1.5 Spectral Indices

Long wavelength radio emission from non-dusty K spectral-type red giants is due to thermal free-free emission in their partially ionized winds while shorter wavelength radio emission emanates from the near static and more ionized lower atmospheric layers. The radio flux density-frequency relationship for these stars is usually found to be intermediate between that expected from the isothermal stellar disk emission, where α follows the Rayleigh-Jeans tail of the Planck function (i.e., $\alpha = +2$), and that from an optically thin plasma (i.e., $\alpha = -0.1$). We have shown in Chapter **??** that the expected radio spectrum from a spherically symmetric isothermal outflow with a constant velocity and ionization fraction varies as $\nu^{0.6}$ (Panagia & Felli, 1975; **?**; **?**). In reality however, thermal gradients will exist in the wind when the heating mechanisms become insufficient to counteract adiabatic and line cooling, so one would expect a temperature decrease in the wind at some point. Also, if the radio emission emanates from the wind acceleration zone then the electron density will not follow $n_e \propto r^{-2}$.

We therefore relax some of the constant property wind model assumptions and assume that the electron density and temperature vary as a function of distance from the star r, and have the power-law form $n_e \propto r^{-p}$ and $T_e \propto r^{-n}$, respectively (e.g., **?**). Finding the spectral index for an outflow with these conditions is non-trivial, so we highlight the main steps required to do so here. We assume the same geometry and notation used for the constant property wind model in section **??** of Chapter **??**, and start by calculating the optical depth at z for a ray with an impact parameter b through the atmosphere

$$\tau_\nu(b,z) = \frac{0.212 Z^2 n_0^2 r_0^{2p-1.35n}}{\nu^{2.1} T_0^{1.35}} \int_{-\infty}^{z} \frac{dz}{(b^2+z^2)^{(2p-1.35n)/2}} \equiv \int_{-\infty}^{z} \frac{C dz}{(b^2+z^2)^{(2p-1.35n)/2}}, \tag{1.1}$$

where T_0 and n_0 are the gas temperature and density, respectively, at the base of the wind, r_0, and C is a constant. We have also assumed that the electron density is the same as the ion density throughout.

The total flux density is found by integrating along the ray and over the entire sky

$$F_\nu = \frac{2\pi}{d^2}\int_{-\infty}^{\infty}\int_0^{\infty} B_\nu(z,b)\exp\left[-\tau_\nu(b,z)\right] b\,db\,dz, \tag{1.2}$$

and so we can now substitute in the Rayleigh Jeans function for B_ν (remembering that T_e is a function of b and z) and Equation 1.1 to get

$$F_\nu = \frac{4\pi k\nu^2 T_0 r_0^n}{d^2c^2}\int_{-\infty}^{\infty}\int_0^{\infty}\frac{b}{(b^2+z^2)^{(n/2)}}\exp\left[-\int_{-\infty}^{z}\frac{C dz}{(b^2+z^2)^{(2p-1.35n)/2}}\right] db\,dz. \tag{1.3}$$

Noting that

$$d\tau = \frac{C}{(z^2+b^2)^{(n/2)}}\frac{dz}{(z^2+b^2)^{(p-2.35/2)}} \tag{1.4}$$

then

$$F_\nu = C\frac{4\pi k\nu^2 T_0 r_0^n}{d^2c^2}\int_0^{\infty}\int_0^{\tau_{\max}}\frac{b}{(b^2+z^2)^{(p-2.35n/2)}}\exp\left(-\tau\right)\,d\tau\,db. \tag{1.5}$$

Therefore, if the spectral index of a stellar outflow can be measured, and if we make an assumption about the property of either the thermal or electron density profile of the wind, then Equation **??** provides us with information on how the other value varies.

The radio spectra for both stars are shown in Figure 1.5, together with the power laws that were fitted to the long wavelength flux densities by minimizing the chi-square error statistic. For α Boo, a power law with $F_\nu \propto \nu^{1.05\pm0.05}$ fits the four longest wavelength data points well. This spectral index is larger than the 0.8 value obtained by Drake & Linsky (1986) whose value was based on a shorter wavelength (2 cm) value and a mean value of four low S/N measurements at 6 cm. α Tau was found to have a larger spectral index and a power law with $S_\nu \propto \nu^{1.58\pm0.25}$ best fitted the three longest wavelength data points. This value is in agreement with Drake & Linsky (1986) who report a value ≥ 0.84 and is lower than the value of 2.18 that can be derived from the shorter wavelength data given in **?**. It should be emphasized that the spectral index for both stars is a lot steeper than that expected from the idealized constant property wind model.

Equation **??** can be used in conjunction with our new spectral index for each star to calculate the density and temperature coefficients that may describe their outflows. The combinations of the electron temperature and density coefficients