Radio Interferometric Studies of Cool Evolved Stellar Winds

A dissertation submitted to the University of Dublin
for the degree of Doctor of Philosophy

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Summary

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Acknowledgements

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List of Publications

Refereed

1. **O'Gorman, E.**, Harper, G. M., Brown, A., Brown, A., Drake, S., and Richards, A. M. S.
   "Multi-wavelength Radio Continuum Emission Studies of Dust-free Red Giants"
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2. Richards, A. M. S., Davis, R. J., Decin, L., Etoka, S., Harper, G. M., Lim, J. J., Garrington, S. T., Gray, M. D., McDonald, I., **O'Gorman, E.**, Wittkowski, M.
   "e-MERLIN resolves Betelgeuse at wavelength 5 cm"
   Monthly Notices of the Royal Astronomical Society Letters, 432, L61 (2013)

3. **O'Gorman, E.**, Harper, G. M., Brown, J. M., Brown, A., Redfield, S., Richter, M. J., and Requena-Torres, M. A.
   "CARMA CO(J = 2 - 1) Observations of the Circumstellar Envelope of Betelgeuse"
   The Astronomical Journal, 144, 36 (2012)

4. Sada, P. V., Deming, D., Jennings, D. E., Jackson, B. K., Hamilton, C. M., Fraine, J., Peterson, S. W., Haase, F., Bays, K., Lunsford, A., and **O'Gorman, E.**
   "Extrasolar Planet Transits Observed at Kitt Peak National Observatory"
   Publications of the Astronomical Society of the Pacific, 124, 212 (2012)

5. Sada, P. V., Deming, D., Jackson, B. K., Jennings, D. E., Peterson, S. W., Haase, F., Bays, K., **O'Gorman, E.**, and Lundsford, A.
   "Recent Transits of the Super-Earth Exoplanet GJ 1214b"
   The Astrophysical Journal Letters, 720, L215 (2010)

Non-Refereed

1. **O'Gorman, E.**, & Harper, G. M.
   "What is Heating Arcturus' Wind?",
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Introduction

Stellar winds in general ism planets

Empirical studies are vital to constrain physical parameters of the mass loss process.

1.1 Motivation for Researching Cool Evolved Stellar Winds

Mass-loss from non-coronal spectral-type K through mid-M evolved stars plays a crucial role in galactic evolution and ultimately provides part of the material required for the next generation of stars and planets. This mass-loss occurs via a cool ($T_e \lesssim 10^4\,K$) wind with terminal velocities typically less than the photospheric escape velocity ($10 \leq v_\infty \leq 50\,\mathrm{km\,s^{-1}}$). The mass-loss rates for the red giants are significant, typically $10^{-9} - 10^{-11}\,M_\odot\,\mathrm{yr}^{-1}$, and are even higher for the more short-lived red supergiants, typically $10^{-4} - 10^{-6}\,M_\odot\,\mathrm{yr}^{-1}$. This implies that a substantial fraction of the star's initial mass can be dispersed to the interstellar medium (ISM) during these post main sequence evolutionary stages (e.g., Schröder & Sedlmayr, 2001). Mass-loss from these stars is therefore a crucial factor governing stellar evolution (Chiosi & Maeder, 1986) and also in explaining the frequency of supernovae in the galaxy (van Loon, 2010). Despite the importance of this phenomenon, and decades of study, the mechanisms that drive winds from evolved spectral-type K through mid-M stars remain unknown (clearly laid out by Holzer & MacGregor 1985 but still unsolved, e.g., Crowley *et al.* 2009). There is insufficient atomic, molecular, or dust opacity to drive a radiation-driven outflow (Jones, 2008; Zuckerman *et al.*, 1995) and acoustic/pulsation models cannot drive the observed mass-loss rates (Sutmann & Cuntz, 1995). Ultraviolet (UV) and optical observations reveal an absence of significant hot wind plasma, and the winds are thus too cool to be Parker-type thermally-driven flows (e.g., Ayres *et al.*, 1981; Haisch *et al.*, 1980; Linsky & Haisch, 1979).

Magnetic fields are most likely involved in the mass-loss process, although current magnetic models are also unable to explain spectral diagnostics. Exquisite high signal-to-noise ratio (S/N) *Hubble Space Telescope* (HST) UV spectra have revealed that the 1-D linear Alfvén wave-driven wind models of the 1980s (e.g., Harper 1988; Hartmann & MacGregor 1980) are untenable (Harper *et al.*, 2001). These models predict chromospheres as integral parts of a turbulent, extended, and heated wind acceleration zone, but the theoretical line profiles and electron densities do not agree with the HST spectra, (e.g., Judge & Carpenter, 1998). A new generation of theoretical models with outflows driven within diverging magnetic flux tubes have now emerged (Falceta-Gonçalves *et al.*, 2006; Suzuki, 2007) but these too are not yet in agreement with observations (Crowley *et al.*, 2009). It has also been suggested that the winds may be driven by some form of magnetic pressure acting on very highly clumped wind material (Eaton, 2008) but Harper (2010) does not find compelling evidence for this hypothesis. Progress in this field continues to be driven by observations which can test existing models and theories, and provide new insights and constraints into the mass-loss problem.

The Advantages of Radio Emission Observations

Understanding the dynamics and thermodynamics of the atmospheres of late-type evolved stars will ultimately lead to a broader understanding of their mass-loss processes. Red supergiants have extended atmospheres which contain a mixture of atoms, ions, molecules, and dust, and are an ideal test bed for ideas and theories of mass-loss. These atmospheres are so extended that the closest red supergiants can be spatially resolved and imaged at centimeter and millimeter-wavelengths, both in continuum emission and in molecular line emission. Such observations can yield direct measurements of the gas temperature, velocity, and atmospheric structure, which can be used to provide essential constraints on the mass-loss process. The red giants [excluding the asymptotic-giant-branch (AGB) red giants] on the other hand have less extended atmospheres which are dust free and contain only small abundances of molecules. They currently[1] cannot be spatially resolved at radio wavelengths, but their partially ionized outflows can still be detected at these wavelengths, providing an area-averaged sweep through the atmospheres of these stars. The lack of spatial resolution prevents the direct measurement of the fundamental atmospheric properties. However, point source radio observations can still be compared against existing atmospheric models based on shorter wavelength observations (e.g., models based on optical and UV observations). Such radio observations sample further out in the star's atmosphere than optical and UV observations and can test the validity of and improve upon existing model atmospheres.

[1]ALMA and e-MERLIN will eventually be capable of spatially resolving the atmospheres of red giants.

1.2 On the Nature of Cool Evolved Stellar Winds

The study of stellar outflows from cool evolved stars began with the discovery of blue-shifted absorption features (i.e., "P Cygni type" profiles) in strong resonance lines from a number of bright red supergiants (Adams & MacCormack, 1935). They attributed these features to gradually expanding envelopes, even though the expansion speed velocity was small ($\sim 5\,\mathrm{km\,s^{-1}}$) and much less than the photospheric escape velocity. Spitzer (1939) analyzed similar data and devised a *fountain model* for the atmospheres of red supergiants. In this model, radiation drives matter upwards from the photosphere until at some height, the ionization state of the matter changes, causing the radiation force to drop so that the matter falls back onto the star. Definitive evidence for mass loss from cool evolved stars came from Deutsch (1956) who observed a system which contained an M5 giant (i.e., α Her) and a G5 dwarf. They found that the same blue-shifted absorption features were present in the spectrum of both stars, which were not present in other single G5 stars. This indicated that both stars were enveloped in material which had been emitted from the M5 giant. The inferred expansion velocity at the distance of the G5 dwarf was sufficient to escape the system, thus confirming that matter was escaping the gravitational potential of the α Her system.

Even though many later studies concluded that evolved late-type stars contained cool ($T_e < 1000\,\mathrm{K}$) extended circumstellar environments (e.g., Bernat & Lambert, 1976; Gehrz & Woolf, 1971; Reimers, 1975; Weymann, 1962), the physical properties of the ouflow between the photosphere and this cool outer environment remained unclear. An important discovery in late-type evolved stellar atmospheres resulted from the first ultraviolet survey of such stars using the *International Ultraviolet Explorer* (IUE; Macchetto & Penston, 1978). The survey revealed a "transition region dividing line" in the giant branch near spectral type K1 and in the supergiant branch near spectral type G5 which separates these stars based on the properties of their atmospheres (Linsky & Haisch, 1979; Simon *et al.*, 1982). In Figure 1.1 we show the approximate location of this dividing line in the H-R diagram. Stars blueward of the dividing line were found to possess chromospheres and transition regions like the Sun, while stars on the red side were found to possess chromospheres and cool winds. X-ray observations showed that

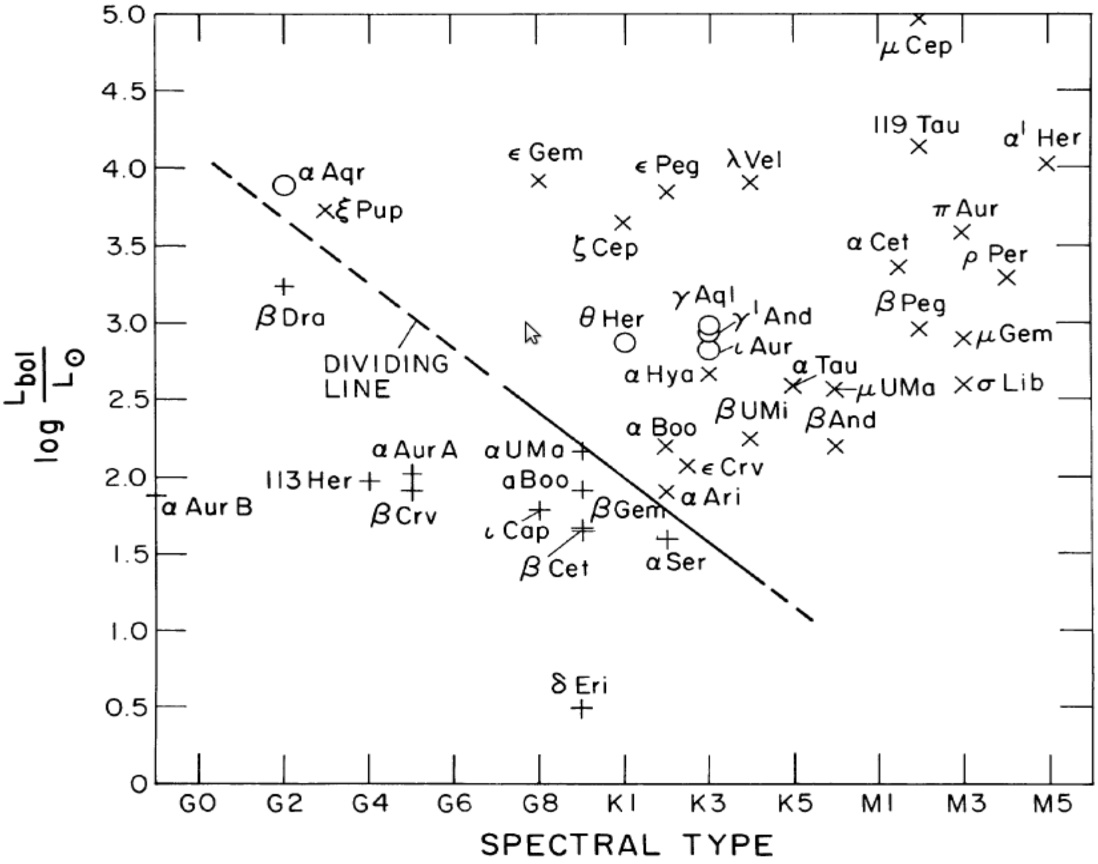

Figure 1.1: A section of the H-R diagram showing the Linsky-Haisch dividing line which was proposed as a sharp division separating coronal (indicated by plus signs) from non-coronal (indicated by crosses) evolved late-type stars. Hybrid atmosphere stars are marked by circles. This image is taken from Drake & Linsky (1986) who carried out a 6 cm survey of late-type evolved stars.

this dividing line extended to coronal emission (Ayres *et al.*, 1981). Around the same time, another class of late-type evolved star emerged which showed signs of possessing both a transition region and a cool wind (e.g., Reimers, 1982). Many of these so-called "hybrid atmosphere" stars now also show evidence for coronal emission, albeit much weaker than on the blue side of the dividing line (Ayres *et al.*, 1997; Dupree *et al.*, 2005).

Following the advent of HST, the important UV diagnostic transitions (e.g., lines from C II, Fe II, Mg II, and O II) could be observed with exquisite detail. The photon-scattering wind produces self-reversals in these chromospheric emission lines and have revealed that, for the most part, the red giant winds accelerate in a quasi-steady manner and are not the result of ballistic ejecta. This is inferred by the increase of wind scattering absorption velocity with optical depth, and thus height in the wind, as shown in Figure 1.2 for λ Vel (Carpenter *et al.*, 1999). The P Cygni-type line profiles, which are indicative of stellar outflows, are also shown in Figure 1.2 for the Mg II and Ca II resonance lines of α Boo (K2 III). For many evolved stars, these disk averaged emission line profiles have also

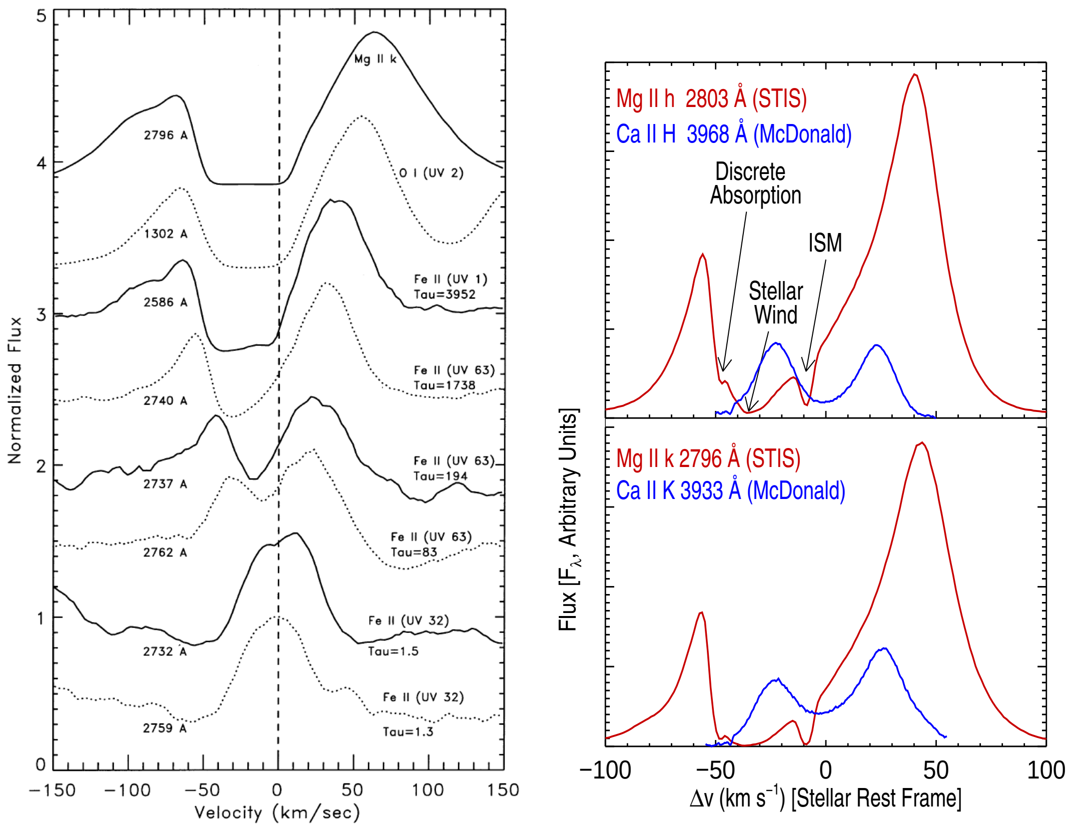

Figure 1.2: *Left: HST* GHRS spectra showing the increase of wind scattering absorption velocity with optical depth for strong chromospheric lines. These data have been taken for λ Vel (a K5 Ib-II star) and shows that its wind accelerates in a quasi-steady manner. *Right:* The Mg II *h* and *k* and Ca II H and K line profiles of α Boo. For many cool evolved stars, these strong resonant lines have often been compared to synthetic profiles to provide estimates of atmospheric properties. Three absorption components are present in the high S/N HST data and are from the ISM, the stellar wind, and an unknown source (See Appendix B).

provided crude estimates of atmospheric properties such as the mass-loss rate and terminal velocity, by comparing them to synthetic profiles based on detailed radiative transfer code (e.g., Robinson *et al.*, 1998).

Chromospheres are the manifestation of surface convection and are found almost exclusively in the cool portion of the H-R diagram (Ayres, 2010a). These non-radiatively heated regions of the inner atmosphere are present in the atmospheres of all late-type evolved stars. The pressure scale height, H, is the height in the atmosphere where the pressure drops by a factor of e^{-1} and is defined as

$$H = \frac{kT_e R_\star^2}{GM_\star}. \tag{1.1}$$

Here, k is the Boltzmann constant, T_e is the electron temperature, $R_\star$ is the stellar radius, G is the gravitational constant, $M_\star$ is the stellar mass, μ is the mean molecular mass, and m_{H} is the mass of a hydrogen atom. The much larger radii of evolved stars means that their *typical* scale height is over two orders of magnitude greater than that of the Sun. It is for this reason that their chromospheres are believed to be much more extended than the Sun's. The red supergiants are now known to have chromospheres which extend out to a few $R_\star$ (Harper *et al.*, 2001; Lim *et al.*, 1998). However, there is still much debate regarding the spatial extent of chromospheres in red giants. Carpenter *et al.* (1985) used measurements of the total emission-line flux and line ratios within the C II multiplet to conclude that coronal giants had thin chromospheres ($\lesssim 0.1\,R_\star$) while the chromospheres in the non-coronal giants were much more extended ($\sim 2.5\,R_\star$). Recently, Berio *et al.* (2011) found that β Ceti, a coronal giant, has a chromosphere which may extend out to $\sim 1.5\,R_\star$, while Luttermoser *et al.* (1994) finds that the chromospheric spatial extend of an M6 giant to be $\leq 1.05\,R_\star$. These findings are in conflict with one another and it is therefore apparent that the spatial extent of the chromosphere in red giants is currently uncertain.

1.3 Basic Concepts of Stellar Winds

he addition of energy above the photosphere is a requirement for a stellar outflow to escape the gravitational potential of a star. This energy input can be either in the form of a heat input (e.g., ambipolar diffusion heating), or in the form of a momentum input (e.g., radiation pressure on gas species). The momentum input is described by Newton's second law, $F = dp/dt$, where F is the outward force and p is the momentum. For this reason, the presence of an outward force is usually called momentum deposition, in contrast to energy deposition. The momentum deposition is governed by the momentum equation

$$F = \boldsymbol{\rho}.d\boldsymbol{v}/dt \tag{1.2}$$

where ρ is the mass density, and v is the velocity. In spherical symmetry, the velocity gradient is

$$\frac{dv(r,t)}{dt} = \frac{\partial v(r,t)}{\partial t} + \frac{\partial v(r,t)}{\partial r}\frac{dr(t)}{dt} = v\frac{dv}{dr}, \tag{1.3}$$

where we have assumed a stationary flow. The momentum equation for a flow being acted on by an outward directed force per unit mass, $f = f(r)$, is then

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM_\star}{r^2} + f \tag{1.4}$$

where each of these terms have units of $\mathrm{cm\,s^{-2}}$ (e.g., Lamers & Cassinelli, 1999). The term on to the left of Equation 1.4 is the acceleration, which is produced by the gas pressure gradient (first term on right), the gravity (second term on right), and some other force(s) which are contained in f. The gas pressure gradient term is directed outwards (positive) because $dP/dr < 0$.

The gas pressure gradient in Equation 1.4 depends on the temperature structure of the outflow, which in turn depends on the heating and cooling. The effects of energy deposition can be expressed via the first law of thermodynamics

$$\frac{du}{dt} = \frac{dq}{dt} - P\left(\frac{d\rho^{-1}}{dt}\right) \tag{1.5}$$

where $u = (3/2)(\mathcal{R}T/\mu)$ is the internal energy of the system per unit mass, q is the net heat gained per unit mass, and the final term is the work done by the gas per unit time per unit mass. The time dependence can be removed using

$d/dt = v\, d/dr$ to give

$$\frac{dq}{dr} = \frac{3}{2}\frac{\mathcal{R}}{\mu}\frac{dT}{dr} + P\frac{d\rho^{-1}}{dr} \tag{1.6}$$

The ideal gas law can be written as

$$\rho = \frac{\mu P}{\mathcal{R}T}, \tag{1.7}$$

and substituting this into the last term in Equation 1.6 gives a desired expression which relates the gas pressure to the heating:

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{5}{2}\frac{\mathcal{R}}{\mu}\frac{dT}{dr} - \frac{dq}{dr}. \tag{1.8}$$

The energy equation for stellar outflows can then be found by replacing the gas pressure term in the momentum equation with an expression which depends on the temperature structure of the outflow and the heating, i.e., combining Equations 1.4 and 1.8:

$$\frac{d}{dr}\left(\frac{v^2}{2} + \frac{5\mathcal{R}T}{2\mu} - \frac{GM_\star}{r}\right) = f(r) + \frac{dq}{dr}. \tag{1.9}$$

The combination of the terms inside the brackets on the left gives the total energy of the system per unit mass, with the first term being the kinetic energy of the flow, the second term being the enthalpy of the gas (the internal kinetic energy plus the capacity to do work), and the third term being the gravitational potential energy. This equation tells us that the change in total energy of the gas as it moves a unit distance outwards from the star, is equal to the momentum input by the force and the heat input.

The energy equation in this form is called the *Bernoulli* equation. Integration gives

$$\begin{aligned} e(r) &= \frac{v^2}{2} + \frac{5\mathcal{R}T}{2\mu} - \frac{GM_\star}{r} \\ &= e(r_0) + W(r) + q(r) \end{aligned} \tag{1.10}$$

which states that the total energy per unit mass, $e(r)$, is equal to the initial energy per unit mass, $e(r_0)$, at the lower boundary r_0, plus the energy added to the wind in the form of the work done by the force, $W(r)$, and the heat deposition,

$q(r)$. In other words, the total energy added to the wind per unit mass is used to increase the kinetic energy and the enthalpy of the wind, and to lift it out of the gravitational potential well. We can also compare the energy of the wind at the photosphere and at infinity, as described by Lamers (1998). At the photosphere, the total energy is negative and is just the gravitational potential energy, because $v_{\rm esc} \gg \mathcal{R}T_\star/\mu$ and $v_{\rm esc} \gg v(R_\star)$, i.e.,

$$e(r_0) \simeq -\frac{GM_\star}{R_\star}. \tag{1.11}$$

At $r \to \infty$ the potential energy and the enthalpy both go to 0, and so the total energy is the kinetic energy,

$$e(r) \simeq \frac{v_{\infty^2}}{2}. \tag{1.12}$$

Subbing Equations 1.25 and 1.26 into Equation 1.10 then gives

$$\frac{v_{\infty^2}}{2} \simeq -\frac{GM_\star}{R_\star} + W(r) + q(r). \tag{1.13}$$

This equation tells us that a wind can only escape the gravitational potential of its star if there is an output force that provides sufficient momentum input or, if there is an energy source that provides sufficient heat input. These energy and momentum inputs are collectively known as wind driving mechanisms and in Section 1.4 we discuss the different mechanisms which occur across the H-R diagram.

1.4 Stellar Wind Driving Mechanisms Across the H-R Diagram

Stellar winds are a ubiquitous phenomenon across most of the H-R diagram. The various types of winds found throughout the H-R diagram can be broadly grouped into the three main categories of hot radiatively driven stellar winds, solar-type winds, and cool evolved stellar winds, as shown in Figure 1.3. The cool evolved stellar winds can also be broadly grouped into the two categories of warm hybrid winds, and cool dense winds, as discussed in Section 1.2.

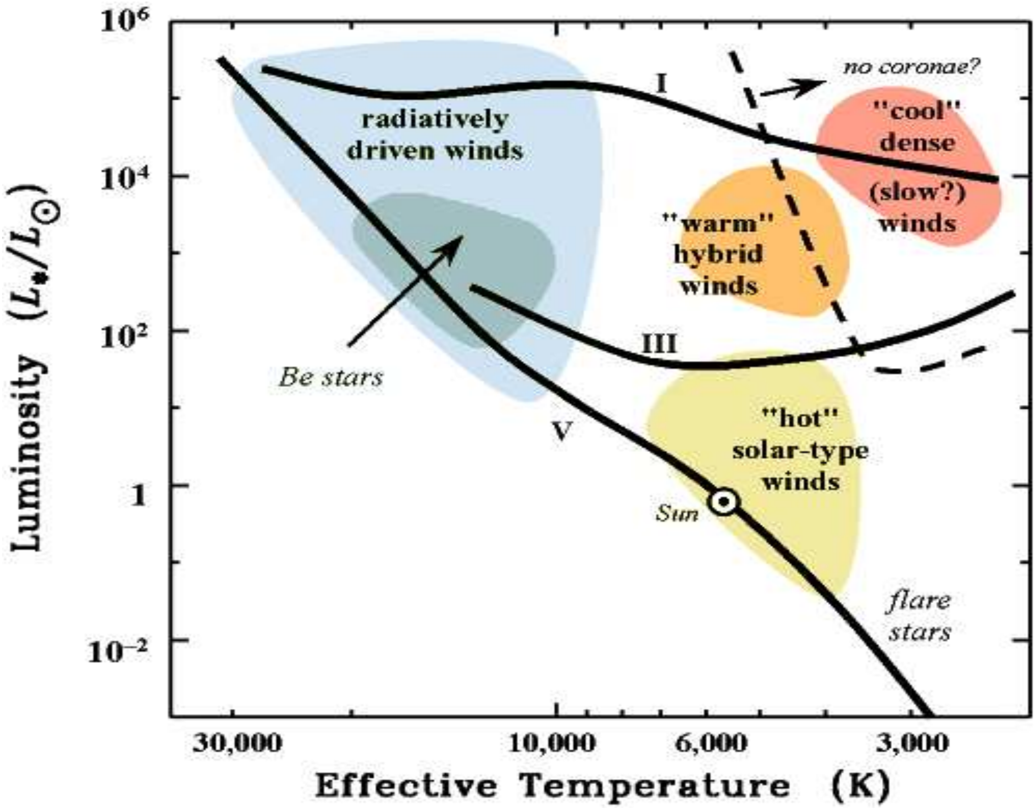

Figure 1.3: Stellar winds across the H-R diagram can be broadly grouped into the three main categories of hot radiatively driven stellar winds (upper left), solar-type winds (center right), and cool evolved stellar winds (upper right). The cool evolved stellar winds can also be broadly grouped into the two categories of coronal (orange group) and noncoronal type (red group). *Image credit:* Steven Cranmer.

1.4.1 Radiatively Driven Winds

Stars earlier than spectral type ~A2 emit their peak radiation in the UV, and so it was not until the early UV rocket observations that the presence of strong winds was confirmed from these stars (e.g., Morton, 1967). The broad P Cygni line profiles observed, indicated mass loss rates as high as $10^{-5}\,M_\odot\,\mathrm{yr}^{-1}$ and wind terminal velocities up to $3500\,\mathrm{km\,s^{-1}}$. Even though the lifetime of these massive hot stars is relatively short at just a few million years, their large mass loss rates can substantially reduce the original stellar mass by a factor of two or more (Owocki, 2004). Indeed, these stars typically end up as "Wolf-Rayet" stars, which often appear to have completely lost their original envelope of hydrogen. These early-type stars do not exhibit the strong sub-surface convection that is present in solar and late-type stars (i.e., cool stars), and are therefore not believed to posses coronae. Their winds are therefore expected to remain at temperatures comparable to the star's surface, and so the gas-pressure term in Equation 1.4 is not sufficient to drive their winds. Instead these stars are known to have a high radiative flux (since this scales as the fourth power of the surface temperature), and Castor *et al.* (1975) showed that this flux can accelerate a time-steady wind, by coupling with optically think atomic lines in regions above the photosphere, where the continuum is optically thin. Therefore, the winds of these stars are effectively driven by the pressure of the star's emitted radiation and so the dominant term in Equation 1.4 is a radiation pressure term contained within f.

1.4.2 Solar Type Winds

Our understanding of the wind driving mechanisms for stars with *hot* solar-type winds is mainly based on solar theory and observations. Equation 1.4 was the cornerstone of many early attempts to explain the dynamics of the solar corona. Chapman & Zirin (1957) assumed a static corona (i.e., the acceleration term was zero) and that the pressure gradient was the only outward force (i.e., $f = 0$). Their results were unphysical, however, as they found that the density goes to infinity at large distances, and that the pressure was many orders of magnitude greater than that of the ISM. Parker (1958) assumed that there was a continuous isothermal outflow of material from the Sun caused only by the thermal pressure gradient term in Equation 1.4 (i.e., $f = 0$). He used the mass continuity equation, and the equation of state, to replace the pressure gradient term with a function depending only on velocity. Parker coined the solution to his simple analytical model as the "solar wind" and the predictions his model made for the solar velocity were confirmed shortly afterwards by some of the first space probes (e.g., Neugebauer & Snyder, 1962). The assumptions of a radially expanding and isothermal outflow are not fully accurate in reality, however, and Parker's solution is an approximate characterization of the observed solar wind.

Coronal winds are generally tenuous, with mass-loss rates being too small to be of evolutionary importance. For example, at the Sun's current rate of mass-loss, about $10^{-14}\,M_{\odot}\,\mathrm{yr}^{-1}$, its mass would be reduced by only $\sim 0.01\%$ during its main sequence lifespan of $10^{9}\,\mathrm{yr}$. Wind velocities are generally high ($200 \rightarrow 800\,\mathrm{km\,s^{-1}}$), except for the very low gravity stars whose velocities are believed to be less Drake & Linsky (1986). Although Parker (1958) showed that the solar wind is a consequence of the thermal pressure gradient of the hot corona, the question of which mechanism drives the solar wind is still controversial, i.e., it is not understood how mechanical energy (convection) is transferred above the solar surface. The dissipation of Alfv́en waves is a reliable candidate as a primary source of coronal heating (Cranmer & Saar, 2011), although other sources of energy and momentum probably exist (e.g., Parker, 1983, 1988). These waves can transfer energy from the surface convection up to the wind acceleration region because they can travel longer distances due to their incompressible nature (e.g., Hollweg, 1973). The dissipation of these waves then transfer momentum and energy to the gas via a cascade from large to small eddies (Verdini & Velli, 2007). Evolved stars blueward of the Linsky-Haisch dividing line also posses cornae and may share a similar mass loss mechanism to that of coronal stars on the main sequence.

1.4.3 Cool Evolved Stellar Winds

Cool evolved stars can generally be grouped into three main categories based on their mass and evolutionary status: (1) massive evolved red supergiants, (2) low and intermediate mass highly evolved stars ($0.8 - 8\,M_{\odot}$) known as AGB stars, and, (3) low and intermediate mass less evolved red giant stars. AGB stars lose a significant fraction of their mass through slow, massive winds at a rate of $10^{-8} \rightarrow 10^{-4}\,M_{\odot}\,\mathrm{yr}^{-1}$ (van Loon *et al.*, 2005). This mass loss occurs as a result of stellar pulsation (Habing, 1996) which levitates atmospheric gas from the stellar surface, followed by acceleration of dust grains by radiation pressure (Gehrz & Woolf, 1971).

The mass loss mechanisms operating in red giants and red supergiants remain largely unknown and the theory governing mass-loss in AGB stars is not appropriate (Josselin & Plez, 2007). Red giants and red supergiants have small amplitude variations and so they do not pulsate in a similar manner to AGB stars. Also, significant amounts of dust are only found at large radii for red supergiants (Danchi *et al.*, 1994), while red giants may have little or no dust in their outflows (Jones, 2008). One of the most plausible mass-loss mechanisms for these stars over the past few decades has been the transfer of energy and momentum to the wind by the dissipation of Alfvén waves. Unlike acoustic waves, Alfvén waves have large damping lengths that can transfer energy and momentum to the gas over many stellar radii. Alfvén wave models were original known to result in high velocity winds, however, unlike what is observed for cool evolved stars. However, Hartmann & MacGregor (1980) showed that these observed low outflow velocities and high mass loss rates could be reproduced if a wave damping mechanism is effective close ($r < 2\,R_\star$) to the star. Hartmann & Avrett (1984) constructed an Alfvén wave model for the red supergiant Betelgeuse which predicted its wind to have an electron temperature of 8000 K at $4\,R_\star$ and remaining above 5000 K out to $10\,R_\star$. However, the radio observations of Lim *et al.* (1998) revealed a much cooler wind cooler wind (see Chapter **??**) in conflict with the models of Hartmann & Avrett (1984). Also, the semi-empirical model of Harper *et al.* (2001) found the wind to be only lightly ionized, which means that Alfvén waves, which couple only to ionized gas, are unlikely to be capable of transferring the required energy and momentum to the winds of red supergiants; although they still may be important for the less massive red giants. One current school of thought is that giant convection cells may initiate the mass-loss process in red supergiants (e.g., Lim *et al.*, 1998), although currently no model exits to explain how such a mechanism would operate.

1.5 Red Giant and Red Supergiant Evolution

Once a star has exhausted the hydrogen in its core, it evolves off the main sequence and spends the majority of its post-main sequence lifetime either as a red giant or a red supergiant. The vast majority of stars are of either low to intermediate mass (i.e., $M_\star \lesssim 8\,M_\odot$), and evolve to become red giants, while the rare massive stars (i.e., $M_\star \gtrsim 8\,M_\odot$) generally evolve to become red supergiants.

These evolved late-type stars contain a condensed core with an extended envelope and have cooler effective temperatures than when on the main sequence. There is currently no consensus in the scientific community on why stars become red giants or red supergiants (e.g., Stancliffe *et al.*, 2009; Sugimoto & Fujimoto, 2000). A common explanation is that the initial expansion is driven by the envelope maintaining thermal equilibrium in response to increasing luminosity from the core. This expansion causes local cooling, allowing heavy elements to recombine, therefore causing an increase in opacity. This increase in opacity traps energy leading to a runaway expansion that brings the star to the red giant or supergiant region of the H-R diagram (Renzini *et al.*, 1992). However, Iben (1993) computed evolutionary models for intermediate mass stars with with the opacity held constant throughout, and showed that these models still became giants. This meant that opacity was not responsible for transition to a red giant.

1.5.1 Change in Atmospheric Dynamics

The expansion of a star's radius as it evolves off the main sequence greatly affects the dynamics of its atmosphere through the change of surface gravity. The decrease in surface gravity causes the density of the winds and density scale height to increase, resulting in very extended atmospheres. The mass-loss rate also increases due to the increase in the stellar surface area ($\propto R_\star^2$) and the increase of the density. In Table 1.1 we describe the typical properties of a 1 and 15 $M_\odot$ star both on the main sequence and as an evolved late-type star. We use these as examples to highlight the changes in the atmospheric dynamics when a star becomes a red giant or red supergiant. For these stars, the massive increase in stellar radius means that the density scale height, H_ρ, as a fraction of the stellar radius, $H_\rho/R_\star \propto T_{\rm eff} R_\star$, is ~ 2 orders of magnitude greater than when on the main sequence. There is also a drastic change in the terminal wind velocity, v_∞. While on the main sequence, hot massive stars have wind velocities that are many times the photospheric escape velocity, $v_{\rm esc}$, while solar type stars have terminal wind speeds close to $v_{\rm esc}$. For evolved late-type stars the terminal wind velocity is generally much less than the photospheric escape velocity, signally that the onset of these winds must be at several stellar radii. The final column in Table 1.1 is a comparison of the integrated mass-loss of these stars while on and off the main sequence. It is clear that during the time spent in these evolved states, t, these stars lose a significant proportion of their initial mass, i.e., $\sim 30\%$ for massive stars and $\sim 10\%$ for the lower to intermediate mass stars.

Table 1.1: Typical properties for main sequence and evolved 15 and $1\,M_\odot$ stars.

Evolutionary Stage[a]	$R_\star$ ($R_\odot$)	$H_\rho/R_\star$	v_∞ ($\mathrm{km\,s^{-1}}$)	$v_{\rm esc}$ ($\mathrm{km\,s^{-1}}$)	t (yr)[b]	$\dot{M}_\star$ ($M_\odot\,\mathrm{yr^{-1}}$)	$t \times \dot{M}_\star$ ($M_\odot$)
MS O/B	5	10^{-4}	3000	1000	10^6	10^{-6}	1
RSG	1000	10^{-2}	20	75	5×10^5	10^{-5}	5
MS F/G	1	10^{-4}	400	600	10^{10}	10^{-14}	10^{-4}
RG	40	10^{-2}	40	100	10^9	10^{-10}	0.1

[a] MS O/B = main sequence spectral type O and B stars. RSG = red supergiant. MS F/G= main sequence spectral type F and G stars. RG = red giant.
[b] The lifetimes for the evolutionary phases of the massive star are taken from Stothers & Chin (1969)

1.5.2 Evolutionary Tracks

1.6 Radio Emission from Stellar Atmospheres

give fundamental frequencies

radio hr diagram gudel

Circumstellar environments (Lamers and Cassinelli CO)

$$F_\nu \approx 0.1 \left(\frac{T_b}{10^6\,K}\right)\left(\frac{\nu}{1\,\mathrm{GHz}}\right)^2\left(\frac{r}{10^{11}\,\mathrm{cm}}\right)^2\left(\frac{1\,\mathrm{pc}}{d}\right)^2 \quad \mathrm{mJy} \tag{1.14}$$

(Güdel, 2002)

1.6.1 Free-free Emission

1.6.2 Molecular Line Emission

1.6.3 Other Emission Mechanisms

Recombination Line Emission

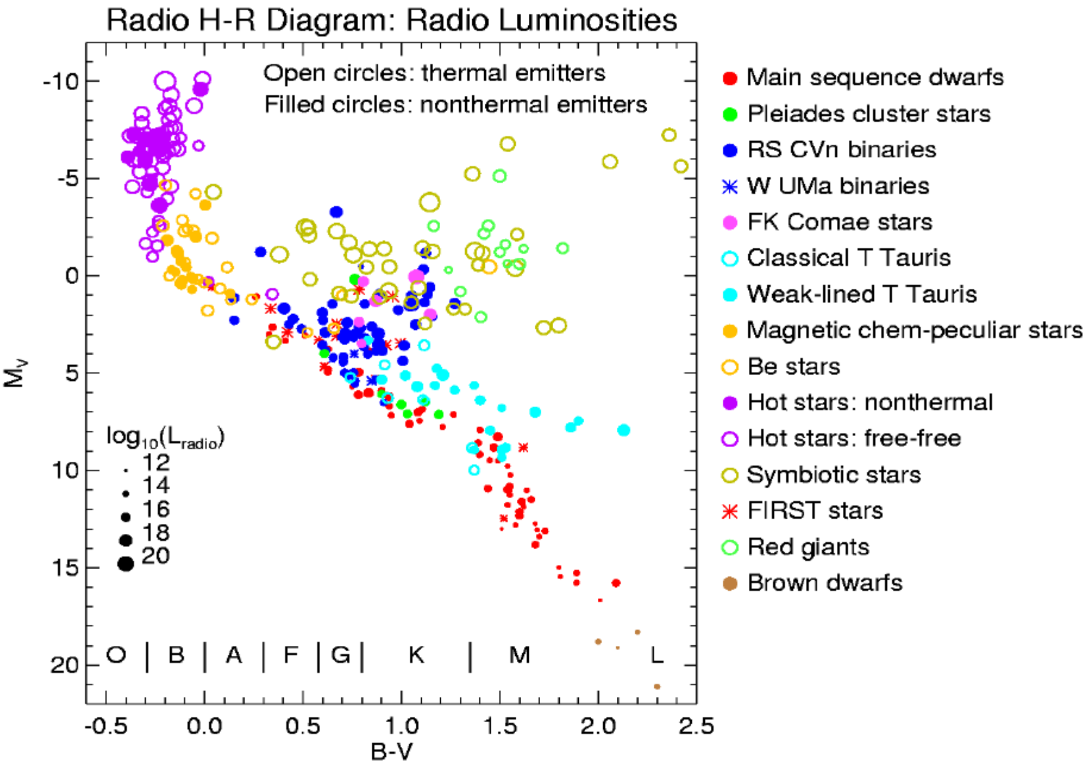

Figure 1.4

Non-Thermal Emission

1.7 Radio Observations of Stellar Atmospheres

In the following sections we present the basic definitions used to describe radio observations of stellar atmospheres. We define the *brightness temperature* which is commonly used in radio astronomy to measure the brightness of a source, along with its relationship to the fundamental quantity measured by a radio telescope, the *flux density*. Focusing on thermal emission, we describe how the flux density varies with frequency when observing both optically thin and optically thick stellar atmospheres.

1.7.1 Brightness Temperature

In thermodynamic equilibrium the spectral distribution or brightness, B_ν, of the radiation of a black body with temperature T_e is given by the Planck law

$$B_\nu(T_e) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT_e} - 1} \tag{1.15}$$

and has units of flux per frequency interval per solid angle. One can easily switch to a wavelength scale using $B_\nu d\nu = B_\lambda d\lambda$. When $h\nu \ll kT_e$ Equation 1.15 becomes the *Rayleigh-Jeans Law*

$$B_\nu(T_e) = I_\nu(T_e) = \frac{2\nu^2 kT_e}{c^2}. \tag{1.16}$$

This equation does not contain Plank's constant and therefore is the classical limit of the Planck Law. We have also included the specific intensity, I_ν, here as it has the same units as the spectral brightness and for blackbody radiation, $I_\nu(T_e) = B_\nu(T_e)$. This equation is valid for all thermal radio sources except in the millimeter or sub-millimeter regime at low temperatures (Rohlfs & Wilson, 1996). In the Rayleigh-Jeans relation, the brightness is strictly proportional to the thermodynamic temperature of the black body. In radio astronomy it is customary to measure the brightness of an object by its *brightness temperature*, T_b. Therefore, the brightness temperature is the temperature at which a blackbody would have to be in order to reproduce the observed brightness of an object at frequency ν and is defined as

$$T_\mathrm{b} = \frac{c^2}{2k\nu^2} I_\nu. \tag{1.17}$$

If $h\nu/kT \ll 1$ and if I_ν is emitted by a blackbody, then T_b is the thermodynamic temperature of the source. If other processes are responsible for the emission or if the frequency is so high that Equation 1.16 is not valid, then T_b is different from the thermodynamic temperature of a black body.

The equation of radiative transfer describes the change in specific intensity of a ray along the line of sight in a slab of material of thickness ds

$$\frac{dI_\nu}{ds} = \varepsilon_\nu - \kappa_\nu I_\nu, \tag{1.18}$$

where ε_ν and κ_ν are the emissivity (in $\mathrm{erg\,s^{-1}\,cm^{-3}\,Hz^{-1}\,sr^{-1}}$) and the absorption/opacity coefficient (in $\mathrm{cm^{-1}}$) of the plasma. In thermodynamic equilibrium the radiation is in complete equilibrium with its surroundings and the brightness distribution is described by the Planck function

$$\frac{dI_\nu}{ds} = 0, \qquad I_\nu = \frac{\varepsilon_\nu}{\kappa_\nu} = B_\nu(T_e). \tag{1.19}$$

Equation 1.18 can be solved by first defining the optical depth, $d\tau_\nu$, as

$$d\tau_\nu = -\kappa_\nu ds, \tag{1.20}$$

and then integrated by parts between 0 to s, and τ to 0, to give

$$I(s) = I(0)e^{-\tau(s)} + \int_{\tau(s)}^{0} e^{-\tau}\frac{\varepsilon_\nu}{\kappa_\nu}d\tau. \tag{1.21}$$

The second term within the integral is known as the source function, S_ν, and this can be taken outside of the integral in the case of a homogeneous source, i.e., one for which both the emissivity and absorption coefficient are constant along the ray path. The solution then to the equation of radiative transfer for a homogeneous source is

$$I_\nu = I_0 e^{-\tau} + \frac{\varepsilon_\nu}{\kappa_\nu}(1 - e^{-\tau}). \tag{1.22}$$

Using Equations 1.17 and 1.19 one obtains

$$T_b = T_0 e^{-\tau} + T_e(1 - e^{-\tau}). \tag{1.23}$$

This equation assumes thermodynamic equilibrium and so only holds for a thermal source. If T_e is replaced with $T_{\mathrm{eff}} = h\nu/k$ then this equation becomes valid for a homogeneous nonthermal source, i.e.,

$$T_b = T_0 e^{-\tau} + T_{\mathrm{eff}}(1 - e^{-\tau}). \tag{1.24}$$

For an isolated thermal source, there are two limiting cases:

$$T_b = T_e \quad \text{(i.e., for optically thick } \tau \gg 1) \tag{1.25}$$

and

$$T_b = \tau T_e \qquad \text{(i.e., for optically thin } \tau \ll 1\text{)}. \tag{1.26}$$

In these equations, T_e can also be replaced by $T_{\rm eff}$ if the radio emission emission is non-thermal. Also, these equations are only valid if the source is spatially resolved. If the source is unresolved then an upper limit to $T_e/T_{\rm eff}$ is found.

1.7.2 Brightness Temperature and Flux Density

The flux density, F_ν, is a fundamental quantity measured by a radio telescope and is usually measured in Janskys (Jy) where 1 Jy$= 1 \times 10^{-26}$ W m^{-2} Hz^{-1}. The observed flux density measured by the radio telescope is

$$F_\nu = \int_\Omega I_\nu \, d\Omega \tag{1.27}$$

where Ω is the solid angle subtended by the star. The radio emission from evolved cool stars is almost purely thermal and so Equation 1.27 becomes

$$F_\nu = \frac{\pi R_\star^2}{d^2} \frac{2k\nu^2 T_b}{c^2}. \tag{1.28}$$

The angular diameter of a star in radians is $\phi_\star = 2R_\star/d$ and so

$$F_\nu = \frac{\pi k \phi_\star^2 T_b}{2\lambda^2} \tag{1.29}$$

If $\phi_\star$ has major and minor axes $\phi_{\rm maj}$ and $\phi_{\rm min}$ then

$$T_b(K) = 1.96 F_\nu(\text{mJy}) \left(\frac{\lambda}{\text{cm}}\right)^2 \left(\frac{\phi_{\rm min}}{\text{arcsec}} \frac{\phi_{\rm min}}{\text{arcsec}}\right)^{-1}. \tag{1.30}$$

Therefore, if an optically thick stellar atmosphere can be spatially resolved (i.e., $\phi_{\rm maj}$ and $\phi_{\rm min}$ can be measured) then the flux density at a particular wavelength tells gives the brightness temperature and therefore the electron temperature. Unfortunately, the number of stars that can have their atmospheres spatially resolved at radio wavelengths is low due to their relatively small angular diameters. However, different layers of stellar atmospheres can still be probed due to the nature of the free-free radio opacity which is discussed in the next section.

1.7.3 Thermal Free-free Radio Opacity

In Section 1.6.1 we derived an expression for the thermal free-free emissivity of an ionized gas. Since we assumed LTE at some temperature T, we can use Kirchoff's law to find the thermal radio free-free opacity (absorption coefficient):

$$\kappa_\nu^{ff} = \frac{\epsilon_\nu^{ff} c^2}{2kT_e\nu^2} \tag{1.31}$$

Substituting in Equation **??** then gives a value for the radio opacity which is corrected for stimulated emission

$$\kappa_\nu^{ff} = \frac{0.018Z^2 n_e n_i g_{ff}(\nu, T_e)}{T_e^{1.5}\nu^2} \tag{1.32}$$

The Gaunt factor is slightly dependent on temperature and frequency and at cm-wavelengths is given by

$$g_{ff}^{cm} = 11.96T_e^{0.15}\nu^{-0.1} \tag{1.33}$$

(Altenhoff *et al.*, 1960), while in the sub-millimeter regime it is slightly different

$$g_{ff}^{sub-mm} = 24.10T_e^{0.26}\nu^{-0.17} \tag{1.34}$$

(Hummer, 1988). The abundant species in the atmospheres of cool evolved stars are either neutral or single ionized so that $Z = 1$ and $n_e = n_i$. Focusing on centimeter wavelengths, the radio opacity is then

$$\kappa_\nu^{ff} = \frac{0.212n_e^2}{T_e^{1.35}\nu^{2.1}} \qquad \mathrm{cm}^{-1}. \tag{1.35}$$

Therefore, the free-free opacity increases towards lower frequencies as $\kappa_\nu^{ff} \propto \nu^{-2.1}$ (or longer wavelengths as $\kappa_\lambda^{ff} \propto \lambda^{2.1}$). This means that the optical depth, $\tau_\lambda = \int \kappa_\lambda dr$, also increases towards longer wavelengths implying that the effective radius (i.e., the radius where $\tau_\lambda = \tau_{\mathrm{radial}}$) will increase with longer wavelengths. As a result, different layers of unresolved stellar atmospheres can be probed by observing them at different radio wavelengths.

In LTE, the solution to the equation of radiative transfer (i.e, Equation 1.22) for a plasma with no background source can be written as

$$I_\nu = B_\nu(1 - e^{-\tau}). \tag{1.36}$$

An example of such a source is an isolated H II region. At long enough wavelengths the H II region becomes opaque so that $\tau_\nu \gg 1$. Equation 1.36 then tells us that the spectrum approaches that of a black body with a flux density varying as $F_\nu \propto \nu^2$. At short wavelengths where $\tau_\nu \ll 1$, the H II region is almost transparent, and the flux density becomes

$$F_\nu \propto \frac{2kT_e\nu^2}{c^2}\tau_\nu \propto \nu^{-0.1}. \tag{1.37}$$

These two scenarios are shown in Figure **??** along with the point where these two slopes intersect which corresponds to the frequency at which $\tau \simeq 1$. When the radio spectrum is plotted on a log-log plot as in Figure 1.5, the spectral slope is referred to as the spectral index, α, and is defined:

$$\alpha = \frac{d\log F_\nu}{d\log\nu}. \tag{1.38}$$

1.7.4 Radio Excess from Stellar Outflows

Cool evolved stars have ionized or partially ionized outflows which emit an excess of continuum emission at long wavelengths. This flux excess is due to thermal free-free emission and is measured relative to the expected photospheric radio flux. If the atmosphere only consisted of a static homogeneous isothermal chromosphere then the radio spectrum would be the summation of the Rayleigh-Jeans tail of the Planck function from the photosphere and the H II spectrum discussed in the previous section. At long wavelengths then, this spectrum would again have a power law of slope $F_\nu \propto \nu^2$. Cool evolved stellar atmospheres cannot in general be described by this simple model because they possess stellar winds which are escaping the gravitational potential of the star. The atmospheric density thus varies with distance from the star. In this section we briefly outline a simple analytical model for the centimeter radio flux for a star with a isothermal, constant velocity and ionization fraction wind. In Chapter **??** we relax some of

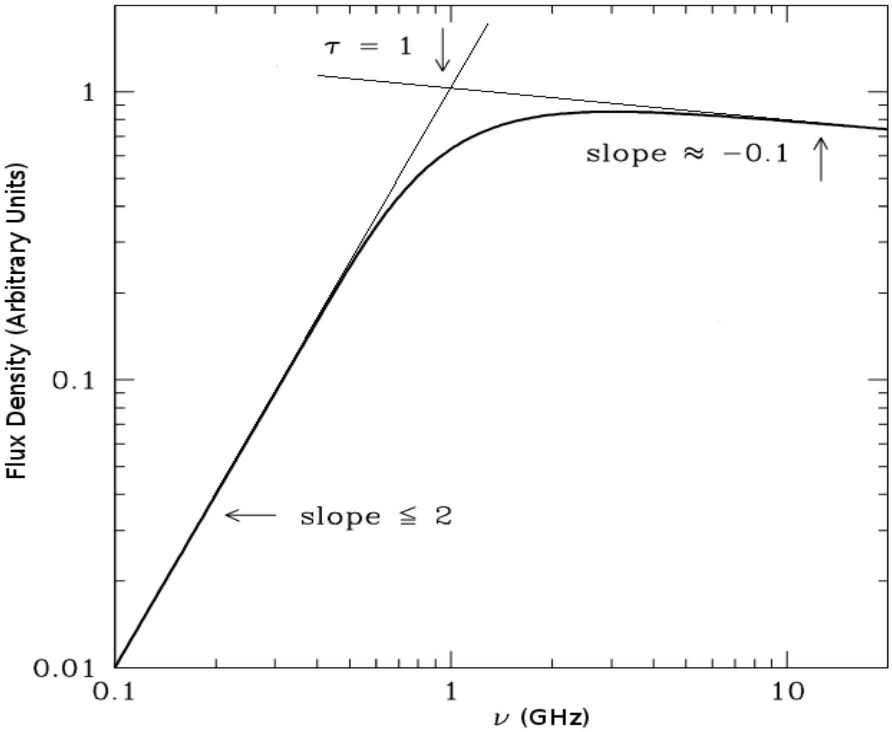

Figure 1.5: The radio spectrum for a hypothetical H II region with no background illuminating source. At long wavelengths the source becomes opaque and has a black body like spectrum with $\alpha = 2$. At short wavelengths where $\tau_\nu \ll 1$, the H II region is almost transparent and $\alpha = -0.1$. Image adapted from NRAO's *Essential Radio Astronomy* course.

these assumptions about the atmosphere's properties to derive a more complete description of the centimeter radio spectrum for these stars.

To calculate the optical depth, we assume spherical geometry and integrate along a ray in the z direction with impact parameter b, as shown in Figure ??. The total optical depth at a frequency ν is then

$$\tau_\nu = \int_{-\infty}^{\infty} \kappa_\nu dz \tag{1.39}$$

where the opacity is defined in Equation 1.35. For a constant velocity the electron

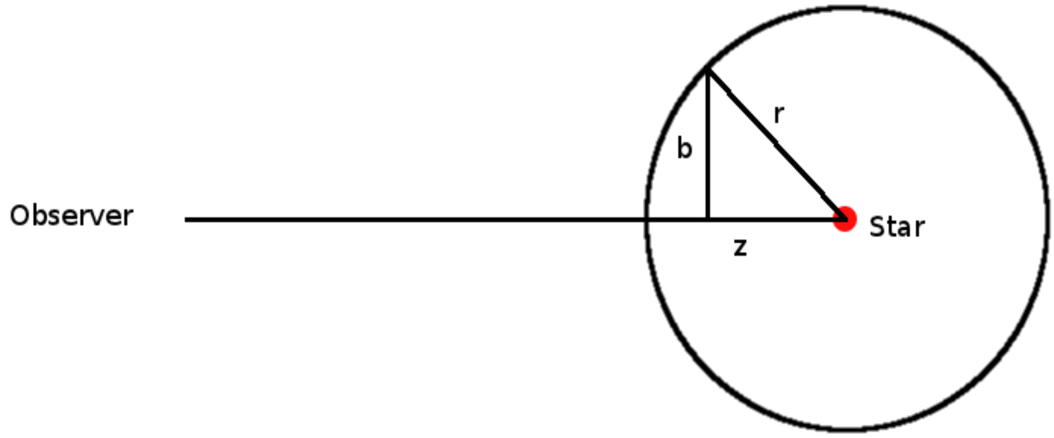

Figure 1.6: In spherical geometry the observer integrates along a ray path in the z direction, with impact parameter b, to calculate the total optical depth τ_ν, at a frequency ν.

density is just $n_e(r) = n_e(r_0)^2(r_0/r)^2$ and so the optical depth can be written as

$$\tau_\nu = \frac{0.212 n_e(r_0)^2 r_0^4}{T^{1.35}\nu^{2.1}} \int_{\infty}^{\infty} \frac{dz}{(b^2+z^2)^2} \tag{1.40}$$

The solution to this integral is given by

$$\int_{-\infty}^{\infty} \frac{dz}{(b^2+z^2)^{A/2}} = b^{1-A}\sqrt{\pi}\frac{\Gamma(A/2-1/2)}{\Gamma(A/2)} \tag{1.41}$$

and so the total optical depth along a ray with impact parameter b is:

$$\tau_\nu = \frac{C}{b^3} \qquad \text{where} \quad C = \frac{0.333 n_e(r_0)^2 r_0^4}{T^{1.35}\nu^{2.1}} \tag{1.42}$$

To calculate the flux density we use Equation 1.27 and assume that the source function is given by the Planck function in the Rayleigh-Jeans approximation:

$$F_\nu = \frac{2\pi}{d^2}\frac{2\nu^2 kT}{c^2}\int_0^{\infty}(1-e^{-C/b^3})b\,db \tag{1.43}$$

This integral can be solved using the following expression

$$\int_0^{\infty} y^{v-1}(1 - e^{-uy^p})dy = \frac{-1}{|p|}u^{-v/p}\Gamma\left(\frac{v}{p}\right), \tag{1.44}$$

which is given in Gradshteyn & Ryzhik (1994). The solution to our integral is then $1.339C^{2/3}$ and so the total flux density can be written as

$$F_\nu = \frac{1.24 \times 10^{-16} n_e(r_0)^{4/3} r_0^{8/3} T^{0.1} \nu^{0.6}}{d^2} \quad \text{Jy}. \tag{1.45}$$

This equation shows that the expected spectral index for an isothermal constant velocity and ionization fraction stellar outflow (i.e., a constant property wind) is $\alpha = 0.6$.

The free-free emission from evolved cool stars is weak (usually less than 1 mJy at $\lambda > 3$ cm) and therefore only a handful of these stars have known radio spectral indices at long wavelengths. The small number of such stars whose spectral indices are known have values which are greater than 0.6 (e.g. Drake & Linsky, 1986) due the assumptions in the constant property wind model being too simplistic. Betelgeuse is by far the best studied evolved cool star at radio wavelengths and its radio spectrum is shown in Figure 1.7 (Newell & Hjellming, 1982). Its spectral index clearly deviates from 0.6. In Chapter **??** we derive a new version of Equation 1.45 which accounts for a thermal gradient in the outflow along with flow acceleration. Nevertheless, Figure 1.7 is a good example of the radio flux excess which is present for all all cool evolved. Even though the observed flux density decreases to longer wavelengths, the excess increases relative to the expected photospheric flux as clearly seen in Figure 1.7.

1.8 Outline and Goals of this Thesis

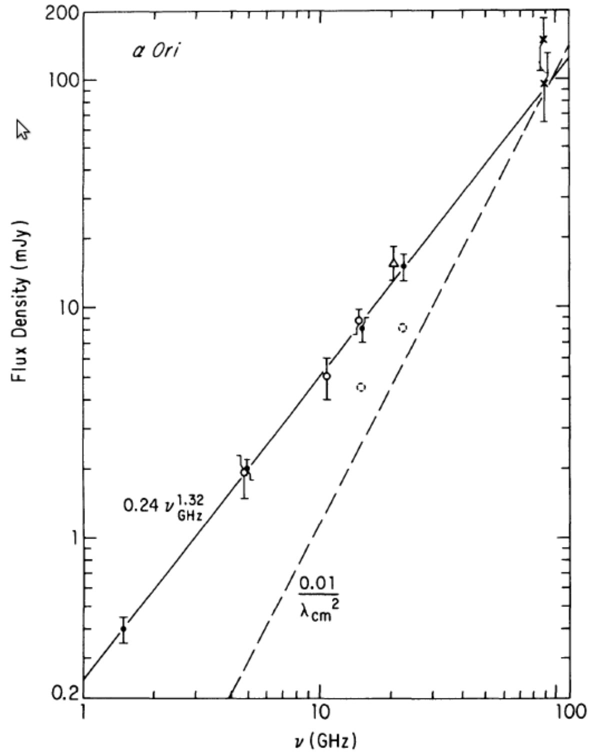

Figure 1.7: Example of a cool evolved star's radio flux excess, which is a direct result of their ionized atmospheres. Here, multi-wavelength radio observation of Betelgeuse are plotted along with a best fit power law indicating a spectral index of $\alpha = 1.32$ (Newell & Hjellming, 1982). Even though the observed flux density decreases at longer wavelengths, the excess increases relative to the expected photospheric flux (dashed line).

A

List of Abbreviations Used in this Thesis.

Table A.1: List of Abbreviations

First entry	Second entry
BIMA	Berkeley Illinois Maryland Association
CARMA	Combined Array for Research in Millimeter-wave Astronomy
CSE	Circumstellar Envelope
DDT	Director's Discretionary Time
e-MERLIN	e-Multi-Element Radio Linked Interferometer Network
FOV	Field of View
GREAT	German Receiver for Astronomy at Terahertz Frequencies
HPBW	Half Power Beamwidth
HST	Hubble Space Telescope
IOTA	Infrared Optical Telescope Array
IR	Infrared
IRAM	Institut de Radioastronomie Millimétrique
IUE	International Ultraviolet Explorer
LSR	Local Standard of Rest
MEM	Maximum Entropy Method
MHD	Magnetohydrodynamic
OVRO	Owens Valley Radio Observatory
RFI	Radio Frequency Interference

Continued on next page

Table A.1 – *Continued from previous page*

First entry	Second entry
S/N	signal-to-noise ratio
SOFIA	Stratospheric Observatory for Infrared Astronomy
SMA	Submillimeter Array
SZA	Sunyaev-Zel'dovich Array
SIS	superconductorinsulatorsuperconductor
UV	Ultraviolet
VLA	Karl G. Jansky Very Large Array
VLBA	Very Long Baseline Array
VLT	Very Large Telescope

B

Discrete Absorption Feature

The temperature equation outlined in Chapter **??** assumes that the wind is homogenous, but this may not be the case for Arcturus. During this study we analyzed STIS spectra of Arcturus from the online StarCAT catalog (Ayres, 2010b). The Mg II h and k lines from data obtained in 2001 show a wind velocity $\sim 30-40$ km s^{-1}, which is similar to that adopted in the Drake models for this star Drake (1985). A narrow discrete absorption feature is found at -49 km s^{-1} in the broad blue-shifted wind absorption component of both lines as shown in Figure B.1. For this discrete feature we find a most probable turbulent velocity of 3.4 km s^{-1} and a Mg column density[1] of 1.4×10^{12} cm^{-2}. A Mg column density of 10^{15} cm^{-2} is required to produce the blueward absorption components in the h and k lines (McClintock *et al.*, 1978). Therefore, this discrete absorption feature accounts for $\sim 0.1\%$ of the total wind column density.

[1]Assuming all Mg to be Mg II

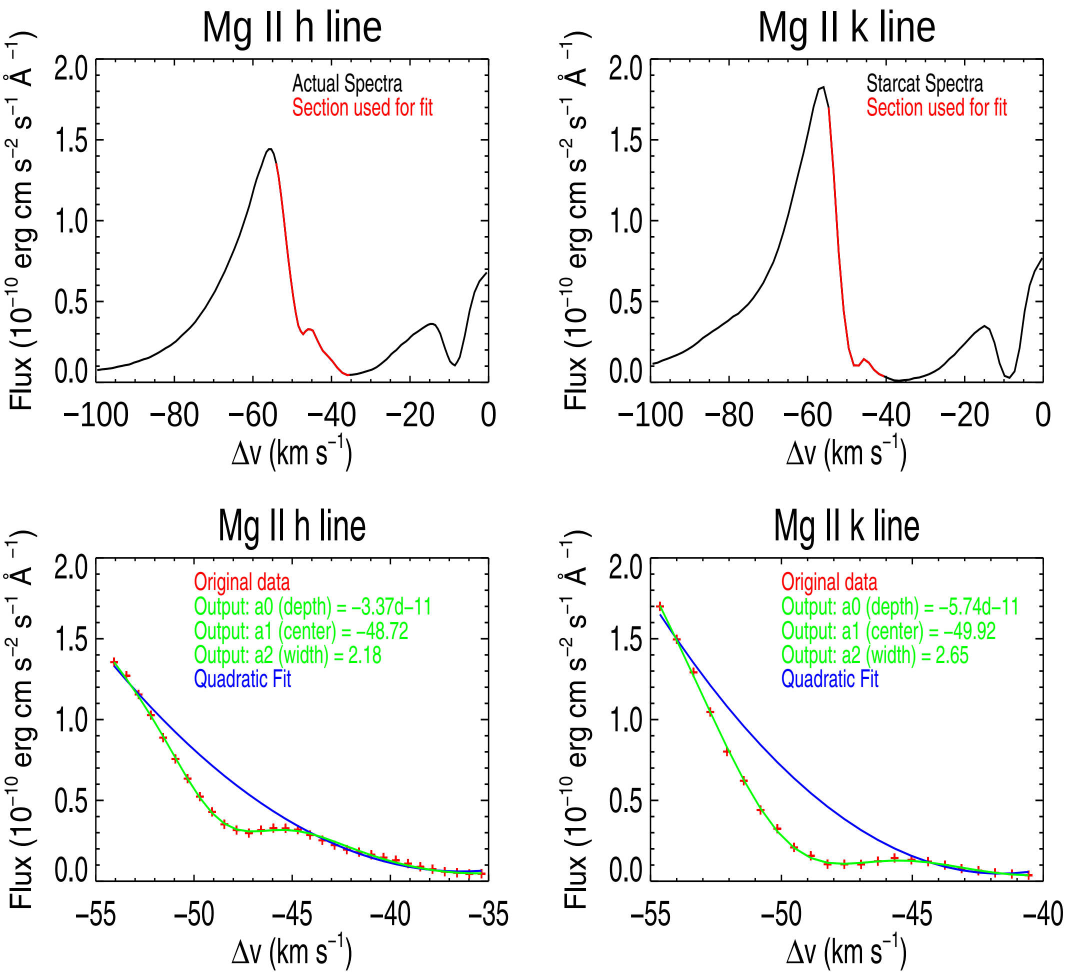

Figure B.1: Analysis on the absorption feature found in the Mg II h and k lines. A function composed of a linear combination of a Gaussian and a quadratic fitted the discrete absorption feature the best. The red data in the upper row shows the data that is used in this analysis.

C
Ambipolar Diffusion Heating

Considering a steady flow accelerating in an inertial frame, the equation of motion can be written as

$$\rho_n \boldsymbol{a}_n = \rho_n \boldsymbol{g} + \boldsymbol{f}_d \tag{C.1}$$

for the neutral species n, and

$$\rho_i \boldsymbol{a}_i = \rho_i \boldsymbol{g} - \boldsymbol{f}_d + \boldsymbol{f}_L \tag{C.2}$$

for the ion species, i. The flow acceleration is defined as $\boldsymbol{a} \equiv (\boldsymbol{v}.\nabla)\boldsymbol{v}$ and the gravitational acceleration is $\boldsymbol{g} \equiv \nabla(GM_\star/r)$. The volumetric drag force of the ions on the neutrals is defined as

$$\boldsymbol{f}_d = \gamma \rho_n \rho_i (\boldsymbol{v}_i - \boldsymbol{v}_n) \tag{C.3}$$

and $\boldsymbol{f}_L$ is the volumetric Lorentz force. The equation of motion for the combined ion-neutral fluid is found by addition of Equations C.1 and C.2

$$\rho \boldsymbol{a} = \rho \boldsymbol{g} + \boldsymbol{f}_L \tag{C.4}$$

where $\rho \equiv \rho_n + \rho_i$ is the mass density without the electrons and $\boldsymbol{a} \equiv (\rho_n \boldsymbol{a}_n + \rho_i \boldsymbol{a}_i)/\rho$ is the total acceleration.

The gravitational acceleration term can then be eliminated from Equations C.1 and C.2 to give

$$\boldsymbol{a}_n - \boldsymbol{a}_i = \left(\frac{1}{\rho_n} + \frac{1}{\rho_i} \right) \boldsymbol{f}_d - \frac{1}{\rho_i} \boldsymbol{f}_L. \tag{C.5}$$

Assuming then that the acceleration of the neutrals and ions are the same we get

$$\boldsymbol{f}_d = \left(\frac{\rho_n}{\rho_n + \rho_i} \right) \boldsymbol{f}_L. \tag{C.6}$$

This equation tells us that for a lightly ionized outflow the drag force is almost equal to the Lorentz force. We can now obtain an expression for the slip velocity, $\boldsymbol{w}$, by subbing this equation into Equation C.3

$$\boldsymbol{w} = \boldsymbol{v}_i - \boldsymbol{v}_n = \frac{\boldsymbol{f}_L}{\gamma \rho_i (\rho_n + \rho_n)}. \tag{C.7}$$

The slip velocity becomes large when the ion density becomes small, but does not become large when the neutral density becomes small because the large density of ions drag the few neutrals that are present along with the rest of the mostly ionized plasma. The heating rate per unit volume due to ambipolar diffusion heating is

$$\Gamma = \boldsymbol{f}_d . \boldsymbol{w} \tag{C.8}$$

and substitution of Equations C.6 and C.7 gives

$$\Gamma = \frac{\rho_n |\boldsymbol{f}_L|^2}{\gamma \rho_i (\rho_n + \rho_i)^2}, \tag{C.9}$$

and so for a completely ionized plasma, $\Gamma = 0$.

In order to calculate the ambipolar diffusion heating, we need to find a value for the ion-neutral momentum transfer coefficient, γ (in units $\mathrm{cm^3\,s^{-1}\,g^{-1}}$) which depends on the collisional coefficient rates, cross sections, slip speed, and gas composition. Shang *et al.* (2002) give the following expression

$$\gamma = \frac{2.13\times10^{14}}{1-0.714x_e}\left(\left[3.23+41.0T_4^{0.5}\times\left(1+1.338\times10^{-3}\frac{w_5^2}{T_4}\right)^{0.5}\right]x_{HI}+0.243\right) \tag{C.10}$$

where T_4 is the temperature in units of $10^4\,\mathrm{K}$, w_5 is the slip speed in units of $\mathrm{km\,s^{-1}}$. We have assumed no molecular hydrogen to be present and the fractional abundance of He, $x_{He} = 0.1$. Subbing Equation C.7 into Equation C.10 gives a quartic equation for γ, i.e.,

$$\begin{aligned}\gamma^4 - (2AE + 2ABx_{HI})\gamma^3 + (A^2E^2 + 2A^2BEx_{HI} + \\ A^2B^2x_{HI}^2 - A^2C^2x_{HI}^2)\gamma^2 - GA^2C^2x_{HI}^2 = 0\end{aligned}$$

where
$A = \frac{2.13\times10^{14}}{1-0.714x_e}$, $B = 3.23$, $C = 41.0T_4^{0.5}$, $D = \frac{1.338\times10^{-3}}{T_4}$, $E = 0.243$, $F = \frac{\boldsymbol{f}_L}{\rho_i(\rho_n+\rho_i)}$, and $G = \frac{DF^2}{1\times10^{10}}$. Finally the radial and azimuthal Lorentz forces and thus the corresponding volumetric ambipolar heating rates can be calculated by using the following expressions for the flow and gravitational accelerations:

$$\boldsymbol{a} = v\frac{dv}{dr}\boldsymbol{r} + \frac{v}{r}\frac{dv}{d\theta}\boldsymbol{\theta} + \frac{v}{r\sin\theta}\frac{dv}{d\phi}\boldsymbol{\phi} \tag{C.11}$$

and

$$\boldsymbol{g} = -\frac{GM_\star}{r^2}\boldsymbol{r} + \frac{1}{r}\frac{d}{d\theta}\left(\frac{GM_\star}{r}\right)\boldsymbol{\theta} + \frac{1}{r\sin\theta}\frac{d}{d\phi}\left(\frac{GM_\star}{r}\right)\boldsymbol{\phi}. \tag{C.12}$$

D
Turbulent Heating

By including time dependence and shear viscosity, μ, into the stellar wind momentum equation of section **??** we get the Navier-Stokes equation

$$\rho \left(\frac{\partial v}{\partial t} + v.\nabla v \right) = -\nabla P + \mu \nabla^2 v + f_{other} \tag{D.1}$$

where f_{other} represents other forces such as gravity, radiation pressure and the unknown wind driving force(s), and ∇^2 is the vector Laplace operator. In most astronomical settings we expect the viscous term to be unimportant. This can be seen by considering the ratio of the inertial term to the viscous term in the Navier-Stokes equation Shu (1992):

$$\frac{\rho v.\nabla v}{\mu \nabla^2 v} \sim \frac{\rho U^2/L}{\mu U/L^2} = \frac{UL}{\nu} \equiv Re, \tag{D.2}$$

where U is a typical flow speed, L is the macroscopic length of the problem, R_e is the Reynolds number, and $\nu \equiv \mu/\rho$ is called the kinematic viscosity. The shear viscosity is defined as

$$\mu \sim m v_T/\sigma \tag{D.3}$$

where v_T is the thermal speed and σ is the typical collision cross section. Therefore the kinematic viscosity has the unit of velocity times length where,

$$\nu \sim v_T l \tag{D.4}$$

and l is the collision mean free path. The Reynolds number can then be written as **?**

$$Re \sim \frac{UL}{v_T l} \gg 1 \quad \text{when} \quad U \sim v_T \tag{D.5}$$

In other words, in stellar winds where the flow speeds are sonic or supersonic, the Reynolds number must be large and viscous forces are much less important than inertial effects. These high [$Re \gg 10^3$ to 10^4 i.e. the Reynolds number values associated with the onset of turbulence **?**] Reynolds numbers then produce turbulent flow resulting in the formation of eddies and other flow instabilities.

The rate at which energy is fed into the largest eddies per unit mass equals

$$\epsilon \sim \frac{U^2}{L/U} = \frac{U^3}{L} \tag{D.6}$$

This energy can neither accumulate nor dissipate viscously so the only other route for it is to be progressively transferred to eddies of smaller and smaller scales. If these smaller eddies have a scale λ and velocity v_λ then the energy that cascades from the large to the small per unit mass is

$$\epsilon \sim v_\lambda^3/\lambda \tag{D.7}$$

Combining equation D.6 and D.7 gives Kolmogorov's law:

$$v_\lambda \sim U \left(\frac{\lambda}{L}\right)^{1/3} \tag{D.8}$$

The process of transferring energy from large eddies down to smaller eddies continues until the scale of the turbulance is small enough for viscous action to become important and dissipation as heat to occur.

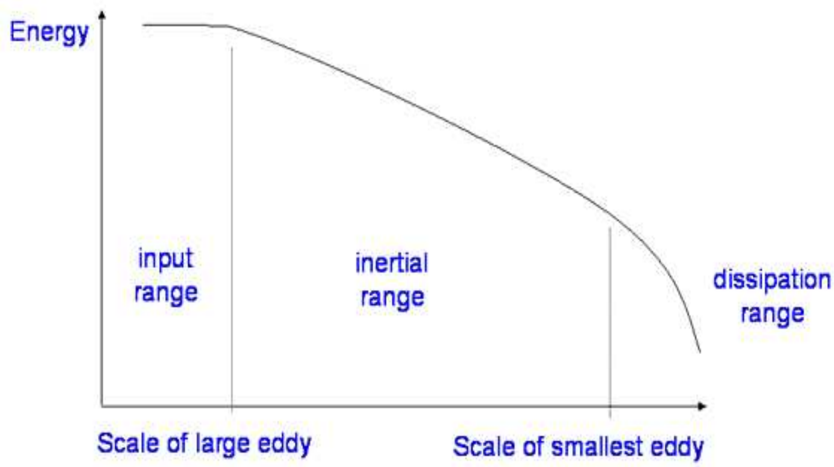

Figure D.1: Kolmogorov spectrum: The energy (turbulent energy) spectrum as a function of eddie size λ that depends on the 1/3 power of λ.

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