Comments: 1. Use arrows on vectors. $\vec{a} \equiv$ vector $a \equiv$ scalar

2. To show $\vec{n}_1$ is not parallel to $\vec{n}_2$, either show $\vec{n}_1 \times \vec{n}_2 = \vec{0}$ or $\vec{n}_1 \neq c\vec{n}_2$ etc.

MATH 202 3. Be able to distinguish equations of lines from planes. Name: SOLUTIONS

Quiz 2 January 27, 2020

Instructions: Five points total.

1. (2 pts.) Find, in both degrees and radians, the angle θ between the vectors

$$\mathbf{v} = \langle 2, 2 \rangle \quad \text{and} \quad \mathbf{w} = \langle -1-\sqrt{3},\ \sqrt{3}-1 \rangle.$$

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}|\,|\vec{w}|} = \frac{\langle 2,2\rangle \cdot \langle -1-\sqrt{3}, \sqrt{3}-1\rangle}{\sqrt{2^2+2^2}\,\sqrt{(-1-\sqrt{3})^2+(\sqrt{3}-1)^2}} = \frac{-2-2\sqrt{3}+2\sqrt{3}-2}{\sqrt{8}\cdot\sqrt{1+2\sqrt{3}+3+3-2\sqrt{3}+1}}$$

$$= \frac{-4}{\sqrt{8}\,\sqrt{8}} = -\frac{4}{8} = -\frac{1}{2}. \qquad \theta = \arccos\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3} = 120^\circ}$$

2. (3 pts.) Justify that the two planes

Plane 1: $y + 2z = 1$
Plane 2: $x + 3y - z = 3$

are **NOT** parallel. Then find the equation of the line ℓ of intersection. (You can give your answer in either vector or parametric form.)

The normal vectors are $\vec{n}_1 = \langle 0,1,2\rangle$ and $\vec{n}_2 = \langle 1,3,-1\rangle$. Since $\vec{n}_1 \neq c\vec{n}_2$ for any non-zero scalar, these vectors, and therefore the planes, are NOT parallel.

For the line ℓ of intersection: By inspection, $P(0,1,0)$ is on both planes.

A direction vector $\vec{v}$ is $\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 1 & 3 & -1 \end{vmatrix} = (-1-6)\hat{i} - (0-2)\hat{j} + (0-1)\hat{k}$

$= \langle -7, 2, -1\rangle$

The vector equation of ℓ is $\langle -7t, 1+2t, -t\rangle$ for $t \in \mathbb{R}$

or the parametric equations are $x(t) = -7t \quad y(t) = 1+2t \quad z(t) = -t \quad t \in \mathbb{R}$