MATH 202 Name :_____

Midterm 2 April 8, 2008

Instructions: Show all work for full credit. Poor or sloppy mathematical notation will be penalized.

1. (10 pts.) Explain why $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ does not exist.

2. (15 pts.)

   (a) (8 pts.) Find the equation of the tangent plane to the elliptic paraboloid $f(x,y) = x^2 + 2y^2$ at the point $(1,1,3)$.

   (b) (7 pts.) Give the value of the best *linear approximation* for $f(.9, 1.01)$.

$f(.9, 1.01) \approx$ _____

3. (12 pts.) Suppose the height in tens of meters of a kite is given by the function

$$h(x, y) = \frac{1}{5 - x} y^2 \text{ tens of } m,$$

and the kite flyer is located at the position $(x, y) = (4, 1)$. (Assume the kite flies directly overhead.)

(a) (4 pts.) In what direction from $(4, 1)$ should the kite flyer move to increase the height of the kite the most?

(b) (4 pts.) If the kite flyer moves in the direction indicated by the vector $\mathbf{v} = (-1, 1)$, what is the rate of change of the kite's height?

(c) (4 pts.) Using your answer to part (b), do you expect the kite to rise or fall as the kite flyer moves in the direction of $\mathbf{v}$?

4. (10 pts.) It is well known that the volume of the solid ball of radius R is given by

$$V = \frac{4}{3}\pi R^3.$$

Set up an appropriate triple integral in *spherical coordinates* that would compute this volume.

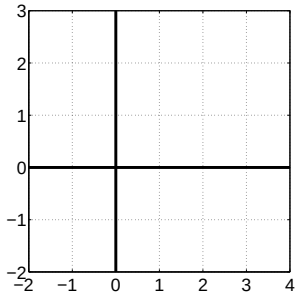
5. (16 pts.) Consider the function $f(x, y) = xy^2 - y^2 - \frac{1}{2}x^2$.

   (a) (8 pts.) Find all critical points of $f(x, y)$.

   (b) (8 pts.) Use the second derivative test to determine if the critical points are local maxima, local minima, saddle points or if there is not enough information to tell.

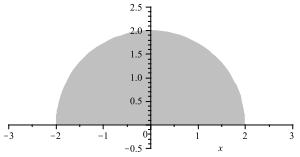
6. (12 pts.) Find the maximum value of $f(x, y) = xy^2$ subject to the constraint $x^2 + y^2 = 1$.

7. (10 pts.) Set up, but do not compute, an iterated integral that evaluates $\iint_D xy\, dA$, where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = x + 1$.



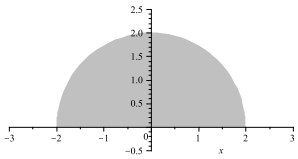
8. (15 pts.) Electric charge is distributed over the semi-circular plate pictured below so that the charge density is $\rho(x, y) = 1 - \sqrt{x^2 + y^2}$ coulombs/cm^2.

   (a) (10 pts.) Find the total electric charge of the semi-circular plate. A complete answer includes units.



   (b) (5 pts.)

      i. (5 pts.) Indicate on the plate the direction of the gradient vector $\nabla\rho(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.



      ii. (Extra credit – 3 pts.) Give the formula for the level curve $\rho(x, y) = k$ that passes through the point $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and sketch this level curve on the plate.