MATH 161 Name :____________________________

Chapter 2 Review Problems October 24, 2012

Chapter 3.1-3.3 Review Problems

Section 2.1: Functions Main ideas: evaluation, domains

1. Evaluate the functions $f(x) = \begin{cases} x^2 - 1, & \text{for } x > 2 \\ \frac{1}{x^2+2}, & \text{for } x \le 2 \end{cases}$, and $g(x) = \dfrac{x^2 - 4}{2x^2 - 9x - 5}$ at $x = -2, 0, \frac{1}{2}, 1, 5$.

2. Give the domain of the following functions:

   (a) $\dfrac{\sqrt{x-2}}{x^2 - 2x - 3}$ (b) $\sqrt[3]{t-1}$ (c) $\sqrt[4]{2y+5}$ (d) $x + \dfrac{1}{x}$

Section 2.2: Graphs of functions Main ideas: sketch by plotting points, domain and range from graphs, vertical line test

1. Sketch the following functions. Give the domain and range.

   (a) $f(x) = 2x^2 - 3$ (b) $g(x) = |x+3|$ (c) $h(x) = \sqrt{2x+1}$ (d) $h(x) = (x+5)^3$

   (e) For each of the functions above, describe the function as a transformation. For example, '*the function is a vertical translation by 10 units up.*'

2. State the *Vertical Line Test* for a function. Now sketch a graph that a) IS a function and b) IS NOT a function.

Section 2.3: Local Minima and Maxima of graphs, Increasing and decreasing functions Main ideas: You will need to know and use the definitions of minima, maxima, and intervals of increase and decrease.

1. (a) Textbook 1-4 (b) Textbook 19, 21 (c) Textbook 31, 33 (d) Textbook 43

Section 2.4: Average rate of change of a function Main ideas: Understand how to compute average rates of change and to interpret them as slopes of secant lines.

1. If an object is dropped from a tall building, the distance it has traveled after t seconds is given by $d(t) = 16t^2$ ft.

   (a) Find its average speed between (i) $t = 1$ second and $t = 5$ seconds (ii) $t = 2$ s and $t = 6$ s

   (b) Sketch a graph of $d(t)$ for $t \ge 0$. Explain the meaning of your answer to (i) and (ii) above. Your answer should include words like 'slope' and include the sketch of some line segments.

Section 2.5: Transformations of Functions Main ideas: Understand horizontal and vertical translations of graphs, dilations/contractions, reflections.

1. A function $f(x)$ is given. Explain how to obtain the formula for the

   (a) y-axis reflection of $f(x)$ (b) the x-axis reflection of $f(x)$

   (c) a horizontal translation of $f(x)$ five units left (d) a vertical translation of $f(x)$ ten units down

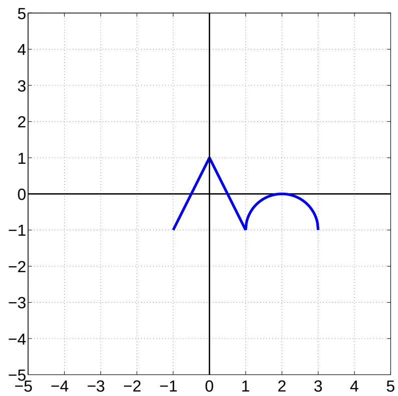
2. Starting with the function $y = x^3$. Graph the following. Include all intercepts on your sketches.

   (a) $y = -(x+2)^3 + 1$ (b) $y = -(-x+2)^3 + 1$ (c) $y = -2(x+1)^3$

3. Starting with the function $y = \sqrt{x}$. Graph the following. Include all intercepts on your sketches.
   (a) $y = -\sqrt{-x+4}+1$ (b) $y = \sqrt{3-x}-1$

4. Starting with the function $g(x) = |x|$. Give the formula for the function obtained by the following sequence of transformations. Then sketch the graphs of these functions.
   (a) a horizontal translation 4 units right, then a y-axis reflection, then a vertical translation 3 units down
   (b) an x-axis translation, followed by a vertical translation 5 units up.

5. Below is the graph of the function $y = f(x)$. Graph
   (a) $y = -f(-x)$ (b) $y = -f(x-3)-1$ (c) $y = f(-x)$ (d) $y = -2f(x)$ (e) $y = |f(x)|$



Section 2.6: Combining Functions Main ideas: Adding, subtracting, multiplying, dividing functions. VERY important: composition of functions.

1. Let $f(x) = |x+1|$, $g(x) = x^3 - 1$, and $h(x) = \sqrt[3]{x+1}$. Compute (if possible)
   (a) $f(g(1))$, $f(g(-1))$, $g(f(1))$, $g(f(-1))$, $g(h(\pi))$, $h(g(\pi))$, $h(g(x))$, $f(g(h(7)))$, $h(g(f(2)))$
   (b) $(f+g)(1)$, $(f-g)(2)$, $(gh)(-1)$, $(\frac{f}{g})(-4)$, $(\frac{f}{h})(26)$
   (c) $(f \circ g)(x)$, $(g \circ g)(x)$, $(h \circ f)(x)$, $(g \circ h)(2)$, $(f \circ h)(x)$
   (d) The domains of $(\frac{g}{f})(x)$, $(g \circ h)(x)$, $(\frac{g}{h})(x)$

Section 2.7: 1-1 functions, inverse functions Main ideas: Horizontal line test; Definition of 1-1 function; Finding inverses of 1-1 functions; Graphing inverses of functions

1. Give the definitions of 1-1 *function* and *inverse of a function* $f(x)$.

2. State the horizontal line test. Determine if the functions below are 1-1 or not.

3. Suppose $f(2) = 3$, $f(4) = 10$, and $f(5) = 2$ and that f^{-1} exists. Find, if possible,
   (a) $f^{-1}(2)$, (b) $f^{-1}(3)$, (c) $f^{-1}(4)$, (d) $f^{-1}(10)$, (e) $(f \circ f)(5)$ (f) $(f \circ f^{-1})(x)$

4. Find the inverse of
   (a) $g(x) = 2x+1$, (b) $h(x) = \dfrac{4x-2}{3x+1}$, (c) $p(t) = 4 - \sqrt[3]{t}$.

5. Sketch a graph of $g(x) = 2x - 1$. Show that $g(x)$ is 1-1. Then sketch a graph of $g^{-1}(x)$. Finally, give a formula for $g^{-1}(x)$.

Section 3.1: Quadratic functions and models Main ideas: Plotting parabolas, putting a quadratic polynomial in 'standard form' by completing the square, find the coordinates of minima and maxima, find the maximum/minimum value

1. Consider the quadratic polynomial functions below. For each of these, (i) find the coordinates of the vertex; (ii) sketch the function; (iii) find the maximum or minimum value; (iv) state the domain and range.

   (a) $f(x) = x^2 + +4x + 1$ (b) $g(x) = -x^2 + 6x + 5$, (c) $h(x) = 3 - 4x - 4x^2$,

2. Find the maximum or minimum value of the function

   (a) $h(t) = 3 - x - \frac{1}{2}x^2$, (b) $h(t) = 100x(x - 150)$

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Section 3.2: Polynomial functions and their graphs Main ideas: Sketching polynomials, determining end behavior, using zeros and sign charts to graph polynomials

1. Determine the 'end behavior' of the following polynomials. Find the zeroes of the polynomials. Finally, sketch the graph of the polynomial. Make sure your graph displays all intercepts and exhibits the correct end behavior.

   (a) $g(x) = -2x^4 - x^3 + 3x^2$ (b) $h(t) = x^3 - 2x^2 - 4x + 8$

Section 3.3: Dividing polynomials Main ideas: Long division of polynomials, synthetic division. Vocabulary: quotient, remainder, divisor, dividend

1. Use long division to perform the following divisions:

   (a) $\dfrac{4x^3 + 2x^2 - 2x - 3}{2x + 1}$ (b) $\dfrac{2x^5 - 7x^4 - 13}{4x^2 - 6x + 8}$ (c) $\dfrac{x^6 + x^4 + x^2 + 1}{x^2 + 1}$

2. Use synthetic division to find the quotient and the remainder.

   (a) $\dfrac{x^2 - 5x + 4}{x - 3}$ (b) $\dfrac{x^3 + 2x^2 + 2x + 1}{x + 2}$