Chapter 5. #141

Answer in back of book is <u>wrong</u>.

$Y_1 \sim Exp(\lambda)$ $\beta = \lambda$ $\qquad f(Y_2 | Y_1 = y_1) = \frac{1}{y_1}$

a) $\mathbb{E}(Y_2) = \frac{\lambda}{2}$

$$\mathbb{E}(Y_2 | Y_1 = y_1) = \int_0^{y_1} y_2 \frac{1}{y_1} \, dy_2 = \frac{1}{y_1} \frac{1}{2} y_1^2 = \frac{1}{2} y_1 \qquad \therefore \mathbb{E}(Y_2 | Y_1) = \frac{1}{2} Y_1$$

$$\mathbb{E}(\mathbb{E}(Y_2|Y_1)) = \mathbb{E}(\tfrac{1}{2} Y_1) = \frac{\lambda}{2}$$

b) $V(Y_2) = \mathbb{E}(V(Y_2|Y_1)) + V(\mathbb{E}(Y_2|Y_1))$

$$V(Y_2|Y_1 = y_1) = \mathbb{E}(Y_2^2 | Y_1 = y_1) - \left[\mathbb{E}(Y_2 | Y_1 = y_1)\right]^2$$

$$= \int_0^{y_1} y_2^2 \frac{1}{y_1} \, dy_2 - \left[\frac{y_1}{2}\right]^2$$

$$= \frac{1}{y_1} \frac{1}{3} y_1^3 - \left[\frac{y_1}{2}\right]^2 = \frac{1}{3} y_1^2 - \frac{1}{4} y_1^2 = \frac{1}{12} y_1^2 \qquad V(Y_2 | Y_1) = \frac{1}{12} Y_1^2$$

$$\therefore V(Y_2) = \mathbb{E}(V(Y_2|Y_1)) + V(\mathbb{E}(Y_2|Y_1))$$

$$= \mathbb{E}\left(\tfrac{1}{12} Y_1^2\right) + V\left(\tfrac{1}{2} Y_1\right)$$

$$= \frac{1}{12} \mathbb{E}(Y_1^2) + \frac{1}{4} V(Y_1) \qquad V(Y_1) = \mathbb{E}(Y_1^2) - \mathbb{E}(Y_1)^2$$

$$= \frac{1}{12}\left[V(Y_1) + \mathbb{E}(Y_1)^2\right] + \frac{1}{4} V(Y_1)$$

$$= \frac{1}{12}\left[(\lambda^2) + \lambda^2\right] + \frac{1}{4}\lambda^2 = \frac{5}{12}\lambda^2$$