HW 6 PROBLEMS

1-8. Hassett, Chapter 3, #6, 7, 8, 9, 11, 12, 14, 15

9. Prove that any bijective function $f : \mathbb{C} \to \mathbb{C}$ is Zariski continuous.

10. Consider the complex affine line $\mathbb{A}^1(\mathbb{C})$. Show that any open subset $U \subset \mathbb{A}^1(\mathbb{C})$ is dense. Then show that the Zariski topology on $\mathbb{A}^1(\mathbb{C})$ is not Hausdorff.

11. Give the Zariski closure of the interval $(0, 1) \subset \mathbb{A}^1(\mathbb{R})$.

12. (a) Let $V \subset \mathbb{A}^2(\mathbb{R})$ be the variety $V(y - x^2)$. Show that the coordinate ring $\mathbb{R}[V]$ is isomorphic to a polynomial ring in one variable over $\mathbb{R}$.
    (b) Let $Z \subset \mathbb{A}^2(\mathbb{R})$ be the variety $V(xy - 1)$. Show that the coordinate ring $\mathbb{R}[Z]$ is *not* isomorphic to a polynomial ring in one variable over $\mathbb{R}$. What does this say about $\mathbb{A}^1(\mathbb{R})$ and Z?

13. Some ring theory problems: Let $f : R \to S$ be a ring homomorphism. Prove the following
    (a) If $J \subset S$ is an ideal of S, then $I = f^{-1}(J)$ is an ideal of R. I is called the *contraction* of J.
    (b) If $J \subset S$ is a prime ideal, show that its contraction is R or a prime ideal of R.
    (c) If $I \subset R$ is an ideal, show that its *extension* $J = f(I)S$ is an ideal of S.
    (d) Give an example to show that if I is a prime ideal of R, then its extension may not be a prime ideal of S.