MATH 161 Name : ______________________________

Review Problems November 26, 2012

Sectons 3.4, 3.7, Chapter 4, Sections 10.1, 10.2, 10.9

Section 3.4: Find real roots of polynomials Main ideas: Rational Root Theorem, Descartes Rule of Signs, Factoring after finding roots

1. Give a list of all possible rational roots of the function $f(x) = -12x^4 + 100x - 2$.
2. Find all rational zeros of
   (a) $f(x) = x^3 - 3x^2 - 4$ (b) $g(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$ (c) $h(x) = 6x^3 + 11x^2 - 3x - 2$
3. Find all real roots of $x^3 + 4x^2 + 3x - 2$. (Textbook 47)
4. Sketch a plot of $x^4 - 5x^3 + 6x^2 + 4x - 8$ (Textbook 61)

Section 3.7: Rational functions Main ideas: Plotting rational functions; finding vertical, horizontal and slant asymptotes; understanding 'end behavior'

Sketch the following. In your analysis, you should
1) Factor numerator (zeros) and denominator (vertical asymptotes).
2) Find $x-$ and y-intercepts.
3) Determine behavior of graph near vertical asymptotes (to ∞ or $-\infty$)
4) Determine the horizontal asymptotes and/or end behavior.
5) Sketch.

1. $R(x) = \dfrac{6x^3 - 2}{2x^3 + 5x^2 + 6x}$ (Textbook 29)
2. $f(x) = \dfrac{x^2 + 2}{x - 1}$ (Textbook 31)
3. $W(x) = \dfrac{x^4 + 2}{x - 1}$ (Variation on Textbook 31)
4. $g(x) = \dfrac{x^2 - 2x + 1}{x^3 - 3x^2}$ (Textbook 63)

Section 4.1: Exponential functions Main ideas: Graphs of exponential functions; compound interest.

1. Sketch (a) $y = -2^{-x} + 5$ (b) $h(x) = -\left(\dfrac{1}{3}\right)^x + 2$.
2. Suppose you invest \$2000 at an interest rate of 5%.
   (i) Compute the amount of money in your account assuming the interest is
   (a) compounded annually (b) compounded semiannually (c) compounded quarterly
   (d) compounded monthly (e) compounded daily
   (ii) Now suppose at the end of 1 year you have \$2016.20 in your account, but that interest was computed using the **simple** interest formula. What the **simple** interest rate for this year?

Section 4.2: The natural exponential e^x Main ideas: Plotting and computing with $y = e^x$, continuously compounded interest.

1. Use a calculator to compute $e^{1.2}$, $e^{-.1}$, e^{-2}, e^3.
2. Sketch a plot of $y = -e^x + e$. Include intercepts and asymptotes.
3. Suppose you invest \$10,000 in an account with interest rate $r = 1.1\%$.
   (a) If the interest is compounded continuously, how much money is in the account after 1 year? 2 years? 3.5 years?
   (b) If the interest is compounded quarterly, how much money is in the account after 1 year? 2 years? 3.5 years?

Section 4.3: Logarithm functions and Section 4.4 Laws of logarithms Main ideas: Plotting and computing with $y = \log x$, $y = \ln x$, understanding logarithms and how to simplify them. Using the laws of logarithms. Change of base formula.

1. Without a calculator simplify
   (a) $\log(100)$ (b) $\log(\frac{2}{200})$ (c) $\log(4) + \log(25)$ (d) $\log(100^x)$ (e) $\log(10^{\ln e})$ (f) $\log(10^{\ln \pi})$
   (g) $\ln(\frac{1}{e})$ (h) $\frac{1}{2}\ln(e^2)$ (i) $\ln(e^{x^2}) - \ln(e)$ (j) $\ln(1)$ (k) $e^{\ln(100)}$ (l) $\ln(12e) - \ln(12)$
   (m) $\log 25 + 4\log 2 - 2\log 2$ (n) $\log_4 16^{100}$ (o) $\log(\log(10^{10,000}))$ (p) $\log_3(\sqrt{27})$ (q) $\log_9(\frac{1}{3})$
   (r) $\log_2(8^5)$ (r) $\log_\pi(\pi)$
2. Without a calculator, answer whether the following expressions are positive, negative, zero, or undefined.
   (a) $\log_2(-.1)$ (b) $\ln(3)$ (c) $\ln(\frac{1}{2})$ (d) $\log(.7)$ (e) $\ln e$
3. Textbook 4.4: 7-51 odd
4. Sketch $y = \ln(x + 1) - 2$. Include intercepts and asymptotes. Give the domain and range.
5. Without a calculator, find integers a, b so that
   (a) $a < \log(76) < b$ (b) $a < \ln(\frac{1}{2}) < b$ (c) $a < \log_4(70) < b$
6. Suppose you invest \$10,000 in an account where the interest is compounded continuously. After two years you have \$10,202.01 in the account. What was the interest rate r?
7. Use a calculator to estimate the following. Round your answer to three decimal places.
   (a) $\log_5(23.2)$ (b) $\log_2(\frac{1}{3})$
8. Write down the three laws of logarithms.
9. Sketch $y = |\ln(x + 3)|$.

Section 4.5: Exponential and Logarithmic Equations Main ideas: Solving equations with the variable in the argument of a logarithm or in the exponent position. Doubling time for investment, population growth.

1. Solve for x. Check your answers.

   (a) $\log(x+2) = -1$ (b) $\log(x^2-1) = 3$ (c) $\ln(x+1) = 2 - \ln(x)$ (d) $\log_2(3x-1) = 2$
   (e) $2\log_2 x = -3$ (f) $e^{2x} - e^x = 6$ (g) $3^{\frac{x}{14}} = .1$ (h) $2^{3x-1} = 3^{x+2}$ (i) $e^{\ln(x^2-3x)} = 4$
   (j) $e^x = -4$

2. Explain in your own words why

   (a) $\log\left(\frac{2}{3}\right) \neq \frac{\log(2)}{\log(3)}$ (b) $\log(2+x) \neq \log(2) + \log(x)$ (c) What are the rules of log's?

3. A student invests \$3000 in an account which pays 3% interest per year. When will there \$5000 in the account, if

   (a) The interest in compounded quarterly? (b) The interest in compounded monthly?

   (c) The interest in compounded continously?

Section 4.6: Modeling with exponential and logarithm functions Main ideas: Doubling/Tripling/Halving times and exponential growth; Relative growth rates.

1. A certain breed of rat was introduced on an island 8 months ago. The current rat population on the island is estimated to be 4100 and doubling every three months.

   (a) What was the initial size of the population?

   (b) Estimate the population one year after the rabbits were introduced to the island.

   (c) Sketch a graph of the rabbit population.

2. Population of California: (Textbook 15) The population of California was 29.76 million in 1990 and 33.87 million in 2000. Assume that the population grows exponentially.

   (a) Find a function that models the population t years after 1990.

   (b) Find the time required for the population to double.

   (c) Use the function from part (a) to predict the population in 2010.

Section 10.1: Systems of linear equations in two variables Main ideas: Solving such systems; counting the number of solutions. Understanding the solutions as intersections of lines.

1. Textbook 33, 35, 37

Section 10.2: Systems of linear equations in several variables Main ideas: Transforming to an equivalent upper triangular system. Unique solutions, No solutions $\equiv$ *inconsistent system*, Infinitely many solutions.

1. Textbook 23, 24, 27, 31

Section 10.9: Solving systems of inequalities Main ideas: Partitioning the plane into regions corresponding to '$>$', '$<$', and '$=$'.

1. Solve

$$x^2 + y^2 < 25$$
$$x + 2y \geq 5$$