MATH 202 Name : SOLUTIONS
Quiz 4 February 27, 2012

Instructions: Point values as indicated. You get one point for taking this quiz.

1. A cannon is fired from ground level (a trench) with an initial velocity of 80 ft per second at an angle of elevation of 45°. Ignoring any effects of air resistance and using that the force due to gravity is 32 $\frac{\text{ft}}{\text{s}^2}$ directly down, compute the following:

   (a) (2 pts.) The vector valued function $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ that gives the path of the cannonball at time t in seconds.

$$\boxed{\vec{r}(t) = \langle 40\sqrt{2}\,t,\ 40\sqrt{2}t - 16t^2 \rangle \text{ ft}}$$

Longer derivation:

$\vec{r}(0) = \langle 0, 0 \rangle$

$\vec{v}(0) = \langle 80\cos \pi/4,\ 80 \sin \pi/4 \rangle = \langle 40\sqrt{2}, 40\sqrt{2} \rangle$

$\vec{a}(t) = -32\hat{j}$

$\vec{v}(t) = -32t\hat{j} + \vec{v}(0)$ ft/s

$= \langle 40\sqrt{2},\ 40\sqrt{2} - 32t \rangle$ ft/s

$\vec{r}(t) = \int \vec{v}(t)\,dt$

$\vec{r}(t) = 40\sqrt{2}t\hat{i} + (40\sqrt{2}t - 16t^2)\hat{j} + \langle 0, 0 \rangle$

$\vec{r}(t) = \langle 40\sqrt{2}\,t,\ 40\sqrt{2}t - 16t^2 \rangle$ ft

   (b) (1 pt.) Find the time t when the cannonball hits the ground.

The cannon ball hits the ground when $y(t) = 0$ in $\vec{r}(t)$.

Solving $y(t) = 40\sqrt{2}t - 16t^2 = 0$

$t(40\sqrt{2} - 16t) = 0$

$t = 0$ ↑ start

$40\sqrt{2} - 16t = 0$

$16t = 40\sqrt{2}$

$t = \frac{40\sqrt{2}}{16} = \frac{5\sqrt{2}}{2}$ seconds

$$\boxed{t = \frac{5\sqrt{2}}{2}\text{ s}}$$

   (c) (1 pt.) Set up, **but do not integrate**, a definite integral that computes the arc length of the trajectory of the cannonball. (Give your answer in simplified form for credit.)

$L = \int_0^{\frac{5\sqrt{2}}{2}} \|\vec{r}\,'(t)\|\,dt$

$\vec{r}\,'(t) = \langle 40\sqrt{2},\ 40\sqrt{2} - 32t \rangle$ ft/s from above

$= 8\langle 5\sqrt{2},\ 5\sqrt{2} - 4t \rangle$ ft/s

$\|\vec{r}\,'(t)\| = 8\sqrt{(5\sqrt{2})^2 + (5\sqrt{2} - 4t)^2}$

$= 8\sqrt{50 + 50 - 40\sqrt{2}t + 16t^2}$

$= 8\sqrt{16t^2 - 40\sqrt{2} + 100}$

$= 16\sqrt{4t^2 - 10\sqrt{2}t + 25}$

$$= \boxed{\int_0^{\frac{5\sqrt{2}}{2}} 16\sqrt{4t^2 - 10\sqrt{2}t + 25}\,dt \text{ ft}}$$