Galois Theory

Extra homework problems

due Thursday, March 12

1. Prove that $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic as fields.

2. Let

$$\sigma(n) = \sum_{d|n} d = \text{sum of divisors of } n, \quad \text{and}$$
$$\tau(n) = \sum_{d|n} 1 = \text{number of of divisors of } n.$$

   (a) Using the fact that σ and τ are multiplicative arithmetic functions, give formulas for these functions evaluated at prime powers p^a and then at positive integers n.

   (b) Find, with proof, the value of

$$\sum_{d|n} \mu\left(\frac{n}{d}\right) \sigma(d).$$

   (c) Find, with proof, the value of

$$\sum_{d|n} \mu\left(\frac{n}{d}\right) \tau(d).$$

   (d) Verify your answers to parts (b) and (c) by computing the sums for some well-chosen (by you) values of n.