Quiz 8 Name: Solutions

Math 202 F01 4/12/06

1. Begin evaluating $\int_C F \cdot d\mathbf{s}$, where $F(x,y) = (xy, y-x)$ and C is the straight-line path from $(4,4)$ to $(5,-2)$. You may leave your answer in a form where only Calculus I/II knowledge is needed to complete the work.

$\vec{r}(t) = (4,4) + t(5-4, -2-4) = (4,4) + t(1,-6) = (4+t, 4-6t)$

$d\vec{s} = \vec{r}'(t)dt = (1,-6)\,dt$ $\qquad 0 \le t \le 1$

$$\int_C F \cdot d\vec{s} = \int_0^1 \left((4+t)(4-6t), (4-6t)-(4+t)\right) \cdot (1,-6)\,dt$$

$$= \int_0^1 (4+t)(4-6t)1 + ((4-6t)-(4+t))(-6) \; dt$$

$$= \int_0^1 16 + 22t - 16t^2 \; dt$$

2. Find a potential function f for the vector field

$$F(x,y) = \left(2 + \frac{1}{xy} - y,\, 2y - x - \frac{\ln x}{y^2}\right),$$

and use it to evaluate $\int_C F \cdot d\mathbf{s}$ where C is parameterized by

$$\mathbf{r}(t) = \left(t^2, \cos(\pi t) + 3\right), \quad 1 \le t \le 2.$$

$\frac{\partial f}{\partial x} = 2 + \frac{1}{xy} - y$

so $f(x,y) = 2x + \frac{\ln x}{y} - xy + C(y)$

so $\frac{\partial f}{\partial y} = -\frac{\ln x}{y^2} - x + \frac{dC}{dy}(y) = 2y - x - \frac{\ln x}{y^2}$

Thus $\frac{dC}{dy}(y) = 2y$, so $C(y) = y^2 + D$

$$\boxed{f(x,y) = 2x + \frac{\ln x}{y} - xy + y^2 + D}$$

$\vec{r}(1) = (1,2)$

$\vec{r}(2) = (4,4)$

so

$\int_C F \cdot d\vec{r} = f(4,4) - f(1,2)$

$= 8 + \frac{\ln 4}{4} - 16 + 16 +$

$-\left(2 + \frac{\ln 1}{2} - 2 + 4\right)$

$= 4 - \frac{\ln 4}{4}$