This refers to these two properties:

$$\sum_{n=0}^{\infty}(a_n + b_n) = \sum_{n=0}^{\infty} a_n + \sum_{n=0}^{\infty} b_n \qquad\qquad \sum_{n=0}^{\infty}(ca_n) = c\sum_{n=0}^{\infty} a_n$$

Finite sums can be reordered at-will, but infinite sums cannot.

IF $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are both convergent,

THEN $\sum_{n=0}^{\infty}(a_n + b_n)$ is also convergent and

$$\sum_{n=0}^{\infty}(a_n + b_n) = \sum_{n=0}^{\infty} a_n + \sum_{n=0}^{\infty} b_n$$

Let $c \in \mathbb{R}$.

IF $\sum_{n=0}^{\infty} a_n$ is convergent,

THEN $\sum_{n=0}^{\infty}(ca_n)$ is also convergent, and

$$\sum_{n=0}^{\infty}(ca_n) = c\sum_{n=0}^{\infty} a_n$$

Proof. for *Theorem 6.1*

$$\begin{aligned}
\sum_{n=0}^{\infty} a_n &= \lim_{k\to\infty} S_k, &&\text{where } S_k = \sum_{n=0}^{\infty} a_n \\
\sum_{n=0}^{\infty} b_n &= \lim_{k\to\infty} T_k, &&\text{where } T_k = \sum_{n=0}^{\infty} b_n \\
\sum_{n=0}^{\infty}(a_n + b_n) &= \lim_{k\to\infty} R_k, &&\text{where } R_k = \sum_{n=0}^{\infty}(a_n + b_n) \\
&\text{By properties of finite sums,} && R_k = S_k + T_k
\end{aligned}$$

By hypothesis, *the first two limits exist.* Then, by the limit laws,

$$\lim_{k\to\infty} R_k \lim_{k\to\infty} S_k + \lim_{k\to\infty} T_k$$

■

This is a template for series—how they're proven.

1. Write the infinite sum as a limit of partial sums—finite sums
2. Use the properties known to be true for finite sums
3. Pass to the limit