MATH 371
Quiz 1

Name : SOLUTIONS

September 27, 2019

Instructions: Give numerical answers unless instructed that a formula suffices. A 'dumb' calculator may be used for routine arithmetic, but nothing else. Good luck.

1. (5 pts.) Answer the following:

   (a) (2 pts.) In order to fulfill general education requirements, a student must take one Math class, one Science class, two English classes, and one History class. The college offers 4 Math general education classes, 5 Science general education classes, 5 English general education classes and 4 History general education classes. How many different ways could a student fulfill this college's general education requirement?

   $$4 \cdot 5 \cdot \binom{5}{2} \cdot 4 = 4 \cdot 5 \cdot 10 \cdot 4 = \boxed{800}$$

   (b) (3 pts.) There are ten quantitative skills which an employee wishes a job applicant to possess. Five of the ten skills are selected at random for a skills test, the applicant is asked to perform them, and passes if they get at least 4 out of five correct. Assuming that the student knows 8 of the ten skills, give a formula that computes the probability that the student passes the skills test (gets *at least 4* skills correct). You **only** need to give the formula here and not its decimal value.

   $$\frac{\binom{8}{4}\binom{2}{1} + \binom{8}{5}\binom{2}{0}}{\binom{10}{5}}$$

2. (10 pts.) Consider the random variable

   Y: *Winnings in dollars in a game of chance*

   and its probability distribution given in the table below:

Y	-2	-1	0	3	5
$p(y)$	.1	.2	.6		.05

   (a) (1 pt.) Assuming the only possible winnings (where a negative value means a loss) in one round of the game are -2, -1, 0, 3, or 5 dollars, find the probability $P(Y = 3)$.

   $$1 - (.1 + .2 + .6 + .05) = 1 - .95 = \boxed{.05}$$

   (b) (4 pts.) Is this game favorable? I.e. Do you expect to win money if you were to play this game repeatedly? To justify your answer, perform the relevant computation and give a brief justification.

   $$\mathbb{E}(Y) = -2(.1) - 1(.2) + 0(.6) + 3(.05) + 5(.05)$$

   $$= 0$$

   Not favorable.

   (c) (5 pts.) Give a general formula for the variance $V = V(Y)$ and the standard deviation $\sigma = \sigma(Y)$ of a random variable. Then compute $V(Y)$ for the Y above.

   $$Var(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2 \qquad \sigma = \sqrt{V(Y)}$$

   $$= (-2)^2(.1) + (-1)^2(.2) + (0)^2(.6) + (3^2)(.05) + (5^2)(.05) - 0^2$$

   $$= \boxed{2.3}$$

3. (10 pts.) Consider the data below collected on placement exams and pass rates of a population of Calculus I students. Five events are labelled including **A**: *Student passes with grade of C or better* and **B**: *Student earns grade less than C.*, etc.

Proportions of Calculus I students

Placement Test Range	(**A**) Passes with C or better	(**B**) Earns grade less than C
(**C**) 78-100	.28	.10
(**D**) 60-78	.20	.25
(**E**) 0-60	.01	.16

(a) (6 pts.) Give the following probabilities:

i. (1 pt.) $P(B \text{ and } D) = P($ Student earns grade less than C and Student scores in range 60-78 on placement test $)$

.25

ii. (2 pts.) $P(A) = P($ Student passes with grade of C or better $)$

$.28 + .20 + .01 = .49$

iii. (3 pts.) $P(A \mid D) = P($ Student passes with grade C or better $\mid$ Student scores in range 60-78 on placement test $)$

$$P(A|D) = \frac{P(A \text{ and } D)}{P(D)} = \frac{.2}{.2+.25} \approx .44$$

(b) (4 pts.) Are the events A and C independent? Prove your answer.

$P(A) = .49 \qquad P(C) = .38$

$P(A)P(C) = .49(.38) = .1862$.

However, $P(A \text{ and } C) = .28$. Since $.1862 \neq .28$, the events are dependent.

4. (6 pts.) A statistician works setting rates for health insurance premiums. This statistician has to determine the annual cost C of a premium. On average, many insured people do not use their health insurance at all, though 35% of the insured people do make claims for an average annual amount of \$1200. Suppose that it costs the insurance company \$15 per person for annual enrollment. How much should the statistician charge for the annual cost C of the premium if the insurance company wants to make \$50 per insured person?

Profit per person $= P = 50 = (C-15) - .35(1200)$

$\Rightarrow C = 50 + 15 + .35(1200) = \boxed{\$485}$

(pagination change)

5. (6 pts.) A population of students contains 55% from UAA and 45% from UAF. It is known that 10% of UAA students and 20% of UAF students favor joining the two universities into a single university. A student selected at random from this population is found to favor joining the two universities. Find the conditional probability that this student is from UAF. Give your answer to 2 decimal places.

UAA: student from UAA UAF: student from UAF

F: Favor single U D: do not favor single U

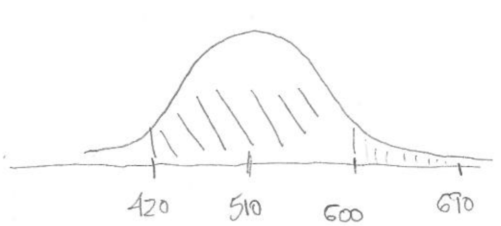
$P(UAA) = .55$ $P(UAF) = .45$ $P(F|UAA) = .10$ $P(F|UAF) = .20$

Find $P(UAF|F)$. $P(UAF|F) = \dfrac{P(F|UAF)P(UAF)}{P(F|UAF)P(UAF) + P(F|UAA)P(UAA)} = \dfrac{.2(.45)}{.2(.45)+.1(.55)}$

$\approx \boxed{.62}$

6. (5 pts.) The College Board finds that student scores on the Mathematics portion of the SAT are approximately normally distributed with a mean $\mu = 510$ and a standard deviation $\sigma = 90$. Use the empirical rule to estimate the percentage of students who score in the range [420, 690] on the Mathematics portion of the SAT. rounding ..

4 Y: student score on SAT, $P(420 \le Y \le 690) = P(\mu - \sigma \le Y \le \mu + 2\sigma) = P(510-\sigma \le Y \le 510+2\sigma)$



By the empirical rule, $510 \pm 90 = [420, 600]$ contains roughly .68 of the population. Area shaded \\\\

Also by the empirical rule, the probability $600 \le Y \le 690$ is approximately $\frac{1}{2}(.95 - .68) = .135$. Summing $.68 + .135$

$= \boxed{.815}$

7. (8 pts. – 4 pts. each) In a round of the Stanley Cup, two teams compete in a best-of-seven series. That is, they play until one team wins four hockey games. Let S be the sample space containing all ways a round in the series can end.

Suppose Team A and B are playing each other. Using notation like $AAAA$ to denote the event that Team A wins the series in exactly four games, or $AABBBB$ to denote the event that Team B wins the series in six games after losing the first two, answer the following.

Let Y be the event

Y: Team A wins the series in exactly 5 *games.*

(a) Write down the atomic events in the sample space S that correspond to event Y. We need 4 A's, 1 B and A at end.

$\boxed{AAABA, \quad AABAA, \quad ABAAA, \quad BAAAA}$

(b) Suppose that Team A is better than Team B and in any game the probability that Team A wins is .6. Compute the probability of event Y. Round your answer to two decimal places

$P(Y) = 4(.6)^4(.4)^1 \approx \boxed{.21}$

↑ sum since mutually exclusive

(c) **Extra Credit:** Using notation as above, define the probability distribution on the random variable

X: the game number on which the round in the series ends

and prove it is a probability distribution (i.e. it sums to one on its support.)

See website