MATH 202 Name :_____
Midterm 1 February 25, 2008

Instructions: Show all work for full credit.

1. (30 pts. — 6 pts. each) Three points, with coordinates

$$A = (1, 1, 0), \quad B = (0, 2, 1), \quad C = (2, 3, 0),$$

are the vertices of a triangle in 3-dimensional space.

(a) What is the length of the side joining A and B?

(b) Give a *unit* normal vector to the plane containing the triangle.

(c) Give an equation of the plane containing the triangle.

(d) What is the angle formed by the sides meeting at A? (You may leave your answer in a form involving inverse trigonometric functions, and you do not need to rationalize denominators.)

(e) What is the area of the triangle?

2. (10 pts.) In the plane, a constant force $\mathbf{F} = 2\mathbf{i} - \mathbf{j}$ N acts on a particle that is moved due east a total of 2 m. Find the work done.

**** Answer key has solution to a different problem. ****

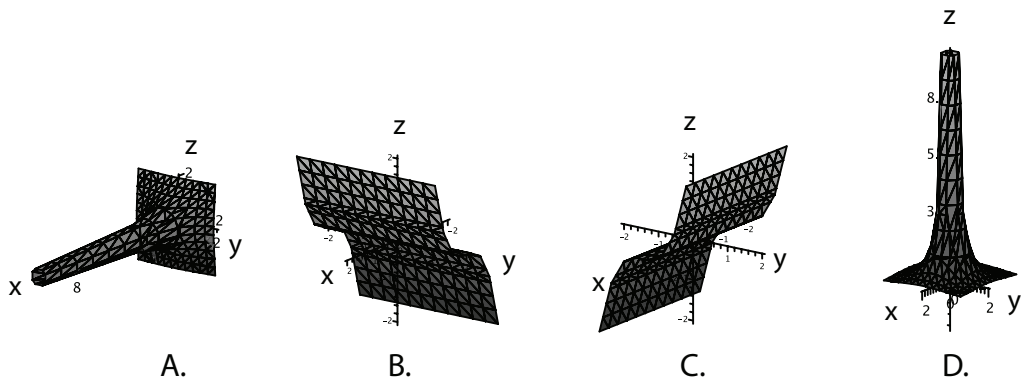
3. (10 pts.) In the plane, a particle moves so that it has constant acceleration $\mathbf{a}(t) = 2\mathbf{j}\ m/s^2$.

   At $t = 0$, it has velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}\ m/s$.

   At time $t = 1$, its position is $\mathbf{r}(1) = 2\mathbf{j}\ m$.

   Give a formula for its position, $\mathbf{r}(t)$, at all times t.

4. (10 pts. — 5 pts. each: 2 for answer, 3 for explanation) Match the equations with the appropriate graph. (Notice that there are more graphs than equations.) Explain your answer.



(a) $z = y^3$

(b) $f(x, y) = \dfrac{1}{x^2 + y^2}$

5. (10 pts. — 5 pts. each) Consider a point $\mathbf{p}$ with rectangular coordinates $(0, -3, 3)$.
Express $\mathbf{p}$ in:

(a) cylindrical coordinates

(b) spherical coordinates

6. (20 pts.) An object moves along a trajectory so that its position, as a function of time, is given by

$$\mathbf{r}(t) = (t^2,\, 2t,\, \ln(t)).$$

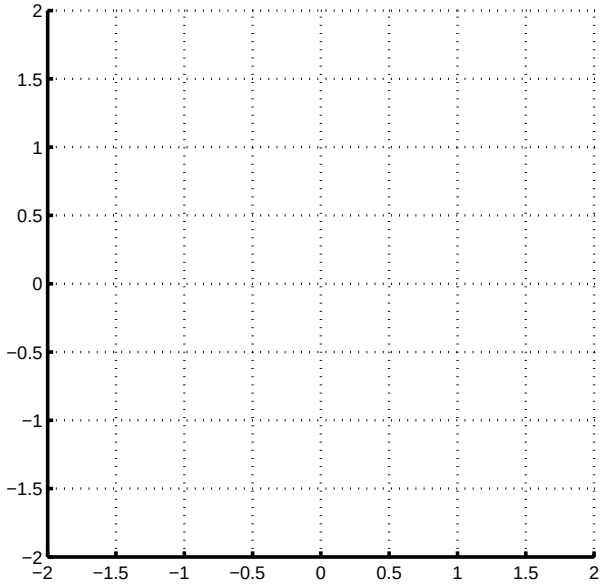
(a) (6 pts.) At what speed is it traveling at time $t = 2$?

(b) (8 pts.) What is the length of its trajectory between times $t = 1$ and $t = 2$?

(c) (6 pts.) Give a parameterization of the line tangent to the trajectory at $\mathbf{r}(2)$.

7. (10 pts.) The temperature (in $^\circ C$) at each point (x, y), $-2 \leq x,\, y, \leq 2$, on a 4×4 metal plate is given by $T(x, y) = 10 - x^2 + y$.

(a) (6 pts.) Draw a contour plot of T that shows the level curves (i.e, isotherms) where $T = 9,\ 10,$ and 11.



(b) (4 pts.) Using the contour plot above, indicate with an 'H' and 'C' the hottest and coldest points on the metal plate (with coordinates $-2 \leq x,\, y \leq 2$).