Exploration of power and treatment effect precision using an adjusted analysis for a randomised controlled trial (Bayesian & frequentist)

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1 Introduction

Many randomised controlled trials (RCTs) are analysed in a simple manner using only the randomised treatment as the independent variable. But when the response outcome is continuous, precision of the treatment effect estimate is improved when adjusting for baseline covariates in a randomised controlled trial. We do not expect covariates to be related to the treatment assignment because of randomisation, but they may be related to the outcome, they are therefore not considered to be confounding. However, differences between the outcome which can be attributed to differences in the covariates can be removed, this results in a more precise estimate of treatment effect. This should be considered more often as sample sizes can be reduced. This is not the case for logistic models (principle of the non collapsibility of the odds ratio).

Here, we perform an investigation of adjusted and unadjusted power in the setting of a RCT. We then choose a sample size and simulate a single RCT. The RCT is then analysed using a frequentist approach and a Bayesian approach.

2 Prepare for stochastic simulations

```
alpha <- 0.05
n.sims <- 200  # simulations
N <- seq(from=100, to=1000, by=50)  # trial sizes to explore
pow.adj1 <- pow.adj2 <- pow <- rep(NA, length(N)) # object to capture summary power estimates
bias <- array(NA, dim=c(length(N),6))  # object to capture summary coeff estimates</pre>
```

3 Function to investigate power for a RCT contingent on the analyses perfored (adjusted and unadjusted)

```
for (j in 1:length(N)){
   n <- N[j]

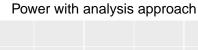
#objects to collect simulations for each sample size before summary stats are calculated
   significant.unadjusted <- significant.adj1 <- significant.adj2 <- rep(NA, n.sims)
   est <- array(NA, dim=c(n.sims,6))

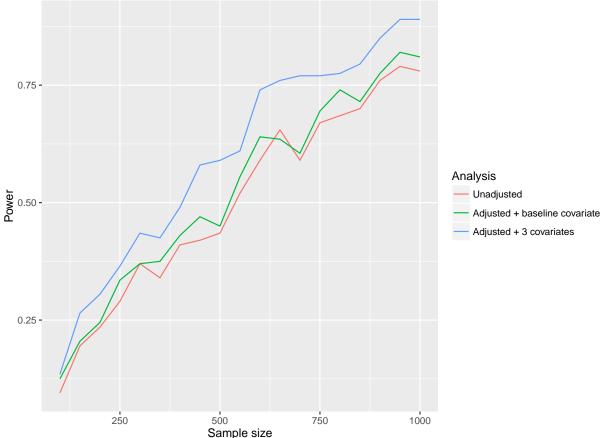
for (i in 1: n.sims){</pre>
```

```
# Hyp the effect of age on outcome, a unit change in outcome over the whole age range,
   # so ~0.02 effect on outcome for each increase in age of 1 year
   effect.of.age <-1/(65-18)
   effect.of.sex <- 5
                                       # Hyp the effect of sex on outcome
   effect.of.treatment <- 1
                                     # Hyp the treatment effect on outcome
   # Hypothesize baseline version of response on outcome,
   # a unit increase results in 0.5 increase in outcome,
   # strong predictor so 5 over the baseline range
   effect.of.baseline <- 1/2
   intercept <- 100</pre>
   noise <- 5
                                       # random error
   #collect the true parmaeters to see how well we recover them
   covp <- c(intercept, effect.of.baseline, effect.of.sex,</pre>
             effect.of.age, effect.of.treatment, effect.of.treatment)
# covariate effects
baseline \leftarrow runif(n,100,110)
                                                  # baseline effect
treat \leftarrow 1*(runif(n)<0.5)
                                                  # randomised treatment effect
sex <- c(rep("F", n/2), rep("M", n/2))
                                                 # Generate sex covariate
age <- sample(x=18:65, size=n, replace=TRUE) # Generate age covariate
# create the outcome, with random error
Y \leftarrow effect.of.sex*(sex=="M") + effect.of.age*age + effect.of.baseline*baseline +
  effect.of.treatment*treat + rnorm(n, intercept, noise)
# prepare for analysis
d <- data.frame(Y=Y, baseline=baseline, treat=treat, sex=sex, age=age)</pre>
dd <- datadist(d, data=d); options(datadist="dd")</pre>
# Unadjusted analysis, extract p-value treatment effect
f0 <- ols(Y ~ treat, d)</pre>
fa <- anova(f0)</pre>
p.value <- as.matrix(fa)[1,5]</pre>
# Adjusted analysis, extract p-value treatment effect adjusted for 1 covariates
fa <- anova( ols(Y ~ baseline + treat, d))</pre>
p.value.adj1 <- as.matrix(fa)[2,5]</pre>
# Adjusted analysis, extract p-value treatment effect adjusted for 3 covariates
f <- ols(Y ~ baseline + sex + age + treat, d)</pre>
fa <- anova(f)
p.value.adj2 <- as.matrix(fa)[4,5]</pre>
# collect p values
significant.unadjusted[i] <- p.value <= alpha</pre>
significant.adj1[i] <- p.value.adj1 <= alpha
significant.adj2[i] <- p.value.adj2 <= alpha</pre>
#get the simulations coefficient estimates for sample size
est[i,]<- c(f$coefficients , f0$coefficients[2][[1]])</pre>
```

Plot the power with sample size relationships

```
df <- data.frame(N, pow, pow.adj1, pow.adj2)</pre>
df2 <- melt(data = df, id.vars = "N") # melt the data to a long format and plot
g <- ggplot(data = df2, aes(x = N, y = value, color= factor(variable,
labels = c("Unadjusted", "Adjusted + baseline covariate", "Adjusted + 3 covariates")))) +
geom_line()
g <- g + ylab("Power") + xlab("Sample size") + labs(color = "Analysis") +
    ggtitle("Power with analysis approach")
print(g)
```





5 Plot the percent relative bias with sample size relationships

% Relative Bias in coefficients

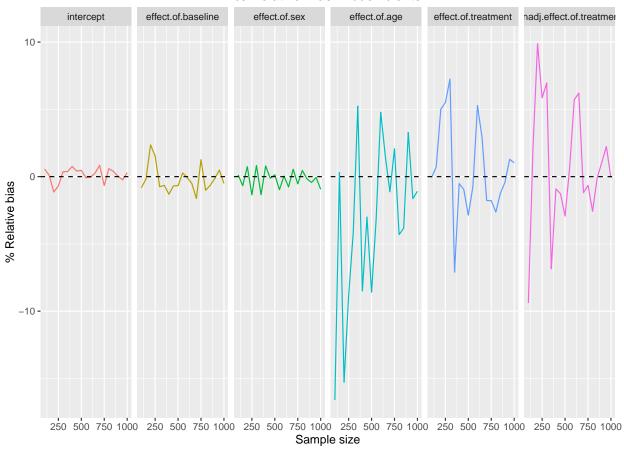


Figure 1: Profiles of percentage relative bias with sample size

Show the % relative bias with sample size relationships

```
x.c <- bias # format all numerical variables
x.c[] <- lapply(x.c, function(.col){</pre>
       if (is.numeric(.col)) return(sprintf("%.1f", .col))
       else return(.col)
   })
print(x.c)
```

```
intercept effect.of.baseline effect.of.sex effect.of.age effect.of.treatment unadj.effect.of.treatment
1
         0.6
                           -0.8
                                           0.1
                                                       -16.6
                                                                             -0.1
                                          -0.7
2
         0.1
                           -0.2
                                                          0.3
                                                                              0.7
3
        -1.1
                             2.4
                                           0.7
                                                        -15.3
                                                                              5.0
4
        -0.7
                                                        -8.9
                                                                              5.5
                            1.5
                                          -1.4
5
         0.4
                           -0.8
                                           0.8
                                                        -4.2
                                                                              7.3
6
         0.4
                                                         5.2
                                                                             -7.1
                           -0.6
                                          -1.3
7
         0.8
                           -1.3
                                           0.8
                                                         -8.5
                                                                             -0.5
         0.4
                           -0.7
                                          -0.1
                                                        -3.0
                                                                             -0.9
8
9
         0.5
                           -0.7
                                           0.1
                                                        -8.6
                                                                             -2.9
                                                         -3.2
10
        -0.1
                            0.3
                                          -1.0
                                                                             -0.7
11
        -0.0
                           -0.1
                                           0.0
                                                         4.8
                                                                              5.3
12
         0.3
                           -0.5
                                          -0.8
                                                         1.5
                                                                              2.9
13
         0.8
                           -1.6
                                           0.5
                                                         -1.1
                                                                             -1.8
14
        -0.7
                            1.3
                                          -0.5
                                                         2.1
                                                                             -1.8
15
         0.6
                                           0.5
                                                                             -2.6
                           -1.0
                                                        -4.3
16
         0.4
                           -0.7
                                          -0.2
                                                         -3.8
                                                                             -1.2
17
         0.0
                           -0.1
                                          -0.4
                                                         3.3
                                                                             -0.4
18
        -0.2
                            0.5
                                          -0.1
                                                         -1.6
                                                                              1.3
19
         0.3
                           -0.5
                                          -0.9
                                                        -1.1
                                                                              1.0
```

2

9

5

-6

-0

-1

-2

0

5

-1

-0

-2

-0

1

2

-0

Based on power estimates of adjusted analyses a trial of 800 patients takes place

```
set.seed(87564)
n <- 800
                                    # sample size
effect.of.age \leftarrow 1/(65-18)
                                    # Hypothesize the effect of age on outcome
effect.of.sex <- 5
                                    # Hypothesize the effect of sex on outcome
effect.of.treatment <- 1</pre>
                                    # Hypothesize the treatment effect on outcome
effect.of.baseline <- 1/2
                                    # Hypothesize the baseline effect on outcome
intercept <- 100
noise <-5
                                    # the standard deviation for the simulated data
baseline \leftarrow runif(n, 100, 110)
                                                   # Generate baseline covariate
treat <- 1*(runif(n)<0.5)
                                                   # randomised treatment effect
sex \leftarrow c(rep("F", n/2), rep("M", n/2))
                                                  # Generate sex covariate
age <- sample(x=18:65, size=n, replace=TRUE)
                                                  # Generate age covariate
```

```
# Create the response outcome, with random error
       Y <- effect.of.sex*(sex=="M") + effect.of.age*age + effect.of.baseline*baseline +
       effect.of.treatment*treat + rnorm(n, intercept, noise)
       #~~~~~~~
       # alternative data creation
       dat <- data.frame( baseline=baseline, treat=treat, sex=sex, age=age)</pre>
       X <- model.matrix(~baseline + treat + sex + age, dat)</pre>
       #the regression slopes
       betas <-c(intercept, effect.of.baseline, effect.of.treatment, effect.of.sex, effect.of.age)
       # the simulated data
       y_norm <- rnorm(800, X%*%betas, noise)</pre>
       #~~~~~~~~
       lm( Y ~ baseline + sex + age + treat)
Call:
lm(formula = Y ~ baseline + sex + age + treat)
Coefficients:
(Intercept)
                baseline
                                 sexM
                                                           treat
                                               age
 96.326964
                0.535192
                             5.183220
                                          0.009781
                                                        1.706045
      lm( y_norm ~ baseline + sex + age + treat)
Call:
lm(formula = y_norm ~ baseline + sex + age + treat)
Coefficients:
(Intercept)
                baseline
                                 sexM
                                               age
                                                          treat
   94.1915
                  0.5508
                               5.8003
                                            0.0232
                                                          1.1279
```

8 Create a data frame

```
d <- data.frame(Y=Y, baseline=baseline, treat=treat, sex=sex, age= age )
dd <- datadist(d, data=d); options(datadist="dd")</pre>
```

9 Bayesian analysis using STAN, the model statement

```
d$sex2 <- as.numeric(ifelse(d$sex=="M", 1,0) )</pre>
 m0 <- map( alist( Y ~ dnorm(mu, sigma),
                  mu \leftarrow b0 + b4*treat,
                  b0 ~ dnorm(0, 100),
                   b4 \sim dnorm(0, 10),
                   sigma ~ dunif(0, 10)),
           data=d )
m1 <- map( alist( Y ~ dnorm(mu, sigma),
                  mu <- b0 + b1*baseline + b2*sex2 + b3*age +
                  b4*treat,
                  b0 ~ dnorm(0, 100),
                  c(b1, b2, b3, b4) ~ dnorm(0, 10),
                  sigma ~ dunif(0, 10)),
          data=d )
# construct centered predictors
# d$baseline <- d$baseline - mean(d$baseline)
# d$age <- d$age - mean(d$age)
     m1 <- map2stan( alist( Y ~ dnorm(mu, sigma),</pre>
                  mu <- b0 + b1*baseline + b2*sex2 + b3*age +
                  b4*treat,
                  b0 ~ dnorm(0, 100),
                  c(b1, b2, b3, b4) \sim dnorm(0, 10),
                  sigma ~ dunif(0, 10) ),
          data=d , chains=3, iter=5000 , warmup=1000 )
```

10 Bayesian analysis using STAN, frequentist and Bayesian results together

```
Mean StdDev
                    2.5% 97.5%
b0
     155.57
             0.29 155.00 156.14
b4
       1.61
              0.42
                    0.80
                           2.42
sigma
       5.88
             0.15
                    5.59
                           6.17
      Mean StdDev lower 0.95 upper 0.95 n_eff Rhat
b0
     95.90
             6.64
                      83.14
                               109.16 4212
      0.54
            0.06
                      0.42
                                 0.66 4244
b1
b2
      5.18 0.35
                      4.49
                                 5.88 8222
b3
      0.01 0.01
                      -0.01
                                 0.04 10026
      1.70 0.35
                      1.02
                                 2.40 8191
sigma 5.05 0.13
                       4.82
                                 5.31 7900
```

11 Model assumption check

```
pairs(m1)
```

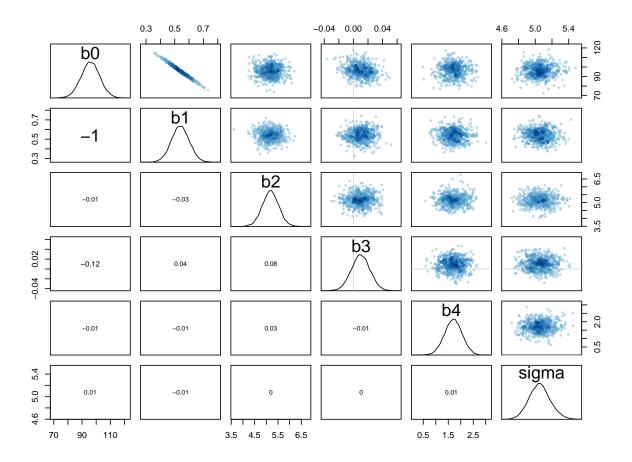
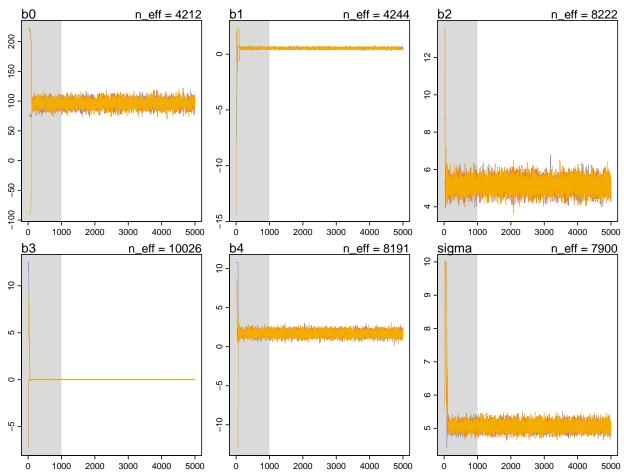


Figure 2: Check chains are mixing

```
post <- extract.samples(m1)
# lapply(post, densityplot)
# par(mfrow=c(3,3))
# for ( i in 1:6 ) { plot(post[i] , type='l', main=names(post)[i]) }
# par(mfrow=c(1,1))

plot(m1)</pre>
```



Plot the predicted effects, counterfactual samples of 100 patients are also simulated and plotted.

Treatment effect

```
nn <- 100 # just plot this number of patient rather than 800
post0 <- extract.samples(m0)</pre>
namez <- "Treatment"</pre>
x \leftarrow c(rep(0,nn/2), rep(1, nn/2))
pred.data1 <- data.frame(</pre>
                                                         # Make predictions for this dataset
     treat= x
# compute counterfactual mean response (mu1)
mu1 <- link( m0 , data=pred.data1 )</pre>
mu.mean \leftarrow apply(mu1, 2, mean)
mu.PI <- apply( mu1 , 2 , PI )</pre>
# simulate counterfactual response outcomes
R.sim1 <- sim(m0), data=pred.data1, n=1e4) # n < posterior samples or error?
R.PI <- apply( R.sim1 , 2 , PI )</pre>
# Need Y to be the same length as x, these values are not plotted
Y <- seq(from=min( R.sim1, na.rm=T), max(R.sim1, na.rm=T), length.out=nn )
```

```
par(mfrow=c(1,2))
# mean and response outcome plot
plot(Y ~ x , type="n", xlab=paste(namez), ylab="Response", xlim=c(-0.2,1.2),
ylim=c(min( R.sim1, na.rm=T)*1, max( R.sim1, na.rm=T)*1), xaxt = "n") # get appropriate
axis(side = 1, at = c(0,1), labels = c(0,1), tck=-.05) # better ticks on x axis
# plot this info
lines( x , mu.mean )
                                                            # plot this info
shade( mu.PI , x , col= col.alpha("blue", 0.15))
shade( R.PI , x , col= col.alpha("purple", 0.15))
points(R.sim1[1,] ~ jitter(x)) # pick one of the many simulations to plot
# mean effect
plot(Y ~ x, type="n", xlab=paste(namez), ylab="Response", ylim=c(155,158), xaxt = "n")
axis(side = 1, at = c(0,1), labels = c(0,1), tck=-.05) # better axes
lines( x , mu.mean )
shade( mu.PI , x , col= col.alpha("blue", 0.15))
grid(nx = NULL, ny = NULL)
mtext(line=-3, text= "Unadjusted treatment effect estimate", outer=T, cex=.7)
# title
foo<-post0$b4
                                # collect the appropriate mcmc samples
mtext( line= -2,
text=paste(namez, "mean effect ", p2(mean(foo)), ", SD=", p3(sd(foo)), ", 95% CI (",
p2(quantile(foo, c(.025))), ", ", p2(quantile(foo, c(0.975))),"), p>0 = ",
mean(foo>0), sep="") ,outer = TRUE, cex = 1)
```

12 Adjusted analysis

```
nn<-100 #* just plot this number of patient rather than 800
par(mfrow=c(1,1))
P.baseline <- median(d$baseline)</pre>
                                                        # Hold baseline
P.treat \leftarrow c(rep(0,nn/2), rep(1, nn/2))
                                                        # vary
P.age <- median(d$age)
                                                        # Hold age at median
P.sex <- 0
                                                        # Hold sex at female 0
namez <- "Treatment"</pre>
x <- P.treat # define the variable here, so less typing later perhaps
pred.data1 <- data.frame(</pre>
                                                        # Make predictions for this dataset
 baseline=P.baseline,
  sex2=P.sex,
  age=P.age,
  treat=x
# compute counterfactual mean response (mu1)
```

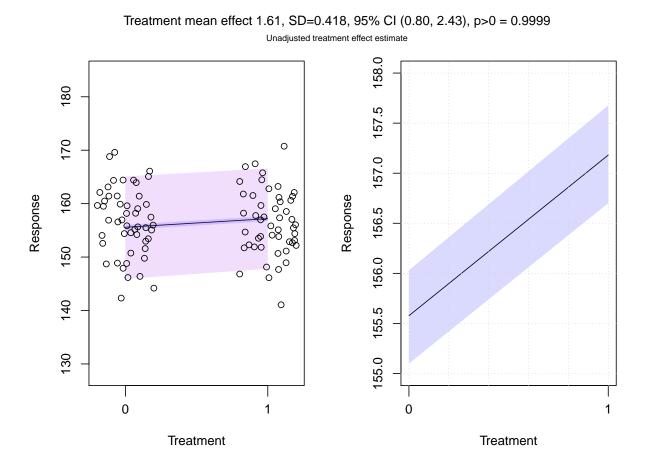


Figure 3: Effects of treatment on outcome, simulated response outcomes left panel, mean response right panel

```
mu1 <- link( m1 , data=pred.data1 )</pre>
mu.mean <- apply( mu1 , 2 , mean )</pre>
mu.PI <- apply( mu1 , 2 , PI )</pre>
# simulate counterfactual response outcomes
R.sim1 \leftarrow sim(m1, data=pred.data1, n=1e4)
R.PI <- apply( R.sim1 , 2 , PI )
\# Need Y to be the same length as x, these values are not plotted
Y <- seq(from=min( R.sim1, na.rm=T), max(R.sim1, na.rm=T), length.out=nn )
par(mfrow=c(1,2))
# mean and response outcome plot
plot( Y ~ x , type="n", xlab=paste(namez), ylab="Response" , xlim=c(-0.2,1.2 ),
ylim=c(min(R.sim1, na.rm=T)*1, max(R.sim1, na.rm=T)*1), xaxt = "n") # get appropriat
axis(side = 1, at = c(0,1), labels = c(0,1), tck=-.05) # better ticks on x axis
# plot this info
lines( x , mu.mean )
                                                             # plot this info
shade( mu.PI , x , col= col.alpha("blue", 0.15))
shade( R.PI , x , col= col.alpha("purple", 0.15))
points(R.sim1[1,] ~ jitter(x)) # pick one of the many simulations to plot
# mean effect
plot(Y ~ x, type="n", xlab=paste(namez), ylab="Response", ylim=c(152,156), xaxt = "n")
axis(side = 1, at = c(0,1), labels = c(0,1), tck=-.05) # better axes
lines( x , mu.mean )
shade( mu.PI , x , col= col.alpha("blue", 0.15))
grid(nx = NULL, ny = NULL)
mtext( line=-3, text=paste(namez ,"effect adjusted to age",
P.age ,"; baseline", p2(P.baseline),"; sex coded", P.sex, sep=" "), outer=T, cex=.7)
# title
foo<-post$b4
                                # collect the appropriate mcmc samples
mtext( line= -2,
text=paste(namez ," mean effect ", p2(mean(foo , na.rm=T)),", SD=", p3(sd(foo,na.rm=T)) ,", 95%
p2(quantile(foo, c(.025), na.rm=T)), ", ", p2(quantile(foo, c(0.975), na.rm=T)),"), p>0 = ",
mean(foo>0), sep="")
,outer = TRUE, cex = 1)
```

Treatment effect adjusted to age 41; baseline 105.03; sex coded 0 Response Response 0 0 o 9 Treatment Treatment

Treatment mean effect 1.70, SD=0.352, 95% CI (1.01, 2.39), p>0=1

Figure 4: Effects of treatment on outcome, simulated response outcomes left panel, mean response right panel

Age in years effect adjusted to treatment 0 ; baseline 105.03 ; sex coded 0 $\,$ Response Response

Figure 5: Effects of age on outcome, simulated response outcomes left panel, mean response right panel

Age in years

Age in years

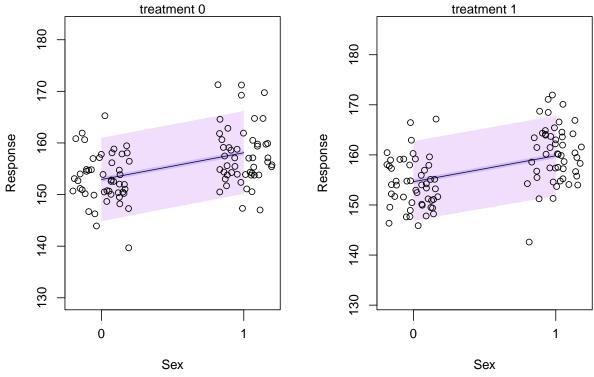
Baseline effect adjusted to treatment 0; age 41; sex coded 0Response Response Baseline Baseline

Baseline mean effect 0.54, 95% CI (0.42, 0.66), p>0=1

Figure 6: Effects of baseline on outcome, simulated response outcomes left panel, mean response right panel

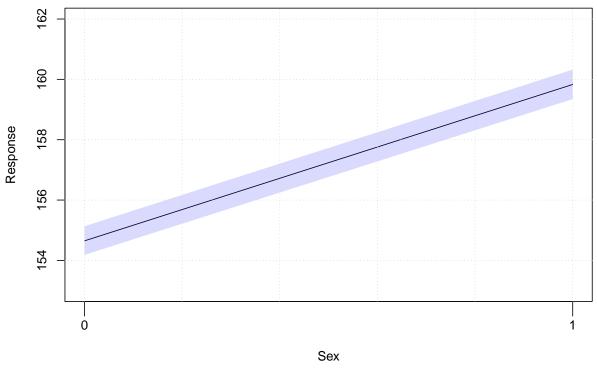
- 13 For completion the effects of the other covariates on outcome; age
- 14 Baseline effect on outcome
- 15 Effect on outcome; sex. There is no real need to show treatment as well, as no interaction was modelled.

Sex mean effect 5.18, 95% CI (4.49, 5.88), p>0=1



Sex effect adjusted to age 41; sex coded 0; baseline 105.0275

Sex mean effect 5.18, 95% CI (4.49, 5.88), p>0 = 1



Sex effect adjusted to age 41; sex coded 0; baseline 105.0275

16 Frequentist analysis, print the regression table (also print Bayesian)

```
precis(m1)
       Mean StdDev lower 0.89 upper 0.89 n_eff Rhat
      95.90
                        85.31
                                  106.44 4212
b0
              6.64
b1
       0.54
              0.06
                         0.44
                                    0.64 4244
                                                  1
b2
       5.18
              0.35
                         4.60
                                    5.72 8222
b3
       0.01
              0.01
                        -0.01
                                    0.03 10026
                                                  1
b4
       1.70
              0.35
                         1.16
                                    2.28 8191
                                                  1
sigma 5.05
              0.13
                         4.85
                                    5.26 7900
       f<- ols( Y ~ baseline + sex + age + treat , d, x=TRUE, y=TRUE)
       print(f)
```

Linear Regression Model

```
ols(formula = Y \sim baseline + sex + age + treat, data = d, x = TRUE,
   y = TRUE)
                Model Likelihood
                                     Discrimination
                   Ratio Test
                                        Indexes
         800
                LR chi2
                           263.60
                                              0.281
                                     R2 adj
sigma 5.0481
                d.f.
                                              0.277
         795
               Pr(> chi2) 0.0000
                                              3.626
                                     g
Residuals
               1Q
                  Median
                                 3Q
                                         Max
-15.8808 -3.3450 -0.1489
                             3.4544 15.8508
          Coef
                  S.E.
                               Pr(>|t|)
                       t
Intercept 96.3270 6.5606 14.68 < 0.0001
           0.5352 0.0621 8.62 < 0.0001
sex=M
           5.1832 0.3576 14.49 < 0.0001
           0.0098 0.0128 0.76 0.4458
age
           1.7060 0.3572 4.78 < 0.0001
treat
```

17 Compute analysis of variance table for the fitted model object with χ^2 statistics.

```
an <- anova(f, main.effect=FALSE, test=c('Chisq')) # try anova(f, test=c('F') ) # F tests
print(an, 'subscripts')</pre>
```

Response: Y

```
d.f. Partial SS MS
                                      Chi-Square P
                1891.44228 1891.44228 74.22
baseline
                                                 <.0001 1
sex
                5353.62495 5353.62495 210.08
                                                 <.0001 2
                             14.83008 0.58
                                                 0.4455 3
                  14.83008
age
            1
                 581.34701 581.34701 22.81
                                                 <.0001 4
treat
            1
                7906.66760 1976.66690 310.27
                                                 <.0001 1-4
REGRESSION
            4
          795 20259.11554
                             25.48316
ERROR
```

Subscripts correspond to:

[1] baseline sex=M age treat

Wald Statistics

18 Plot the partial effect of each variable.

```
par(mfrow=c(3,1))
plot(an,
what=c("proportion chisq"),
xlab=NULL, pch=16,
rm.totals=TRUE, rm.ia=FALSE, rm.other=NULL,
sort=c("descending"), margin=NULL, pl=TRUE,
trans=NULL, ntrans=40 )

plot(an,
what=c("chisqminusdf"))

plot(an,
what=c('proportion R2'))
```

19 Compute predicted values and confidence limits

```
plot(Predict(f), anova=an, pval=TRUE)
```

The key result, computed predicted values and confidence limits for the treatment effect on outcome. Notice the larger SE for the unadjusted analysis.

```
summary(f, treat=c(0,1), est.all=FALSE)

Effects Response : Y

Factor Low High Diff. Effect S.E. Lower 0.95 Upper 0.95
treat 0 1 1 1.706 0.35719 1.0049 2.4072

ggplot(Predict(f, treat), anova=an, pval=TRUE)
```

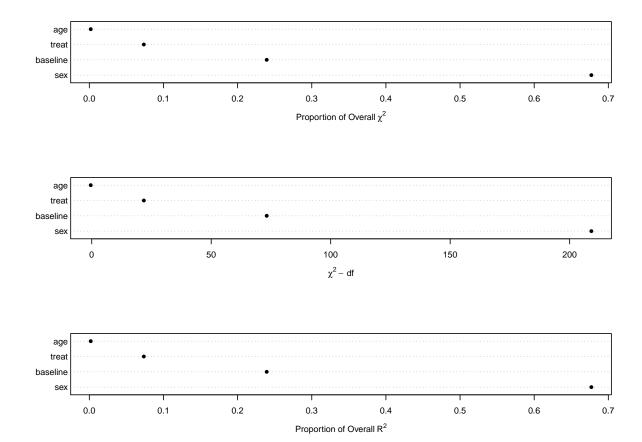


Figure 7: The default for plot(anova()) is to display the Wald Chi2 statistic minus its degrees of freedom for assessing the partial effect of each variable. Even though this is not scaled [0,1] it is probably the best method in general because it penalizes a variable requiring a large number of parameters to achieve the Chi2. If a predictor interacts with any other predictor(s), the Chi2 and partial R2 measures combine the appropriate interaction effects with main effects. For example if the model was $y \sim$ biomarker * treatment the statistic for treatment is the combined effects of treatment as a main effect plus the effect modification that treatment provides for the biomarker effect. This is an assessment of whether there is a difference between the treatment for any biomarker score.

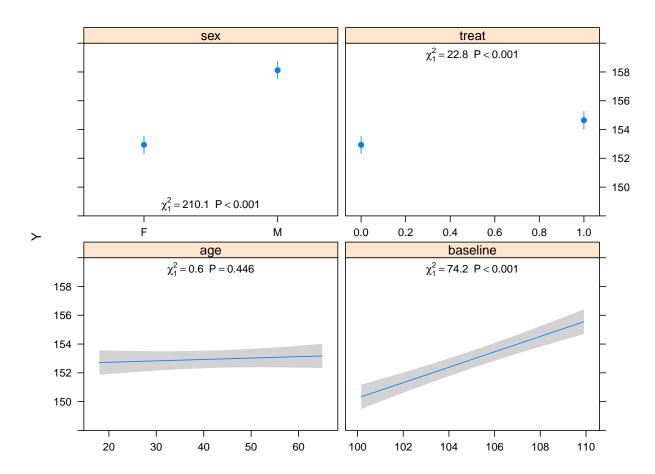
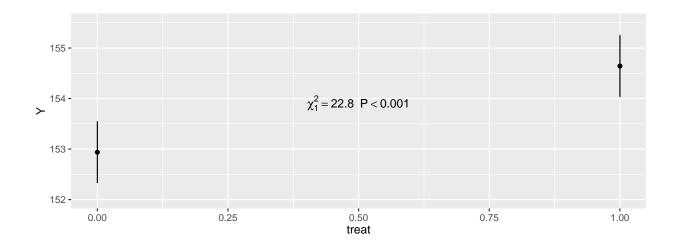


Figure 8: Show predicted values and confidence bands



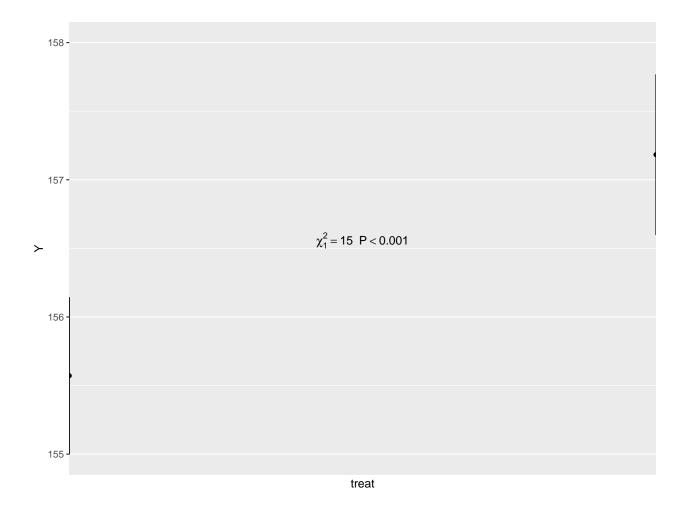
Adjusted to:baseline=105 sex=F age=41

```
#univariate
f0 <- ols( Y ~ treat , d, x=TRUE, y=TRUE)
summary(f0, treat=c(0,1))</pre>
```

Effects Response : Y

Factor Low High Diff. Effect S.E. Lower 0.95 Upper 0.95 treat 0 1 1 1.6114 0.41636 0.79413 2.4287

```
an0 <- anova(f0, main.effect=FALSE, test=c('Chisq'))
ggplot(Predict(f0), anova=an0, pval=TRUE)</pre>
```



21 Plot the other covariate effects for information. Sex main effect on outcome.

```
summary(f, sex=c("M","F"), est.all=FALSE)

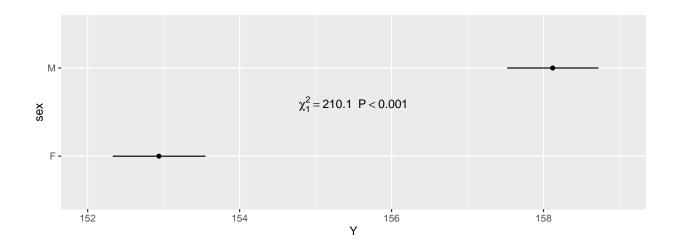
Effects Response : Y

Factor Low High Diff. Effect S.E. Lower 0.95 Upper 0.95
sex - M:F 1 2 NA 5.1832 0.3576 4.4813 5.8852

ggplot(Predict(f, sex), anova=an, pval=TRUE)
```

22 Show age main effect on outcome.

```
summary(f, age=c(18,65), est.all=FALSE)
```



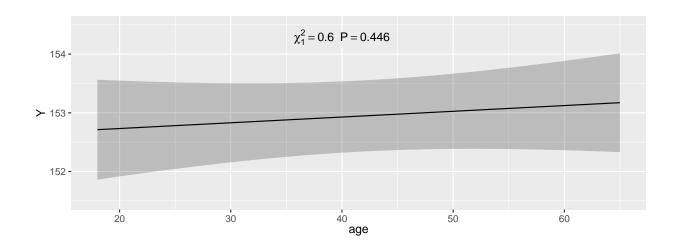
Adjusted to:baseline=105 age=41 treat=0

Figure 9: Sex main effect on outcome

Effects Response : Y

Factor Low High Diff. Effect S.E. Lower 0.95 Upper 0.95 age 18 65 47 0.45971 0.60262 -0.7232 1.6426

ggplot(Predict(f, age), anova=an, pval=TRUE)



Adjusted to:baseline=105 sex=F treat=0

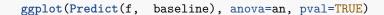
Figure 10: Age main effect on outcome

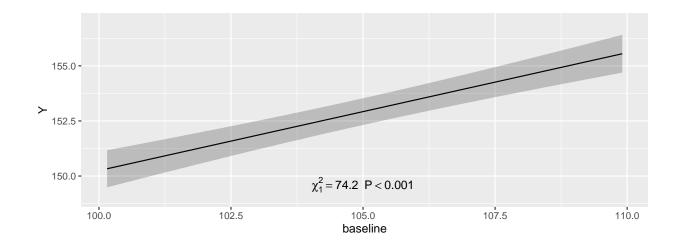
23 Show baseline main effect on outcome.

```
summary(f, baseline=c(-1,1), est.all=FALSE)
```

Effects Response : Y

Factor Low High Diff. Effect S.E. Lower 0.95 Upper 0.95 baseline -1 1 2 1.0704 0.12424 0.8265 1.3143





Adjusted to:sex=F age=41 treat=0

Figure 11: Baseline main effect on outcome

24 Reproduce the regression table using the summary function

```
summary(f, baseline=c(0,1), sex=c("M","F"), age=c(0,1), treat=c(0,1))
            Effects
                                 Response : Y
          Low High Diff. Effect
                                           Lower 0.95 Upper 0.95
Factor
                                  S.E.
baseline
                         0.5351900 0.062121 0.413250 0.65713
age
              1
                    1
                        0.0097812\ 0.012822\ -0.015387\ 0.03495
treat
          0
             1
                   1
                        1.7060000 0.357190 1.004900 2.40720
sex - M:F 1
                        5.1832000 0.357600 4.481300 5.88520
                  NA
```

25 The contrast function, match the regression table for treatment effect

The contrast function, treatment effect when covariates held at median, however this does not matter as there is no interaction in the model

```
contrast(f,list(treat=1,
                      age=median(age), baseline=median(baseline), sex="F"),
                list(treat=0,
                     age=median(age), baseline=median(baseline), sex="F"))
  age baseline sex Contrast
                              S.E.
                                      Lower
                                                Upper
                                                        t Pr(>|t|)
1 41 105.0275 F 1.706045 0.35719 1.004898 2.407192 4.78
Error d.f.= 795
Confidence intervals are 0.95 individual intervals
         contrast(f, list(treat=1),list(treat=0))
  baseline sex age Contrast
                              S.E.
                                                Upper
                                                        t Pr(>|t|)
                                      Lower
           F 41 1.706045 0.35719 1.004898 2.407192 4.78
Error d.f.= 795
Confidence intervals are 0.95 individual intervals
```

27 Assumption check

```
r <- residuals(f)
plot(r); abline(h=0)</pre>
```

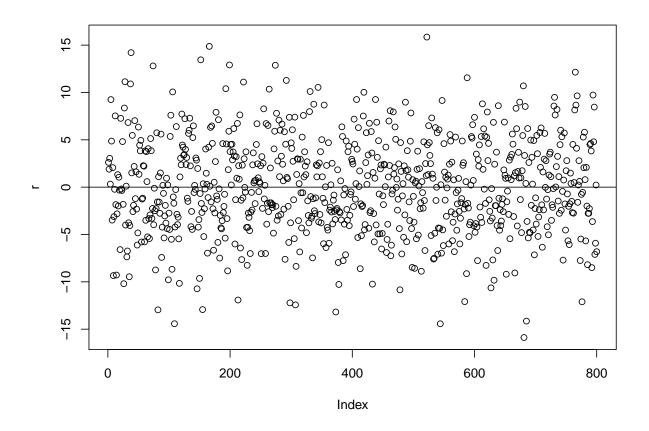


Figure 12: Check OLS assumptions



Normal Q-Q Plot

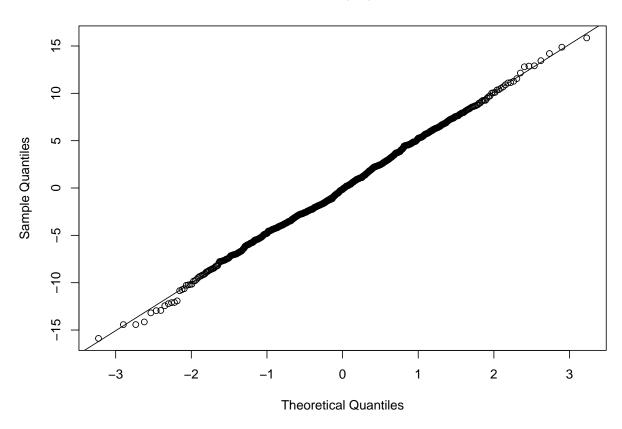


Figure 13: Check OLS assumptions

```
# resid(f, "dfbetas")
# which.influence(f)
```

28 References

Statistical Rethinking 1st Edition, Richard McElearth post treatment bias p151, WAIC p199 and simulate p129

http://the statsgeek.com/2014/02/01/adjusting-for-baseline-covariates-in-randomized-controlled-trials/http://egap.org/content/power-analysis-simulations-r

http://stackoverflow.com/questions/14554558/simulate-a-linear-model-100-times

http://stats.stackexchange.com/questions/155246/which-variable-relative-importance-method-to-use

29 Computing Environment

R version 3.2.2 (2015-08-14)

Platform: x86_64-w64-mingw32/x64 (64-bit) Running under: Windows 8 x64 (build 9200)

locale:

- [1] LC_COLLATE=English_United Kingdom.1252 LC_CTYPE=English_United Kingdom.1252
- [3] LC_MONETARY=English_United Kingdom.1252 LC_NUMERIC=C
- [5] LC_TIME=English_United Kingdom.1252

attached base packages:

[1] grid	parallel stats	graphics	grDevices utils	datasets	methods	base	
other ottoch	od nackamos:						

other attached packages:

[1]	VennDiagram_1.6.17	<pre>futile.logger_1.4.3</pre>	rethinking_1.58	rstan_2.10.1	StanHeaders_2.
[6]	reshape_0.8.5	rms_4.5-0	SparseM_1.7	Hmisc_3.17-4	ggplot2_2.1.0
[11]	Formula_1.2-1	survival_2.39-5	lattice_0.20-33	knitr_1.13	

loaded via a namespace (and not attached):

	1				
[1]	zoo_1.7-13	reshape2_1.4.1	splines_3.2.2	colorspace_1.2-6	htmltools_0.3.
[6]	stats4_3.2.2	100_0.1.6	yaml_2.1.13	chron_2.3-47	foreign_0.8-66
[11]	RColorBrewer_1.1-2	lambda.r_1.1.9	matrixStats_0.50.2	multcomp_1.4-6	plyr_1.8.4
[16]	stringr_1.0.0	MatrixModels_0.4-1	munsell_0.4.3	gtable_0.2.0	mvtnorm_1.0-5
[21]	codetools_0.2-14	coda_0.18-1	evaluate_0.9	labeling_0.3	latticeExtra_0
[26]	inline_0.3.14	quantreg_5.26	TH.data_1.0-7	Rcpp_0.12.6	KernSmooth_2.23
[31]	acepack_1.3-3.3	scales_0.4.0	formatR_1.4	<pre>gridExtra_2.2.1</pre>	digest_0.6.9
[36]	stringi_1.1.1	polspline_1.1.12	tools_3.2.2	sandwich_2.3-4	magrittr_1.5
[41]	<pre>futile.options_1.0.0</pre>	cluster_2.0.3	MASS_7.3-45	Matrix_1.2-2	data.table_1.9
[46]	rmarkdown_1.0	rpart_4.1-10	nnet_7.3-12	nlme_3.1-128	

^{[1] &}quot;~/GIT\\programs-master"

This took 744.85 seconds to execute.