Oblig 7 eaflund / Eirik

2.23

a) $\int_{-3}^{+1} (x^3 - 3x^2 + 2x - 1)\,\delta(x+2)\,dx = \underset{-3}{\overset{1}{I}}(x)$

observer integrasjons intervallet $-2 \in [-3, 1]$

da har vi at: $\underset{-3}{\overset{1}{I}}(x) = f(-2)\int_{-3}^{1} \delta(x+2)dx = f(2) = -25$

b) $I = \int_{0}^{\infty} [\cos(3x) + 2]\,\delta(x - \pi)\,dx$, tilsvarende for grensene

$= f(\pi)\int_{0}^{\infty} \delta(x-\pi)\,\pi = f(\pi) = 1$

c) $\int_{-1}^{1} e^{(|x|+3)}\,\delta(x-2)\,dx = 0$ fordi $\delta(x = 2)$ intervallet er utenfor integrasjons intervallet $[-1, 1]$

2.24

$\int_{-\infty}^{\infty} f(x)\,D_1(x)\,dx = \int_{-\infty}^{\infty} f(x)\,D_2(x)\,dx$

antar $\delta(x-0)$

a) vis at $\delta(cx) = \frac{1}{|c|}\delta(x)$

Bruker relasjonen $\int_{-\infty}^{\infty} f(x)\,\delta(x-a)\,dx = f(a)$

Siden det er en skalar c i deltafunksjonen så gjør jeg et variabelskifte $dx = \frac{1}{c}\,d\sigma$

$f(0)\int_{-\infty}^{\infty} \delta(x)\,\frac{1}{c}\,d\sigma \qquad f(0)\,\frac{1}{c}\int_{-\infty}^{\infty} \delta(x)\,d\sigma$ Siden $a = 0$

Som oppfyller betingelsen $-\infty < a < \infty$ Så er integralet

$\frac{1}{|c|} f(0) = \frac{1}{|c|}\delta(x)$

b) $\Theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$

$$\int_{-1}^{1} f(x) \frac{d\theta}{dx} dx = f(0) \int_{-1}^{1} d\theta = f(0)[\Theta(1) - \Theta(-1)] = f(0)$$

vi har at $\frac{d}{dx}\theta = \delta(x)$ siden $\Delta\theta = f(0)$

2.26 Vis at $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx} dk$

Plancherels teorem: $\int_{-\infty}^{\infty} f(x) e^{-2\pi i \xi x} dx = \hat{f}(\xi)$

vi har at $\int_{-\infty}^{\infty} f(x)\,\delta(x) = f(0)$

$$\Delta k = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta(x) e^{-ikx} dx = \frac{1}{\sqrt{2\pi}} \text{ fra tidligere oblig en}$$

$$\delta(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Delta k e^{ikx} dx = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx} dx \text{ (invers)}$$

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dx$$

2.27 $V(x) = -\alpha[\delta(x+a) + \delta(x-a)]$

tester $x=0$, utenfor potensialet

$V(0)$: $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) = E\psi(x)$ relasjoner $k = \frac{\sqrt{-2mE}}{\hbar}$

gir $\psi(x) = \begin{cases} Ae^{-kx}, & x>a \\ Be^{-kx} + Ce^{kx} & x \in (0,a) \\ Be^{kx} + Ce^{-kx} & x \in (-a,0) \\ Ae^{kx} & x \in (-\infty,-a) \end{cases}$

$\left.\begin{matrix} x>a \\ x\in(0,a) \end{matrix}\right\}* \quad \left.\begin{matrix} x \geq a \\ x<a \end{matrix}\right\} x=a$

$\left.\begin{matrix} x\in(-a,0) \\ x\in(-\infty,-a) \end{matrix}\right\}** \quad \left.\begin{matrix} x>-a \\ x<-a \end{matrix}\right\} x=-a$

* Elimineres konstanter ved Borns betingelse

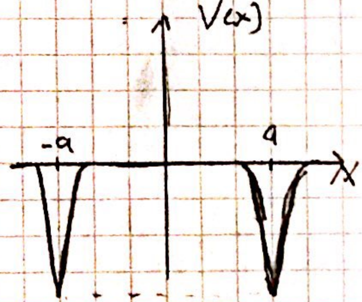
$Ae^{-kx} = Be^{-kx} + Ce^{kx} \quad |: e^{-kx}$

$A = B + Ce^{2kx}$

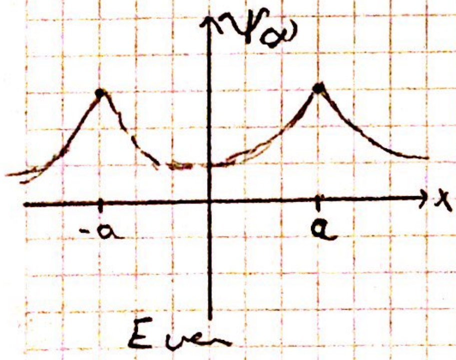
** $Be^{kx} + Ce^{-kx} = Ae^{kx} \quad |: e^{kx}$

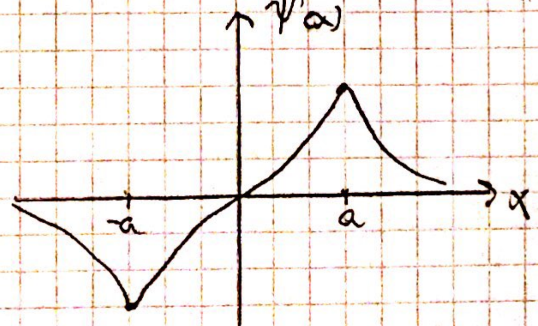
$B = C$

vi får da $\psi(x)_{even} = \begin{cases} B(1+e^{2ka})e^{-kx} & x>a \\ B(e^{-kx} + e^{kx}) & x \in (-a,a) \\ B(1+e^{2ka})e^{kx} & x<-a \end{cases}$



$\psi(x)_{odd} = \begin{cases} B(1-e^{2ka})e^{-kx} & x>a \\ B(e^{-kx} - e^{kx}) & x \in (-a,a) \\ -B(1-e^{2ka})e^{kx} & x<-a \end{cases}$





Even

$\alpha = \frac{\hbar^2}{4ma} = 2$ og $\alpha = \frac{\hbar^2}{ma} = 1$

Så: 1 bound state for $\alpha \leq \frac{\hbar^2}{2ma}$ $\wedge$ 2 if $\alpha > \frac{\hbar^2}{2ma}$