MATH 253
February 13, 2020

Name: ______________________________

**Instructions.** (100 points) You have 60 minutes. Closed book, closed notes, and no calculators allowed. *Show all your work* in order to receive credit. Bald answers will receive little, if any, credit.

($15^{\text{pts}}$) **1.** Consider the three points $P(1,3,-1)$, $Q(3,2,4)$, $R(2,3,-3)$ in $\mathbb{R}^3$.

(a) (5 pts) Find the equation of the plane $S$ containing the three points.

(b) (5 pts) By finding an appropriate point $B$ on the line $\vec{\ell}(t) = \langle 1-3t,\ 2t,\ 2t+1\rangle$, $t \in \mathbb{R}$, show that the line $\vec{\ell}(t)$ does **not** lie in the plane $S$.

(c) (5 pts) Using your answer to part (b), find the distance between your point $B$ and the plane $S$.

(12 pts) **2.** Consider a particle moving in the plane with velocity:

$$\mathbf{v}(t) = \cos t\,\mathbf{i} + te^t\mathbf{j}.$$

(a) (8 pts) Find the position function at all times if $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$.

(b) (4 pts) Find the acceleration function at all times.

(10 pts) **3.** A wrench 15 cm long lies along the positive $y$-axis and grips a bolt at the origin (0,0,0). A force $\mathbf{F}$ of 10 N is applied in the direction of $\langle 0, 1, -\sqrt{3}\rangle$.

(a) (4 pts) Give the coordinates of the force vector $\mathbf{F}$.

(b) (4 pts) What is the magnitude of the torque $\boldsymbol{\tau}$? Give units with your answer.

(c) (2 pts) In what direction does the torque vector $\boldsymbol{\tau}$ point? Place a ✓ by the correct answer. You need not justify your answer.

| | | |
|---|---|---|
| (A) $\mathbf{i}$ | (B) $\mathbf{j}$ | (C) $\mathbf{k}$ |
| (D) $-\mathbf{i}$ | (E) $-\mathbf{j}$ | (F) $-\mathbf{k}$ |

(22pts) **4.** Consider the equations of two planes:

$$\begin{array}{lrcrcl} \text{Plane 1:} & 6x & & + \; 6z & = & 5 \\ \text{Plane 2:} & x + \; y & & & = & 9 \end{array}$$

(a) (3 pts) Show that the two planes are not parallel.

(b) (5 pts) Find the angle $\theta$ between the two planes.

(c) (7 pts) Give the **vector** equation of the line $\vec{\ell}(t)$ passing through $P(4, -2, 1)$ and parallel to the line of intersection of Plane 1 and Plane 2.

(d) (7 pts) Give the vector projection $\text{proj}_{\overrightarrow{n_2}} \vec{v}$ of $\vec{v} = \langle -2, 5, 1 \rangle$ onto the normal vector $\overrightarrow{n_2}$ of Plane 2.

(21pts) **5.** Consider a space curve parameterized by:

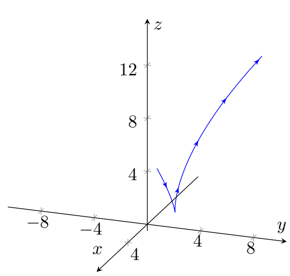
$$\mathbf{r}(t) = \left\langle t^2 - 3, t^3 + 1, \frac{5t^2\sqrt{2}}{2} \right\rangle \quad , \quad t \in \mathbb{R}.$$

(a) (7 pts) Find **parametric** equations for the tangent line to the curve at the point $\left(1, 9, 10\sqrt{2}\right)$.

(b) (5 pts) Compute the unit tangent vector $\mathbf{T}(t)$ for all times $t > 0$. Simplify your answer.

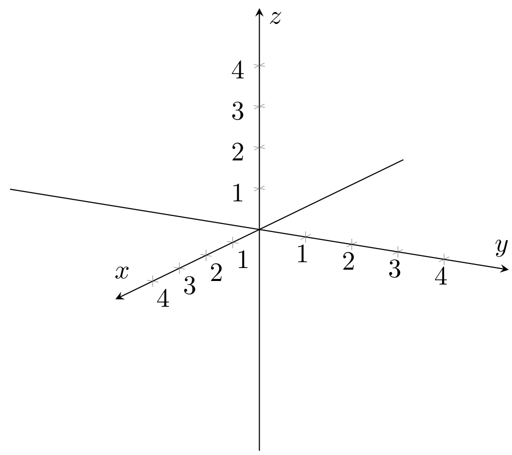
(c) (6 pts) Briefly explain why the unit tangent vector $\mathbf{T}(t)$ is undefined at $t = 0$. Then find the limit as $t \to 0^+$ of $\mathbf{T}(t)$.

(d) (3 pts) The graph of the space curve is shown below for $-1 \le t \le 2$. Indicate the position on the curve when $t = 0$. Justify your answer.

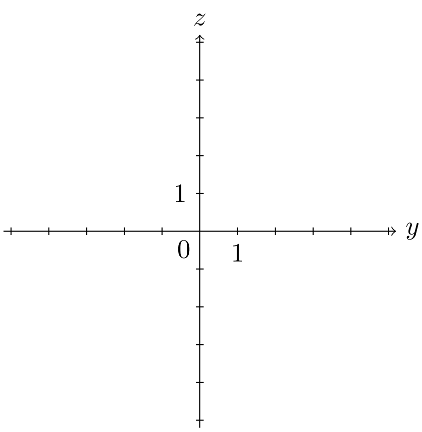


(10pts) **6.** Draw the requested sketches.

(a) (5 pts) Sketch $x^2 - y + 4z^2 = 0$ in 3D. Include the coordinates of **at least three well-chosen** points with your sketch.




(b) (5 pts) Consider the surface $y^2 - z^2 = 1$.
Sketch the trace $x = 2$, then describe in words what the surface looks like.




trace: $x = 2$

(10pts) **7.** Find the arc length of the curve described by $\mathbf{r}(t) = \left\langle \sin t, 2t^{\frac{3}{2}}, \cos t \right\rangle$ for $1 \leq t \leq 2$.