Homework 3 extra problem

due: February 11, 2019

$\sqrt{2}$. Prove that the following are equivalent for a field L:

(a) Every polynomial of positive degree over L has a root in L.

(b) Every polynomial in $L[x]$ has all its roots in L.

(c) The only irreducible polynomials over L are the linear ones: $ax + b$, $a \neq 0, a, b \in L$.

(d) If M is an algebraic extension of L, then $M = L$.

Note that if any of these conditions hold, then L is algebraically closed.