MATH253
Fall 2019

Midterm Exam 1

Name: _____

Instructions. (100 points) You have 90 minutes. Closed book, closed notes, and no calculators allowed. *Show all your work* in order to receive full credit.

(17 pts) **1.** Consider points $A(4,-3,2)$ and $B(2,1,c)$ and vectors $\mathbf{u}=\langle 1,-2,3\rangle$ and $\mathbf{v}=\langle -1,-1,2\rangle$.

(a) (4 pts) Find the vector projection of $\mathbf{u}$ along $\mathbf{v}$.

(b) (4 pts) Find the area of the parallelogram with adjacent sides $\mathbf{u}$ and $\mathbf{v}$.

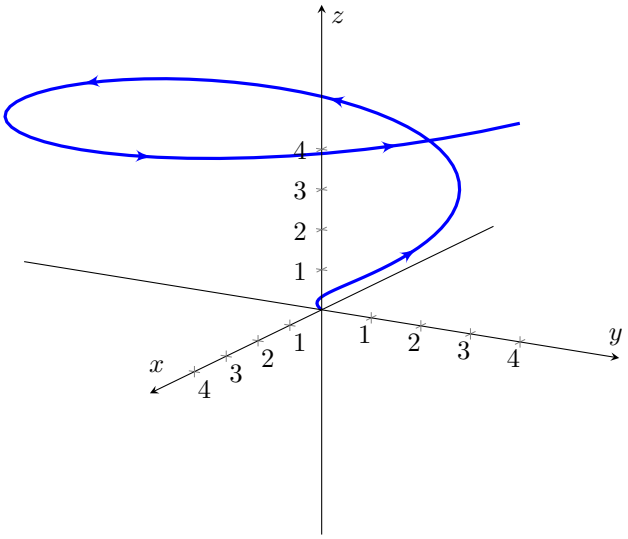
(c) (3 pts) Find all values of c such that the length of $\overrightarrow{AB}$ equals 5.

(d) (3 pts) Find all values of c such that $\overrightarrow{AB}$ is parallel to $\mathbf{u}$.

(e) (3 pts) Find all values of c such that $\overrightarrow{AB}$ is orthogonal to $\mathbf{v}$.

(15$^{\text{pts}}$) **2.** Below is a sketch of the space curve:

$$\mathbf{r}(t) = \langle t\cos t, t\sin t, t\rangle \quad , \quad 0 \le t \le \frac{7\pi}{3}.$$



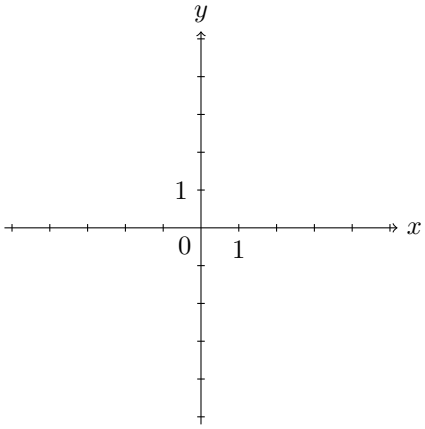
(a) (8 pts) Draw on the above the position and velocity vectors for $t = \dfrac{3\pi}{2}$.

(b) (4 pts) Find the speed at time t and simplify your result.

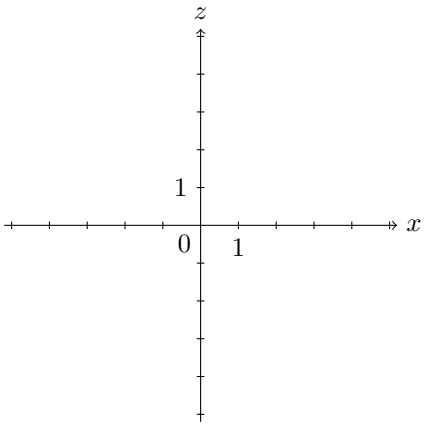
(c) (3 pts) At what time(s) is the acceleration horizontal (i.e. normal to $\mathbf{k}$)?

(15$^{\text{pts}}$) **3.** Time to sketch some surfaces!

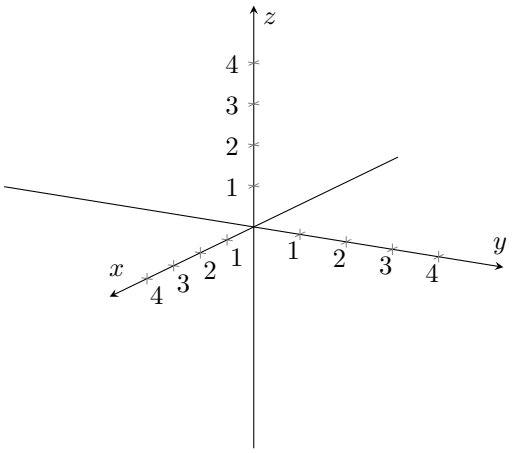
(a) (10 pts) For $x^2 + \dfrac{y^2}{4} - z^2 = -1$, sketch the given traces, then the surface in 3D.



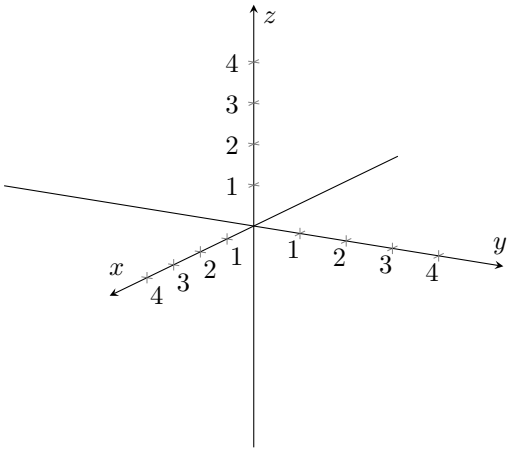
traces: $z = 0, \pm\sqrt{5}$



trace: $y = 0$



(b) (5 pts) Sketch the surface $y = z^2 + 1$.



(21^{pts}) **4.** Consider the following point, line, and plane:

$$A = (3, -2, 5),$$
$$\vec{\ell}(t) = \langle 1 - 2t, t, 3 + 4t \rangle,$$
$$P: \quad 2x - 3y + z = -4,$$

(a) (4 pts) Give the equation of a plane parallel to the plane P that passes through A.

(b) (3 pts) Find the point of intersection of the line $\vec{\ell}(t)$ and the plane P.

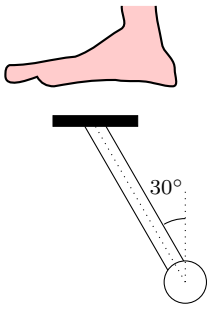
(c) (4 pts) Find the angle the line $\vec{\ell}(t)$ makes with the normal to the plane P. (Your answer may involve an inverse trigonometric function.)

(d) (6 pts) Find an equation for the plane containing the point A and the line $\vec{\ell}(t)$.

(e) (4 pts) Find the distance from the point A to the plane P.

(9pts) **5.** A bicycle pedal is attached to a 17 cm crank. When the crank is at an angle of 30° with the vertical (as shown) a foot applies a downward force of 200 N.

(a) (4 pts) What is the resulting torque? Give your answer as a vector.



(b) (3 pts) What is the magnitude of the torque? Indicate units.

(c) (2 pts) What is the direction of the torque vector? (Into the page $\otimes$, or out of the page $\odot$, in the figure).

(8pts) **6.** An object moves in the plane with acceleration

$$\mathbf{a}(t) = \left\langle \frac{1}{t^2}, \frac{t}{(1+t^2)^2} \right\rangle.$$

At time $t = 1$ it is located at the point $(1, 0)$ and has velocity $\langle 2, 1\rangle$. Find a function $\mathbf{r}(t)$ giving its position at all times $t > 0$.

(15^{pts}) **7.** A particle moves with *velocity* $\mathbf{v}(t) = \langle t^2, 2t, 2\rangle$.

(a) (5 pts) Find the distance the particle travels between times $t = 1$ and 2.

(b) (5 pts) Calculate the curvature of the trajectory at time $t = 1$.

(c) (5 pts) Find the unit tangent vector $\mathbf{T}(t)$ and the tangential component of acceleration $a_{\mathbf{T}}$ at $t = 1$.