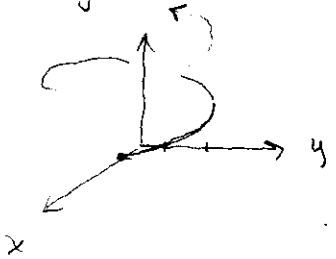
MATH 202 Name : SOLUTIONS

Quiz 4 February 19, 2008

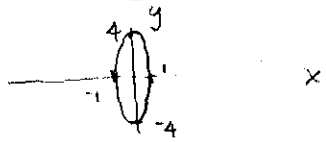
Instructions: This quiz is worth five points. You get one point for taking this quiz.

1. (2 pts.) Sketch and describe the vector-valued function $\mathbf{r}(t) = (\cos t, 4\sin t, t)$ for $t \geq 0$. A complete answer should include the equation (in terms of x and y) of the projection of this curve in the xy-plane.

Since $x = \cos t$, $y = 4\sin t$, notice $x^2 + \frac{y^2}{16} = 1$ $\rightsquigarrow$ the equation of an ellipse with intercepts $(\pm 1, 0)$, $(0, \pm 4)$ in the xy-plane



An elliptical helix



2. (2 pts.) Find the parametric equations for the tangent line to the curve $\mathbf{r}(t)$ at the point $P_0(0, 4, \frac{\pi}{2})$.

$\vec{r}(t) = (\cos t, 4\sin t, t)$ $\quad t \geq 0$

The tangent vector is $\vec{r}\,'(t) = (-\sin t, 4\cos t, 1)$ $\quad \forall\, t \geq 0$

The direction vector $\vec{v}$ for the tangent line is $\vec{r}\,'(\frac{\pi}{2}) = (-\sin\frac{\pi}{2}, 4\cos\frac{\pi}{2}, 1)$

$= (-1, 0, 1)$

I've used above that $P_0(0, 4, \frac{\pi}{2})$ is $\vec{r}(\frac{\pi}{2})$ $\quad \uparrow \; t = \frac{\pi}{2}$

The vector equation of the tangent line is $(0, 4, \frac{\pi}{2}) + s(-1, 0, 1)$ $\quad s \in \mathbb{R}$

The parametric equations are

$$x(s) = -s, \quad y(s) = 4, \quad z(s) = \frac{\pi}{2} + s \qquad \text{for } s \in \mathbb{R}$$

Note Looking at the sketch above it makes sense that $\frac{dx}{dt} = -1$, $\frac{dy}{dt} = 0$

x is decreasing

y is changing directions from + to −