p-series For which values of $p > 0$ is the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent? Use integral test.

Let $f(x) = \dfrac{1}{x^p}$, then f is continuous, positive, and decreasing.

By integral test, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p} \sim \int_1^{\infty} \frac{1}{x^p} dx$.

We know that $\displaystyle\int_1^{\infty} \frac{1}{x^p} dx$ is convergent iff $p > 1$. Thus, $\boxed{\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ is convergent iff } p > 1}$.

other Is $\sum_{n=2}^{\infty} frac1n \ln n$ convergent? It's positive, decreasing, continuous for $x \geq 2$.

By integral test, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n} \sim \int_2^{\infty} \frac{1}{x \ln x} dx$. We can find the antiderivative for this function.

$$\begin{aligned}
\int_2^{\infty} \frac{1}{x \ln x} dx &= \lim_{b \to \infty} \left[\int_2^b \frac{1}{x \ln x} dx \right] = \lim_{b \to \infty} \left[\ln \ln x \Big|_2^b \right] \\
&= \lim_{b \to \infty} [\ln \ln b - \ln \ln 2] = \infty
\end{aligned}$$

So by the integral test, the series diverges.