Estimating Population Size

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Contents

Contents

List of Figures

List of Tables

In any war, it is always of value to one side to have good intelligence on the weapons resources of the other side. During the Second World War, for example, Allied military planners eagerly searched for ways to accurately estimate the Axis production of tanks, aircrafts and numerous other weapons platforms. In the specific case of German tanks, a very clever way to do that was based on using either the stamped serial numbers or gearbox markings on captured Mark I and Mark V tanks, respectively. This type of problem has far wider applications.

We can experimentally see how well the formula that was used works in practice with a Monte Carlo Simulation. That is, the program first randomly picks a value for N (integer between 100 and 1000) with a value for the the sample size as a percentage of N.

The program then generates, randomly, n different integers in the interval 1 to N, the maximum of those integers is then used in the estimation formula to estimate a value for N. This estimate can be compared to the actual value of N to determine how well the formula has performed. We investigate samples that are 2%, 5%, 10% and 20% of the population.

Function to implement Monte Carlo simulation using formula

```
runSim <- function(nSim = 1000, sample.size.perc = 10/100) {

    # set up an array to store parameter estimates
    estArray <- array(0, dim = c(nSim, 3))

    for (i in 1:nSim) {

        N <- sample(100:1000, 1)  # select a population size

        n <- round(sample.size.perc * N, 0)  # the sample size

        sample2 <- sample(N, n)  # select a sample from the population

        est <- (n + 1)/n * max(sample2) - 1  # apply formula

        est <- round(est, 0)  # estimate of pop size

        estArray[i, 1] <- (est - N)/N * 100  # percentage error is captured
        estArray[i, 2] <- N
        estArray[i, 3] <- sample.size.perc

    }

    list(estArray = estArray)

}
```

Sample size 2% of population

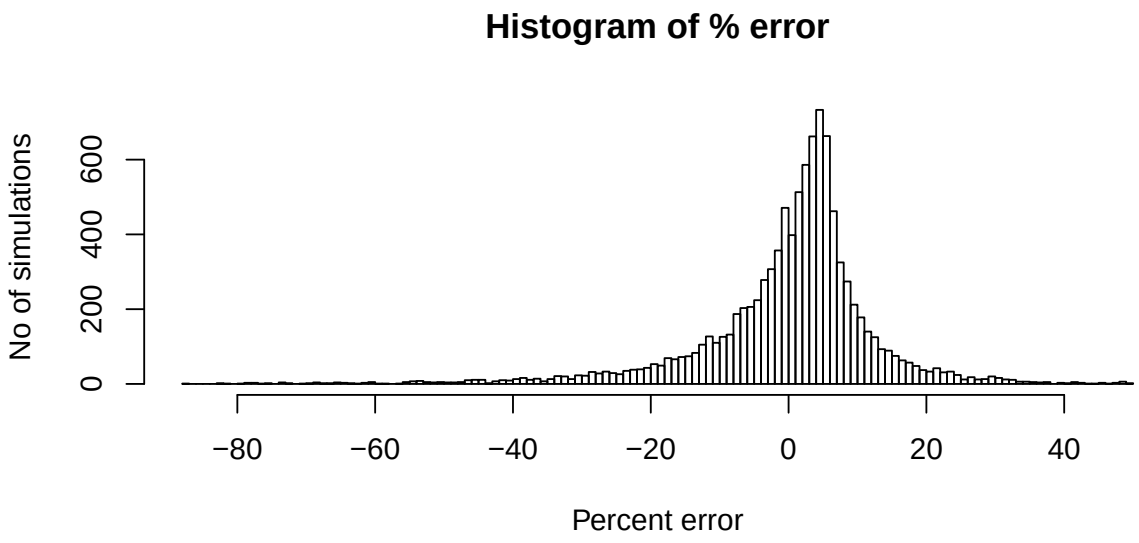
```
x <- runSim(nSim = 10000, sample.size.perc = 2/100)  # run simulation
apply(x$estArray, 2, mean, na.rm = TRUE)[1]  # mean % error
```

```
[1] 0.01385591
```

```
# median and 95% confidence intervals for % error
quantile(x$estArray[, 1], prob = c(0.025, 0.5, 0.975))
```

```
      2.5%        50%      97.5%
-33.333333   2.290867  22.222222
```

```
hist(x$estArray[, 1], breaks = 100, main = "Histogram of % error",
    xlab = "Percent error", ylab = "No of simulations")
```



Sample size 5% of population

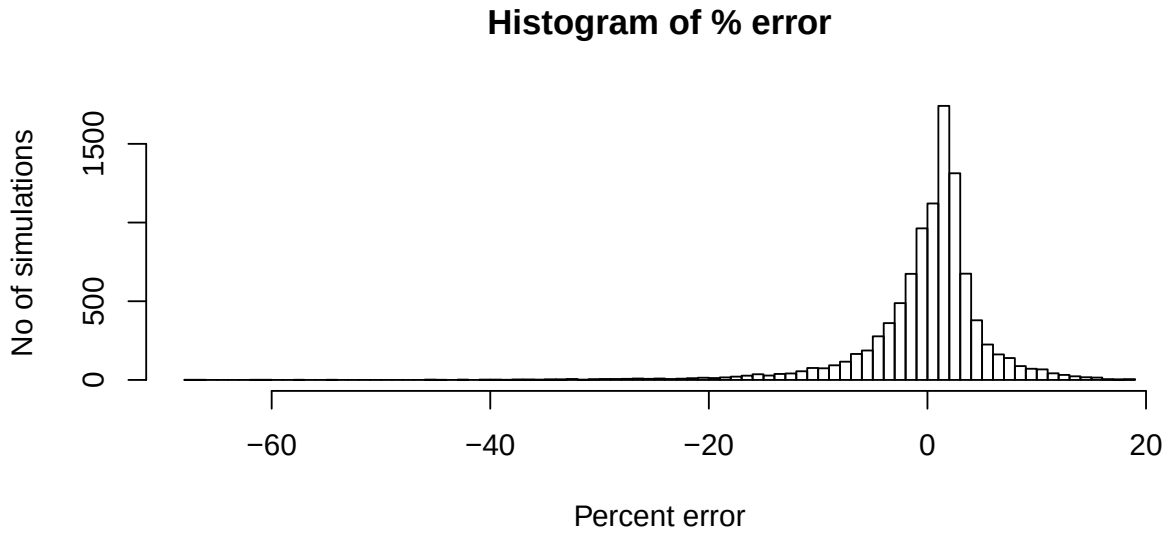
```
x <- runSim(nSim = 10000, sample.size.perc = 5/100)  # run simulation
apply(x$estArray, 2, mean, na.rm = TRUE)[1]  # mean % error
```

```
[1] -0.0187242
```

```
# median and 95% confidence intervals for % error
quantile(x$estArray[, 1], prob = c(0.025, 0.5, 0.975))
```

```
      2.5%        50%      97.5%
-14.525259   1.003344   9.524408
```

```
hist(x$estArray[, 1], breaks = 100, main = "Histogram of % error",
    xlab = "Percent error", ylab = "No of simulations")
```



Sample size 10% of population

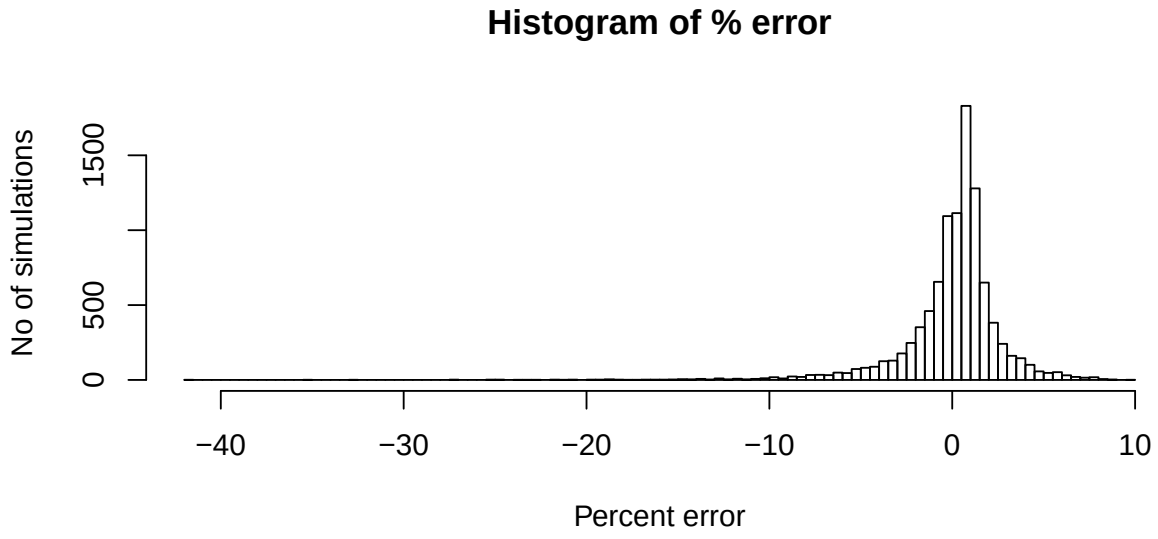
```
x <- runSim(nSim = 10000, sample.size.perc = 10/100)  # run simulation
apply(x$estArray, 2, mean, na.rm = TRUE)[1]  # mean % error
```

```
[1] 0.04915648
```

```
# median and 95% confidence intervals for % error
quantile(x$estArray[, 1], prob = c(0.025, 0.5, 0.975))
```

```
      2.5%        50%      97.5%
-6.8185347  0.5107252  4.5005653
```

```
hist(x$estArray[, 1], breaks = 100, main = "Histogram of % error",
    xlab = "Percent error", ylab = "No of simulations")
```



Sample size 20% of population

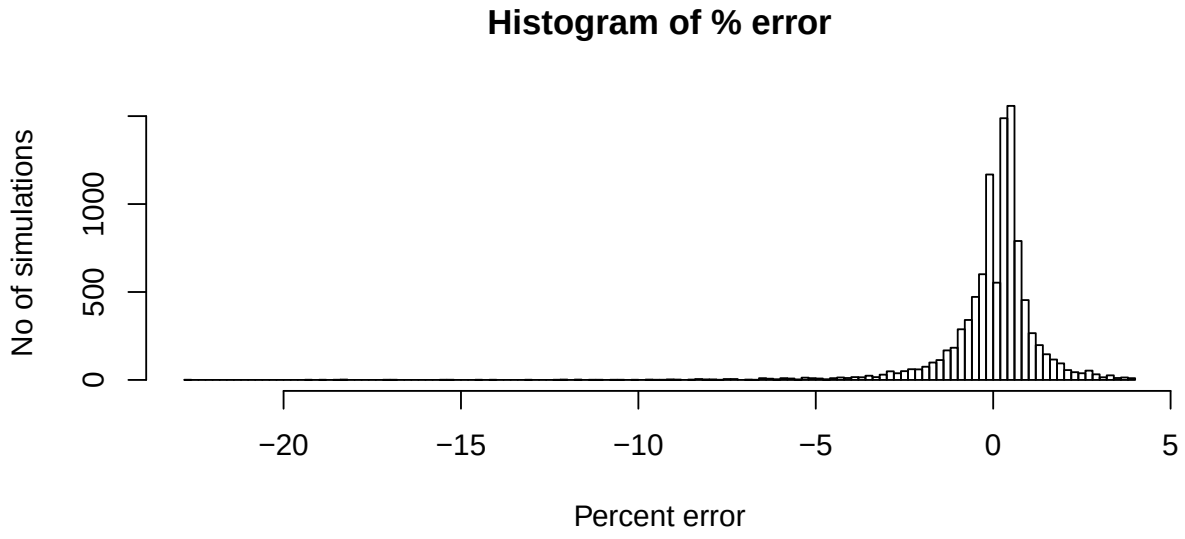
```
x <- runSim(nSim = 10000, sample.size.perc = 20/100)  # run simulation
apply(x$estArray, 2, mean, na.rm = TRUE)[1]  # mean % error
```

```
[1] 0.01389191
```

```
# median and 95% confidence intervals for % error
quantile(x$estArray[, 1], prob = c(0.025, 0.5, 0.975))
```

```
      2.5%        50%      97.5%
-3.1392783  0.2453988  2.1739130
```

```
hist(x$estArray[, 1], breaks = 100, main = "Histogram of % error",
    xlab = "Percent error", ylab = "No of simulations")
```



Computing Environment

```
sessionInfo()
```

```
R version 3.6.0 (2019-04-26)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 17134)

Matrix products: default

locale:
[1] LC_COLLATE=English_United Kingdom.1252
[2] LC_CTYPE=English_United Kingdom.1252
[3] LC_MONETARY=English_United Kingdom.1252
[4] LC_NUMERIC=C
[5] LC_TIME=English_United Kingdom.1252

attached base packages:
[1] stats     graphics  grDevices utils     datasets
[6] methods   base

other attached packages:
[1] knitr_1.23

loaded via a namespace (and not attached):
 [1] compiler_3.6.0  magrittr_1.5    formatR_1.7
 [4] tools_3.6.0     htmltools_0.3.6 yaml_2.2.0
 [7] Rcpp_1.0.1      stringi_1.4.3   rmarkdown_1.13
[10] stringr_1.4.0   xfun_0.7        digest_0.6.19
[13] evaluate_0.14
```

This took 1.83 seconds to execute.