3. Consider the jointly distributed random variables (X, Y) with joint density function

$$f(x, y) = \begin{cases} ce^{-y}, & \text{for } 0 \le x \le e^2 - 1,\ 0 \le y \le \ln(x+1) \\ 0, & \text{otherwise.} \end{cases}$$

(a) Draw the *support* of the joint density function $f(x, y)$; that is, the region S where $f(x, y) > 0$. Then find the value of c so that $f(x, y)$ is a valid density function on S.

(b) Set up an integral to find the marginal density $f_Y(y)$ and include the domain of this function.

(c) Verify that your marginal density $f_Y(y)$ is correct by integrating it on the support of Y.

(d) Find the value of the conditional probability $P(X \ge 4 \mid Y = \ln(3))$. *Answer:* $\dfrac{e^2 - 5}{e^2 - 3} \approx .54$.