SOLUTIONS TO §4.8 # 5, 6, 7 from HW # 12

#5. $q(x) = \begin{cases} x^3 + 3x^2 + 1 & -1 \le x \le 0 \\ -x^3 + kx^2 + 1 & 0 \le x \le 1 \end{cases}$ Find k so that $q(x)$ is a spline.

Solution: There is only 1 knot $x = 0$. Require: $q(x)$ to be continuous and twice differentiable at $x = 0$. For ease, let $q_L(x) = x^3 + 3x^2 + 1$ $q_R(x) = -x^3 + kx^2 + 1$

continuous at $x = 0$: $q_L(0) = 1 = q_R(0)$ ✓

$q_L'(0) = q_R'(x)$: $q_L'(x) = 3x^2 + 6x$ $q_R(x) = -x^3 + kx^2 + 1$

$q_L'(0) = 0$ $q_R'(x) = -3x^2 + 2kx$

$q_R'(0) = 0$

$q_L'(0) = q_R'(0)$ ✓

2nd derivatives: $q_L''(x) = 6x + 6$ $q_R''(x) = -6x + 2k$

At $x=0$, $q_L''(x) = q_R''(x) \Rightarrow 6 = 2k$ or $\boxed{k = 3}$ □

#6. Natural spline. nodes $= 1/2, 5/8, 3/4, 7/8, 1$ $n = 4$ $f(x) = \frac{1}{x}$ so the

function values $= 2, 8/5, 4/3, 8/7, 1$

index: 0 1 2 3 4

Auxiliary equations:

$c_{-1} = 2c_0 - c_1$ $c_4 = y_4/6$

$c_0 = \frac{y_0}{6}$ $c_5 = 2c_4 - c_3$

Matrix Equations for c_1, c_2, c_3

$$\begin{pmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} y_1 - y_0/6 \\ y_2 \\ y_3 - y_4/6 \end{pmatrix}$$

Simplifying:

$c_{-1} = 2c_0 - c_1$ $c_4 = 1/6 = .1\overline{6}$

$\boxed{c_0 = 2/6 = 1/3}$ $\boxed{c_5 = 2c_4 - c_3}$

$$\begin{pmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 8/5 - 2/6 \\ 4/3 \\ 8/7 - 1/6 \end{pmatrix} \approx \begin{pmatrix} 1.2\overline{6} \\ 1.\overline{3} \\ .9762 \end{pmatrix}$$

$c_1 \approx .2615$ $c_2 \approx .2207$ $c_3 \approx .1889$

Solving: $c_{-1} \approx .4052$ $c_5 \approx .1445$

$$c = \begin{bmatrix} .4052 & 1/3 & .2615 & .2207 & .1889 & .1\overline{6} & .1445 \end{bmatrix}$$

c_{-1} c_0 c_1 c_2 c_3 c_4 c_5

□

#7. Redo with complete splines

i:	0	1	2	3	4
x_i	1/2	5/8	3/4	7/8	1
y_i	2	8/5	4/3	8/7	1

$h = 1/8 = .125$

$f(x) = x^{-1} \quad f'(x) = -1x^{-2}$

$f'(1/2) = -4 \quad f'(1) = -1$

Auxiliary Equations:

$$c_{-1} = c_1 - \tfrac{1}{3} h f'(x_0) = c_1 - \tfrac{1}{3}\left(\tfrac{1}{8}\right)(-4) = c_1 + \tfrac{1}{6}$$

$$c_5 = c_3 + \tfrac{h}{3} f'(x_n) = c_3 + \tfrac{1}{3}\left(\tfrac{1}{8}\right)(-1)(-1) = c_3 - \tfrac{1}{24}$$

Matrix equations: 5x5 system

$$\begin{pmatrix} 4 & 2 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} y_0 + \frac{1}{3}\left(\frac{1}{8}\right)(-4) \\ y_1 \\ y_2 \\ y_3 \\ y_n - \frac{1}{3}\left(\frac{1}{8}\right)(-1) \end{pmatrix} = \begin{pmatrix} 2 - \frac{1}{6} \\ 8/5 \\ 4/3 \\ 8/7 \\ 1 + 1/24 \end{pmatrix} \begin{matrix} \rightarrow 1.8\bar{3} \\ \\ \\ \\ \rightarrow 1.0417 \end{matrix}$$

Solve gives in order

$$\left[\; .4299 \quad .3267 \quad .2633 \quad .2202 \quad .1892 \quad .1658 \quad .1475 \;\right]$$

$$\underset{c_{-1}}{''} \qquad\qquad\qquad\qquad\qquad\qquad\qquad \underset{c_5}{''}$$

My plot is perfect, but this answer does not agree with the solutions manual.