for the ring axioms is the set of integers $< \mathbb{Z}, +, \cdot >$, together with the operations of addition and multiplication. While this may be a bit of an overstatement, you may think of rings as invented as a way to study very general properties of sets with two operations that interact via a distributive law.

A field $< F, +, \cdot >$ has the most structure of any algebraic object we study in the course. The fields $\mathbb{R}$ and $\mathbb{C}$ are well-known from courses in real and complex analysis. Perhaps less familiar, but of utmost importance to algebraists, is the field of rational numbers $\mathbb{Q}$.

In this survey course, we undertake an in-depth study of groups, rings, and fields. The emphasis will be on rings and fields, and a solid understanding of group field from undergraduate study will be assumed.

Course Logistics: MATH 631 is a four-hour graduate course with lectures three days a week and a dedicated office hour on Monday. It forms the basis for the graduate qualifying exam for the Master's degree in Mathematics.

The pace of the course will be fast, and students should be prepared to fill in spotty knowledge or incomplete background material on their own. Unfortunately, students often enroll in a graduate algebra course with very different backgrounds in algebra. This course presumes a thorough knowledge of Chapters 1-3 in the textbook (basics on groups), a thorough knowledge of Chapter 7 (definition and first examples of rings), and some familiarity with Chapter 8 (Euclidean domains, PIDs, UFDs) and Chapter 9 (basics of polynomial rings).

In class, lectures will highlight the main ideas and theorems for the topic under study, and will provide insight for good ways to think about objects or why these algebraic structures are so interesting. As it will not be possible to cover all topics in complete detail in lecture, students are responsible for reading each chapter in the text carefully, and fully digesting ideas and examples in the text, regardless of whether such material was covered in class. You should expect homework problems on material in the text that has been omitted from lecture.

One of the most important skills to learn in a beginning graduate mathematics class is the ability to write cogent proofs and to give convincing arguments for theorems. Homework will be assigned regularly and collected once a week, typically on Wednesday. Please feel free to work with other students to discuss ideas for proofs, but your write-up should be completely your own. One goal for this course is for you to develop an independent and mature sense of when you have given a correct and well-written proof of a theorem.

When individual homework is collected, most of the individual problems will be graded. A homework grade on a scale of $0 - 10$ will be recorded for your assignment, reflecting my assessment of both the completeness and the correctness of the assignment. Late homeworks will not be accepted.

In addition, for each assignment each student will be assigned two problems (or so) chosen by your instructor, and will be responsible for drawing up complete solution in TEX. Each week a different student will be in charge of overseeing that these problems are compiled together for submission. A template format will be made available for you, and office hours or class time if necessary will be used to discuss the basics of TEX. You will also receive an individual grade on your contribution to these solutions.

Each Monday, at a time to be determined by the class, there will be an office hour (recitation) exclusively for students in Abstract Algebra. If there is enough interest, a room will be reserved for this purpose, and we will meet to discuss solutions to homework problems. Homeworks will generally be collected on Wednesdays, giving students time to digest ideas and make improvements to proofs before handing in assignments.

Both the midterm and the final exam in MATH 631 will be two hours in length. The format of these exams is modeled after a comprehensive exam in Algebra: some problems are mandatory, and others may be chosen from a list of options. The exact time and date of the midterm will be determined by the class (two hours are needed), and the target week is Nov 3–7. The final exam is scheduled for 8:00 - 10:00 on Monday, December 15.

Other Policies:

Course accommodations: If you need course adaptations or accommodations because of a disability, please inform your instructor during the first week of the semester, after consulting with the Office of Disability Services, 203 Whitaker (474-7403).

University and Department Policies: Your work in this course is governed by the UAF Honor Code. The Department of Mathematics and Statistics has specific policies on incompletes, late withdrawals, and early final exams, some of which are listed below. A complete listing can be found at

`http://www.dms.uaf.edu/dms/Policies.html`.

Prerequisites: The prerequisite for MATH 631 is an undergraduate course in Abstract Algebra with a grade of C or better. Students not meeting this prerequisite are not eligible to take this course and will be dropped. The undergraduate algebra course MATH 405W (formerly MATH 308W) is offered each spring.

Late Withdrawal: This semester the last day for withdrawing with a 'W' appearing on your transcript is October 31. If, in my opinion, a student is not participating adequately in the class, I may elect to drop this student.

Graded Coursework: Please keep all graded work for MATH 631 until final grades have been assigned.

Academic Honesty: Academic dishonesty, including cheating and plagiarism, will not be tolerated. It is a violation of the Student Code of Conduct and will be punished according to UAF procedures.

Grade Bands: A, A- (90 - 100%), B+, B, B- (80 - 89%), C+, C, C- (70 - 79%), D+, D, D- (60 - 69%), F (0 - 59%). On rare occasion, I may lower the thresholds. Also, in an effort to reward the student who makes significant improvement over the course of the term, a stellar grade on the final may overcome a deficiency on the midterm and improve a student's final grade.

Courtesies: As a courtesy to your instructor and fellow students, please arrive to class on time, turn your cell phones and iPods off during class, pay attention in class, and take notes.

Tentative Schedule:

	Monday		Wednesday		Friday
				9/4:	§1.1-1.6 Groups
9/8:	§0.2-0.3 $\mathbb{Z}, \mathbb{Z}/n\mathbb{Z}$, Euler φ	9/10:	§1.7 Group actions	9/12:	§2.1-2.2 Subgroups, $N(G)$, $C(G)$, $C(x)$
9/15:	§2.3 Cyclic groups	9/17:	probably no class	9/19:	§2.3, 2.4 Cyclic groups, generators and relations
9/22:	§2.5, 3.1, Lattices, Quotient groups	9/24:	§3.2-3.3 Lagrange's Theorem, Isomorphism theorems	9/26:	§3.3, 3.4 Isomorphism theorems, simple and solveable groups
9/29:	§4.1-4.2 Group actions	10/1:	§4.3 Class equation	10/3:	probably no class
10/6:	§4.5 Sylow theorems	10/8:	§4.5 Sylow theorems	10/10:	§5.1-5.2 Fund thm of fg Abelian groups
10/13:	§7.1-7.3 Rings	10/15:	§7.4-7.5 Ideals	10/17:	§7.6 Chinese remainder theorem
10/20:	§8.1-8.2 PIDs, EDs	10/22:	§8.2-8.3 PIDs, UFDs	10/24:	§9.1 $k[x]$, $F[x]$
10/27:	§9.3 Gauss lemma	10/29:	§9.4-9.5 Irreducibility criterion Finite subgroups of fields	10/31:	§10.1-10.2 Modules
11/3:	§11.1-11.2 Vector spaces	11/5:	exam (tentative)	11/7:	§13.1 Field extensions
11/10:	§13.2 Algebraic extensions	11/12:	§13.3 Constructibility	11/14:	§13.4 Splitting fields
11/17:	§13.5 (In)Separable extensions	11/19:	§13.6 Cyclotomic extensions	11/21:	§14.1 Intro to Galois theory
11/24:	§14.2 Galois correspondence	11/25:	§14.3 Finite fields	11/27:	§14.2 Extensions
12/1:	§14.5 Cyclotomic extensions	12/3:	§14.6 Galois groups	12/5:	§14.7 Solvable extensions Quintic insolvability
12/8:	slack	12/10:	slack	12/12:	slack
12/15:	Final Exam 8-10am				