PA114 – Waves and Quanta

Unit 4 – Workshop - Answers

1

Picture the Problem We can use Equation 36-15 with $Z = 2$ to explain how it is that every other line of the Pickering series is very close to a line in the Balmer series. We can use the relationship between the energy difference between two quantum states and the wavelength of the photon emitted during a transition from the higher state to the lower state to find the wavelength of the photon corresponding to a transition from the $n = 6$ to the $n = 4$ level of He^+.

(*a*) From Equation 36-15, the energy levels of an atom are given by:

$$E_n = -Z^2 \frac{E_0}{n^2}$$

where E_0 is the Rydberg constant (13.6 eV).

For He^+, $Z = 2$ and:

$$E_n = -4\frac{E_0}{n^2}$$

Because of this, an energy level with even principal quantum number n in He^+ will have the same energy as a level with quantum number $n/2$ in H. Therefore, a transition between levels with principal quantum numbers $2m$ and $2p$ in He^+ will have almost the same energy as a transition between level m and p in H. In particular, transitions from $2m$ to $2p = 4$ in He^+ will have the same energy as transitions from m to $n = 2$ in H (the Balmer series).

(*b*) Transitions between these energy levels result in the emission or absorption of a photon whose wavelength is given by:

$$\lambda = \frac{hc}{E_6 - E_4} \qquad (1)$$

Evaluate E_6 and E_4:

$$E_6 = -4\left(\frac{13.6\text{ eV}}{6^2}\right) = -1.51\text{ eV}$$

and

$$E_4 = -4\left(\frac{13.6\text{ eV}}{4^2}\right) = -3.40\text{ eV}$$

Substitute for E_6 and E_4 in equation (1) and evaluate λ:

$$\lambda = \frac{1240\text{ eV}\cdot\text{nm}}{-1.51\text{ eV} - (-3.40\text{ eV})} = \boxed{656\text{ nm}}$$

which is the same as the $n = 3$ to $n = 2$ transition in H.

2 **Picture the Problem** The separation of the nuclei when they are just touching is the sum of their radii, which is given by $R = R_0 A^{1/3}$.

The electrostatic potential energy of the system is given by:

$$U = k\frac{q_1 q_2}{R} = k\frac{(Z_{\text{Ba}}e)(Z_{\text{Kr}}e)}{R_{\text{Ba}} + R_{\text{Kr}}}$$

where R is the distance from the center of the ^{144}Ba nucleus to the center of the ^{89}Kr nucleus.

The radii of the nuclei are:

$$R_{\text{Ba}} = R_{\text{o}} A_{\text{Ba}}^{1/3} \text{ and } R_{\text{Kr}} = R_{\text{o}} A_{\text{Kr}}^{1/3}$$

Substitute for R_{Ba} and R_{Kr} and simplify to obtain:

$$U = ke^2 \frac{(Z_{\text{Ba}})(Z_{\text{Kr}})}{R_{\text{o}} A_{\text{Ba}}^{1/3} + R_{\text{o}} A_{\text{Kr}}^{1/3}} = \frac{ke^2}{R_{\text{o}}} \frac{(Z_{\text{Ba}})(Z_{\text{Kr}})}{\left(A_{\text{Ba}}^{1/3} + A_{\text{Kr}}^{1/3}\right)}$$

Substitute numerical values and evaluate U:

$$U = \frac{\left(8.99\times10^9\,\text{Nm}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(1.2\times10^{-15}\,\text{m}\right)} \frac{(56)(36)}{\left(144^{1/3} + 89^{1/3}\right)}$$

$$= 3.98\times10^{-11}\,\text{J} \times \frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} = \boxed{249\,\text{MeV}}$$

3

Picture the Problem The required mass is given by $M = (5 \text{ counts/min})/R$, where R is the current counting rate per gram of carbon. We can use the assumed age of the casket to find the number of half-lives that have elapsed and $R = \left(\frac{1}{2}\right)^n R_0$ to find the current counting rate per gram of ^{14}C.

The mass of carbon required is:

$$M = \frac{5\,\text{counts/min}}{R} \qquad (1)$$

Because there were about 15.0 decays per minute per gram of the living wood, the counting rate per gram is:

$$R = \left(\tfrac{1}{2}\right)^n R_0 = \left(\tfrac{1}{2}\right)^n (15\,\text{counts/min}\cdot\text{g})$$

We can find n from the assumed age of the casket and the half-life of ^{14}C:

$$n = \frac{18{,}000\,\text{y}}{5730\,\text{y}} = 3.141$$

Substitute for n and evaluate R:

$$R = \left(\tfrac{1}{2}\right)^{3.141}(15\,\text{counts/min}\cdot\text{g}) = 1.70\,\text{counts/min}\cdot\text{g}$$

Substitute for R in equation (1) and evaluate M:

$$M = \frac{5\,\text{counts/min}}{1.70\,\text{counts/min}\cdot\text{g}} = \boxed{2.94\,\text{g}}$$

4 **Picture the Problem** We can use the energy released in the reactions of Problem 50, together with the 17.6 MeV released in the reaction described in this problem, to find the energy released using 5 ^{2}H nuclei. Finding the number of D atoms in 4 L of H_2O, we can then find the energy produced if all of the ^{2}H nuclei undergo fusion.

Find the energy released using 5 ^{2}H nuclei:

$$Q = 3.27\,\text{MeV} + 4.03\,\text{MeV} + 17.6\,\text{MeV} = 24.9\,\text{MeV}$$

The number of H atoms in 4 L of H_2O is:

$$N_\text{H} = 2\left(\frac{m_{\text{H}_2\text{O}}}{18\,\text{g/mol}}\right)N_\text{A}$$

Substitute numerical values and evaluate N_H:

$$N_\text{H} = 2\left(\frac{4\,\text{kg}}{18\,\text{g/mol}}\right)\left(6.02\times10^{23}\,\text{atoms/mol}\right) = 2.676\times10^{26}$$

The number of D atoms in 4 L of H_2O is:

$$N_\text{D} = \left(1.5\times10^{-4}\right)N_\text{H} = \left(1.5\times10^{-4}\right)\left(2.676\times10^{26}\right) = 4.01\times10^{22}$$

The energy produced is given by:

$$E = \frac{N_\text{D}}{5}Q$$

Substitute numerical values and evaluate E:

$$E = \frac{4.01\times10^{22}}{5}(24.9\,\text{MeV}) = 1.997\times10^{23}\,\text{MeV}\times\frac{1.60\times10^{-19}\,\text{J}}{\text{eV}} = \boxed{3.20\times10^{10}\,\text{J}}$$

Concept Questions:

The half-life of 14C is much less than the age of the Universe, yet 14C is found in Nature. Why?

Sustained production from cosmic rays that provide neutrons to convert nitrogen-14 to carbon-14.

What effect would a long-term variation in cosmic-ray activity have on the accuracy of 14C dating?

Only if no correction is applied. Measurements of abundance of carbon-14 and berrylium-10 can be used to build a record of cosmic activity.

The idea of radiocarbon dating is straightforward, but there are complications. Which of the following effects might complicate 14C dating:

A. Contamination with, for example, modern carbon in the laboratory.
B. Fossil fuels are ancient and so when burned emit CO_2 that has very little carbon-14, thus diluting the relative abundance of carbon-14 in the atmosphere.