Hw #7 Solution keys.

13.6 # 30 (2pts)

$\vec{PQ} = -\frac{\pi}{2}\vec{i} + \pi\vec{j}$, $u = -\frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{j}$

$\nabla f = 2\cos 2x \cos y\, \vec{i} - \sin 2x \sin y\, \vec{j}$

$\nabla f(\pi, 0) = 2i$

$D_u f = \nabla f \cdot u = -\frac{2}{\sqrt{5}} = \boxed{-\frac{2\sqrt{5}}{5}}$

13.6 #50. (3 pts)

a) $f(x,y) = \frac{8y}{1+x^2+y^2} = 2$, so

$4y = 1 + x^2 + y^2$

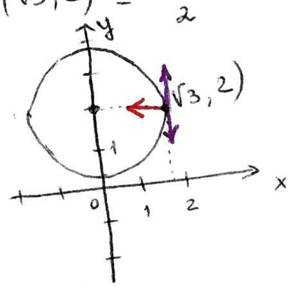
$y^2 - 4y + 4 + x^2 = 4 - 1$

$(y-2)^2 + x^2 = 3$

Circle : center $(0,2)$
radius: $\sqrt{3}$

b) $\nabla f = \frac{-16xy}{(1+x^2+y^2)^2} i + \frac{8+8x^2-8y^2}{(1+x^2+y^2)} j$

$\nabla f(\sqrt{3}, 2) = -\frac{\sqrt{3}}{2} i$



c) The directional derivative of f is 0 in the direction $\pm j$ ($\updownarrow$ -on the graph)

(In order for the directional derivative to be zero $\nabla f(\sqrt{3},2)$ and the unit vector must be orthogonal.

13.6 # 56 (2pts)

a) $\nabla f(x,y) = i - 2y j$

$\nabla f(4,-1) = i + 2j$

b) $\|\nabla f(4,-1)\| = \sqrt{5}$

$\frac{1}{\sqrt{5}}(i+2j)$ is a unit vector normal to the level curve $x - y^2 = 3$ at $(4,-1)$

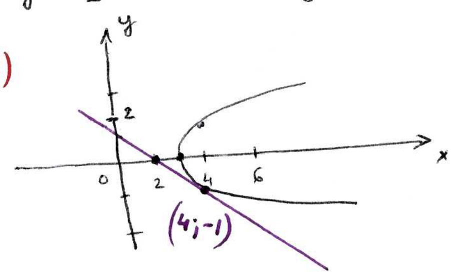
c) The vector $2i - j$ is tangent to the level curve.

Slope $= -\frac{1}{2}$

$y + 1 = -\frac{1}{2}(x-4)$

$y = -\frac{1}{2}x + 1$ Tangent line

d)



13.6 # 66 (2pts)

$h(x,y) = 5000 - 0.001x^2 - 0.004y^2$

$\nabla h = -0.002x i - 0.008y j$

$\nabla h(500, 300) = -i - 2.4j$ or

$5\nabla h = -(5i + 12j)$

13.6 # 70 (2 pts)

$T(x,y) = 100 - x^2 - 2y^2$, $P = (4,3)$

$\frac{dx}{dt} = -2x$

$x(t) = C_1 e^{-2t}$

$4 = x(0) = C_1$

$x(t) = 4e^{-2t}$

$\frac{dy}{dt} = -4y$

$y(t) = C_2 \cdot e^{-4t}$

$3 = y(0) = C_2$

$y(t) = 3e^{-4t}$

$\frac{3x^2}{16} = e^{-4t} = y \Rightarrow y = \frac{3}{16}x^2$

13.7 # 20 (2pts)

$g(x,y) = \arctan \frac{y}{x}$, $(1,0,0)$

$G(x,y,z) = \arctan\left(\frac{y}{x}\right) - z$

The surface is level surface

$G(x,y,z) = 0$

The normal vector is $\underline{\vec{n} = \nabla G(1,0,0)}$,

$$\nabla G(x,y,z) = \left\langle \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \left(\frac{-y}{x^2}\right), \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}, -1 \right\rangle$$

$\nabla G(1,0,0) = \langle 0, 1, -1 \rangle$

Equation $\vec{x} \cdot \vec{n} = \vec{p} \cdot \vec{n}$

So: $0 \cdot x + 1 \cdot y - 1 \cdot z = \langle 1,0,0 \rangle \cdot \langle 0,1,-1 \rangle = 0$

$$\boxed{y - z = 0}$$

13.7 # 24 (2pts)

$F(x,y,z) = e^x (\sin y + 1) - z$

$F_x(x,y,z) = e^x(\sin y + 1)$; $F_x(0, \frac{\pi}{2}, 2) = 2$

$F_y(x,y,z) = e^x \cos y$; $F_y(0, \frac{\pi}{2}, 2) = 0$

$F_z(x,y,z) = -1$; $F_z(0, \frac{\pi}{2}, 2) = -1$

$$2x - z = -2$$

13.7 # 54 (2pts)

$F(x,y,z) = 4x^2 + 4xy - 2y^2 + 8x - 5y - 4 - z$

$\nabla F(x,y,z) = (8x + 4y + 8)\vec{i} + (4x - 4y - 5)\vec{j} - k$

$$\left.\begin{cases} 8x + 4y + 8 = 0 \\ 4x - 4y - 5 = 0 \\ \hline 12x \quad + 3 = 0 \end{cases}\right\} \Rightarrow x = \frac{-1}{4} \Rightarrow y = -\frac{3}{2}, \quad z = -\frac{5}{4}$$

Point $\left(-\frac{1}{4}, -\frac{3}{2}, -\frac{5}{4}\right)$