**Instructions:** Show all work for full credit. Poor notation or sloppy work will be penalized. Point values as indicated.

1. (10 pts. – 5 pts. each) Simplify. Your answer should have no negative exponents. You may assume all variables are positive.

    (a) $\dfrac{\sqrt[3]{16x^{17}y^2}}{\sqrt[3]{2x^2y^{-4}}}$

    (b) $\left(\dfrac{8x^{-4}y^2}{27x^2y^{-1}}\right)^{\frac{-2}{3}}$

2. (12 pts. – 6 pts. each) Multiply using a special product formula.

    (a) $(\sqrt{y} - 2)(\sqrt{y} + 2)$

    (b) $(x - 2y)^3$

3. (10 pts. − 5 pts. each) Factor completely.

(a) $(x^2 - 3)^2 - 4(x^2 - 3) - 5$

(b) $3x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 6x^{\frac{-1}{2}}$

4. (7 pts.) Rationalize the denominator. Then cancel any common terms.

$$\frac{h}{\sqrt{x+h} - \sqrt{x}}$$

5. (10 pts.)

(a) (3 pts.) Give the formula for simple interest. Define all quantities in your answer.

(b) (7 pts.) Cal invests $10,000$ in a Certificate of Deposit (CD) at an interest rate of $4\%$. After $t$ years, $1200$ in simple interest has been earned. Find the number $t$ of years that this money was invested.

6. (16 pts. – 4 pts. each) Find all real solutions. If there are no real solutions, write "No real solutions." Check your answers, if necessary.

(a) $2a^2 - a = 7$

(d) $x^2 + 4 = 0$

(b) $\dfrac{3}{x} + \dfrac{5}{x+2} = 2$

(e) $\sqrt{x+5} = x - 1$

7. (15 pts. – 5 pts. each) Solve each inequality. Give your answer either in interval notation or using inequality notation.

(a) $3x + 2 \leq 7x - 1$

(b) $\left| \dfrac{3x - 1}{2} \right| > 7$

(c) Use a number line to solve this inequality. Include test points as part of your answer. Place your answer in interval notation on the line provided.

$$\dfrac{2x - 3}{x + 1} \leq 1$$

Answer: _____

8. (10 pts. – 5 pts. each) Consider the two points $P(1, -2)$ and $Q(5, 1)$.

    (a) Find the distance $d(P, Q)$ between the two points.

    (b) Find the equation of the circle with center $P$ and containing the point $Q$.

9. (10 pts.) Amy buys an old car. The value $V$ in dollars of this car after $t$ years is given by the linear function

$$V = -100t + 1000 \text{ dollars}$$

for \$1000 which depreciates at a rate of \$100 per year.

    (a) (6 pts.) Find the $t$ intercept of this line.
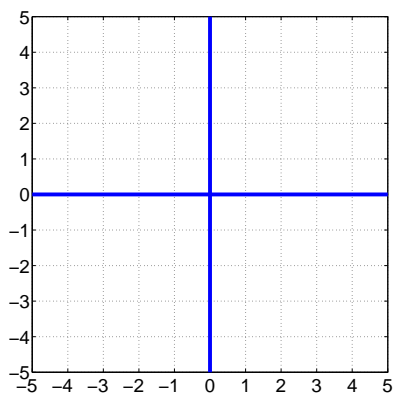
    (b) (4 pts.) Explain the meaning of the $t$ intercept.

**Instructions:** Show all work for full credit. Poor notation or sloppy work will be penalized. Point values as indicated.
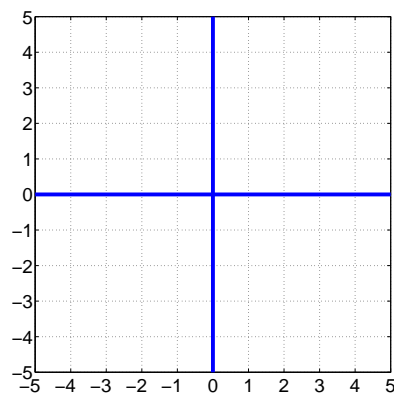
1. (9 pts. total – 3 pts. each)

   (a) Give the definition of a function.

   (b) Sketch a graph of something which is NOT a function.

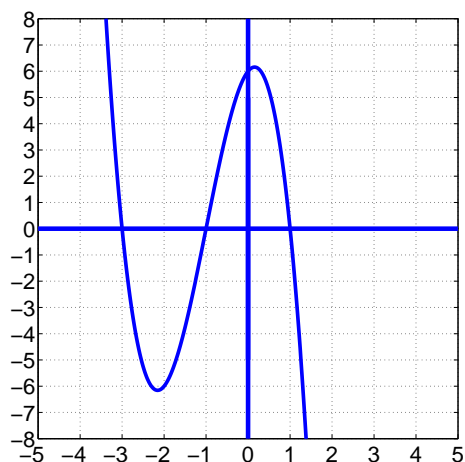   (c) Now sketch the graph of something which is a function.





2. (10 pts.) Divide $P(x) = 2x^3 - 7x^2 + 5$ by $D(x) = x - 3$. Give both the quotient $Q(x)$ and remainder $R(x)$.

The quotient $Q(x)$ is _____ .        The remainder $R(x)$ is _____ .

3. (20 pts. – No partial credit.)

   Consider the graph of the following polynomial $P(x)$.



(a) (2 pts.) What is the degree of the polynomial? _____

(b) (2 pts.) Is the leading coefficient positive or negative? _____

(c) (2 pts.) Fill in the blanks.

   As $x \to \infty$,  $P(x) \to$ _____.          As $x \to -\infty$,  $P(x) \to$ _____.

(d) (2 pts.) Is $P(x)$ 1-1? Justify your answer.

(e) (2 pts.) Give the *definition* that a function $f(x)$ is 1-1. (You must be precise for credit.)

(f) (3 pts.) What are the zeros of $P(x)$? _____

(g) (2 pts.) What is the $y$-intercept of $P(x)$? _____

(h) (5 pts.) Using your answers to the previous questions, give an equation for $P(x)$. (You may leave your answer in factored form.)

4. (21 pts. – No partial credit. (a) - (c) are 2 pts. each. (d) - (h) are 3 pts. each.)

   Consider the following graph with functions $f(x)$, $g(x)$ as labeled.
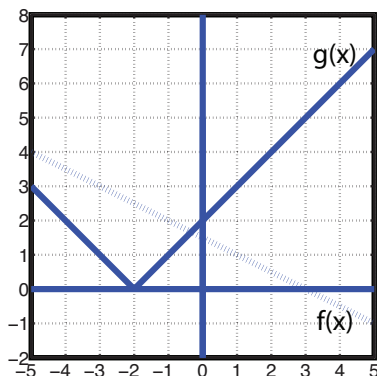


   Compute, if possible, the following quantities. If there is not enough information to compute the quantity, write "IMPOSSIBLE" or "UNDEFINED" as appropriate.

   (a) $(g \circ g)(1) =$

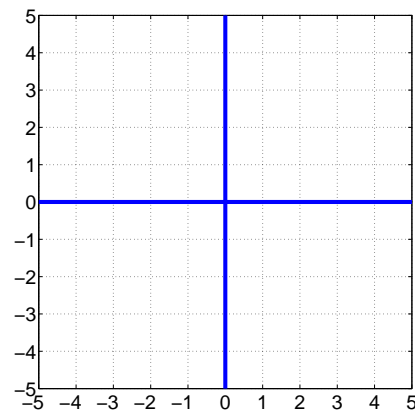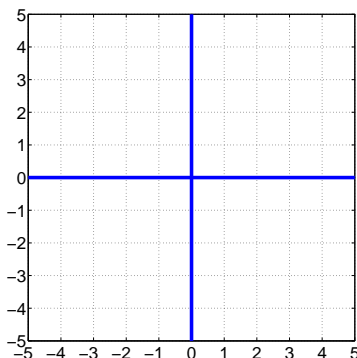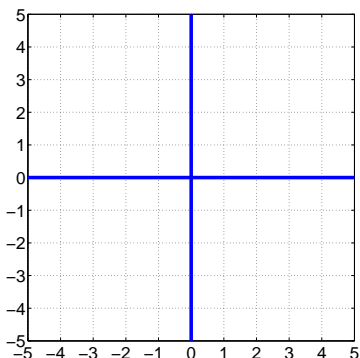   (b) $\left(\dfrac{f}{g}\right)(-2) =$

   (c) $g(f(3)) =$

   (d) $\left(\dfrac{g}{f}\right)(-1) =$

   (e) $f^{-1}(2) =$

   (f) $g^{-1}(1) =$

   (g) $(f - g)(1) =$

   (h) The average rate of change of $g(x)$ from $x = -3$ to $x = -1$.

5. (10 pts.) Starting with the function $y = \sqrt{x}$, sketch a graph of $y = \sqrt{-x+4} - 2$ on the axes below. You have been given three sets of axes here to use for a sequence of transformations. Please place your final answer on the axes on right. A complete answer has both $x$- and $y$-intercepts labeled.







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Final Answer:   $y = \sqrt{-x+4} - 2$

6. (20 pts.) If a ball is thrown directly upward with an initial velocity of 40 ft/s, its height in feet after $t$ seconds is given by

$$h(t) = 40t - 16t^2 \text{ feet}$$

   (a) (5 pts.) Compute $h(2)$ and explain its meaning. Include units in your answer.

   (b) (10 pts.) What is the maximum height attained by the ball? and at what time $t$ does the ball reach this height?

   (c) (5 pts.) At what time does the ball hit the ground?

7. (10 pts.) It is possible to check that $g(x) = \dfrac{4x - 1}{x + 3}$ is a 1-1 function. (You can trust me on this.)

   (a) (7 pts.) Compute the inverse function $g^{-1}(x)$.

   (b) (3 pts.) Without performing any calculations at all, give $g\big(g^{-1}(\sqrt{\pi + 2})\big)$. Explain briefly how you got your answer *without computation*.

Name :_____

**Instructions:** Show all work for full credit. Poor notation or sloppy work will be penalized. Point values as indicated.

1. (5 pts.) Find **ONE** rational root of $f(x) = x^3 + x^2 - 4x - 4$. You must show your work for credit, including showing that your answer is indeed a root of $f(x)$.

Answer: $x =$ _____

2. (10 pts.) Consider the graph of a function $f(x)$ below. Fill in the blanks based on the graph.
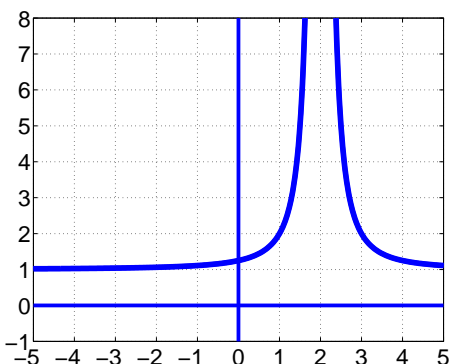


(a) Give the equation of the vertical asymptote.

Ans: _____

(b) As $x \to 2^-$, $f(x) \to$ _____

(c) Give the equation of the horizontal asymptote.

Ans: _____

3. (5 pts.) The function $h(x) = \dfrac{-2x^2 + 1}{3x^2 - 1}$ has a horizontal asymptote. What is the equation of this asymptote? Give your answer in $y = mx + b$ form.

Answer: The equation is _____

4. (24 pts. No partial credit.) Simplify.

(a) $\log(.01)$

(g) $\ln(\frac{1}{e})$

(b) $\log(2) + \log(50)$

(h) $\ln(13e^2) - \ln(13)$

(c) $3^{\log_3(6)}$

(i) $3\log 2 + \log 50 - 2\log 2$

(d) $e^{\ln(\log 10)}$

(j) $\log_3 9^{100}$

(e) $\log(100^x)$

(k) $\log(\log(10^{1000}))$

(f) $\ln(1)$

(l) $\log_2(\sqrt{8})$

5. (16 pts. – 4 pts. each) Solve the following equations. Check your answers in (c) and (d).

(a) $5 + 2\log(4x) = 11$

(c) $\log_2 x = 2 - \log_2(x+3)$

(b) $e^{\ln(x+1)} = 5$

(d) $e^{2x} - 2e^x + 1 = 0$

6. (10 pts.) Solve the system of linear equations for $x$ and $y$. (The solution is unique.)

$$3x + 2y = 14$$
$$x - y = 3$$

7. (9 pts.) A student invests $4000 in an account, and wants it to grow to $5000 in ten years. What rate of return $r$ must the student realize, if interest is compounded continuously? Round your answer to one decimal place. (An acceptable answer looks like 5.1%.)

8. (9 pts.) How long will it take for an investment of $10,000 to reach a value of $15,000, if the interest rate is 2.5% year compounded quarterly? Round your answer to one decimal place.

9. (12 pts.) Solve the following equations. Round your answers to two decimal places.

(a) $10^{x+1} = 2^{3x-1}$

(b) $\dfrac{10^{x+1}}{10^{2x-3}} = 5$