1983) so a small uncertainty in the magnetic field strength can lead to a large uncertainty in the mass-loss rate. Relaxing some of these simplifications such as purely radial flows or non-assumption of the WKB approximation (Charbonneau & MacGregor, 1995) may also also lead to better agreement with our radio data.

1.5 Spectral Indices

Long wavelength radio emission from non-dusty K spectral-type red giants is due to thermal free-free emission in their partially ionized winds while shorter wavelength radio emission emanates from the near static and more ionized lower atmospheric layers. The radio flux density-frequency relationship for these stars is usually found to be intermediate between that expected from the isothermal stellar disk emission, where α follows the Rayleigh-Jeans tail of the Planck function (i.e., $\alpha = +2$), and that from an optically thin plasma (i.e., $\alpha = -0.1$). We have shown in Chapter **??** that the expected radio spectrum from a spherically symmetric isothermal outflow with a constant velocity and ionization fraction varies as $\nu^{0.6}$ (Olnon, 1975; Panagia & Felli, 1975; Wright & Barlow, 1975). In reality however, thermal gradients will exist in the wind when the heating mechanisms become insufficient to counteract adiabatic and line cooling, so one would expect a temperature decrease in the wind at some point. Also, if the radio emission emanates from the wind acceleration zone then the electron density will not follow $n_e \propto r^{-2}$.

We therefore relax some of the constant property wind model assumptions and assume that the electron density and temperature vary as a function of distance from the star r, and have the power-law form $n_e \propto r^{-p}$ and $T_e \propto r^{-n}$, respectively (e.g., Seaquist & Taylor, 1987). Finding the spectral index for an outflow with these conditions is non-trivial, so we highlight the main steps required to do so here. We assume the same geometry and notation used for the constant property wind model in section **??** of Chapter **??**, and start by calculating the optical depth along a ray at position z with an impact parameter b through the atmosphere

$$\tau_\nu(b,z) = \frac{0.212 Z^2 n_0^2 r_0^{2p-1.35n}}{\nu^{2.1} T_0^{1.35}} \int_{-\infty}^{z} \frac{dz}{(b^2+z^2)^{(2p-1.35n)/2}} \equiv \int_{-\infty}^{z} \frac{C dz}{(b^2+z^2)^{(2p-1.35n)/2}}, \tag{1.1}$$

where T_0 and n_0 are the gas temperature and density, respectively, at the base of the wind, r_0, and C is a constant representing everything outside of the first integral. We have also assumed that the electron density is the same as the ion density throughout.

The total flux density is found by integrating along the entire ray and over the entire sky

$$F_\nu = \frac{2\pi}{d^2} \int_{z=-\infty}^{\infty} \int_{b=0}^{\infty} B_\nu(z, b) \exp\left[-\tau_\nu(b, z)\right] b\, db\, dz, \tag{1.2}$$

and so we can now substitute in the Rayleigh Jeans function for B_ν (remembering that T is a function of b and z) and Equation 1.1 to get

$$F_\nu = \frac{4\pi k \nu^2 T_0 r_0^n}{d^2 c^2} \int_{z=-\infty}^{\infty} \int_{b=0}^{\infty} \frac{b}{(b^2+z^2)^{n/2}} \exp\left[-\int_{-\infty}^{z} \frac{C dz}{(b^2+z^2)^{(2p-1.35n)/2}}\right] db\, dz. \tag{1.3}$$

Noting that

$$\frac{dz}{(b^2+z^2)^{n/2}} = \frac{(b^2+z^2)^{(2p-0.35)/2} d\tau_\nu}{C}, \tag{1.4}$$

then

$$F_\nu = \frac{4\pi k \nu^2 T_0 r_0^n}{C d^2 c^2} \int_{b=0}^{\infty} \int_{\tau=0}^{\tau_{\max}} b(b^2+z^2)^{(2p-0.35)/2} \exp\left(-\tau\right)\, d\tau\, db, \tag{1.5}$$

where $\tau_{\max}$ is the total optical depth. We now define everything outside of the integral as the constant, D, and integrate over τ to get

$$F_\nu = D \int_0^{\infty} b(b^2+z^2)^{(2p-0.35)/2} \left[1 - e^{-\tau_{\max}}\right] db. \tag{1.6}$$

The total optical depth along a ray, $\tau_{\max}$, can be found using changing the limits in Equation 1.1, using the relationship

$$\int_{-\infty}^{\infty} \frac{1}{(b^2+z^2)^{t/2}} dz = b^{1-t} \sqrt{\pi} \left[\frac{\Gamma(0.5t - 0.5)}{\Gamma(0.5t)}\right] \tag{1.7}$$

and setting $t = (2p - 1.35n)$. Here Γ is the gamma function i.e., $\Gamma(y) = \int_0^\infty u^{y-1} e^{-u} du$. The total optical depth along a ray with impact parameter b is then

$$\tau_{\max} = G b^{1-2p+1.35n} \tag{1.8}$$

where G is a constant that incorporates ν, i.e.,

$$G = \frac{0.3757 Z^2 n_0^2 r_0^{2p-1.35n} \Gamma(p - 0.675n - 0.5)}{\nu^{2.1} T_0^{1.35} \Gamma(p + 0.675n)}. \tag{1.9}$$

Equation 1.6 can now be written as

$$F_\nu = D \int_0^\infty b(b^2 + z^2)^{(2p-0.35)/2}(1 - e^{-(Gb^{1-2p+1.35n})})db \tag{1.10}$$

The ν dependence in Equation 1.10 can now be taken outside of the integral using the following relationship from Gradshteyn & Ryzhik (1994, 5th Edition: p. 386, Eq: 3.478-2)

$$\int_0^\infty y^{v-1}[1 - \exp(-uy^p)]dy = \frac{-1}{|p|} u^{-v/p} \Gamma\left(\frac{v}{p}\right). \tag{1.11}$$