MATH 253 Name: SOLUTIONS

Quiz 5 February 28, 2020

Instructions: Five points total.

1. (2 pts.) Suppose that $f(x,y) = \ln(3x+2y)$ where $x(s,t) = s\sin t$ and $y(s,t) = t\cos s$. Use notation correctly for full credit.

   Find $\dfrac{\partial f}{\partial t}$ and $\dfrac{\partial f}{\partial t}\left(\pi, \dfrac{\pi}{2}\right)$.

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

$$= \boxed{\frac{3}{(3x+2y)} \cdot s\cos t + \frac{2}{(3x+2y)} \cdot \cos(s)}$$

At $s = \pi$, $t = \pi/2$, we find

$$x(\pi, \pi/2) = \pi \cdot (\sin(\pi/2)) = \pi$$

$$y(\pi, \pi/2) = \pi/2 \cos(\pi) = -\pi/2$$

$$\frac{\partial f}{\partial t}(\pi, \pi/2) = \frac{3}{3(\pi) + 2(-\pi/2)} \cdot \pi\cos(\pi/2) + \frac{2}{3(\pi)+2(-\pi/2)} \cdot \cos(\pi)$$

$$= 0 + \frac{2}{2\pi}(-1) = \boxed{-1/\pi}$$

2. (3 pts.) Consider the function $f(x,y) = ye^x$.

   (a) Find the directional derivative $D_{\mathbf{u}}f$ at the point $P(2,0)$ in the direction of $\mathbf{v} = \langle -6, 8\rangle$.

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{36+64}}\langle -6, 8\rangle = \left\langle \frac{-6}{10}, \frac{8}{10}\right\rangle = \left\langle \frac{-3}{5}, \frac{4}{5}\right\rangle$$

$\nabla f = \langle f_x, f_y\rangle = \langle ye^x, e^x\rangle$ and $\nabla f(2,0) = \langle 0 \cdot e^2, e^2\rangle = \langle 0, e^2\rangle$

$$D_{\vec{u}} f(2,0) = \nabla f(2,0) \cdot \vec{u} = \langle 0, e^2\rangle \cdot \left\langle \frac{-3}{5}, \frac{4}{5}\right\rangle = \boxed{\frac{4}{5}e^2}$$

increasing

   (b) In what direction should you move from $(2,0)$ to maximize $f(x,y)$?

$$\boxed{\nabla f(2,0) = \langle 0, e^2\rangle}$$