Cake Puzzle

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Introduction

Lady Frederica von Battenberg has baked a long, thin rectangular cake for her daughter Victoria. She has picked two random points on the top of the cake on which she has placed two candles. She hands the cake knife to Victoria, who now proceeds to pick a random point along the length of the cake, and cut across the cake at that point. Now that the cake has been cut in two, what is the chance that both pieces of the cake have a candle on them? (Presumably Victoria does this blindfold and carefully).

Function to implement Monte Carlo simulation

```
runSim <- function(nSim = 1000) {
    # set up an array to store parameter estimates
    estArray <- array(0, dim = c(nSim, 1))

    for (i in 1:nSim) {

        A <- runif(1)
        B <- runif(1)
        C <- runif(1)

        if ((B > A && B < C) | (A > B && A < C)) {
            res = 1
        } else {
            res = 0
        }

        estArray[i] <- res
    }
    list(estArray = estArray)
}


x <- runSim(nSim = 99999)
mean(x$estArray, na.rm = TRUE)
```

```
[1] 0.3372334
```

Analytical solution

Simulation is a little overkill for this problem, but a good programming excercise. We can use permutations.

Denote the location of the candles as A and C and the location of the cut B. Remember that repetition is allowed in permutations unlike in combinations.

${}^{n}p_{r}$: which means the number of permutations of n items taken r items at a time.

For example; given 3 letters ABC find ${}^{3}p_{3}$

ABC, BAC, BCA, CBA, CAB, ACB

which mean that there are 6 ways, in other words ${}^{3}p_{3}= 6$.

Lets look at the permutation function using factorials:

n!/(n-r)! = 3!/(3-3)! = 3x2x1/0! = 6/1 = 6.

We can see that B is in the middle of 'ABC, BAC, BCA, CBA, CAB, ACB' 2 out of 6 times. So the chance that both pieces of the cake have a candle on them is 1/3.

Reference

New Scientist Quick quiz #9, 29 June 2019, page 50.

Computing Environment

```
sessionInfo()
```

```
R version 3.6.1 (2019-07-05)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 17134)

Matrix products: default

locale:
[1] LC_COLLATE=English_United Kingdom.1252
[2] LC_CTYPE=English_United Kingdom.1252
[3] LC_MONETARY=English_United Kingdom.1252
[4] LC_NUMERIC=C
[5] LC_TIME=English_United Kingdom.1252

attached base packages:
[1] stats     graphics  grDevices utils     datasets
[6] methods   base

other attached packages:
[1] knitr_1.23

loaded via a namespace (and not attached):
 [1] compiler_3.6.1  magrittr_1.5    formatR_1.7
 [4] tools_3.6.1     htmltools_0.3.6 yaml_2.2.0
 [7] Rcpp_1.0.1      stringi_1.4.3   rmarkdown_1.14
[10] stringr_1.4.0   xfun_0.8        digest_0.6.20
[13] evaluate_0.14
```

This took 0.56 seconds to execute.