MATH 161
November 26, 2012

Name : SOLUTIONS

These problems will be collected at the end of class today. You will definitely need to study more topics than this activity covers. This is for you to get graded feedback from your instructor. Point values for each problem are indicated. The total value of this exercise is 25 points.

(4 pts.)

1. Use the laws of logarithms to expand the following expressions:

(a) $\log \sqrt[3]{\dfrac{x+3}{x^4(x+2)}}$

$= \log(x+3)^{\frac{1}{3}} - \log\left(x^4(x+2)\right)^{\frac{1}{3}}$

$= \frac{1}{3}\log(x+3) - \frac{4}{3}\log x - \frac{1}{3}\log(x+2)$

(b) $\log \sqrt[5]{\dfrac{y^2}{1-2y^3}}$ $= \frac{2}{5}\log y - \frac{1}{5}\log(1-2y^3)$

(4 pts.)

2. Combine into a single logarithm

(a) $\ln x - 2\ln(x^2+1) + \frac{1}{2}\ln(3-x^4)$

$= \ln\left(\dfrac{x(3-x^4)^{\frac{1}{2}}}{(x^2+1)^2}\right)$

$= \ln\left(\dfrac{x\sqrt{3-x^4}}{(x^2+1)^2}\right)$

(b) $\log(x^2+5) - \frac{1}{4}\log(x^3+7) - 2\log(x^2+5)$

$= -\frac{1}{4}\log(x^3+7) - \log(x^2+5)$

$= -\log(x^3+7)^{\frac{1}{4}} - \log(x^2+5)$

$= \log \dfrac{1}{\sqrt[4]{x^3+7}\,(x^2+5)}$

(4 pts.)

3. Solve

(a) $\log(x+2) + \log(x+1) = 1$

$\log\left((x+2)(x+1)\right) = 1$

$\log(x^2+3x+2) = 1$

$x^2+3x+2 = 10$

$x^2+3x-8 = 0$

$x = \dfrac{-3 \pm \sqrt{9-4(1)(-8)}}{2}$

$= \dfrac{-3 \pm \sqrt{9+32}}{2}$

$x = \dfrac{-3+\sqrt{41}}{2}, \dfrac{-3-\sqrt{41}}{2}$

≈ 1.7016

Does check!

does not check

(b) $4 + 3\log(2x) = 16$

$3\log(2x) = 12$

$\log(2x) = 4$

$10^4 = 2x$

$10000 = 2x$

$\boxed{5000 = x}$

Checks.

(c) $e^{2x+1} = e^5$

$2x+1 = 5$

$2x = 4$

$\boxed{x = 2}$

(d) $e^{2x} - 6e^x = 7$

$e^{2x} - 6e^x - 7 = 0$

$(e^x - 7)(e^x + 1) = 0$

$e^x = 7 \quad e^x = -1$

$\boxed{x = \ln 7} \approx 1.946$

$x = \ln(-1)$ Nonsense.

(8 pts.) 4. You invest \$5000 in an account and want it to grow to \$6000 in five years.

(a) What rate r of return must you realize, if the interest on this account is compounded quarterly? Round your answer to one decimal place. (For example, 5.1% is an acceptable answer.)

$P = 5000 \quad A(5) = 6000$

$A(t) = 5000\left(1 + \frac{r}{4}\right)^{4t}$

Putting this together

$6000 = 5000\left(1 + \frac{r}{4}\right)^{4(5)}$

$6000 = 5000\left(1 + \frac{r}{4}\right)^{20}$

$1.2 = \left(1 + \frac{r}{4}\right)^{20}$

Take 20th root

$\sqrt[20]{1.2} = 1 + \frac{r}{4}$

$r = 4\left(\sqrt[20]{1.2} - 1\right) \approx .036631$

$\boxed{r = 3.7\%}$ rounded

(b) What rate r of return must you realize, if the interest on this account is compounded continuously?

$P = 5000 \quad A = 5000e^{rt}$

$A(5) = 6000$

Thus, $6000 = 5000\,e^{r(5)}$

$1.2 = e^{5r}$

$\ln(1.2) = 5r$

$\frac{1}{5}\ln(1.2) = r$

$r = \frac{1}{5}\ln(1.2) \approx .03646$

$\boxed{r = 3.6\%}$

(5pts) 5. How long will it take for an investment of \$1000 to double in value, if the interest rate is 3% per year, compounded quarterly?

$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{.03}{4}\right)^{4t} = 1000\left(1.0075\right)^{4t}$

Doubling means: FIND t when $A(t) = \$2000$.

$2000 = 1000\left(1.0075\right)^{4t}$

$2 = \left(1.0075\right)^{4t}$

$\log 2 = 4t \log(1.0075)$

$\frac{\log 2}{4\log(1.0075)} = t$

$\boxed{t \approx 23.19 \text{ years}}$