Oblig 4. eaflund / Eirik Lund , ønsker tilbakemelding. ☑

2.1 $\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-i\frac{E_n t}{\hbar}}$

$\Psi(x,t) = \psi(x) e^{-i\frac{(E_0 + i\Gamma)t}{\hbar}}$

$\Psi(x,t) = \psi(x) e^{-\frac{iE_0 t}{\hbar}} e^{\frac{\Gamma t}{\hbar}}$

Normalisering gir: $\int |\Psi(x,t)|^2 dx = 1 \Rightarrow e^{\frac{2\Gamma t}{\hbar}} \int |\psi(x)|^2 dx = 1$

da har vi at $\Gamma = 0$ for at normaliseringen skal være gyldig. $1 \cdot \int |\psi(x)|^2 \Leftrightarrow \int |\psi^* \psi \, e^0| = |\psi|^2$

og da har vi også at E_n må være reell.

b) Eq 2.5: $\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$

$E_1: \frac{-\hbar^2}{2m} \frac{\partial^2 \Psi_1(x,t)}{\partial x^2} + V(x,t)\Psi_1(x,t) = i\hbar \frac{\partial \Psi_1(x,t)}{\partial t}$

$E_2: \frac{-\hbar^2}{2m} \frac{\partial^2 \Psi_2(x,t)}{\partial x^2} + V(x,t)\Psi_2(x,t) = i\hbar \frac{\partial \Psi_2(x,t)}{\partial t}$

x gen: $\Psi(x,t) = c_n \Psi_n(x,t)$

$E_1 + E_2$ (*) innsatt 2.5: $c_1\left[\frac{-\hbar^2}{2m} \frac{\partial^2 \Psi_1(x,t)}{\partial x^2} + V(x,t)\Psi_1(x,t)\right] +$

$c_2\left[\frac{-\hbar^2}{2m} \frac{\partial^2 \Psi_2(x,t)}{\partial x^2} + V(x,t)\Psi_2(x,t)\right] = c_1\left[i\hbar \frac{\partial \Psi_1(x,t)}{\partial t}\right] + c_2[\ldots\Psi_2]$

Sorterer for operator:

$\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}\left(c_1\Psi_1(x,t) + c_2\Psi_2(x,t)\right] + V(x,t)\left[c_1\Psi_1(x,t) + c_2\Psi_2(x,t)\right]$

$= i\hbar \frac{\partial}{\partial t}\left[c_1\Psi_1(x,t) + c_2\Psi_2(x,t)\right]$

Setter inn: $\Psi(x,t) = c_1\Psi_1(x,t) + c_2\Psi_2(x,t)$

$\Rightarrow \frac{-\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$

Som er vårt utgangspunkt.

c) $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(-x) = E\psi(-x)$ <u>even</u> k

$\psi(x) = a\psi(x)$, $|a| = 1$ hvor $a = \begin{cases} +1 & \text{even} \\ -1 & \text{odd} \end{cases}$

even: $-\frac{\hbar^2}{2m}\frac{\partial^2 a\psi(-x)}{\partial x^2} + V(-x)\,a\psi(-x) = E_a\,\psi(-x) = *$

odd $-\frac{\hbar^2}{2m}\frac{\partial^2 a\psi(x)}{\partial x^2} + V(x)\,a\psi(x) = E_a\,\psi(x) = -*$

<u>2.5</u> $\Psi(x,0) = A[\psi_1(x) + \psi_2(x)]$

a) starter med normalisering for å løse A.

$1 = \int_{-\infty}^{\infty} |\Psi(x,0)|^2 dx = |A|^2 \int (\psi_1^*(x) + \psi_2^*(x))(\psi_1(x) + \psi_2(x))dx$

$|A|^2 \int_{-\infty}^{\infty} \psi_1^2(x) + \psi_2^2(x)\, dx = |A|^2 \left(\int_{-\infty} |\psi_1(x)|^2 dx + \int_{-\infty} |\psi_2(x)|^2 dx \right)$

$2|A|^2 = 1 \qquad A = \frac{1}{\sqrt{2}}$

b) relasjon : $\psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi x}{a}\right)$

det gir : $\Psi(x,t) = \frac{\sqrt{2}}{2}\left[\psi_1(x)e^{i\omega t} + \psi_2(x)e^{-i4\omega t}\right]$

$|\Psi(x,t)|^2 = A^2\left(c_1\psi_1(x)e^{+i\omega t} + c_2\psi_2(x)e^{+4i\omega t}\right)\left(c_1\psi_1(x)e^{-i\omega t} + c_2\psi_2(x)e^{-4i\omega t}\right)$

$= A^2\left[(c_1\psi_1(x))^2 + (c_2\psi_2(x))^2 + 2c_1c_2\psi_1\psi_2\cos\left(\frac{(E_2-E_1)t}{\hbar}\right)\right]$

$|c_1|^2 = |c_2|^2 \qquad |\Psi(x,t)|^2 = \frac{1}{2}\left(\psi_1^2(x) + \psi_2^2(x) + 2\psi_1\psi_2\cos(3\omega t)\right)$

c) $\langle x \rangle = |A|^2 \int x\,(\Psi_1^2(x) + \Psi_2^2(x) + 2\Psi_1\Psi_2 \cos(3\omega t))\,dx$

minner om $\Psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi x}{a}\right)$

$\Psi_1^2 = \frac{2}{a}\sin^2\left(\frac{\pi x}{a}\right) \qquad \Psi_2^2(x) = \frac{2}{a}\sin^2\left(\frac{2\pi x}{a}\right)$

$\langle x \rangle = \frac{1}{2}\int_0^a x\,|\Psi(x,t)|^2 = \frac{a}{2} - \frac{16a}{9\pi^2}\cos(3\omega t)$

$\omega = \frac{\pi^2\hbar}{2ma^2}$ vi har at $\frac{a}{2} \gg \frac{16a}{9\pi^2}\cos(3\omega t)$ så ser vi

at partikkelen oscillerer rundt midtpunktet $\frac{a}{2}$ og med en

amplitude på $\frac{16a}{9\pi^2}$

d) $\langle p \rangle = m\frac{\partial}{\partial t}\langle x \rangle = -i\hbar\int\left(\Psi^*\frac{\partial\Psi}{\partial}\right)dx$

$\langle p \rangle = m\frac{\partial}{\partial t}\left(\frac{a}{2} - \frac{16a}{9\pi^2}\cos(3\omega t)\right)$

$\langle p \rangle = m\left(-\frac{16a}{9\pi^2} - \sin(3\omega t)\,\omega\right)$

$\langle p \rangle = \frac{\cancel{m}\,\cancel{16}\,\cancel{a}}{9\cancel{\pi^2}}\,\frac{\cancel{\pi^2}\hbar}{\cancel{2}\cancel{m}a^{\cancel{2}}}\sin(3\omega t) = \frac{8\hbar}{3a}\sin(3\omega t)$

e) $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$ har: $\hat{H}\Psi = E\Psi$

$\langle H \rangle = \int \Psi^*\hat{H}\Psi\,dx = E\int|\Psi|^2 dx = E\int|\Psi|^2 = E$

$\langle H \rangle = \frac{1}{2}(E_1 + E_2)$

minner om: $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$, $E_1 = \frac{\pi^2\hbar^2}{2ma^2} \wedge E_2 = \frac{4\pi^2\hbar^2}{2ma^2}$

$\langle H \rangle = \frac{5\pi^2\hbar^2}{4ma^2}$ hver av leddene er vektet med

lik sannsynlighet

2.6 $\Psi(x,0) = A[\psi_1(x) + e^{i\varphi}\psi_2(x)]$

finner A ved å normalisere: $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$

litt som forrige oppgave gir dette

$4|A|^2 \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 2 \qquad 2|A|^2 = 2 \qquad A = \frac{\sqrt{2}}{2}$

$$\Psi_n(x,t) = A\left(\psi_1(x) e^{-i\frac{E_1 t}{\hbar}} + \psi_2(x) e^{-i\frac{E_2 t}{\hbar} + i\varphi}\right)$$
$$= \frac{\sqrt{2}}{2}\left(\psi_1(x) e^{-i\omega t} + \psi_2(x) e^{i(\varphi - 4\omega t)}\right)$$

$$\langle x \rangle = \int x |\Psi_n(x,t)|^2 dx = \frac{1}{2}\int x(\psi^* \psi) dx$$

$$\langle x \rangle = \frac{1}{2}\int_0^a x[\psi_1^2(x) + \psi_2^2(x) + 2\psi_1\psi_2 \cos(\varphi - 3\omega t)] dx$$

$$\langle x \rangle = \frac{a}{2} - \frac{16a}{9\pi^2}\cos(\varphi - 3\omega t)$$

undersøker ved $\varphi = \frac{\pi}{2}$

$$\langle x \rangle_{\frac{\pi}{2}} = \frac{a}{2} - \frac{16a}{9\pi^2}\cos\left(\frac{\pi}{2} - 3\omega t\right) \Rightarrow \frac{a}{2} - \frac{16a}{9\pi^2}\sin(3\omega t)$$

for $\varphi = \pi$

$$\langle x \rangle_{\pi} = \frac{a}{2} - \frac{16a}{9\pi^2}\cos(\pi - 3\omega t) \Rightarrow \frac{a}{2} - \frac{16a}{9\pi^2}\cos(3\omega t)$$

Vi ser at amplituden endres ikke, men at fase endring for oscillasjon ved $\varphi = \frac{\pi}{2}$.