MATH 202
Quiz 1

Name: SOLUTIONS

January 22, 2020

Instructions: Five points total. Problem 2b is worth two points.

1. An object is located at the point $P(3, -1, 0)$, but is constrained so that it can only move in the straight-line direction toward the point $Q(2, 1, 1)$.

   (a) Give, in coordinate form, a vector $\mathbf{v}$ representing the direction in which the object can move.

   $\vec{v} = \vec{PQ} = \langle 2-3, 1-(-1), 1-0 \rangle = \langle -1, 2, 1 \rangle$

Extra comments:

1) Use hats on $\hat{i}, \hat{j}, \hat{k}$.

2) A unit vector $\vec{u}$ has length 1, $\frac{1}{|\vec{v}|}\vec{v}$ is a unit vector.

   $\mathbf{v} = \boxed{\langle -1, 2, 1 \rangle}$

   (b) Give, in coordinate form, a *unit* vector $\mathbf{u}$ pointing in the direction that the object can move.

   $|\vec{v}| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$

   $\vec{u} = \frac{1}{|\vec{v}|} v = \boxed{\langle \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle}$

   $\mathbf{u} =$ __________

2. (a) Determine if the vectors $\mathbf{v_1} = (-1, 3, 7)$ and $\mathbf{v_2} = (-2, -3, 1)$ are perpendicular.

   $\vec{V_1} \cdot \vec{V_2} = \langle -1, 3, 7 \rangle \cdot \langle -2, -3, 1 \rangle = 2 - 9 + 7 = 0$

   Therefore, perpendicular.

   (b) Find a vector $\mathbf{a}$ that is perpendicular to the plane containing the vectors $\mathbf{v_1}$ and $\mathbf{v_2}$.

   $$\vec{V_1} \times \vec{V_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 7 \\ -2 & -3 & 1 \end{vmatrix} = (3+21)\hat{i} - (-1+14)\hat{j} + (3+6)\hat{k}$$

   $$= 24\hat{i} - 13\hat{j} + 9\hat{k}$$

   $\mathbf{a} = \langle 24, -13, 9 \rangle$ or any non-zero scalar multiple.