MATH 371
Practice problem 1

Name : Solutions

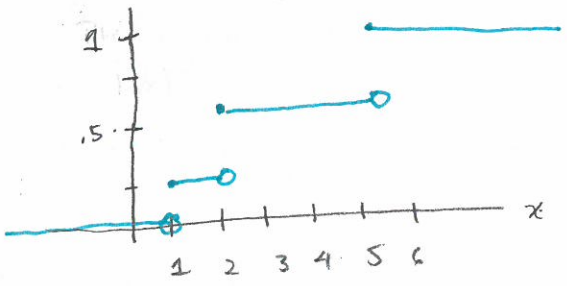
November 1, 2019

This one would have made it onto the test if there were more time.

1. A 3-side die has the numbers 1, 2 and 5 on it. These occur with probabilities .25, .35 and .4 respectively.

   (a) Give a formula for the c.d.f. (the distribution function) of X: *number rolled on this 3-sided die.*

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ .25 & \text{if } 1 \le x < 2 \\ .6 & \text{if } 2 \le x < 5 \\ 1 & \text{if } x \ge 5 \end{cases}$$



   (b) Use the c.d.f. to find the probability that you roll a number between 2 and 5 inclusive. Check your answer.

$P(2 \le X \le 5) = P(5) - P(1.9) = 1 - .25 = .75$

$P(1.9)$: any value x with $1 \le x < 2$

Check: $P(2 \le X \le 5) = p(2) + p(5) = .35 + .4 = .75$ agrees!

   (c) Suppose you have to pay \$2 to enter a round of a game involving this die. After paying \$2 to enter, you earn 50 cents for each dot that shows up on the die. Letting C be the earnings in one round of this game, find the expected value and variance of C.

$C = .5X - 2$

$E(C) = E(.5X - 2) = .5\,E(X) - 2$

$= \frac{1}{2}(2.95) - 2$

$= \boxed{-.525}$

lose

$E(X) = 1(.25) + 2(.35) + 5(.4)$

$= 2.95$

$Var(C) = Var(.5X - 2)$

$= \left(\frac{1}{2}\right)^2 Var(X)$

$= \frac{1}{4}(2.9475)$

$\approx \boxed{.74}$

$Var(X) = E(X^2) - (2.95)^2$

$= 1^2(.25) + 2^2(.35) + 5^2(.4) - (2.95)^2$

$= 11.65 - (2.95)^2 = 2.9475$