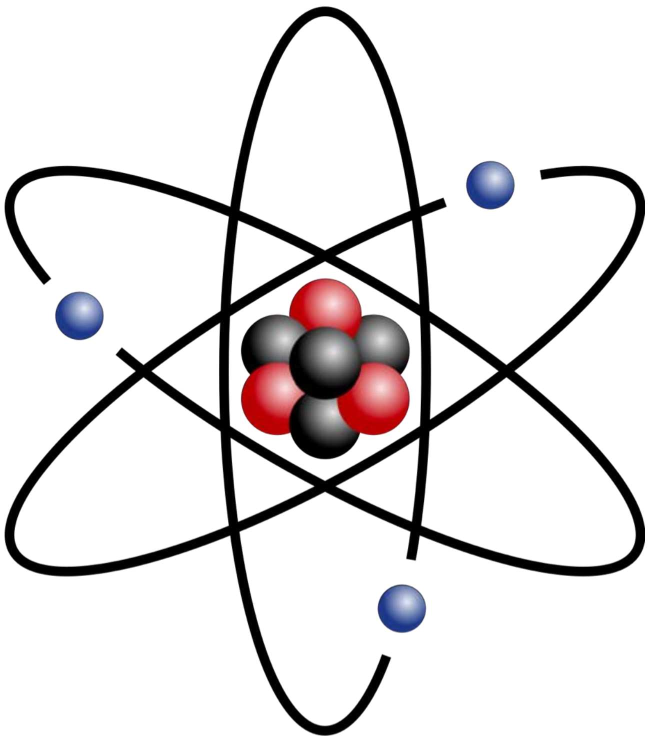
PA1140: Waves and Quanta

Unit 4: Atoms and Nuclei

Tipler, Chapters 36 (36-1 to 36-2) & 40

University of Leicester



Dr Eloise Marais (Michael Atiyah Annex, 101)

Course Rep Nominations for 2019/2020 are Now Open

Course reps are important! They have direct input to the teaching strategy in the department and they influence the way the course is run

If students would like to contribute please nominate yourself before **15 March**

NOMINATIONS

Open until Friday 15th March

ELECTIONS

Tuesday 19th- Thursday 21st March

FIND OUT IF YOU'VE WON

Friday 22nd March

www.leicesterunion.com/represent/course-reps

Lecture 1 Follow Up

Why is angular momentum quantized?

Bohr postulated that angular momentum of an electron orbiting a nucleus is quantized for his model to work.

This was later explained by de Broglie

According to de Broglie, an electron moving in a circular orbit behaves like a particle wave

Reminder of the expression for the de Broglie wavelength:

$$\lambda = \frac{h}{p}$$

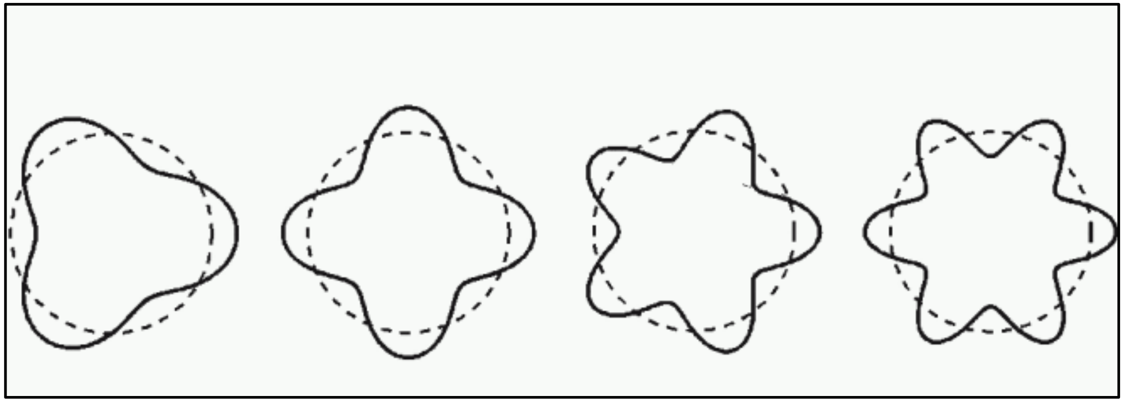
where p is the momentum of the electron

Lecture 1 Follow Up (contd)

For any electron moving in circular orbit n of radius r_n, the total distance is equal to the circumference of an orbit, $2\pi r_n$

This can be illustrated as follows:

Standing wave on a circle for 3, 4, 5, 6 cycles.



Conceptually, only a discrete set of rates are possible, since the original wave function has to return to its starting point after each rotation

Lecture 1 Follow Up (contd)

Mathematically, this is expressed following these steps:

Relate circumference to wavelength:

$$2\pi r_n = n\lambda \qquad \textbf{Equation (1)}$$

Define de Broglie wavelength ($\lambda = \frac{h}{p}$) in terms of angular momentum:

$$\lambda = \frac{h}{mv_n} \qquad \textbf{Equation (2)}$$

where mv_n is the momentum of an electron revolving in orbit n

Substitute equation (1) into equation (2) to get an expression for quantized angular momentum

$$\boxed{mv_n r_n = n\frac{h}{2\pi}}$$

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Reminder of Bohr's 3rd Postulate:

Only orbits are allowed if angular momentum of the electron is an integral multiple of $h/2\pi$ (quantized angular momentum).

$$\boxed{\hbar = \frac{h}{2\pi} = 1.055\times10^{-34} Js = 6.582\times10^{-16}\ \mathrm{eV\ s}}$$

where $\hbar$ is the reduced Planck's constant

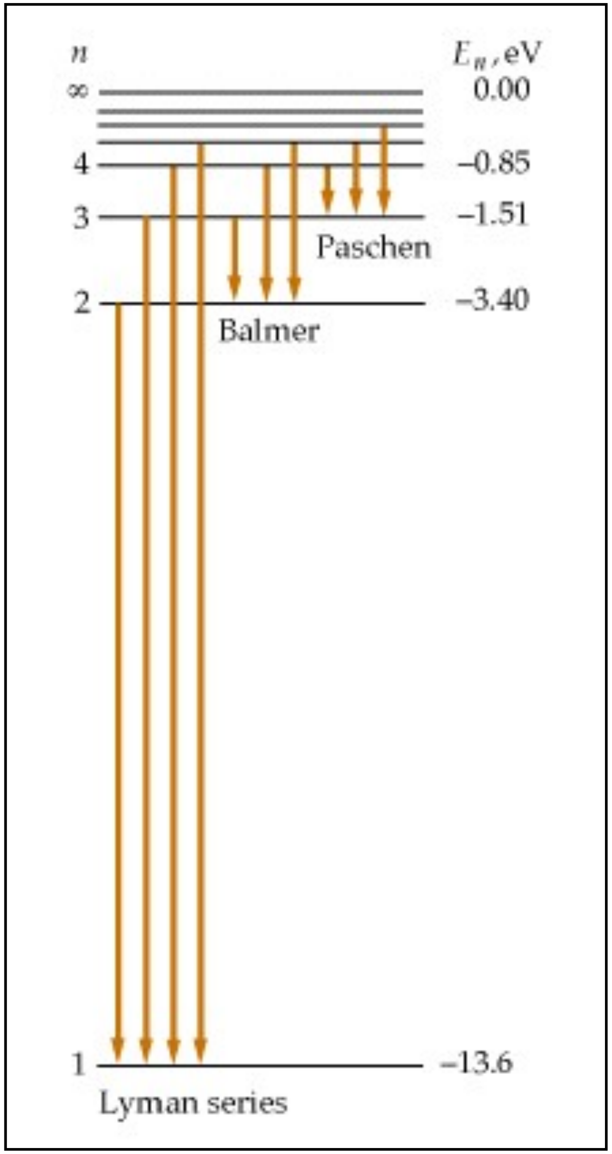
The magnitude of a circular orbit is $\mathrm{L} = mvr$, so the 3rd postulate is:

$$\boxed{mv_n r_n = n\hbar}$$

where n = 1, 2, 3,….
(the quantum number of the state)

Practice Problem: Atomic Spectra

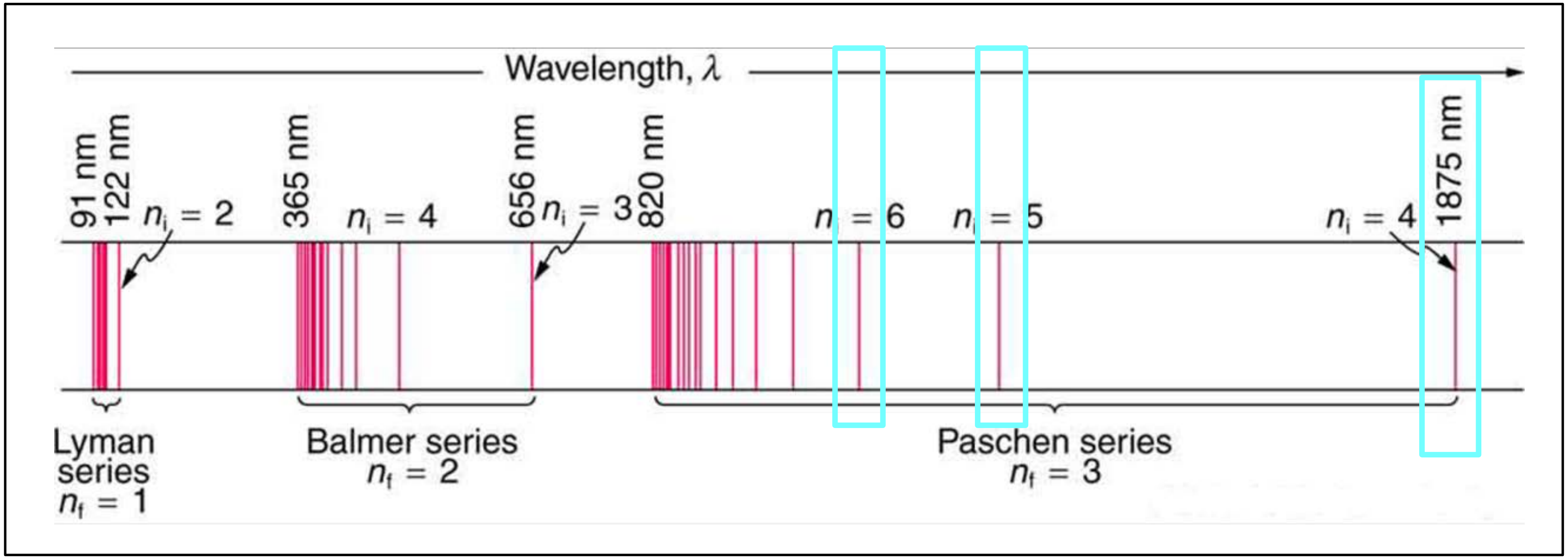
Hydrogen emission lines



a) Find the photon energy and wavelength for the series limit (shortest wavelength) in the Paschen series (n_2 = 3).

b) Calculate the wavelengths for the three longest wavelengths in this series and indicate their positions on a horizontal linear scale.

Let’s check our answer:



Practice Problem: Ionization Energy

Calculate the ionization energy for:

a) the hydrogen atom

b) He^{+}

c) Li^{2+}

Practice Problem: ionization energy

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The Periodic Table

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57-71	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89-103	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
		57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
		89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

Lecture 2: Properties of Nuclei & Radioactivity

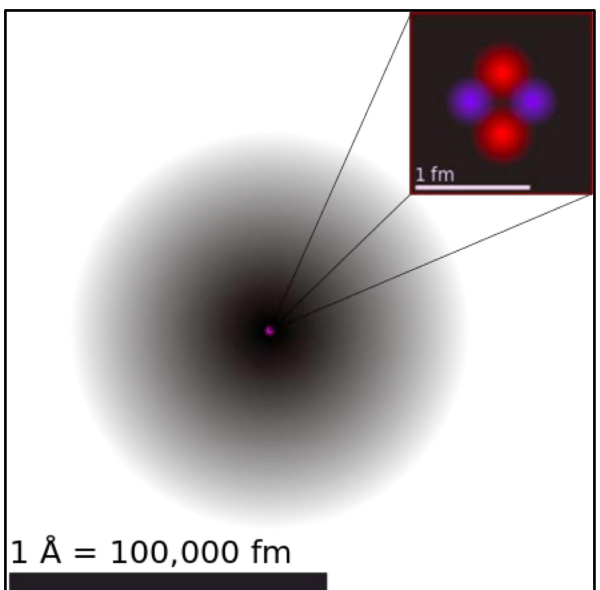
Relevant Terminology

- **Nucleus**: incudes protons and neutrons (masses are about equal)
- Number of **protons**: **Z** or atomic number
- Number of **neutrons**: **N**
- N = Z for lighter nuclei
  N > Z for heavier nuclei (difference increases as mass increases)
- Number of **nucleons**: **A** or nucleon or mass number (A = N + Z)
- **Nuclide**: name given to a nuclear species
- **Atomic symbol**: unique name for an atom (H for hydrogen)
- **Isotope**: 2 or more nuclides with the same Z, but different N (and A)

Strong Nuclear Force

- One of four fundamental forces
- Nucleons exert strong attractive forces on other nucleons
- Stronger than electrostatic force of repulsion between protons
- Much stronger than gravitational forces (negligible)

Nucleus of He



- About the same between 2 neutrons, 2 protons, a neutron and proton
- Decrease with distance to zero at a distance of a few fentometer (1 fm = 10^{-15} m)

2 protons stay together due to nuclear force

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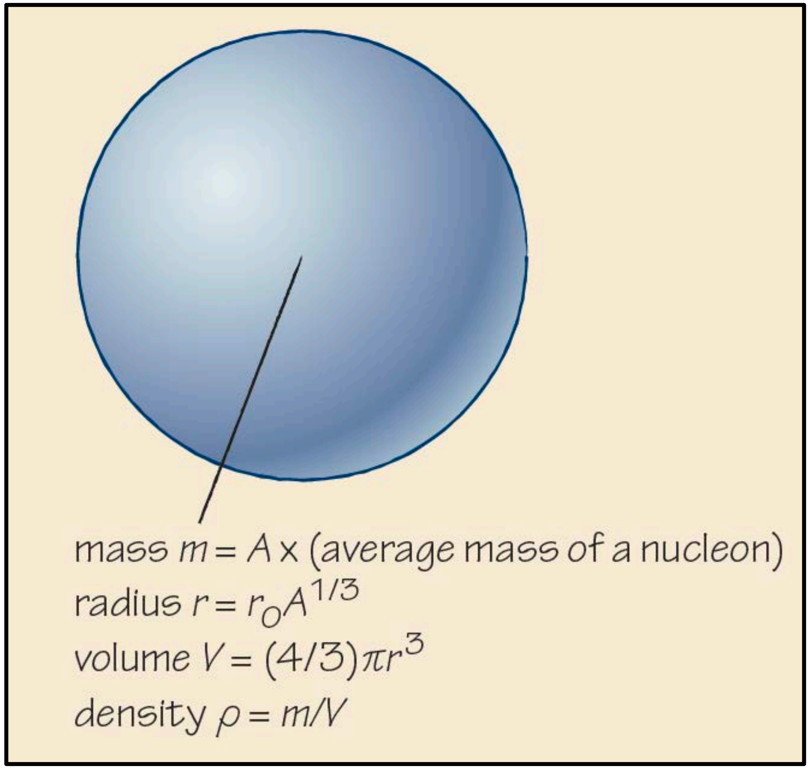
Nuclear Size, Shape, and Density

Assuming spherical shape, nuclear **radii** can be calculated as follows:

$$R = r_0 A^{1/3}$$

where $A = N + Z$ is the atomic mass

and $r_0 \sim 1.2 \text{ fm}$ (empirical constant)



$$R \propto A^{1/3}$$

So: $V \propto A$

Nuclear **mass**:

$$m = A \times m_{nucleus}$$

Because $m_{nucleus} \propto A$, **densities of all nuclei are roughly the same**

Practice Problem: Density of nuclear matter

Let’s prove that densities of nuclear matter is roughly the same.

N and Z Numbers

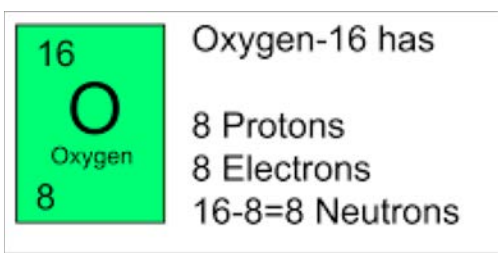
Light nuclei:

- stable for $N \approx Z$

Heavier nuclei:

- instability caused by electrostatic repulsion between protons
- instability minimized by having more neutrons than protons

Example:

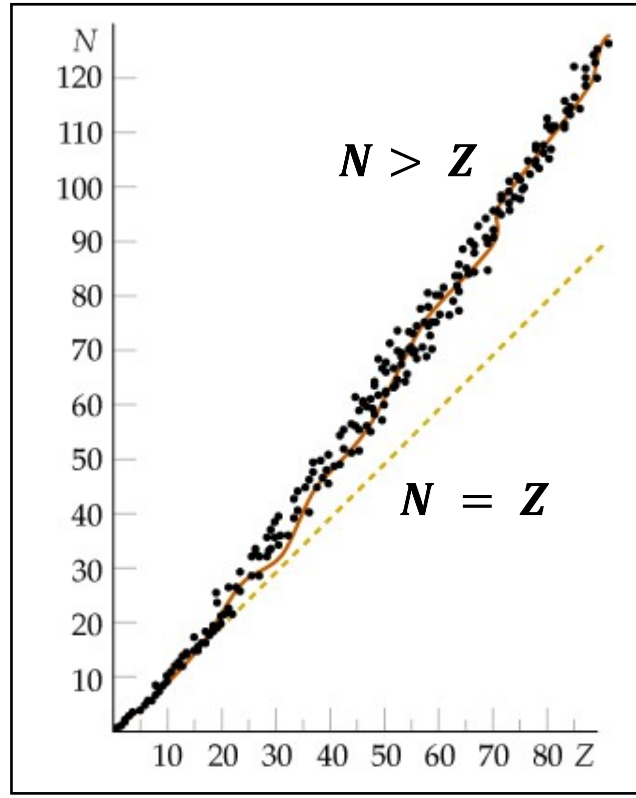


$$N = Z = 8$$

$$^{238}_{92}\mathrm{U}$$

$$N = 146$$
$$Z = 92$$

Neutrons versus protons



Mass and Binding Energy

Mass of nucleus = sum of parts – E_b/c^2

$\boldsymbol{E_b}$: **binding energy** (energy needed to break all bonds)

When 2 or more nucleons fuse together to form a nucleus: energy is released and mass decreases (opposite when break apart)

atomic mass units (a.m.u): mass convention for atoms and nuclei

1 a.m.u = one-twelfth the mass of carbon-12

Rest energy of 1 a.m.u (using $E = mc^2$), and $m_{a.m.u}$ = 1.66054 x 10^{-27} kg:

$$(1u)c^2 = 931.5 \text{ MeV}$$

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Worked Example

^{4}He includes 2 protons and 2 neutrons. The mass of ^{4}He measured with a mass spectrometer is **4.002603 u**. The mass of a proton (^{1}H) is 1.007825 u and of a neutron is 1.008665 u.

The mass of 2 protons and 2 neutrons = **4.03298 u**

(*which is greater than the measured mass of* ^{4}He)

We can use this to estimate the binding energy, E_b:

$$E_b = (4.03298u\ c^2 - 4.002603u\ c^2)\frac{931.5\ \text{MeV}}{1\ u\ c^2}$$

$$E_b = 28.3\ \text{MeV}$$

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Binding Energy (contd)

General expression for E_b:

$$\boxed{E_b = (ZM_H + NM_N - M_A)c^2}$$

where: M_H is the mass of a proton
M_N is the mass of a neutron
M_A is the final atomic mass

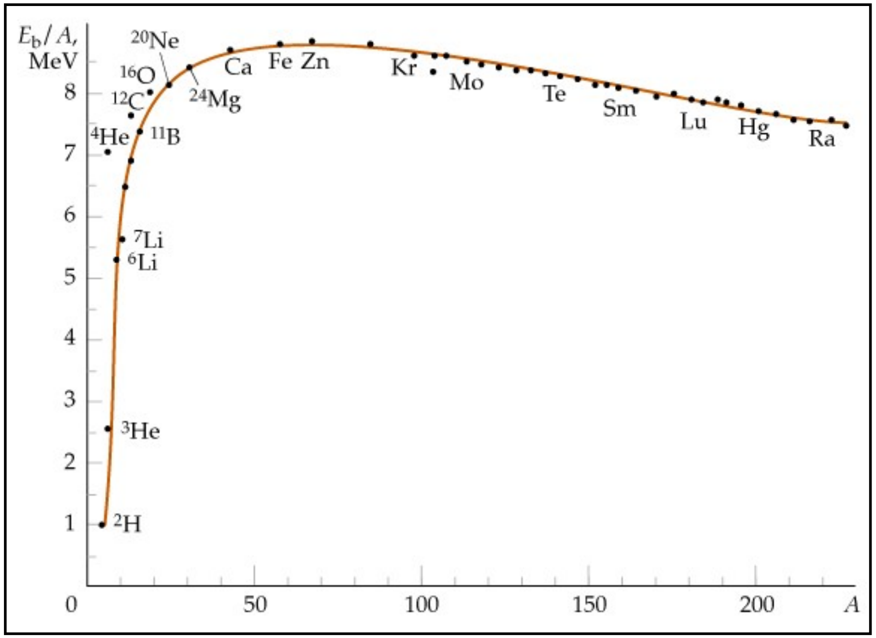
Electrons not included. 1000 times lighter than proton:

Mass of proton: of the order 10^{-27} kg

Mass of electron: of the order 10^{-30} kg

Binding Energy (contd)

Binding energy vs mass number



A < 20: Increase in nearest neighbours and in number of bonds per nucleon

50 < A < 200: Saturation of nuclear forces

A > 200: Coulomb repulsion so great that nucleus becomes unstable.

A > 300: Nucleus unstable and undergoes spontaneous fission.

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Radioactivity

Relevant Terminology

- **Radioactive nuclei**: decay into other nuclei by emitting particles (photons, electrons, neutrons, particles)
- $\boldsymbol{\alpha}$ **decay**: release of ^{4}He atom (i.e. α particle)
- $\boldsymbol{\beta}$ **decay**: release of electrons (β^{-}) or positrons (β^{+})
- **positron**: positively charged particle with the same mass as an electron
- $\boldsymbol{\gamma}$ **decay**: release of photons

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