MATH 253

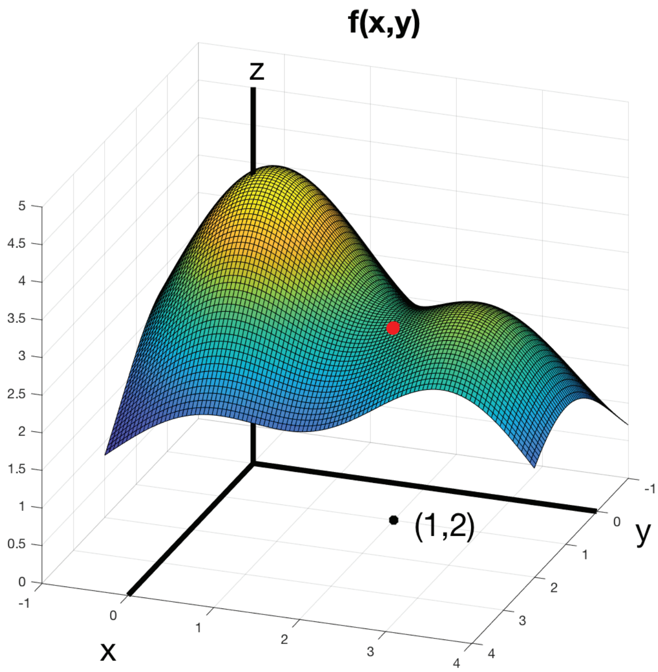
April 2, 2020

Name: ______________________________

Instructions. (100 points) You have 120 minutes to scan, complete, and upload this exam. In other words, you have up to a maximum of two hours for this exam. Closed book, closed notes, no internet, no calculators, and no help allowed. No cheating of any kind. **Show all your work** in order to receive credit. Incomplete answers with little work shown will be graded harshly.

(6^{pts}) **1.** Find the directional derivative $D_{\vec{u}}(1,0)$ of $h(x,y) = x\sin(xy)$ in the direction of $\vec{v} = \langle 3, 3\rangle$.

(8^{pts}) **2.** The graph of $f(x,y)$ is shown in the figure below with the red point denoting $(1,2,f(1,2))$.



(a) (4 pts) Is $\dfrac{\partial f}{\partial x}(1,2)$ negative, zero, or positive? Explain carefully.

(b) (4 pts) Is $\dfrac{\partial f}{\partial y}(1,2)$ negative, zero, or positive? Explain carefully.

(12^{pts}) **3.** Compute the integral

$$I = \int_0^3 \int_{y^2}^9 \frac{1}{x\sqrt{x}+1}\, dx\, dy$$

by drawing the region of integration and then reversing the order of integration.

(12$^{\text{pts}}$) **4.** Consider the function $f(x,y) = x^2y + y^2 - 4xy + 3y$.

(a) (5 pts) Show that the point $(2, 1/2)$ is a critical point for $f(x,y)$.

(b) (7 pts) Use the second derivative test to classify $(2, 1/2)$ as a local minimum, local maximum or saddle point of $f(x,y)$.

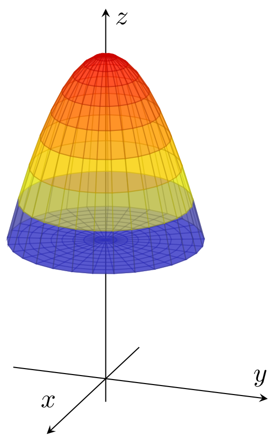
(8$^{\text{pts}}$) **5.** Find an equation of the tangent plane to the surface

$$x^2 \sin z + yz - \ln y - 2x = 4$$

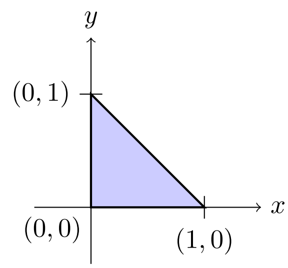
at the point $(-2, 1, 0)$.

(16$^{\text{pts}}$) **6.** Set up, but **DO NOT INTEGRATE**, double integrals for the computations below. A complete answer has limits of integration and the integrand is simplified completely.

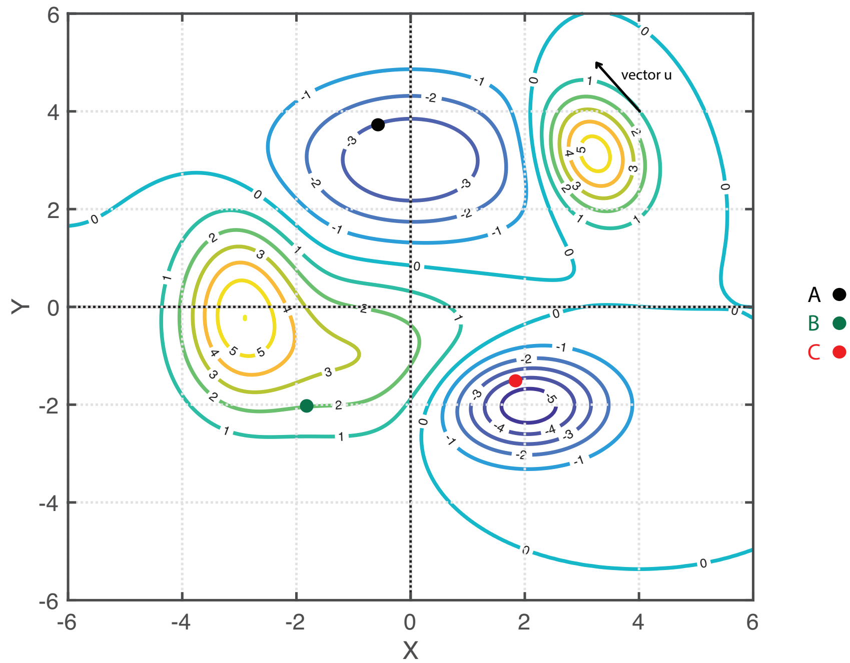
(a) (8 pts) Compute the volume of the solid that lies below the paraboloid $z = 7 - x^2 - y^2$ and above the plane $z = 3$. **Use polar coordinates and DO NOT EVALUATE**.



(b) (8 pts) Compute the surface area of the part of the plane $2x+y+z=4$ that lies above the triangular region in the xy-plane bounded by vertices $(0,0)$, $(1,0)$, and $(0,1)$. **Use rectangular coordinates, and DO NOT EVALUATE**.



(14^pts) **7.** Consider the contour plot of a function $f(x,y)$ below where $f(x,y)$ gives the temperature in degrees Celsius. Points A, B and C are shown in the figure, and a vector $\mathbf{u}$ too.



A is black
B is green
C is red

u is the vector shown

(a) (4 pts) The magnitude of the gradient vector is largest at which of the three points (A, B, or C)? Why?

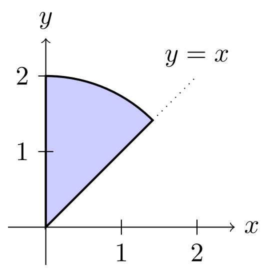
(b) (4 pts) A cold-seeking particle is located at C (red dot). Which direction (roughly) should it move to decrease its temperature the most. Draw an arrow on the contour plot to indicate this, or if you do not have a printer, simply make a cartoon drawing that shows where your arrow would be. Explain your answer briefly.

(c) (3 pts) Consider the point $(3, 3)$.
Is the value $f_{xx}(3, 3)$ negative, positive, or zero? (Circle one.) Why?

(d) (3 pts) What is the value of the directional derivative $D_{\vec{u}}(4, 4)$ where $\vec{u}$ is the vector shown in the figure?

(6^{pts}) **8.** Show that $\displaystyle\lim_{(x,y)\to(2,-1)} \frac{xy+2}{x^2-y-5}$ does not exist.

(8^{pts}) **9.** Compute the total charge on the lamina pictured below, if the charge density is given by $\sigma(x,y) = 3y$ coulombs/ in^2. Include units in your final answer.



(10pts) **10.** Use the method of Lagrange multipliers to find the absolute maximum and absolute minimum of the function $f(x, y) = y^2 - x^2$ subject to the constraint $g(x, y) = 4x^2 + y^2 - 36 = 0$.