

HW #6 Solution keys.

13.2 #28

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - 4y^4}{x^2 + 2y^2} = \lim_{(x,y) \rightarrow (0,0)} \frac{(x^2 - 2y^2)(x^2 + 2y^2)}{x^2 + 2y^2} = \lim_{(x,y) \rightarrow (0,0)} (x^2 - 2y^2) = 0$$

13.2 #30

$$\begin{aligned} \lim_{(x,y) \rightarrow (2,1)} \frac{x-y-1}{\sqrt{x-y}-1} \cdot \frac{\sqrt{x-y}+1}{\sqrt{x-y}+1} &= \lim_{(x,y) \rightarrow (2,1)} \frac{(x-y-1)(\sqrt{x-y}+1)}{(x-y)-1} = \\ &= \lim_{(x,y) \rightarrow (2,1)} (\sqrt{x-y}+1) = 2 \end{aligned}$$

13.2 #32

The limit doesn't exist because along the line $x=y$ you have:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x}{x^2 - y^2} = \lim_{(x,x) \rightarrow (0,0)} \frac{x}{x^2 - x^2} = \lim_{(x,x) \rightarrow (0,0)} \frac{x}{0}$$

Because the denominator is zero, the limit doesn't exist.

13.4 #38

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad ; \quad R = \frac{R_1 R_2}{R_1 + R_2} \quad ; \quad \begin{aligned} dR_1 &= \Delta R_1 = 0.5 \\ dR_2 &= \Delta R_2 = -2 \end{aligned}$$

$$\Delta R \approx dR = \frac{\partial R}{\partial R_1} dR_1 + \frac{\partial R}{\partial R_2} dR_2 = \frac{R_2^2}{(R_1 + R_2)^2} \Delta R_1 + \frac{R_1^2}{(R_1 + R_2)^2} \Delta R_2$$

When $R_1 = 10$ and $R_2 = 15$, we have

$$\Delta R \approx \frac{15^2}{(10+15)^2} (0.5) + \frac{10^2}{(10+15)^2} (-2) = -0.14 \text{ ohm.}$$