

# Functional Sensitivity

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Program is provided which Fits 11 models to single predictor and use sum of square of residual as model selection criteria

## 1 Functional Sensitivity function

```
func.sens <- function (x, y, model, spec, print.plot=1) {  
  
  # Define models  
  
  if (model %in% 1 ) {mod="Linear Y=a+bX"}  
  if (model %in% 2 ) {mod="Exponential Y=exp(a+bX)"}  
  if (model %in% 3 ) {mod="Reciprocal-Y Y=1/(a+bX)"}  
  if (model %in% 4 ) {mod="Reciprocal-X Y=a+b/X"}  
  if (model %in% 5 ) {mod="Double Reciprocal Y=1/(a+b/X)"}  
  if (model %in% 6 ) {mod="Logarithmic-X Y=a+b(log(X))"}  
  if (model %in% 7 ) {mod="Multiplicative Y=aX^b"}  
  if (model %in% 8 ) {mod="Square Root-X Y=a+b(sqrt(X))"}  
  if (model %in% 9 ) {mod="Square Root-Y Y=(a+bX)^2"}  
  if (model %in% 10) {mod="S-curve Y=exp(a+b/X)"}  
  if (model %in% 11) {mod="Square X and Y Y^2=a+X^2/b"}  
  
  # transformation of data for 11 models  
  
  ty1 <- y;      tx1 <- x  
  ty2 <- log(y); tx2 <- x  
  ty3 <- 1/y;    tx3 <- x  
  ty4 <- y;      tx4 <- 1/x  
  ty5 <- 1/y;    tx5 <- 1/x  
  ty6 <- y;      tx6 <- log(x)  
  ty7 <- log(y); tx7 <- log(x)  
  ty8 <- y;      tx8 <- sqrt(x)  
  ty9 <- sqrt(y); tx9 <- x  
  ty10 <- log(y); tx10 <- 1/x  
  ty11 <- y^2;   tx11 <- x^2  
  
  # save the original data  
  
  x1 <- x  
  y1 <- y  
  
  # transform using the selected model (1 - 11)  
  
  x <- eval(parse(text=(paste("tx", model, sep="")) ))  
  y <- eval(parse(text=(paste("ty", model, sep="")) ))  
}
```

```

# summary statistics of the independent variable

n <- length(x)
txbar <- mean(x)
txstd <- sd(x)

# run regression on the transformed data grab slope intercept

f <- lm(y~x)
intercept <- coef(f)[1][[1]]
slope <- coef(f)[2][[1]]

# R2 on the transformed x and y, an idea but not used

v1 <- unlist(cor.test(x,y)$estimate^2)[1][[1]]

# obtain the predictions

p <- predict.lm(f, interval="confidence")

# transform the predicted values & 95%CI back to original scale

if (model %in% c(2,7,10)) {p <- exp(p)}
if (model %in% c(3,5) ) {p <- 1/p}
if (model %in% c(9) ) {p <- p^2}
if (model %in% c(11) ) {p <- p^.5}

# calculate residual squared
# residuals original y and transformed back predicted values

r <- (y1-p[,1])^2

# R2 on the original x and transformed back predicted y, an idea but not used

v2 <- unlist(cor.test(x1,p[,1])$estimate^2)[1][[1]]

# residual sum of squares, this will be used to judge best model

ssr <- sum(r, na.rm=T)

# transform the response that we will read back

tyspec <- spec
if (model %in% c(2,7,10)) {tyspec <- log(tyspec)}
if (model %in% c(3,5) ) {tyspec <- 1/tyspec}
if (model %in% c(9) ) {tyspec <- sqrt(tyspec)}
if (model %in% c(11) ) {tyspec <- tyspec ^2}

# grab the residual standard deviation

rsd2<-as.data.frame(anova(f))[2,3]^0.5

# read back on transformed scale

```

```

mse <- rsd2^2
t <- qt(0.975, n-2)
a <- t^2*mse/((n-1)*txstd^2)-slope^2
b <- 2*slope*(tyspec-intercept-slope*txbar)
c <- t^2*mse/n-(tyspec-intercept-slope*txbar) ^2
txpre <- (tyspec-intercept)/slope
txup <- (-b+sqrt(b^2-4*a*c))/(2*a) + txbar
txlow <- (-b-sqrt(b^2-4*a*c))/(2*a) + txbar

# transform the read back estimates to the original scale

if (model %in% c(4,5,10)) {txpre <- 1/txpre; txup <- 1/txup; txlow <- 1/txlow}
if (model %in% c(6,7) ) {txpre <- exp(txpre); txup <- exp(txup); txlow <- exp(txlow)}
if (model %in% c(8) ) {txpre <- txpre^2; txup <- txup^2; txlow <- txlow^2}
if (model %in% c(11) ) {txpre <- txpre^.5; txup <- txup^.5; txlow <- txlow^.5}

# ensure order of limits is correct

limits <- sort(c(txlow,txup))
txlow <- limits[1]
txup <- limits[2]

# help with plotting

ymin <- min(y)
ymax <- max(y)
ystep <- (ymax-ymin)/8
ymin1 <- ymin-ystep
ymax1 <- ymax+ystep

xmin <- min(x)
xmax <- max(x)
xstep <- (xmax-xmin)/8
xmin1 <- xmin-xstep
xmax1 <- xmax+xstep

# put all pertinent data together, original data and predicted with 95%CI

foo <- data.frame(cbind(x=x1,obsy=y1, pred= p[,1], p2a=p[,2], p3=p[,3]))
foo <- foo[order(foo$obsy),]

# plot and present the estimated read back

p1 <- ggplot(foo, aes(x=x,y=pred)) +
  geom_line( ) +
  geom_ribbon(data=foo , aes(ymin= p2a,ymax= p3),alpha=0.2,fill="blue") +
  geom_point(data=foo, aes(x=x ,y=obsy))
scale_x_continuous(limits = c(xmin1, xmax1))
scale_y_continuous(limits = c(xmin1, xmax1))

p <- p1 + geom_hline(yintercept=spec, colour="#990000", linetype="dashed")

p <- p + theme(axis.text.x = element_text(angle = 90, hjust = 1, size=13,color="darkred"))

```

```

p <- p + scale_color_manual(values=c("Red", "blue"))
p <- p + theme_bw()
p <- p + xlab('Independent variable')
p <- p + ylab('Dependent variable')

p <- p + ggtitle(paste("Model for the curve #", model, " ", mod, " ", " ", "\nExploration of model fitting
# ", R2 transformed x, y ", p2(v1), ", R2 x, trans. back pred. y", p2(v2),
sep=" ")) + theme(plot.title = element_text(lineheight=1, face="bold", color="black", size=11))

p <- p + labs(x = "Independent variable", y = "Response")
p <- p + theme(axis.title.y = element_text(size = rel(1.1), angle = 90))
p <- p + theme(axis.title.x = element_text(size = rel(1.1), angle = 00))

if (print.plot==1) {print(p)}

return(ssr)
}

```

## 2 Test data 1

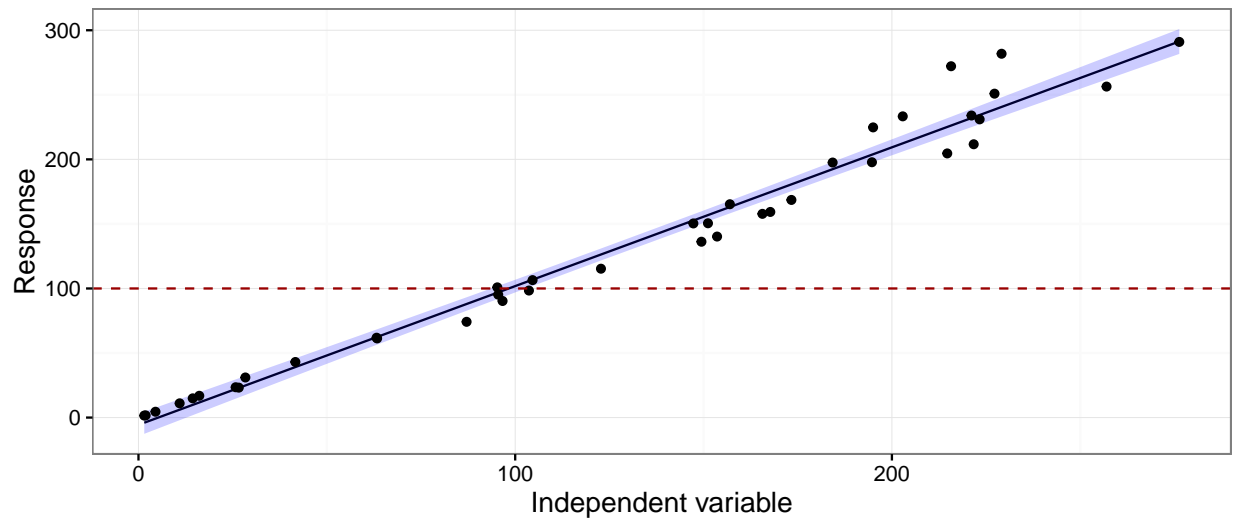
```
require(ggplot2)

toread <- "sample _NAME_ COL1 COL2
A046 conc 122.766 115.304
A048 conc 221.714 211.704
A049 conc 223.287 231.001
A050 conc 276.281 290.975
A051 conc 229.109 281.813
A052 conc 156.985 165.255
A054 conc 147.296 150.360
A058 conc 87.098 74.180
A062 conc 103.648 98.389
A063 conc 173.321 168.583
A069 conc 214.687 204.615
H098 conc 195.010 224.759
H099 conc 202.874 233.319
H100 conc 227.235 250.898
H101 conc 221.122 233.964
H102 conc 256.975 256.384
H105 conc 215.708 272.139
H112 conc 151.172 150.507
H113 conc 167.717 159.274
H114 conc 165.622 157.793
M023 conc 1.537 1.541
M024 conc 194.695 197.779
M025 conc 95.258 100.862
M026 conc 10.932 11.033
M027 conc 14.404 14.902
M030 conc 4.521 4.530
M031 conc 16.144 16.884
M032 conc 96.648 90.294
M036 conc 28.378 31.051
M037 conc 26.629 23.187
M038 conc 63.352 61.304
M044 conc 41.647 43.043
N008 conc 25.798 23.489
N011 conc 1.907 1.705
T079 conc 63.212 61.854
T081 conc 95.499 95.207
T082 conc 104.613 106.458
T085 conc 149.441 136.205
T086 conc 153.592 140.220
T090 conc 184.267 197.540"

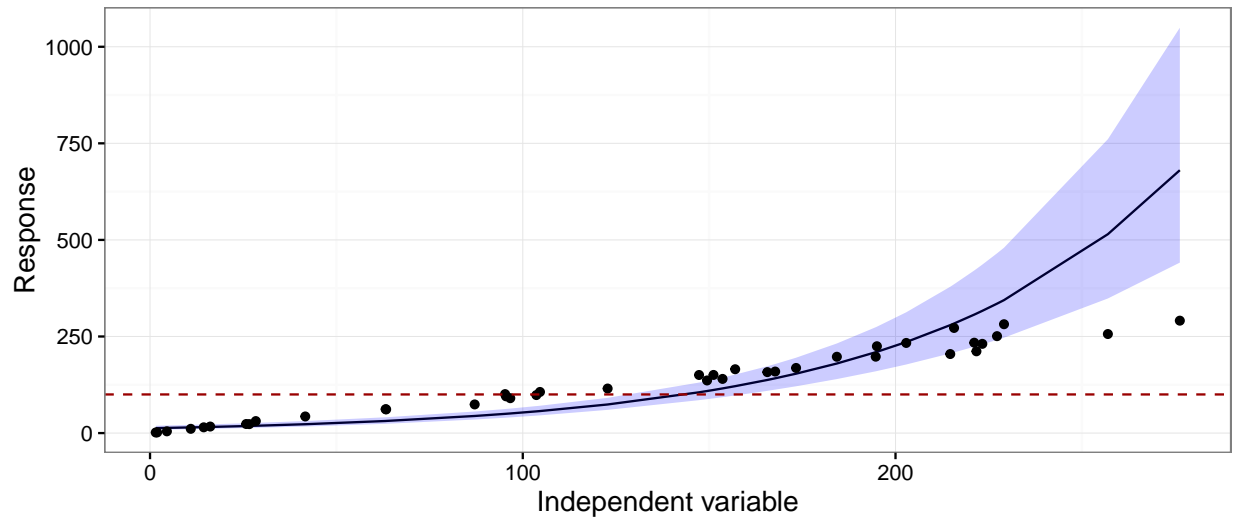
survey <- read.table(textConnection(toread), header = TRUE)

for (j in 1:11) {
  func.sens(x=survey$COL1, y=survey$COL2, model=j, spec=100)
}
```

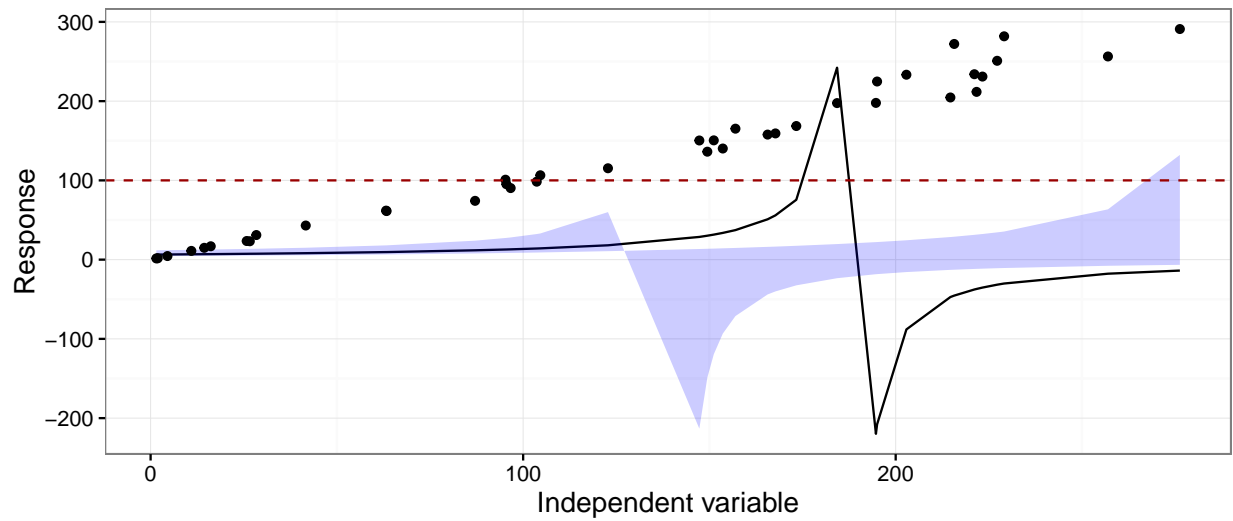
Model for the curve # 1 Linear  $Y=a+bX$  ,  
 Exploration of model fitting at response of 100 , 98.2904 and 95% CI: ( 93.5925 , 102.8185 )  
 Residual sum of squares 8000.1736 , Residual standard deviation 14.5097



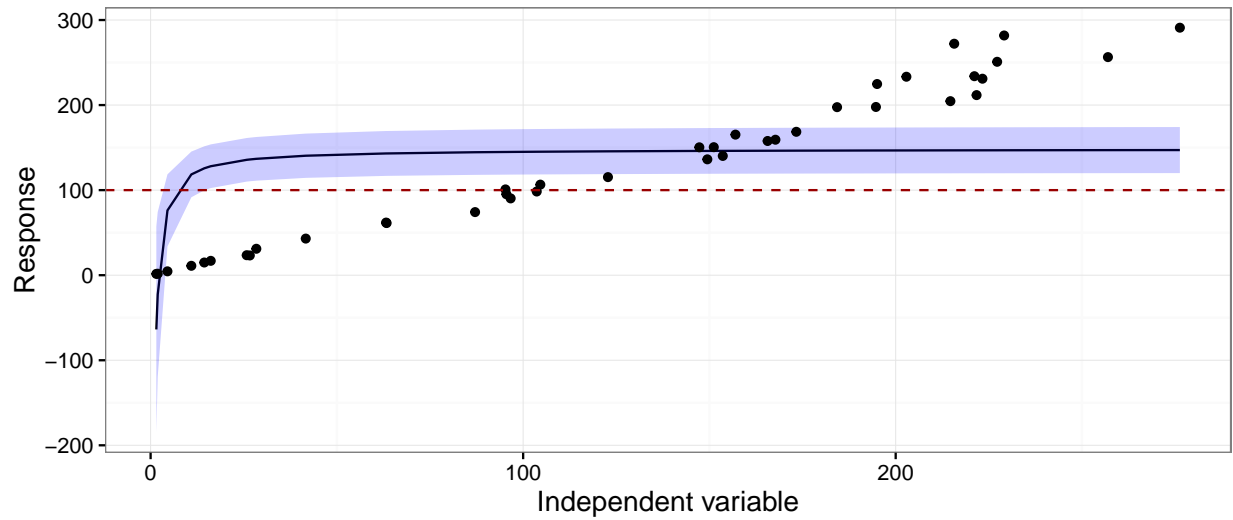
Model for the curve # 2 Exponential  $Y=\exp(a+bX)$  ,  
 Exploration of model fitting at response of 100 , 143.5647 and 95% CI: ( 129.1227 , 158.9999 )  
 Residual sum of squares 281528.4841 , Residual standard deviation 0.6519



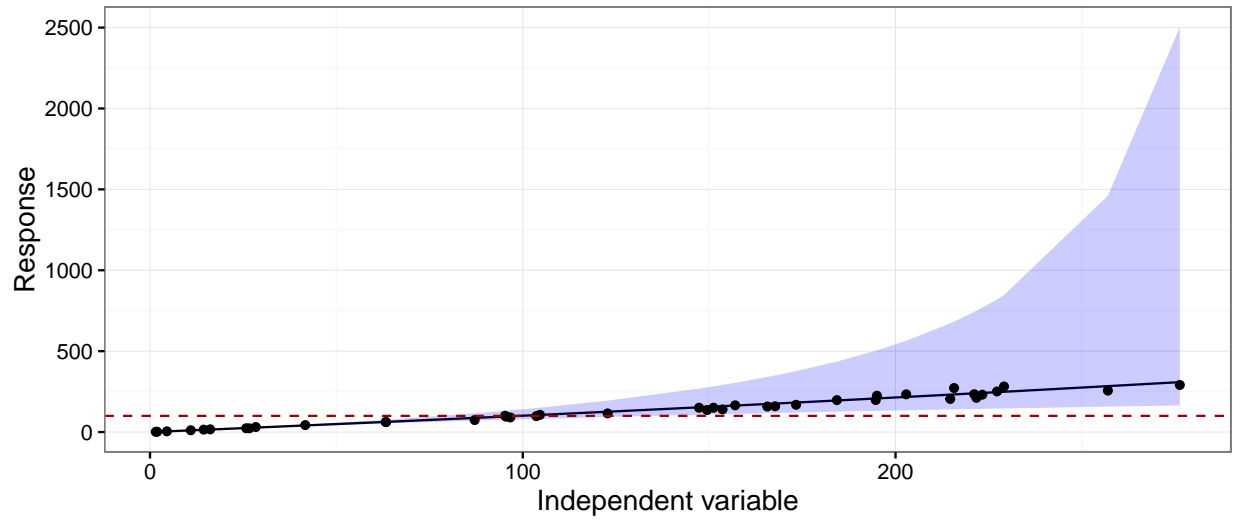
Model for the curve # 3 Reciprocal-Y  $Y=1/(a+bX)$  ,  
 Exploration of model fitting at response of 100 , 177.2108 and 95% CI: ( 130.8675 , 270.4737 )  
 Residual sum of squares 1338842.6841 , Residual standard deviation 0.1204



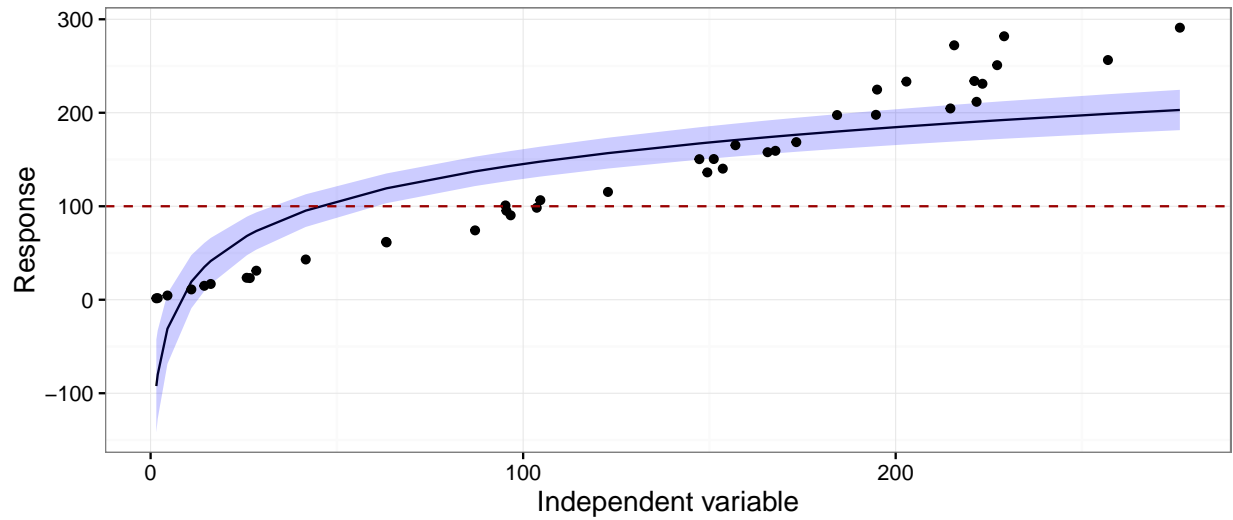
Model for the curve # 4 Reciprocal-X  $Y=a+b/X$  ,  
 Exploration of model fitting at response of 100 , 6.7509 and 95% CI: ( 2.9322 , 14.5270 )  
 Residual sum of squares 242215.6512 , Residual standard deviation 79.8379



Model for the curve # 5 Double Reciprocal  $Y=1/(a+b/X)$  ,  
 Exploration of model fitting at response of 100 , 98.9373 and 95% CI: ( 78.2508 , 135.0492 )  
 Residual sum of squares 7991.2452 , Residual standard deviation 0.0084

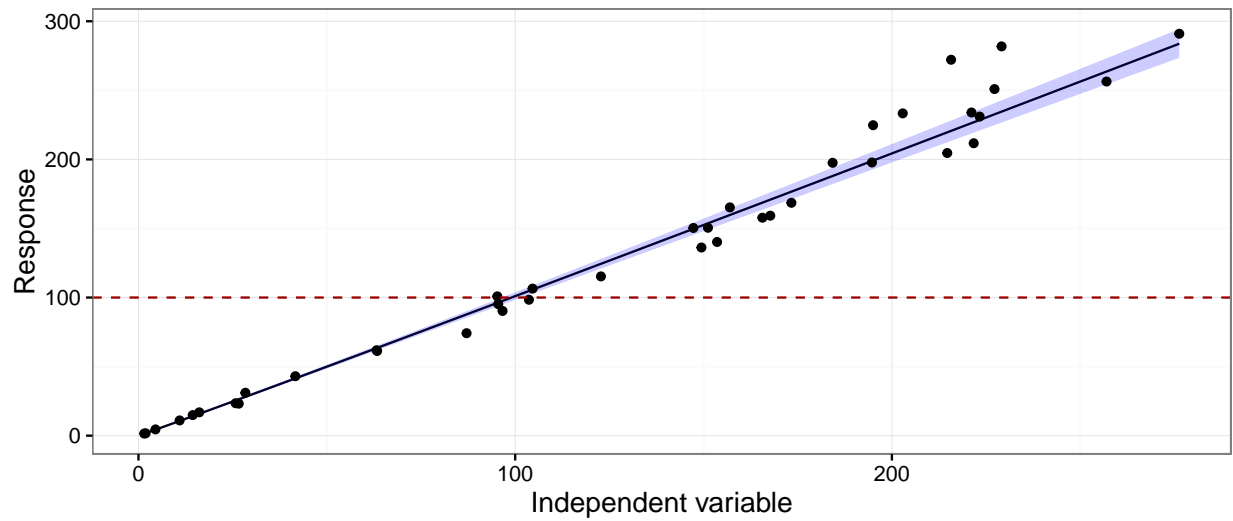


Model for the curve # 6 Logarithmic-X  $Y=a+b(\log(X))$  ,  
 Exploration of model fitting at response of 100 , 45.2021 and 95% CI: ( 32.3587 , 59.9250 )  
 Residual sum of squares 91062.5663 , Residual standard deviation 48.9529

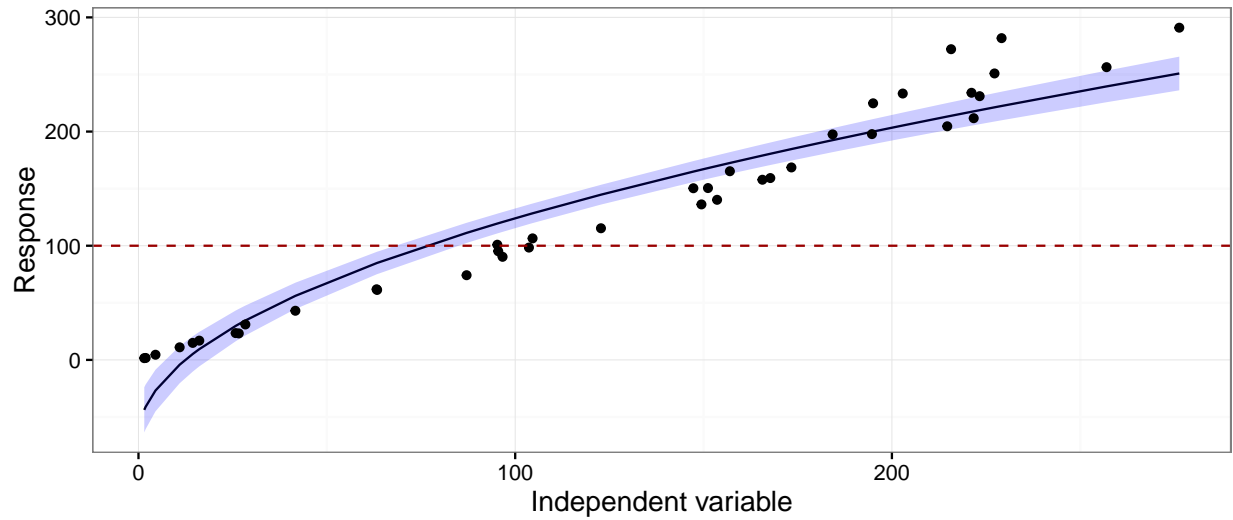




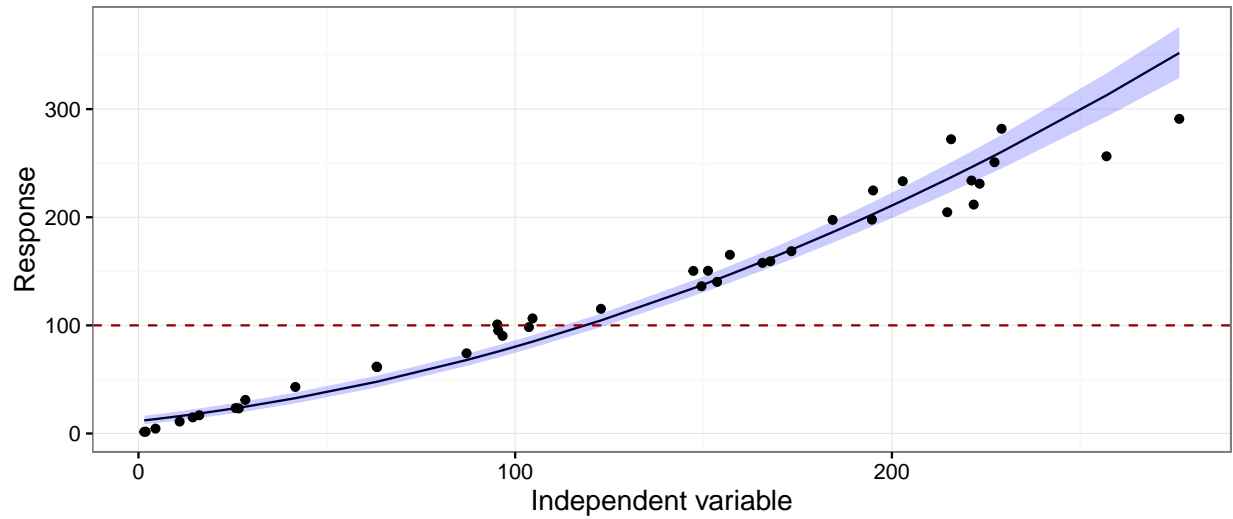
**Model for the curve # 7 Multiplicative  $Y=aX^b$  ,**  
**Exploration of model fitting at response of 100 , 99.0181 and 95% CI: ( 96.4279 , 101.6956 )**  
**Residual sum of squares 8546.8865 , Residual standard deviation 0.0833**



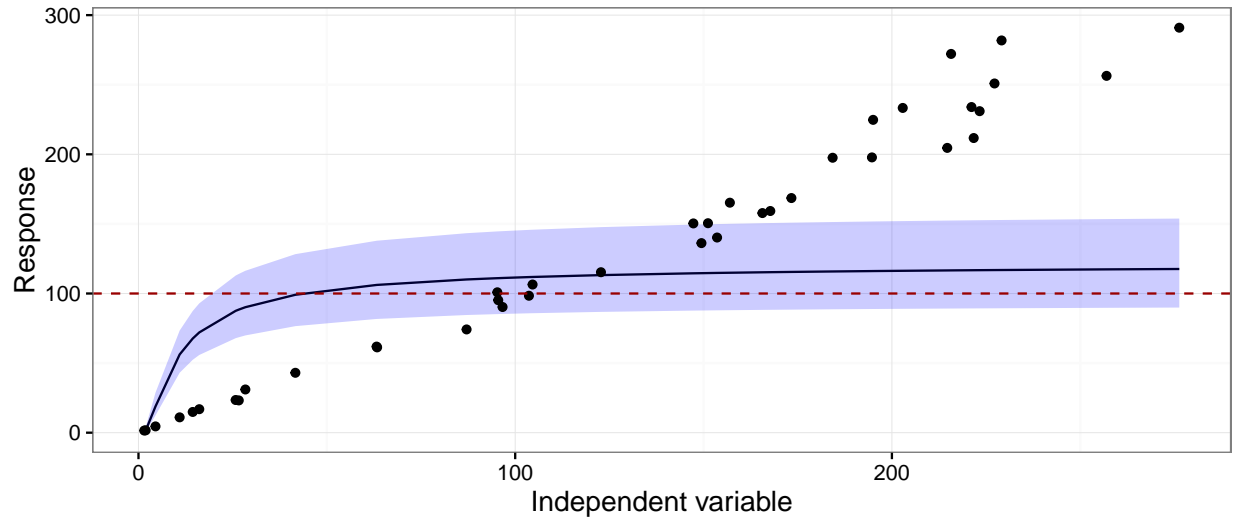
**Model for the curve # 8 Square Root-X  $Y=a+b(\sqrt{x})$  ,**  
**Exploration of model fitting at response of 100 , 76.4871 and 95% CI: ( 68.0042 , 84.8241 )**  
**Residual sum of squares 27263.3368 , Residual standard deviation 26.7854**



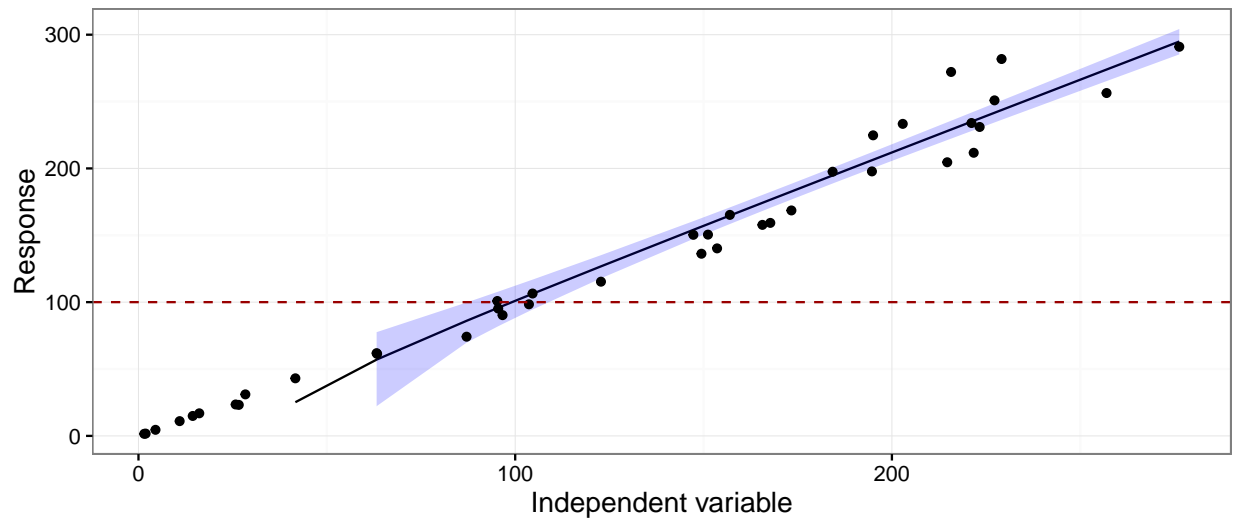
Model for the curve # 9 Square Root- $Y = (a+bX)^2$  ,  
 Exploration of model fitting at response of 100 , 118.7073 and 95% CI: ( 113.1718 , 124.1565 )  
 Residual sum of squares 15909.0752 , Residual standard deviation 0.9452



Model for the curve # 10 S-curve  $Y = \exp(a+b/X)$  ,  
 Exploration of model fitting at response of 100 , 43.6212 and 95% CI: ( -97.5223 , 18.8301 )  
 Residual sum of squares 243260.6736 , Residual standard deviation 0.7919



Model for the curve # 11 Square X and Y  $Y^2=a+X^2/b$  ,  
Exploration of model fitting at response of 100 , 99.1323 and 95% CI: ( 87.3551 , 108.7806 )  
Residual sum of squares 8064.4991 , Residual standard deviation 6389.2725



### 3 Test data 2

```
n <- 150
x99 <- seq( from=10.001 , to=100, length.out=n)

#y99 <- abs((x99)+rnorm(n,0,4 ))^0.5 # spec^2
#y99 <- abs((x99)+rnorm(n,0,4 ))^2   # spec^0.5
#y99 <- abs((x99)+rnorm(n,0,4 ))^-1  # spec^-1
#y99 <- abs((x99)+rnorm(n,0,4 ))^-0.1 # spec^-(1/.1) == spec^-10 [model 7]
y99 <- abs((x99)+rnorm(n,0,10))      # linear

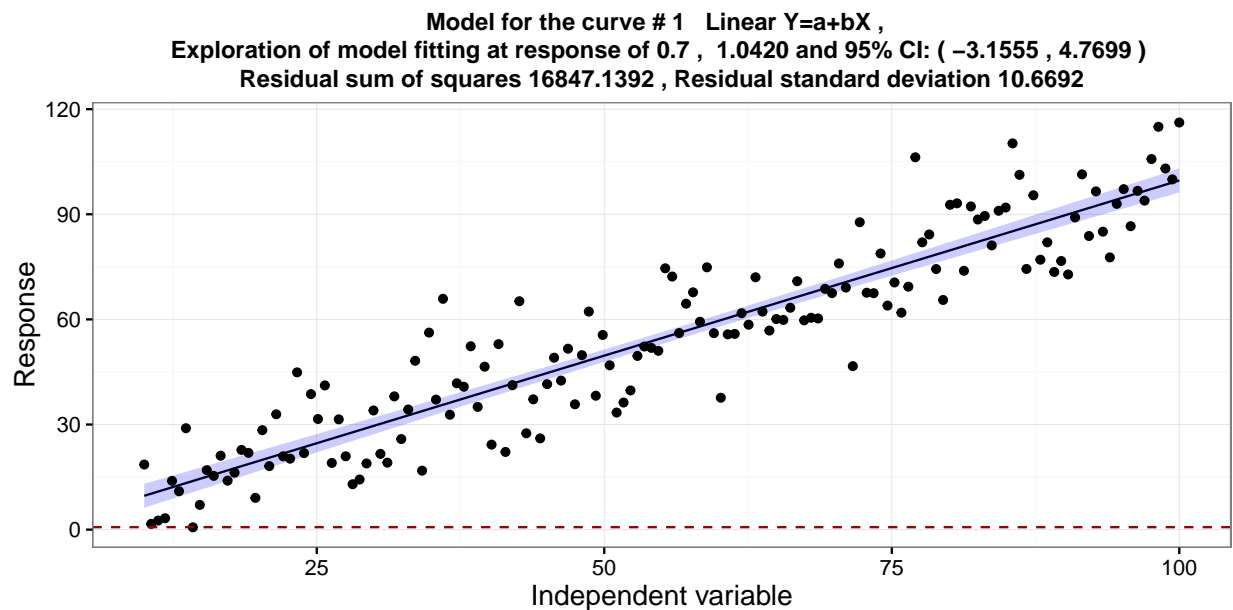
ssr <- rep(NA,11)

for (j in 1:11) {

  res <- func.sens(x=x99, y=y99, model=j, spec=.7, print.plot=0) # don't print
  ssr[j] <- res

}

func.sens(x=x99, y=y99, model=which(ssr==min(ssr)), spec=.7, print.plot = 1)
```



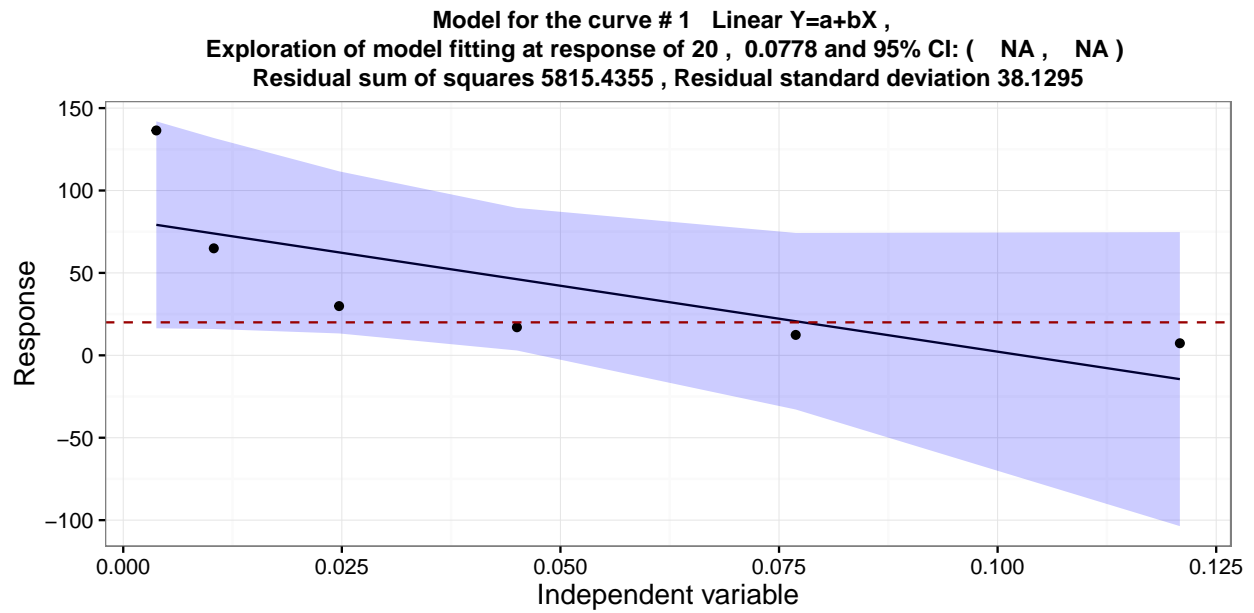
```
[1] 16847.14
```

```
cat(paste0("The best fitting model is #", which(ssr==min(ssr))))
```

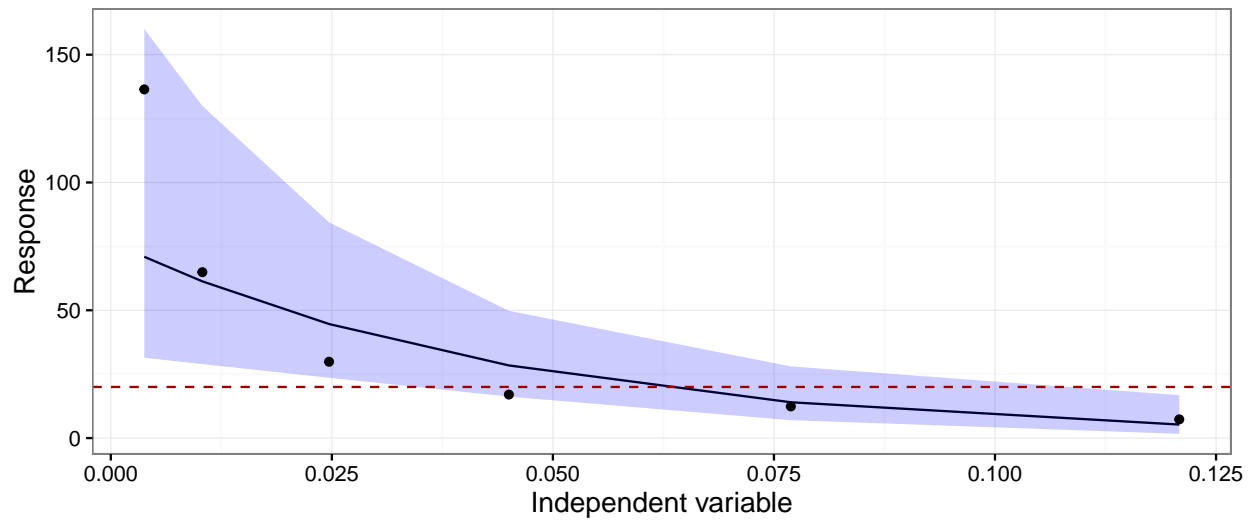
The best fitting model is #1

## 4 Test data 3

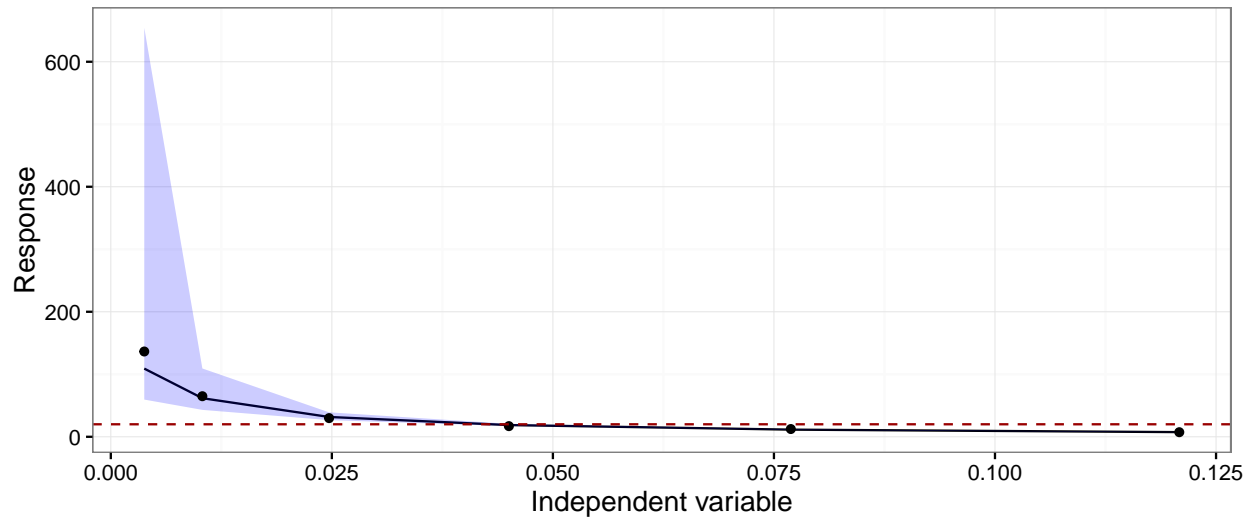
```
mu<-c(  
  0.1208444444,  
  0.0769101124,  
  0.0450227273,  
  0.0246853933,  
  0.0103555556,  
  0.0037840909)  
  
cv<-c(  
  7.3282535623,  
  12.384142008,  
  17.022255039,  
  29.872271843,  
  64.921152057,  
  136.43305684)  
  
for (j in 1:11) {  
  func.sens(x=mu, y=cv, model=j, spec=20)  
}
```



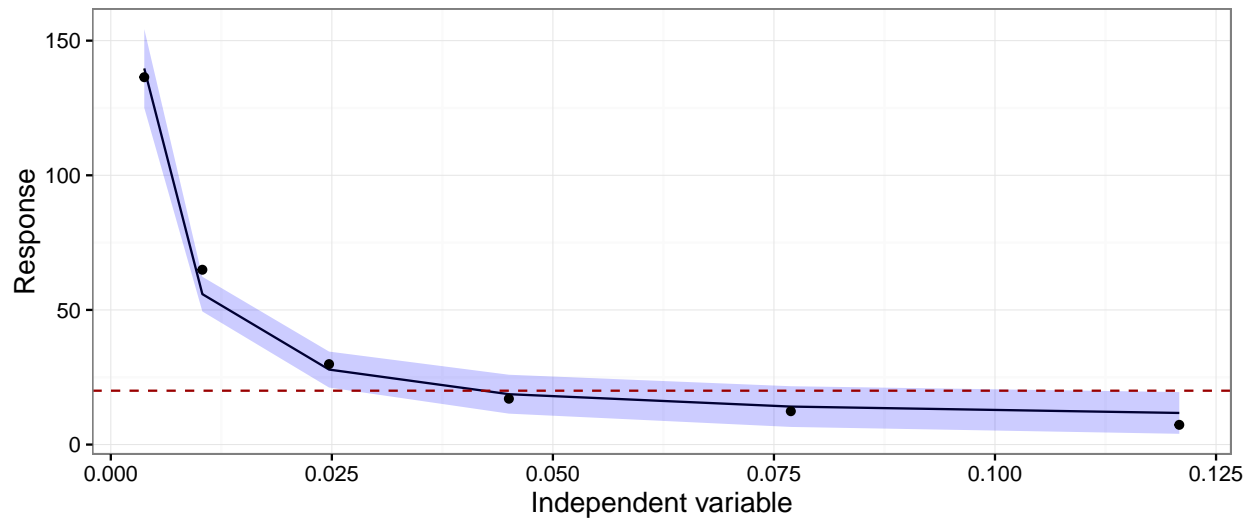
**Model for the curve # 2 Exponential  $Y=\exp(a+bX)$  ,**  
**Exploration of model fitting at response of 20 , 0.0609 and 95% CI: ( 0.0345 , 0.1044 )**  
**Residual sum of squares 4657.3113 , Residual standard deviation 0.4941**



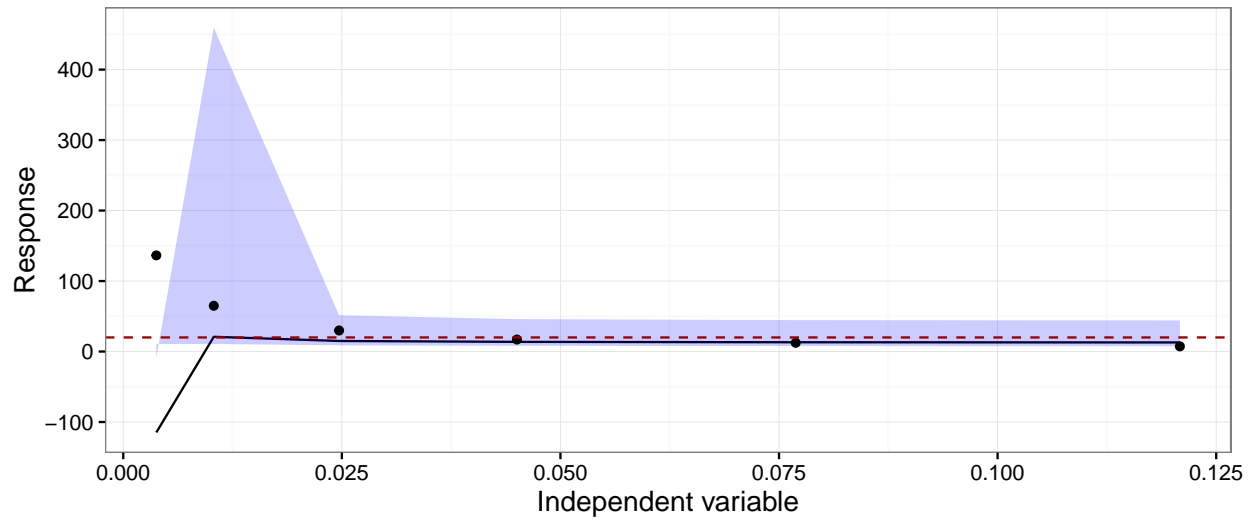
**Model for the curve # 3 Reciprocal- $Y=1/(a+bX)$  ,**  
**Exploration of model fitting at response of 20 , 0.0419 and 95% CI: ( 0.0369 , 0.0468 )**  
**Residual sum of squares 765.1815 , Residual standard deviation 0.0046**



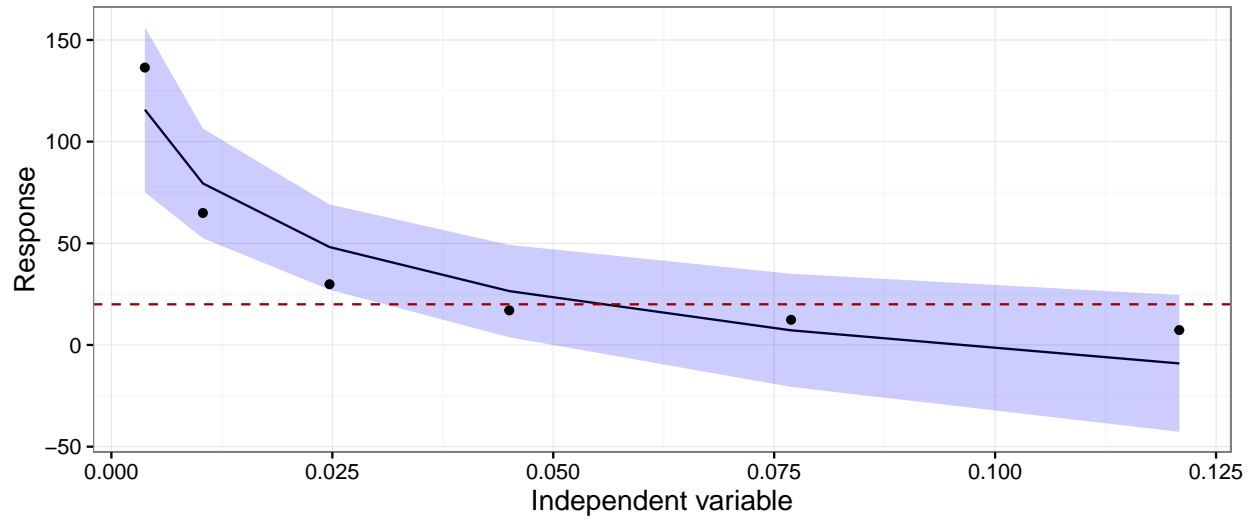
**Model for the curve # 4 Reciprocal- $X$   $Y=a+b/X$  ,**  
**Exploration of model fitting at response of 20 , 0.0403 and 95% CI: ( 0.0261 , 0.1067 )**  
**Residual sum of squares 121.8580 , Residual standard deviation 5.5195**



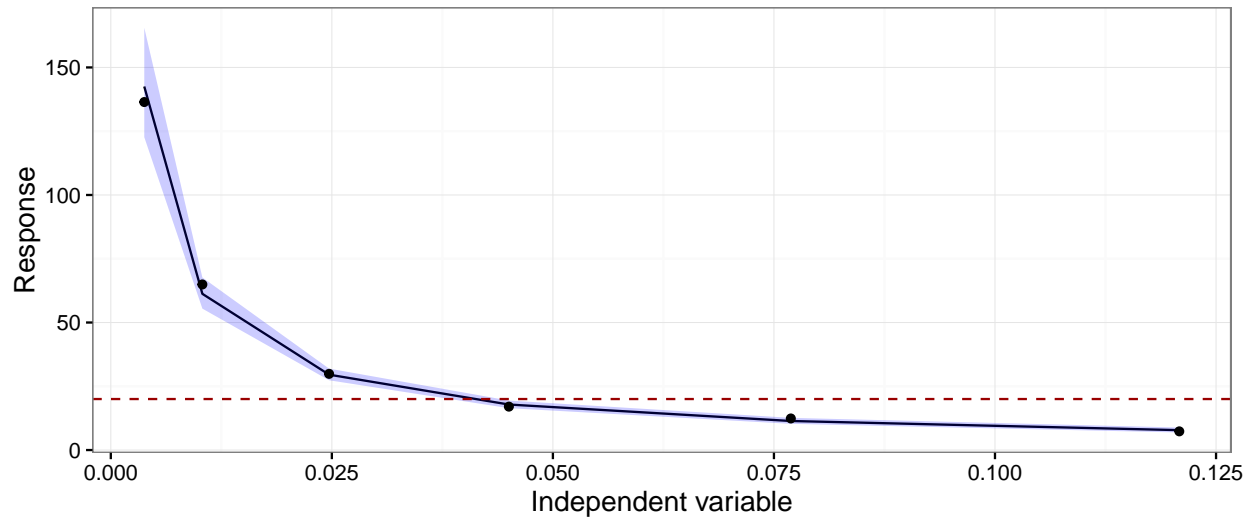
**Model for the curve # 5 Double Reciprocal  $Y=1/(a+b/X)$  ,**  
**Exploration of model fitting at response of 20 , 0.0111 and 95% CI: ( NA , NA )**  
**Residual sum of squares 65339.8134 , Residual standard deviation 0.0391**



**Model for the curve # 6 Logarithmic- $Y=a+b(\log(X))$  ,**  
**Exploration of model fitting at response of 20 , 0.0539 and 95% CI: ( 0.0301 , 0.1490 )**  
**Residual sum of squares 1359.8382 , Residual standard deviation 18.4380**

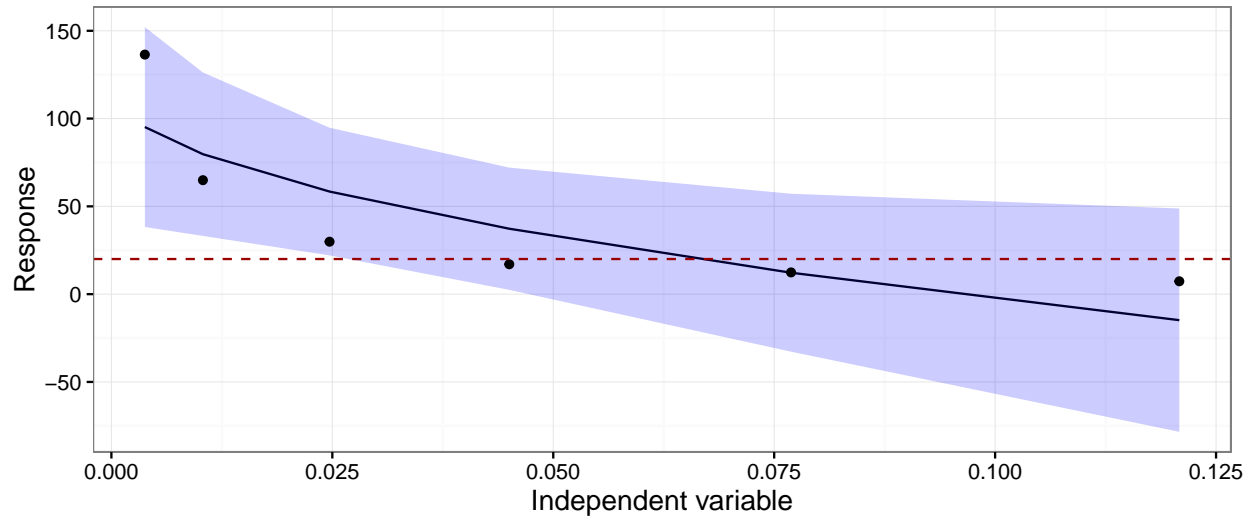


**Model for the curve # 7 Multiplicative  $Y=aX^b$  ,**  
**Exploration of model fitting at response of 20 , 0.0393 and 95% CI: ( 0.0358 , 0.0434 )**  
**Residual sum of squares 52.6646 , Residual standard deviation 0.0680**

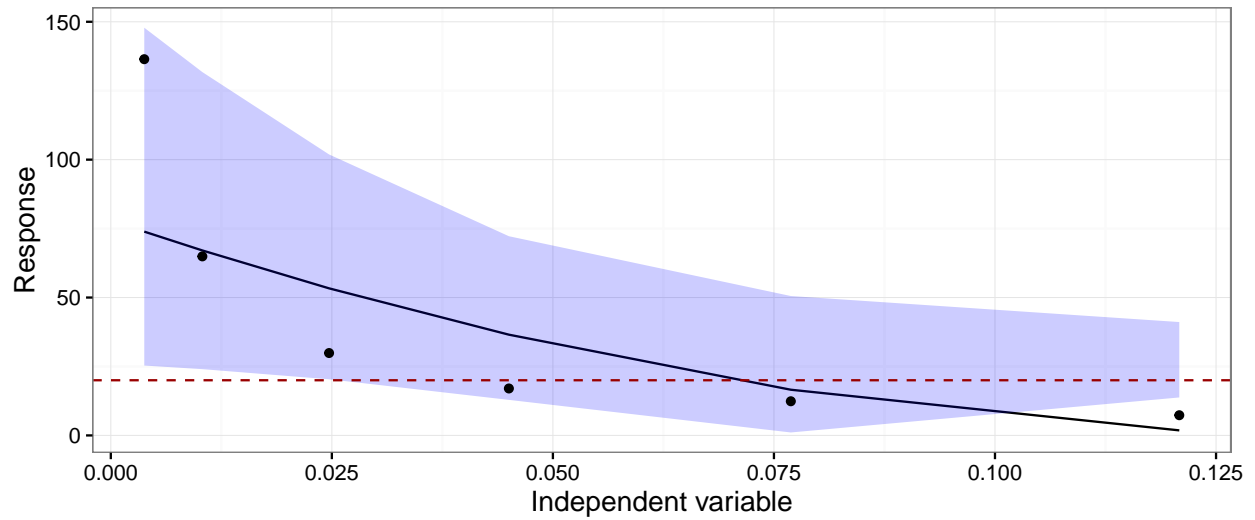




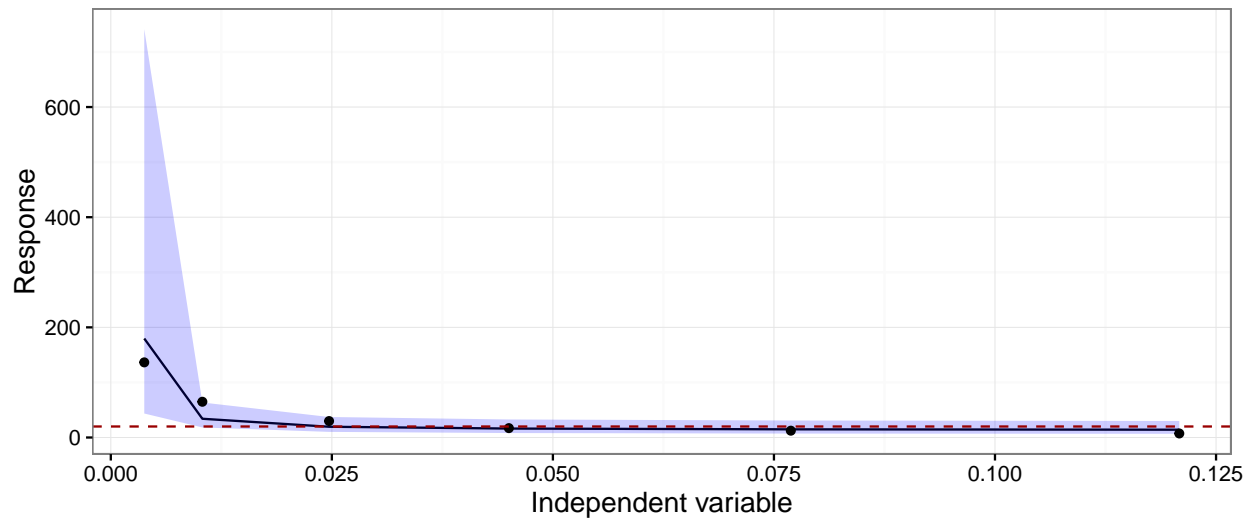
**Model for the curve # 8 Square Root- $X$   $Y=a+b(\sqrt{x})$  ,**  
**Exploration of model fitting at response of 20 , 0.0661 and 95% CI: ( 0.0271 , 0.8248 )**  
**Residual sum of squares 3635.4993 , Residual standard deviation 30.1476**



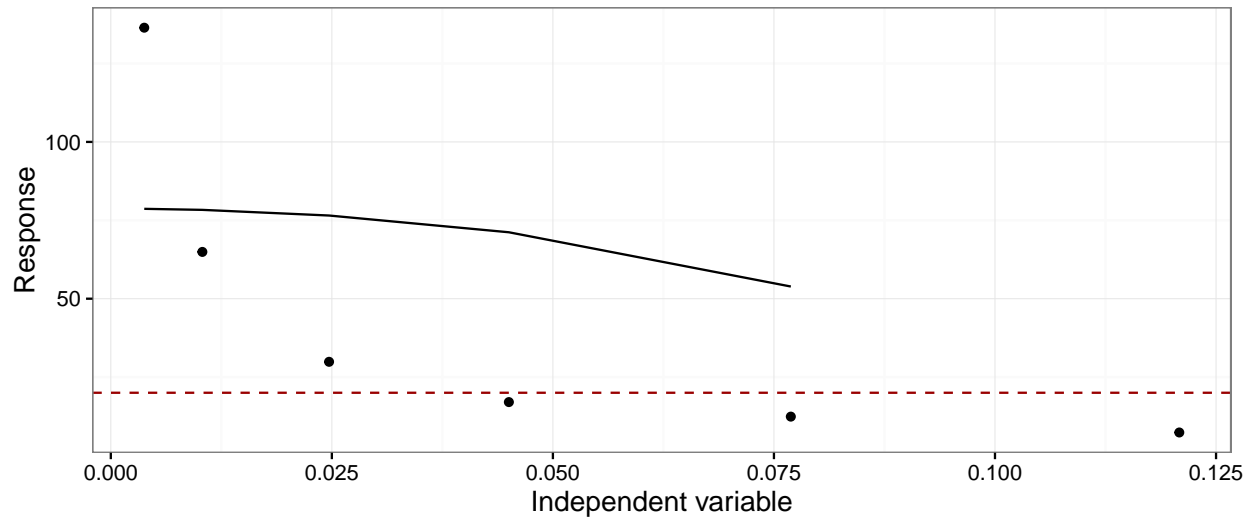
**Model for the curve # 9 Square Root- $Y$   $Y=(a+bX)^2$  ,**  
**Exploration of model fitting at response of 20 , 0.0704 and 95% CI: ( 0.0259 , 0.8071 )**  
**Residual sum of squares 4893.6728 , Residual standard deviation 2.1637**



**Model for the curve # 10 S-curve  $Y=\exp(a+b/X)$  ,**  
**Exploration of model fitting at response of 20 , 0.0233 and 95% CI: ( -0.0132 , 0.0092 )**  
**Residual sum of squares 2987.2487 , Residual standard deviation 0.5349**



**Model for the curve # 11 Square X and Y  $Y^2=a+X^2/b$  ,**  
**Exploration of model fitting at response of 20 , 0.1020 and 95% CI: ( NA , NA )**  
**Residual sum of squares 10349.7798 , Residual standard deviation 7366.8110**



## 5 Computing Environment

R version 3.2.2 (2015-08-14)

Platform: x86\_64-w64-mingw32/x64 (64-bit)

Running under: Windows 8 x64 (build 9200)

locale:

```
[1] LC_COLLATE=English_United Kingdom.1252
[2] LC_CTYPE=English_United Kingdom.1252
[3] LC_MONETARY=English_United Kingdom.1252
[4] LC_NUMERIC=C
[5] LC_TIME=English_United Kingdom.1252
```

attached base packages:

```
[1] stats      graphics  grDevices  utils      datasets  methods
[7] base
```

other attached packages:

```
[1] ggplot2_2.1.0 knitr_1.13
```

loaded via a namespace (and not attached):

```
[1] Rcpp_0.12.6      digest_0.6.9     plyr_1.8.4
[4] grid_3.2.2       gtable_0.2.0     formatR_1.4
[7] magrittr_1.5     evaluate_0.9     scales_0.4.0
[10] stringi_1.1.1    rmarkdown_1.0    labeling_0.3
[13] tools_3.2.2      stringr_1.0.0    munsell_0.4.3
[16] yaml_2.1.13      colorspace_1.2-6 htmltools_0.3.5
```

```
[1] "~/X/DEVELOPMENT/FUNCTIONAL SENSITIVITY MODEL SELECTION"
```

This took 8.64 seconds to execute.