MATH 253
Spring 2019

Midterm Exam 1

Name: ____________________________

Instructions. (100 points) You have 120 minutes. Closed book, closed notes, and no calculators allowed. *Show all your work* in order to receive full credit.

(7^{pts}) **1.** Consider the point $A(1, -2, 0)$ and the line

$$x - 2 = \frac{y+1}{3} = \frac{z-1}{2}$$

(a) (4 pts) Find the equation of the plane containing A and the line.

(b) (3 pts) Find the distance from A to the line.

(8^{pts}) **2.** Consider the space curve parametrized by:

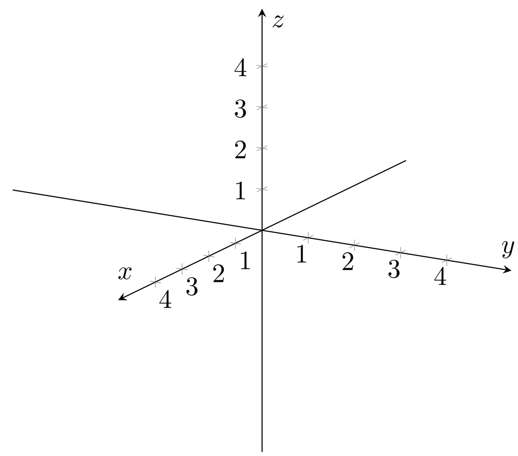
$$\mathbf{r}(t) = \langle \cos t, \cos t + 3\sin t, 3\sin t \rangle \,.$$

(a) (4 pts) Show that $\mathbf{r}(t)$ is a parametrization of the intersection of the surfaces $x - y + z = 0$ and $9x^2 + z^2 = 9$.

(b) (4 pts) Show that the tangent line to $\mathbf{r}(t)$ at $t = \dfrac{3\pi}{4}$ is parallel to $\langle 1, 4, 3 \rangle$.

(6pts) **3.** Rewrite the following equation in standard form then sketch the surface.

$$9x^2 + 36y^2 + 4z^2 - 18x + 8z = 23$$



(0pts) **4.** Consider the following planes.

$$\begin{array}{ll} \text{plane 1:} & x - y + 4z = 5 \\ \text{plane 2:} & 3x - y - z = 2 \end{array}$$

(a) (2 pts) Show that the planes are orthogonal.

(b) (6 pts) Find parametric equations for the line of intersection of the two planes.

(15^{pts}) **5.** Consider the following space curves:

$$\mathbf{r_1}(t) = \left\langle 2t - 3, t^2 - 5t + 3, t^3 - 2 \right\rangle \quad , \quad \mathbf{r_2}(t) = \left\langle -t + 2, t - 4, 3t^2 + 2t + 1 \right\rangle$$

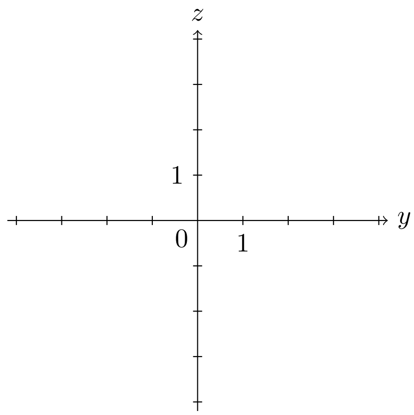
(a) (6 pts) Find any intersection point(s) of the space curves.

(b) (4 pts) Find the unit tangent vector $\mathbf{T_1}(t)$ for the space curve $\mathbf{r_1}(t)$ at time t.

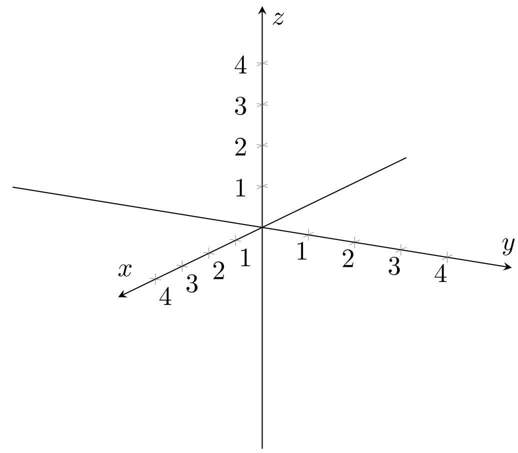
(c) (5 pts) Find the curvature of the space curve $\mathbf{r_2}(t)$ at $t = -1$.

(15pts) **6.** For each equation, name the type of surface, sketch the given trace in 2D then the surface in 3D.

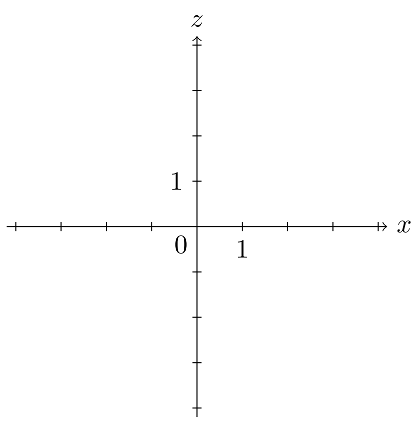
(a) (5 pts) $x^2 - y^2 + 4z^2 = 0$ Type of surface: ______________________________



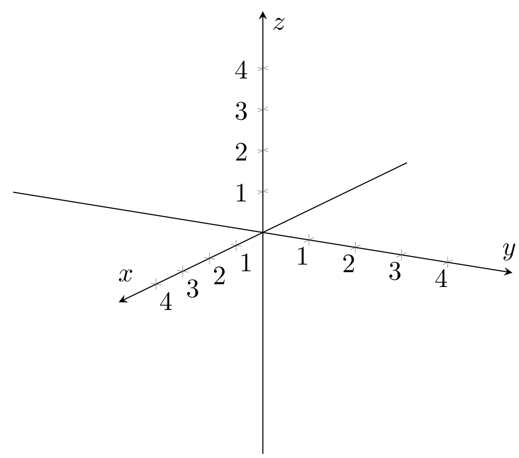
trace: $x = -2$



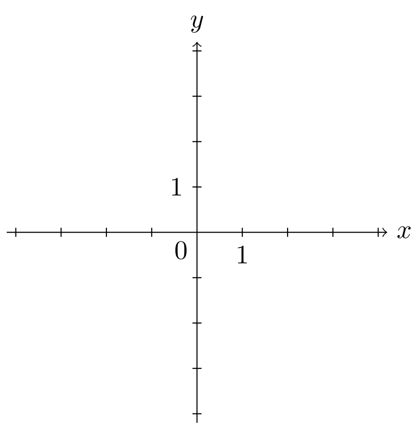
(b) (5 pts) $x = y^2 + z^2$ Type of surface: ______________________________



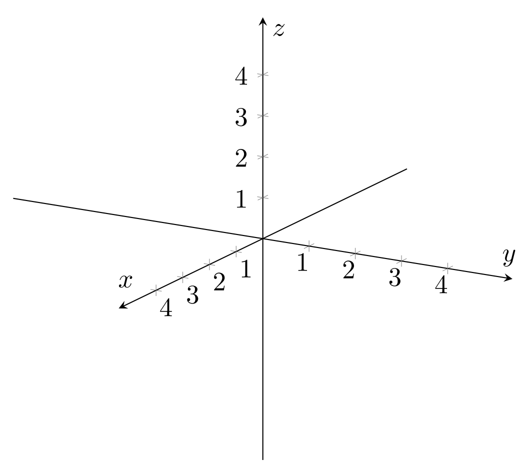
trace: $y = 1$



(c) (5 pts) $x^2 + y^2 = z^2 - 3$ Type of surface: ______________________________



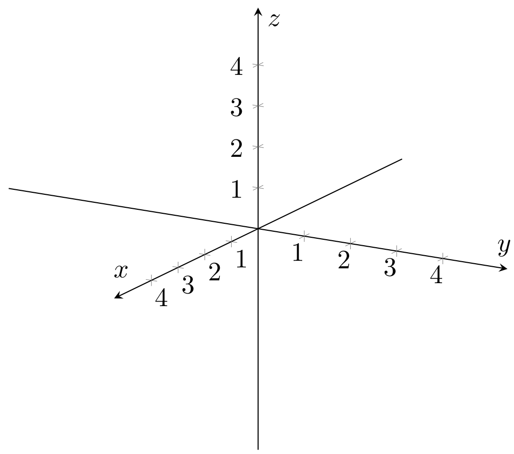
trace: $z = 2$



(9pts) **7.** Let $\mathbf{a} = \langle -1, 3, c\rangle$ and $\mathbf{b} = \langle 2, 1, 4\rangle$.

(a) (2 pts) For what value(s) of c will the angle between $\mathbf{a}$ and $\mathbf{b}$ be obtuse (i.e. greater than $90°$)?

(b) (3 pts) Sketch $\mathbf{a}$ and $\mathbf{b}$ in standard position for $c = -1$.



(c) (4 pts) Find the vector projection of $\mathbf{b}$ along $\mathbf{a}$ for $c = -1$ and sketch it on the above set of axes (make sure to label it).

(15pts) **8.** Consider a particle moving in space with ***velocity*** (measured in m/s):

$$\overrightarrow{v}(t) = (t^2 - 4)\vec{\imath} + 3\vec{\jmath} + 3t\sqrt{2}\vec{k}.$$

(a) (6 pts) Find the position vector $\overrightarrow{r}(t)$ of the particle at time t if $\overrightarrow{r}(1) = 2\vec{\imath} - \vec{\jmath}$.

Recall the velocity (in m/s):

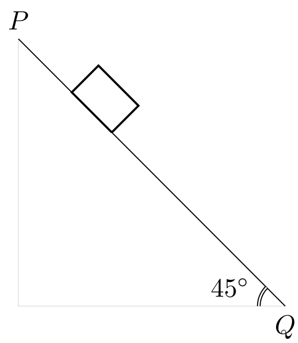
$$\vec{v}(t) = (t^2 - 4)\vec{\imath} + 3\vec{\jmath} + 3t\sqrt{2}\vec{k}.$$

(b) (6 pts) Find the distance traveled by the particle (i.e. the arc length) between $t = 0$ s and $t = 3$ s.

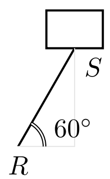
(c) (3 pts) Find the tangential component of the acceleration at time t.

(9pts) **9.** Throughout this problem assume no friction, use 10 m/s^2 as an approximation for the acceleration due to gravity, and don't forget units in your answers. We will consider an ice block of mass 30 kg.

(a) (4 pts) The ice block is brought down along a ramp between P and Q which is at a 45° angle with the horizontal. Find the work done by gravity to move the block down the incline if $\left\|\overrightarrow{PQ}\right\| = 20$ m.



(b) (5 pts) Find the direction ($\odot$ or $\otimes$) and the magnitude of the torque when the weight of the ice block is used at S to rotate an axis placed at R if $\left\|\overrightarrow{RS}\right\| = 6$ m and $\overrightarrow{RS}$ is at a 60° angle with the horizontal.



(8pts) **10.** A golf ball takes off from the ground in "Calculus III conditions"[1] with an initial speed of 200 ft/s and at an angle of 50° with the horizontal on a flat terrain. Show that the total horizontal distance traveled by the golf ball is

$$x_{\max} = 1250 \sin 100° \text{ ft.}$$

[1] I.e. the acceleration is constant and only due to gravity at 32 ft/s^2. That is we ignore ball spin, air resistance, etc.