HW 3

The due date for these problems is Tuesday, February 12 at the beginning of class.

1. p. 5, 5.

2. p. 5, 7. Just rigorously prove the part $\lim_{n \to \infty} d(a_n, b_n)$ exists.

   Jeremiah outlined a proof in class, but there is one detail that needs a bit more attention. You need to use the definition of absolute value $|X| < a \iff -a < X < a$, which contains *two* inequalities.

3. Prove (or disprove) the following:

   Let p be a prime and x a p-adic integer with $x = \ldots a_k a_{k-1} \ldots a_2 a_1 a_{0 \wedge}$. If the p-adic expansion of x is eventually periodic, then x represents a rational number with denominator relatively prime to p. (Note: this holds more generally for any p-adic numbr.)

HW 1 feedback: Very good solutions. Just keep in mind that our answers were ten 'digit' approximations to square roots. For the record, I got

$$3032431212_\wedge \qquad \text{and} \qquad 1761192486_\wedge$$

for the 5-adic approximation to a square root of -1 and the 11-adic approximation to the square root of 3 respectively.