MATH 202 Name : SOLUTIONS

Quiz 8 November 6, 2009

Instructions: You get one point for taking this quiz. Note: this quiz had two pages.

1. (2 pts.) Let $f(x, y) = 4x + 6y$. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y)$ subject to the constraint $x^2 + y^2 = 13$.

Solve

$\nabla f = \lambda \nabla g$ $x^2+y^2=13$

(1.) $4 = \lambda 2x$ (2) $6 = \lambda 2y$ (3.) $x^2+y^2=13$

$2 = \lambda x$ $3 = \lambda y$ $x^2+y^2=13$

Note: $\lambda \neq 0$, since $4 \neq \lambda 2x = 0$.

Dividing by λ implies

$x = \frac{2}{\lambda}$ $y = \frac{3}{\lambda}$

Plugging into (3):

$\left(\frac{2}{\lambda}\right)^2 + \left(\frac{3}{\lambda}\right)^2 = 13$ or $13 = 13\lambda^2$

$1 = \lambda^2$

$\lambda = \pm 1$

If $\lambda = 1$:

$x = 2$, $y = 3$

critical point $(2,3)$

If $\lambda = -1$,

$x = -2$, $y = -3$

Critical point $(-2,-3)$

Evaluate $f(x,y)$ at critical points:

$f(2,3) = 4(2) + 6(3) = 26$

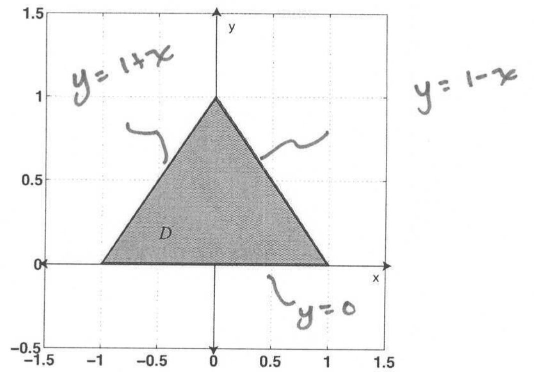
$f(-2,-3) = 4(-2) + 6(-3) = -26$

$$\boxed{\begin{array}{l}\text{Max value} = 26 \\ \text{Min Value} = -26\end{array}}$$

at $(2,3)$

at $(-2,-3)$

2. (2 pts.) Consider the region D of integration shown in the figure below.



(a) (.5 pts.) Give equations for the three boundary curves defining D. (Labeling the figure above is probably easiest.)

(b) (1.5 pts.) Find the value of the double integral $\iint_D x + 2y\, dA$.

Note: "dy dx" order is easier!

$$\iint_D x+2y \, dA = \int_0^1 \int_{y-1}^{1-y} x+2y \, dx\, dy$$

If $y = 1+x$, $x = y-1$

If $y = 1-x$, $x = 1-y$

$$= \int_0^1 \frac{1}{2}x^2 + 2xy \Big|_{y-1}^{1-y} dy$$

$$= \int_0^1 \left[\left(\frac{1}{2}(1-y)^2 + 2(1-y)y\right] - \left[\frac{1}{2}(y-1)^2 + 2(y-1)y\right] dy$$

$$= \int_0^1 \left[\frac{1}{2} - y + \frac{1}{2}y^2 + 2y - 2y^2\right] - \left[\frac{1}{2}y^2 - y + \frac{1}{2} + 2y^2 - 2y\right] dy$$

all cancel

$$= \int_0^1 2y - 2y^2 - 2y^2 + 2y \, dy$$

$$= \int_0^1 4y - 4y^2 \, dy$$

$$= 2y^2 - \frac{4}{3}y^3 \Big|_0^1$$

$$= \left(2 - \frac{4}{3}\right) - (0) = \boxed{\frac{2}{3}}$$