

Instructions: This quiz is worth five points. You get one point for taking this quiz.

1. (2 pts.)

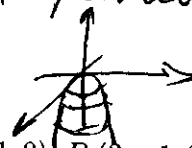
(a) Write the equation $z = -3x^2 - 3y^2$ in spherical and cylindrical coordinates.

$$\begin{aligned} \begin{cases} z = z \\ x = r \cos \theta \\ y = r \sin \theta \end{cases} & \quad \begin{aligned} z &= -3r^2 \cos^2 \theta - 3r^2 \sin^2 \theta \\ &= -3r^2 (\cos^2 \theta + \sin^2 \theta) = -3r^2 \\ \rho \cos \varphi &= -3\rho^2 \sin^2 \varphi \cos^2 \theta - 3\rho^2 \sin^2 \varphi \sin^2 \theta \\ &= -3\rho^2 \sin^2 \varphi \\ \rho &= -\frac{1}{3} \frac{\cos \varphi}{\sin^2 \varphi} \end{aligned} \\ \begin{cases} x = \rho \sin \varphi \cos \theta \\ y = \rho \sin \varphi \sin \theta \\ z = \rho \cos \varphi \end{cases} & \quad \begin{aligned} &\text{in cylindrical coordinates: } z = -3r^2 \\ &\text{in spherical coordinates: } \rho = -\frac{1}{3} \frac{\cos \varphi}{\sin^2 \varphi} \end{aligned} \end{aligned}$$

(b) Describe the surface $z = -3x^2 - 3y^2$. (You may sketch it, but be sure to describe the cross-sections.)

$$\begin{aligned} z = k: & -3x^2 - 3y^2 = k & x^2 + y^2 = -\frac{k}{3} & \text{circles} \\ x = k: & z = -3y^2 - 3k^2 & & \text{downward parabolas} \\ y = k: & z = -3x^2 - 3k^2 & & \text{downward parabolas} \end{aligned}$$

elliptic paraboloid



2. (2 pts.) Find the equation of the plane passing through the points $P_1(1, 1, 0)$, $P_2(2, -1, 0)$, and $P_3(1, 2, -1)$.

$$\begin{aligned} &\text{Diagram showing vectors } \vec{P_1 P_2} \text{ and } \vec{P_1 P_3} \text{ originating from } P_1. \\ &\vec{P_1 P_2} = \langle 1, -2, 0 \rangle \\ &\vec{P_1 P_3} = \langle 0, 1, -1 \rangle \\ &\vec{P_1 P_2} \times \vec{P_1 P_3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 0 \\ 0 & 1 & -1 \end{vmatrix} \\ &= 2\mathbf{i} + \mathbf{j} + \mathbf{k} \\ &2(x-1) + (y-1) + z = 0 \\ &\boxed{2x + y + z = 3} \end{aligned}$$