Sectioon 14.2 #44

$$V = \int_0^5 \int_0^{\pi} \sin^2 x \, dx \, dy$$

14.2 #46

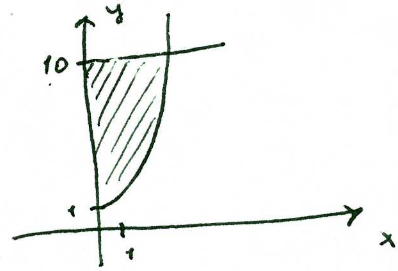
$$V = \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \left([18 - x^2 - y^2] - [x^2 + y^2] \right) dy \, dx =$$

$$= 4 \int_0^3 \int_0^{\sqrt{9-x^2}} \left(18 - 2x^2 - 2y^2 \right) dy \, dx$$

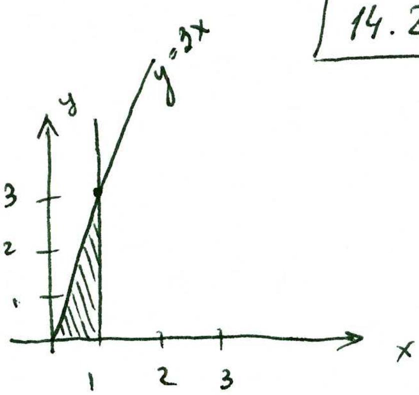
14.2 #54

$$\int_0^{\ln} \int_{e^x}^{10} \frac{1}{\ln y} \, dy \, dx = \int_1^{10} \int_0^{\ln y} \frac{1}{\ln y} \, dx \, dy = \int_1^{10} \left[\frac{x}{\ln y} \right]_0^{\ln y} dy =$$

$$= \int_1^{10} dy = [y]_1^{10} = 9$$



14.2 #56



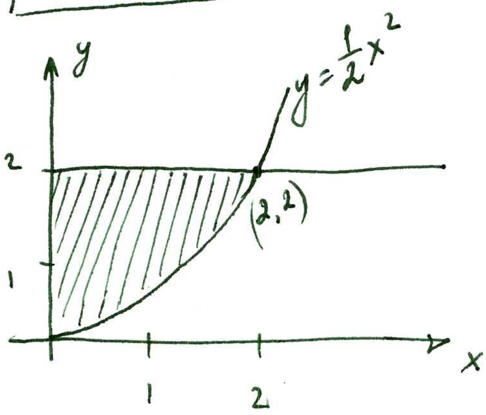
$$\int_0^3 \int_{y/3}^{1} \frac{1}{1+x^4} \, dx \, dy = \int_0^1 \int_0^{3x} \frac{1}{1+x^4} \, dy \, dx =$$

$$= \int_0^1 \left[\frac{y}{1+x^4} \right]_0^{3x} dx = \int_0^1 \frac{3x}{1+x^4} \, dx = \frac{3}{2} \arctan x^2 \Big|_0^1 =$$

$$= \frac{3}{2} \cdot \frac{\pi}{4} = \frac{3\pi}{8}$$

$\textcircled{2}$

$\boxed{14.2 \quad \# 58}$



$$\int_0^2 \int_{(1/2)x^2}^2 \sqrt{y} \cos y \, dy \, dx = \int_0^2 \int_0^{\sqrt{2y}} \sqrt{y} \cos y \, dx \, dy =$$

$$= \int_0^2 \sqrt{2y} \sqrt{y} \cos y \, dy = \sqrt{2} \int_0^2 y \cos y \, dy =$$

$$= \sqrt{2} \left[\cos y + y \sin y \right]_0^2 = \sqrt{2} \left[\cos 2 + 2 \sin 2 - 1 \right]$$

Section 14.3 # 36

$$V = \iint_R \ln(x^2 + y^2) \, dA = \int_0^{2\pi} \int_1^2 (\ln r^2) r \, dr \, d\theta = 2 \int_0^{2\pi} \int_1^2 r \ln r \, dr \, d\theta =$$

$$= 2 \int_0^{2\pi} \left[\frac{r^2}{4} (-1 + 2 \ln r) \right]_1^2 d\theta = 2 \int_0^{2\pi} \left(\ln 4 - \frac{3}{4} \right) d\theta = 4\pi \left(\ln 4 - \frac{3}{4} \right)$$

$\boxed{14.3 \quad \# 38}$

$$V = \int_0^{2\pi} \int_1^4 \sqrt{16 - r^2} \, r \, dr \, d\theta = \int_0^{2\pi} \left[-\frac{1}{3} \left(\sqrt{16 - r^2} \right)^3 \right]_1^4 d\theta = \int_0^{2\pi} 5\sqrt{15} \, d\theta =$$

$$= 10\sqrt{15} \, \pi$$

Section 14.4 # 50

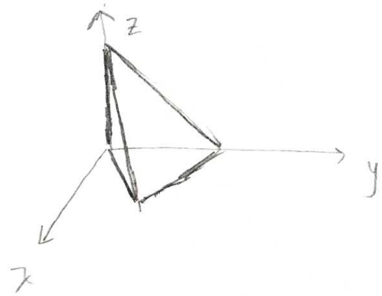
a) $\rho(x,y) = ky$: $\bar{y}$ will increase

b) $\rho(x,y) = k|2-x|$: $\bar{y}$ will decrease

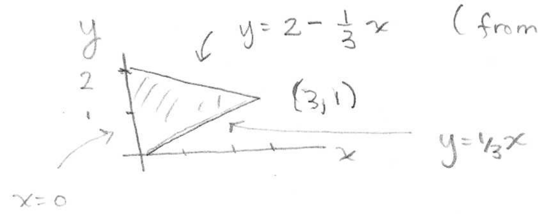
c) $\rho(x,y) = kxy$ both $\bar{x}$ & $\bar{y}$ will increase

d) $\rho(x,y) = k(4-x)(4-y)$ both $\bar{x}$ and $\bar{y}$ will decrease

Find the volume of the tetrahedron bounded by the plane $x+3y+z=6$, $x=3y$, $x=0$, and $z=0$.



Region R of integration:



(from intersection of plane with xy-plane)

$$\iint_R 6-3y-x \, dA = \int_0^3 \int_{\frac{x}{3}}^{2-\frac{x}{3}} 6-3y-x \, dy\, dx$$

$$= \int_0^3 \left[(6-x)y - \frac{3}{2}y^2\right|_{\frac{1}{3}x}^{2-\frac{1}{3}x} dx = \int_0^3 \left[(6-x)(2-\tfrac{1}{3}x) - \tfrac{3}{2}(2-\tfrac{x}{3})^2\right] - \left[(6-x)(\tfrac{x}{3}) - \tfrac{3}{2}(\tfrac{x}{3})^2\right] dx$$

$$= \int_0^3 \underbrace{12 + \frac{1}{3}x^2 - 4x - \frac{3}{2}\left(4 - \frac{4}{3}x + \frac{x^2}{9}\right)} - \left(2x - \frac{x^2}{3} - \frac{x^2}{6}\right) dx$$

$$= \int_0^3 -\frac{1}{6}x^2 - 2x + 6 - 2x + \frac{5}{6}x^2 \, dx$$

$$= \int_0^3 \frac{2}{3}x^2 - 4x + 6 \, dx = \frac{2}{9}x^3 - 2x^2 + 6x \Big|_0^3 = \left(\frac{2}{9}\cdot 27 - 2\cdot 9 + 18\right) - 0$$

$$= 6 - 18 + 18 = \boxed{6}$$