SOLUTIONS TO PRACTICE PROBLEMS FOR THE FINAL EXAM

1. (a) $(e-1)/2$. (b) $(1.136, 1.652)$. **2.** $1/2$.

3. (a) $3\pi/2$. (b) $\left(\frac{\pi}{8}\right)\ln 5$. **4.** $1/12$; $(2/5, 1/5, 2/5)$.

5.

$$(a) \quad \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} (x+2y+z)\, dzdydx,$$

$$(b) \quad \int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r(r\cos\theta + 2r\sin\theta + z)\, dzdrd\theta,$$

$$(c) \quad \int_0^{2\pi} \int_0^{\pi/4} \int_0^2 \rho^3(\sin\varphi\cos\theta + 2\sin\varphi\sin\theta + \cos\varphi)\sin\varphi\, d\rho d\varphi d\theta.$$

6. (a) (a) $2ma^3/3$. (b) $\pi/14$.

7.

$$\int_0^{\pi/2} \int_1^2 \int_0^{\sqrt{4-r^2}} r(z^2+r)\, dzdrd\theta.$$

8.

$$\int_0^{2\pi} \int_0^2 \int_{-\sqrt{16-r^2+6r\sin\theta}}^{\sqrt{16-r^2+6r\sin\theta}} r\, dzdrd\theta$$

9. (i) $\pi - 9/2$. (ii) $-3\pi/4$. The force field is not conservative because $\int_c \vec{F}\cdot d\vec{r}$ is not path independnt.

10. $4/5$.

11. (a) 2π, $\vec{F}$ is not conservative. (b) curl $\vec{F} = 0$. This does not contradict Green's Theorem because the partial derivatives of $\vec{F}$ are not continuous at the origin.

12. (a) 10. (b) 17.

13. No, since div(curl $\vec{G}$) must be 0 but div$(2x\,\vec{i} + 3yz\,\vec{j} - xz^2\,\vec{k}) = 2 + 3z - 2xz \neq 0$.

14. (a) $(2a/\pi, 2a/\pi)$. (b) $(3a/8, 3a/8, 3a/8)$. **15.** $500,000\pi$.

16. 11π. **17.** $-1/2$. **18.** 4π.