PA1140: Waves and Quanta

Unit 4: Atoms and Nuclei

Tipler, Chapters 36 (36-1 to 36-2) & 40

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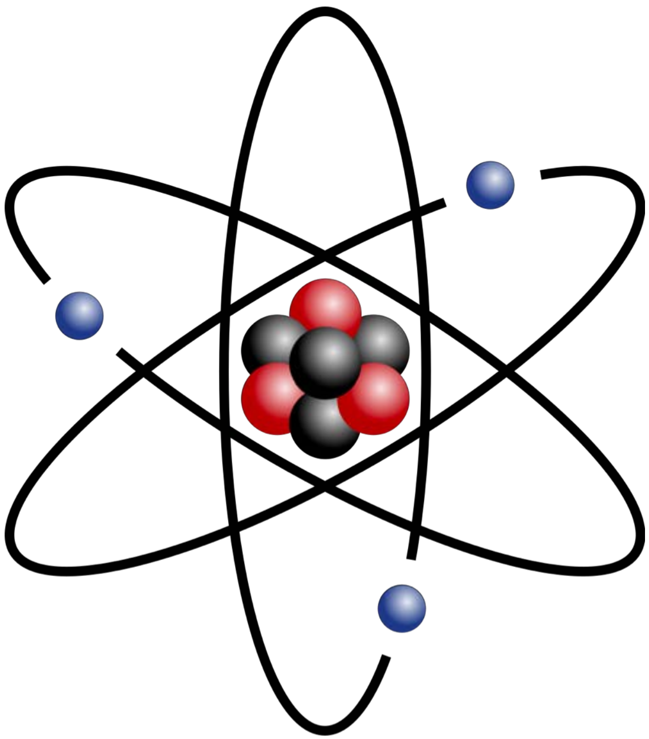


Dr Eloise Marais (Michael Atiyah Annex, 101)

Unit 4: Atoms and Nuclei

- The Atom
- Atomic Spectra
- The Bohr Model
- Properties of Nuclei
- Mass and Binding Energy
- Radioactivity
- Nuclear Reactions
- Fission
- Fusion

Lecture 1: The Atom

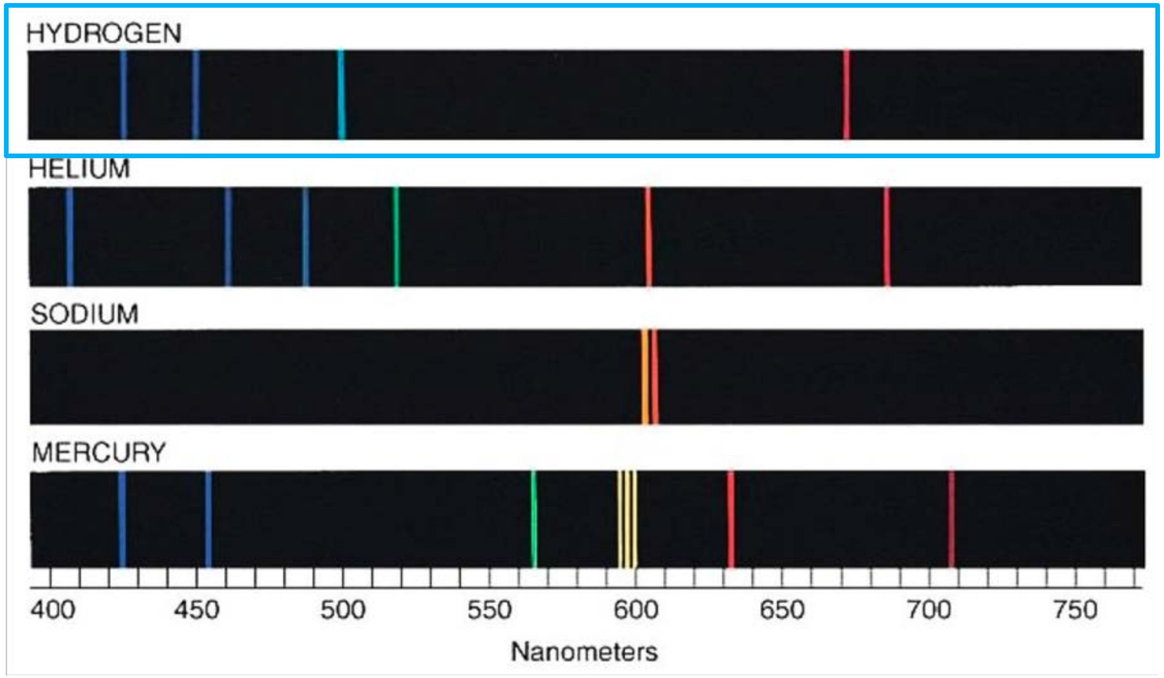
Relevant Terminology

- **Atomic number** (Z): number of protons
  matches location on periodic table
  Z = 1 for hydrogen
  Z = 2 for helium
  Z = 3 for lithium etc.
- Number of protons = number of electrons (for uncharged atoms)
- **Nucleus**: comprised of protons and neutrons
- **Nucleus charge**: $+Ze$ (where e = $1.60217662 \times 10^{-19}$ C; elementary charge)
- **Ions**: atoms that gain or lose an electron and so have a positive or negative charge

Atomic Spectra

Each atom emits a unique light spectrum (fingerprint of an atom)

Observed emission spectra of atoms:



Why is this? What properties of the atom cause this? Can we understand this by building a model?

Atomic Spectra

Representation of wavelengths of lines in the visible (400-800 nm) for hydrogen: (found by Balmer in 1884):

$$\boxed{\lambda = (364.6\ \mathrm{nm})\left(\frac{m^2}{m^2-4}\right)}$$

where m = 3, 4, 5,….

Rydberg-Ritz formula is a more general formula for multiple elements across a wider wavelength range:

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)}$$

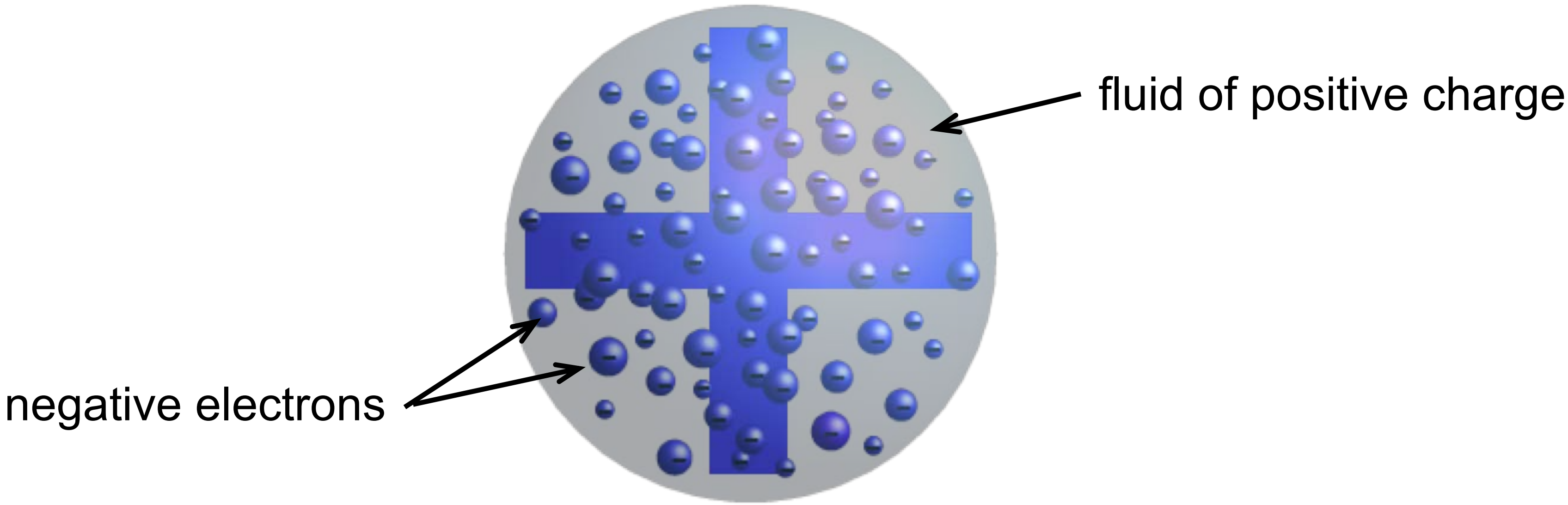
where: n_1 and n_2 are integers

$n_1 > n_2$

R: Rydberg constant for each element

Models of the Atom: Thompson's Plum Pudding Model

Electrons embedded in a fluid. Fluid has a distributed positive charge:

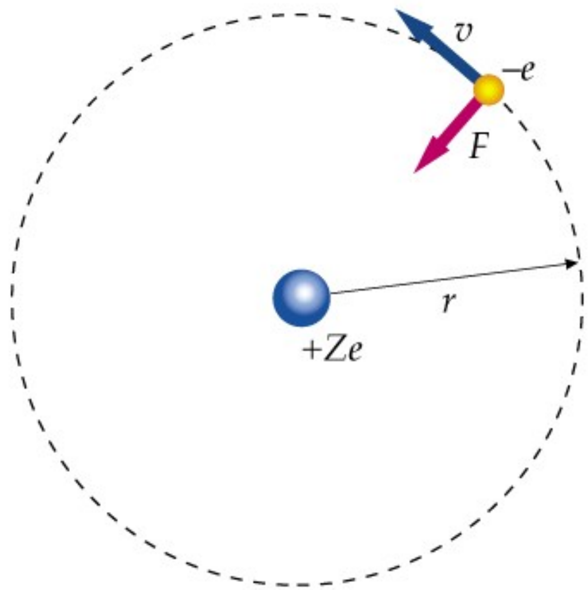


Disobeys classical physics: electric forces alone cannot produce a stable equilibrium

Disproved in Rutherford's laboratory: showed that atom contains small, massive, positively charged nucleus

The Bohr Model

Electron moves in a circular/elliptical orbit around a positive nucleus:



v: speed

$-e$: charge of electron

F: attractive electron force

r: radius

$+Ze$: charge of nucleus

Centripetal force acting on the orbiting electron is the electrostatic force of attraction (Coulomb force):

$$\boxed{F = \frac{k|q_1 q_2|}{r^2}}$$

where k = 8.99 x 10^9 N m^2 C^{-2} (Coulomb's constant)

$$\boxed{F = \frac{kZe^2}{r^2} = \frac{mv^2}{r}}$$

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Energy for a Circular Orbit

Potential energy (U) of electron of charge $-e$ at distance r from nucleus of positive charge Ze:

$$\boxed{U = \frac{k|q_1 q_2|}{r} = \frac{kZe(-e)}{r} = -\frac{kZe^2}{r}}$$

Kinetic energy (K) obtained using Newton's 2nd Law and the centripetal force:

$$\boxed{K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{kZe^2}{r}}$$

Note: $U = -2K$ (general result for orbiting particles with $F \propto 1/r^2$):

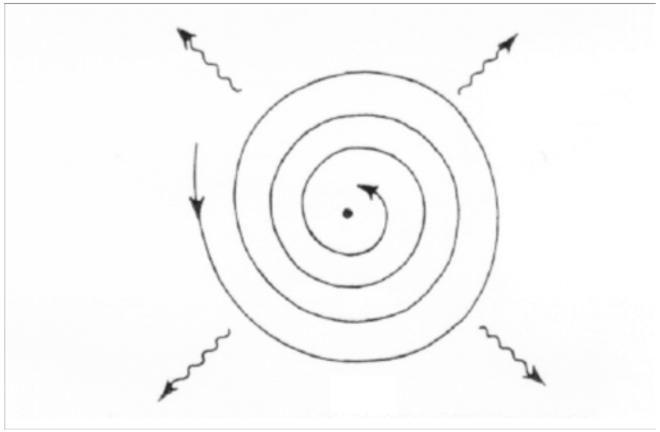
$$\boxed{E = K + U = -\frac{1}{2}\frac{kZe^2}{r}}$$

Why The Bohr Model Fails

Mechanically stable, because Coulomb force provides centripetal force to keep electron in orbit.

But, according to electromagnetic theory, this atom is **electronically unstable**.

Electron must accelerate when moving in a circle and so would **emit electromagnetic radiation** of frequency equal to its motion, **lose energy**, and spiral into the nucleus in a very short time.



The atom could not exist using this model.

Bohr's Postulates

1st Postulate. Electron in hydrogen atom can move only in certain non-radiating, circular orbits called **stationary states**

2nd Postulate. An electron can make a transition from a stationary state of higher energy, E_2, to a state of lower energy, E_1, leading to emission of a photon of frequency f.

$$\boxed{f = \frac{E_i - E_f}{h} = \frac{\Delta E}{h}}$$

where h = 6.626 x 10^{-34} m^2 kg s^{-1}
(Planck's constant; also in units of J s)

We can use total energy expression to obtain an expression for f in terms of radii of final and initial orbits, r_f and r_i:

$$\boxed{f = \frac{E_i - E_f}{h} = \frac{1}{2}\frac{kZe^2}{h}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$$

Bohr's Postulates (contd)

3rd Postulate. Only orbits are allowed if angular momentum of the electron is an integral multiple of $h/2\pi$ (quantized angular momentum).

$$\boxed{\hbar = \frac{h}{2\pi} = 1.055\times10^{-34} Js = 6.582\times10^{-16}\ \mathrm{eV\ s}}$$

where $\hbar$ is the reduced Planck's constant

The magnitude of a circular orbit is $\mathrm{L} = mvr$, so the 3rd postulate is:

$$\boxed{mv_n r_n = n\hbar}$$

where n = 1, 2, 3,….
(the quantum number of the state)

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Radii of Bohr Orbit

Derivation on board leading to:

$$\boxed{r_n = n^2 \frac{a_0}{Z}}$$

a_0 is a physical constant: the orbital radius of the electron of a hydrogen atom that has $n = 1$

$$\boxed{a_0 = \frac{\hbar^2}{mke^2} = \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.0529\ \mathrm{nm}}$$

Frequency of a Line

Line frequency in terms of energy levels (or orbits) n

Derivation on board leading to:

$$\boxed{f = \frac{mk^2Z^2e^4}{4\pi\hbar^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

Use this and the relationship between frequency and wavelength ($f = c/\lambda$) and assume $Z = 1$ to obtain the Rydberg constant (R)

$$\boxed{R = \frac{mk^2Z^2e^4}{4\pi\hbar^3 c} = \frac{me^4}{8\varepsilon_0^2 h^3 c} = 1.096776\times10^7\ \mathrm{m}^{-1}}$$

Atomic Energy Levels

Mechanical energy of a hydrogen atom ($Z = 1$) is related to the radius of a circular orbit, so we can obtain an expression for the energy of orbit n for hydrogen

Derivation on board leading to:

$$\boxed{E_n = -\frac{1}{2}\frac{mk^2Z^2e^4}{n^2\hbar^2} = -Z^2\frac{E_0}{n^2}}$$

where: n = 1, 2, 3,…., and

$$E_0 = \frac{mk^2e^4}{2\hbar^2} = \frac{1}{2}\frac{ke^2}{a_0} = 13.6\ \text{eV}$$

Atomic Energy Levels (contd)

Energies E_n are quantized allowed energies for hydrogen ($Z = 1$)

Transitions between these allowed energies result in emission or absorption of a photon with frequency:

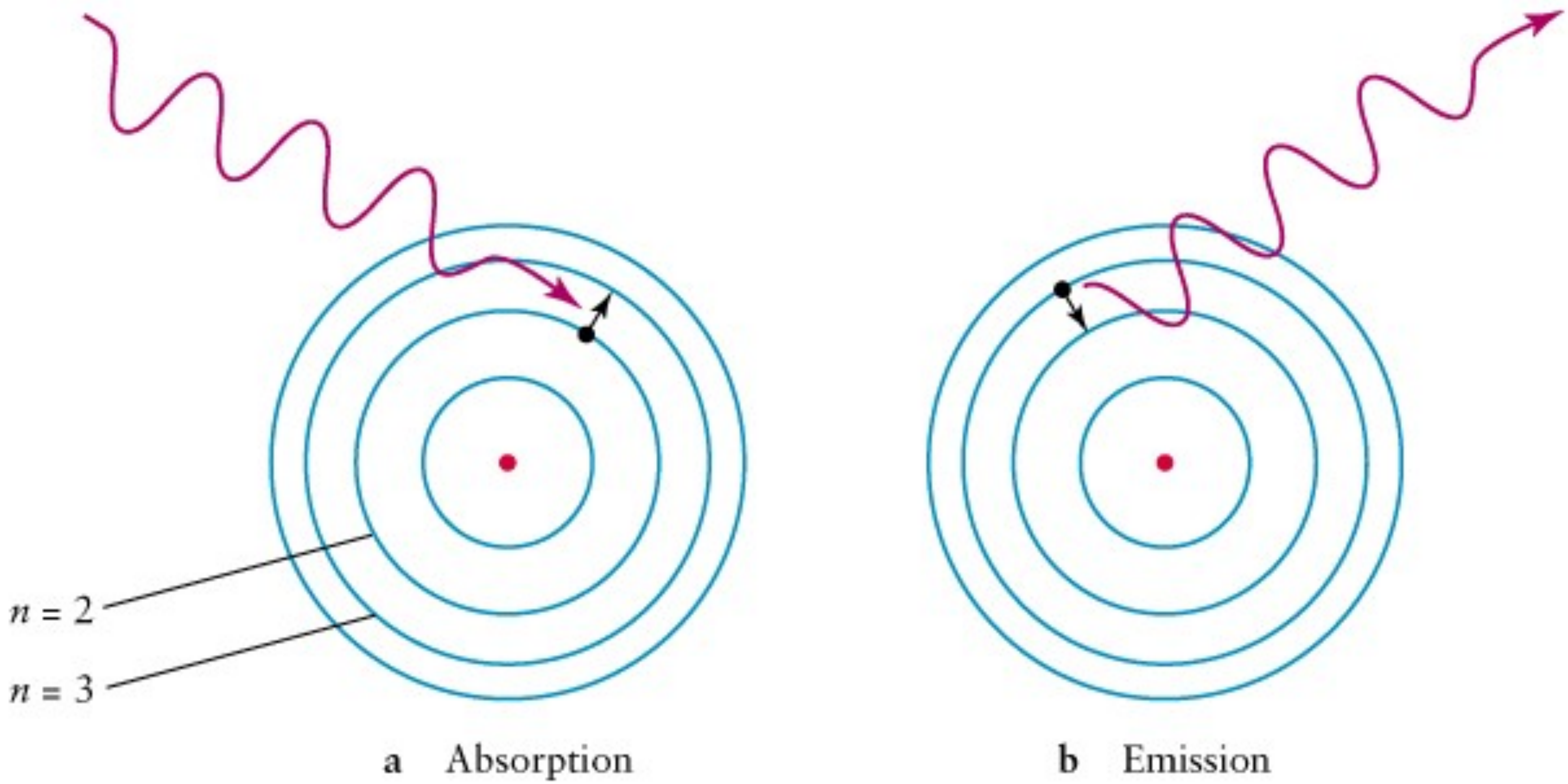
$$f = \frac{E_i - E_f}{h}$$

And of wavelength, λ, of:

$$\lambda = \frac{c}{f} = \frac{hc}{E_i - E_f}$$ where $hc = 1240 \text{ eV nm}$

f and λ of emitted and absorbed radiation are quantized (consistent with line spectrum).

Quantized Energy Transitions

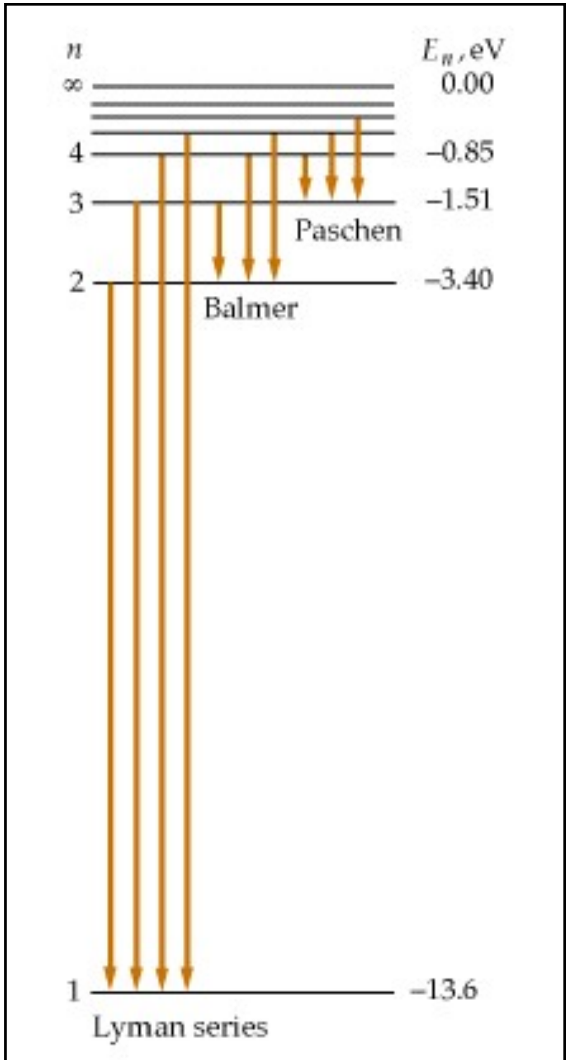


Electron transitions from lower to higher state (requires energy)

Electron transitions from higher to lower state (releases energy)

Energy Transitions

Hydrogen emission lines



Arrows in opposite direction for absorption

$E_1 = -13.6\ \text{eV}$ (ground state or lowest energy state of hydrogen)

As $n \to \infty$, $E_n \to 0$ (as $E_n \propto 1/n^2$)

Figure shows transitions from higher to lower states:

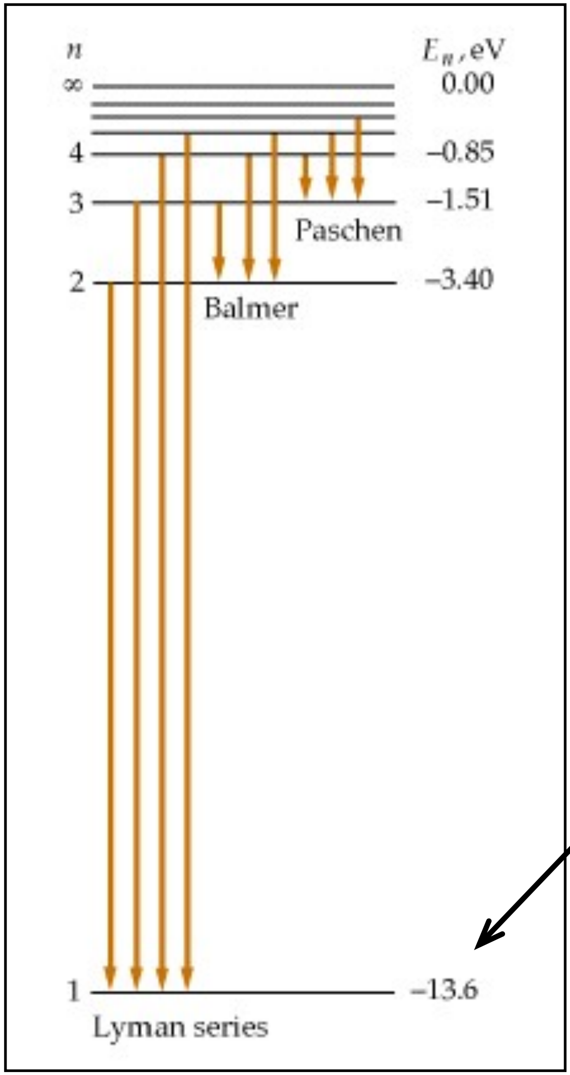
Balmer lines: $n_f = 2;\ n_i = 3, 4, 5, \ldots$

Paschen lines: $n_f = 3;\ n_i = 4, 5, 6, \ldots$

Lyman lines: $n_f = 1;\ n_i = 2, 3, 4, \ldots$

Electron Removal

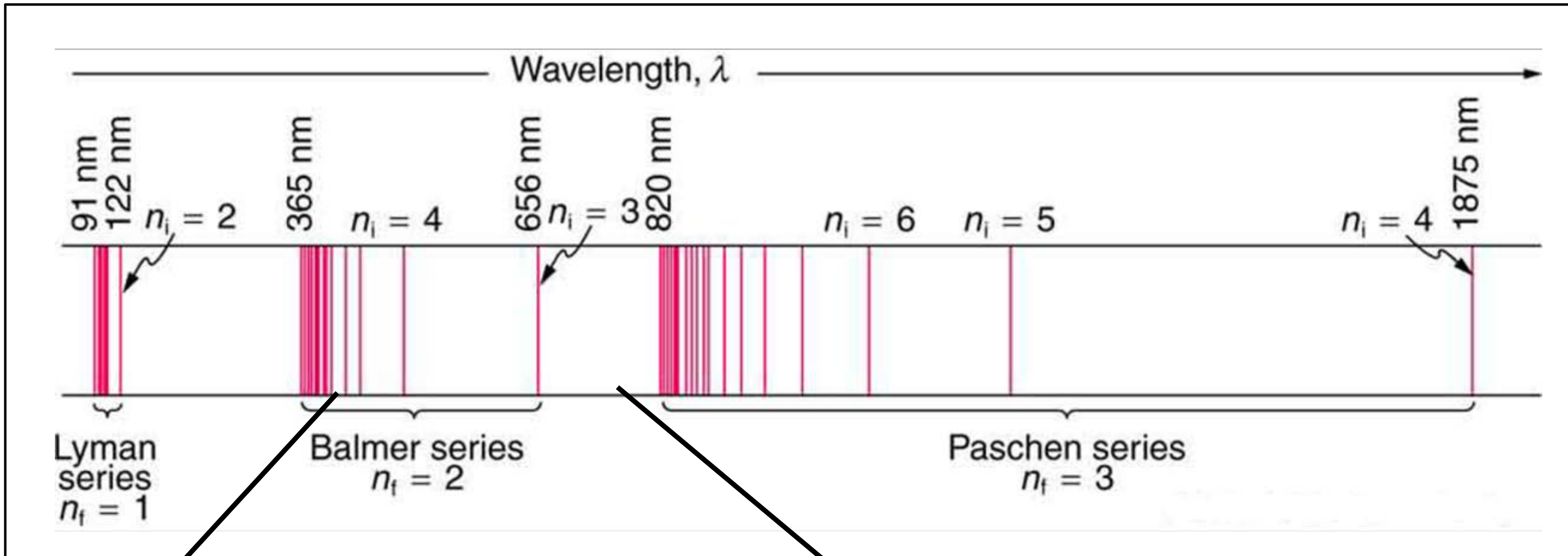
Hydrogen emission lines



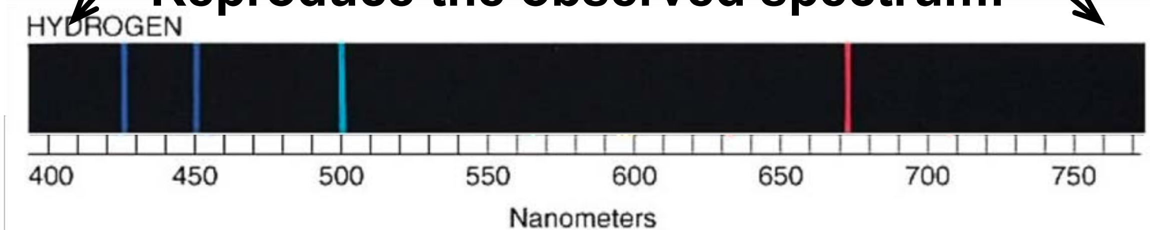
- **Ionization**: removal of one electron from an atom
- **Ionization energy**: minimum energy required for ionization (ground state ionization energy for hydrogen is 13.6 eV)

Energy Transitions

Build the emission lines observed for hydrogen:



Reproduce the observed spectrum:



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