MATH 161
Exam 3

Name : Solutions
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Instructions: Show all work for full credit. Poor notation or sloppy work will be penalized. Point values as indicated.

1. (5 pts.) Find **ONE** rational root of $f(x) = x^3 + x^2 - 4x - 4$. You must show your work for credit, including showing that your answer is indeed a root of $f(x)$.

Candidates: $x = 1, -1, 2, -2, 4, -4$

Testing one finds $x = -1, 2, -2$ are roots

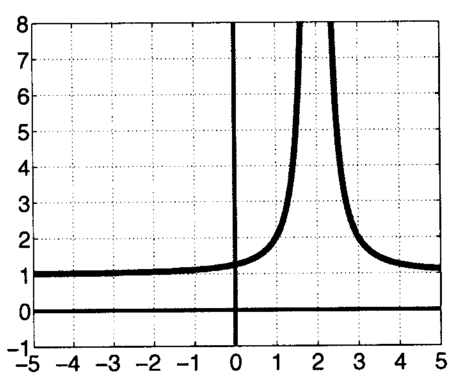
$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = -1 + 1 + 4 - 4 = 0$

$f(2) = (2)^3 + (2)^2 - 4(2) - 4 = 8 + 4 - 8 - 4 = 0$

$f(-2) = (-2)^3 + (-2)^2 - 4(-2) - 4$
$= -8 + 4 + 8 - 4 = 0$

Answer: $x =$ Any of $-1, 2, -2$ are correct.

2. (10 pts.) Consider the graph of a function $f(x)$ below. Fill in the blanks based on the graph.



(a) Give the equation of the vertical asymptote.

Ans: $x = 2$ 3 pts

(b) As $x \to 2^-$, $f(x) \to$ $+\infty$ 3 pts

(c) Give the equation of the horizontal asymptote.

Ans: $y = 1$ 4 pts.

3. (5 pts.) The function $h(x) = \dfrac{-2x^2 + 1}{3x^2 - 1}$ has a horizontal asymptote. What is the equation of this asymptote?

Give your answer in $y = mx + b$ form.

Answer: The equation is $y = \frac{-2}{3}$

4. (24 pts. No partial credit.) Simplify.

(a) $\log(.01)$ $= \boxed{-2}$

(b) $\log(2) + \log(50)$

$= \log(100) = \boxed{2}$

(c) $3^{\log_3(6)}$ $= \boxed{6}$

(d) $e^{\ln(\log 10)}$ $= \log 10 = \boxed{1}$

(e) $\log(100^x)$ $= \boxed{2x}$

(f) $\ln(1)$ $= 0$

(g) $\ln(\frac{1}{e})$ $= \boxed{-1}$

(h) $\ln(13e^2) - \ln(13)$

$= \ln\left(\frac{13e^2}{13}\right) = \ln e^2 = \boxed{2}$

(i) $3\log 2 + \log 50 - 2\log 2$

$= \log 2^3 + \log 50 - \log 4$

$= \log\left(\frac{8 \cdot 50}{4}\right) = \log(100) = \boxed{2}$

(j) $\log_3 9^{100}$ $= \log_3(3^2)^{100} = \log_3 3^{200}$

$= \boxed{200}$

(k) $\log(\log(10^{1000}))$

$= \log(1000) = \boxed{3}$

(l) $\log_2(\sqrt{8})$ $= \log_2\left(8^{\frac{1}{2}}\right) = \log_2(2^3)^{\frac{1}{2}}$

$= \log_2 2^{\frac{3}{2}} = \boxed{\frac{3}{2}}$

5. (16 pts. – 4 pts. each) Solve the following equations. Check your answers in (c) and (d).

(a) $5 + 2\log(4x) = 11$

$2\log(4x) = 6$

$\log(4x) = 3$

$10^3 = 4x$

$1000 = 4x$

$\boxed{250 = x}$

(b) $e^{\ln(x+1)} = 5$

$x + 1 = 5$

$\boxed{x = 4}$

(c) $\log_2 x = 2 - \log_2(x+3)$

$\log_2 x + \log_2(x+3) = 2$

$\log_2(x^2 + 3x) = 2$

$x^2 + 3x = 2^2$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4$ $\quad x = 1$

Check: $x = -4$ fails

$\log(-4)$ DNE

$x = 1$: $\log_2(1) \stackrel{?}{=} 2 - \log_2(1+3)$

$0 \stackrel{?}{=} 2 - \log_2 4$

$0 \stackrel{?}{=} 2 - 2$ $\quad 0 = 0$ ✓

$\boxed{x = 1}$

(d) $e^{2x} - 2e^x + 1 = 0$

$(e^x - 1)^2 = 0$

$e^x = 1$

$x = 0$

Check: $e^{2(0)} - 2e^0 + 1 \stackrel{?}{=} 0$

$1 - 2 + 1 \stackrel{?}{=} 0$

$0 = 0$ ✓

$\boxed{x = 0}$

6. (10 pts.) Solve the system of linear equations for x and y. (The solution is unique.)

$3x + 2y = 14$
$x - y = 3$

Multiply 2nd equation by 2 then add

$$\begin{array}{rrcl} & 3x + 2y & = & 14 \\ + & 2x - 2y & = & 6 \\ \hline & 5x & = & 20 \end{array}$$

$\boxed{x = 4}$

Substitute into 2nd equation.

$x - y = 3$

$4 - y = 3$

$\boxed{y = 1}$

$\boxed{x = 4, \quad y = 1}$

7. (9 pts.) A student invests \$4000 in an account, and wants it to grow to \$5000 in ten years. What rate of return r must the student realize, if interest is compounded continuously? Round your answer to one decimal place. (An acceptable answer looks like 5.1%.)

$A(t) = 4000e^{rt}$

$5000 = A(10)$

$5000 = 4000e^{r(10)}$

$\frac{5000}{4000} = e^{10r}$

$1.25 = e^{10r}$

$\ln 1.25 = 10r$

$\frac{\ln 1.25}{10} = r$

$r \approx .022$ or $\boxed{2.2\%}$

8. (9 pts.) How long will it take for an investment of \$10,000 to reach a value of \$15,000, if the interest rate is 2.5% year compounded quarterly? Round your answer to one decimal place.

$A(t) = 10,000\left(1 + \frac{.025}{4}\right)^{4t}$

$A(t) = 10,000\,(1.00625)^{4t}$

Q: Find t, when $A(t) = 15,000$.

$15,000 = 10000\,(1.00625)^{4t}$

$\frac{15,000}{10,000} = (1.00625)^{4t}$

$\frac{3}{2} = 1.00625^{4t}$

$\log(1.5) = 4t \log(1.00625)$

$\frac{\log(1.5)}{4\log(1.00625)} = t$

$t \approx 16.269 \approx \boxed{16.3 \text{ years}}$

9. (12 pts.) Solve the following equations. Round your answers to two decimal places.

(a) $10^{x+1} = 2^{3x-1}$

$\log 10^{x+1} = \log 2^{3x-1}$

$x + 1 = (3x-1)\log 2$

$x + 1 = 3x \log 2 - \log 2$

$x - 3x\log 2 = -1 - \log 2$

$x(1 - 3\log 2) = -1 - \log 2$

$x = \frac{-(1 + \log 2)}{(1 - 3\log 2)} \approx \boxed{-13.43}$

(b) $\frac{10^{x+1}}{10^{2x-3}} = 5$

$10^{(x+1)-(2x-3)} = 5$

$10^{x+1-2x+3} = 5$

$10^{-x+4} = 5$

$\log 10^{-x+4} = \log 5$

$-x + 4 = \log 5$

$x = 4 - \log 5 \approx \boxed{+3.30}$

Note: Both of these problems could be on the non-calculator portion of the exam.