MATH 371
Review problems

1. Consider the jointly continuous uniformly distributed random variables $(X, Y)$ on the domain bounded by $x = 0$, $y = 2$, $xy = 4$, $x = 4$, and $y = 0$. (It is **easy** to check your answers without integrating.)

   (a) Draw the *support* of the joint density function $f(x, y)$; that is, the region $S$ where $f(x, y) > 0$.

   (b) Find the value of $c$ so that $f(x, y)$ is a valid density function on $S$.

   (c) Set up an integral to find the marginal density $f_X(x)$ and include the domain of this function.

   (d) Set up an integral to find the marginal density $f_Y(y)$ and include the domain of this function.

   (e) Set up an integral that computes the conditional probability $P(X \geq 1 \mid Y = \frac{3}{2})$.

   (f) Set up a computation that computes the conditional probability that $P(X \geq 1 \mid Y \geq \frac{1}{2})$.

2. In a large calculus class of 200 students, 40 earn an A on a test, 60 earn a B, and the remaining students earn a C, D, or F. Suppose a random sample of size 25 is taken.

   (a) Find the probability that five students in the sample earned an A on the exam.

   (b) Find the marginal probability function for the variable
   $A$: number of students who earned an A on the exam.

   (c) Write down a formula that computes the probability of the event
   $E$: Between 2 and 5 students in the sample earn a B on the exam, given that 10 students in the sample earned an A.

   (d) Give the probability function for the jointly distributed random variables $(A, B)$. Are $A$ and $B$ independent?