MATH 371 Name : SOLUTIONS

Quiz 2 October 3, 2008

Instructions: Round all answers to two significant digits (i.e. two decimal places, or if your answer is very small, give the first two non-zero digits following the decimal point). There are 10 points on this quiz. Good luck.

1. (2 pts.) Suppose you are dealt a hand of six cards from a 52 card deck.

   What is the probability of getting exactly one pair?

$$\frac{\binom{13}{1}\binom{4}{2}\binom{12}{4}\binom{4}{1}^4}{\binom{52}{6}} \approx .49$$

2. (2 pts.) A population of voters contains 60% Republicans and 40% Democrats. It is reported that 59% of Republicans plan to vote for the McCain-Palin ticket and 45% of the Democrats plan to vote for the McCain-Palin ticket in November.

   A person chosen at random for the population is found to favor McCain-Palin. What is the conditional probability that this person is a Democrat? (A complete answer will define notation for the various events being considered.)

$P(R) = .6 \qquad P(D) = .4$

$P(MP|R) = .59 \qquad P(MP|D) = .45$

$$P(D|MP) = \frac{P(MP|D)\,P(D)}{P(MP|D)\,P(D) + P(MP|R)\,P(R)} = \frac{.45(.4)}{.45(.4) + .59(.6)} \approx .34$$

3. (2 pts.) A student studies for a multiple choice quiz where each question has five possible answers. Suppose the student knows the answer to twelve out of the fifteen questions that the teacher suggested for study. For the quiz, the teacher selects ten questions at random from the fifteen on the list given to the students for review. What is the probability the student can solve exactly eight questions on the quiz?

$$\frac{\binom{12}{8}\binom{3}{2}}{\binom{15}{10}} \approx .49$$

4. (3 pts.) A survey of young children is undertaken to determine which super-hero (Superman or Batman) is preferred. The results indicate that 56% of children prefer Superman (of the two). Suppose three different children are selected at random and define the random variable:

Y : the number of the three children who prefer Superman

(a) What is the range of Y? That is, what are the possible values of $y \in \mathbf{R}$ so that the value of the probability function $p(y) \neq 0$.

$y = 0, 1, 2,$ or 3

(b) Give the values of the probability function $p(y)$. (It is enough to write down only those values for which $p(y) \neq 0$.)

$p(0) = \binom{3}{0}(.56)^0(.44)^3 = .44^3 \approx .085$

$p(1) = \binom{3}{1}(.56)(.44)^2 \approx .325$

$p(2) = \binom{3}{2}(.56)^2(.44) \approx .414$

$p(3) = (.56)^3 \approx .176$

(c) Give the expected value $E(Y)$ of Y.

$E(Y) = 0(.085) + 1(.325) + 2(.414) + 3(.176) \approx 1.681$

5. (1 pt.) The probability distribution for a discrete random variable X is given in the table below:

x	p(x)
1	.5
2	.3
5	.2

(For all other values of x, $p(x) = 0$.)

Give the variance $\mathrm{Var}(X)$.

$E(X) = 1(.5) + 2(.3) + 5(.2) = 2.1$

$\mathrm{Var}(X) = E(X^2) - E(X)^2$

$= 1^2(.5) + 2^2(.3) + 5^2(.2) - (2.1)^2$

$= 2.29$

$E((X-2.1)^2) = (1-2.1)^2(.5) + (2-2.1)^2(.3) + (5-2.1)^2(.2) \approx 2.29$