

Instructions: Each problem is worth one point. There is no partial credit on this quiz. You get one point for taking this quiz.

1. Show that the following limit does ~~not~~ exist:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^3 + y^3}$$

~~It did not exist!~~ **It did exist!**

For practice, try $\lim_{(x,y) \rightarrow (0,0)} \frac{-x^3 + y}{x^3 + y}$ and $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4}$

2. Consider the function $f(x, y) = \ln(x^2 + y^2 + 1)$.

- (a) Compute the partial derivative $\frac{\partial f}{\partial x}$.

$$\frac{\partial f}{\partial x} = \frac{1}{x^2 + y^2 + 1} \cdot 2x = \frac{2x}{x^2 + y^2 + 1}$$

- (b) Is the function f increasing, decreasing, or constant in the positive x -direction at the point $P(0, 1)$. Explain briefly.

$$\frac{\partial f}{\partial x}(0, 1) = \frac{2(0)}{0^2 + 1^2 + 1} = 0 \quad \therefore \text{constant, in the positive } x\text{-direction}$$

3. Give the formula for the differential dV for the volume of a box with square base $10 \times 10 \text{ in}^2$ and height $h = 20 \text{ in}$.

$$V = x^2 h$$

$$dV = V_x(10, 20) dx + V_h(10, 20) dh$$

$$dV \approx 400 dx + 100 dh$$

$$V_x = 2xh$$

$$V_x(10, 20) = 2 \cdot 10 \cdot 20 = 400$$

$$V_h = x^2$$

$$V_h(10, 20) = 10^2 = 100$$