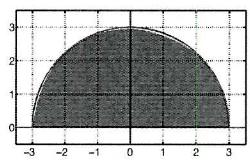
MATH 202 Name : SOLUTIONS

Quiz 8 April 9, 2012

Instructions: Point values as indicated. You get one point for taking this quiz.

1. On a semicircular lamina (pictured below), the density $\rho(x, y)$ at any point (x, y) is equal to its distance from the origin: $\rho(x, y) = \sqrt{x^2 + y^2}$.



mass $m = \iint_R \rho(x,y)\, dA \qquad \bar{y} = \frac{M_x}{m} = \frac{\iint_R y\,\rho(x,y)\, dA}{m}$

(a) (1 pt.) Find the mass of the lamina.

$m = \iint_R \sqrt{x^2+y^2}\, dA$ In polar coordinates, $m = \int_0^{\pi}\int_0^{3} r\,(r\,dr\,d\theta)$

$= \int_0^{\pi} \frac{1}{3} r^3 \Big|_0^3 \, d\theta = \int_0^{\pi} 9\, d\theta = \boxed{9\pi}$

(b) (2 pts.) Find the center of mass $(\bar{x}, \bar{y})$ of the lamina.

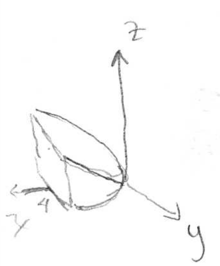
$\bar{x} = 0$ by symmetry. $M_x = \iint_R y\sqrt{x^2+y^2}\, dA$. In polar coordinates,

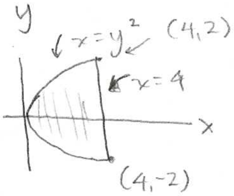
$M_x = \int_0^{\pi}\int_0^{3} r\sin\theta\,(r)(r\,dr\,d\theta) = \int_0^{\pi}\int_0^{3} r^3 \sin\theta\, dr\,d\theta = \int_0^{\pi} \sin\theta\, \frac{1}{4} r^4 \Big|_0^3 \, d\theta$

$= \int_0^{\pi} \frac{81}{4}\sin\theta\, d\theta = -\frac{81}{4}\cos\theta \Big|_0^{\pi} = -\frac{81}{4}\cos\pi + \frac{81}{4}\cos 0 = 2\cdot\frac{81}{4} = \frac{81}{2}$

Therefore, $\bar{y} = \frac{81/2}{9\pi} = \boxed{\frac{9}{2\pi}}$ $\qquad (\bar{x}, \bar{y}) = \left(0, \frac{9}{2\pi}\right)$

2. (1 pt. – no partial credit) Set up, **but do not integrate**, a triple integral the computes the volume of the solid enclosed by the cylinder $\underbrace{x = y^2}_{\text{sides}}$ and the planes $\underbrace{x = z}_{\text{top}}$, $\underbrace{z = 0}_{\text{bottom}}$ and $\underbrace{x = 4}_{\text{front}}$. (Use the back of this quiz for your work.)





$\int_{-2}^{2}\int_{y^2}^{4}\int_{0}^{x} dz\,dx\,dy = \int_{-2}^{2}\int_{y^2}^{4} x\,dx\,dy$