MATH 371

Still more probability problems....

1. The proportions of blood phenotypes, A, B, AB, O, of the population of students at UAF is given by .43, .12, .07, .38 respectively. A single student is chosen at random from the population.

   (a) What is the probability that the student chosen at random has either type AB or type O blood?

   .07+.38 = .45

   (b) What is the probability that the student chosen at random does not have type O blood?

   1-.38 = .62

2. A woman has her purse snatched by two teenagers. She is subsequently shown a police lineup consisting of five male suspects, including the two perpetrators. Consider the experiment *Woman picks two men out of lineup.*

   (a) List the sample space for this experiment. (Clearly define your notation in your answer.)

   There will be $\binom{5}{2} = 10$ points in the sample space. Using P_1, P_2 for the perps and M_1, M_2, M_3 for the three other men in the lineup, we have:

$$S = \big\{\{P_1, P_2\}, \{P_1, M_1\}, \{P_1, M_2\}, \{P_1, M_3\}, \{P_2, M_1\},$$
$$\{P_2, M_2\}, \{P_2, M_3\}, \{M_1, M_2\}, \{M_1, M_3\}, \{M_2, M_3\}\big\}.$$

   (b) Give the probability of event A: Woman makes at least one incorrect identification.

   $1 - \frac{1}{10} = .9$

3. A fashionable country club has 100 members, 30 of whom are lawyers. Rumor has it that 25 of the club members are liars and that 55 are neither lawyers nor liars.

	Lawyers	Non-lawyers	tot
Non-liars	20	55	75
Liars	10	15	25
tot	30	70	

   (a) How many members are liars, but not lawyers? 15

   (b) What proportion of the lawyers are liars? $10/30 = \frac{1}{3}$

4. At the Pump House restaurant, a diner can choose from among three appetizers, five entrees, two beverages, and four desserts. How many different dinners are available if a dinner consists of an appetizer, entree, beverage, and dessert?

   $3 \cdot 5 \cdot 2 \cdot 4 = 120$

5. BP will send ten employees to three cities: five employees will travel to San Francisco, four employees will travel to Dallas, and one will travel to Chicago. In how many distinct ways can this be accomplished?

   $\binom{10}{5\ 4\ 1} = \frac{10!}{5!4!1!} = 1260$. Equivalently, $1260 = \binom{10}{5}\binom{5}{4}\binom{1}{1}$.

6. A six member committee consisting of three men and three women is to be chosen from a group of 25 men and 30 women. How many different committees can be formed?

   $\binom{25}{3}\binom{30}{3} = 9,338,000$

7. How many different six character codes can be formed if the first three characters are letters and the last three characters are digits and no repetitions are allowed? What if repetitions are allowed?

   a. $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8$ b. $26^3 \cdot 10^3$

8. Two cards are drawn from a 52 card playing deck. What is the probability that a jack and a queen are drawn? (Give your answer to six decimal places.)

   $\frac{\binom{4}{1}^2}{\binom{52}{2}} \approx .012$

9. A novice soccer player must master ten skills to be allowed to play on a local team. Suppose this soccer player masters eight of the skills before the try-outs. At the try-out, the coach selects three of the skills at random to test the soccer players. What is the probability that the student can perform all three skills during the try-out? (Give your answers to three decimal places.)

   $\frac{\binom{8}{3}}{\binom{10}{3}} = \frac{7}{15} \approx .467$

10. Calculate the probabilities of the following five card poker hands.

    (a) One pair (Answer = .42)

    (b) Two pairs (Answer = .048)

    (c) Three-of-a-kind (Answer = .021)

    (d) Full House [three-of-a-kind and one pair] (Answer = .0014)

    (e) Straight [five cards having consecutive denominations, but not all in the same suit] (Answer = .00392 if Ace is high or low; Answer = .00353, if Ace is high) [Hint: for the numerator first calculate the number of ways to get five cards having consecutive denominations. Then subtract the number of those that are of the same suit.]

11. An urn contains eight chips, numbered 1 to 8. A sample of three is drawn without replacement. What is the probability that the largest chip in the sample is a "5"? (Answer = .11)

    $\frac{1\binom{4}{2}}{\binom{8}{3}} = \frac{6}{\frac{8!}{3!5!}} = \frac{3}{28} \approx .107$

12. (The Birthday Problem.) Suppose there are k people in a room. What is the probability that two of them have the same birthday? (Ignore Leap Year.)

    (a) Suppose $k = 15$. What is the probability two of them have the same birthday? (Answer = .2529)

    (b) Suppose $k = 23$. What is the probability two of them have the same birthday? (Answer = .5073)

    (c) Suppose $k = 50$. What is the probability two of them have the same birthday? (Answer = .9704)

    (d) If there are k people in a room, what is the probability two of them have the same birthday? (Give a formula that involves k.)

13. A student needs money to stay in school. This is possible if she finds a part-time job or is awarded a scholarship. She estimates the probability that she will get a job as .75, that she gets a scholarship as .23, and that both happen as .15. What is the probability that she stay in school?

    .75 + .23 - .15 = .83