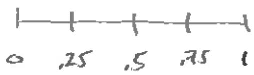
§5.2 #1-3   §5.3 1-3, 9, 11

§5.2 #1. $I=\int_0^1 x(1-x^2)\,dx$ Find $T_4(f)$ and $T_4^C(f)$.



$$T_4(f)=\frac{1}{2}(h)\left[f(0)+2f(.25)+2f(.5)+2f(.75)+f(1)\right]$$

$$=\frac{1}{2}\left(\frac{1}{4}\right)\left[0+2\left(\frac{15}{64}\right)+2\left(\frac{3}{8}\right)+2(21/64)+0\right]=15/64\approx\boxed{.2344}$$

$$T_4^C(f)=T_4-\frac{1}{12}h^2(f'(1)-f'(0))$$ $f(x)=-x^3+x$ $f'(x)=-3x^2+1$ $f'(0)=1$ $f'(1)=-2$

$$=\frac{15}{64}-\frac{1}{12}\left(\frac{1}{4}\right)^2(-2-1)=15/64+3/192=\frac{1}{4}=\boxed{.25}$$ ← Exact

#2. $I=\int_0^1\frac{dx}{\sqrt{1+x^4}}$ $f(x)=(1+x^4)^{-1/2}$ $f'(x)=-\frac{1}{2}(1+x^4)^{-3/2}4x^2=-2x^2(1+x^4)^{-3/2}$

$\therefore f'(1)=-2(2)^{-3/2}=-\frac{1}{\sqrt{2}}$ $f'(0)=0$

$$T_4(f)=\frac{1}{2}\left(\frac{1}{4}\right)\left[f(0)+2f(.25)+2f(.5)+2f(.75)+f(1)\right]$$

$$\approx\frac{1}{8}\left[1+2(.9981)+2(.9701)+2(.8716)+1/\sqrt{2}\right]\approx\boxed{.9233}$$

$$T_4^C(f)=T_4(f)-\frac{1}{12}(h)^2(f'(1)-f'(0))\approx .9233-\frac{1}{12}\frac{1}{16}\left(-\frac{1}{\sqrt{2}}\right)=.9233+\frac{1}{192\sqrt{2}}\approx\boxed{.9270}$$

↑ good

#3. $I=\int_0^1\ln(1+x)\,dx=2\ln 2-1$ $f(x)=\ln(1+x)$ $f'(x)=\frac{1}{1+x}$

$f'(1)=\frac{1}{2}$ $f'(0)=1$

$$T_4(f)=\frac{1}{2}\left(\frac{1}{4}\right)\left[f(0)+2f(.25)+2f(.5)+2f(.75)+f(1)\right]$$

$$=\frac{1}{8}\left[0+2\ln(1.25)+2\ln(1.5)+2\ln(1.75)+\ln 2\right]\approx .3837$$

$$T_4^C(f)=T_4(f)-\frac{1}{12}h^2(f'(1)-f'(0))\approx .3837-\frac{1}{12}\left(\frac{1}{16}\right)\left(\frac{1}{2}-1\right)=.3837+\frac{1}{384}\approx .3863$$

↑ close to $2\ln 2-1$

§5.3

#1. $I = \int_0^1 -x^3 + x \, dx$ Since the integrand is a cubic, Simpson's Rule is exact.

$h = 1/4$ $S_4(f) = \frac{h}{3}\left(f(0) + 4f(.25) + 2f(.5) + 4f(.75) + f(1)\right)$

$= \frac{1}{12}\left(0 + 4f(.25) + 2f(.5) + 4f(.75) + 0\right] = .25$

#2. $I = \int_0^1 \frac{1}{\sqrt{1+x^2}} dx$ See last page too

$S_4(f) = \frac{1}{12}\left[f(0) + 4f(.25) + 2f(.5) + 4f(.75) + f(1)\right]$

$\approx \frac{1}{12}\left[1 + 4(.9981) + 2(.9701) + 4(.8716) + 1/\sqrt{2}\right] \approx .9272$

#3. $I = \int_0^1 \ln(1+x) dx$ $S_4(f) = \frac{1}{12}\left[0 + 4\ln(1.25) + 2\ln(1.5) + 4\ln(1.75) + \ln 2\right] \approx .3863$

Error $< 10^{-3}$: $I - S_n(f) = \frac{-(b-a)h^4}{180} f^{(4)}(\xi)$ $f(x) = \ln(1+x)$, $f'(x) = (1+x)^{-1}$ $f^{(2)}(x) = -1(1+x)^{-2}$

$f^{(3)}(x) = 2(1+x)^{-3}$ $f^{(4)}(x) = -6(1+x)^{-4}$

Thus, $|I - S_n(f)| \leq \left|\frac{-h^4}{180}\right| \|f^{(4)}\|_{I,\infty} = \frac{+h^4}{180} \left\|\frac{-6}{(1+x)^4}\right\|_{\infty,[0,1]} = \frac{h^4}{180}\frac{6}{(1+0)^4} = \frac{h^4}{30}$

$\frac{h^4}{30} < 10^{-3}$ if $h^4 < 30 \cdot 10^{-3} \Rightarrow h < \sqrt[4]{30 \cdot 10^{-3}} \approx \boxed{.4162}$

$\frac{h^4}{30} < 10^{-6} \Rightarrow h < \sqrt[4]{30 \cdot 10^{-6}} \approx \boxed{.0740}$

For trapezoid, $I(f) - T_n(f) = \frac{-(b-a)}{12} h^2 f''(\xi_n)$ so $|I(f) - T_n(f)| < \frac{1}{12}h^2 \|f^2\|_{\infty,5} = \frac{1}{12}h^2 \left\|\frac{-1}{(1+x)^2}\right\|$

$= \frac{1}{12}h^2(1) = \frac{h^2}{12}$ $\frac{h^2}{12} < 10^{-3} \Rightarrow h < \sqrt{12 \cdot 10^{-3}} \approx \boxed{.1095}$

$\frac{h^2}{12} < 10^{-6} \Rightarrow h < \sqrt{12 \cdot 10^{-6}} \approx \boxed{.0035}$

#9. $\ln x = \int_1^x \frac{1}{t} dt$

Simpson's: $|\ln x - S_n(f)| = \frac{-(x-1)}{180} h^4 \|f^4\|_{\infty,[1/2,1]}$ for $f = t^{-1}$ and $x \in [1/2, 1]$

If $f(t) = t^{-1}$, $f'(t) = -t^{-2}$, $f''(t) = 2t^{-3}$, $f^{(3)}(t) = -6t^{-4}$ $f^4(t) = 24t^{-5} = \frac{24}{t^5}$

$\|f^4\|_{\infty,[1/2,1]} = \frac{24}{(1/2)^5} = 24 \cdot 32 = 768.$ Note: $|x-1|$ is largest at $x = 1/2$,

$\Rightarrow$

$|\ln x - S_n(f)| \le \frac{1}{180} (1/2) h^4 \cdot 768 = \frac{768}{360} h^4 = \frac{32}{15} h^4$

Thus, $\frac{32}{15} h^4 < 10^{-6}$ if $h^4 < \frac{15}{32} \cdot 10^{-6}$ $\quad h < \sqrt[4]{\frac{15}{32} \cdot 10^{-6}} \approx .0261658\ldots$

$h = \frac{b-a}{n} \le \frac{1/2}{n} \Rightarrow n > \frac{1}{2} \frac{1}{.02616\ldots} \approx 19.1 \quad n \ge 20$

$h = \frac{b-a}{n} \le \frac{1/2}{n} \le \sqrt[4]{15/32 \cdot 10^{-16}} \Rightarrow n > \frac{1}{2} \frac{1}{\sqrt[4]{15/32 \cdot 10^{-16}}} \approx 6042.7 \quad n \ge 6043$

Trapezoid:

$|\ln x - T_n(f)| \le \left| \frac{-(b-a)}{12} h^2 \|f''\|_{\infty,[1/2,1]} \right| \le \frac{1/2}{12} h^2 \|2t^{-3}\|_{\infty,[1/2,1]} = \frac{1}{24} h^2 \left\| \frac{2}{(1/2)^3} \right| = \frac{16}{24} h^2 = \frac{2}{3} h^2$

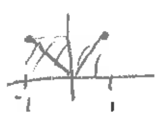
Require $\frac{2}{3} h^2 < 10^{-6} \Rightarrow h < \sqrt{\frac{3}{2 \cdot 10^6}} = \sqrt{\frac{3}{2}} \frac{1}{10^3} \approx .0012247$

If $h \le \frac{1/2}{n} = \frac{1}{2n}$, then $\frac{1}{2n} < .0012247\ldots$ ~~if~~ $n > \frac{1}{2(.0012247)} \approx 408.2 \quad \underline{n \ge 409}$

If $\frac{1}{2n} < \sqrt{3/2} \frac{1}{10^8}$, then $n > \frac{1}{2\sqrt{3/2}} \cdot 10^8 \approx 4082482.9 \quad n \ge 4{,}082{,}483$!

Simpson's is <u>WAY</u> better

II. $\int_{-1}^{1} |x|\,dx = I(f) = 2\left(\frac{1}{2}\right)(1)(1) = \boxed{1}$ ← true value

$T_1(f)$:  $\quad T_1(f) = 2(1) = 2$

$T_1^c(f) = T_1(f) - \frac{1}{12} h^2 (f'(1) - f'(-1)) = 2 - \frac{1}{12}(1)^2(1 - (-1)) = 2 - \frac{2}{12} = 5/6 = 1.8\bar{3}$

$S_2(f) = \frac{h}{3}[f(-1) + 4f(0) + f(1)] = \frac{1}{3}[1 + 0 + 1] = \frac{2}{3} = .6$

$\int_{-1}^{1} x^2\,dx = 2\int_0^1 x^2\,dx = 2/3$

<u>All</u> are bad.

Presumably, the problem should have been for

$\int_0^1 |x|\,dx$. ??