

**Instructions:** Five points total.

1. (1 pt.) Describe carefully, but in your own words, why the definition of arc length on a curve  $\mathbf{r}(t)$  from time  $t = a$  to  $t = b$  is given by the formula below:

$$L = s = \int_a^b |\mathbf{r}'(t)| dt.$$

Speed · time = distance

The integrand is the "instantaneous" distance traveled along  $\mathbf{r}(t)$ .

By integrating this from times  $t=a$  until  $t=b$ , you get the distance along  $\mathbf{r}(t)$ , or arc length.

2. (4 pts.) Consider the surface defined by

$$4x^2 - y^2 + 2z^2 + 4 = 0.$$

(Hint: Before answering these questions, you should probably convert this equation to standard form. The space below is for scratch work.)

If you like standard form.

$$4 = -4x^2 + y^2 - 2z^2$$

$$1 = -x^2 + \frac{y^2}{4} - \frac{z^2}{2}$$

$$z=0 : 1 = -x^2 + \frac{y^2}{4}$$

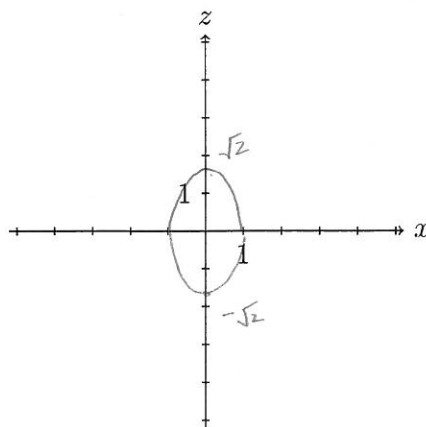
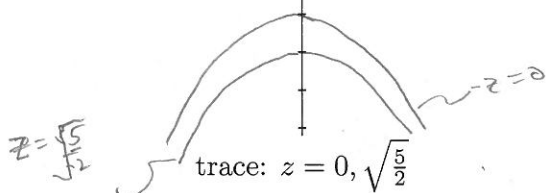
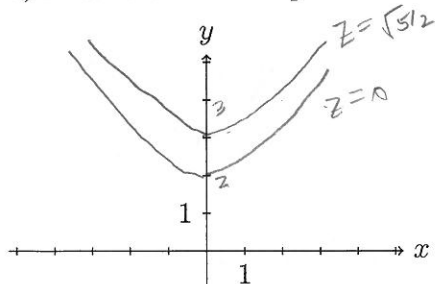
$$z = \sqrt{5/2} : 1 = -x^2 + \frac{y^2}{4} - \frac{(\sqrt{5/2})^2}{2}$$

$$1 = -x^2 + \frac{y^2}{4} - \frac{5}{4}$$

$$z = \sqrt{5/2} \downarrow$$

$$\frac{9}{4} = -x^2 + \frac{y^2}{4}$$

- a) Draw the traces requested on the axes below.



trace:  $y = \sqrt{8}$

$y = \sqrt{8} :$

$$1 = -x^2 + \frac{8}{4} - \frac{z^2}{2}$$

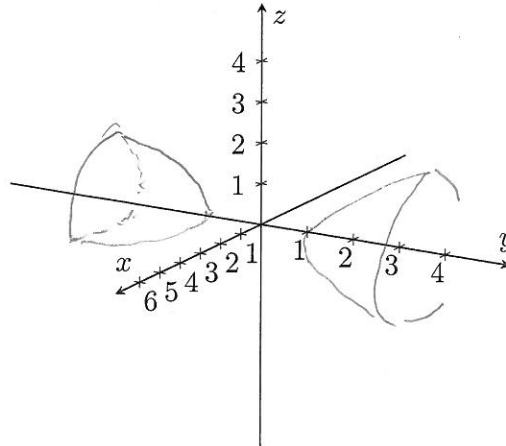
$$x^2 + \frac{z^2}{2} = 1$$

Part b) is on the next page.

$$4x^2 - y^2 + 2z^2 + 4 = 0.$$

b) Sketch the surface on the axes below. (Give it a name if you can.)

Hyperboloid of 2  
Sheets



(space for scratch work)