Cake Puzzle

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Contents

Contents

List of Figures

List of Tables

Function to implement Monte Carlo simulation

```
runSim <- function(nSim = 1000) {
    # set up an array to store parameter estimates
    estArray <- array(0, dim = c(nSim, 1))

    for (i in 1:nSim) {

        A <- runif(1)
        B <- runif(1)
        C <- runif(1)

        if ((B > A && B < C) | (A > B && A < C)) {
            res = 1
        } else {
            res = 0
        }

        estArray[i] <- res
    }
    list(estArray = estArray)
}


x <- runSim(nSim = 99999)
mean(x$estArray, na.rm = TRUE)
```

```
[1] 0.3372334
```

Analytical solution

Simulation is a little overkill for this problem, but a good programming excercise. We can use permutations.

Denote the location of the candles as A and C and the location of the cut B. Remember that repetition is allowed in permutations unlike in combinations.

${}^{n}p_{r}$: which means the number of permutations of n items taken r items at a time.

For example; given 3 letters ABC find ${}^{3}p_{3}$

ABC, BAC, BCA, CBA, CAB, ACB

which mean that there are 6 ways, in other words ${}^{3}p_{3}= 6$.

Lets look at the permutation function using factorials:

n!/(n-r)! = 3!/(3-3)! = 3x2x1/0! = 6/1 = 6.

We can see that B is in the middle of 'ABC, BAC, BCA, CBA, CAB, ACB' 2 out of 6 times. So the chance that both pieces of the cake have a candle on them is 1/3.

Computing Environment

```
sessionInfo()
```

```
R version 3.6.1 (2019-07-05)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 17134)

Matrix products: default

locale:
[1] LC_COLLATE=English_United Kingdom.1252
[2] LC_CTYPE=English_United Kingdom.1252
[3] LC_MONETARY=English_United Kingdom.1252
[4] LC_NUMERIC=C
[5] LC_TIME=English_United Kingdom.1252

attached base packages:
[1] stats     graphics  grDevices utils     datasets
[6] methods   base

other attached packages:
[1] knitr_1.23

loaded via a namespace (and not attached):
 [1] compiler_3.6.1  magrittr_1.5    formatR_1.7
 [4] tools_3.6.1     htmltools_0.3.6 yaml_2.2.0
 [7] Rcpp_1.0.1      stringi_1.4.3   rmarkdown_1.14
[10] stringr_1.4.0   xfun_0.8        digest_0.6.20
[13] evaluate_0.14
```

This took 0.58 seconds to execute.