Exam 2 Name:_____

Math 202 F01 04/03/06

1. (15 pts.) A solid hemisphere of radius 5 and uniform density lies so that its flat circular base is on the xy-plane, with center at the origin. By symmetry, it is easy to see that the center of mass $(\bar{x}, \bar{y}, \bar{z})$ has $\bar{x} = 0$ and $\bar{y} = 0$. Using *spherical coordinates*, give an expression for $\bar{z}$ involving integrals. Do *not* evaluate any integrals. Leave your answer in a form where only the evaluation of integrals remains to obtain a numerical answer. (Partial credit will be given for answers in other coordinate systems).

2. (15 pts.) Find all points satisfying the constraint $x^2 + y^2 = 1$ that maximize the function $f(x, y) = x^3 y$.

3. (15 pts.) Consider the function $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2$.

   (a) Find all critical points of f.

   (b) Use the second derivative test to determine as much as possible about the critical points you found in part (a).

4. (12 pts.) A metal plate is shaped like the region in the xy-plane bounded by

$$y = -x/2, \quad y = \sqrt{x}, \quad \text{and } x = 4,$$

   with x, y, z measured in cm. Electric charge is distributed over the plate, with charge density $\rho(x, y) = x^2y$ coulombs/cm^2. What is the total charge on the plate? Specify units.

5. (15 pts.) The density of food available in a fish tank is given by

$$\rho(x, y, z) = xy^2 e^{10-z} \text{ calories/m}^3,$$

where x, y, z are measured in m. A particular fish is located at the point $(4, 1, 9)$, and is interested in swimming in whatever direction will most rapidly increase the density of food in its surroundings.

(a) In what direction should the fish swim?

(b) If the fish swims in the direction you specify in part (a), at what rate will the food density change? Specify units.

(c) If, due to barriers in its path, the fish is instead forced to swim in the direction given by the vector $(1, -1, 0)$, at what rate will the food density it experiences change?

6. (10 pts.) Give the linear approximation of $f(x, y) = \sqrt{x + e^{-2y}}$ valid near the point (8,0).

7. (8 pts.) Explain why $\lim_{(x,y)\to(0,0)} \dfrac{y^3 - x^2}{x^2 + y^2}$ does not exist.

8. (10 pts.) Reverse the order of integration in the following:

$$\int_{-2}^{0} \int_{4x}^{x^3} f(x, y)\, dy\, dx$$