PA1140: Waves and Quanta

Unit 4: Atoms and Nuclei

Tipler 6th ed, Chapters 36 (36-1 to 36-2) & 40

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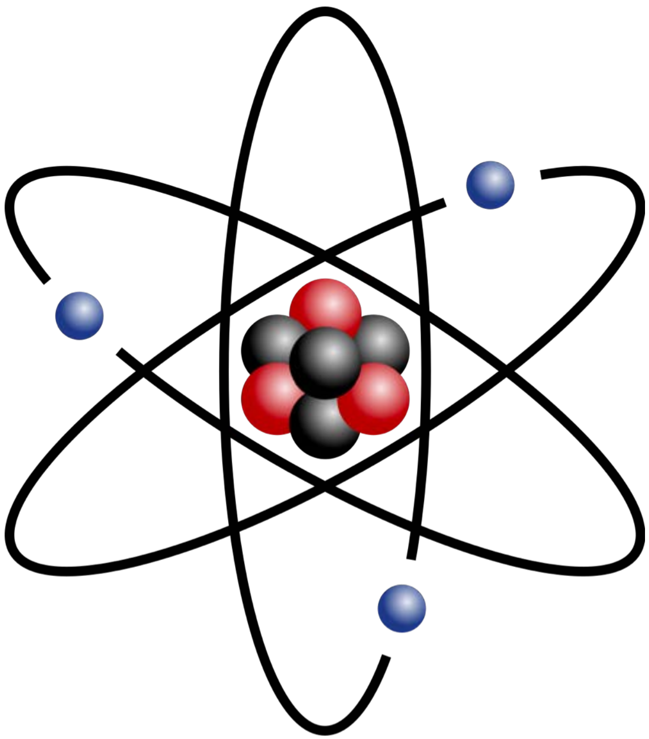

Dr Eloise Marais (Michael Atiyah Annex, 101)

Lecture 2 Recap

Properties of Nuclei

Nuclear Radii:

$$\boxed{R = R_0 A^{1/3}}$$

Binding Energy:

$$\boxed{E_b = (ZM_H + NM_N - M_A)c^2}$$

Radioactivity

Exponential Decay of a Radioactive Sample:

$$\boxed{N = N_0 e^{-\lambda t}}$$

Decay Rate or Activity of a Sample:

$$\boxed{R = R_0 e^{-\lambda t}}$$

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RADIOACTIVE DECAY (contd)

Lifetime

The mean **lifetime**, τ, of a sample is the reciprocal of the decay constant:

$$\boxed{\tau = \frac{1}{\lambda}}$$

After a time equal to the mean lifetime (τ):

$$\left.\begin{aligned} N &= e^{-1}N_0 = 0.37N_0 \\ R &= e^{-1}R_0 = 0.37R_0 \end{aligned}\right\}$$

N and R are 37% of their original values

Lifetime (τ) is also referred to as the **e-folding time**

Half-life

Half-life, $t_{1/2}$, is the time it takes for the number of nuclei (the sample) and the decay rate to decrease by half:

$$N = \frac{N_0}{2} \qquad R = \frac{R_0}{2}$$

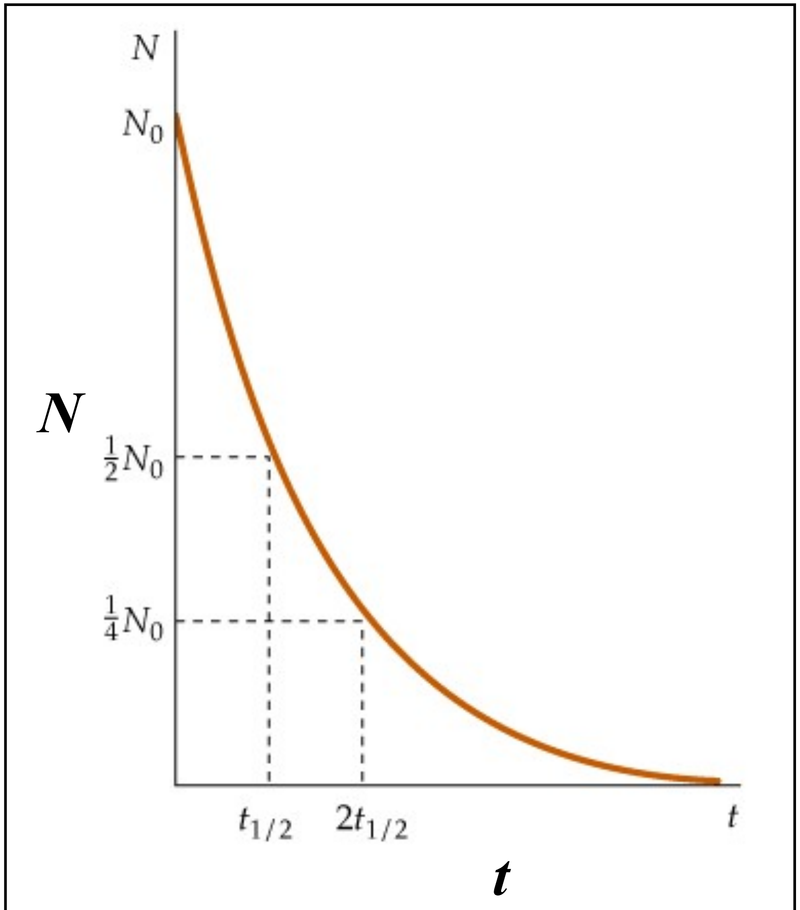
We can obtain an expression for $t_{1/2}$ in terms of the decay constant:

Derivation on board leading to:

$$\boxed{t_{1/2} = \frac{ln2}{\lambda} = \tau ln2 = 0.693\tau}$$

Half-life

Number of particles vs time



After each half-life, the number of remaining nuclei have decreased by half

The decay rate $R = \lambda N$ has the same dependence as N

Plot as a function of R by multiplying the y-axis by λ.

Half-life

We can obtain a general expression for the decay rate after n half-lives:

$$\boxed{R_n = \left(\frac{1}{2}\right)^n R_0}$$

Half-lives vary from 1 μs for very unstable nuclei to 10^{10} y for very stable nuclei

SI unit of radioactive decay: **becquerel** (**Bq**):
(1 Bq = 1 decay s^{-1})

Historical unit is the **curie** (**Ci**):

1 Ci = 3.7 x 10^{10} decays/s = 3.7 x 10^{10} Bq
(*i.e. the rate at which radiation is emitted by 1 g of radium*)

Marie Curie

Beta Decay

Occurs for parent nuclei with too many or too few neutrons for stability (e.g. nitrogen-13 from lightning or carbon-14 from cosmic rays).

Involves conversion of proton into neutron or vice versa.

Leads to change in nuclide type. New nuclide has a more stable ratio of protons and neutrons.

Includes release of a β **particle** (electron or positron) and **antineutrino** or **neutrino.**

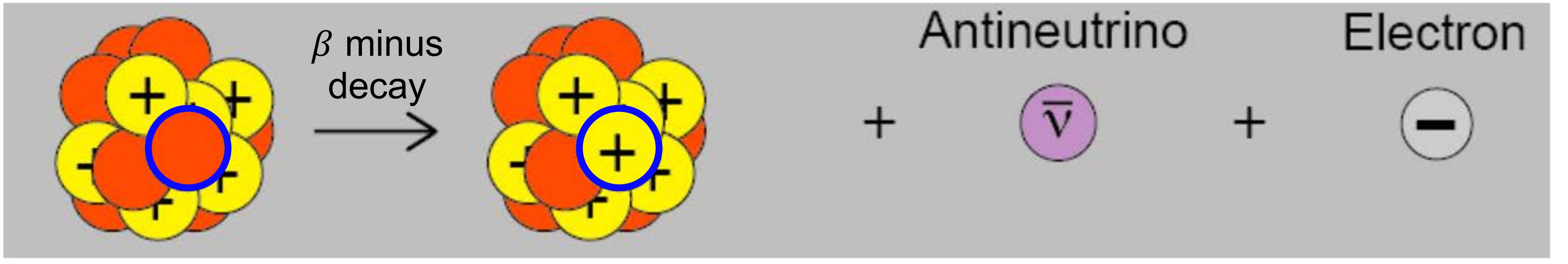
β particles and neutrinos don't exist in the nucleus, but are created in the decay process.

A remains the same, whereas Z either increases or decreases by 1.

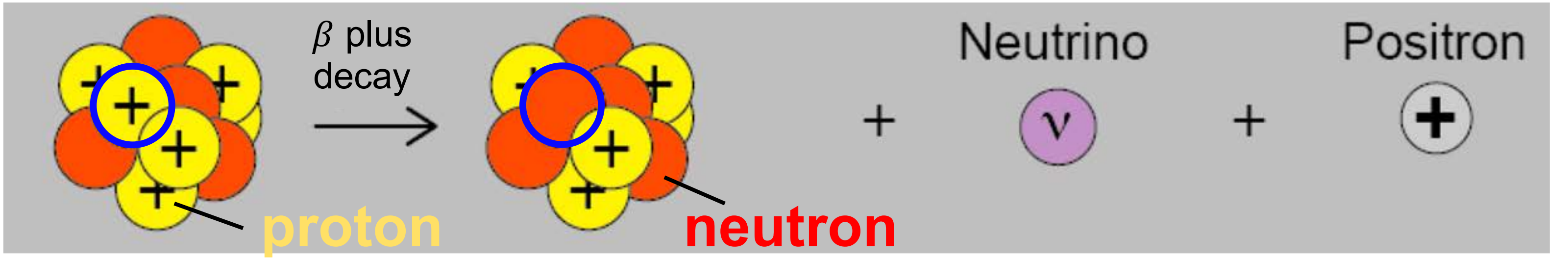
Energy of β decay is 0.782 MeV (the difference between the rest energy of a neutron and the rest energy of a proton and electron)

Beta Decay

Beta minus decay:



Beta plus decay:



β minus	β plus
Neutron → proton	Proton → neutron
Electron & antineutrino emitted	Positron & neutrino emitted
N decreases; Z increases	N increases; Z decreases

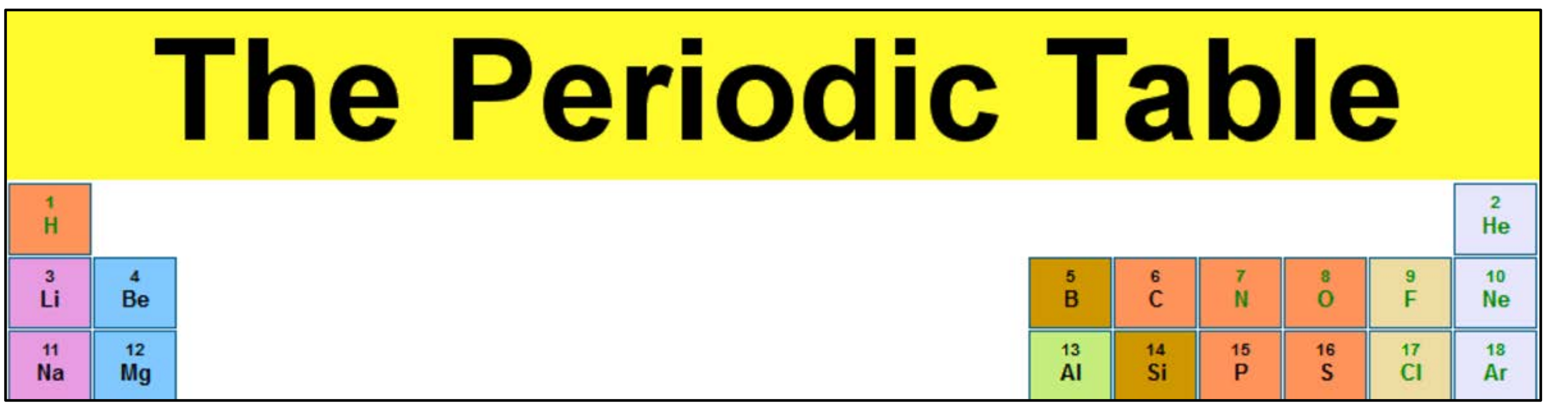
Beta Decay

Via what β decay will carbon-14 and magnesium-23 decay?
What are the decay products?

$${}^{14}_{6}\mathrm{C} \rightarrow$$

$${}^{23}_{12}\mathrm{Mg} \rightarrow$$

Find name of the new nuclides in the periodic table:



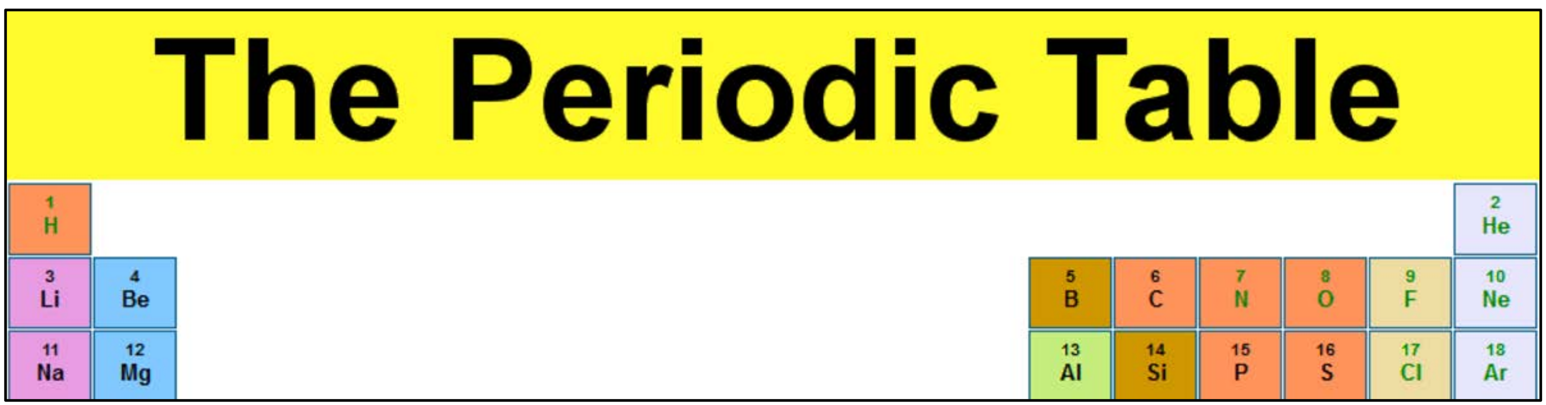
Beta Decay

Via what β decay will carbon-14 and magnesium-23 decay?
What are the decay products?

$$ {}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}} $$

$$ {}^{23}_{12}\mathrm{Mg} \rightarrow {}^{23}_{11}\mathrm{Na} + \mathrm{e}^{+} + \nu_{\mathrm{e}} $$

Find name of the new nuclides in the periodic table:



Application of β decay: Radiocarbon Dating

Application of understanding of beta decay to determine the age of an object containing organic material

We already know the decay products of ^{14}C:

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}$$

$\boldsymbol{t_{1/2}}$ for this decay is **5730 years**
(takes 5730 years to reach half original number of carbon-14 atoms)

Should expect concentration of ^{14}C to decrease over time, but it's generated via reaction between ^{14}N and a neutron:

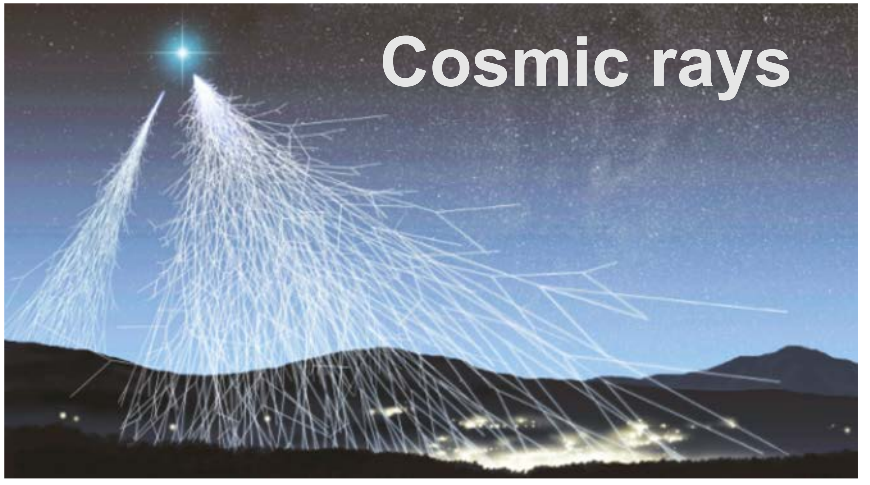
$$\mathrm{n} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + \mathrm{p}$$

n = neutron; p = proton

^{14}N is abundant in the Earth's atmosphere (~79% of air)

What is the neutron source?

Application of β decay: Radiocarbon Dating



$$\mathrm{n} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + \mathrm{p}$$

High-energy radiation originating from the Sun or outside our solar system.

Composed mostly of protons and α particles (^{4}He) that are converted to neutrons and other particles when they collide with atoms and molecules in the Earth's atmosphere

Reaction to form ^{14}C occurs mostly 9-15 km overhead
(upper troposphere/lower stratosphere and aircraft cruising altitude)

Application of β decay: Radiocarbon Dating

^{14}C and ^{12}C (the most abundant carbon isotope) have the same reactivity

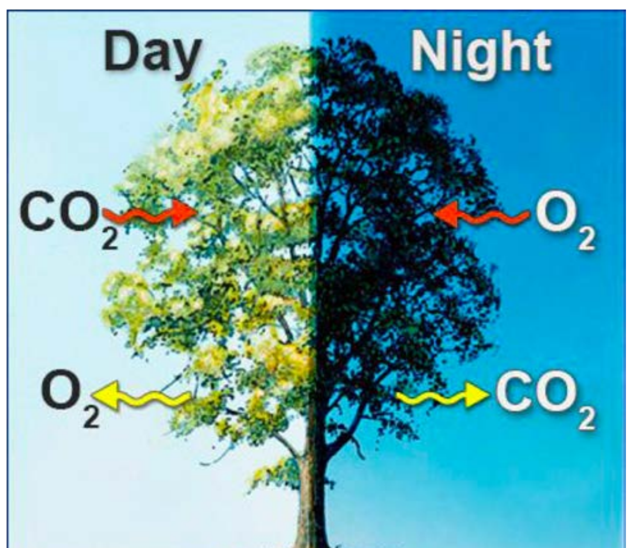
^{14}C and ^{12}C react with oxygen (O_2) in the atmosphere to form carbon dioxide ($^{12}CO_2$ from ^{12}C and $^{14}CO_2$ from ^{14}C)

The relative abundance of $^{14}CO_2$ and $^{12}CO_2$ ($^{14}CO_2/^{12}CO_2$) is ~1.3×10^{-12}

Living organisms like plants continually exchange CO_2 with the atmosphere, so have the same ratio of $^{14}CO_2$-to-$^{12}CO_2$):

Photosynthesis

Carbon assimilation during the day

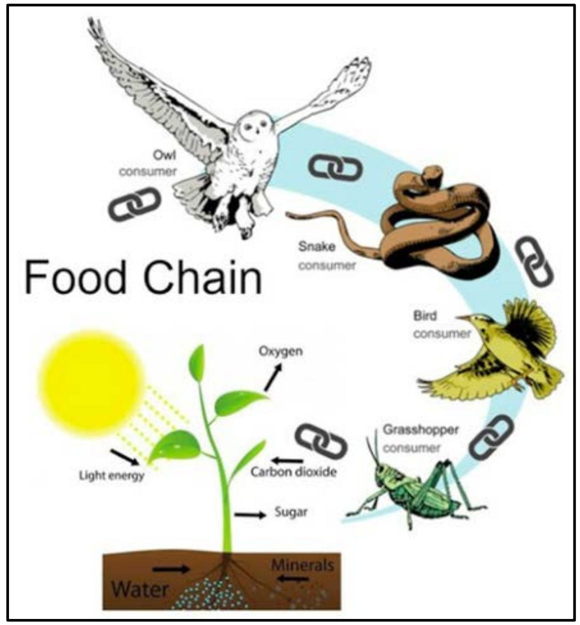


Respiration

Respire CO_2 at night (and during the day)

Application of β decay: Radiocarbon Dating

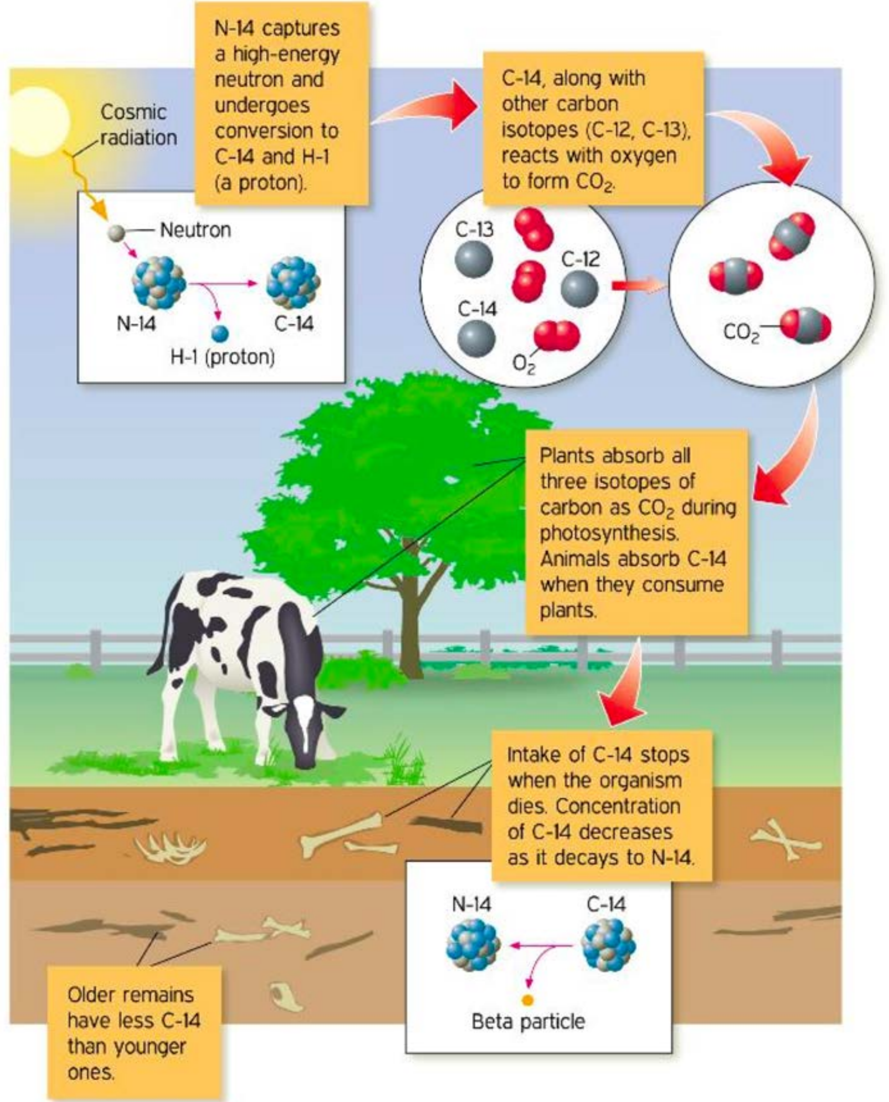
Animals eat the plants, animals eat the animals that ate the plants, and so on up the food chain so that radiocarbon becomes distributed throughout the biosphere.



When these organisms die, they no longer take up ^{14}C, so $^{14}C/^{12}C$ decreases (^{12}C is stable, ^{14}C undergoes β decay)

Oldest samples that can be reliably measured are ~50,000 years old (~8 half-lives)

Application of β decay: Radiocarbon Dating



Our Impact on Radiocarbon Dating

We're adding CO_2 to the atmosphere from burning fossil fuels (coal, natural gas, oil that are millions of years old)

What is the effect on atmospheric $^{14}C/^{12}C$?

What is the effect on carbon dating?

Practice Problems: Carbon Dating

(a) A non-living wood sample contains 10 g of carbon and shows a ^{14}C decay rate of 100 counts/min. Counts is the number of decays. How old is it?
The activity of a living carbon-containing organism is about 15 decays per minute per gram of carbon and the half-life of ^{14}C is 5730 years

(b) What decay rate would you expect from 15 g of 10,000 year-old wood?

Practice Problems: Carbon Dating

(a) A non-living wood sample contains 10 g of carbon and shows a ^{14}C decay rate of 100 counts/min. Counts is the number of decays. How old is it?
The activity of a living carbon-containing organism is about 15 decays per minute per gram of carbon and the half-life of ^{14}C is 5730 years

Answer: 3352 years

(b) What decay rate would you expect from 15 g of 10,000 year-old wood?

Answer: 67 min^{-1}

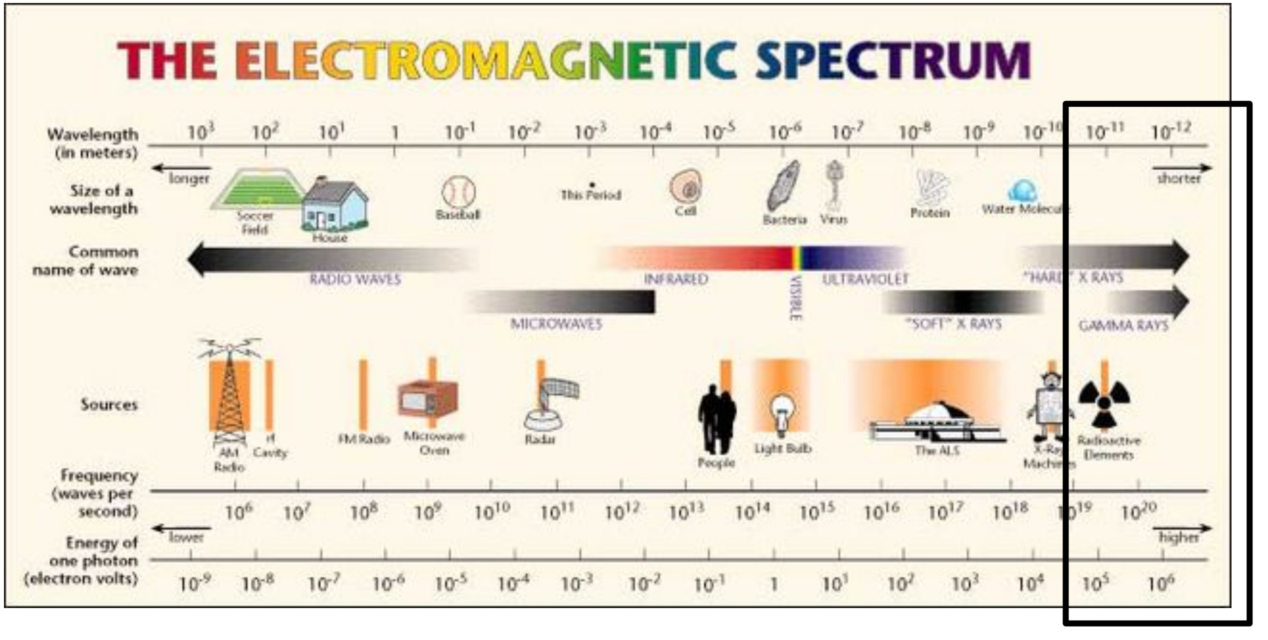
Gamma Decay

Nucleus in an excited state decays to a lower energy state by emitting a photon

A and Z remain the same

Spacing of nuclear energy levels is of the order 1 MeV, so wavelength of emitted photons are order 1 pm (10^{-12} m)

$$\boxed{\lambda = \frac{hc}{\Delta E}}$$



Gamma Decay

Gamma decay often occurs after α or β decay

Radioactive parent nucleus decays by β decay to excited state, then daughter nucleus decays to its ground state by γ emission

Lifetime of nuclei that undergo γ decay is very short (10^{-11} s is possible)

Example of γ decay:

$$^{125}_{53}\mathrm{I}^* \rightarrow {}^{125}_{53}\mathrm{I} + \gamma$$

* denotes isotope in an excited state

Gamma rays given off during nuclear fission (nuclear reactors and nuclear explosions).

Gamma rays also come from interaction of the Earth's atmosphere with cosmic rays.

Gamma rays are ionizing and so are biologically hazardous
(can pass through the body, so pose a shielding challenge in nuclear reactors)

Alpha Decay

Tends to occur for very heavy nuclei (Z > 83) (and where N > Z)

Coulomb repulsion between protons so great that becomes unstable

Daughter nucleus mass number A is reduced by 4 relative to parent nucleus (removal of an α particle or ^{4}He).

Example:

$$^{232}\mathrm{Th} \rightarrow {}^{228}\mathrm{Ra} + \alpha = {}^{228}\mathrm{Ra} + {}^{4}\mathrm{He}$$

Mass of the parent nucleus exceeds mass of decay products (daughter nucleus and α particle)

Use the mass difference between reactants and products to calculate binding energy (E_b; lecture 2)

Nuclear Reactions

Two nuclei or a nucleus and subatomic particle (proton, neutron) collide to produce one or more new nuclides

Q value: amount of energy released or absorbed during a nuclear reaction:

$$\boxed{Q = -\Delta m c^2}$$

where $\Delta m = \text{mass of products} - \text{mass of reactants}$

Location of Q appears in a nuclear reaction:

$$^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + {}^{1}\text{H} + Q$$

$\boldsymbol{Q > 0}$ **Exothermic** reaction: energy is released

$\boldsymbol{Q < 0}$ **Endothermic** reaction: energy is required

Practice Problem: Q Factor

Find the Q values for the reactions below.
Are these exothermic or endothermic?

$$^{2}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + {}^{1}\mathrm{H} + Q$$

$$^{10}\mathrm{B} + \mathrm{n} \rightarrow {}^{4}\mathrm{He} + {}^{7}\mathrm{Li} + Q$$

Masses from Tipler Table 40-1

^{1}H = 1.007825 u
n = 1.008665 u
^{2}H = 2.014102 u
^{3}He = 3.016030 u
^{4}He = 4.002603 u
^{7}Li = 7.016004 u
^{10}B =10.012939 u

Reminder:

$$(1u)c^{2} = 931.5\ \mathrm{MeV}$$

Practice Problem: Q Factor

Find the Q values for the reactions below.
Are these exothermic or endothermic?

$$^{2}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + {}^{1}\mathrm{H} + Q$$

Answer: Q = 4.05 MeV (exothermic)

$$^{10}\mathrm{B} + \mathrm{n} \rightarrow {}^{4}\mathrm{He} + {}^{7}\mathrm{Li} + Q$$

Answer: Q = -2.78 MeV (endothermic)

Fission

Occurs when a very heavy nucleus breaks apart into medium-mass (lighter) nuclei. Energy is released

Needs to be initiated by absorption of a neutron

Fission occurs when the excitation energy produced when a nucleus captures a neutron exceeds the critical energy (minimum energy needed for fission to occur)

This is why ^{235}U undergoes fission, but ^{238}U does not:

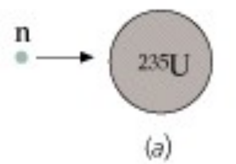
$^{236}\mathrm{U}$ critical energy: 5.3 MeV
$^{235}\mathrm{U} + \mathrm{n}$ excitation energy: 6.4 MeV

Excitation > critical, therefore **fission occurs**

$^{239}\mathrm{U}$ critical energy: 5.9 MeV
$^{238}\mathrm{U} + \mathrm{n}$ excitation energy: 5.2 MeV

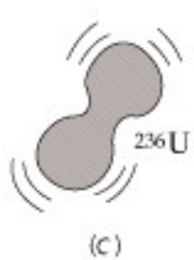
Excitation < critical, so **fission won't occur**

Fission of uranium-235 (^{235}U)

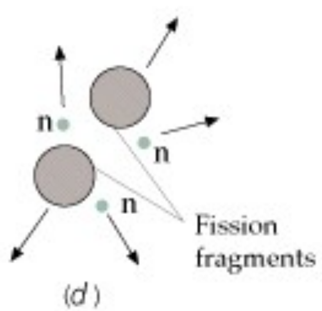


Absorption of neutron by ^{235}U

Forms ^{236}U in an excited state



Oscillation of ^{236}U becomes unstable



Nucleus splits into 2 nuclei and neutrons are emitted that initiate further fission (chain reaction ensues)

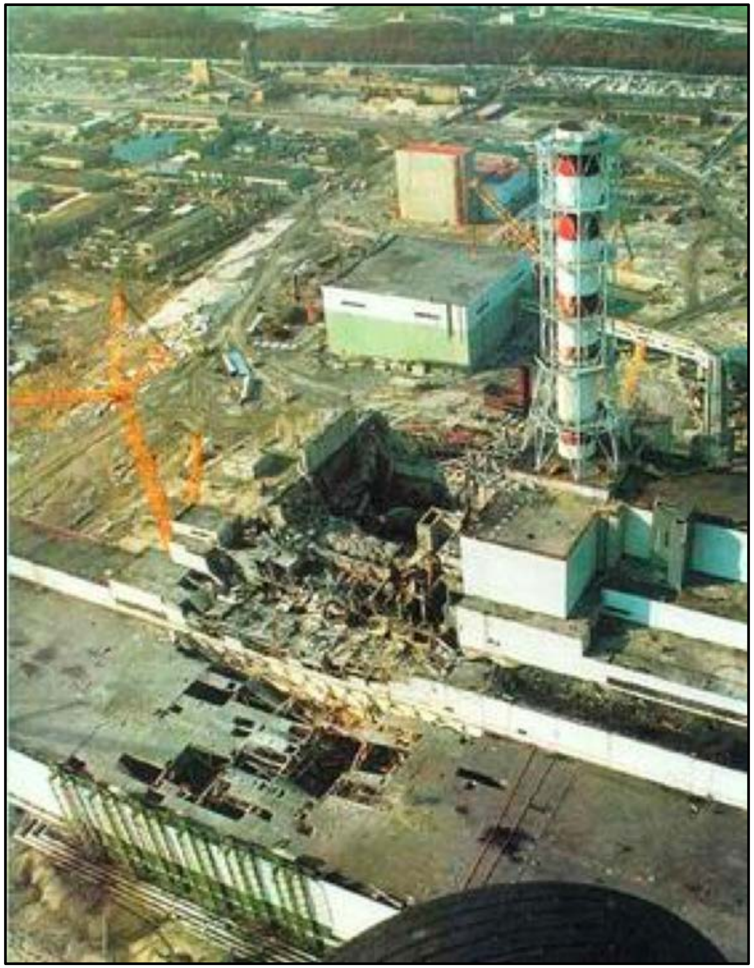
Coulomb repulsion force drives fragments apart

Typical fission reaction:

$$n + {}^{235}U \rightarrow {}^{141}Ba + {}^{92}Kr + 3n$$

Fission of uranium-235 (^{235}U) (contd)

Chernobyl disaster (1986):
Nuclear explosion due to runaway chain reaction



Fusion

Two light molecules fuse together to form a nucleus of greater mass

Example:

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + \mathrm{n} + 17.6\ \mathrm{MeV}$$

Greater amount of energy produced per unit mass than fission

Promising source of energy:

High efficiency

Abundance of light nuclei

Less dangerous than fission reactors

Fusion (contd)

Technology not yet a reality for commercial electricity generation

Issues and challenges:

- Coulomb repulsion is very large between ^2H and ^3H nuclei
- Need KE > 1 MeV to overcome Coulomb repulsion
- Can obtain these energies by speeding up particles in an accelerator, but scattering is more likely than fusion
- Particles must be heated to temperatures of order 10^8 K to form a plasma (ionization of atomic nuclei)
- Confinement hard to achieve in fusion reactor. Confined in Sun by enormous gravitational field.
- Ultimately, fusion requires more energy than is recovered

Fusion Research-Grade Reactors

Joint European Torus (JET) Fusion Reactor in the UK