MATH 202 Name : SOLUTIONS

Quiz 6 October 20, 2009

Instructions: You get one point for taking this quiz.

$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$

1. Let $z = \arctan\left(\frac{y}{x}\right)$, where $x = e^t$ and $y = 1 - e^{-t}$.

(a) (1.5 pts.) Use the Chain Rule to find the derivative $\frac{dz}{dt}$ at $t = 0$.

$z = \arctan\left(\frac{y}{x}\right)$, then $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

$$= \underbrace{\left[\frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot -\frac{y}{x^2}\right]}_{\frac{\partial z}{\partial x}} \underbrace{e^t}_{\frac{dx}{dt}} + \underbrace{\left[\frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}\right]}_{\frac{\partial z}{\partial y}} \underbrace{(+e^{-t})}_{\frac{dy}{dt}}$$

$$= \left[\frac{-y}{x^2+y^2}\right] e^t + \left[\frac{x}{x^2+y^2}\right] e^{-t}$$

At $t=0$, $x = 1$, $y = 0$

$\therefore \left.\frac{dz}{dt}\right|_{t=0} = \left[\frac{-0}{1^2+0^2}\right]e^0 + \left[\frac{1}{1^2+0^2}\right]e^{-0}$

$= 0 + 1 = \boxed{1}$

$\left.\frac{dz}{dt}\right|_{t=0} =$ $\boxed{1}$

(b) (.5 pt.) Is the function $z = f(x, y)$ increasing or decreasing (or neither) at $t = 0$? Explain briefly.

Since $1 > 0$, $f(x,y)$ is increasing.

2. (2 pts.) Find the directional derivative $D_{\mathbf{u}}f(\pi, 0)$ of $f(x, y) = \sin(xy) + x$ in the direction of $\mathbf{v} = (1, 1)$.

unit vector $\vec{u} = \frac{1}{\sqrt{2}}(1,1)$

$f_x(x,y) = y\cos(xy) + 1$

$f_y(x,y) = x\cos(xy)$

$\nabla f(x,y) = (y\cos(xy) + 1, \ x\cos(xy)) \Rightarrow \nabla f(\pi, 0) =$ [next line]

$= (0\cos(\pi \cdot 0) + 1, \ \pi\cos(\pi \cdot 0))$

$= (1, \pi)$

$\therefore D_{\vec{u}}f(\pi, 0) = \nabla f(\pi,0) \cdot \frac{1}{\sqrt{2}}(1,1) = \frac{1}{\sqrt{2}}(1(1) + \pi(1)) = \boxed{\frac{1+\pi}{\sqrt{2}}}$

$\frac{1+\pi}{\sqrt{2}}$