HW # 27 (4.5) 20, 38, 70, 76, 78, 80 (4.6) 6, 12, 14, 16

(4.5)

20) $\left(\frac{1}{4}\right)^x = 75$

$4^{-x} = 75$

$\log 4^{-x} = \log 75$

$-x = \frac{\log 75}{\log 4} \Rightarrow x = \frac{-\log 75}{\log 4}$ $\boxed{\approx -3.1144}$

38) $\ln(2+x) = 1$

$2 + x = e^1$

$x \boxed{= e - 2 \approx 0.7183}$

70) $x^2e^x - 2e^x < 0$

$e^x(x^2 - 2) < 0$

$e^x(x - \sqrt{2})(x + \sqrt{2}) < 0$

Interval	$(-\infty, -\sqrt{2})$	$(-\sqrt{2}, \sqrt{2})$	$(\sqrt{2}, \infty)$
sign of e^x	+	+	+
sign of $(x-\sqrt{2})$	−	−	−
sign of $(x+\sqrt{2})$	−	+	+
sign of $e^x(x+\sqrt{2})(x-\sqrt{2})$	+	−	+

So, $\boxed{-\sqrt{2} < x < \sqrt{2}}$

76) $A(2) = 6500e^{0.06(2)}$ $\boxed{\approx \$7328.73}$

b $8000 = 6500e^{0.06t}$

$\frac{16}{13} = e^{0.06t}$

$\ln\frac{16}{13} = 0.06t \qquad t = \frac{\ln\frac{16}{13}}{0.06}$ $\boxed{\approx 3.46}$ so, it doubles every $3\frac{1}{4}$ years.

78) $5000 = 4000\left(1 + \frac{0.0975}{2}\right)^{2t}$

$1.25 = (1.04875)^{2t}$

$\log 1.25 = 2t \log 1.04875 \qquad t = \frac{\log 1.25}{2\log 1.04875}$ $\boxed{\approx 2.344}$ about $2\frac{1}{3}$ years.

80) $1435.77 = 1000\left(1 + \frac{r}{2}\right)^{2(4)}$

$1.43577 = \left(1 + \frac{r}{2}\right)^8$

$\sqrt[8]{1.43577} = 1 + \frac{r}{2}$

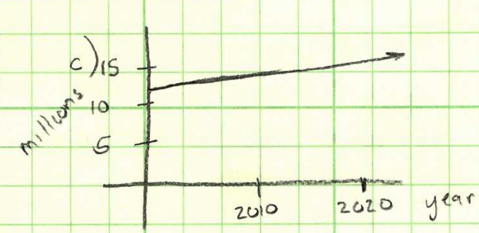
$\sqrt[8]{1.43577} - 1 = \frac{r}{2} \qquad r = 2\left(\sqrt[8]{1.43577} - 1\right)$ $\boxed{\approx 0.0925}$ so rate is about 9.25%

4.6 6, 12, 14, 16

6) $t=0$ in year 2000, n in millions ∴ $n_0 = 12$ $r = 0.012$

so, an exponential model is $\boxed{n(t) = 12e^{0.012t}}$

c)



b) in 2005, $t = 5$

$n(5) = 12e^{0.012(5)}$

$= 12.742$ or $\boxed{12{,}742{,}000 \text{ fish}}$

12) From the graph given, we see that $\boxed{\text{initial frog population was } 100}$

b) using $n(t) = n_0 e^{rt}$; $n_0 = 100$ given population at 2 years = 225,

$n(2) = 255 = 100\, e^{r(2)}$

$r = \frac{1}{2} \ln \frac{255}{100} \approx 0.4055$ so a model is $\boxed{n(t) = 100e^{0.4055t}}$

c) $n(15) = 100e^{0.4055(15)}$ $\boxed{\approx 43{,}812 \text{ frogs}}$

d) $n(t) = 100e^{0.4055t} = 75{,}000$

$e^{0.4055t} = 750$

$t = \frac{\ln 750}{0.4055}$ $\boxed{\approx 16.32}$ so, it'll take about $16\frac{1}{3}$ years for the population to be 75000

14) $n(t) = n_0 e^{rt}$

a) $n(2) = 400$, $n(6) = 25{,}600$

$n_0 e^{2r} = 400$; $n_0 e^{6r} = 25{,}600$

$\frac{n_0 e^{6r}}{n_0 e^{2r}} = \frac{25600}{600} = 64 \Rightarrow e^{4r} = 64$

$4r = \ln 64$

$r = \frac{1}{4} \ln 64$ $\boxed{\approx 1.04}$ so, growth rate is 104%

b) $n(t) = n_0 e^{\frac{1}{2}\ln 8 (t)}$ $r = \frac{1}{4}\ln 64 = \frac{1}{2} \ln 8$

$400 = n_0 e^{\ln 8} \Rightarrow n_0 = \frac{400}{e^{\ln 8}} = 400/8$ $\boxed{= 50}$ so initial size was 50

c) $n_0 = 50$ $r = 1.04$

$\boxed{n(t) = 50e^{1.04t}}$

d) $n(4.5) = 50e^{1.04(4.5)} = 50e^{4.68}$

$\boxed{\approx 5388.5}$

e) $n(t) = 50{,}000 = 50e^{1.04t}$

$1000 = e^{1.04t}$

$\ln 1000 = 1.04t$ $t = \frac{\ln 1000}{1.04}$ $\boxed{\approx 6.64}$

16) a) $2n_0 = n_0 e^{0.02t}$

$2 = e^{0.02t}$

$\ln 2 = 0.02t$ $t = 50 \ln 2 \approx 34.65$

$t = 34.65 + 1995 = 2029.65$

so, $\boxed{\text{it will double by } 2029}$

b) $3n_0 = n_0 e^{0.02t}$

$3 = e^{0.02t}$

$\ln 3 = 0.02t$ $t = 50 \ln 3 \approx 54.93$

$t = 54.93 + 1995 = 2049.93$

$\boxed{\text{tripled by year } 2050}$