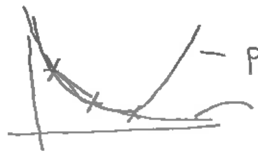
SOLUTIONS HW #8

3.10 2a, b, 5   4.1 #1-3, 6

§3.10 2a. Roughly $.6235 \pm .7818i$, $-.9010 \pm .4339i$, $-.2235 \pm .9749i$

b. Roughly 1.2852, $-.8813$, $.6714 \pm .7849i$, $-.3733 \pm .8296i$

§4.1 1.   2. $p_2(x) = 3x^2 - 2x$ 3.  $p_2(x) = 8/3\,x^2 - 6x + 13/3$, $\frac{1}{x}$ 6. $p_3(x) = x^3$ of course

§3.10 #5 3pts.   Code: 3 pts   other problems worth one point

total 12 pts.

↑

I was looking for careful consideration of this problem.

HW #9   Solutions §4.1 #7, 8 §4.2 2a, b, c, 8, 11 §4.3 1, 2, 3, 8   Comment code

§4.1 #7. A unique quadratic polynomial passes through 3 points; $p_2(x) = f(x) = x^2 + 2x$

For the cubic $f(x) = p_3(x)$

#8. Proved in class.

§4.2 2a. $f(x) = \sqrt{x}$ $x_i = [0, 1, 4]$ $c_0 = 0$, $c_1 = 1$, $c_2 = -1/6$

b. $f(x) = \ln x$ $x_i = [1, 3/2, 2]$ $c_0 = 0$, $c_1 \approx .8109$, $c_2 \approx -.2356$

c. $f(x) = \sin \pi x$ $x_i = [0, .25, .5, .75, 1]$ $c_0 = 0$, $c_1 =$

K	x_K	f_0	f_1	f_2	f_3	
0	0	0				
			$2\sqrt{2}$			
1	1/4	$\sqrt{2}/2$		$18 - 8\sqrt{2}$	$32 - \frac{64}{3}\sqrt{2}$	$64 - \frac{128}{3}\sqrt{2}$
			$2(2-\sqrt{2})$	$-16 + 8\sqrt{2}$	$-32 + \frac{64}{3}\sqrt{2}$	
2	1/2	1	$-4 + 2\sqrt{2}$			
				$8 - 8\sqrt{2}$		
3	3/4	$\sqrt{2}/2$				
			$-2\sqrt{2}$			
4	1	0				

decimal approx: 0, 2.8284, −3.3137, −1.8301, 3.6602

§4.2 #8. $x_i = [300, 500, 700]$ $\quad p_2(x) = -.000000875(x-300)(x-500) + .000125x(x-300)$

$+ .084$

$p_2(400)$, $p_2(600)$ are good approximations

11. a. $t = [220, 260, 300]$ $\quad p(t) = y = [17.188 \quad 35.42 \quad 66.98]$

Only the coefficients are given $\quad c_0 = 17.188 \quad c_1 = .4568 \quad c_2 = .0042$

For $t = [220, 240, 260, 280, 300]$

$c_0 = 17.1880 \quad c_1 = .3891 \quad c_2 = .0033 \quad \underbrace{c_3 = .000013375 \quad c_4 = .000000229167}_{\text{So small!}}$

Both are good approximations

§4.3 1. $f(x) = \sqrt{x}$ $\quad n=2$ $\quad x = [.25 \ .625 \ 1]$ $\quad I = [1/4, 1]$ $\quad 1/4 \quad 5/8 \quad 1$

$$|f(x) - p_2(x)| = |\sqrt{x} - p_2(x)| \le \frac{1}{9\sqrt{3}} h^3 \max_{\xi \in I} |f^{3}(\xi)| = \frac{1}{9\sqrt{3}} \left(\frac{3}{8}\right)^3 12 \approx \boxed{.4059}$$

$$f(x) = x^{+1/2} \quad f'(x) = +1/2\,(x)^{-1/2} \quad f^{(2)}(x) = \frac{1}{4}x^{-3/2} \quad f^{(3)}(x) = -\frac{3}{8}x^{-5/2}$$

$$\|f^{(3)}\|_{\infty, I} = +\frac{3}{8}\left(\frac{1}{4}\right)^{-5/2} = \frac{3}{8}(2)^5 = \frac{3}{8}.32 = 12$$

2. $f(x) = \frac{1}{x}$ $\quad I = [\frac{1}{2}, 1]$ $\quad x = [.5 \ .75 \ 1]$

$$|f(x) - p_2(x)| \le \frac{1}{9\sqrt{3}} h^3 \|f^3\|_{\infty, I} = \frac{1}{9\sqrt{3}}(.25)^3 \cdot 96 \approx \boxed{.096225}$$

$$f(x) = x^{-1} \quad f'(x) = -x^{-2} \quad f^{(2)}(x) = 2x^{-3} \quad f^{(3)}(x) = -6x^{-4} \Rightarrow \|f^{(3)}\|_{\infty, I}$$

$$= \|-6x^{-4}\|_{\infty, [\frac{1}{2}, 1]} = 6 \cdot 2^4 = 96$$

3 a. $|\sqrt{x} - p_3(x)| \le \frac{1}{24} h^4 \|f^{(4)}\|_{\infty, [1/4, 1]}$ $\quad h = .25$ $\quad f^{(4)}(x) = \frac{15}{16}x^{-7/2}$ $\quad \|f^{(4)}\|_{\infty, I} = \frac{15}{16}\left(\frac{1}{4}\right)^{-7/2}$

$= \frac{15}{16} 2^7 = 120$

$$\le \frac{1}{24}(.25)^4 120 \approx \boxed{.0195}$$

b. $|\frac{1}{x} - p_3(x)| \le \frac{1}{24} h^4 \|f^{(4)}\|_{\infty, [.5, 1]}$ $\quad h = \frac{1}{6}$ $\quad f^{(4)}(x) = 24x^{-5}$ $\quad \|f^{(4)}\|_{\infty, [\frac{1}{2}, 1]} = 24 \cdot 2^5$

$$\le \frac{1}{24}\left(\frac{1}{6}\right)^4 24.32 = \frac{32}{6^4} = \frac{2}{3^4} \approx \boxed{.0247}$$

§ 4.3

#8 $f(x) = e^x$ $\quad I = [-1,1]$ $\quad$ nodes $x_0, \dots, x_n$ $\Rightarrow$ $n+1$ nodes

Linear Interpolation error

$$|f(x) - p_1(x)| \le \frac{1}{8}(x_{i+1} - x_i)^2 \, \|f^{(2)}\|_{\infty, [x_i, x_{i+1}]} \qquad f^{(2)}(x) = e^x \qquad \|f^{(2)}\|_I = e$$

$$\le \frac{1}{8} h^2 e = \frac{1}{8}\left(\frac{2}{n}\right)^2 e \qquad h = \frac{2}{n}$$

Thus, require $\frac{1}{8}\left(\frac{2}{n}\right)^2 e < 10^{-6}$ $\Rightarrow$ $n^2 > \frac{1}{2} e \cdot 10^6$ or $n > 10^3 \sqrt{\frac{e}{2}} \approx 1165.8$

$\Rightarrow$ $n \ge 1166$ or 1167 nodes

Quadratic Interpolation

$$|f(x) - p_2(x)| \le \frac{1}{9\sqrt{3}} h^3 \, \|f^{(3)}\|_{\infty, [-1,1]} \qquad \|f^{(3)}\|_{\infty, [-1,1]} = e \text{ too.}$$

$$= \frac{1}{9\sqrt{3}} h^3 e \qquad h = \frac{2}{n}$$

$$= \frac{1}{9\sqrt{3}}\left(\frac{2}{n}\right)^3 e$$

Require $\frac{e}{9\sqrt{3}}\left(\frac{2}{n}\right)^3 < 10^{-6}$ $\Rightarrow$ $n^3 > \frac{8e}{9\sqrt{3}} \cdot 10^6$ or $n > 2\sqrt[3]{\frac{e}{9\sqrt{3}}} \cdot 10^2 = 200\sqrt[3]{\frac{e}{9\sqrt{3}}}$

≈ 111.7

$n \ge 112$ $\quad$ 113 points technically