PA1140: Waves and Quanta

Unit 4: Atoms and Nuclei

Tipler, Chapters 36 (36-1 to 36-2) & 40

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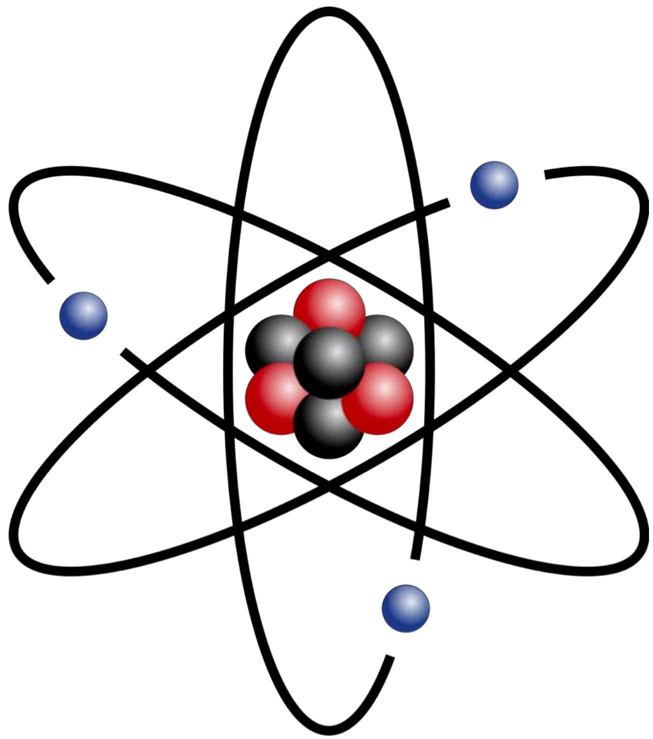

Dr Eloise Marais (Michael Atiyah Annex, 101)

Lecture 3: Radioactivity

Relevant Terminology

- **Radioactive nuclei**: decay into other nuclei by emitting particles (photons, electrons, neutrons, other subatomic particles)
- $\boldsymbol{\alpha}$ **decay**: release of ^{4}He atom (i.e. α particle)
- $\boldsymbol{\beta}$ **decay**: release of electrons (β^-) or positrons (β^+)
- **positron**: positively charged particle with the same mass as an electron
- $\boldsymbol{\gamma}$ **decay**: release of photons
- **Parent nucleus**: original nucleus undergoing decay
- **Daughter nucleus**: nucleus formed as a result of decay

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Radioactive Decay

Amount of a radioactive sample **decreases exponentially** with time (*statistical process that predicts average decay of sample, rather than of individual nuclides*)

Radioactive decay is **independent of pressure and temperature** (*nucleus is well shielded from others by the electrons*)

Change in number N of radioactive nuclei between time t and $t + dt$:

$$dN = -\lambda N dt$$

where λ is the decay constant in s^{-1} (*not to be confused with wavelength*)

$$\frac{dN}{dt} \propto N$$

characteristic of exponential decay

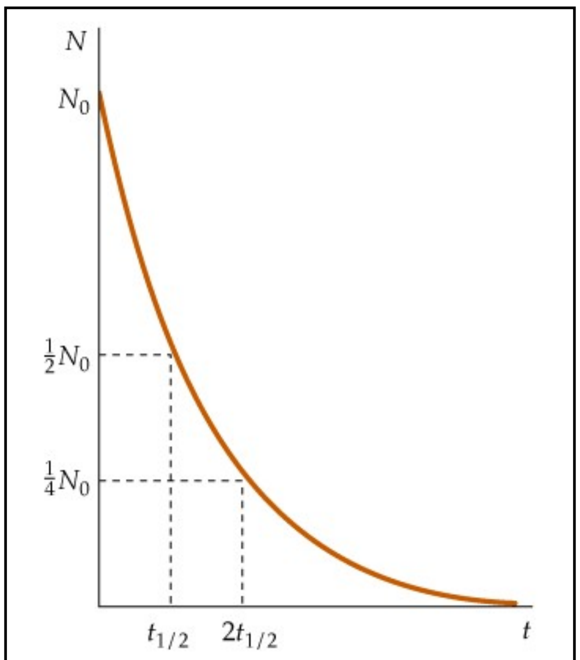
Radioactive Decay (contd)

Solving for N gives:

Derivation on board leading to:

$$\boxed{N = N_0 e^{-\lambda t}}$$

Number of particles vs time



The number of decays per second R is:

$$R = -\frac{dN}{dt}$$

$$\boxed{R = R_0 e^{-\lambda t}}$$

where $R_0 = \lambda N_0$ = 15 min^{-1} g^{-1} is the decay rate at $t = 0$

R also called **decay rate** or **activity of a sample**

R can be determined experimentally

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Lifetime and Half-life

The mean lifetime, τ, of a sample is the inverse of the decay constant:

$$\boxed{\tau = \frac{1}{\lambda}}$$

τ is the time required for the particle to decay to ${}^{1}/_{e}\, N_0$ or 37% of N_0
(*also referred to as e-folding time*)

Half-life, $t_{1/2}$, is the time it takes for N to reach $\frac{1}{2}N_0$:

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$e^{\lambda t_{1/2}} = 2$$

$$\boxed{t_{1/2} = \frac{ln2}{\lambda} = \tau ln2 = 0.639\tau}$$

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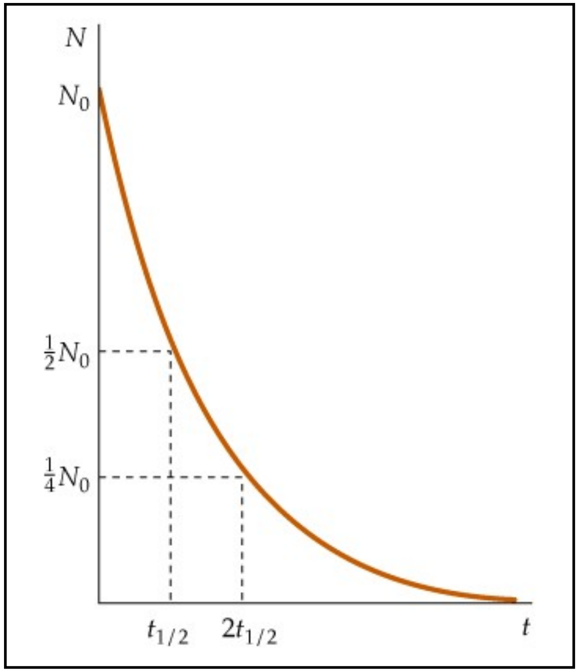
Lifetime and Half-life (contd)

Number of particles vs time



To plot instead as R, multiply N by lambda

At each half-life time interval, N and R decrease to half their previous value

After n half-lives:

$$\boxed{R_n = \left(\frac{1}{2}\right)^n R_0}$$

SI unit of radioactive decay: **becquerel** (**Bq**):

1 Bq = 1 decay per second

Historical unit is the curie (Ci):

1 Ci = 3.7×10^{10} decays/s = 3.7×10^{10} Bq

(*i.e. the rate at which radiation is emitted by 1 g of radium*)

Beta Decay

Occurs for nuclei with too few or too many neutrons.

Involves conversion of proton into neutron and vice versa.

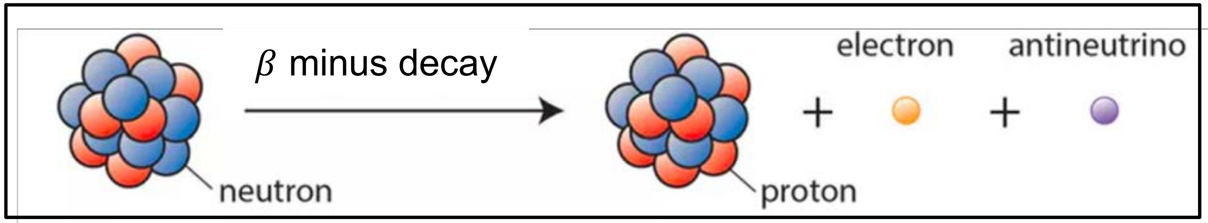
Leads to change in nuclide type. New nuclide has a more stable ratio of protons and neutrons

Beta minus decay:

Neutron becomes a proton

Electron and antineutrino emitted

N decreases, Z increases



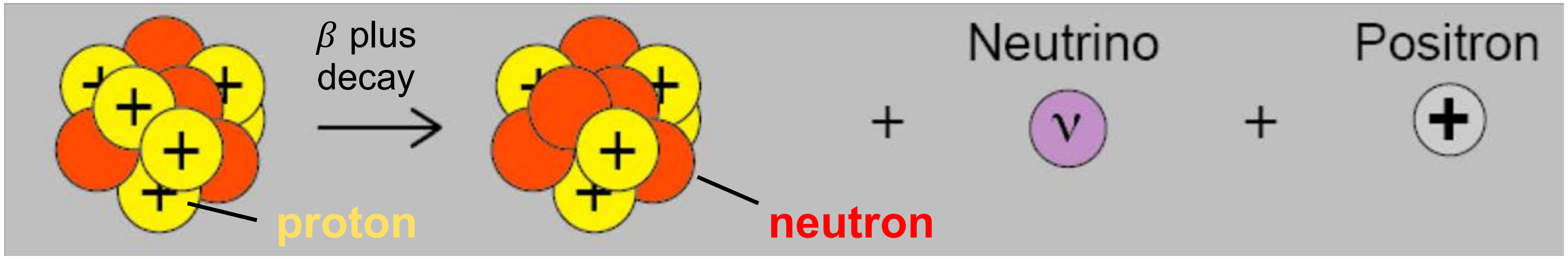
Beta Decay (contd)

Beta plus decay:

Proton becomes a neutron.

Positron and neutrino emitted.

N increases, Z decreases



Beta particle (electron or positron) and antineutrino or neutrino don't exist in the nucleus, but are created in the decay process.

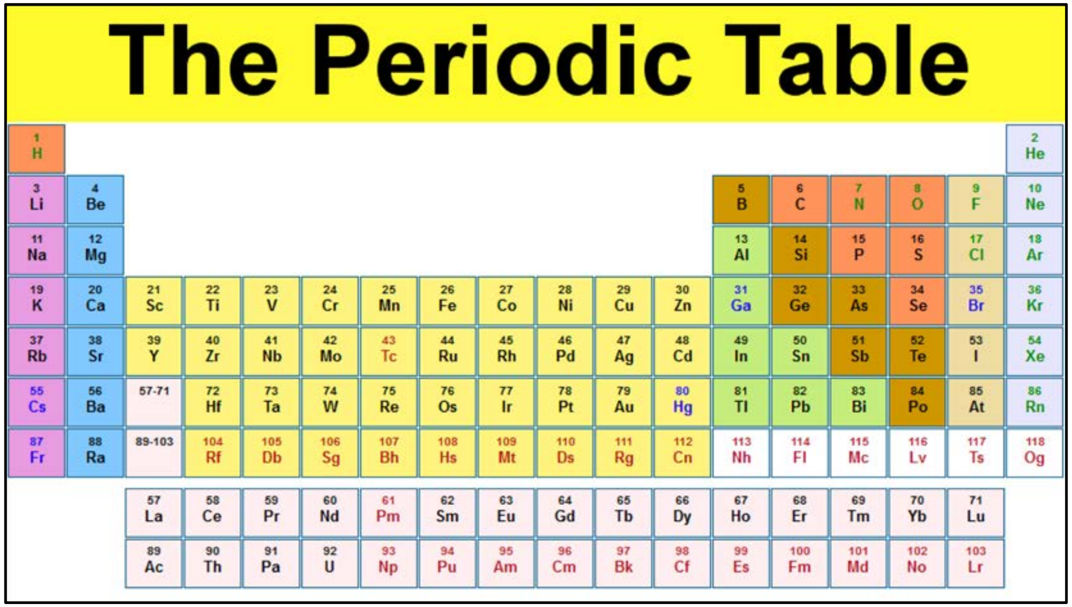
Beta Decay

Via what route β decay will carbon-14 and magnesium-23 decay?
What are the decay products?

$$^{14}_{6}\mathrm{C} \rightarrow$$

$$^{23}_{12}\mathrm{Mg} \rightarrow$$

Name new nuclides by identifying location on the periodic table:



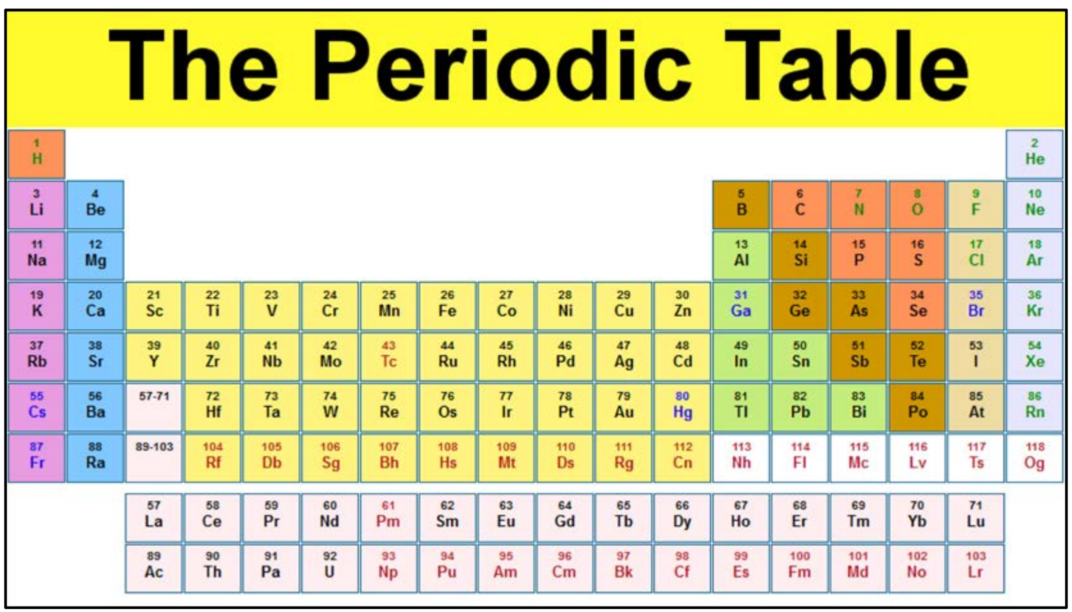
Beta Decay

Via what route β decay will carbon-14 and magnesium-23 decay?
What are the decay products?

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^{-} + \bar{\nu}_e$$

$$^{23}_{12}\mathrm{Mg} \rightarrow {}^{23}_{11}\mathrm{Na} + e^{+} + \nu_e$$

Name new nuclides by identifying location on the periodic table:



Radioactive Carbon Dating

Application of understanding of beta decay to determine the age of an object containing organic material

We already know the decay products of ^{14}C:

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^{-} + \bar{\nu}_e$$

$\boldsymbol{t_{1/2}} =$ **5730 years**

(takes 5730 years to reach half original number of carbon-14 atoms)

Should expect concentration of ^{14}C to reduce over time, but it's generated via reaction between ^{14}N and a neutron:

$$\mathrm{n} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + \mathrm{p}$$

n = neutron; p = proton

What is the neutron source?

Radioactive Carbon Dating (contd)

$$n + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + p$$

Cosmic rays

High-energy radiation originating from the Sun or outside our solar system.

Composed mostly of protons and alpha parties (^{4}He).

These are converted to neutrons and other particles when they collide with atoms and molecules in the Earth's atmosphere

Reaction to form ^{14}C occurs mostly in the upper troposphere/lower stratosphere (9-15 km aloft)

^{14}N is abundant in the Earth's atmosphere:
79% of air is nitrogen and 99.6% of nitrogen is present as the ^{14}N isotope

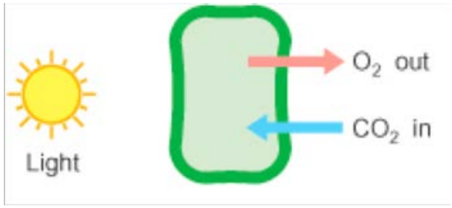
Radioactive Carbon Dating (contd)

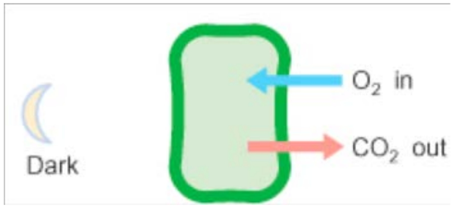
Once formed, ^{14}C reacts with oxygen in the atmosphere to form carbon dioxide ($^{14}CO_2$)

Our atmosphere is comprised of $^{14}CO_2$ and $^{12}CO_2$ with a relative abundance ($^{14}C/^{12}C$) of ~1.3×10^{-12}

Living organisms like plants continually exchange CO_2 with the atmosphere and so have the same $^{14}C/^{12}C$ as atmospheric CO_2

Photosynthesis





Respiration

Plants assimilated carbon via photosynthesis. Animals eat the plants, animals eat the animals that ate the plants, and so radiocarbon becomes distributed throughout the biosphere.

When organism dies, it no longer takes up ^{14}C, so $^{14}C/^{12}C$ decreases (^{12}C is stable)

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Radioactive Carbon Dating (contd)

Oldest samples that can be reliably measured are ~50,000 years old (~8 half-lives)

We're adding CO_2 to the atmosphere from burning fossil fuels (coal, natural gas)

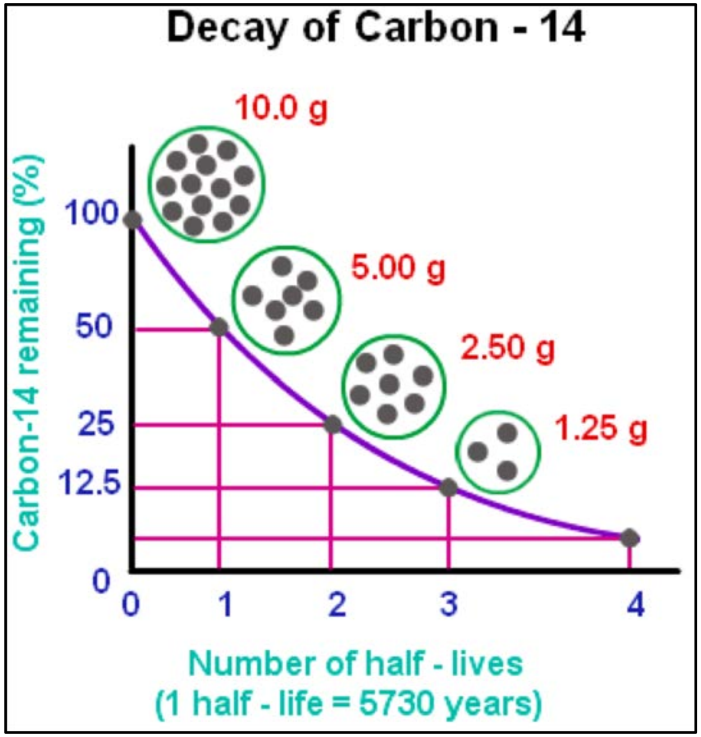
What is the effect on atmospheric $^{14}C/^{12}C$?

What the effect on carbon dating?

Practice Problems: Carbon Dating

(a) A non-living wood sample contains 10 g of carbon and shows a ^{14}C decay rate of 100 counts/min. How old is it?

(b) What decay rate would you expect from 15 g of 10,000 year-old wood?



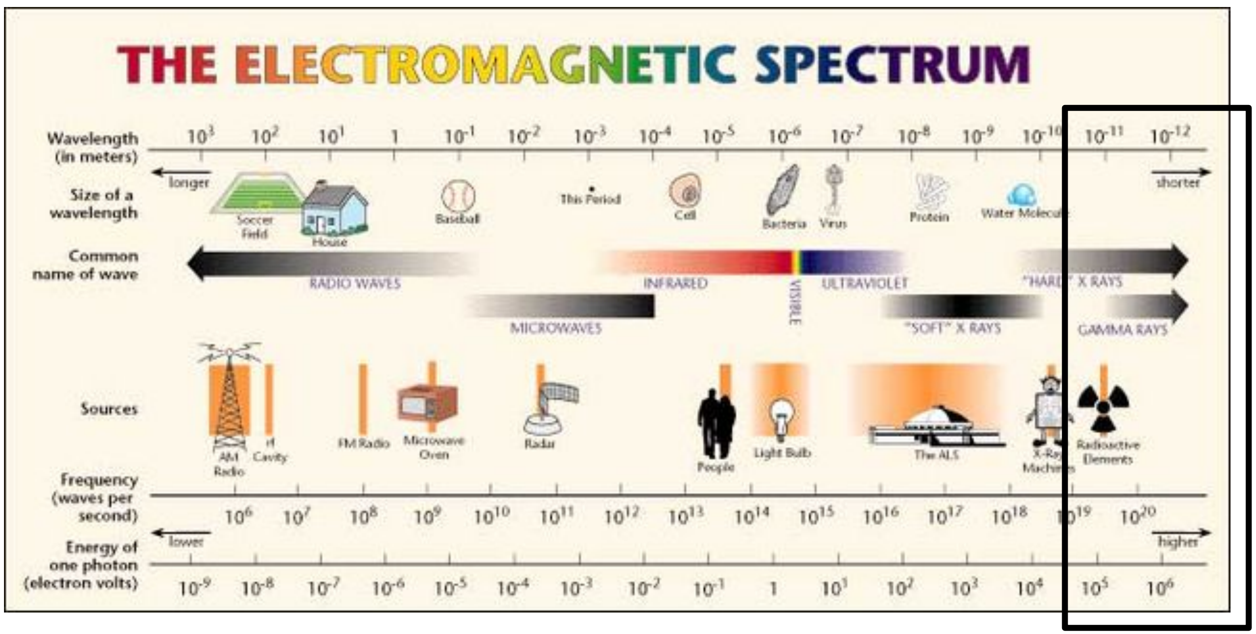
Reminder:
^{14}C has a half-life of 5730 years and exists in living material in a ratio $^{14}C/^{12}C$ of 1.3×10^{-12}, giving a decay rate at $t = 0$ of 15 min^{-1} g^{-1}.

Gamma Decay

Nucleus in an excited state decays to lower energy state by emitting a photon

A and Z remain the same

Spacing of nuclear energy levels range from a few keV to ~8 MeV, so wavelength of emitted photons are order 1 pm (10^{-12} m)



Gamma Decay (contd)

Gamma decay often occurs after β decay when excited daughter nucleus decays to its ground state

Lifetime of nuclei that can undergo β decay is very short (10^{-11} s is possible)

Example of β decay:

$$^{125}_{53}\mathrm{I}^* \rightarrow {}^{125}_{53}\mathrm{I} + \gamma$$

* denotes isotope in an excited state

Gamma rays are given off during nuclear fission (occurs in nuclear reactors and nuclear explosions).

Gamma rays also come from interaction of the Earth's atmosphere with cosmic rays.

Gamma rays are ionizing and so are biologically hazardous (can pass through the body, so pose a shielding challenge in nuclear reactors)

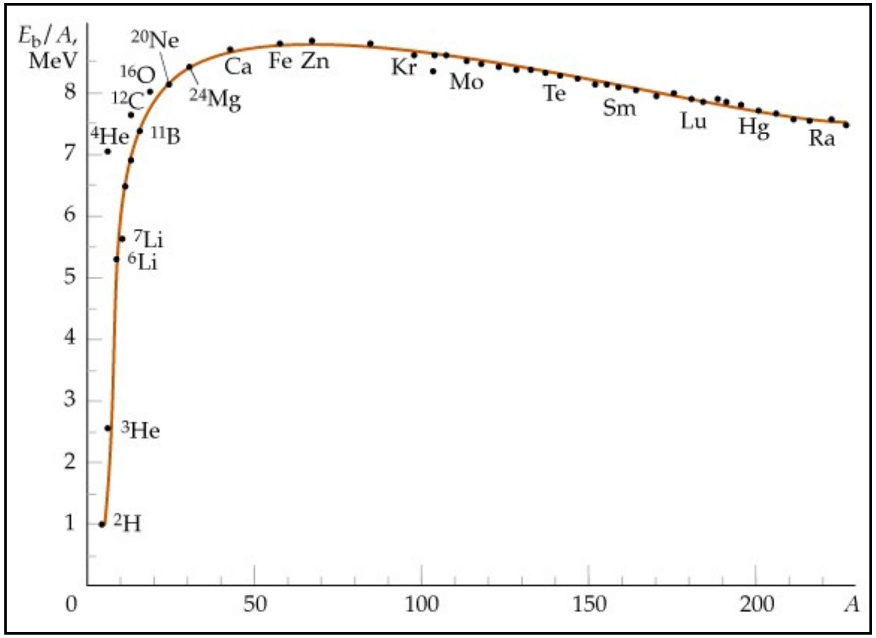
Alpha Decay

Tends to occur for very heavy nuclei (Z > 83) (and where N > Z)

Binding energy per nucleon (A) is no longer a minimum.
Coulomb repulsion so great that becomes unstable.

Daughter nucleus has atomic number, A, that is reduced by 4 relative to parent nucleus (removal of an α particle. i.e. ^{4}He).

Binding energy vs mass number



Example:

$$^{232}\mathrm{Th} \rightarrow {}^{228}\mathrm{Ra} + \alpha$$

or

$$^{232}\mathrm{Th} \rightarrow {}^{228}\mathrm{Ra} + {}^{4}\mathrm{He}$$

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Nuclear Reactions

Two nuclei or a nucleus and subatomic particle (proton, neutron) collide to produce one or more new nuclides

Q (or **Q factor**): amount of energy released or absorbed during a nuclear reaction

$$\boxed{Q = -\Delta mc^2}$$

where $\Delta m = \text{mass of products} - \text{mass of reactants}$

Where Q appears in a nuclear reaction:

$$^{2}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + {}^{1}\mathrm{H} + Q$$

$\boldsymbol{Q > 0}$ **Exothermic** reaction: energy is released

$\boldsymbol{Q < 0}$ **Endothermic** reaction: energy is required

Practice Problem: Q Factor

Find the Q values for the reactions below.
Are these exothermic or endothermic?

$$^{2}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + {}^{1}\mathrm{H} + Q$$

$$^{10}\mathrm{B} + \mathrm{n} \rightarrow {}^{4}\mathrm{He} + {}^{7}\mathrm{Li} + Q$$

Masses from Tipler Table 40-1

^{1}H = 1.007825 u
n = 1.008665 u
^{2}H = 2.014102 u
^{3}He = 3.016030 u
^{4}He = 4.002603 u
^{7}Li = 7.016004 u
^{10}B =10.012939 u

Reminder:

$$(1u)c^2 = 931.5\ \mathrm{MeV}$$

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Fission

Occurs when very heavy nucleus breaks apart into medium-mass (lighter) nuclei and energy is released

Requires initiation by absorption of a neutron

For fission to occur, the excitation energy (addition of a discrete amount of energy for change to occur) produced when a nucleus captures a neutron must be greater than the critical energy (minimum excitation energy required for fission to occur)

This is why ^{235}U undergoes fission, but ^{238}U does not:

$^{236}\mathrm{U}$ critical energy: 5.3 MeV
$^{235}\mathrm{U} + \mathrm{n}$ excitation energy: 6.4 MeV

Excitation > critical,
therefore **fission occurs**

$^{239}\mathrm{U}$ critical energy: 5.9 MeV
$^{238}\mathrm{U} + \mathrm{n}$ excitation energy: 5.2 MeV

Excitation < critical,
so **fission won't occur**

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Fission of uranium-235 (^{235}U)

(a)

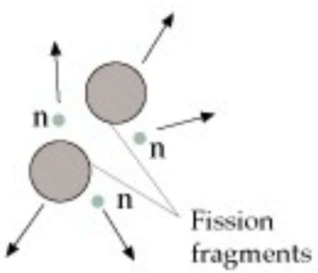
Absorption of neutron by ^{235}U

(b)

Forms ^{236}U in an excited state

(c)

Oscillation of ^{236}U becomes unstable



(d)

Nucleus splits into 2 nuclei and neutrons are emitted that initiate further fission (chain reaction ensues)

Coulomb repulsion force drives fragments apart

Typical fission reaction:

$$n + 235U \rightarrow {}^{141}Ba + {}^{92}Kr + 3n$$

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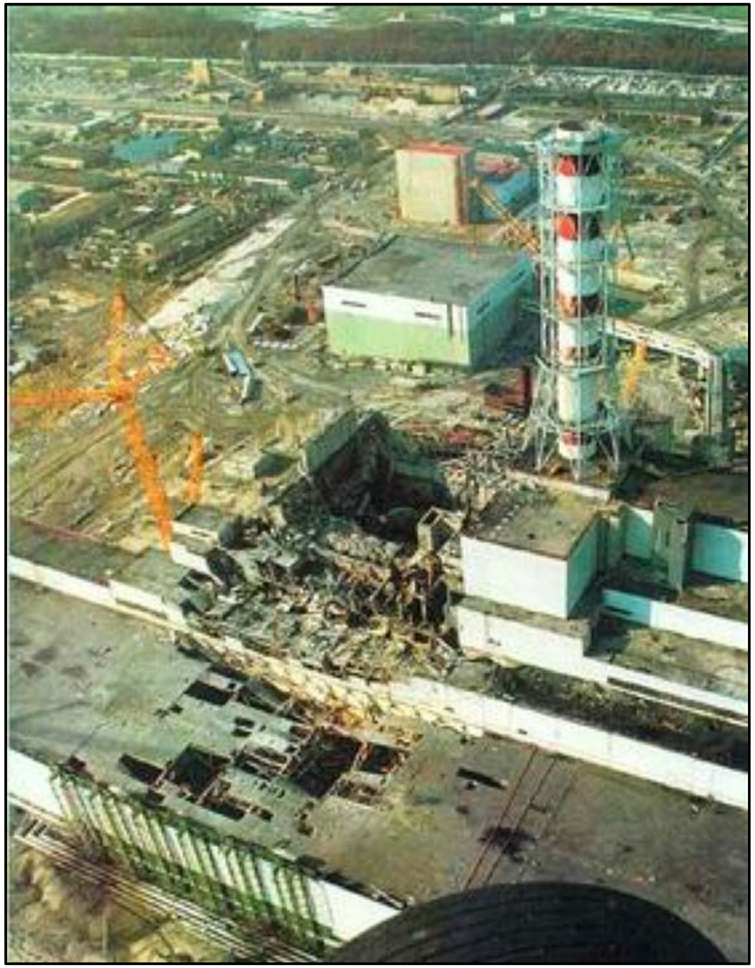
Fission of uranium-235 (^{235}U) (contd)

Chernobyl disaster (1986):
Nuclear explosion due to runaway chain reaction



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