Oblig 11 Eirik Dal / eafdmd

4.24

a) fra kompendiet: klassisk angularmoment er
$L^2 = \ell(\ell+1)\hbar^2$ hvor $\ell \in 0, 1, 2, \dots$
vi har og relasjonen $L^2 = a^2 m^2 v^2$
Hamilton $H = 2(\frac{1}{2} m v^2)$, $v^2 = \frac{L^2}{a^2 m}$
$H = \frac{\ell(\ell+1)\hbar^2}{m a^2} \Rightarrow E_n = \frac{n(n+1)\hbar^2}{m a^2}$

b) $\psi_{nm}(\theta, \varphi) = Y_n^m(\theta, \varphi)$ normalisert
~~egenfunksjon.~~

vi har fra ligning 4.119 at for hver gitte
n så er det 2n+1 forskjellige
m verdier.

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4. 2d

a) $[S_x, S_y] = S_x S_y - S_y S_x$ **Del 2** *

$$= \frac{\hbar^2}{4}\left\{\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} - \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\right\}$$

$$= \frac{\hbar^2}{4}\left\{\begin{pmatrix}i&0\\0&-i\end{pmatrix} - \begin{pmatrix}-i&0\\0&i\end{pmatrix}\right\}$$

$$= \frac{\hbar^2}{4}\begin{pmatrix}2i&0\\0&-2i\end{pmatrix}$$

$$= \frac{\hbar^2}{2}\begin{pmatrix}i&0\\0&-i\end{pmatrix}$$

$$= i\hbar\frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = i\hbar S_z$$

tilsvarende har vi for $[S_y, S_z] = \frac{\hbar^2}{4}\{\sigma_y\sigma_z - \sigma_z\sigma_y\}$

$$= \frac{\hbar}{2} i\hbar\,\sigma_x$$

$$= i\hbar S_x$$

$$[S_z, S_x] = \frac{\hbar^2}{4}\{\sigma_z\sigma_x - \sigma_x\sigma_z\}$$

$$= \frac{\hbar}{2} i\hbar\,\sigma_y = i\hbar S_y$$

b) $\sigma_x\sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$\sigma_y\sigma_y = \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$\sigma_x\sigma_x = \sigma_y\sigma_y = \sigma_z\sigma_z = 1$

$\sigma_x\sigma_y = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = \begin{pmatrix}i&0\\0&-i\end{pmatrix} = i\sigma_z$

det følger videre av kommutering at $\sigma_y\sigma_z = i\sigma_x$
og at $\sigma_z\sigma_x = i\sigma_y$

ved $\sigma_y\sigma_x = -i\sigma_z$ som vi har fra * del 2
da følger $\sigma_z\sigma_y = -i\sigma_x$ og $\sigma_x\sigma_z = -i\sigma_y$

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4.27 $\quad \chi = A\begin{pmatrix}3i\\4\end{pmatrix}$

a) $\chi^\dagger\chi = \chi^T\chi = A^2(9+16) \equiv 1 \qquad A=\sqrt{\frac{1}{25}} = \frac{1}{5}$

b) $\langle S_x\rangle = \chi^T S_x \chi = A^2\frac{\hbar}{2}(-3i\ 4)\,\sigma_x\begin{pmatrix}3i\\4\end{pmatrix}$

$$= \frac{1}{25}\frac{\hbar}{2}(-3i\ 4)\begin{pmatrix}4\\3i\end{pmatrix}$$

$$= \frac{1}{50}\hbar(-12i+12i) = 0$$

$\langle S_y\rangle = \chi^T S_y \chi = A^2\frac{\hbar}{2}(-3i\ 4)\,\sigma_y\begin{pmatrix}3i\\4\end{pmatrix}$

$$= \frac{1}{50}\hbar(-3i\ 4)\begin{pmatrix}-4i\\-3\end{pmatrix}$$

$$= \frac{1}{50}\hbar(-12-12) = -\frac{12}{25}\hbar$$

$\langle S_z\rangle = \chi^T S_z \chi = \frac{1}{50}\hbar(-3i\ 4)\begin{pmatrix}3i\\-4\end{pmatrix}$

$$= \frac{\hbar}{50}(9-16) = \frac{-7}{50}\hbar$$

c) $\sigma S_x = \sqrt{\langle S_x^2\rangle - \langle S_x\rangle^2}$

$$= \sqrt{\frac{\hbar^2}{4} - 0}$$

$\sigma S_x = \frac{\hbar}{2}$

$\sigma S_y = \sqrt{\frac{\hbar^2}{4} - \left(-\frac{12}{25}\hbar\right)^2}$

$\sigma S_y = \frac{7}{50}\hbar$

$\sigma S_z = \sqrt{\frac{\hbar^2}{4} - \left(\frac{-7}{50}\hbar\right)^2} = \frac{12}{25}\hbar$

observere at $\sigma S_y = -\langle S_z\rangle$ og at $\sigma S_z = -\langle S_y\rangle$

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4.27 forts

d) Eq. 4.100 : $\sigma_{L_x} \sigma_{L_y} \geq \frac{\hbar}{2} |\langle L_z \rangle|$

$$\sigma_{S_x} \sigma_{S_y} \geq \frac{\hbar}{2} |\langle S_z \rangle|$$

$$\frac{\hbar}{2} \cdot \frac{7}{50}\hbar \geq \frac{\hbar}{2} \frac{7}{50}\hbar \rightarrow \sigma_{S_x} \sigma_{S_y} = \frac{\hbar}{2} |\langle S_z \rangle|$$

$$\sigma_{S_y} \sigma_{S_z} \geq \frac{\hbar}{2} |\langle S_x \rangle|$$

$$\frac{7}{50}\hbar \cdot \frac{24}{50}\hbar \geq \frac{\hbar}{2} \cdot 0 \rightarrow \sigma_{S_y} \sigma_{S_z} > \frac{\hbar}{2} |\langle S_x \rangle|$$

$$\sigma_{S_z} \sigma_{S_x} \geq \frac{\hbar}{2} |\langle S_y \rangle|$$

$$\frac{\cancel{24}}{50}\hbar \cdot \frac{\hbar}{\cancel{2}} \geq \frac{\hbar}{2} \frac{12}{25}\hbar \rightarrow \sigma_{S_z} \sigma_{S_x} = \frac{\hbar}{2} |\langle S_y \rangle|$$

Alle var innenfor betingelsene.

4.31 $\quad \chi_+ = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\chi_0 = c_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ og $\chi_- = c_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (≈ 4.149)

$\chi = c_1 \chi_+ + c_2 \chi_0 + c_3 \chi_-$, vi har 3 egentilstander

$S^2 \chi_j = \hbar^2 s(s+1) \chi_j = 2\hbar^2 \chi_j \quad$ for $j = 1, 0$ og -1

$S^2 = 2\hbar^2 [\chi_j, \chi_0, \chi_-] \qquad$ eig(σ_z)

bruker relasjonen $S_z \chi_j = j\hbar \chi_j$

$$S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Ved ligning 4.136 : $S_\pm |s\, m\rangle = \hbar \sqrt{s(s+1) - m(m \pm 1)} |s\, (m \pm 1)\rangle$

$S_+ \chi_1 = 0 \qquad S_+ \chi_0 = \hbar \sqrt{2} \chi_1 \qquad S_+ \chi_{-1} = \sqrt{2} \hbar \chi_0$

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$$S_+ = \sqrt{2}\hbar \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad S_- = \sqrt{2}\hbar \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S_x = \frac{1}{2}(S_+ + S_-) = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad (4.146)$$

$$S_y = \frac{1}{2i}(S_+ - S_-) = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$