H/w #12

15.1 #58

$$F(x,y,z) = y^2z^3 i + 2xyz^3 j + 3xy^2z^2 k$$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2z^3 & 2yxz^3 & 3xy^2z^2 \end{vmatrix} = 0$$

Conservative

$$f(x,y,z) = xy^2z^3 + K$$

15.1 #64

$$F(x,y) = xe^x i + ye^y i$$

$$\text{div } F(x,y) = \frac{\partial}{\partial x}(xe^x) + \frac{\partial}{\partial y}(ye^y) = xe^x + e^x + ye^y + e^y$$

#90

$$\text{div}(\text{curl } F) = 0$$

Let $F = Mi + Nj + Pk$

$$\text{curl } F = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right) i - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right) j + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) k$$

$$\text{div}(\text{curl } F) = \frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right] - \frac{\partial}{\partial y}\left[\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right] + \frac{\partial}{\partial z}\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right]$$

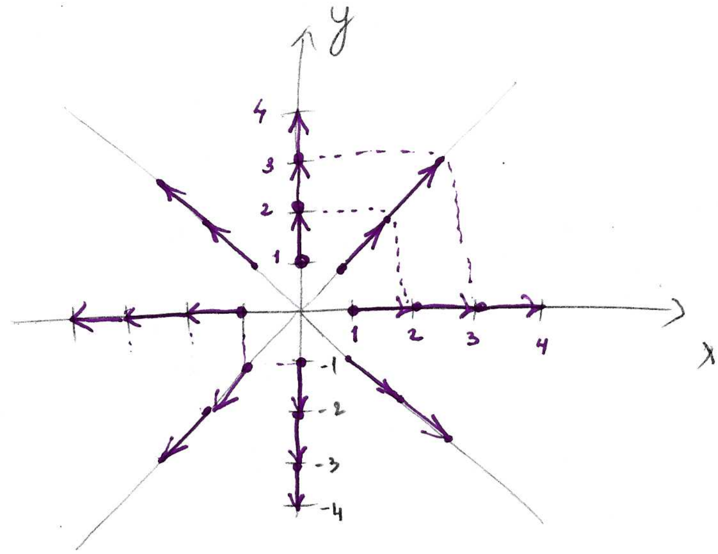
$$= \frac{\partial^2 P}{\partial x \partial y} - \frac{\partial^2 N}{\partial x \partial z} - \frac{\partial^2 P}{\partial y \partial x} + \frac{\partial^2 M}{\partial y \partial z} + \frac{\partial^2 N}{\partial z \partial x} - \frac{\partial^2 M}{\partial z \partial y} = 0$$

(because the mixed partials are equal)

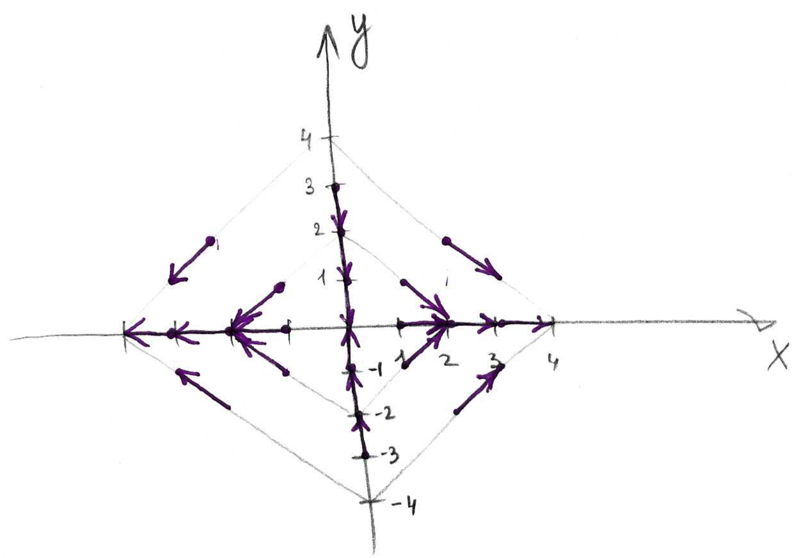
Sect. 15.1 # 94

H/w #12 contin.

a) $F(x,y) = \dfrac{xi + yj}{\sqrt{x^2+y^2}}$



b) $G(x,y) = \dfrac{xi - yj}{\sqrt{x^2+y^2}}$



c) All the vectors are unit vectors.
Those of F point away from origin.
Answers will vary.

Hw # 13

15.2 # 36

Find the work done by the force field F on a participle moving along the given path.

$F(x,y) = x^2 i - xy j$

$C: x = \cos^3 t \ , \ y = \sin^3 t$ from $(1,0)$ to $(0,1)$

$\overline{r(t)} = \cos^3 t i + \sin^3 t j \ , \ 0 \le t \le \frac{\pi}{2}$

$r'(t) = -3\cos^2 t \sin t i + 3\sin^2 t \cos t j$

$F(t) = \cos^6 t i - \cos^3 t \sin^3 t j$

$F.r' = -3\cos^8 t \sin t - 3\cos^4 t \sin^5 t = -3\cos^4 t \sin t (\cos^4 t + \sin^4 t)$

$= -3\cos^4 t \sin t [\cos^4 t + (1-\cos^2 t)^2] =$

$= -3\cos^4 t \sin t (2\cos^4 t - 2\cos^2 t + 1) =$

$= -6\cos^8 t \sin t + 6\cos^6 t \sin t - 3\cos^4 t \sin t$

$$\text{Work} = \int_C F.dr = \int_0^{\pi/2} [-6\cos^8 t \sin t + 6\cos^6 t \sin t - 3\cos^4 t \sin t] dt$$

$$= \left[\frac{2\cos^9 t}{3} - \frac{6\cos^7 t}{7} + \frac{3\cos^5 t}{5}\right]\Big|_0^{\pi/2} = -\frac{43}{105}$$

15.2 # 40

$F(x,y,z) = yz i + xz j + xy k$

$C:$ line from $(0,0,0)$ to $(5,3,2)$

$r(t) = 5t i + 3t j + 2t k \ , \ 0 \le t \le 1$

$r'(t) = 5i + 3j + 2k$

$F(t) = 6t^2 i + 10t^2 j + 15t^2 k$

$F.r' = 90t^2$

$$\text{Work} = \int_C F\,dr = \int_0^1 90t^2\,dt = 30$$