MATH 371
Exam 1 BONUS

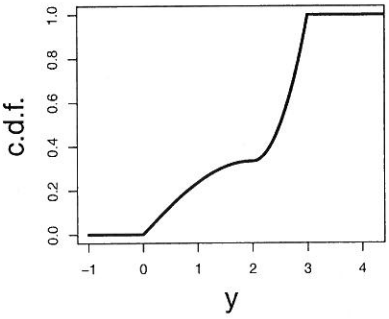
Name : SOLUTIONS

October 30, 2019

Instructions: You may consult tables on the inside cover of your textbook and in the appendices, but indicate in your solution that you have done so by writing 'Table.' No calculators are allowed on this exam, but you should perform routine arithmetic. Bald answers will receive very limited, if any, credit. Ten points total. Good luck.

1. Consider the graph below of the c.d.f. $F(y)$ (*a.k.a.* the distribution function) for a random variable Y. Its equation is also given.

$$F(y) = \begin{cases} 0, & \text{if } y < 0 \\ \frac{1}{3}\sin(\frac{\pi}{4}y), & \text{if } 0 \le y \le 2 \\ \frac{1}{3} + \frac{2}{3}(y-2)^3, & \text{if } 2 < y \le 3 \\ 1, & \text{if } y > 3. \end{cases}$$



Easy plug in.

Trick: $F(2) = \frac{1}{3} + \frac{2}{3}(2-2)^2 = \frac{1}{3}$

(a) Find the exact value of the probability $P(Y \ge 2)$.

$P(Y \ge 2) = 1 - F(2) = 1 - \frac{1}{3}\sin\left(\frac{\pi}{4}2\right) = 1 - \frac{1}{3}\sin\left(\frac{\pi}{2}\right) = \boxed{\frac{2}{3}}$

Since F is continuous

(b) Give a formula for the density function $f(y)$ for Y, giving its values for all real numbers y.

$f(y) = F'(y) = \begin{cases} 0 & \text{if } y < 0 \\ \frac{\pi}{12}\cos(\frac{\pi}{4}y) & \text{if } 0 \le y \le 2 \\ 2(y-2)^2 & \text{if } 2 < y \le 3 \end{cases}$ and 0 if $y > 3$.

(c) Find the probability $P(1 \le Y \le 4)$.

$P(1 \le Y \le 4) = F(4) - F(1) = 1 - \frac{1}{3}\sin\left(\frac{\pi}{4}(1)\right) = 1 - \frac{1}{3}\cdot\frac{\sqrt{2}}{2} = \boxed{1 - \frac{\sqrt{2}}{6}}$

2. Answer the following.

(a) Waiting times for the next large earthquake in the Aleutians are modeled with an exponential random variable Y: `time in months until the next large earthquake` with mean 10. Find the probability that the next large earthquake in the Aleutians occurs between 8 and 12 months from now. Simplify your answer so that the only remaining part of the computation is calculator entry.

$Y \sim \text{Exp}(10)$ $\quad P(8 \le Y \le 12) = \int_8^{12} \frac{1}{10} e^{-y/10}\, dy = -e^{-y/10}\Big|_8^{12} = -e^{-12/10} - (-e^{-8/10})$

$= \boxed{e^{-4/5} - e^{-1.2}}$

(b) In a small community, signups during for open enrollment for annual health insurance policies occur at a rate of 7.2 per week. Find the probability that between 5 and 9 signups occur in a one week period.

Pois(7.2)! $\quad P(5 \le x \le 9) = P(x \le 9) - P(x \le 4) \overset{\text{Table}}{=} .810 - .156 = \boxed{.654}$

discrete so this is not $P(X \le 5)$

$P(x \le 5) \ne P(x \le 4)$