Math 221 Tangent Line Worksheet

Objectives

- Show how to include an image using Prefigure code.
- Show displayed and alligned math as well as some minor math typesetting issues that come from xml syntax (see the code for details).

In this activity students are asked to decide when to switch the tangent line approximation they use to approximate square roots.

Set-up. Recall from class that we can approximate square roots of numbers that aren't perfect squares using the tangent line approximation near some number a

$$L_a(x) = f(a) + f'(a)(x - a)$$

where $f(x) = \sqrt{x}$.

For instance, to approximate $\sqrt{5}$ we decided to use the tangent line approximation at the nearest perfect square, which is given by

$$\begin{aligned} L_4(x) &= \sqrt{4} + \frac{1}{2\sqrt{4}}(x-4) \\ &= 2 + \frac{1}{4}(x-4) \\ &= 1 + \frac{1}{4}x. \end{aligned}$$

Thus, we can estimate $\sqrt{5} = f(5) \approx L_4(5) = 2.25$.

Now, what if there isn't a "nearest" perfect square? For instance, which tangent line should we use to approximate $\sqrt{6.5}$ (6.5 is exactly halfway between 4 and 9)? The following exercise will help us figure out the answer.

1.

(a) Use Figure **??** to give intervals of x-values on which $L_4(x)$ and $L_9(x)$ give more accurate estimates of $\sqrt{x}$. That is, find some x-value, s, such that

$$|\sqrt{x} - L_4(x)| < |\sqrt{x} - L_9(x)|$$

whenever $x < s$ and

$$|\sqrt{x} - L_9(x)| < |\sqrt{x} - L_4(x)|$$

whenever $x > s$.

(b) Use algebra to justify your answer above.

2. Another exercise.

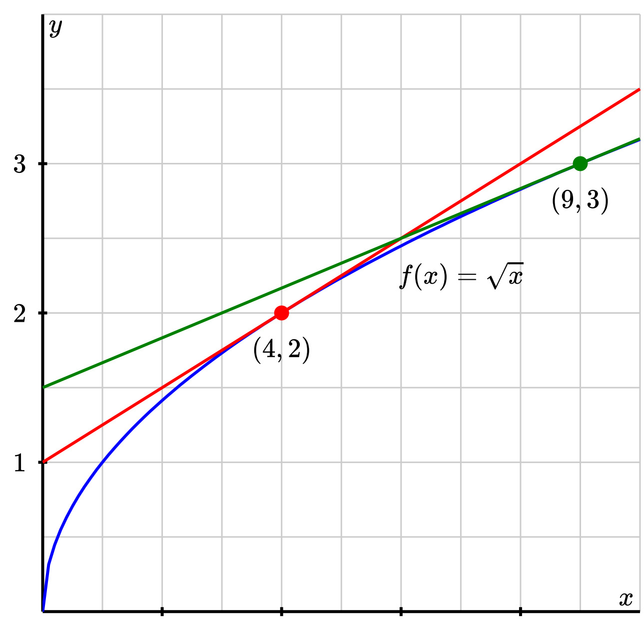

Figure 1 The graph of $f(x) = \sqrt{x}$ and its tangent line approximations at $x = 4$ and $x = 9$.

Interactive Remarks

Remark 2 Note the issues in the code where Latex syntax conflicted with xml. There are a few instances when these happen.

Remark 3 Prefigure is really more intuitive to use than Tikz and generates more accessible images. Full documentation is available at `PrefigureDocumentation` and a very convenient tool for testing your figures is available at `PrefigurePlayground`.