MATH 202 Name : SOLUTIONS

Quiz 5 October 19, 2009

Instructions: You get one point for taking this quiz. Each problem is worth one point and there is no partial credit on this quiz.

1. Consider the function $f(x, y) = e^{-xy} \cos x$.

   (a) Compute the value of $f(\pi, 0)$.

   $f(\pi,0) = e^{-(\pi)(0)} \cos(\pi)$

   $= 1(-1) = \boxed{-1}$

   (b) Find the equation of the tangent plane to the surface defined by $f(x, y)$ at the point $P(\pi, 0, f(\pi, 0))$.

   $P = (\pi, 0, -1)$ from (a).

   $f_x(x,y) = e^{-xy}(-\sin x) + e^{-xy}(-y)\cos x$

   $= e^{-xy}[-\sin x - y\cos x]$

   $\therefore f_x(\pi,0) = e^{-\pi(0)}[-\sin\pi - (0)\cos(\pi)]$

   $= 1[0-0] = \underline{0}$

   $f_y(x,y) = -xe^{-xy}\cos x \quad \therefore f_y(\pi,0) =$

   $= -\pi e^{-(\pi)(0)}\cos\pi = \underline{\pi}$

   $z = f(\pi,0) + f_x(\pi,0)(x-\pi) + f_y(\pi,0)(y-0)$

   $= -1 + 0(x-\pi) + \pi(y-0)$

   $\boxed{z = -1 + \pi y}$

2. Consider the function $g(x, y) = \sqrt{x + \cos^2 y}$.

   (a) Give the best linear approximation $L(x, y)$ to function $g(x, y)$ at the point $(0, 0)$.

   This means give the equation of the tangent plane.

   $g(x,y) = (x+\cos^2 y)^{1/2} \Rightarrow g_x(x,y) = \frac{1}{2}(x+\cos^2 y)^{-1/2}(1) = \frac{1}{2\sqrt{x+\cos^2 y}}$

   $g_y(x,y) = \frac{1}{2}(x+\cos^2 y)^{-1/2}(2\cos y(-\sin y))$

   $= \frac{-\cos y \sin y}{\sqrt{x+\cos^2 y}}$

   $g(0,0) = \sqrt{0+\cos^2(0)}$

   $= 1$

   Moreover, $g_x(0,0) = \frac{1}{2\sqrt{0+\cos^2(0)}} = \frac{1}{2}$

   and $g_y(0,0) = \frac{-\cos(0)\sin(0)}{\sqrt{0+\cos^2(0)}}$

   $= \underline{0}$

   Thus, $L(x,y) = g(0,0) + g_x(0,0)x + g_y(0,0)y$

   $\boxed{= 1 + \frac{1}{2}x}$

   (b) Use $L(x, y)$ to estimate $g(.01, -.02)$.

   $L(.01, -.02) \simeq g(.01, -.02)$

   $= 1 + \frac{1}{2}(.01) = \boxed{1.005}$