MATH 202
Quiz 7

Name :________________________

March 26, 2008

Instructions: This quiz is worth five points. You get one point for taking this quiz.

1. (2 pts.) Consider the function $g(x, y) = \sin(xy)$. It is possible to check that $(1, \frac{3\pi}{2})$ is a critical point of $g(x, y)$. (Do not do this check.)

   (a) Without using calculus at all, but instead using your understanding of the function $z = \sin(xy)$, is the point $(1, \frac{3\pi}{2})$ a local maximum, a local minimum, or a saddle point? Explain briefly.

   It is a local minimum. $\sin(\frac{3\pi}{2}) = -1$.
   If I change x or y a little, then $g(x,y) = \sin(xy) > -1$

   (b) Now use the second derivative test to determine if the critical point $(1, \frac{3\pi}{2})$ is a local maximum, a local minimum, or a saddle point.

   $g_{xx} = -y^2 \sin(xy) = \frac{9\pi^2}{4}$

   $g_{yy} = -x^2 \sin(xy) = 1$

   $g_{xy} = \cos(xy) - yx\sin(xy) = \frac{3\pi}{2}$

   $D = g_{xx}(1, \frac{3\pi}{2})\, g_{yy}(1, \frac{3\pi}{2}) - \left(g_{xy}(1, \frac{3\pi}{2})\right)^2 = 0$

   The test gives no information

2. (2 pts.) Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 2xy$ subject to the constraints $x^2 + y^2 = 1$ and $x \geq 0$. Your final answer should include both the maximum and minimum values and the coordinates of the points (x, y) where these extreme values occur. (To eliminate some computation, please only consider those points for which $x \geq 0$.)

$\nabla f(x,y) = \langle 2y, 2x \rangle$

$\nabla g(x,y) = \langle 2x, 2y \rangle$

$$\begin{cases} \nabla f(x,y) = \lambda \nabla g(x,y) \\ g(x,y) = k \end{cases}$$

$$\begin{cases} 2y = \lambda 2x \\ 2x = \lambda 2y \\ x^2 + y^2 = 1 \end{cases}$$

1st eq: $\lambda = \frac{y}{x}$

$$\begin{cases} 2x = \frac{y}{x} 2y \\ x^2 + y^2 = 1 \end{cases}$$

$$\begin{cases} y^2 = x^2 \\ x^2 + y^2 = 1 \end{cases}$$

$2x^2 = 1$

$x = \frac{\sqrt{2}}{2}$

$y = \pm\frac{\sqrt{2}}{2}$

$\boxed{\text{max: } f(\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}) = 1; \quad \text{min: } f(\tfrac{\sqrt{2}}{2}, -\tfrac{\sqrt{2}}{2}) = -1}$