MATH 202 Name : SOLUTIONS

Quiz 6 March 9, 2012

Instructions: Point values as indicated. There is no partial credit on problems 1 and 2. You get one point for taking this quiz.

1. (1 pt.) Find $\frac{\partial w}{\partial t}$ for $w = \cos(3x + 2y)$ where $x = s + t$, $y = s - t$ at the point $s = \frac{\pi}{2}$ and $t = 0$.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

$$= \left[-\sin(3x+2y)(3)\right](1) + \left[-\sin(3x+2y)(2)\right](-1)$$

$$= -3\sin(3x+2y) + 2\sin(3x+2y)$$

$$= -\sin(3x+2y)$$

at $s = \pi/2$, $t = 0$

$x = \pi/2$, $y = \pi/2$

so $(3x+2y) = \frac{3\pi}{2} + \pi = \frac{5\pi}{2}$

Therefore, $\frac{\partial w}{\partial t}(\pi/2, 0) = -\sin\left(\frac{5\pi}{2}\right)$

$$= \boxed{-1}$$

2. (1 pt.) Find the directional derivative $D_{\mathbf{u}}f(\pi, 0)$ for the function $f(x, y) = e^{xy} + \sin(x)$ at the point $(\pi, 0)$ in the direction of $\mathbf{v} = \langle -3, 4\rangle$.

unit vector in direction of $\vec{v}$ is $\vec{u} = \langle -\frac{3}{5}, \frac{4}{5}\rangle$

$\nabla f(x,y) = \langle ye^{xy} + \cos(x), xe^{xy}\rangle$

$\nabla f(\pi, 0) = \langle 0 + \cos(\pi), \pi e^0\rangle$

$= \langle -1, \pi\rangle$

$D_{\vec{u}} f(\pi, 0) = \langle -1, \pi\rangle \cdot \langle -\frac{3}{5}, \frac{4}{5}\rangle$

$$= \boxed{\frac{1}{5}(3 + 4\pi)}$$

3. (2 pts.) Find the equation of the tangent plane to the surface $z = \arctan\left(\frac{y}{x}\right)$ at the point $(1, 0, 0)$.

Consider $G(x,y,z) = \arctan\left(\frac{y}{x}\right) - z$. The surface is a level surface $G(x,y,z) = 0$. Thus, the normal vector is $\vec{n} = \nabla G(1,0,0)$.

$$\nabla G(x,y,z) = \left\langle \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{-y}{x^2}, \frac{1}{\left(1+\left(\frac{y}{x}\right)^2\right)}\cdot\frac{1}{x}, -1\right\rangle = \left\langle \frac{y}{x^2+y^2}, \frac{1}{x}\cdot\frac{1}{1+\frac{y^2}{x^2}}, -1\right\rangle$$

so $\nabla G(1,0,0) = \langle 0, 1, -1\rangle$

Equation: $\vec{X}\cdot\vec{n} = \vec{p}\cdot\vec{n}$

$y - z = \langle 1, 0, 0\rangle \cdot \langle 0, 1, -1\rangle$

$= 0$

$$\boxed{y - z = 0}$$