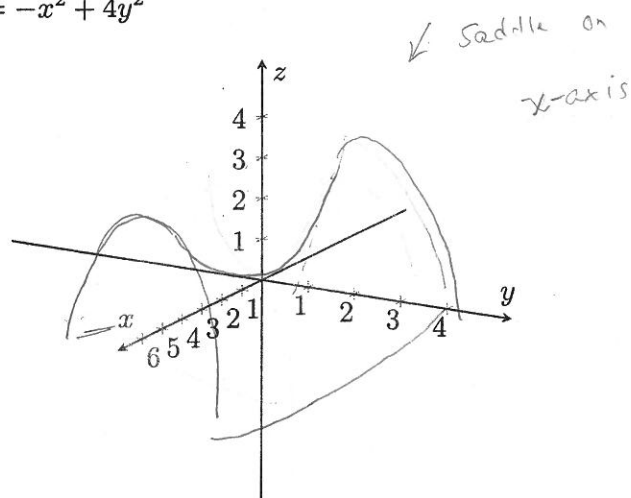


Instructions: Five points total.

1. (4 pts.) Consider the function of two variables $f(x, y) = -x^2 + 4y^2$



- (a) (1 pt.) Sketch the surface on the axes to the right.

- (b) (3 pts.) Find the equation of the tangent plane to $f(x, y)$ at the point $(a, b) = (2, 1)$. Simplify your answer.

$$z = f_x(2, 1)(x-2) + f_y(2, 1)(y-1) + f(2, 1) \quad f_x = -2x \quad f_x(2, 1) = -4$$

$$f_y = 8y \quad f_y(2, 1) = 8$$

$$z = -4(x-2) + 8(y-1) + 0$$

$$f(2, 1) = -(2)^2 + 4(1)^2 = 0$$

$$4x - 8y + z = +8 - 8$$

$$\boxed{4x - 8y + z = 0}$$

$$z = 8y - 4x$$

2. (1 pt.) Use **implicit differentiation** to solve for $\frac{\partial z}{\partial y}$ in the implicitly defined surface

$$\tan(xz) + xyz = 1$$

$$\frac{\partial}{\partial y} [\sec^2(xz) \cdot x \frac{\partial z}{\partial y} + x[y \frac{\partial z}{\partial y} + 1(z)]] = 0$$

$$\frac{\partial}{\partial y} [x \sec^2(xz) + xy] = -x$$

$$\frac{\partial z}{\partial y} = \frac{-xz}{x \sec^2(xy) + xy} = \frac{-z}{y + \sec^2(xy)} \quad x \neq 0 \quad y \neq \sec^2(xy)$$