4.42 The distribution function is $F(y)=\dfrac{y-\theta_1}{\theta_2-\theta_1}$, for $\theta_1 \le y \le \theta_2$. For $F(\phi_{.5})=.5$, then $\phi_{.5} = \theta_1 + .5(\theta_2 - \theta_1) = .5(\theta_2 + \theta_1)$. This is also the mean if the distribution.

4.43 Let $A = \pi R^2$, where R has a uniform distribution on the interval (0, 1). Then,

$$E(A) = \pi E(R^2) = \pi\int_0^1 r^2 dr = \frac{\pi}{3}$$

$$V(A) = \pi^2 V(R^2) = \pi^2[E(R^4) - \left(\frac{1}{3}\right)^2] = \pi^2\left[\int_0^1 r^4 dr - \left(\frac{1}{3}\right)^2\right] = \pi^2\left[\frac{1}{5} - \left(\frac{1}{3}\right)^2\right] = \frac{4\pi^2}{45}.$$

4.44 **a.** Y has a uniform distribution (constant density function), so $k = 1/4$.

b. $$F(y) = \begin{cases} 0 & y < -2 \\ \int_{-2}^{y} \frac{1}{4} dy = \frac{y+2}{4} & -2 \le y \le 2 \\ 1 & y > 2 \end{cases}$$

4.45 Let Y = low bid (in thousands of dollars) on the next intrastate shipping contract. Then, Y is uniform on the interval (20, 25).
a. $P(Y < 22) = 2/5 = .4$
b. $P(Y > 24) = 1/5 = .2$.

4.46 Mean of the uniform: $(25 + 20)/2 = 22.5$.

4.47 The density for Y = delivery time is $f(y) = \frac{1}{4}$, $1 \le y \le 5$. Also, $E(Y) = 3$, $V(Y) = 4/3$.
a. $P(Y > 2) = 3/4$.
b. $E(C) = E(c_0 + c_1 Y^2) = c_0 + c_1 E(Y^2) = c_0 + c_1[V(Y) + (E(Y))^2] = c_0 + c_1[4/3 + 9]$

4.48 Let Y = location of the selected point. Then, Y has a uniform distribution on the interval (0, 500).
a. $P(475 \le Y \le 500) = 1/20$
b. $P(0 \le Y \le 25) = 1/20$
c. $P(0 < Y < 250) = 1/2$.

4.49 If Y has a uniform distribution on the interval (0, 1), then $P(Y > 1/4) = 3/4$.

4.50 Let Y = time when the phone call comes in. Then, Y has a uniform distribution on the interval (0, 5). The probability is $P(0 < Y < 1) + P(3 < Y < 4) = .4$.

4.51 Let Y = cycle time. Thus, Y has a uniform distribution on the interval (50, 70). Then,

$P(Y > 65 \mid Y > 55) = P(Y > 65)/P(Y > 55) = .25/(.75) = 1/3$.

4.52 Mean and variance of a uniform distribution: $\mu = 60$, $\sigma^2 = (70–50)^2/12 = 100/3$.

4.53 Let Y = time when the defective circuit board was produced. Then, Y has an approximate uniform distribution on the interval (0, 8).
a. $P(0 < Y < 1) = 1/8$.
b. $P(7 < Y < 8) = 1/8$
c. $P(4 < Y < 5 \mid Y > 4) = P(4 < Y < 5)/P(Y > 4) = (1/8)/(1/2) = 1/4$.

4.54 Let Y = amount of measurement error. Then, Y is uniform on the interval (–.05, .05).
a. $P(-.01 < Y < .01) = .2$
b. $E(Y) = 0$, $V(Y) = (.05 + .05)^2/12 = .00083$.

4.55 Let Y = amount of measurement error. Then, Y is uniform on the interval (–.02, .05).
a. $P(-.01 < Y < .01) = 2/7$
b. $E(Y) = (-.02 + .05)/2 = .015$, $V(Y) = (.05 + .02)^2/12 = .00041$.

4.56 From Example 4.7, the arrival time Y has a uniform distribution on the interval (0, 30). Then, $P(25 < Y < 30 \mid Y > 10) = 1/6/(2/3) = 1/4$.

4.57 The volume of a sphere is given by $(4/3)\pi r^3 = (1/6)\pi d^3$, where r is the radius and d is the diameter. Let D = diameter such that D is uniform distribution on the interval (.01, .05).

Thus, $E(\frac{\pi}{6}D^3) = \frac{\pi}{6}\int_{.01}^{.05} d^3 \frac{1}{4} dd = .0000065\pi$. By similar logic used in Ex. 4.43, it can be found that $V(\frac{\pi}{6}D^3) = .0003525\pi^2$.

4.58 **a.** $P(0 \le Z \le 1.2) = .5 - .1151 = .3849$
b. $P(-.9 \le Z \le 0) = .5 - .1841 - .3159$.
c. $P(.3 \le Z \le 1.56) = .3821 - .0594 = .3227$.
d. $P(-.2 \le Z \le .2) = 1 - 2(.4207) = .1586$.
e. $P(-1.56 \le Z \le -.2) = .4207 - .0594 = .3613$
f. $P(0 \le Z \le 1.2) = .38493$. The desired probability is for a standard normal.

4.59 **a.** $z_0 = 0$.
b. $z_0 = 1.10$
c. $z_0 = 1.645$
d. $z_0 = 2.576$

4.60 The parameter σ must be positive, otherwise the density function could obtain a negative value (a violation).

4.61 Since the density function is symmetric about the parameter μ, $P(Y < \mu) = P(Y > \mu) = .5$. Thus, μ is the median of the distribution, regardless of the value of σ.

4.62 **a.** $P(Z^2 < 1) = P(-1 < Z < 1) = .6826$.
b. $P(Z^2 < 3.84146) = P(-1.96 < Z < 1.96) = .95$.

4.63 **a.** Note that the value 17 is (17 – 16)/1 = 1 standard deviation above the mean.
So, $P(Z > 1) = .1587$.
b. The same answer is obtained.

4.64 **a.** Note that the value 450 is (450 – 400)/20 = 2.5 standard deviations above the mean.
So, $P(Z > 2.5) = .0062$.
b. The probability is .00618.
c. The top scale is for the standard normal and the bottom scale is for a normal distribution with mean 400 and standard deviation 20.

4.65 For the standard normal, $P(Z > z_0) = .1$ if $z_0 = 1.28$. So, $y_0 = 400 + 1.28(20) = \$425.60$.

4.66 Let Y = bearing diameter, so Y is normal with $\mu = 3.0005$ and $\sigma = .0010$. Thus,
Fraction of scrap = $P(Y > 3.002) + P(Y < 2.998) = P(Z > 1.5) + P(Z < -2.5) = .0730$.

4.67 In order to minimize the scrap fraction, we need the maximum amount in the specifications interval. Since the normal distribution is symmetric, the mean diameter should be set to be the midpoint of the interval, or $\mu = 3.000$ in.

4.68 The GPA 3.0 is (3.0 – 2.4)/.8 = .75 standard deviations above the mean. So, $P(Z > .75) = .2266$.

4.69 The z–score for 1.9 is $(1.9 - 2.4)/.8 = -.625$. Thus, $P(Z < -.625) = .2660$.

4.70 From Ex. 4.68, the proportion of students with a GPA greater than 3.0 is .2266. Let X = # in the sample with a GPA greater than 3.0. Thus, X is binomial with $n = 3$ and $p = .2266$. Then, $P(X = 3) = (.2266)^3 = .0116$.

4.71 Let Y = the measured resistance of a randomly selected wire.
a. $P(.12 \le Y \le .14) = P(\frac{.12-.13}{.005} \le Z \le \frac{.14-.13}{.005}) = P(-2 \le Z \le 2) = .9544$.
b. Let X = # of wires that do not meet specifications. Then, X is binomial with $n = 4$ and $p = .9544$. Thus, $P(X = 4) = (.9544)^4 = .8297$.

4.72 Let Y = interest rate forecast, so Y has a normal distribution with $\mu = .07$ and $\sigma = .026$.
a. $P(Y > .11) = P(Z > \frac{.11-.07}{.026}) = P(Z > 1.54) = .0618$.
b. $P(Y < .09) = P(Z > \frac{.09-.07}{.026}) = P(Z > .77) = .7794$.

4.73 Let Y = width of a bolt of fabric, so Y has a normal distribution with $\mu = 950$ mm and $\sigma = 10$ mm.
a. $P(947 \le Y \le 958) = P(\frac{947-950}{10} \le Z \le \frac{958-950}{10}) = P(-.3 \le Z \le .8) = .406$
b. It is necessary that $P(Y \le c) = .8531$. Note that for the standard normal, we find that $P(Z \le z_0) = .8531$ when $z_0 = 1.05$. So, $c = 950 + (1.05)(10) = 960.5$ mm.