HW #21 (3.4) 70 (3.7) 8, 10, 12, 14, 16, 18, 20, 34, 36

(3.4) 70) $P(x) = x^8 - x^5 + x^4 - x^3 + x^2 - x + 1$; $P(-x) = x^8 + x^5 + x^4 + x^3 + x^2 + x + 1$

since $P(x)$ has 2 variations in sign $\Rightarrow$ 2 or 0 positive reals
since $P(-x)$ has no variation in sign $\Rightarrow$ no negative zeros

$\therefore$ P has 6, 4, 2, or 0 real zeros

(3.7) 8) $r(x) = \frac{4x+1}{x-2}$

a)

x	r(x)
1.5	-14
1.9	-86
1.99	-896
1.999	-8996

x	r(x)
2.5	22
2.1	94
2.01	904
2.001	9004

x	r(x)
10	5.125
50	4.188
100	4.092
1000	4.009

x	r(x)
-10	3.25
-50	3.827
-100	3.912
-1000	3.991

b) $r(x) \to -\infty$ as $x \to 2^-$
$r(x) \to \infty$ as $x \to 2^+$

c) r has horizontal asymptote $y = 4$.

10) $r(x) = \frac{3x^2+1}{(x-2)^2}$

a)

x	r(x)
1.5	31
1.9	1183
1.99	128,803
1.999	12,988,003

x	r(x)
2.5	79
2.1	1423
2.01	131,203
2.001	13,012,003

x	r(x)
10	4.703
50	3.256
100	3.124
1000	3.012

x	r(x)
-10	2.09
-50	2.774
-100	2.884
-1000	2.988

b) $r(x) \to \infty$ as $x \to 2$

c) r has horizontal asymptote $y = 3$

12) $s(x) = \frac{3x}{x-5}$

$s(0) = \frac{3(0)}{(0)-5} = 0$ x-intercept: when numerator $= 0$

$3x = 0$
$x = 0$

y-intercept $= 0$ x-intercept $= 0$

14) $r(x) = \frac{2}{x^2+3x-4}$

y-intercept: $r(0) = \frac{2}{(0)^2-3(0)-4} = \frac{2}{-4} = -\frac{1}{2}$ x-intercept: numerator never $= 0$

y-intercept $= -\frac{1}{2}$, no x-intercept

16, 18, 20, 36, 38

16) $r(x) = \dfrac{x^3+8}{x^2+4}$

y-intercept: $r(0) = \dfrac{(0)+8}{(0)+4} = 2$

x-intercept: $0 = \dfrac{x^3+8}{x^2+4}$

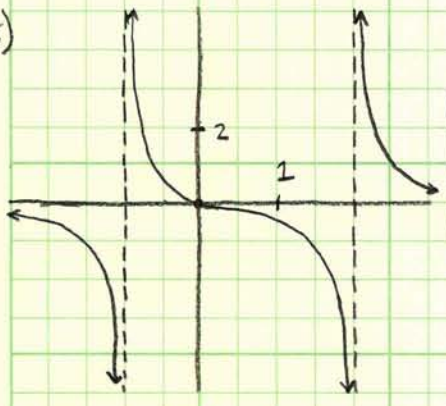
$0 = x^3+8$

$0 = (x+2)(x^2-2x+4)$

$x = -2$ $\quad x = 1 \pm i\sqrt{3}$ (not real solution)

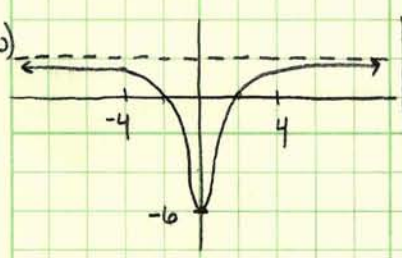
y-intercept = 2, x-intercept = -2

18)



x-intercept: $x = 0$
y-intercept: $y = 0$
horizontal asymptote: $y = 0$
vertical asymptote: $x = -1, 2$

20)

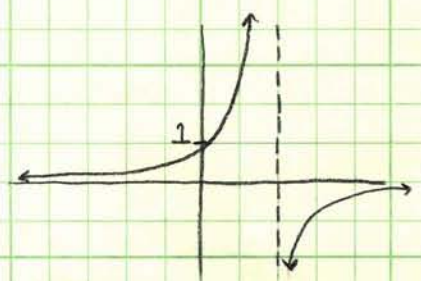


x-intercept: $x = \mp 2$
y-intercept: $y = -6$
horizontal asymptote: $y = 2$
vertical asymptote: none

36) $s(x) = \dfrac{-2}{x-2}$ $\quad f(x) = \dfrac{1}{x}$

$s(x) = \dfrac{-2}{x-2} = -2\left(\dfrac{1}{x-2}\right) = -2f(x-2)$

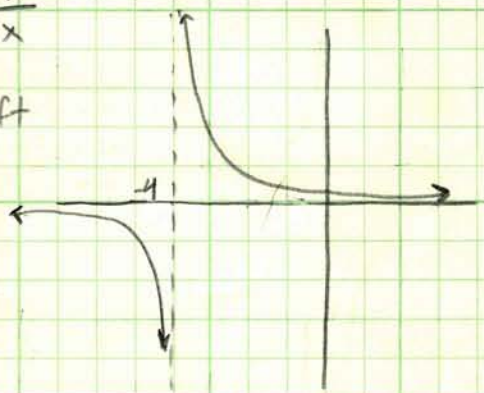
so, shift graph of $f(x)$ 2 unit right,
stretch by a factor of 2,
and reflect about the x-axis



vertical asymptote: $x = 2$
horizontal asymptote: $y = 0$

34) $r(x) = \dfrac{1}{x+4} = f(x+4)$, $\quad f(x) = \dfrac{1}{x}$

so, shift $f(x)$ 4 units left



vertical asymptote: $x = -4$
horizontal asymptote: $y = 0$