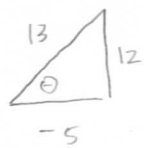
Quiz 9 Solutions

1. $\theta = \arccos(-5/13)$ a) θ is in $\boxed{Q\,II}$ since $\cos\theta = \frac{-5}{13} < 0$

b)



$\boxed{\sin\theta = \frac{12}{13}}$ $\boxed{\tan\theta = -\frac{12}{5}}$ Q II !

2. Domain $\mathbb{R}^2$ is open, simply-connected ✓

$\vec{F}(x,y) = \langle xy^2, -x^2 \rangle$ a) $P = xy^2 \quad \frac{\partial P}{\partial y} = 2xy \neq \frac{\partial Q}{\partial x} = -2x$ $\boxed{\text{NOT CONSERVATIVE}}$

ii) $\vec{r}(t) = (1-t)\langle 0,0\rangle + t\langle 3,2\rangle \quad 0 \le t \le 1$

$= \langle 3t, 2t \rangle \quad 0 \le t \le 1$

$\vec{r}\,'(t) = \langle 3, 2 \rangle$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \langle (3t)(2t)^2, -(3t)^2 \rangle \cdot \langle 3,2 \rangle \, dt$$

$$= \int_0^1 \langle 12t^3, -9t^2 \rangle \cdot \langle 3, 2 \rangle \, dt = \int_0^1 36t^3 - 18t^2 \, dt$$

$$= 9t^4 - 6t^3 \Big|_0^1 = \boxed{3}$$

b) $\vec{G}(x,y) = \langle ye^x + \sin y, e^x + x\cos y \rangle$

$P = ye^x + \sin y \quad \frac{\partial P}{\partial y} = e^x + \cos y$

$Q = e^x + x\cos y \quad \frac{\partial Q}{\partial x} = e^x + \cos y$

$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \Rightarrow \boxed{\text{conservative}}$

ii) Find the potential function $g(x,y)$.

Guess: $\boxed{g(x,y) = ye^x + x\sin y + C}$ check: $\frac{\partial g}{\partial x} = ye^x + \sin y \quad \frac{\partial g}{\partial y} = e^x + x\cos y$ ✓

otherwise:

$P = \frac{\partial g}{\partial x} = ye^x + \sin y \Rightarrow g(x,y) = \int ye^x + \sin y \, dx = ye^x + x\sin y + c(y)$ function of y!

If $g(x,y) = ye^x + x\sin y + c(y)$, then $\frac{\partial g}{\partial y} = e^x + x\cos y + c'(y) = Q$

$= e^x + x\cos y$ Regain $\Rightarrow c(y) = C$

Again $\boxed{g(x,y) = ye^x + x\sin y + C}$

Line Segment joining $a = \langle 0, \frac{3}{2} \rangle$ to $\langle 0, \frac{13\pi}{6} \rangle = b$

$$\int_C \vec{F} \cdot d\vec{r} = f\left(\langle 0, \tfrac{13\pi}{6} \rangle\right) - f\left(\langle 0, \tfrac{3\pi}{2} \rangle\right)$$

$$= \left(\frac{13\pi}{6} e^0 + 0 \sin\left(\frac{3\pi}{6}\right)\right) - \left(\frac{3\pi}{2} e^0 + 0 \sin\left(\frac{3\pi}{2}\right)\right)$$

$$= \frac{13\pi}{6} - \frac{3}{2}\pi = \left(\frac{13-9}{6}\right)\pi = \frac{4}{6}\pi = \boxed{\frac{2}{3}\pi}$$