MATH 631: Algebra I

MWF 3:30 - 4:30, Problem Session TBD
Zoom and hopefully face-to-face
https://eallman.github.io/classes/631/631_2020.html

Instructor: Elizabeth S. Allman, Chapman 308B, e.allman@alaska.edu and 474-2479.

Covid-19 modifications. See information below. Most importantly, *everything* is in flux.

Office Hours: TBD and by appointment. These office hours may change at a later date, depending on student demand and scheduling concerns. Please note that the best way to reach me is by e-mail.

Prerequisities: an undergraduate course in Abstract Algebra

Textbook: *Abstract Algebra* by D. Dummit and R. Foote. 3rd edition, Wiley, 2004.

Grading: There will be two midterms and a cumulative final exam. See details below. Grades will be assigned using the following weights:

Homework	10 %
Midterms ($\times$ 2)	25 %
Final Exam	40 %

The midterms are tentatively scheduled for the weeks of October 8-12 and November 12-16. Finding a two hour time slot for these is one of our first organizational tasks. The final exam will take place as published in the schedule of courses, on Wednesday, December 9 from 2:30 - 5:30. Makeup exams will not be given.

Content: Broadly speaking, the field of Abstract Algebra studies sets together with operations $+, \cdot$ on the elements of these sets. The main objects of study are *groups*, *rings*, and *fields*. A group $< G, \cdot >$ with a single binary operation that satisfies a few axioms is the most general algebraic object familiar to you from undergraduate study. Groups arise naturally in many fields of mathematics, and ones familiar to you might include those arising from a geometric setting (dihedral groups D_{2n}), the permutation groups (S_n and A_n), ones of importance in number theory ($< \mathbb{Z}, + >$, $\mathbb{Z}/n\mathbb{Z}$), and matrix groups ($GL_n(\mathbb{R})$, $SL_n(\mathbb{R})$, etc.)

A more structured algebraic object, called a *ring* $< R, +, \cdot >$, is a non-empty set endowed with two binary operations that satisfies certain axioms. The model for the ring axioms is the set of integers $< \mathbb{Z}, +, \cdot >$, together with the operations of addition and multiplication which are linked together via the distributive law. A field $< F, +, \cdot >$ has the most structure of any algebraic object we study in the course. The fields $\mathbb{R}$ and $\mathbb{C}$ are well-known from courses in real and complex analysis. Perhaps less familiar, but of utmost importance to algebraists, is the field of rational numbers $\mathbb{Q}$.

In this survey course, we undertake an in-depth study of groups, rings, and fields. A solid understanding of group theory from undergraduate study will be assumed so that there will be more time for rings and fields.

Course Logistics: MATH 631 is a four hour graduate course with lectures three days a week and a problem session on a day to be determined. It forms the basis for the graduate qualifying exam for the Master's degree in Mathematics.

The pace of the course will be fast, and students should be prepared to fill in spotty knowledge or incomplete background material on their own. Unfortunately, students often enroll in a graduate algebra course with very different backgrounds in algebra. This course presumes a thorough knowledge of Chapters 1-3 in the textbook (basics on groups), a thorough knowledge of Chapter 7 (definition and first examples of rings), and some familiarity with Chapter 8 (Euclidean domains, PIDs, UFDs) and Chapter 9 (basics of polynomial rings).

In class, lectures will highlight the main ideas and theorems for the topic under study, and will provide insight for good ways to think about objects or why these algebraic structures are so interesting. As it will not be possible to cover all topics in complete detail in lecture, students are responsible for reading each

chapter in the text carefully, and fully digesting ideas and examples in the text, regardless of whether such material was covered in class. You should expect homework problems on material in the text that has been omitted from lecture.

One of the most important skills to learn in a beginning graduate mathematics class is the ability to write cogent proofs and to give convincing arguments for theorems. Homework will be assigned regularly and collected once a week, typically on Wednesday, at the beginning of class. Please feel free to work with other students to discuss ideas for proofs, but your write-up should be completely your own. One goal for this course is for you to develop an independent and mature sense of when you have given a correct and well-written proof of a theorem.

Homework: A quick internet search will show that homework solutions to virtually all the problems in Dummit and Foote are available online. You could either google key words in the problem to find a specific solution or even, for $19.99 a month (!), have access to a complete solutions manual on Chegg. For this reason, solutions to homework problems will make up only a small portion of your final grade.

This is not to say that homework is not important; indeed, it is probably the *single most important effort* you can make to gain a firm and deep understanding of algebra (or indeed any field of mathematics). Doing problems is the key to learning and understanding mathematical ideas and is what makes mathematics fun. As graduate students, you should understand that you alone are responsible for your learning and that motivation and time management are key attributes of the successful mathematician.

Homework will be assessed in two ways this term. First, on its due date an electronic (LaTeX-ed) homework pdf file from each student will be deposited in a class repository. This will be checked for completeness, and a few problems will be marked by me so that you get personalized feedback on your proof writing skills and understanding. Separately, each student will submit a few of their LaTeX-ed solutions in a text file. These solutions should be in a separate LaTeX file and a template will be made available. These solutions will be collated and used both for my review and for *peer review* of proof writing. Of course, they will be marked for correctness too and the marked assignment will be made available online.

There will also be several class periods devoted to student presentations of homework solutions. Particularly in this age of widespread use of electronic media, it is essential that the fundamental skill of communicating mathematics orally not be neglected. The goal is to help students develop into good teachers and mathematical expositors. While it is impossible to draw up a definitive list describing a good expositor or exposition, adjectives like 'clear,' 'correct' 'motivated,' 'engaging,' 'stimulating,' etc. ideally will come to mind.

Course Outline:

For a tentative schedule of topics and midterm dates, see the table below. The crux of the course will cover Chapters/Sections 0-4, 5.2, (groups); 7-9 (rings); 11-12 (*fast* modules). We will discuss Chapter 13 on fields or another topic of interest too, time permitting.

Tentative Schedule

Dates	M1	M2	W	F	Comments
Week 1: Aug 24 - 28	Chapters 0, 1, start 2 Organization				
Week 2: Aug 31 - Sept 4	Problems	Chapter 2			
Week 3: Sept 7-11	✗	Problems	Chapter 2/3		Labor day holiday Monday
Week 4: Sept 14-18	Chapter 3	Problems	Chapter 4		
Week 5: Sept 21-25	Chapter 4	Problems	Chapter 4		
Week 6: Sept 28 - Oct 2	Section 5.2	Student HW presentations	Chapter 6	slack	
Week 7: Oct 5-9	Chapter 7, Problems Midterm 1				Date/Time TBA
Week 8: Oct 12-16	CRT, Chapter 8	Problems	Chapter 8		
Week 9: Oct 19-23	Chapter 9	Problems	Chapter 9		
Week 10: Oct 26-30	slack	Problems	slack		
Week 11: Nov 2-6	Chapter 10/11/12, Problems				
Week 12: Nov 9-13	Chapter 10/11/12, Problems				
Week 13: Nov 16-20	Chapter 10/11/12, Problems Midterm 2				Date/Time TBA
Week 13.5: Nov 23-27	Student HW presentations		✗	✗	Thanksgiving
Week 14: Nov 30 - Dec 4	????				
Final Exam	Wednesday, December 9 2:30 - 5:30				

Covid-19 adaptations: Most importantly, please do your best to stay healthy AND as a subsidiary goal to learn graduate algebra. These are extraordinary times, but the mental discipline of learning algebra under these circumstances can be both rewarding and helpful. To the extent possible in a 'hybrid' format, I will offer MATH 631 as usual. Of course, sickness or increased illness in the community may change our plan, so I ask your forbearance while we try to muddle through with remote learning.

Please **do** keep up with current UAF policies on the pandemic:

"Students should keep up-to-date on the universitys policies, practices, and mandates related to COVID-19 by regularly checking this website:
`https://sites.google.com/alaska.edu/coronavirus/uaf/uaf-students?authuser=0`
Further, students are expected to adhere to the universitys policies, practices, and mandates and are subject to disciplinary actions if they do not comply."

Things you should do ASAP:

1. Sign into Blackboard so you can see that our class is accessible to you.
2. Create either a Dropbox or Google Drive directory using the format

   MATH_631_Lastname_Firstname

   and share this directory with me. Please do not share a drive, but only a directory. This is where your HWs will be returned to you.
3. Start your first HW assignment which is due Wed Sept 2.
4. Join our class shared directory in response to an invitation from me.

Other Policies:

Course accommodations: If you need course adaptations or accommodations because of a disability, please inform your instructor during the first week of the semester, after consulting with the Office of Disability Services, 203 Whitaker (474-7403).

University and Department Policies: Your work in this course is governed by the UAF Honor Code. The Department of Mathematics and Statistics has specific policies on incompletes, late withdrawals, and early final exams, some of which are listed below. A complete listing can be found at

`https://www.uaf.edu/dms/policies/`.

Prerequisites: The prerequisite for MATH 631 is an undergraduate course in Abstract Algebra.

Late Withdrawal: This semester the last day for withdrawing with a 'W' appearing on your transcript is Friday, October 30. If, in my opinion, a student is not participating adequately in the class, I may elect to drop this student.

Graded Coursework: Please keep all graded work for MATH 631 until final grades have been assigned.

Academic Honesty: Academic dishonesty, including cheating and plagiarism, will not be tolerated. It is a violation of the Student Code of Conduct and will be punished according to UAF procedures.

Grade Bands: A, A- (90 - 100%), B+,B, B- (80 - 89%), C+, C, C- (70 - 79%), D+, D, D- (60 - 69%), F (0 - 59%). On rare occasion, I may lower the thresholds. Also, in an effort to reward the student who makes significant improvement over the course of the term, a stellar grade on the final may overcome a deficiency on the midterm and improve a student's final grade.

Courtesies: As a courtesy to your instructor and fellow students, please arrive to class on time, turn your cell phones and electronic gadgets off during class, and pay attention in class.