MATH 371
Midterm 1

Name : SOLUTIONS

October 3, 2017

Instructions: Give numerical answers unless instructed that a formula suffices. Good luck.

1. (15 pts. – 5 pts. each) Give numerical answers to the following.

   (a) As a promotion, Amazon will give away 200 CD's of your choosing for first prize, a \$500 gift certificate for second prize, and free shipping on your next order for third prize. If the winners are drawn at random from 100 entrants, how many ways can the prizes be awarded?

   $100(99)(98) = 970{,}200$

   (b) Suppose on five consecutive days, an "instant winner" lottery ticket is purchased and the probability of winning on any given day is .1. What is the probability of purchasing exactly two winning tickets?

   Binom $(5, .1)$ $\quad p(2) = \binom{5}{2}(.1)^2(.9)^3 = .0729$

   (c) Customers arrive at a Fred Meyer checkout counter at a mean rate of 11 per hour. Give the probability that 10 or more customers arrive in a given hour.

   Poisson $\lambda = 11$

   $P(Y \geq 10) = 1 - P(Y \leq 9) = 1 - .341 = .659$ $\quad$ Table p. 846

2. (12 pts.) In a small pond with 50 fish, ten are tagged for a study. Suppose a state biologist catches seven fish at random from this pond. (No fish is caught twice.)

   (a) (5 pts.) Give a formula for the probability that exactly two of the seven fish caught are tagged.

   $$\frac{\binom{10}{2}\binom{40}{5}}{\binom{50}{7}}$$

   (b) (2 pts.) What type of probability distribution did you use to answer (i). Explain briefly.

   Hypergeometric $\quad p = P(\text{tagged})$ is not constant. The sample size n is somewhat large wrt $N = 50$.

   (c) (5 pts.) What is the expected value for the number of tagged fish out of the seven caught?

   $n\left(\frac{r}{N}\right) = 7\left(\frac{10}{50}\right) = \frac{7}{5} = 1.4$

3. (20 pts.) Consider an experiment in which two numbers are selected *consecutively* and at random from the set $\{1, 2, 3\}$ *with replacement.* (This means that outcomes like 1, 1 are possible as well as 3, 1.)

(a) (2 pts.) Give the sample set S for this experiment and briefly explain why S is a *discrete* sample space.

$$S = \left\{ \begin{matrix} 1,1 & 2,1 & 3,1 \\ 1,2 & 2,2 & 3,2 \\ 1,3 & 2,3 & 3,3 \end{matrix} \right\}$$

S is finite $\Rightarrow$ discrete

(b) (5 pts.) Now let X denote the sum of the two numbers selected.

Give the probability distribution $p(x)$ of X. Be sure your answer includes the domain of $p(x)$.

The sum can only equal $x = 2, 3, 4, 5,$ or 6

x	2	3	4	5	6
$p(x)$	$\frac{1}{9}$	$2/9$	$3/9 = 1/3$	$2/9$	$1/9$

(c) (5 pts.) Suppose you play the following game: If the sum of the two numbers is 4 or less, you win the dollar value of the sum. However, if the sum is 5 or more, you lose the dollar value of the sum. Consider the random variable

Y: total earnings

in playing this game.

Calculate the expected value $E(Y)$. Should you play this game? Explain briefly.

$$\mathbb{E}(Y) = 2\left(\tfrac{1}{9}\right) + 3\left(\tfrac{2}{9}\right) + 4\left(\tfrac{1}{3}\right) - 5\left(\tfrac{2}{9}\right) - 6\left(\tfrac{1}{9}\right) = \frac{2+6+12-10-6}{9}$$

$$= \frac{4}{9} \approx +.44$$

i.e. you win 44¢ per game on average

Yes. You should play.

(d) (4 pts.) Now compute the $E(Y^2)$, the second moment about the origin.

$$\mathbb{E}(Y^2) = (2)^2\tfrac{1}{9} + (3)^2\left(\tfrac{2}{9}\right) + (4)^2\left(\tfrac{1}{3}\right) + (-5)^2\tfrac{2}{9} + (-6)^2\left(\tfrac{1}{9}\right)$$

$$= \tfrac{1}{9}\left[4 + 18 + 48 + 50 + 36\right] = \frac{156}{9} = 17.\bar{3}$$

(e) (4 pts.) Use your answer to the last problem to give the variance of your earnings $V(Y)$.

$$V(Y) = \mathbb{E}(Y^2) - \left[\mathbb{E}(Y)\right]^2$$

$$= \frac{156}{9} - \left(+\frac{4}{9}\right)^2$$

$$= \frac{156(9) - 16}{81}$$

$$= \frac{1388}{81} \approx 17.14$$

4. (10 pts. – 5 pts. each) A survey is conducted to determine the attitudes of males and females towards President Trump's handling of US relations with North Korea. Males make up 80% of the respondents. Suppose that if a respondent is male, he approves of the Trump's handing of this delicate situation with probability .75 and if the respondent is female, she approves of Trump's handling of US relations with North Korea with probability .4. Give formulas for the probabilities you calculate below and numerical answers.

(a) What is the probability a respondent (either male or female) approves of President Trump's handing of the crisis with North Korea?

$P(M) = .8 \quad P(F) = .2 \quad P(A|M) = .75 \quad P(A|F) = .4$ A: Approves

$$P(A) = P(A|M)P(M) + P(A|F)P(F)$$
$$= .75(.8) + .4(.2)$$
$$= .68$$

(b) What is the probability that a respondent is female, given that she approves of President Trump's handling of US relations with North Korea? Round to three digits.

$$P(F|A) = \frac{P(A|F)P(F)}{P(A|F)P(F) + P(A|M)P(M)} = \frac{.4(.2)}{.68} \approx .118$$

5. (12 pts. – 4pts. each) Consider a discrete random variable X with moment generating function $m(t) = (.4e^t + .6)^{25}$.

(a) What is the probability distribution of X? Give any parameters necessary for answering this questions.

$X \sim \text{Binom}(25, .4) \quad n = 25 \quad p = .4$

(b) Use the moment generating function to find the expected value $E(X)$. You can check your answer easily, but credit is only awarded for using the moment generating function.

$$m(t) = (.4e^t + .6)^{25}$$
$$m'(t) = 25(.4e^t + .6)^{24}(.4e^t)$$
$$\mathbb{E}(X) = m'(0) = 25(.4e^0 + .6)^{24}(.4e^0)$$
$$= 25(1)(.4)$$
$$= 10$$

(c) Give the variance of the random variable $W = -2X + 10$. (Do not use the mgf here.)

$$V(W) = V(-2X + 10) = (-2)^2 V(X) = 4npq = 4(25)(.4)(.6) = 24$$

6. (15 pts.) A large number of adults, some with children under the age of 26 and some without, were surveyed and asked if they thought that the price of health insurance was: A: Too High, B: About Right, C: Too Low. The results are tallied below.

		Cost of Health Insurance		
Offspring Information		Too High (A)	About Right (B)	Too Low (C)
Child over 26	(D)	.20	.09	.01
No child over 26	(E)	.41	.21	.08

(a) (~~3~~ 5 pts.) Give the probabilities of the following events.

$P(A) = P(\text{insurance cost is TOO HIGH}) =$ .61.

$P(B) = P(\text{insurance cost is ABOUT RIGHT}) =$ .3.

$P(D) = P(\text{Adult in survey had a CHILD OVER 26}) =$ .3.

(b) (5 pts.) Are the events A and D independent? Prove your answer.

$P(A) = .61 \neq P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{.2}{.3} = .\bar{6}$

Dependent

(c) (5 pts.) Are the events B and D independent? Prove your answer.

$P(B) = .3 = P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{.09}{.3} = .3$

Therefore, independent

7. (16 pts) In a recent telephone poll of a large number of individuals, 20% of the population expressed dissatisfaction with the legalization of marijuana. Let Y denote the number of calls until the first person is found who is disapproves of legalizing marijuana. Give your answers to four decimal places.

(a) (5 pts.) Compute the probability that $Y \geq 3$.

$Y \sim$ Geometric $p(S) = P(\text{disapproves}) = .2$

$P(Y \geq 3) = 1 - (P(Y=1) + P(Y=2)) = 1 - (.2) - (.8)(.2) = 1 - .2 - .16 = .64$

(b) (5 pts.) Compute the probability that $Y < 21$.

$P(Y < 21) = 1 - P(Y \geq 21) = 1 - \sum_{y=21}^{\infty} (.8)^{y-1}(.2) = 1 - (.2)\sum_{y=21}^{\infty}(.8)^{y-1}$

$= 1 - (.2)\frac{(.8)^{20}}{(1-.8)} = 1 - (.8)^{20} = 1 - .0115 = .9885$

(c) Suppose you wanted to survey individuals until you found 30 people who disapprove of legalizing marijuana.

i. (4 pts.) How many individuals should you expect to survey?

$r = 30 \quad p = .2 \qquad \mathbb{E}(Y) = \frac{r}{p} = \frac{30}{.2} = 150$

ii. (2 pts.) What type of discrete random variable did you use to answer (i)? (No explanation needed.)

Negative Binomial