MATH 161 Name : SOLUTIONS

These problems will be collected at the end of class today. You will definitely need to study more topics than this activity covers. This is for you to get graded feedback from your instructor. Point values for each problem are indicated. The total value of this exercise is 25 points.

1. (9 pts.) A manufacturer finds that the revenue generated by selling x pairs of skis is given by the function $R(x) = 90x - .45x^2$ dollars.

   (a) (1 pt.) How much revenue is generated if 10 pairs of skis are sold?

   $R(10) = 90(10) - .45(10)^2 = 900 - .45(100) = 900 - 45 = \boxed{\$855}$

   (b) (4 pts.) What is the maximum revenue, and how many pairs of skis should be sold to achieve this maximum revenue?

   Complete the square to put in standard form.

   $R(x) = -.45x^2 + 90x$

   $= -.45(x^2 - 200x)$

   $= -.45(x^2 - 200x + 100^2 - 100^2)$

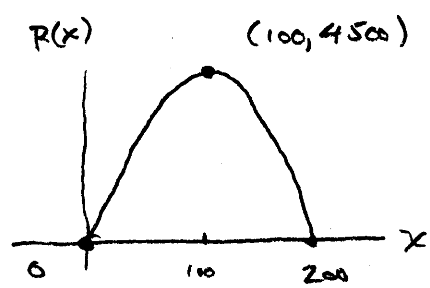
   $= -.45(x-100)^2 + .45(100)^2$

   $= -.45(x-100)^2 + 4500$

   Maximum revenue: \$4500 Number of skis sold: 100

   (c) (2 pts.) Use your knowledge of graphing parabolas to sketch a graph of $R(x)$ for those values of x when $R(x) \geq 0$. Label all intercepts and local maxima/minima.

   Find x-intercepts: $0 = 90x - .45x^2 = .45x(200 - x)$. Therefore, intercepts are $x = 0, 200$.



   (d) (2 pts.) Restricting the domain to those values of x where $R(x) \geq 0$, give the intervals where $R(x)$ is increasing and $R(x)$ is decreasing.

   Interval(s) of Increase: $(0, 100)$ Interval(s) of Decrease: $(100, 200)$

2. (8 pts.) Consider the polynomial $P(x) = x^3 - 2x^2 - 3x$.

(a) (2 pts.) Find the zeros of $P(x)$.

$0 = x^3 - 2x^2 - 3x$

$0 = x(x^2 - 2x - 3)$

$0 = x(x-3)(x+1)$

The zeros are $x = 0, 3, -1$

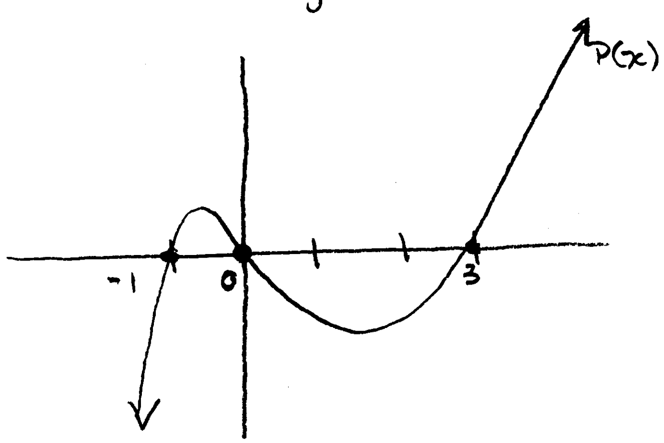
(b) (2 pts.) Describe the 'end behavior' of $P(x)$. That is,

As $x \to \infty, P(x) \to$ $+\infty$.

As $x \to -\infty, P(x) \to$ $-\infty$.

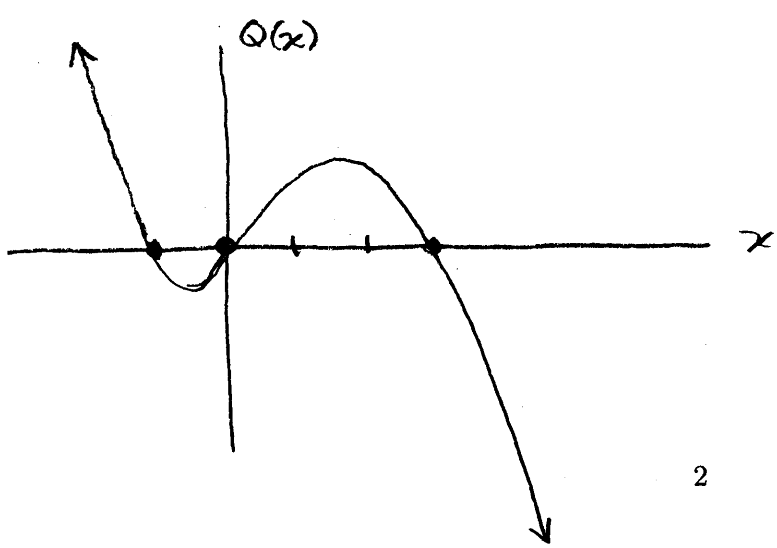
(c) (2 pts.) Use your answers to (a) and (b) to sketch $P(x)$. Label all intercepts.

y-intercept is $P(0) = 0$



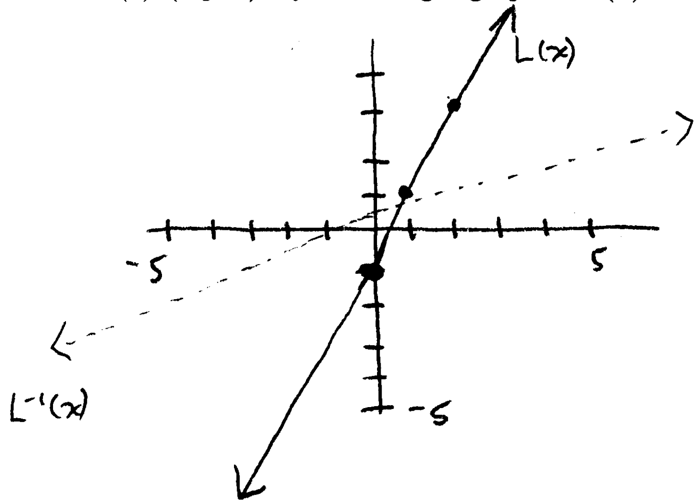
(d) (2 pts.) Now sketch $Q(x) = -x^3 + 2x^2 + 3x$.

Q(x) is the x-axis reflection of P(x).



3. (8 pts.) Consider the linear function $L(x) = 2x - 1$.

(a) (2 pts.) By sketching a graph of $L(x)$ show that $L(x)$ is 1-1. Justify your answer.



L(x) passes the horizontal line test.

(b) (2 pts.) Find the inverse function $L^{-1}(x)$.

If $y = 2x - 1$, then switching x and y we get

$x = 2y - 1$

$x + 1 = 2y$

$\frac{1}{2}x + \frac{1}{2} = y$

$$\boxed{L^{-1}(x) = \frac{1}{2}x + \frac{1}{2}}$$

(c) (1 pt.) Check your answer to (b) by computing $L(L^{-1}(x))$.

$$L(L^{-1}(x)) = 2\left(\frac{1}{2}x + \frac{1}{2}\right) - 1$$

$$= x + 1 - 1$$

$$= x!$$

(d) (1 pt.) What is $L^{-1}(L(x))$? $= x$

(e) (2 pts.) On the axes you drew in part (a), graph $L^{-1}(x)$. (Label your functions please.)

See above.