

Instructions: Do these problems and redo ALL exam problems. A concise set of review problems for all topics is available on the web too.

1. **Home affordability:** You realize that you can pay \$500 per month towards a mortgage after a \$50,000 down payment. If you take out a 30 year loan at an interest rate of 5% per year on the unpaid balance, with interest computations made at the end of each month,

- a) How expensive a house can you afford? (Remember to include the down payment and the loan amount.)
b) How much do you pay in interest over the life of the loan?

a) Present value of an Annuity with $R=500$, $i = \frac{r}{n} = \frac{.05}{12}$, $n=360$

$$P = 500 \left(\frac{1 - \left(1 + \frac{.05}{12}\right)^{-360}}{.05/12} \right) \approx 93140.81 = \$93,140.81$$

↑
loan amount

You can afford \$50,000 + \$93,140.81 or

\$193,140.81 house

b) Future value of an Annuity with $R=500$, $i = \frac{.05}{12}$, $n=360$

$$S = 500 \left(\frac{\left(1 + \frac{.05}{12}\right)^{360} - 1}{.05/12} \right) \approx \text{span style="border: 1px solid black; padding: 2px;">\$416,129.32}$$

Therefore, the interest payment is \$322,988.51

↑
Answer:

5.2 #27. $\$167,712 - \$203,390$

5.2: 22, 27 5.3: 18, 22, 28

3. Solve the following equations or inequalities. Check your answers when appropriate.

(a) $\sqrt{x-1} = 3-x$

$$\begin{aligned} (x-1) &= (3-x)^2 && \text{check: } x=2 \checkmark \\ x-1 &= 9-6x+x^2 && \sqrt{2-1} \stackrel{?}{=} 3-2 \\ 0 &= x^2-7x+10 && 1=1 \checkmark \\ 0 &= (x-2)(x-5) && x=5: \quad \sqrt{5-1} \stackrel{?}{=} 3-5 \\ x &= \boxed{2, 5} && 2 \stackrel{?}{=} -2 \quad \text{No} \end{aligned}$$

(c) $\log_2(x-6) = 3 - \log_2(x+1)$

$$\begin{aligned} \log_2((x-6)(x+1)) &= 3 \\ \log_2(x^2-5x-6) &= 3 \\ x^2-5x-6 &= 8 \\ x^2-5x-14 &= 0 \\ (x-7)(x+2) &= 0 \\ x &= 7, -2 \end{aligned}$$

check:

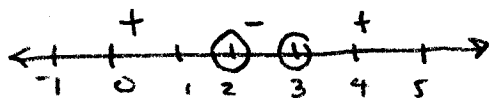
$$\begin{aligned} x=7: & \quad \log_2(7-6) \stackrel{?}{=} 3 - \log_2(8) \\ 0 &= 0 \checkmark \end{aligned}$$

$$\begin{aligned} x=-2: & \quad \log_2(-2-6) \text{ undefined} \\ & \quad \times \end{aligned}$$

$$\boxed{x=7}$$

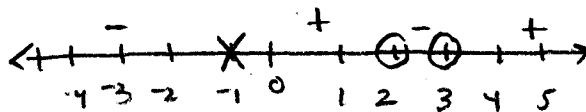
(b) $x^2 \leq 5x-6$

$$\begin{aligned} x^2-5x+6 &\leq 0 && \text{Test points} \\ \text{Zeros: } x^2-5x+6 &= 0 && x=0: +6 < 0 \\ (x-3)(x-2) &= 0 && x=\frac{3}{2}, \left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) + 6 \\ x &= 2, 3 && = \frac{9}{4} - \frac{15}{2} + 6 = \frac{3-30+24}{4} = -\frac{3}{4} < 0 \end{aligned}$$



$$\begin{aligned} x=4: & \quad 4^2 - 5(4) + 6 \\ &= 16 - 20 + 6 = 2 > 0 \end{aligned}$$

Answer: $[-2, 3]$



Answer:

$$(-\infty, -1) \cup [2, 3]$$

4. Solve the following equations. Round your answers to two decimal places.

(a) $10^{x+1} = 2^{3x-1}$

on exam

(b) $\frac{10^{x+1}}{10^{2x-3}} = 5$

on exam

(c) $e^{3x+1} = 5^{-x}$

$$\ln e^{3x+1} = \ln 5^{-x}$$

$$3x+1 = -x \ln 5$$

$$3x + x \ln 5 = -1$$

$$x(3 + \ln 5) = -1$$

$$x = \frac{-1}{3 + \ln 5} \approx -0.22$$

(d) $\frac{e^{2x+7}}{e^{x-1}} = 5$

$$e^{2x+7-(x-1)} = 5$$

$$e^{2x+7-x+1} = 5$$

$$e^{x+8} = 5$$

$$x+8 = \ln 5$$

$$x = \ln 5 - 8 \approx -6.39$$

2. Simplify.

$$(a) \log(.001) = \boxed{-3}$$

$$(g) \ln\left(\frac{1}{e^2}\right) = \boxed{-2}$$

$$(b) \log(4) + \log(25) = \boxed{2}$$

$$(h) \ln(10e^5) - \ln(10) = \boxed{5}$$

$$(c) 5^{\log_5(7)} = \boxed{7}$$

$$(i) 2\log 5 + 5\log 2 + \log 25 - \log 5 - \log 4$$

$$\begin{aligned} &= \log 5 + \log 2^5 + \log 25 - \log 4 \\ &= \log\left(\frac{5^1 \cdot 2^2 \cdot 2^5}{2^2}\right) = \log(5^3 \cdot 2^3) \\ &= \log 10^3 = \boxed{3} \end{aligned}$$

$$(d) e^{\ln(\log 100)} = \boxed{2}$$

$$(j) \log_5 25^{42} = \boxed{84}$$

$$(e) \log(100^x) = \boxed{2x}$$

$$\begin{aligned} (k) \log(\log(100^{500})) &= \log(\log(10^2)^{500}) \\ &= \log(\log 10^{1000}) = \log(1000) \\ &= \boxed{3} \end{aligned}$$

$$(f) \log(1) = \boxed{0}$$

$$(l) \log_5(\sqrt[4]{125}) = x$$

$$5^x = \sqrt[4]{125}$$

$$5^x = 5^{3/4}$$

$$x = \boxed{3/4}$$

5.2 #22. \$150,000 rollover

$$A = 150,000 \left(1 + \frac{.08}{4}\right)^{80} \approx \$487,543.92$$

\$3000 per quarter

$$S = 3000 \left(\frac{\left(1 + \frac{.08}{4}\right)^{80} - 1}{.08/4} \right) \approx \$581,315.87$$

$$\text{Total savings} = \boxed{\$1,068,859.79}$$

5.3 #18. $n = 9.25(12) = 111$ payments $r = .0625$ $m = 12$ $i = \frac{.0625}{12}$

Use amortization formula:

$$P = 42,000$$

$$R = \frac{Pi}{(1 - (1+i)^{-n})} = \frac{42,000 \left(\frac{.0625}{12}\right)}{1 - \left(1 + \frac{.0625}{12}\right)^{-111}} \approx \boxed{\$499.19}$$

#22. $n = 25(12) = 300$ $P = 96,000$ $m = 12$ $i = \frac{.09}{12}$

Use amortization formula

$$R = \frac{96,000 \left(\frac{.09}{12}\right)}{1 - \left(1 + \frac{.09}{12}\right)^{-300}} \approx \$805.63$$

#28. done in class