MATH 253
Quiz 4

Name: SOLUTIONS

February 5, 2021

**Instructions:** Five points total. Show all work for credit. **GS:** Scan TWO pages for your solutions.

2.5 pts. 1. Find the arc length to the helix $\mathbf{r}(t) = \langle 5\cos(2t), 5\sin(2t), 2t\rangle$ as $t$ varies from $t = 0$ to $t = 2\pi$. **Simplify** your answer for full credit.

Compute $\int_0^{2\pi} |\vec{r}\,'(t)|\, dt$

$\vec{r}\,'(t) = \langle -10\sin(2t), 10\cos(2t), 2\rangle$

$$|\vec{r}\,'(t)| = \sqrt{(-10\sin(2t))^2 + (10\cos(2t))^2 + 4}$$

$$= \sqrt{100\left(\sin^2(2t) + \cos^2(2t)\right) + 4}$$

$$= \sqrt{104}$$

$104 = 4 \cdot 26$

$$= 2\sqrt{26}$$

Therefore,

$$L = \int_0^{2\pi} 2\sqrt{26}\, dt = 4\pi\sqrt{26}$$

*Answer:* Arc length = $\boxed{4\pi\sqrt{26}}$.

2.5 pts.

2. Consider the space curve with vector equation

$$\langle 3+t^2,\ 7+t^3,\ 0\rangle .$$

Give a formula for the curvature function $\kappa(t)$.

We will use:

$$\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$$

$$\vec{r}\,'(t) = \langle 2t, 3t^2, 0\rangle$$

$$\vec{r}\,''(t) = \langle 2, 6t, 0\rangle$$

$$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2t & 3t^2 & 0 \\ 2 & 6t & 0 \end{vmatrix} = \left[(2t)(6t) - 2(3t^2)\right]\hat{k}$$

$$= (12t^2 - 6t^2)\hat{k} = 6t^2\hat{k} = \langle 0, 0, 6t^2\rangle$$

The magnitude $|\vec{r}\,'(t) \times \vec{r}\,''(t)| = 6t^2$.

The magnitude $|\vec{r}\,'(t)| = |\langle 2t, 3t^2, 0\rangle| = \sqrt{(2t)^2 + (3t^2)^2 + 0^2}$

$$= \sqrt{4t^2 + 9t^4}$$

Thus, $\kappa(t) = \dfrac{6t^2}{(4t^2+9t^4)^{3/2}}$

$t \neq 0$

↑

No deduction of points, but please notice.

*Answer:* $\kappa(t) = \dfrac{6t^2}{(4t^2+9t^4)^{3/2}} = \dfrac{6t^2}{|t|\,(4+9t^2)^{3/2}} = \dfrac{6t^2\sqrt{4+9t^2}}{|t|\,(4+9t^2)}$.