MATH 202 Name : SOLUTIONS

Quiz 1 September 14, 2009

Instructions: All questions are worth 1 point. You get one point for taking this quiz.

1. An object is located at the point $P(1,1,1)$, but is constrained so that it can only move in the straight-line direction toward the point $B(2,-1,0)$.

   (a) Give, in coordinate form, a vector $\mathbf{v}$ representing the direction in which the object can move.

   $\vec{PB} = (2-1, -1-1, 0-1) = \boxed{(1,-2,-1)}$

   (b) Give, in coordinate form, a *unit* vector pointing in the direction that the object can move.

   $\vec{u} = \dfrac{\vec{PB}}{\|\vec{PB}\|} = \dfrac{(1,-2,-1)}{\sqrt{(-1)^2+2^2+(-1)^2}} = \boxed{\dfrac{1}{\sqrt{6}}(1,-2,-1)}$

   $= \left(\dfrac{1}{\sqrt{6}}, \dfrac{-2}{\sqrt{6}}, \dfrac{-1}{\sqrt{6}}\right)$

2. (a) Determine if the vectors $\mathbf{v}_1 = (3,-1,1)$ and $\mathbf{v}_2 = (2,4,-2)$ are perpendicular.

   $\vec{v}_1 \cdot \vec{v}_2 = 3(2) + (-1)(4) + (1)(-2) = 6-4-2 = 0$

   Since the dot product is 0, $\boxed{\vec{v}_1 \text{ is orthogonal to } \vec{v}_2.}$

   (b) Find the angle θ between the vectors $\mathbf{a} = (1,1)$ and $\mathbf{b} = (-1-\sqrt{3}, \sqrt{3}-1)$. Give your answer in radians.

   $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|}$. Thus, $\vec{a}\cdot\vec{b} = (-1-\sqrt{3}) + (\sqrt{3}-1) = -2$,

   $\|\vec{a}\| = \sqrt{1^2+1^2} = \sqrt{2}$,

   $\|\vec{b}\| = \sqrt{(-1-\sqrt{3})^2 + (\sqrt{3}-1)^2} = \sqrt{1+2\sqrt{3}+3+3-2\sqrt{3}+1}$

   $= \sqrt{8} = 2\sqrt{2}$

   and so $\cos\theta = \dfrac{-2}{\sqrt{2}\,2\sqrt{2}} = \dfrac{-2}{4} = \dfrac{-1}{2}$.

   Finally, $\theta = \arccos\left(\dfrac{-1}{2}\right) = \boxed{\dfrac{2\pi}{3}}$