Oblig 3, Eirik Lund / eirlund

4.5 Vis at dispersjonsrelasjonen for frie r. e⁻ bølger er:

a) $\omega(k) = c\sqrt{k^2 + \left(\frac{mc}{\hbar}\right)^2}$

Relasjoner: De Broglie; $E^2 = (mc^2)^2 + (pc)^2$ *

$E = \hbar\omega$ & $p = \hbar k$

$$* \quad \cancel{\hbar^2}\omega^2 = \frac{m^2c^4}{\hbar^2} + \frac{p^2c^2}{\hbar^2}$$

$$\omega^2 = \frac{m^2c^4}{\hbar^2} + \hbar^2 k^2 c^2$$

$$\omega = c\sqrt{\left(\frac{mc}{\hbar}\right)^2 + k^2}$$

b) fasehastighet $v_f = \frac{\omega}{k} = \frac{c}{k}\cdot\sqrt{k^2 + \left(\frac{mc}{\hbar}\right)^2}$

$$v_f = c\sqrt{1 + \left(\frac{mc}{\hbar k}\right)^2}$$

$$v_g = \frac{\partial\omega}{\partial k} = c\cdot\frac{1}{2}\sqrt{k^2 + \left(\frac{mc}{\hbar}\right)^2}^{\,-\frac{1}{2}}\cdot 2k = ck\sqrt{k^2 + \left(\frac{mc}{\hbar}\right)^2}^{\,-\frac{1}{2}}$$

$$v_g \cdot v_f = \frac{ck}{\sqrt{k^2 + \left(\frac{mc}{\hbar}\right)^2}} \cdot \frac{c}{k}\sqrt{k^2 + \left(\frac{mc}{\hbar}\right)^2} = c^2$$

c) bølgen / pakken reiser samlet med en hastighet som bølgefronten $\leq c$, men komponenter av pakken reiser i ulike hastigheter og bølgene blir strukket ut. Hastighetsøkningen skyldes resonans.

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4.6

a) En partikkel med masse 1g

uskarphet $\Delta v = 10^{-6}\ \frac{m}{s}$

Relasjon: $\Delta x \Delta p = \hbar$ $\quad \Delta p = \hbar \Delta k$ $\quad k = \frac{p}{\hbar}$

$\Delta x\, m \Delta v = \frac{h}{n 2\pi}$ $\quad \Delta x = \frac{h}{n 2\pi m \Delta v}$ $= \frac{2.11 \cdot 10^{-26}}{n = 1}$

b) e^- med Energi 10keV i et område $\Delta x = 0.1\,nm$

$\Delta x \Delta p \geq \hbar$ $\quad \Delta k = \frac{n}{c} \Delta \omega = \frac{n 2\pi E}{h c}$

$\Delta x = \sqrt{\frac{1}{\Delta k^2}}$

$\Delta p = \frac{\hbar}{2} \sqrt{\frac{n^2 4\pi^2 E^2}{h^2 c^2}} = n \cdot 2.67 \cdot 10^{-24}$

c) uskarphet

$\Delta E = \frac{n p^2}{2m} = n \cdot 3.914 \cdot 10^{-18}$

1.3 $\quad p(x) = A e^{-\lambda(x-a)^2}$

a) løs for A.

$$1 = \int_{-\infty}^{\infty} p(x)\,dx \qquad A\int_{-\infty}^{\infty} e^{-\lambda(x-a)^2}\,dx = A\sqrt{\frac{\pi}{\lambda}}$$

$$A\sqrt{\frac{\pi}{\lambda}} = 1 \qquad A = \sqrt{\frac{\lambda}{\pi}} \quad (*)$$

b)

$$\langle x\rangle = \int_{-\infty}^{\infty} x\cdot A e^{-\lambda(x-a)^2}\,dx \qquad A\int_{-\infty}^{\infty} x e^{-\lambda(x-a)^2} \qquad u = x-a$$

$$A\int_{-\infty}^{\infty} (\cancel{u}+a)\, e^{-\lambda u^2}\,du \qquad Aa\int e^{-\lambda u^2}\,du \overset{(*)}{=} \sqrt{\frac{\lambda}{\pi}}\, a\sqrt{\frac{\pi}{\lambda}}$$

$$\langle x\rangle = a$$

$$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2 A e^{-\lambda(x-a)^2}\,dx \qquad u = x-a \quad (+)$$

$$A\int_{-\infty}^{\infty} (u+a)^2 e^{-\lambda u^2}\,du \Rightarrow A\int_{-\infty}^{\infty} (u^2 + 2ua + a^2)\, e^{-\lambda u^2}\,du$$

Substituerer in + $\Rightarrow A\int_{-\infty}^{\infty} (u^2 + a^2)\, e^{-\lambda u^2}\,du$

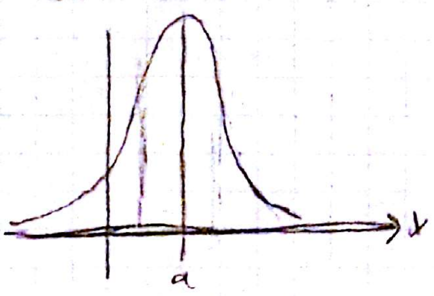
$$(*) \; \sqrt{\frac{\lambda}{\pi}}\left[\frac{(1+2a^2)\lambda}{2\lambda^2}\sqrt{\frac{\pi}{\lambda}}\right]$$

$$\langle x^2\rangle = \frac{1}{2\lambda} + \frac{2a^2\cancel{\lambda^2}}{2\lambda^2} = \frac{1}{2\lambda} + a^2$$

$$\sigma = \sqrt{\sigma^2} \qquad \sigma^2 = \frac{1}{2\lambda} + a^2 - a^2 = \frac{1}{2\lambda}$$

$$\sigma = \frac{1}{\sqrt{2\lambda}}$$

c)



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1.5 Bølge 1. $\Psi(x,t) = Ae^{-\lambda|x| - i\omega t}$

a) Normaliser

konjugert

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1 \quad \text{Vi har at } |\Psi(x,t)|^2 = \Psi \cdot \Psi^*$$

$$Ae^{-\lambda|x|} e^{-i\omega t} \cdot Ae^{-\lambda|x|} e^{i\omega t}$$ imaginære leddet blir $e^0 = 1$

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = |A|^2 \left[\int_{-\infty}^{0} e^{2\lambda x} dx + \int_{0}^{\infty} e^{-2\lambda x} dx\right]$$

$$\Rightarrow 2|A|^2 \int_0^{\infty} e^{-2\lambda x} dx = 2|A|^2 \left[\frac{1}{-2\lambda} e^{-2\lambda x}\right]_0^{\infty}$$

$$\Rightarrow 2|A|^2 \left[0 - \left(-\frac{1}{2\lambda}\right)\right] \Rightarrow \frac{|A|^2}{\lambda} = 1 \Leftrightarrow A = \sqrt{\lambda} \quad (*)$$

- $\langle x \rangle = 0$

$$\langle x^2 \rangle = 2|A|^2 \int_0^{\infty} x^2 e^{-2\lambda x} dx \overset{*}{\Rightarrow} 2\lambda \int_0^{\infty} x^2 e^{-2\lambda x} dx$$

$u = 2\lambda x$ Gammafunksjon $\Gamma(2) = \int_0^{\infty} x^{2-1} e^{-x} dx = (2-1)!$

$$\langle x^2 \rangle = 2\lambda \int_0^{\infty} \frac{u^2}{(2\lambda)^3} e^{-u} du$$

$$\langle x^2 \rangle = \frac{1}{4\lambda^2} \int_0^{\infty} u^2 e^{-u}, \quad \Gamma(3) = \int_0^{\infty} u^{3-1} e^{-u} du = (3-1)!$$

$$\langle x^2 \rangle = \frac{2!}{4\lambda^2} = \frac{1}{2\lambda^2}$$

b) $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{1}{2\lambda^2} - 0 \qquad \sigma = \frac{1}{\sqrt{2}\lambda}$

4.9 $\Psi(x,t) = Ae^{-a\left[\frac{mx^2}{\hbar} + it\right]}$

a) Løs for A.

Bruker normalisering. først rydder i imaginære leddet.

$$|\Psi(x,t)|^2 = \Psi \Psi^*$$

$$\Rightarrow |A|^2 \int_{-\infty}^{\infty} e^{-\frac{2max^2}{\hbar}} dx \quad , \quad |A|^2 \sqrt{\frac{\pi \hbar}{2ma}} = 1$$

$$A = \left(\frac{2ma}{\pi \hbar}\right)^{1/4}$$

b) potensiell energi tilfredstil. Schrödinger.

$$\Psi(x) = \left(\frac{2ma}{\pi \hbar}\right)^{1/4} e^{-\frac{amx^2}{\hbar}}$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x)}{\partial x^2} = \frac{2ma}{\hbar^2}(\hbar - 2amx^2) e^{-\frac{amx^2}{\hbar}} = -a(\hbar - 2amx^2) e^{-\frac{amx^2}{\hbar}}$$

Energi. $i\hbar \frac{\partial E}{\partial t} = \frac{\partial e^{-iat}}{\partial t} \quad E = \hbar a$

$$V(x)\Psi(x) = E\Psi(x) + a(\hbar - 2amx^2) e^{-\frac{amx^2}{\hbar}} \quad \Big| : \Psi(x)$$

$$V(x) = 2ma^2x^2$$

c) $\langle x \rangle = 0$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \Psi^2 dx \Rightarrow |A|^2 \int_{-\infty}^{\infty} x^2 e^{-\frac{2amx^2}{\hbar}} \quad u = \frac{amx^2}{\hbar}$$

$$|A|^2 \int_{-\infty}^{\infty} \frac{u\hbar}{am} e^{-u} \Rightarrow 2\frac{|A|^2\hbar}{am} \int_0^{\infty} u e^{-u} du = \frac{\hbar}{4am}$$

$$\langle p \rangle = -i\hbar \int_{-\infty}^{\infty} \Psi^* \frac{\partial \Psi}{\partial x} dx = 0$$

$$\langle p^2 \rangle = -i\hbar^2 \int_{-\infty}^{\infty} \Psi^* \frac{\partial^2 \Psi}{\partial x^2} dx = \hbar m a$$

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{\hbar}{4am} \quad \sigma_x = \sqrt{\frac{\hbar}{4am}} = \frac{1}{2}\sqrt{\frac{\hbar}{am}}$$

$$\sigma_p^2 = \langle p^2 \rangle - \langle p \rangle^2 = \hbar m a - 0 \quad \sigma_p = \sqrt{\hbar m a}$$