Oblig 6 Fys 2140 Eirik [illegible] / eafkud

Griffiths: 1.14 2.15 2.18 og 2.19

1.14 $P_{ab}(t)$, prob find $P \in (a<x<b)$ at time t.

a) $P_{ab} = \int_a^b |\Psi^* \Psi| \, dx$

$$\frac{dP_{ab}}{dt} = \int_a^b \left[\frac{\partial \Psi^*}{\partial t}\Psi + \frac{\partial \Psi}{\partial t}\Psi^*\right] dx \quad \text{linear operator.}$$

$$= \int_a^b \left\{-\frac{i\hbar}{2m}\frac{\partial^2 \Psi^*}{\partial x^2} - \frac{1}{i\hbar} V \Psi^*\right\}\Psi \, dx + \int_a^b \left\{\frac{i\hbar}{2m}\frac{\partial^2 \Psi}{\partial x^2} + \frac{1}{i\hbar} V\Psi\right\}\Psi^* dx$$

$$= J(x,t) = \frac{i\hbar}{2m}\left(\Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x}\right)$$

b) $\frac{\partial P_{ab}}{\partial t} = J(a,t) - J(b,t)$ "ΔP_{ab}"

$$\Psi(x,t) = \left(\frac{2am}{\pi\hbar}\right)^{1/4} e^{-a[(mx^2/\hbar) + it]}$$

$$\frac{\partial \Psi}{\partial x} = -\frac{1}{\pi^{1/4}}\left(\frac{2am}{\hbar}\right)^{5/4} x e^{-a[(mx^2/\hbar) + it]}$$

$$J(x,t) = -\frac{i\hbar}{2m}\frac{1}{\pi^{1/4}}\left(\frac{2am}{\hbar}\right)^{5/4} x e^{-a[(mx^2/\hbar) + it]}\left(\frac{2am}{\pi\hbar}\right)^{1/4} e^{-a[(mx^2/\hbar) + it]^*}$$

$$+ \text{ " } \quad \text{———} \quad \text{ '' } \quad {}^* \quad \text{———} \quad {}^* \text{ " }$$

$$J(x,t) = 0$$

Scanned by CamScanner

2.15

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$$

$$P_{ab} = \int_a^b \psi_0^2(x)\,dx, \qquad \left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\int_{-\sqrt{\frac{2E}{m\omega^2}}}^{\sqrt{\frac{2E}{m\omega^2}}} e^{-\frac{m\omega x^2}{\hbar}}$$

$E = \frac{\hbar\omega}{2}$ for grunntilstand

gir klassisk: $\left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \int_{-\sqrt{\frac{\hbar}{m\omega}}}^{\sqrt{\frac{\hbar}{m\omega}}} e^{-\frac{m\omega x^2}{\hbar}}\,dx$

$\xi = \sqrt{\frac{m\omega}{\hbar}}\,x$

$P_{ab} = \frac{1}{\sqrt{\pi}}\int_{-1}^{1} e^{-\xi^2}$ denne er likt som feilfunksjon

matlab gir erf(1) $\approx$ 0,843

utsiden blir da $1 - P_{classical} = 0.157$

2.18 $\quad Ae^{ikx} + Be^{-ikx} = (A+B)\cos kx + i(A-B)\sin kx$

$= C\cos kx + D\sin kx$

$C = (A+B)$

$D = i(A-B)$

Scanned by CamScanner

2.19

vi har $J(x,t) = \frac{i\hbar}{2m}\left(\frac{\partial \psi^*}{\partial x}\psi - \frac{\partial \psi}{\partial x}\psi^*\right)$

Stasjonær tilstand:

$\psi(x,t) = Ae^{ikx}e^{-\frac{i\hbar k^2 t}{2m}}$

$\frac{\partial \psi}{\partial x} = ikA^{ikx}e^{-\frac{i\hbar k^2 t}{2m}} = ik\psi$

$J(x,t) = \frac{i\hbar k}{2m}(-i|A|^2 - i|A|^2) = \frac{\hbar k}{m}|A|^2$

vi strømmer imaginært pga positiv kurvert : x-ret

Scanned by CamScanner