PA1140: Waves and Quanta

Unit 4: Atoms and Nuclei

Tipler, Chapters 36 (36-1 to 36-2) & 40

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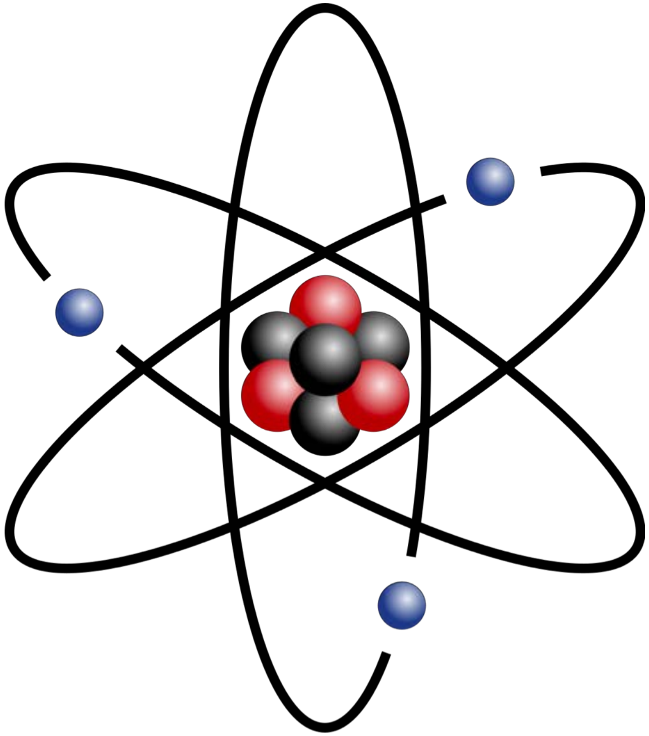


Dr Eloise Marais (Michael Atiyah Annex, 101)

Fusion

Two light molecules fuse together to form a nucleus of greater mass

Example:

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + \mathrm{n} + 17.6\ \mathrm{MeV}$$

Greater amount of energy produced per unit mass than fission

Promising source of energy:

High efficiency

Abundance of light nuclei

Less dangerous than fission reactors

Fusion (contd)

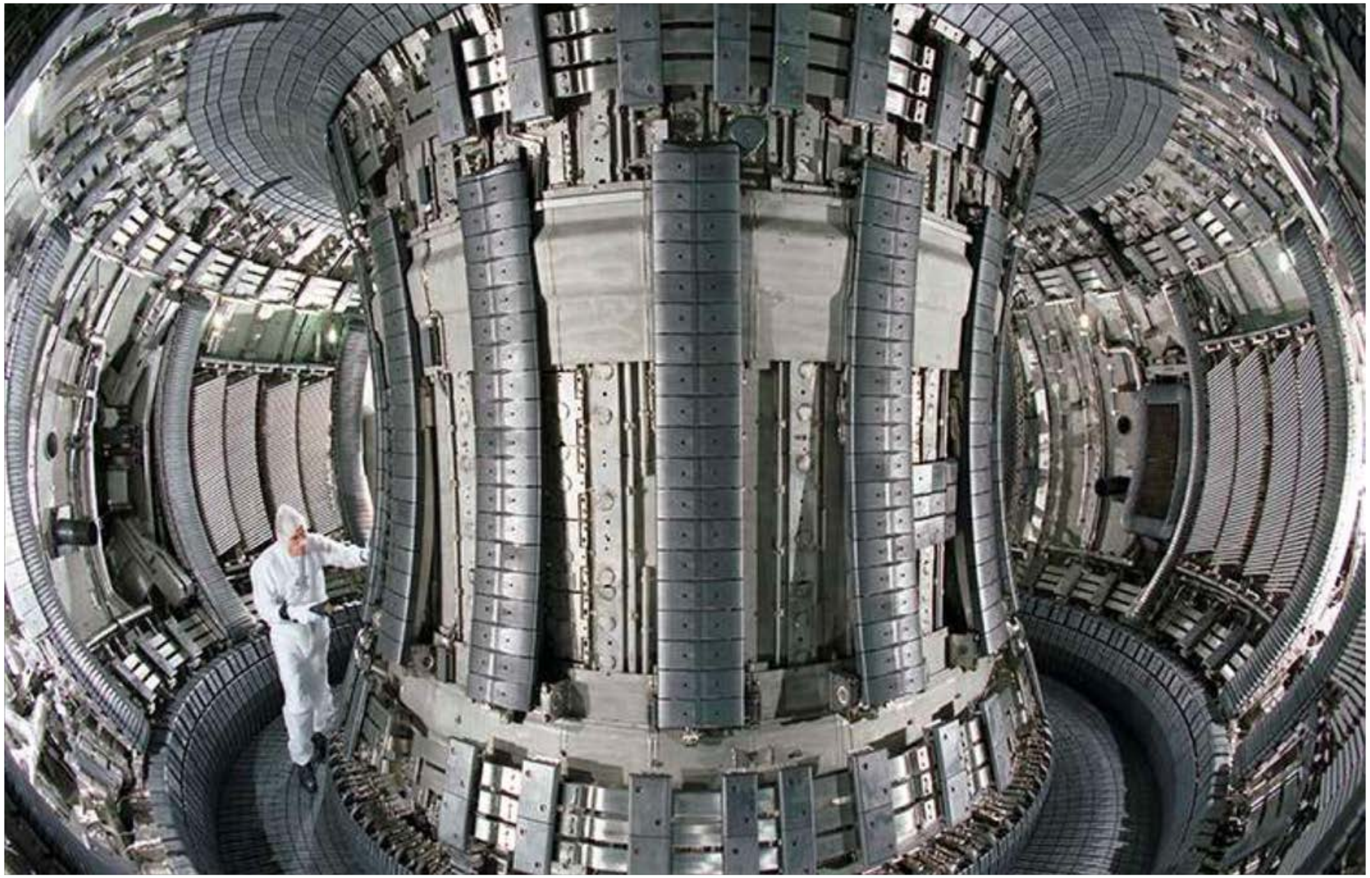
Technology not yet a reality for commercial electricity generation

A few issues and challenges:

- Coulomb repulsion is very large between ^{2}H and ^{3}H nuclei
- Need KE > 1 MeV to overcome Coulomb repulsion
- Can obtain these energies by speeding up particles in an accelerator, but scattering is more likely than fusion
- Particles must be heated to temperatures of order 10^8 K to form a plasma (ionization of atomic nuclei)
- Confinement hard to achieve in fusion reactor. Confined in Sun by enormous gravitational field.
- Ultimately, fusion requires more energy than is recovered

Fusion Research-Grade Reactors

Joint European Torus (JET) Fusion Reactor in the UK



Unit 4: Review

- The Atom
- Atomic Spectra
- The Bohr Model
- Properties of Nuclei
- Mass and Binding Energy
- Radioactivity
- Nuclear Reactions
- Fission
- Fusion

Atomic Spectra and the Bohr Model

Rydberg-Ritz formula:

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)}$$

where: n_1 and n_2 are integers

$n_1 > n_2$

$R = 1.096776 \times 10^7 \text{ m}^{-1}$ for hydrogen

Energy for a Circular Orbit:

$$\boxed{E = -\frac{1}{2}\frac{kZe^2}{r}}$$

Bohr's Postulates:

1. Electron can move only in circular orbits called **stationary states**

2. Electron can make transition from high to low energy orbit leading to emission of photon

3. Angular momentum of an electron orbiting a nucleus is quantized

Atomic Spectra and the Bohr Model

Photon frequency:

$$\boxed{f = \frac{E_i - E_f}{h} = \frac{\Delta E}{h}}$$

Photon wavelength:

$$\boxed{\lambda = \frac{c}{f} = \frac{hc}{E_i - E_f}}$$

Energy of orbit of hydrogen atom:

$$\boxed{E_n = -Z^2 \frac{E_0}{n^2}}$$

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Properties of Nuclei

Nuclear Radii:

$$\boxed{R = r_0 A^{1/3}}$$

Binding Energy:

$$\boxed{E_b = (ZM_H + NM_N - M_A)c^2}$$

Radioactivity

Exponential Decay of a Radioactive Sample:

$$\boxed{N = N_0 e^{-\lambda t}}$$

Decay Rate or Activity of a Sample:

At time t:

$$\boxed{R = R_0 e^{-\lambda t}}$$

After n half-lives:

$$\boxed{R_n = \left(\frac{1}{2}\right)^n R_0}$$

Lifetime or e-folding time:

$$\boxed{\tau = \frac{1}{\lambda}}$$

Half-life:

$$\boxed{t_{1/2} = 0.639\tau}$$

Radioactive Decay

Beta Decay:

Conversion of proton to neutron (plus) or neutron to proton (minus)

Application to determine age of organic material (radiocarbon dating)

Gamma Decay:

Excited state nucleus decays to ground state

Alpha Decay:

Decay of heavier nuclei leading to formation of ^{4}He and daughter nucleus A reduced by 4 relative to parent nucleus

Nuclear Reactions

Energy released or absorbed during a nuclear reaction:

$$\boxed{Q = -\Delta mc^2}$$

$\boldsymbol{Q} > \mathbf{0}$: exothermic
$\boldsymbol{Q} < \mathbf{0}$: endothermic

Fission:

Heavy nucleus reacts with neutron and breaks apart into two medium-mass nuclei

Fusion:

Two light nuclei fuse together to form a nucleus of greater mass

Practice Problem 1

The wavelength of the emission lines produced by the hydrogen atom are given by the formula

$$\frac{1}{\lambda} = 1.096776\times10^{-2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)\mathrm{nm}^{-1}$$

a) What are the wavelengths of the first two lines in the Balmer series that involve transitions to level $n = 2$?

b) Calculate the energy in eV required to raise an electron from the ground state to level 2.

c) Electrons of energy 12.9 eV are fired at hydrogen atoms in a gas discharge tube. If initially all atoms are in the ground state, what is the highest level to which the electrons in the atom can be excited? What is the longest wavelength of the possible level transitions that may then follow?

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Practice Problem 2

Substitute the missing atomic number, mass numbers, and/or symbols in the following nuclear reactions and radioactive decays:

$$\boxed{{}^{59}_{27}\mathrm{Co} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{61}_{29}\mathrm{Cu} + ?}$$

$$? + {}^{63}_{29}\mathrm{Cu} \rightarrow {}^{64}_{30}\mathrm{Zn} + \mathrm{n}$$

$$\mathrm{p} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + ?$$

$$? + {}^{235}_{92}\mathrm{U} \rightarrow {}^{93}_{37}\mathrm{Rb} + {}^{141}_{55}\mathrm{Cs} + 2\mathrm{n}$$

$$\boxed{\mathrm{n} + {}^{238}_{92}\mathrm{U} \rightarrow {}^{239}_{93}\mathrm{Np} + \mathrm{e}^{-} + ?}$$

$${}^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + ? + \overline{\nu}$$

$$\boxed{{}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + ? + \nu}$$

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Practice Problem 3

A radioactive nucleus with decay constant λ is produced in a nuclear reactor at a rate R_0 nuclei s^{-1}. Assuming that the number of nuclei initially present is zero, the number of nuclei N after time t is given by the expression:

$$N = \frac{R_0}{\lambda}\left(1 - e^{-\lambda t}\right)$$

The rate of production of ^{22}Na in a reactor is 10^{15} nuclei s^{-1}. Production continues for a period of one year. What is the decay rate of the ^{22}Na sample a further one year after completion of irradiation? The half-life of ^{22}Na is 2.6 years.

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