Useful Formulas

- Tschebysheff's Theorem: Let Y be a random variable with mean μ and finite variance σ^2. Then for any constant $k > 0$,
$$P(\,|Y-\mu| < k\sigma\,) \geq 1 - \frac{1}{k^2},$$
or, equivalently,
$$P(\,|Y-\mu| \geq k\sigma\,) \leq \frac{1}{k^2}.$$

- $\mathbb{E}(Y_1) = \mathbb{E}\big(\mathbb{E}(Y_1 \mid Y_2)\big) \qquad V(Y_1) = \mathbb{E}\big(V(Y_1 \mid Y_2)\big) + V\big(\mathbb{E}(Y_1 \mid Y_2)\big)$

- Let $Y_1, \dots Y_n$ be random variables with mean μ_i, $i = 1, \dots n$, and $X_1, \dots X_m$ be random variables with mean η_j, $j = 1, \dots m$. Let $U = \sum_{i-1}^{n} a_i Y_i$ and $W = \sum_{j=1}^{m} b_j X_j$. Then
$$V(U) = \sum_{i-1}^{n} a_i^2 V(Y_i) + 2 \sum\sum_{i<j} a_i a_j \operatorname{Cov}(Y_i, Y_j)$$
$$\operatorname{Cov}(U, W) = \sum_{i=1}^{n}\sum_{j=1}^{m} a_i b_j \operatorname{Cov}(Y_i, X_j)$$

- Variance and covariances of $(X_1, \dots, X_k) \sim \text{Multinomial}(n; p_1, \dots, p_k)$:
$$\mathbb{E}(Y_i) = np_i,\; V(Y_i) = np_i(1-p_i) \qquad \operatorname{Cov}(Y_i, Y_y) = -np_i p_j,\; i \neq j$$