MATH 371 Name : SOLUTIONS

Quiz 5 November 24, 2008

Instructions: Round all answers to two significant digits (i.e. two decimal places, or if your answer is very small, give the first two non-zero digits following the decimal point). Also, you must show your work (integration, etc) to get full credit. There are 10 points on this quiz. Good luck.

1. (2 pts.) Suppose

   X_1: *gender coded by* $M = 0$, $F = 1$

   X_2: *the number of times a person takes a driver's test before passing*

   have joint probability function given by

		X_2 1	2	3	
X_1	$M = 0$	.23	.19	.01	.43
	$F = 1$	.33	.19	.05	
		.56			

   Determine, with proof, if the variables X_1 and X_2 are independent.

   $p_1(0) = P(X_1 = 0) = .43$

   $p_2(1) = P(X_2 = 0) = .56$

   $p_1(0)\,p_2(1) = (.43)(.56) = .2408 \neq .23 = p(0,1)$

   $\Rightarrow$ DEPENDENT

2. (3 pts.) Suppose that X_1 and X_2 are independent binomial random variables drawn from the same binomial probability distribution, $X_1, X_2 \sim \text{Binom}(10, .3)$.

   (a) Give the expected value of the average $\overline{X} = \dfrac{X_1 + X_2}{2}$.

   2pts. $\mathbb{E}(\bar{X}) = \mathbb{E}\left(\frac{X_1 + X_2}{2}\right) = \frac{1}{2}\left(\mathbb{E}(X_1) + \mathbb{E}(X_2)\right) = \frac{1}{2}\left(10(.3)(2)\right) = \boxed{3}$

   or from Monday's class, $\mathbb{E}(\bar{X}) = \mu = np = 3$.

   (b) Give the covariance $\text{Cov}(X_1, X_2)$.

   1 pt. Since X_1 and X_2 are independent, $\text{Cov}(X_1, X_2) = 0$.

3. (2 pts.) On a special holiday, 30% of revelers drink beer with dinner, 42% drink wine with dinner, 3% drink other sorts of spirits with dinner, and 25% drink no alcohol with dinner. Suppose a random sample of size 10 is drawn from this population of revelers. What is the probability that 4 drink beer, 3 drink wine, 1 drinks spirits, and 2 do not drink alcohol with dinner?

Multinomial

$$p(4,3,1,2) = \frac{10!}{4!\,3!\,1!\,2!}(.3)^4(.42)^3(.03)^1(.25)^2$$

$$\approx .014$$

4. (3 pts.) Let Y_1 and Y_2 denote the lifetime in years of two electronic components in an iPod and suppose the joint density function is given by

$$f(y_1, y_2) = \begin{cases} \frac{1}{210} y_1(y_2^2+1), & 0 \le y_1 \le 3,\ 0 \le y_2 \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Set up, but do not evaluate, an integral to compute the expected value of the product Y_1Y_2, $E(Y_1, Y_2)$.

1 pt.

$$\int_0^5\int_0^3 y_1 y_2 \frac{1}{210} y_1 (y_2^2+1)\,dy_1\,dy_2 = E(Y_1Y_2)$$

$$= \frac{1}{210}\int_0^5\int_0^3 y_1^2 y_2 (y_2^2+1)\,dy_1\,dy_2$$

(b) Are Y_1 and Y_2 independent? Explain briefly.

2 pts.

Yes. The domain of $f(y_1, y_2)$ is a rectangle and $f(y_1,y_2) = \left[\frac{1}{210} y_1\right]\left[y_2^2+1\right] = g(y_1)h(y_2)$ is a factorization of the joint dencity function

$\therefore$ Y_1 and Y_2 are independent.