MATH 632: An Introduction to Algebraic Geometry

T Th 8:00 – 9:30
Gruening 401
http://www.dms.uaf.edu/∼eallman/classes/632/632-2013.html

Instructor: Elizabeth S. Allman

Contact Details: Chapman 308B, e.allman@alaska.edu and 474-2479.

Office Hours: M 2:00, W 9:00, Th 9:30, and by appointment. These hours may change.

Prerequisites: MATH 631 (or permission of instructor) with a grade of C- or better.

Textbook: *An Introduction to Algebraic Geometry*, 1st ed., by B. Hassett, Cambridge University Press

Midterm: Thursday, October 24

Final Exam Date: December 17, 8:00 – 10:00

Course Overview and Goals:

Algebraic geometry is concerned with the *zero set of polynomials*. For instance, the real algebraic variety defined by the equation

$$x^2 + y^2 + 1 = 0$$

is the empty set, since no 2-tuples of real numbers satisfy that polynomial equation. Over the larger, algebraically closed field of complex numbers $\mathbb{C}$, the zero set defines an algebraic variety with representative points $(\pm i, 0)$, $(0, \pm i)$, and $(\pm\sqrt{3}, 2i)$ and $(\frac{\sqrt{3}}{2} + \frac{1}{2}i, \frac{\sqrt{3}}{2} - \frac{1}{2}i)$. As this example shows, finding and counting roots of polynomials depends intimately on the field of definition of the variety.

As the name suggests, algebraic geometry is a fascinating intermingling of algebra and geometry, using tools from both areas to understand solutions to polynomial equations. Whereas any linear algebra class teaches you to solve a system of linear equations in n unknowns, we may view algebraic geometry as the generalization of such ideas to systems of non-linear polynomial equations. Some of the elegance and simplicity of linear algebra persists, yet the higher degree of the polynomial equations in this setting requires great subtlety and delicacy to solve completely.

Remarkably, in the last 40 years algorithms have been introduced to solve systems of higher degree polynomials equations, and tremendous strides have been made towards using computers to solve them. (It quickly becomes clear giving complete solutions to even simple polynomials equations or systems of equations is virtually impossible by hand.) Thus, we will learn and use computational algebra software packages like `Singular` and `Macaulay2` extensively in homework. A quick Google search will turn up the webpage for these software packages, and you should download and install them as soon as possible.

Course Mechanics:

Class meetings will be run as interactive lectures, to the extent possible given the enrollment. That means that while I will be presenting material at the board, and you will be taking notes, I will also be asking for suggestions, ideas, and questions about the material as we go along. I don't expect 'correct' answers, but I do expect you to be actively following and participating — that makes the class more interesting for us all.

As some topics in MATH 632 require more background then you received in MATH 631, there may be some review of polynomials rings and other special rings, together with coverage of some additional topics.

Class attendance is expected, although I will not formally take roll. If you miss a class, you should get notes from another student. Homework assignments will be mentioned in class, with the particular problems posted on the course web page.

Homework will be assigned regularly and collected once a week on Wednesday. There will rarely be time at the beginning of class for simple questions on homework, so you should expect to get your homework questions answered during office hours, or ideally during our extra weekly session to discuss problems. There is an emphasis on proof-writing in algebra courses, and substantial time and effort must be expended to develop clear and lucid proof-writing techniques.

I encourage you to work with others on the homework, and to share ideas for solutions, but you must *write up solutions independently*. You will learn nothing from simply copying a solution. Even though you may find you can't do every problem without help or hints, as a group we should be able to give proofs for all problems.

An ambitious and tentative outline of the topics we will cover is given in the table below.

Week	Tuesday	Thursday
0		9/5 Chap 1
1	9/10 Chap 1 + 2	9/12 Chap 2
2	9/17 Chap 2 + 3	9/19 Chap 3
3	9/24 Chap 3	9/26 Chap 3
4	10/1 Chap 4	10/3 Chap 4
5	10/8 Chap 4	10/10 Rest
6	10/15 Chap 6	10/17 Chap 6
7	10/22 No class	10/24 Exam
8	10/29 Chap 6	10/31 Chap 7
9	11/5 Chap 7	11/7 Chap 8
10	11/12 Chap 8	11/14 Chap 8
11	11/19 Chap 9	11/21 Chap 9
12	11/26 Chap 9	11/28
13	12/3 Chap 9	12/5 Chap 10
14	12/10 Chap 10	12/12 Chap 10

Homework will be accepted until 5pm on its due date, either at my office or in my mailbox in the math department office. I will not accept *any* late homework that has not been cleared ahead of time or is not due to a genuine emergency (e.g., a death in the family).

Finally, 'homework' will likely include some project-like problems, and students will be expected to present solutions to the whole class on such extensive problems.

Missed examinations or homework papers that are not approved in advance will result in an 'F' on that work. No make-ups will be given except in extreme circumstances (e.g., family death, documented illness, etc.). Notifying me by email or a note that you will miss an exam is not sufficient for advance approval; you must speak with me to be excused.

Grades:

As should be clear from the above, the largest contribution to your final grade will be from your graded work on homework assignments. This is an elective class, and I expect students to work maturely and hard to learn some algebraic geometry. This includes filling

in gaps in background independently, when necessary. Grades will be assigned using the following weights:

Homework	60 %
Class Participation	10 %
Midterm	15 %
Final Exam	15 %

Grade Bands: A, A- (90 - 100%); B+, B, B- (80 - 89%); C+, C, C- (70 - 79%); D+, D, D- (60 - F%); 69 (0 - 59%). On rare occasion, I may lower the thresholds. Also, in an effort to reward the student who makes significant improvement over the course of the term, marked improvement over the semester may overcome earlier deficiencies and result in a better final grade.

Other Policies:

Course accommodations: If you need course adaptations or accommodations because of a disability, please inform your instructor during the first week of the semester, after consulting with the Office of Disability Services, 203 Whitaker (474-7403).

University and Department Policies: Your work in this course is governed by the UAF Honor Code. The Department of Mathematics and Statistics has specific policies on incompletes, late withdrawals, and early final exams, some of which are listed below. A complete listing can be found at

`http://www.dms.uaf.edu/dms/Policies.html`.

Prerequisites: The prerequisite for MATH 632 is MATH 631 (or permission of instructor) with a grade of C- or better. Students not meeting this prerequisite are not eligible to take this course and will be dropped.

Late Withdrawal: This semester the last day for withdrawing with a 'W' appearing on your transcript is November 1. If, in my opinion, a student is not participating adequately in the class, I may elect to drop this student.

Graded Coursework: Please keep all graded work for MATH 632 until final grades have been assigned.

Academic Honesty: Academic dishonesty, including cheating and plagiarism, will not be tolerated. It is a violation of the Student Code of Conduct and will be punished according to UAF procedures.

Courtesies: As a courtesy to your instructor and fellow students, please arrive to class on time, turn your cell phones and iPods off during class, and pay attention in class.