MATH253X-F01
Fall 2019

Midterm Exam 2

Name: ______________________________

Instructions. (100 points) You have 60 minutes. Closed book, closed notes, no calculator. *Show all your work* in order to receive full credit.

(6^{pts}) **1.** Consider the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2+3y^2}{3x^2+y^2}.$$

Either show it does not exist, or give strong evidence for suspecting it does.

(10pts) **2.** The following table gives some information about a function $f(x, y)$:

(x, y)	f	f_x	f_y
$(-1, 3)$	3	2	-1
$(0, 1)$	-5	-1	3
$(3, 4)$	1	4	-2

(a) (5 pts) Use the chain rule to compute $\frac{dg}{dt}(0)$ where:

$$g(t) = f(t^2 - t + 3, 2e^{-3t} + 2).$$

(b) (5 pts) Give an equation for the linear (tangent plane) approximation to f at the point $(-1, 3)$, and use it to estimate $f(-1.1, 3.2)$.

(12^{pts}) **3.** Evaluate the integral

$$\int_0^4 \int_{\sqrt{y}}^2 e^{\left(x^3+1\right)} dx\, dy$$

fully, by first drawing the region of integration, and then reversing the order of integration.

(12pts) **4.** Find and classify (using the Second Derivatives Test) all critical points of

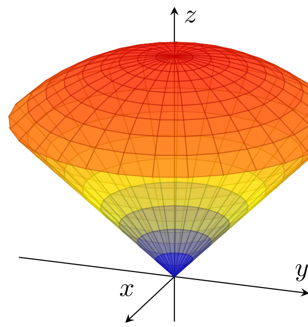
$$f(x, y) = x^2y - 2xy + y^2 - 3y + 1.$$

(8pts) **5.** Give an equation for the tangent plane to the surface

$$\frac{xy}{y+z} + e^{-z}\ln(x+2y) = 3$$

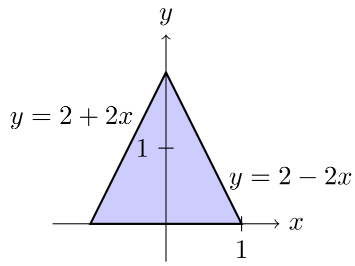
at the point $(3, -1, 0)$.

(10pts) **6.** Use polar coordinates to find the volume of the solid bounded by the cone $z = \sqrt{x^2 + y^2}$ and the top half of the sphere $x^2 + y^2 + z^2 = 6$.



(16pts) **7.** A flat triangular plate is bounded by the lines $y = 2 - 2x$, $y = 2 + 2x$ and the x-axis, where x, y are in m. The mass density is given by

$$\rho(x, y) = y^2 \ \text{kg/m}^2.$$



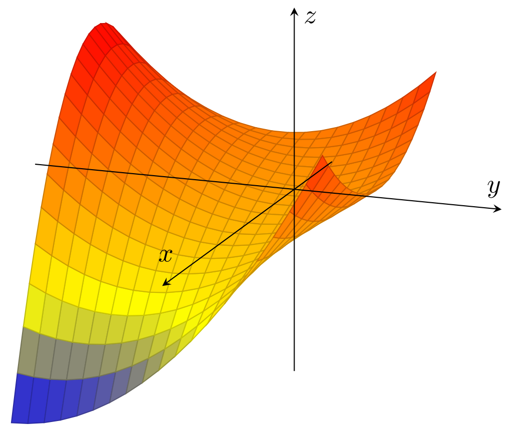
From the symmetry of the plate and the density, you can see that the center of mass of the plate must be on the y-axis, so $\bar{x} = 0$.

(a) (8 pts) Give an expression involving integrals for $\bar{y}$, including appropriate limits of integration.

(b) (8 pts) The total mass of the plate is $m = \dfrac{4}{3}$ kg. Use this to calculate $\bar{y}$.

(10^{pts}) **8.** Use Lagrange multipliers to find the maximum product of two positive numbers satisfying $x^2 + y = 6$.

(16^{pts}) **9.** Let $f(x,y) = x^2y - x + y^2$.



(a) (5 pts) Compute the directional derivative of f when moving in the direction of $-\mathbf{j}$ when you are at the point $(1,-1)$. Interpret your result in terms of change in values of f.

(b) (5 pts) Give the direction and magnitude of maximum decrease of f when at the point $(1,-1)$.

(c) (6 pts) Fully set up bounds and integrand for computing the surface area of f over the region $[-1,2]\times[-2,1]$. DO NOT EVALUATE.