MATH 371 Name : SOLUTIONS

Quiz 3 October 15, 2008

Instructions: Round all answers to two significant digits (i.e. two decimal places, or if your answer is very small, give the first two non-zero digits following the decimal point). There are 10 points on this quiz. Good luck.

1. (3 pts.) In a certain state, a growing number of individuals pursuing job opportunities in the field of information services are choosing training at technical colleges over the more traditional 4-year college degree training. A group of nine candidates for three local positions in information technology consisted of five who attended a technical college and four who attended a traditional four-year college. All nine candidates appear to be equally qualified, so three candidates are selected at random to fill three open positions.

   Let Y be the number of technical college trained candidates who are hired.

   (a) (2 pts.) What type of random variable is Y. Explain briefly. Then list the parameters for Y.

   Y is hypergeometric $N=9$ $n=3$ $r=5$ success = attended technical college

   Y: # of successes in a random sample of size 3

   (b) (1 pt.) Find the probability that two or more technical college trained candidates are hired.

   $$P(Y \geq 2) = p(2) + p(3) = \frac{\binom{5}{2}\binom{4}{1}}{\binom{9}{3}} + \frac{\binom{5}{3}\binom{4}{0}}{\binom{9}{3}} \approx .4762 + .1190 \approx .595 \approx \boxed{.60}$$

2. (4 pts. – 2 pts. each)

   (a) Give an example of a binomial random variable X. Carefully describe the parameters needed to define X, and the reasons why your random variable is binomial.

   Varies

   (b) Give, with proof, a formula for the expected value of X. (Giving the formula is easy; giving the proof is the substance of this question.)

   $E(X) = np$ See class notes for a longer proof

Proof: $$\sum_{y=0}^{n} y \binom{n}{y} p^y q^{n-y} = np \sum_{y=1}^{n} \binom{n-1}{y-1} p^{y-1} q^{(n-1)-(y-1)}$$ since $n\binom{n-1}{y-1} = y\binom{n}{y}$

$$= np \underbrace{\sum_{x=0}^{n-1} \binom{n-1}{x} p^x q^{(n-1)-x}}_{=1}$$ since $\sum p(x)$ for $X \sim \text{Binom}(n-1$

$$= np(1)$$

3. (1 pt.) After a long-term study, it is found that about .2 accidents take place at the corner of Geist Road and University Avenue per year.

   What is the probability that at two or more accidents occur in a year at this intersection?

   Y: Poisson with $\lambda = .2$

   $P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - .982 \approx .018$

   Table p 843

4. (2 pts.) Employees at a nuclear power plant facility are being tested for indications of radiation exposure. The power plant is requested to send three employees that show evidence of radiation exposure to a hospital for further testing. Suppose 25% of the employees have positive indications of radiation exposure.

   (a) What is the probability that ten individuals must be tested to find three positives?

   This is negative binomial with $p = .25$, Y: # tested until $r = 3$ positive found.

   $p(10) = \binom{9}{2}(.25)^3(.75)^7 \approx .075$

   (b) What number of employees would you expect to test for positive indications of radiation exposure, if the power plant is to find three employees with radiation exposure.

   $\mathbb{E}(Y) = \frac{r}{p} = \frac{3}{.25} = \boxed{12}$