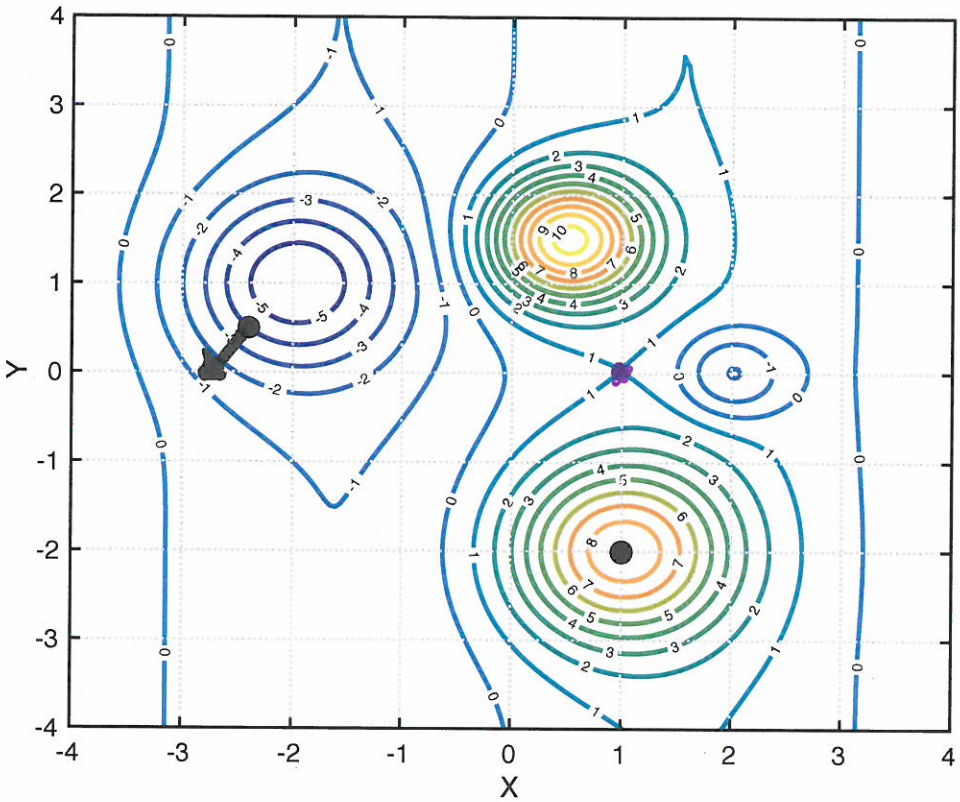
MATH 253
Quiz 6

Name: SOLUTIONS

March 4, 2020

Instructions: Six points total.

1. (4 pts.) Consider the contour plot for the smooth function $z = f(x, y)$ displayed below.



$\nabla f(-2.4, .5)$ must 1) be orthogonal to level curve; 2) point in direction of (maximal) increase.

(a) At the black point $P(-2.4, .5)$ shown, draw a vector pointing in the direction of $\nabla f(-2.4, .5)$.

(b) Consider the black point $Q(1, -2)$ shown in the contour plot.

Estimate $f_x(1, -2)$ $= 0$ It's a local max.

Give, as best you can, the equation of the tangent plane at $Q(1, -2)$. Briefly justify your answer.

$\boxed{z = 9}$ The value of $f(1,-2)$ was not given, but $f(1,-2) > 8$.

(c) The function $f(x, y)$ has (at least) one saddle point at (a, b). Give the coordinates (a, b) for this saddle point and then **justify** why this is a saddle point for $f(x, y)$.

At $(1, 0)$, there is a saddle point. (Drawn in purple above)

Why? $\frac{\partial^2 f}{\partial x^2}(1,0) < 0$, $\frac{\partial^2 f}{\partial y^2}(1,0) > 0$ or, more informally,

$f(x,y)$ increases moving vertically $(\pm y)$ from $(1,0)$

$f(x,y)$ decreases moving horizontally $(\pm x)$ from $(1,0)$

2. (2 pts.) Give the equation of the tangent plane at the point $P(3,1,2)$ to the surface defined implicitly by

$$\frac{x^2}{9} - y^2 + \frac{z^2}{4} = 1$$

Hyperboloid of one sheet

(y-axis)

$\vec{n} = \nabla f(3,1,2)$

$\nabla f = \langle \frac{2x}{9}, -2y, \frac{z}{2} \rangle$

$\nabla f(3,1,2) = \langle \frac{2}{3}, -2, 1 \rangle$

Take $\vec{n} = 3\nabla f(3,1,2)$

$= \langle 2, -6, 3 \rangle$

Plane Equation:

$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

$2x - 6y + 3z = \langle 2, -6, 3 \rangle \cdot \langle 3, 1, 2 \rangle$

$2x - 6y + 3z = 6 - 6 + 6$

$$\boxed{2x - 6y + 3z = 6}$$