Quiz 9 Name: Solutions
Math 202 F01 4/20/05

1. For $F(x,y,z) = (xz + y, x + yz, x^2 + y^2)$:

   (a) Compute curl $F(1,0,0)$.

   $$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz+y & x+yz & x^2+y^2 \end{vmatrix} = (2y - y, -(2x - x), 1 - 1) = (y, -x, 0)$$

   $$\nabla \times F(1,0,0) = (0,-1,0)$$

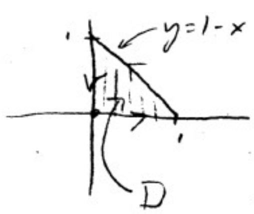
   (b) Assuming F represents a velocity field for a fluid flowing, interpret the result in part (a). (What does it tell you about the vector field?) A small "paddle ball" placed at $(1,0,0)$ will rotate as shown around an axis in the negative y direction

2. Use Green's theorem to evaluate

$$\oint_C F \cdot d\mathbf{s},$$

where $F(x,y) = (y^2, xy)$ and C is the boundary of the triangle with vertices $(0,0)$, $(1,0)$, and $(0,1)$, traced in a counterclockwise direction.

$$\oint_{C=\partial D} y^2\,dx + xy\,dy = \iint_D \frac{\partial xy}{\partial x} - \frac{\partial y^2}{\partial y}\,dA = \iint_D y - 2y\,dA = \iint_D -y\,dA$$



$$= \int_0^1 \int_0^{1-x} -y\,dy\,dx = \int_0^1 -\frac{y^2}{2}\Big|_0^{1-x} dx = \int_0^1 -\frac{(1-x)^2}{2}\,dx$$

$$= \frac{(1-x)^3}{6}\Big|_0^1 = 0 - \frac{1}{6} = -\frac{1}{6}$$