Féidearthachtaí as Cuimse
Infinite Possibilities

Machine Learning

Lecture 3: Decision Trees

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Slides adapted from Sarah Jane Delany and book slides from: Fundamentals of Machine Learning for Predictive Data Analytics. Kelleher, Mac Namee and D'Arcy

Overview

- The idea behind decision trees
- Shannon's entropy model
- Splitting criteria in decision trees
- The ID3 algorithm in decision trees
- Handling various feature types in decision trees
- Improving decision tree training

Administrivia

- Lectures:
  - Bojan: PT – Thursday: 6pm to 8pm
  - Bujar: FT – Thursday: 11am to 1pm
- Labs:
  - Bojan: PT – Thursday: 8pm to 10pm
  - Bujar: FT – Thursday: 2pm to 4pm

- All notes, lecture recordings, tutorials, lab work, and assignments will be available on Brightspace.

- For all module queries please contact: bojan.bozic@tudublin.ie, and bujar.raufi@tudublin.ie

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Big idea!

- Guess who?

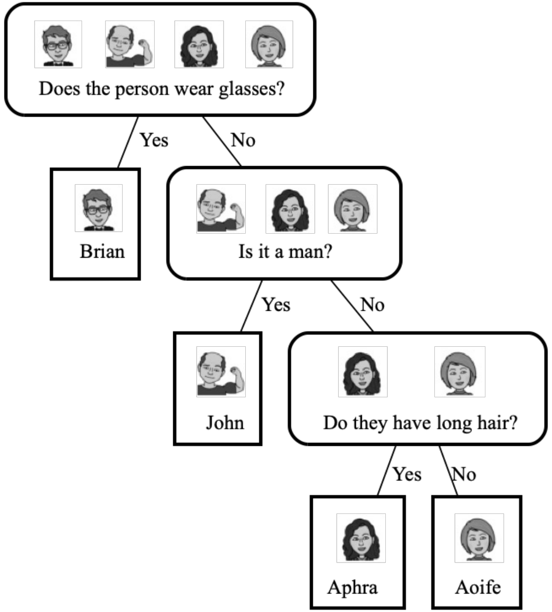
Brian John Aphra Aoife

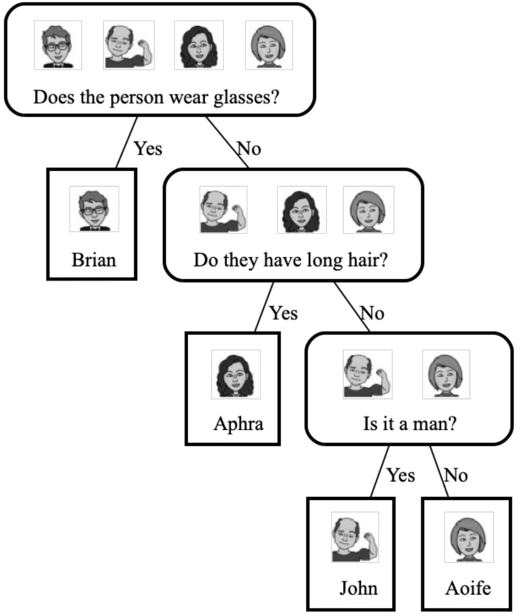
Q1: Does the person wear glasses?

Q2: Is the person a man?

Q3: Does the person have long hair?

Big idea

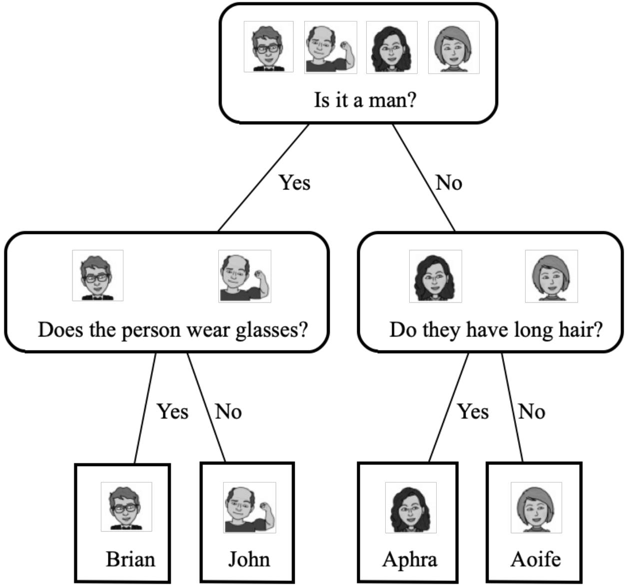




- What average number of questions do you have to ask for Q2?!?!

$$q = \frac{1+2+3+3}{4} = 2.2$$

Big idea



- What average number of questions do you have to ask for Q1?!?!

$$q = \frac{2+2+2+2}{4} = 2$$

Big idea

- Getting an answer to Q2 (Is it a man?) gives more information than an answer to other questions.
- Information given is about how the domain is split up after the answer is received and the likelihood of each answer.
- Information-based learning uses this idea…
- Algorithms determine which descriptive features provide the most info about the target feature
- Predictions are made by sequentially testing the features in order of informativeness.

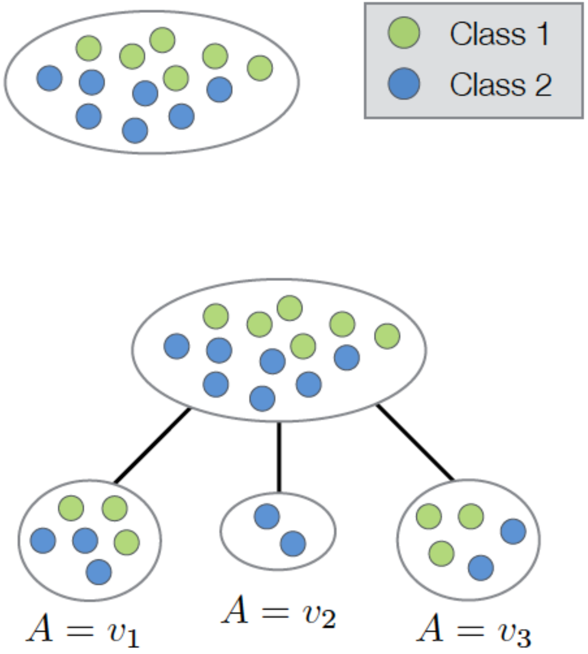
Man	Long Hair	Glasses	Name
Yes	No	Yes	Brian
Yes	No	No	John
No	Yes	No	Aphra
No	No	No	Aoife

How do we quantify this?!?!

Decision Tree Training

1) Initially, all examples in the training set are placed at the root node of the tree.

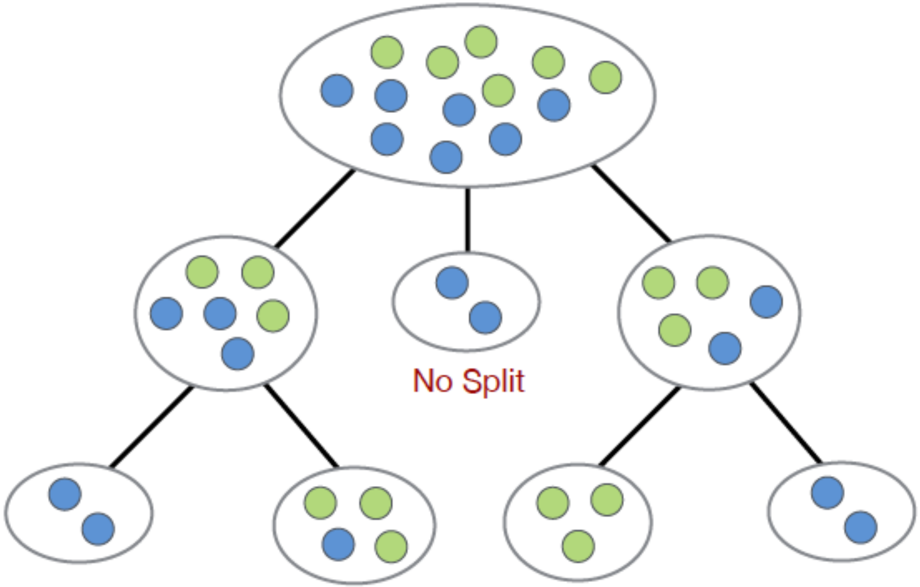
2) A test is performed on a feature (A) to split the examples at the root node into two or more subsets of examples at interior nodes.



Decision rules

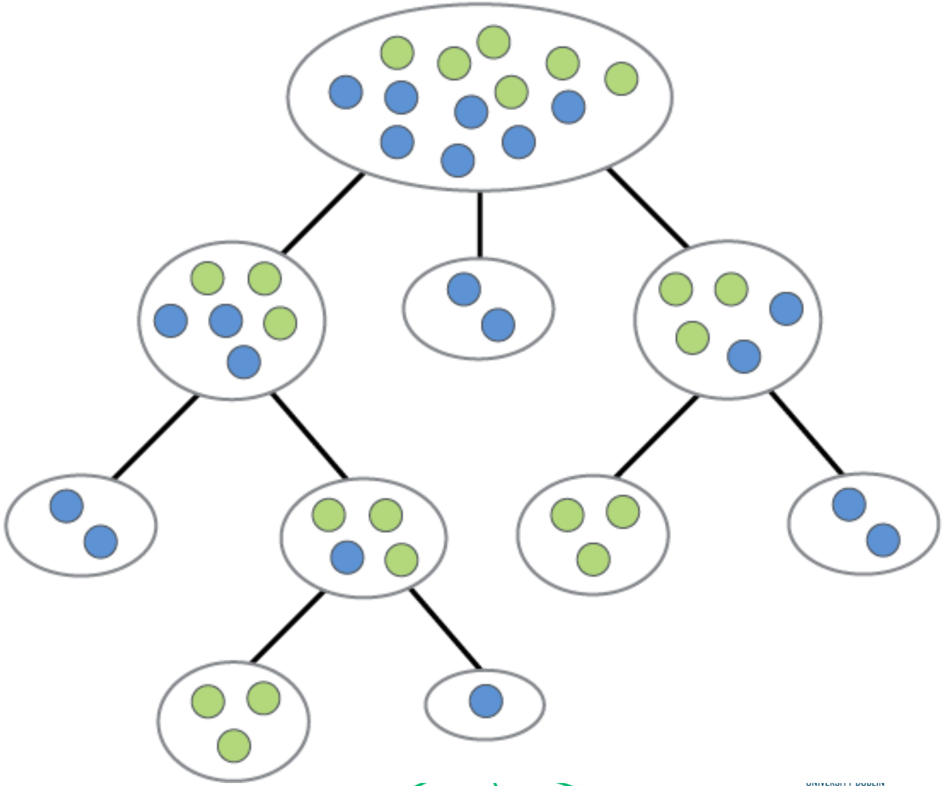
Decision Tree Training

3. The same process is now applied to each interior node, except at leaf nodes, where all examples have the same class.



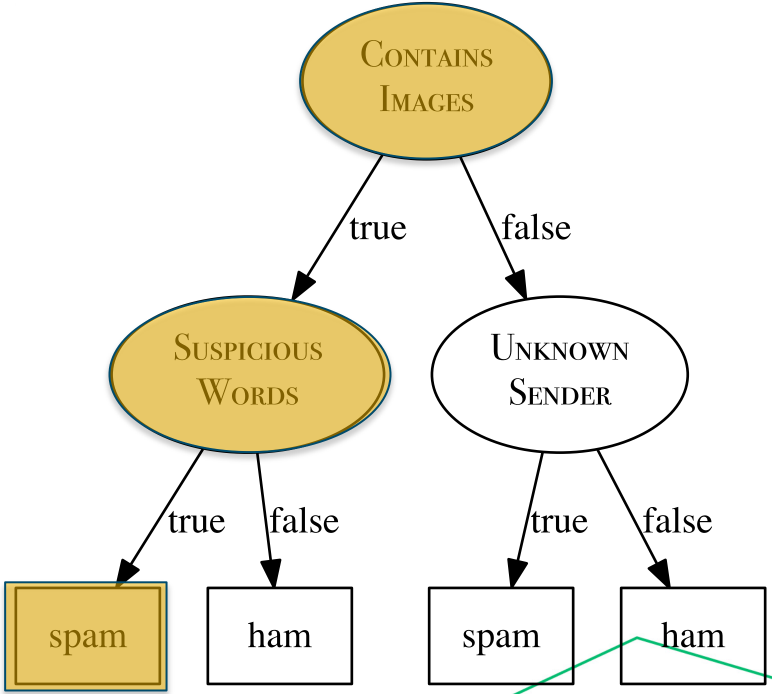
Decision Tree Training

4. Repeat until all leaf nodes in the tree have examples with the same class.



Decision Tree Classification

ID	Suspicious Words	Unknown Sender	Contains Images	Class
376	true	false	true	spam
489	true	true	false	spam
541	true	true	false	spam
693	false	true	true	ham
782	false	false	false	ham
976	false	false	false	ham

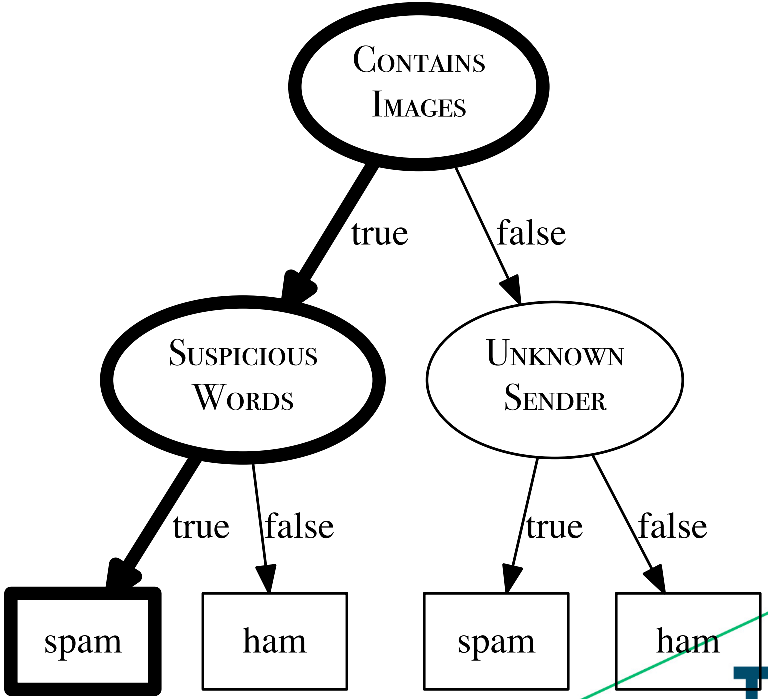


- To classify a query example:
- Test the value of the feature at the node and follow the relevant branch until a leaf node is reached

Decision Tree Classification

Query example:

- Suspicious Words: true
- Unknown Sender: true
- Contains Images: true



Decision Tree Classification

ID	Suspicious Words	Unknown Sender	Contains Images	Class
376	true	false	true	spam
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693	false	true	true	ham
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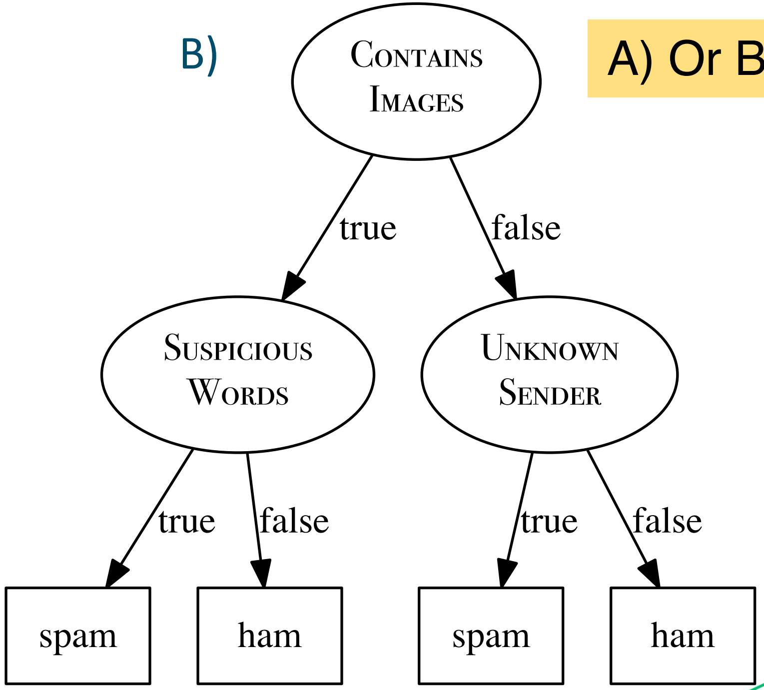
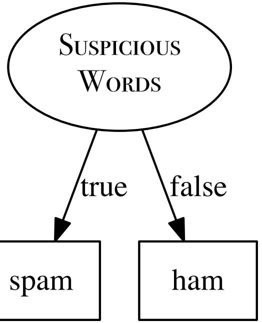
976	false	false	false	ham

B)



A) Or B)

A)



Both trees are consistent with the examples in the training data

Decision Tree Classification

ID	Suspicious Words	Unknown Sender	Contains Images	Class
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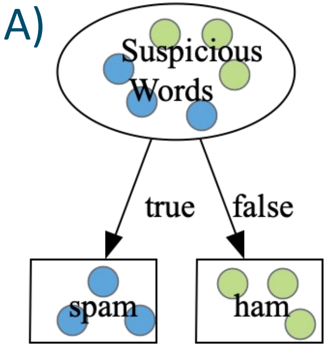

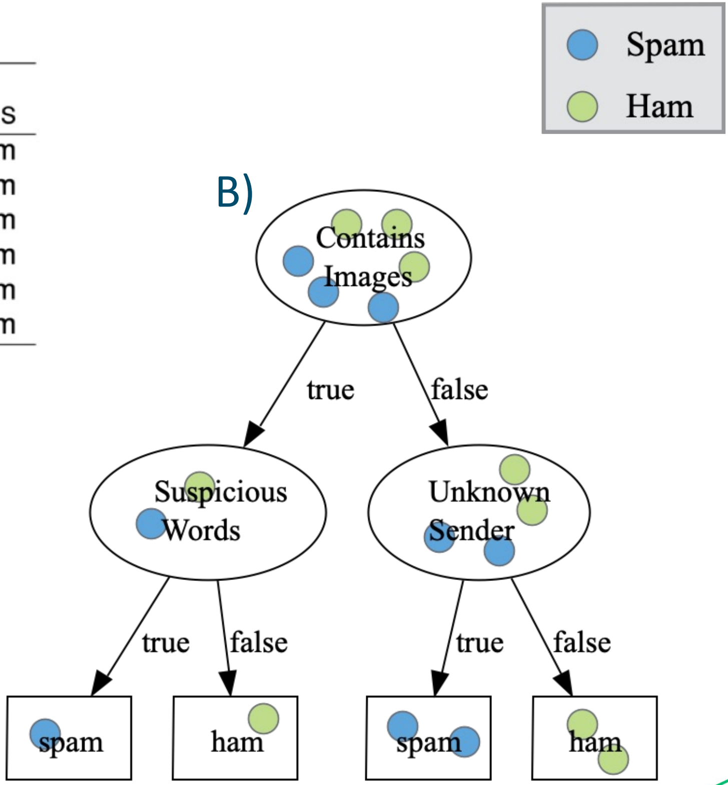

A node is **pure** if all examples at that node have the same label

Decision Tree Inductive Bias

- Preference Bias: Choose decision trees that have fewer tests, i.e. shallower trees
- Pure nodes provide more information about the value of the target feature for a query.
- Descriptive features that split the dataset into pure sets provide information about the target feature and are considered more informative.
- Testing the informative features early on in the tree can result in shallower trees.
- Claude Shannon's entropy model is a computational metric of the purity of a set.

Entropy ~ the uncertainty associated with guessing the result if you were to make a random selection from a set

Entropy

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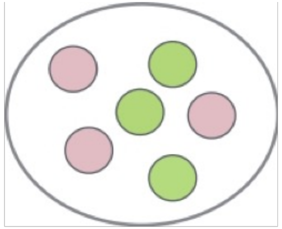
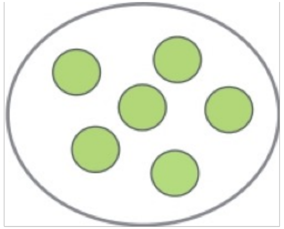


Node has high uncertainty
→ High entropy



Node has low uncertainty
→ Low entropy

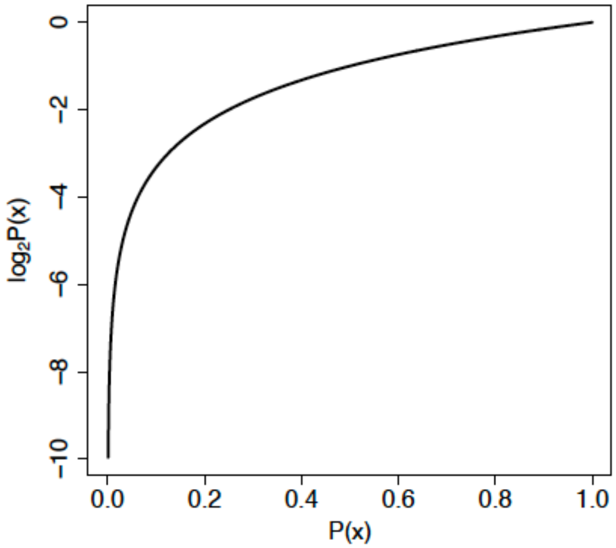
- Entropy is related to the probability of an outcome.
- High probability—> Low entropy
- Low probability —> High entropy

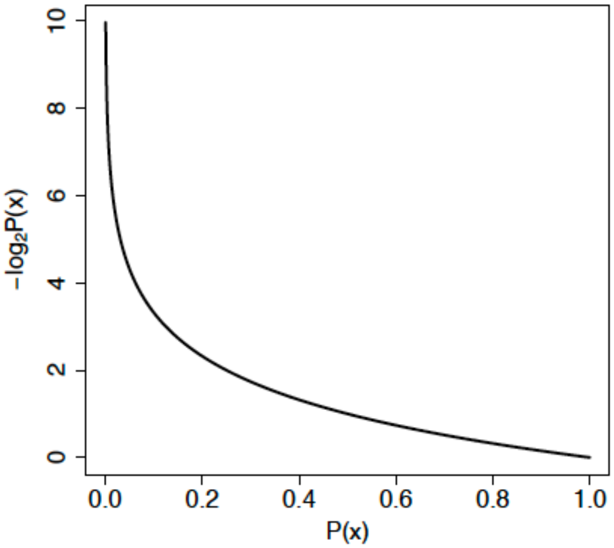
Entropy

- The log of a probability multiplied by -1 gives the mapping
  - High probability —> low entropy
  - Low probability —> high entropy

$$log_b(a) = x \text{ where } b^x = a$$

$$log_2(0.5) = -1 \text{ because } 2^{-1} = 0.5$$
$$log_2(1) = 0 \text{ because } 2^0 = 1$$
$$log_2(8) = 3 \text{ because } 2^3 = 8$$
$$log_5(25) = 2 \text{ because } 5^2 = 25$$





Entropy

- Entropy of a dataset of examples D with labels $\{t_1, t_2, t_3, \cdots, t_l\}$

$$H(D) = -\sum_{i=1}^{l} (P(t_i) \times log_2(P(t_i)))$$

Where $P(t_i)$ is the probability of randomly selecting an example with label t_i.

NB: Define $\log_2(0)=0$

$$p(t_1) = 6/6 = 1.0 \quad p(t_2) = 0/6 = 0$$

$$H(D) = -((1 \times \log_2(1)) + (0 \times \log_2(0))) = -(0+0) = 0$$

$$p(t_1) = 0/6 = 0 \quad p(t_2) = 6/6 = 1.0$$

$$H(D) = -((0 \times \log_2(0)) + (1 \times \log_2(1))) = -(0+0) = 0$$

$$p(t_1) = 3/6 = 0.5 \quad p(t_2) = 3/6 = 0.5$$

$$H(D) = -((0.5 \times \log_2(0.5)) + (0.5 \times \log_2(0.5))) = -(-0.5 - 0.5) = 1$$

Entropy examples

- The entropy of a set of 52 playing cards:

$$H(D) = -\sum_{i=1}^{52} P(card = i) \times log_2(P(card = i))$$

$$= -\sum_{i=1}^{52} \frac{1}{52} \times log_2(\frac{1}{52}) = 5.7$$

- The entropy of a set of 52 playing cards distinguishing cards only by suit:

$$H(D) = -\sum_{i=1}^{4} P(suit = i) \times log_2(P(suit = i))$$

$$= -\sum_{i=1}^{4} \frac{13}{52} \times log_2(\frac{13}{52}) = 2$$

Entropy

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376	true	false	true	spam
489	true	true	false	spam
541	true	true	false	spam
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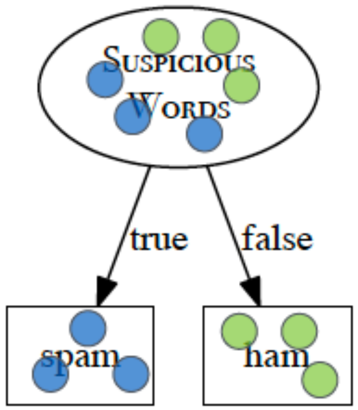
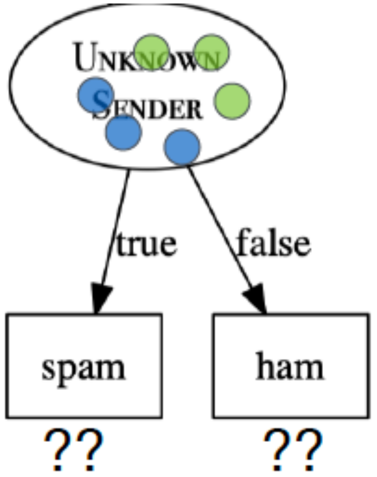
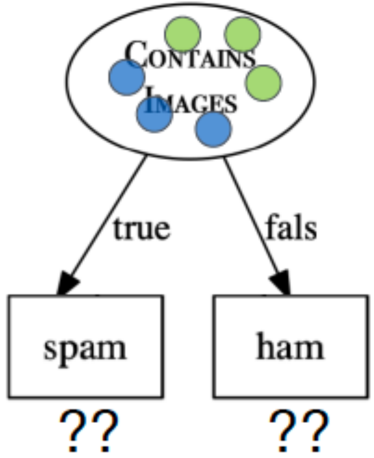
782	false	false	false	ham
976	false	false	false	ham

Information Gain ~ a measure of the reduction in the overall entropy of a set that is achieved by testing on a feature







Information gain

- IG for descriptive feature d that splits a dataset D of examples into subsets or partitions $\{D_1, D_2, \ldots, D_k\}$.

$$IG(d, D) = (\text{original entropy}) - (\text{entropy after split})$$

$$IG(d, D) = H(D) - rem(d, D)$$

The entropy remaining after the dataset is split using descriptive feature *d*

$$H(D) = -\sum_{i=1}^{l} (P(t_i) \times log_s(P(t_i)))$$

The entropy on the full dataset wrt the target feature *t*

$$rem(d, \mathcal{D}) = \sum_{i}^{k} \underbrace{\frac{|\mathcal{D}_i|}{|\mathcal{D}|}}_{\text{weighting}} \times \underbrace{H(\mathcal{D}_i)}_{\substack{\text{entropy of} \\ \text{partition } \mathcal{D}_i}}$$

Each partition is weighted in proportion to its size

Information gain: example

- **Calculate $H(D)$:** Entropy of dataset wrt target feature.

ID	Suspicious Words	Unknown Sender	Contains Images	Class
376	true	false	true	spam
489	true	true	false	spam
541	true	true	false	spam
693	false	true	true	ham
782	false	false	false	ham
976	false	false	false	ham

$$
\begin{aligned}
H(D) &= -\sum_{l \in \{\text{spam},\text{ham}\}} (P(t_l) \times log_2(P(t_l))) \\
&= -((P(t=\text{spam}) \times log_2(P(t=\text{spam})) \\
&\quad + (P(t=\text{ham}) \times log_2(P(t=\text{ham}))) \\
&= -\left(\left({}^3/_6 \times log_2({}^3/_6)\right) + \left({}^3/_6 \times log_2({}^3/_6)\right)\right) = 1
\end{aligned}
$$

Information gain: example

- **Calculate $\mathbf{rem}(\boldsymbol{SW}, \boldsymbol{D})$:** Entropy remaining after splitting on $\boldsymbol{SW}$ descriptive feature.

ID	Suspicious Words	Unknown Sender	Contains Images	Class
376	true	false	true	spam
489	true	true	false	spam
541	true	true	false	spam
693	false	true	true	ham
782	false	false	false	ham
976	false	false	false	ham

$$rem(\mathrm{SW}, D) = \sum_{i \in \{true, false\}} \left(\frac{|D_i|}{|D|}\right) \times H(D_i)$$

$$rem\left(\mathrm{SW}, D\right) = \left(\frac{|D_{true}|}{|D|} \times H\left(D_{true}\right)\right) + \left(\frac{|D_{false}|}{|D|} \times H\left(D_{false}\right)\right)$$

$$= \left({}^{3}/_{6} \times \left(-\sum_{c \in \{spam, ham\}} P(t_c) \times log_2(P(t_c))\right)\right) + \left({}^{3}/_{6} \times \left(-\sum_{c \in \{spam, ham\}} P(t_c) \times log_2(P(t_c))\right)\right)$$

$$= \left({}^{3}/_{6} \times \left(-\left(\left({}^{3}/_{3} \times log_2({}^{3}/_{3})\right) + \left({}^{0}/_{3} \times log_2({}^{0}/_{3})\right)\right)\right)\right)$$

$$+ \left({}^{3}/_{6} \times \left(-\left(\left({}^{0}/_{3} \times log_2({}^{0}/_{3})\right) + \left({}^{3}/_{3} \times log_2({}^{3}/_{3})\right)\right)\right)\right)$$

$$= 0$$

Information gain: example

- **Calculate $\mathbf{rem}(\mathbf{US}, \boldsymbol{D})$:** Entropy remaining after splitting on $\boldsymbol{US}$ descriptive feature.

ID	Suspicious Words	Unknown Sender	Contains Images	Class
376	true	false	true	spam
489	true	true	false	spam
541	true	true	false	spam
693	false	true	true	ham
782	false	false	false	ham
976	false	false	false	ham

$$rem(\text{US}, D) = \sum_{i \in \{true, false\}} \left(\frac{|D_i|}{|D|}\right) \times H(D_i)$$

$$rem(\text{US}, D) = \left(\frac{|D_{true}|}{|D|} \times H(D_{true})\right) + \left(\frac{|D_{false}|}{|D|} \times H(D_{false})\right)$$

$$= ({}^3/_6 \times (-\sum_{c \in \{spam, ham\}} P(t_c) \times log_2(P(t_c))) + ({}^3/_6 \times (-\sum_{c \in \{spam, ham\}} P(t_c) \times log_2(P(t_c)))$$

$$= ({}^3/_6 \times (-(({}^2/_3 \times log_2({}^2/_3)) + ({}^1/_3 \times log_2({}^1/_3)))))$$
$$+ ({}^3/_6 \times (-(({}^1/_3 \times log_2({}^1/_3)) + ({}^2/_3 \times log_2({}^2/_3)))))$$

$$= 0.9183$$

Information gain: example

- **Calculate $\mathbf{rem}(\mathbf{CI}, \boldsymbol{D})$:** Entropy remaining after splitting on $\boldsymbol{CI}$ descriptive feature.

ID	Suspicious Words	Unknown Sender	Contains Images	Class
376	true	false	true	spam
489	true	true	false	spam
541	true	true	false	spam
693	false	true	true	ham
782	false	false	false	ham
976	false	false	false	ham

$$rem(\mathrm{CI}, D) = \sum_{i \in \{true, false\}} \left(\frac{|D_i|}{|D|}\right) \times H(D_i)$$

$$rem(\mathrm{CI}, D) = \left(\frac{|D_{true}|}{|D|} \times H(D_{true})\right) + \left(\frac{|D_{false}|}{|D|} \times H(D_{false})\right)$$

$$= ({}^2/_6 \times (-\sum_{c \in \{spam, ham\}} P(t_c) \times log_2(P(t_c))) + ({}^4/_6 \times (-\sum_{c \in \{spam, ham\}} P(t_c) \times log_2(P(t_c)))$$

$$= ({}^2/_6 \times (-(({}^1/_2 \times log_2({}^1/_2)) + ({}^1/_2 \times log_2({}^1/_2)))))$$

$$+ ({}^4/_6 \times (-(({}^2/_4 \times log_2({}^2/_4)) + ({}^2/_4 \times log_2({}^2/_4)))))$$

$$= 1$$

Information gain: example

$$IG(d, D) = H(D) - rem(d, D)$$

$$\begin{aligned} IG(\text{SW}, D) &= H(D) - rem(\text{SW}, D) \\ &= 1 - 0 = 1 \end{aligned}$$

$$\begin{aligned} IG(\text{US}, D) &= H(D) - rem(\text{US}, D) \\ &= 1 - 0.9183 = 0.0817 \end{aligned}$$

$$\begin{aligned} IG(\text{CI}, D) &= H(D) - rem(\text{CI}, D) \\ &= 1 - 1 = 0 \end{aligned}$$

Which feature should we split on?!?!?!

- This result matches our intuitions - Suspicious Words is the best feature to split on.

ID3 algorithm

- ID3 (Iterative Dichotomizer 3)
- Attempts to create the shallowest tree that is consistent with the dataset.
- Builds the tree in a recursive, depth-first manner, beginning at the root node and working down to the leaf nodes.

ID3 algorithm

Set of training examples D

Set of descriptive features d

IF all examples in D belong to the same class C ***THEN***

Return a leaf node and label it with class C

IF no features left in D ***THEN***

Return a leaf node and label it with majority class C of D

IF no examples left in D ***THEN***

Return a leaf node and label the it with majority class C of examples at the immediate parent node

ELSE

Select a feature d_i from d based on some feature selection criterion

Generate a tree node with d_i as the test feature

FOR EACH value v_j of d_i

Let $D_j \subset D$ contains all examples with $d_i = v_j$

Build a subtree by applying $ID3(D_j)$

Stop growing the current path by adding a leaf node.

Extend the current path by adding an interior node and growing its branches

ID3 example

ID	STREAM	SLOPE	ELEVATION	VEGETATION
1	false	steep	high	chaparral
2	true	moderate	low	riparian
3	true	steep	medium	riparian
4	false	steep	medium	chaparral
5	false	flat	high	conifer
6	true	steep	highest	conifer
7	true	steep	high	chaparral

Ecological modelling: Predicting vegetation based on features from aerial maps, which inputs to animal management.

$$H(\mathcal{D}) = -\sum_{l \in \left\{\begin{matrix}\text{chaparral,}\\ \text{riparian,}\\ \text{conifer}\end{matrix}\right\}} P(\text{Vegetation} = l) \times log_2\left(P(\text{Vegetation} = l)\right)$$

$$= -\left(\left({}^3/_7 \times log_2({}^3/_7)\right) + \left({}^2/_7 \times log_2({}^2/_7)\right) + \left({}^2/_7 \times log_2({}^2/_7)\right)\right)$$

$$= 1.5567$$

ID3 example

ID	STREAM	SLOPE	ELEVATION	VEGETATION
1	false	steep	high	chaparral
2	true	moderate	low	riparian
3	true	steep	medium	riparian
4	false	steep	medium	chaparral
5	false	flat	high	conifer
6	true	steep	highest	conifer
7	true	steep	high	chaparral

Split By Feature		Part.	Instances	Partition Entropy	Rem.	Info. Gain
STREAM	'true'	$\mathcal{D}_1$	$\mathbf{d}_2, \mathbf{d}_3, \mathbf{d}_6, \mathbf{d}_7$	1.5	1.2507	0.3060
	'false'	$\mathcal{D}_2$	$\mathbf{d}_1, \mathbf{d}_4, \mathbf{d}_5$	0.9183		
SLOPE	'flat'	$\mathcal{D}_3$	$\mathbf{d}_5$	0	0.9793	0.5774
	'moderate'	$\mathcal{D}_4$	$\mathbf{d}_2$	0		
	'steep'	$\mathcal{D}_5$	$\mathbf{d}_1, \mathbf{d}_3, \mathbf{d}_4, \mathbf{d}_6, \mathbf{d}_7$	1.3710		
ELEVATION	'low'	$\mathcal{D}_6$	$\mathbf{d}_2$	0	0.6793	0.8774
	'medium'	$\mathcal{D}_7$	$\mathbf{d}_3, \mathbf{d}_4$	1.0		
	'high'	$\mathcal{D}_8$	$\mathbf{d}_1, \mathbf{d}_5, \mathbf{d}_7$	0.9183		
	'highest'	$\mathcal{D}_9$	$\mathbf{d}_6$	0		

$H(D) = 1.5567$

What feature should be at the root of the tree?

ID3 example

Elevation

Pure set → Convert to leaf node

low

D6	ID	Stream	Slope	Vegetation
	2	true	moderate	riparian

medium

D7	ID	Stream	Slope	Vegetation
	3	true	steep	riparian
	4	false	steep	chaparral

high

D8	ID	Stream	Slope	Vegetation
	1	false	steep	chaparral
	5	false	flat	conifer
	7	true	steep	chaparral

highest

Pure set → Convert to leaf node

D9	ID	Stream	Slope	Vegetation
	6	true	steep	conifer

$$
\begin{aligned}
&H(\mathcal{D}_7) \\
&= - \sum_{l \in \left\{\begin{array}{l}\text{chaparral,}\\ \text{riparian,}\\ \text{conifer}\end{array}\right\}} P(\text{Veg} = l) \times log_2\left(P(\text{Veg} = l)\right) \\
&= -\left(\left({}^{1}/_{2} \times log_2({}^{1}/_{2})\right) + \left({}^{1}/_{2} \times log_2({}^{1}/_{2})\right) + \left({}^{0}/_{2} \times log_2({}^{0}/_{2})\right)\right) \\
&= 1.0
\end{aligned}
$$

ID	STREAM	SLOPE	ELEVATION	VEGETATION
1	false	steep	high	chaparral
2	true	moderate	low	riparian
3	true	steep	medium	riparian
4	false	steep	medium	chaparral
5	false	flat	high	conifer
6	true	steep	highest	conifer
7	true	steep	high	chaparral

ID3 example

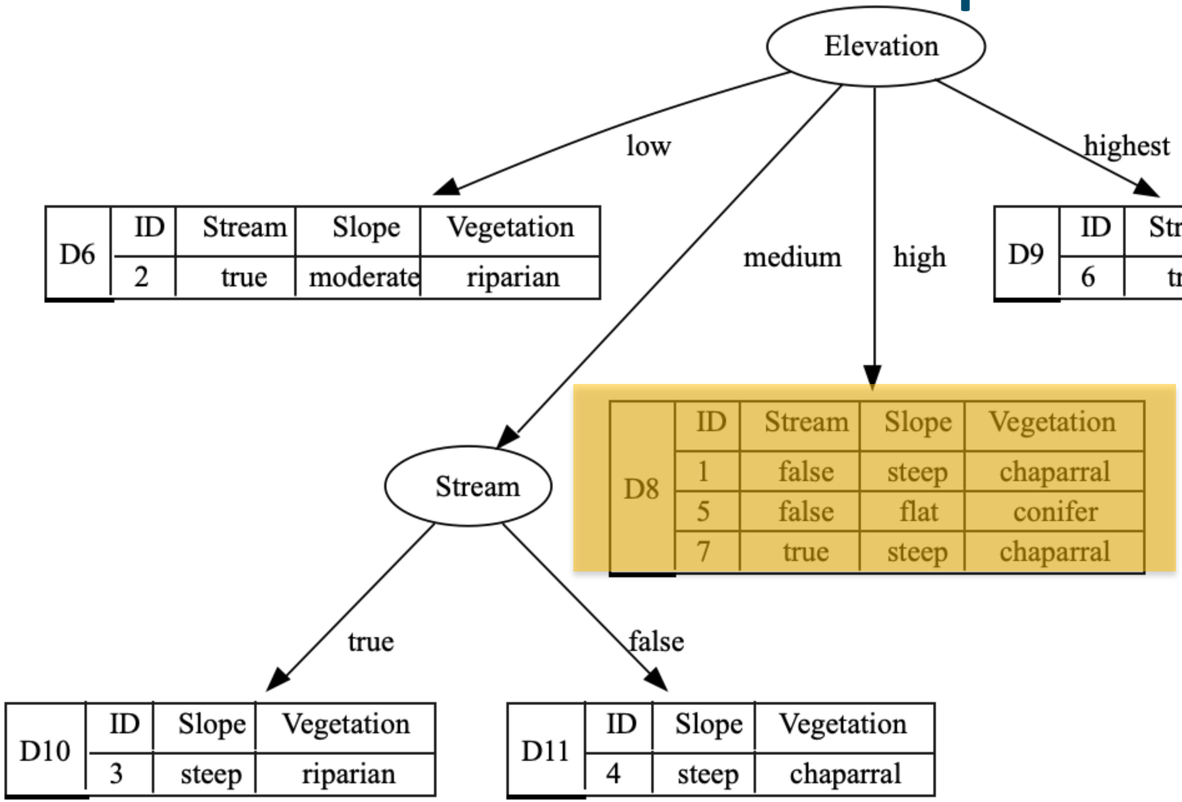
	ID	STREAM	SLOPE	VEGETATION
D7	3	true	steep	riparian
	4	false	steep	chaparral

$$H(D_7) = 1.0$$

Split By Feature	Level	Part.	Instances	Partition Entropy	Rem.	Info. Gain
STREAM	'true'	$\mathcal{D}_{10}$	$\mathbf{d}_3$	0	0	1.0
	'false'	$\mathcal{D}_{11}$	$\mathbf{d}_4$	0		
SLOPE	'flat'	$\mathcal{D}_{12}$		0	1.0	0
	'moderate'	$\mathcal{D}_{13}$		0		
	'steep'	$\mathcal{D}_{14}$	$\mathbf{d}_3, \mathbf{d}_4$	1.0		

What feature should be split on?

ID3 example



D6

ID	Stream	Slope	Vegetation
2	true	moderate	riparian

D9

ID	Stream	Slope	Vegetation
6	true	steep	conifer

D8

ID	Stream	Slope	Vegetation
1	false	steep	chaparral
5	false	flat	conifer
7	true	steep	chaparral

D10

ID	Slope	Vegetation
3	steep	riparian

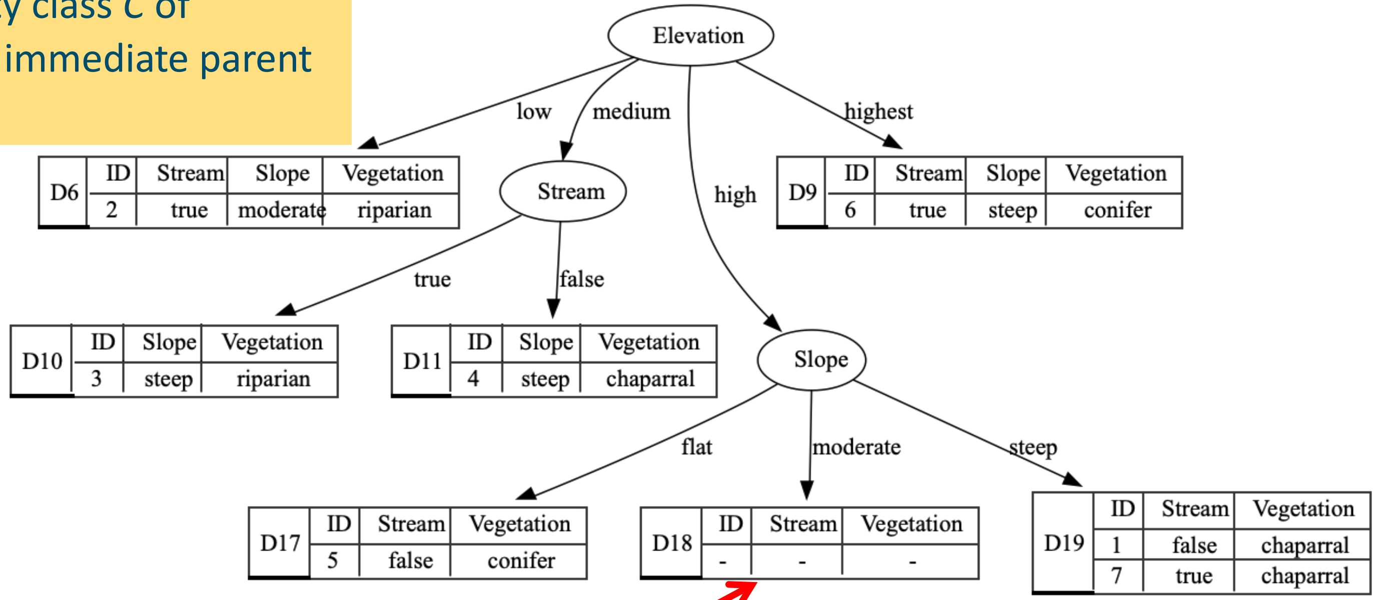
D11

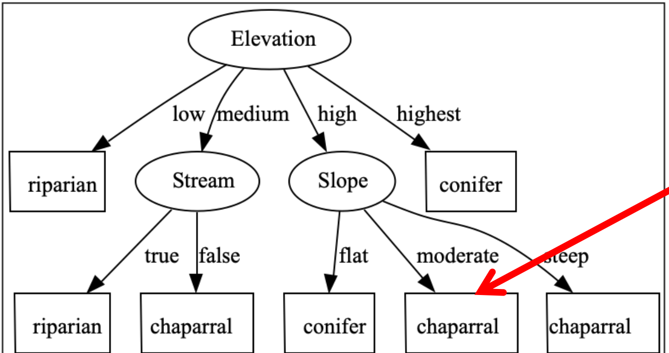
ID	Slope	Vegetation
4	steep	chaparral

What feature should be split on D_8?

IF no examples left in *D* ***THEN***
– Return a leaf node and label it with the majority class *C* of examples at the immediate parent node.

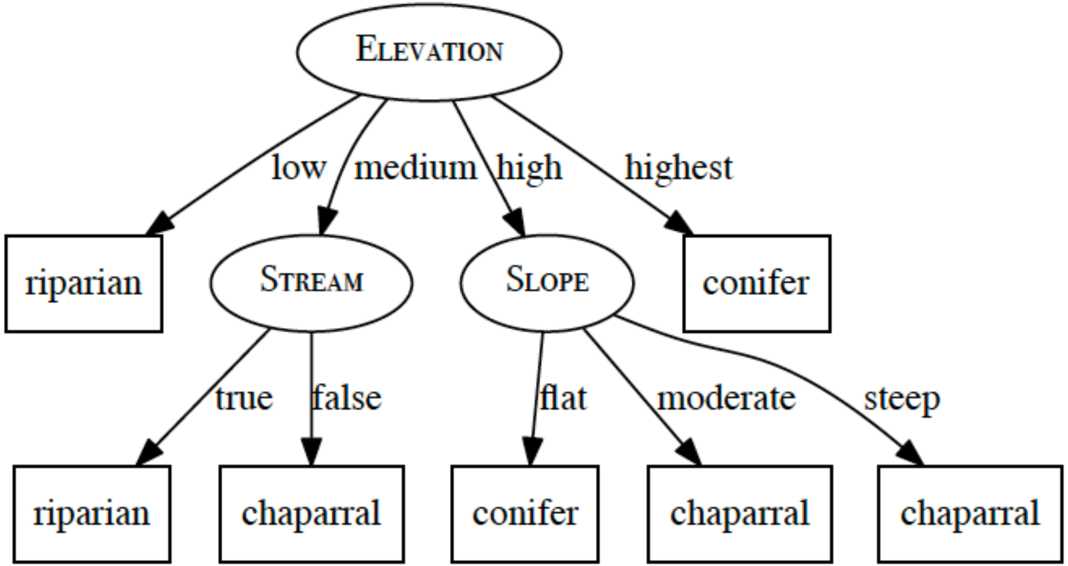
ID3 example





D8	ID	Stream	Slope	Vegetation
	1	false	steep	chaparral
	5	false	flat	conifer
	7	true	steep	chaparral

Using the tree for prediction



- What prediction would the tree return for the query?

Stream = ‘true’, Slope = ‘moderate’, Elevation = ‘high’

ID3 algorithm

Set of training examples D

Set of descriptive features d

IF all examples in D belong to the same class C ***THEN***

Return a leaf node and label it with class C

IF no features left in D ***THEN***

Return a leaf node and label it with majority class C of D

IF no examples left in D ***THEN***

Return a leaf node and label the it with majority class C of examples at th immediate parent node

ELSE

Select a feature d_i from d based on some feature selection criterion

Generate a tree node with d_i as the test feature

FOR EACH value v_j of d_i

Let $D_j \subset D$ contains all examples with $d_i = v_j$

Build a subtree by applying $ID3(D_j)$

Stop growing the current path by adding a leaf node.

Extend the current path by adding an interior node and growing its branches

Different feature selection criteria

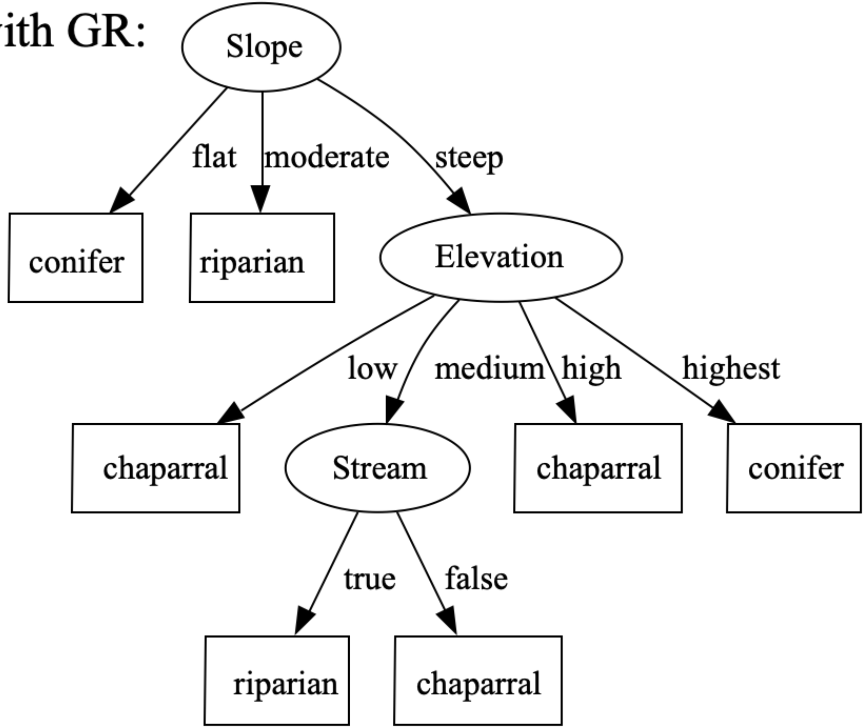
- Information Gain prefers features with many labels as it will split the data into small subsets, which will tend to be pure, irrespective of any correlation between the feature and the target.
- Information Gain Ratio:
- Divide the IG of a feature d by the amount of information used to determine the value of the feature (i.e. the entropy of the dataset wrt the feature d) .

$$GR(d, \mathcal{D}) = \frac{IG(d, \mathcal{D})}{-\sum_{l \in labels(d)} (P(d = l) \times log_2(P(d = l)))}$$

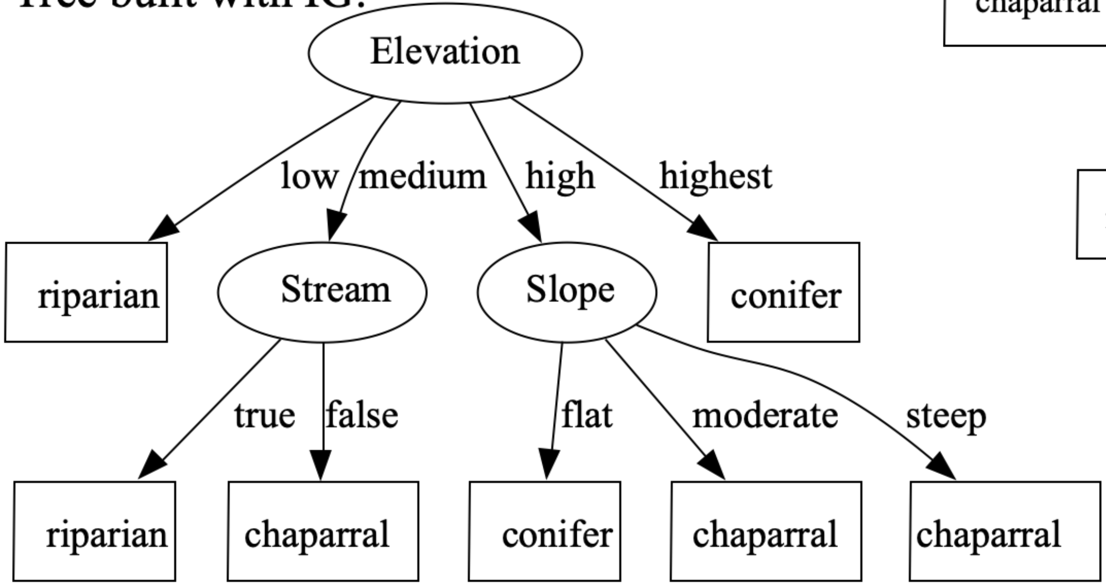
- GR addresses the bias IG has towards features with large numbers of values as the divisor biases away from these types of features

$$GR(\textsc{Stream}, \mathcal{D}) = \frac{0.3060}{0.9852} = 0.3106$$
$$GR(\textsc{Slope}, \mathcal{D}) = \frac{0.5774}{1.1488} = 0.5026$$
$$GR(\textsc{Elevation}, \mathcal{D}) = \frac{0.8774}{1.8424} = 0.4762$$

Tree built with GR:



Tree built with IG:



$$IG(\textsc{Stream}, \mathcal{D}) = 0.3060$$
$$IG(\textsc{Slope}, \mathcal{D}) = 0.5774$$
$$IG(\textsc{Elevation}, \mathcal{D}) = 0.8774$$

Different feature selection criteria

- Gini index

$$Gini(t, \mathcal{D}) = 1 - \sum_{l \in labels(t)} P(t = l)^2$$

- where *P(t=l)* is prob of an instance having target label *l*
- Gini index can be thought of as calculating how often you would misclassify an instance in a dataset if you classified it based on the distribution of target labels in the dataset •
- IG can be calculated by replacing entropy with the Gini index
- CART algorithm (variant of ID3) uses the Gini index

Handling continuous descriptive features

- Turn them into boolean features based on a threshold
- To find the threshold:
  1. Sort according to feature values
  2. Adjacent instances that have different classifications are potential thresholds
  3. Compute IG for each potential threshold
  4. Select one with the highest IG as the actual threshold
- New dynamically created boolean feature competes with other features for selection as the splitting feature for a node
- Repeat as needed as the tree is built.

Example: Continous descriptive features

ID	STREAM	SLOPE	ELEVATION	VEGETATION
1	false	steep	3 900	chapparal
2	true	moderate	300	riparian
3	true	steep	1 500	riparian
4	false	steep	1 200	chapparal
5	false	flat	4 450	conifer
6	true	steep	5 000	conifer
7	true	steep	3 000	chapparal

3

Split by Threshold	Part.	Instances	Partition Entropy	Rem.	Info. Gain
≥750	$\mathcal{D}_1$	$\mathbf{d}_2$	0.0	1.2507	0.3060
	$\mathcal{D}_2$	$\mathbf{d}_4, \mathbf{d}_3, \mathbf{d}_7, \mathbf{d}_1, \mathbf{d}_5, \mathbf{d}_6$	1.4591		
≥1 350	$\mathcal{D}_3$	$\mathbf{d}_2, \mathbf{d}_4$	1.0	1.3728	0.1839
	$\mathcal{D}_4$	$\mathbf{d}_3, \mathbf{d}_7, \mathbf{d}_1, \mathbf{d}_5, \mathbf{d}_6$	1.5219		
≥2 250	$\mathcal{D}_5$	$\mathbf{d}_2, \mathbf{d}_4, \mathbf{d}_3$	0.9183	0.9650	0.5917
	$\mathcal{D}_6$	$\mathbf{d}_7, \mathbf{d}_1, \mathbf{d}_5, \mathbf{d}_6$	1.0		
≥4 175	$\mathcal{D}_7$	$\mathbf{d}_2, \mathbf{d}_4, \mathbf{d}_3, \mathbf{d}_7, \mathbf{d}_1$	0.9710	0.6935	0.8631
	$\mathcal{D}_8$	$\mathbf{d}_5, \mathbf{d}_6$	0.0		

1

ID	STREAM	SLOPE	ELEVATION	VEGETATION
1	false	steep	3 900	chapparal
2	true	moderate	300	riparian
3	true	steep	1 500	riparian
4	false	steep	1 200	chapparal
5	false	flat	4 450	conifer
6	true	steep	5 000	conifer
7	true	steep	3 000	chapparal

750

1350

2250

4175

2

Selected threshold

4

Predicting continuous targets

Regression Trees

- The output value is typically the mean of the target feature values of examples in the leaf node
  => error = predicted value - actual target value
- The tree should be built so that the variance of the target feature values at the leaf node is minimised.
- The measure of impurity at a node is variance.

$$var(t, \mathcal{D}) = \frac{\sum_{i=1}^{n} (t_i - \bar{t})^2}{n-1}$$

where n training examples at the node, t_i is the target feature value of example i , and $\bar{t}$ is the mean of target values of n examples.

ID3 algorithm for continuous targets

Set of training examples D

Set of descriptive features d

IF all examples in D belong to the same class C ***THEN***

Return a leaf node and label it with class C

???

IF no features left in D ***THEN***

Return a leaf node and label it the with average target value of D

IF no examples left in D ***THEN***

Return a leaf node and label the it with average target value of examples at the immediate parent node

$var(d, \mathfrak{D})$

ELSE

Select a feature d_i from d based on some feature selection criterion

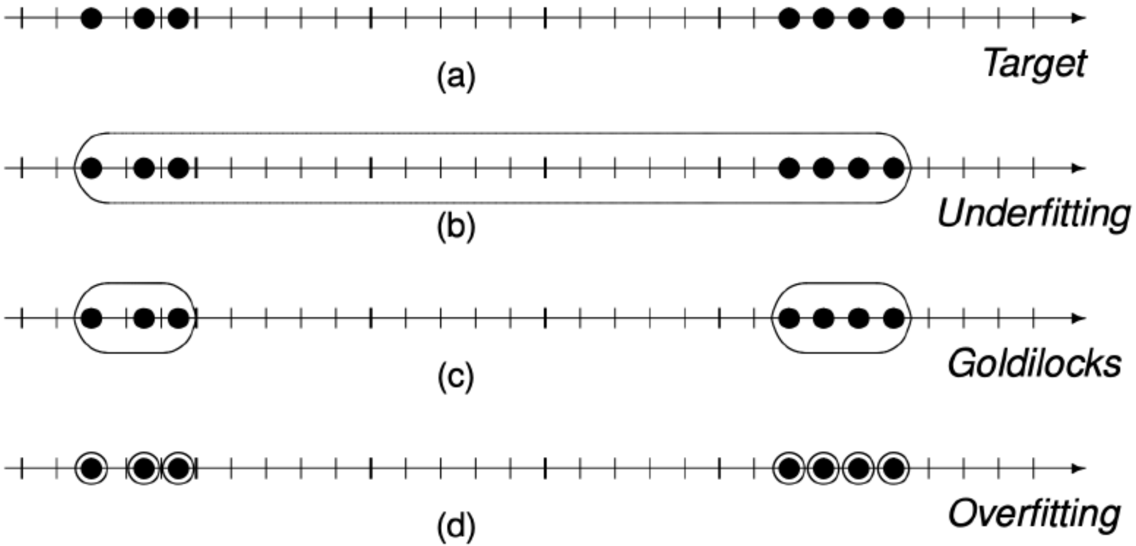
Generate a tree node with d_i as the test feature

FOR EACH value v_j of d_i

Let $D_j \subset D$ contains all examples with $d_i = v_j$

Build a subtree by applying $ID3(D_j)$

Partitioning using variance



- To prevent (d) use an early stopping criterion
  - Stop partitioning if n < some threshold, usually 5% of overall dataset size

Tree pruning

- Decision trees have a natural tendency to segregate noisy data and create leaf nodes around these instances.
- Overfitting in a decision tree involves splitting data at an irrelevant feature.
- The likelihood of over-fitting occurring increases as a tree gets deeper as predictions are based on smaller and smaller subsets
- Pruning the tree identifies and removes sub-trees that are likely to be due to noise and sample variance
  - replace subtree with leaf node covering data partition at that point
  - may result in a tree not being consistent with training data but will promote generalisation.

Pruning

- Pre-pruning involve Early Stopping Criteria
  - simple approaches e.g. n < some threshold; IG < some threshold (critical value pruning); tree depth > some threshold;
  - statistical significance tests, e.g. pruning.
  - Computationally efficient but can miss interactions between features that emerge within subtrees.
- Post-pruning involves growing tree to completion and then checking each branch for tuning
  - recommended approach is to compare the error rate when subtree is included and excluded on an independent validation set.

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Summary

- A decision tree is an eager learning algorithm where the model is induced from data in the form of decision rules.
- A decision tree model makes predictions based on a sequence of tests on the descriptive features of a query
- Advantages:
  - Interpretable
  - can handle both categorical and continuous descriptive features (C4.6 algorithm)
  - relatively robust to noise if pruning is used
- Disadvantages:
  - can become large when dealing with continuous features
  - can overfit if there is a lot of features (high dimensionality)
  - require retraining when modelling concepts that change over time, concept drift

Questions?