Oblig 10 Eirik Lund /eaflund

4.11

a) $R_{20}(r) = \frac{c_0}{2a}\left(1 - \frac{r}{2a}\right) e^{-\frac{r}{2a}}$

$n = 2$ og $L = 0$ $\quad \rho = \frac{r}{an}$

$c_{j+1} = \frac{2(j+L+1-n)}{(j+1)(j+2L+2)} c_j$

for $j = 0$, $\quad c_1 = \frac{2(1+(-2))c_0}{1 \cdot 2} = -c_0$

$u(\rho) = c_0 \rho^{L+1} e^{\rho} \qquad v(\rho) = c_0 e^{2\rho}$

$R_{20}(r) = \frac{1}{r} u_{20}(r) = \frac{1}{r} \rho e^{-\rho} v_{20}(r)$

$R_{20}(r) = \frac{1}{\cancel{r}} \frac{\cancel{r}}{2a} e^{-\frac{r}{2a}} \left(1 - \frac{r}{2a}\right) c_0$

$\int_0^\infty |R_{20}|^2 r^2 dr = \frac{a}{2} c_0^2 \equiv 1 \qquad c_0 = \sqrt{\frac{2}{a}}$

$R_{20}(r) = \frac{1}{2a}\sqrt{\frac{2}{a}} e^{-\frac{r}{2a}}\left(1 - \frac{r}{2a}\right)$

$R_{20}(r) = \frac{1}{\sqrt{2}} a^{-3/2} e^{-\frac{r}{2a}}\left(1 - \frac{r}{2a}\right)$

Siden $L = 0$ så er $\psi_{200} = R_{20}(r) Y_0^0(\theta, \varphi)$

Table 4.3 : $Y_0^0 = \left(\frac{1}{4\pi}\right)^{1/2}$

$\underline{\text{Så}} \Rightarrow \psi_{200} = \frac{1}{2\sqrt{2\pi}} a^{-3/2} e^{-\frac{r}{2a}}\left(1 - \frac{r}{2a}\right)$

4.11

b) $R_{21} = \frac{c_0}{4a^2} r e^{-\frac{r}{2a}}$

$n=2, \ l=1$

$$c_{j+1} = \frac{2(j+l+1-n)}{(j+1)(j+2l+2)} c_j$$

$$c_{0+1} = \frac{2(0+1+1-2)}{(1)(0+2+2)} c_0 = 0$$

da blir $R_{21}(r) = \frac{1}{r} u_{21}(\rho)$ $\qquad R_{21} = \frac{1}{r}\rho^2 e^{-\rho} v_{21}(\rho)$

$R_{21}(r) = \frac{1}{r}\left(\frac{r}{2a}\right)^2 e^{-\frac{r}{2a}} c_0$ $\qquad R_{21} = \frac{r}{(2a)^2} e^{-\frac{r}{2a}} c_0$

$$\int_0^\infty |R_{21}|^2 r^2 dr = c_0^2 \frac{3a}{2} \equiv 1 \qquad c_0 = \sqrt{\frac{2}{3a}}$$

$R_{21}(r) = \frac{1}{4a^2}\sqrt{\frac{2}{3a}}\, r\, e^{-\frac{r}{2a}}$ $\qquad R_{21}(r) = \frac{1}{2\sqrt{6}} a^{-5/2} r e^{-\frac{r}{2a}}$

Siden vi har $l=1$:

$$Y_1^{+1} = -\left(\frac{3}{8\pi}\right)^{1/2} \sin\theta\, e^{i\varphi}$$

$$Y_1^{-1} = \left(\frac{3}{8\pi}\right)^{1/2} \sin\theta\, e^{-i\varphi}$$

$$Y_1^{0} = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$$

$$\psi_{21+1} = R_{21}(r)\, Y_1^1(\theta,\varphi) = -\frac{1}{8\sqrt{\pi}} \frac{r}{a^{5/2}} \sin\theta\, e^{-\frac{r}{2a}+i\varphi}$$

$$\psi_{21-1} = R_{21}(r)\, Y_1^{-1}(\theta,\varphi) = \frac{1}{8\sqrt{\pi}} \frac{r}{a^{5/2}} \sin\theta\, e^{-\frac{r}{2a}-i\varphi}$$

$$\psi_{210} = R_{21}(r)\, Y_1^0 = \frac{1}{4\sqrt{2\pi}} \frac{r}{a^{5/2}} \cos\theta\, e^{-\frac{r}{2a}}$$

4.14

a.) $[L_z, x] = i\hbar y$

$$[L_z, x] = [xp_y - yp_x, x]$$
$$= [xp_y, x] - [yp_x, x]$$
$$= 0 - y[p_x, x] = i\hbar y$$

$[L_z, y] = -i\hbar x$

$$[L_z, y] = [xp_y - yp_x, y]$$
$$= [xp_y, y] - [yp_x, y]$$
$$= x[p_y, y] - 0 = -i\hbar x$$

$$[L_z, z] = [xp_y - yp_x, z]$$
$$= [xp_y, z] - [yp_x, z]$$
$$= 0 - 0 = 0$$

$$[L_z, p_x] = [xp_y - yp_x, p_x]$$
$$= [xp_y, p_x] - [yp_x, p_x]$$
$$p_y[x, p_x] - 0 = i\hbar p_y$$

$$[L_z, p_y] = [xp_y - yp_x, p_y]$$
$$= [xp_y, p_y] - [yp_x, p_y]$$
$$= 0 - p_x[y, p_y] = -i\hbar p_x$$

$$[L_z, p_z] = [xp_y - yp_x, p_z]$$

$$= [xp_y, p_z] - [yp_x, p_z] = 0$$

4.19

b) $[L_z, L_x] = i\hbar L_y$

$= [xP_y - yP_x \,,\, yP_z - zP_y]$

$= [xP_y, yP_z] - [yP_x, -zP_y]$

$- [xP_y, -zP_y] - [yP_x, yP_z]$

$= -P_y[-x, +z] - y[P_x, P_z]$

$= -i\hbar x P_z + i\hbar P_x z = i\hbar (zP_x - xP_z)$

$[L_z, L_x] = i\hbar L_y$

c) $[L_z, r^2] = [L_z, x^2] + [L_z, y^2] + [L_z, z^2]$

$= [L_z, x]x + x[L_z, x] + [L_z, y]y + y[L_z, y]$

$+0 \leftarrow$ fra 4.19 a) $[L_z, z] = 0$

$= i\hbar y x + x i\hbar y - i\hbar x y - y i\hbar x$

$= 2i\hbar yx - 2i\hbar yx = 0$

$[L_z, P^2] = [L_z, P_x^2] + [L_z, P_y^2] + [L_z, P_z^2]$

$= [L_z, P_x]P_x + P_x[L_z, P_x] + [L_z, P_y]P_y$

$+ P_y[L_z, P_y] + 0$

$= 2i\hbar P_y P_x - 2i\hbar P_x P_y = 0$

d) $H = \left(\frac{P^2}{2m}\right) + V$, vi vet fra Symmetri at L_x og L_y kommuterer med r^2 og P^2

for å slippe å skrive alt på nytt så kan vi se P^2 i Hamilton operatoren, $P^2 = P_x^2 + P_y^2 + P_z^2$

Så vet vi at vi får lik kommutering som ovenfor. potensialet er kun avhengig av radiusen.

4.22

a) $L_+ Y_l^l$ L_+ er heve/senke operatorer for angular momentum tilstander

Y_l^l er den normaliserte angular bølge funksjonen

$L_+ Y_l^l = 0$ er ved toppen av øverste angular momentum tilstand ref figur 4.8 i boken. I denne tilstanden så har kommet til hver egenverdien $(\mu + \hbar)$ bar fått en tilstand mer enn det eksisterer tilstander, slik at vi må ha er max tilstand hvor $L_+ Y_l^l = 0$

b) 4.130 og $L_z Y_l^l = \hbar l Y_l^l$ for å bestemme $Y_l^l(\theta, \varphi)$

$$L_\pm = \pm \hbar e^{\pm i\varphi}\left(\frac{\partial}{\partial\theta} \pm i \cot\theta \frac{\partial}{\partial\varphi}\right)$$

$$L_z Y_l^l = \frac{\cancel{\hbar}}{i}\frac{\partial}{\partial\varphi} Y_l^l = \cancel{\hbar} l Y_l^l \quad | \cdot i \qquad \frac{\partial}{\partial\varphi} Y_l^l = i l Y_l^l$$

$$\frac{\partial Y_l^l}{\partial\varphi} = i l Y_l^l \qquad \text{så dus } \frac{d u'}{\partial\varphi} = i l$$

da er $Y_l^l = e^{il\varphi}$ egenfunksjon $f(\theta) e^{il\varphi}$

fra a)

$$L_+ Y_l^l = \hbar e^{i\varphi}\left(\frac{\partial}{\partial\theta} + i\cot\theta \frac{\partial}{\partial\varphi}\right) f(\theta) e^{il\varphi}$$

$$0 = \hbar e^{i\varphi}\left[\frac{\partial}{\partial\theta} f(\theta) e^{il\varphi} + i \cot(\theta) \frac{\partial}{\partial\varphi} f(\theta) e^{il\varphi}\right]$$

$$\frac{df}{d\theta} = l \cot\theta f \qquad \frac{df}{df} = l \cot\theta \, d\theta$$

$$\int \frac{df}{f} = \int l \cot\theta \, d\theta \qquad \ln f = l \int \frac{\cos\theta}{\sin\theta} d\theta \qquad u = \sin\theta, \; u' = \cos\theta$$

$$\ln f = l \int \frac{1}{u} du \qquad \ln f = l \ln(\sin\theta) + C$$

$$\ln f = \ln(\sin^l\theta) + C \qquad \ln f - \ln(\sin^l\theta) = C$$

$$\ln\left(\frac{f}{\sin^l\theta}\right) = C \qquad f(\theta) = C \sin^l\theta \qquad f(\theta) = A \sin^l\theta$$

$$Y_l^l = A(e^{i\varphi}\sin\theta)^l$$

b) forts

$$Y_L^L(\theta,\phi) = A\,(e^{i\varphi}\sin\theta)^L$$

c) $$\iint |Y_L^L|^2\, d\theta\, d\varphi \Rightarrow \int_0^{2\pi}\int_0^{\pi} A^2 \sin^{2L}\theta \sin\theta\, d\theta\, d\phi$$

$$A^2\, 2\pi \int_0^{\pi} \sin^{(2L+1)}(\theta)\, d\theta = 2\pi A^2 \cdot \frac{2L}{2L+1} \int_0^{\pi} \sin^{2L-1}\theta\, d\theta \ldots$$

$$4\pi A^2 \frac{(2^L L!)^2}{(2L+1)!} \leftarrow \text{Wolfram}$$

$$4\pi A^2 \frac{(2^L L!)^2}{(2L+1)!} = 1 \qquad A = \frac{1}{(2^L L!)^2}\sqrt{\frac{(2L+1)!}{4\pi}}$$

$$A = \frac{1}{2^{L+1} L!}\sqrt{\frac{(2L+1)!}{\pi}}$$