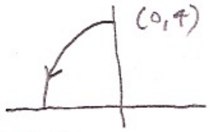
MATH 202 Name : SOLUTIONS

Quiz 9 December 4, 2009

Instructions: You get one point for taking this quiz.

1. (1 pt.) Evaluate the line integral $\int_C x^2 y\, ds$, where C is the quarter arc of a circle of radius 4 traversed in the counter-clockwise direction from $(0,4)$ to $(-4,0)$.



(0,4)

(-4,0)

$\vec{r}(t) = (4\cos t, 4\sin t)$

$\frac{\pi}{2} \le t \le \pi$

$\vec{r}\,'(t) = (-4\sin t, 4\cos t)$

$|\vec{r}\,'(t)| = \sqrt{(-4\sin t)^2 + (4\cos t)^2} = \sqrt{16(\sin^2 t + \cos^2 t)} = \sqrt{16} = 4$

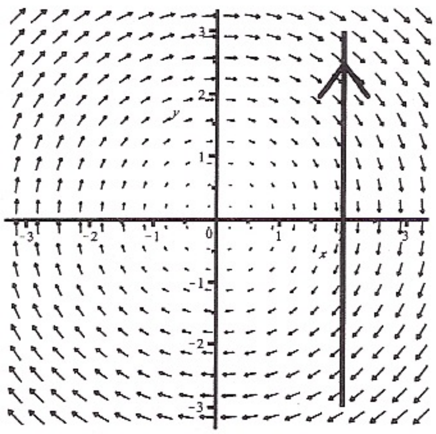
$$\int_C x^2 y\, ds = \int_{\pi/2}^{\pi} (4\cos t)^2 (4\sin t)\, 4\, dt$$

$$= \int_{\pi/2}^{\pi} 256 \cos^2 t \sin t\, dt = 256 \left. -\frac{\cos^3 t}{3} \right|_{\pi/2}^{\pi}$$

$$= 256\left[-\frac{\cos^3 \pi}{3} - -\frac{\cos^3 \pi/2}{3}\right] = 256\left(-\frac{(-1)^3}{3}\right)$$

$$= \boxed{\frac{256}{3}}$$

2. (1 pt.) Consider the force field $\mathbf{F}$ and the straight line C shown. The line C has an orientation from the point $(2,-3)$ to the point $(2,3)$.



Is the value of the line integral $\int_C \mathbf{F} \cdot dr$ positive, negative, or zero? Explain your answer. (Credit only for a correct explanation.)

Negative. $\vec{F} \cdot d\vec{r} < 0$ since the component of $\vec{F}$ along $\vec{r}(t)$ is negative. ($\vec{F}$ points "opposite" the direction of $\vec{r}(t)$)

Notice for any point $P = (x(t_p), y(t_p))$, $\vec{F}(x(t_p), y(t_p)) \cdot \vec{r}\,'(t_p) < 0$ since $\frac{\pi}{2} < \theta < \pi$

3. (2 pts.) Consider the force field given by $\mathbf{F}(x, y) = (e^{x-1}, xy)$ Newtons. Compute the work done by the force field in moving a particle along a curve C given parametrically by $\mathbf{r}(t) = (t^2, t^3)$ for $0 \le t \le 1$. Assume the position coordinates are all measured in meters, and include units in your final answer.

$$W = \int_C \vec{F} \cdot d\vec{s} = \int_0^1 (e^{t^2-1}, t^2 t^3) \cdot (2t, 3t^2)\, dt$$

where $\vec{r}\,'(t) = (2t, 3t^2)$

$$= \int_0^1 2t e^{t^2-1} + 3t^7 \, dt$$

$$= e^{t^2-1} + \frac{3}{8} t^8 \Big|_0^1$$

$$= (e^{1^2-1} + \frac{3}{8}) - (e^{0^2-1} + 0)$$

$$= 1 + \frac{3}{8} - e^{-1}$$

$$= \boxed{\frac{11}{8} + \frac{1}{e} \text{ Newton m}}$$