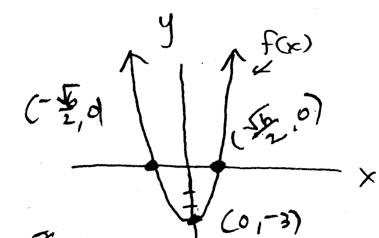
Review Solutions

2.1: 1. $f(-2)=\frac{1}{6}$, $f(0)=\frac{1}{2}$, $f(\frac{1}{2})=\frac{1}{(\frac{1}{2})^2+2}=\frac{4}{9}$, $f(1)=\frac{1}{3}$, $f(5)=24$

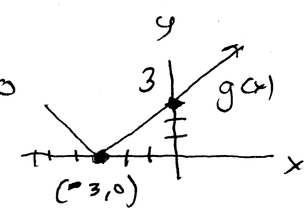
$g(-2)=0$, $g(0)=\frac{4}{5}$, $g(\frac{1}{2})=\frac{5}{12}$, $g(1)=\frac{1}{4}$, $g(5)=$ undefined (denominator $=0$)

2. a. $x \geq 2$ and $x \neq 3$; $(2,3)\cup(3,\infty)$ b. all real numbers $(-\infty,\infty)$

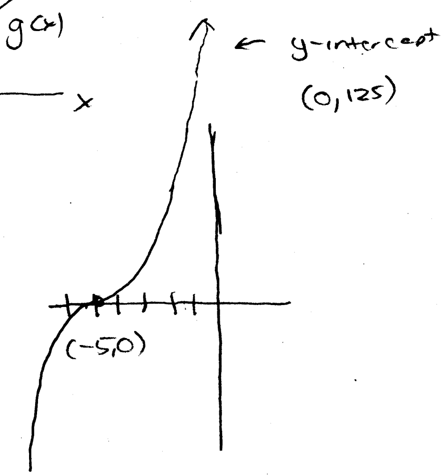
c. $y \geq -\frac{5}{2}$ d. $x \neq 0$



2.2: 1. a $f(x)=2x^2-3$ domain $(-\infty,+\infty)$; range $y \geq -3$



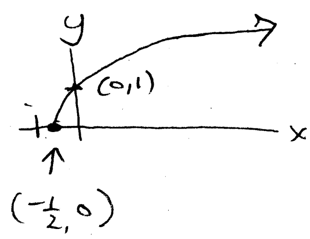
b. $g(x)=|x+3|$ domain $(-\infty,+\infty)$; range $y \geq 0$



c. $h(x)=\sqrt{2x+1}$

domain $x \geq -\frac{1}{2}$

range $y \geq 0$



d. $h(x)=(x+5)^3$ domain $(-\infty,+\infty)$; range $(-\infty,\infty)$

e. a) narrowing of x^2 followed by a vertical translation down 2 units

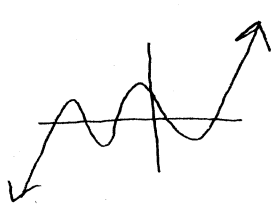
b) Horizontal translation 3 units left of $y=|x|$

c) Dilation (Narrowing) by 2 units, Horizontal translation $\frac{1}{2}$ unit left.

$$y=\sqrt{x} \mapsto y=\sqrt{2x} \mapsto y=\sqrt{2(x+\frac{1}{2})} = y=\sqrt{2x+1}$$

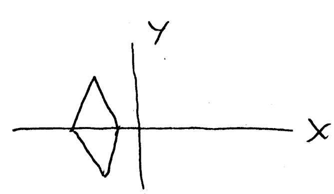
2. f(x) is a function if every vertical line intersects its graph at most once.
(i.e. every x corresponds to one y and not two.)

a)



passes the vertical line test

b)

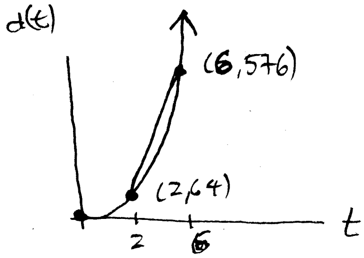


fails the vertical line test

2.3. See textbook.

2.4. 1 a) (i) average value $= \dfrac{d(5)-d(1)}{5-1} = \dfrac{400-16}{4} = \dfrac{384}{4} = 96$ ft/s.

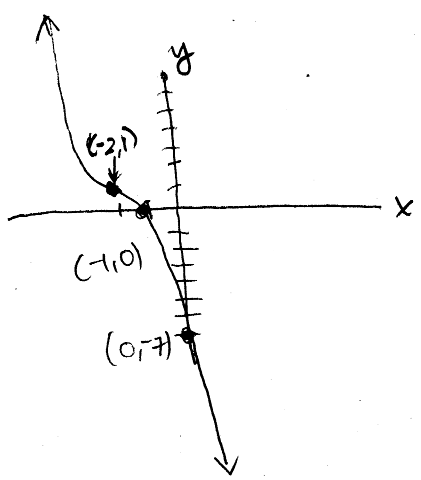
(ii) $\dfrac{d(6)-d(2)}{6-2} = \dfrac{576-64}{4} = \dfrac{512}{4} = 128$ ft/s



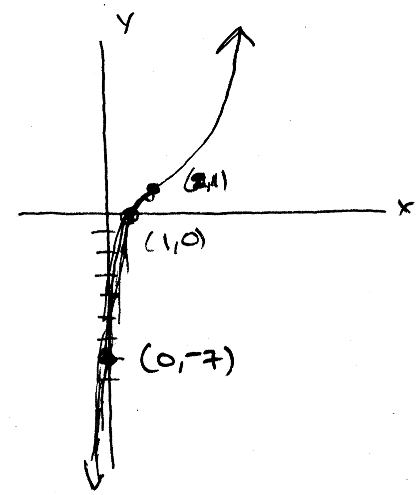
The average speed is the slope of the line segment joining the two points. Here (2,64) and (6,576) are pictured.

2.5: 1a. $y=f(-x)$ b. $y=-f(x)$ c. $y=f(x+5)$ d. $y=f(x)-10$

2a.

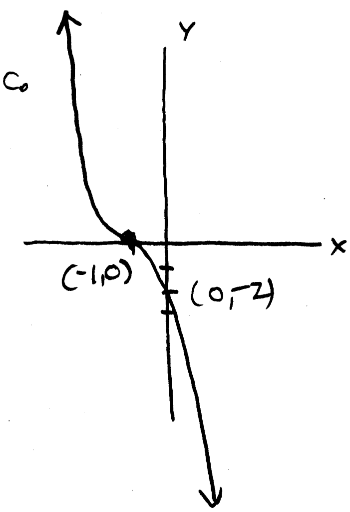


b.

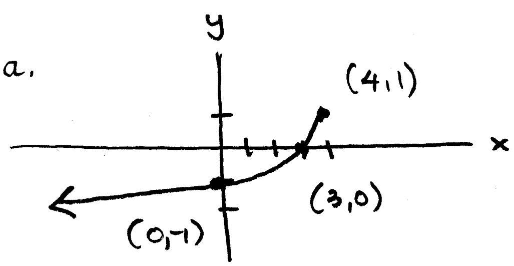


x-axis reflection of (a)

c.

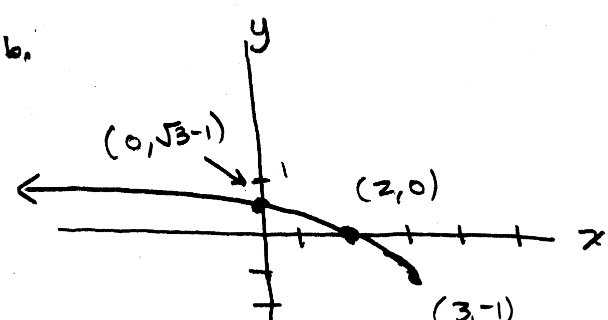


3. a.

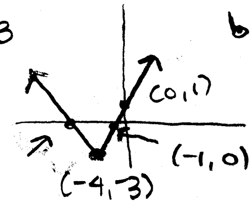


$y = -\sqrt{-x+4}+1$

b.

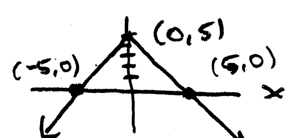


4. a. $y = |-x-4| - 3$
$= |x+4| - 3$

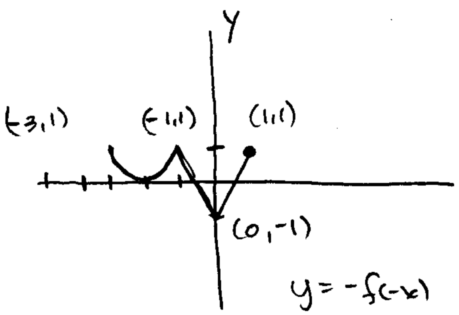


b. Show read x-axis reflection ...

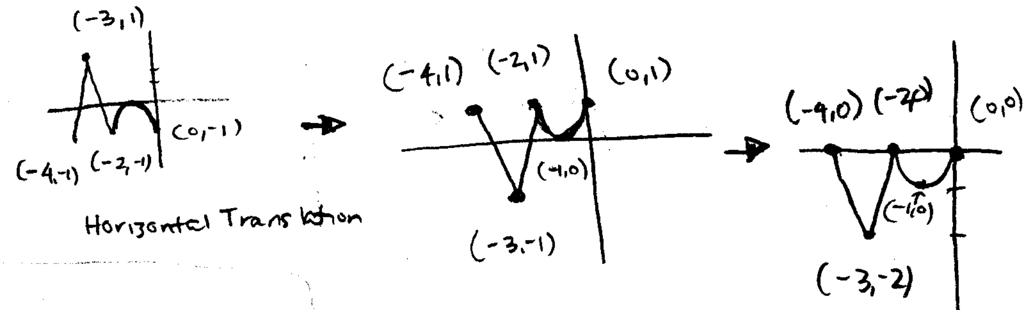
$y = -|x|+5$



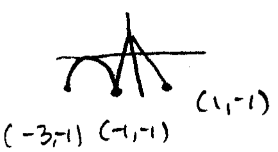
5. a) y- and x-axis reflection



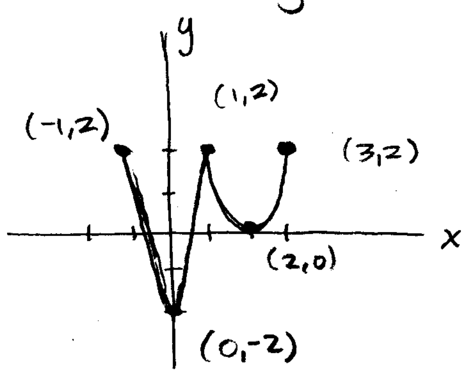
b) $y = -f(x-3)-1$ Horizontal translation 3 units left, x-axis Refl. VT 1 unit down



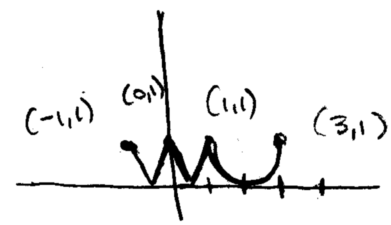
c) y-axis reflection



(d) x-axis reflection and dilation by 2



(e) Absolute Value



2.6: 1a. $f(g(1)) = 1$, $f(g(-1)) = 1$, $g(f(1)) = 7$, $g(f(-1)) = -1$, $g(h(\pi)) = \pi$, $h(g(x)) = x$

$f(g(h(7))) = 8$, $h(g(f(2))) = 3$

b. $(f+g)(1) = 2$, $(f-g)(2) = -4$, $(gh)(-1) = -1$, $(\frac{f}{g})(-4) = \frac{3}{-65} = \frac{-3}{65}$, $(\frac{f}{h})(26) = 9$

c. $(f \circ g)(x) = |x^3|$, $(g \circ g)(x) = x^9 - 3x^6 + 3x^3 - 2$, $(h \circ f)(x) = \sqrt[3]{|x+1|+1}$

$(g \circ h)(2) = 2$, $(f \circ h)(x) = |\sqrt[3]{x+1} + 1|$

d. $(\frac{g}{f})(x)$ has domain $x \neq -1$, $(g \circ h)(x)$ has domain $(-\infty, \infty)$, $(\frac{g}{h})(x)$ has domain $x \neq -1$

2.7: 1. $f(x)$ is 1-1 if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. $f^{-1}(x)$ is a function such that

$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

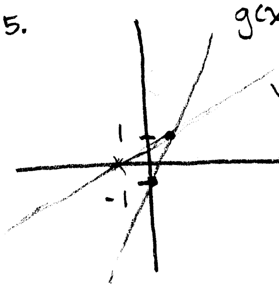
2. HLT: A function is 1-1 if every horizontal line intersects its graph at most once.

3a. $f^{-1}(2) = 5$ b. $f^{-1}(3) = 2$ c. $f^{-1}(4) = ?$ can not determine d. $f^{-1}(10) = 4$. $(f \circ f)(5) = 3$

f. $(f \circ f^{-1})(x) = x$

4. a. $g^{-1}(x) = +\frac{1}{2}x - \frac{1}{2}$ b. $h^{-1}(x) = \dfrac{-(y+2)}{3y-4}$ c. $p^{-1}(t) = -(t-4)^3$

5. $g(x) = 2x - 1$



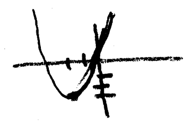
$g^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$
(reflection about $y = x$)

$g(x)$ is 1-1 since it passes the horizontal line test.

from definition:
$g(x_1) = g(x_2) \Rightarrow 2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$

3.1. a. $f(x) = x^2 + 4x + 1 = (x+2)^2 - 3$ Vertex: $(-2, -3)$, Minimum value $= -3$

domain: $(-\infty, +\infty)$ range: $y \geq -3$



x-intercepts:

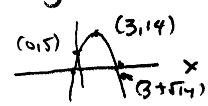
$\dfrac{-4 \pm \sqrt{16-4}}{2} = \dfrac{-4 \pm \sqrt{12}}{2}$

$= -2 \pm \sqrt{3}$

b. $g(x) = -x^2 + 6x + 5 = -(x-3)^2 + 14$ Vertex $(3, 14)$

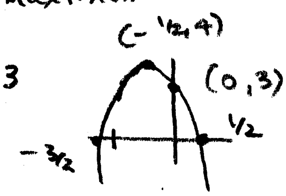
Max value $= 14$ domain $(-\infty, +\infty)$, range $y \leq 14$

intercepts: $x = 3 + \sqrt{14}, 3 - \sqrt{14}$



c. $h(x) = 3 - 4x - 4x^2 = -4x^2 - 4x + 3$
$= -4(x + \frac{1}{2})^2 + 4$ Vertex $(-\frac{1}{2}, 4)$, maximum value $+4$; domain $(-\infty, \infty)$; Range $y \leq 4$

x-intercepts: $x = -\frac{3}{2}, \frac{1}{2}$ y-int: $y = 3$

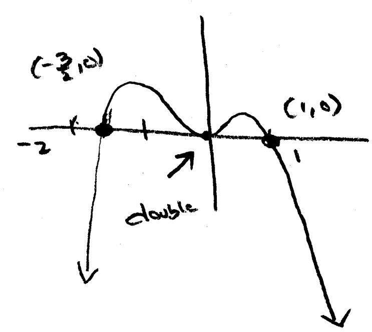


2. a. Max value $= \frac{7}{2}$ b. Min value $= -562{,}500$

3.2: a. $g(x) = -2x^4 - x^3 + 3x^2 = x^2(-2x^2 - x + 3)$

Zeros: $x = 0, 1, -3$ As $x \to \infty$, $g(x) \to -\infty$

As $x \to -\infty$, $g(x) \to -\infty$



b. $h(t) = t^3 - 2t^2 - 4t + 8$ (t not x)

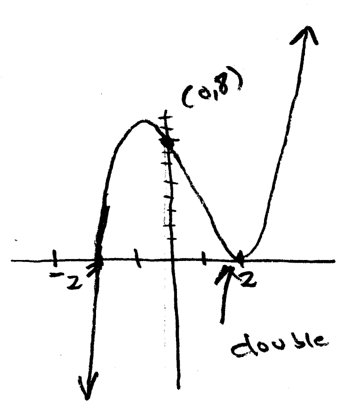
Zeros: $0 = t^3 - 2t^2 - 4t + 8$

$0 = t^2(t-2) - 4(t-2)$

$0 = (t-2)(t^2-4)$

$0 = (t-2)^2(t+2)$

$t = 2, -2$



As $t \to \infty$, $h(t) \to \infty$
As $t \to -\infty$, $h(t) \to -\infty$

$\boxed{3.4}$ Check by multiplying

1.	Quotient	Remainder	
1a.	$2x^2-1$	-2	
b	$\frac{1}{2}x^3 - x^2 - \frac{5}{2}x - \frac{7}{4}$	$1 + \frac{19}{2}x$	
c.	x^4+1	0	$\rightarrow$ evenly divides! $x^6+x^4+x^2+1 = (x^4+1)(x^2+1)$
2 a.	$x-2$	-2	
b.	x^2+2	-3	