HW #7 26, 34, 40, 44, 56, 68, 78, 80, 84, 108

26) $2(7x-3) \le 12x+16$

$7x - 3 \le 6x + 8$

$x \le 11$

$\boxed{\text{interval: } (-\infty, 11]}$

graph:


34) $-\frac{1}{2} \le \frac{4-3x}{5} \le \frac{1}{4}$ * everything by LCD $\Rightarrow 20$

$-10 \le 4(4-3x) \le 5$

$-10 \le 16 - 12x \le 5$

$-26 \le -12x \le -11$

$\frac{13}{6} \ge x \ge \frac{11}{12}$

$\boxed{\text{Interval: } [\frac{11}{12}, \frac{13}{6}]}$

graph


40) $x^2 + 5x + 6 > 0$

$(x+3)(x+2) > 0$ so, changes sign at $x = -3$ and $x = 2$

| interval | $(-\infty, -3)$ | $(-3, -2)$ | $(-2, \infty$ |
|---|---|---|---|
| sign of $x+3$ | $-$ | $+$ | $+$ |
| sign of $x+2$ | $-$ | $-$ | $+$ |
| sign of $(x+3)(x+2)$ | $+$ | $-$ | $+$ |

so, the solution set is $\{x \mid x < -3 \text{ or } -2 < x\}$

$\boxed{\text{interval: } (-\infty, -3) \cup (-2, \infty)}$

graph:


44) $5x^2 + 3x \ge 3x^2 + 2$

$2x^2 + 3x - 2 \ge 0$

$(2x-1)(x+2) \ge 0$ so, changes sign at $x = \frac{1}{2}$ and $x = -2$

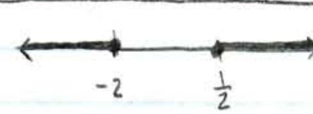
| Interval | $(-\infty, -2)$ | $(-2, \frac{1}{2})$ | $(\frac{1}{2}, \infty)$ |
|---|---|---|---|
| sign of $2x-1$ | $-$ | $-$ | $+$ |
| sign of $x+2$ | $-$ | $+$ | $+$ |
| sign of $(2x-1)(x+2)$ | $+$ | $-$ | $+$ |

so the solution set is ...

$\{x \mid x \le -2 \text{ or } \frac{1}{2} \le x\}$

$\boxed{\text{interval: } (-\infty, -2] \cup [\frac{1}{2}, \infty)}$

graph:



56) $16x \le x^3$

$x^3 - 16x \ge 0 \Rightarrow x(x^2 - 16) \ge 0 \Rightarrow x(x+4)(x-4) \ge 0$

so, changes sign at $x = 0$, $x = 4$, $x = -4$

| interval | $(-\infty, -4)$ | $(-4, 0)$ | $(0, 4)$ | $(4, \infty)$ |
|---|---|---|---|---|
| sign of $x+4$ | $-$ | $+$ | $+$ | $+$ |
| sign of $x$ | $-$ | $-$ | $+$ | $+$ |
| sign of $x-4$ | $-$ | $-$ | $-$ | $+$ |
| sign of $x(x-4)(x+4)$ | $-$ | $+$ | $-$ | $+$ |

so, $\{x \mid -4 \le x \le 0 \text{ or } 4 \le x\}$

$\boxed{\text{interval: } [-4, 0] \cup [4, \infty)}$

graph


68) $\frac{x}{2} \ge \frac{5}{x+1} + 4$

$\frac{x}{2} - \frac{5}{x+1} - 4 \ge 0$

$\frac{x(x+1)}{2(x+1)} - \frac{2 \cdot 5}{2(x+1)} - \frac{4 \cdot 2(x+1)}{2(x+1)} \ge 0 \Rightarrow \frac{x^2 + x - 10 - 8x - 8}{2(x+1)} \ge 0$

$\Rightarrow \frac{x^2 - 7x - 18}{2(x+1)} \ge 0 \Rightarrow \frac{(x-9)(x+2)}{2(x+1)} \ge 0$ so, changes sign when $x = 9, -2, -1$

| Interval | $(-\infty, -2)$ | $(-2, -1)$ | $(-1, 9)$ | $(9, \infty)$ |
|---|---|---|---|---|
| sign of $x-9$ | $-$ | $-$ | $-$ | $+$ |
| sign of $x+2$ | $-$ | $+$ | $+$ | $+$ |
| sign of $x+1$ | $-$ | $-$ | $+$ | $+$ |
| sign of $\frac{(x-9)(x+2)}{(x+1)}$ | $-$ | $+$ | $-$ | $+$ |

so, $\{x \mid -2 \le x < -1, 9 \le x\}$

$\boxed{\text{interval: } [-2, -1) \cup [9, \infty)}$

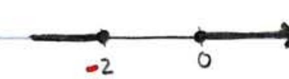
graph


78) $|x+1| \ge 1 \Rightarrow x + 1 \ge 1 \Rightarrow x \ge 0$

$\Rightarrow x + 1 \le -1 \Rightarrow x \le -2$

$\boxed{\text{interval: } (-\infty, -2] \cup [0, \infty)}$

graph:



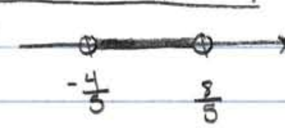

80) $|5x - 2| < 6 \Rightarrow -6 < 5x - 2 < 6$

$\Rightarrow -4 < 5x < 8$

$\Rightarrow -\frac{4}{5} < x < \frac{8}{5}$

$\boxed{\text{interval: } (-\frac{4}{5}, \frac{8}{5})}$

graph:




check #64

84) $\left|\frac{x+1}{2}\right| \geq 4 \Rightarrow \left|\frac{1}{2}(x+1)\right| \geq 4 \Rightarrow \frac{1}{2}|x+1| \geq 4$

$\Rightarrow |x+1| \geq 8 \Rightarrow -8 \geq x+1 \geq 8$

$\Rightarrow -9 \geq x \geq 7$ $\boxed{\text{interval: } (-\infty,-9] \cup [7,\infty)}$

graph:


108) $x$ = # of minutes for long-distance calls per month

Plan A: $25 + 0.05x = C$ Plan B: $5 + 0.12x = C$

so solve for $25 + 0.05x > 5 + 0.12x$

$20 > 0.07x$

$285.7 > x$

so Plan B is better before 286 minutes are used.