

Radio Interferometric Studies of Cool Evolved Stellar Outflows

A dissertation submitted to the University of Dublin
for the degree of Doctor of Philosophy

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Summary

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Acknowledgements

Some sincere acknowledgements...

List of Publications

Refereed

1. **O’Gorman, E.**, Harper, G. M., Brown, J. M., Brown, A., Redfield, S., Richter, M. J., and Requena-Torres, M. A.
“CARMA CO(J = 2 - 1) Observations of the Circumstellar Envelope of Betelgeuse”
The Astronomical Journal, 144, 36 (2012)
2. Sada, P. V., Deming, D., Jennings, D. E., Jackson, B. K., Hamilton, C. M., Fraine, J., Peterson, S. W., Haase, F., Bays, K., Lunsford, A., and **O’Gorman, E.**
“Extrasolar Planet Transits Observed at Kitt Peak National Observatory”
Publications of the Astronomical Society of the Pacific, 124, 212 (2012)
3. Sada, P. V., Deming, D., Jackson, B. K., Jennings, D. E., Peterson, S. W., Haase, F., Bays, K., **O’Gorman, E.**, and Lundsford, A.
“Recent Transits of the Super-Earth Exoplanet GJ 1214b”
The Astrophysical Journal Letters, 720, L215 (2010)

Non-Refereed

1. **O’Gorman, E.**, & Harper, G. M.
“What is Heating Arcturus’ Wind?”,
Proceedings of the 16th Cambridge Workshop on Cool Stars, Stellar Systems and the Sun. Astronomical Society of the Pacific Conference Series, 448, 691 (2011)

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1

Introduction

Here is the introduction of the thesis, complete with a few references ([Prothero & Buell, 2007](#); [Sagan, 1997](#)). Section [3.1](#) contains Equation [1.1](#), Section [3.2](#) has Figure [1.1](#) and Section [3.3](#) has Table [1.1](#). Chapter [2](#) has pretty much nothing in it.

1.1 First Section

This section has an equation. Here it is:

$$L_{\odot} = 4\pi R_{\odot}^2 \sigma T_e^4 \tag{1.1}$$

which is a nice way of describing the luminosity.

1.2 Second Section

So this section has a figure in it¹. That figure depicts the basic structure of a red giant.

¹And also a footnote.

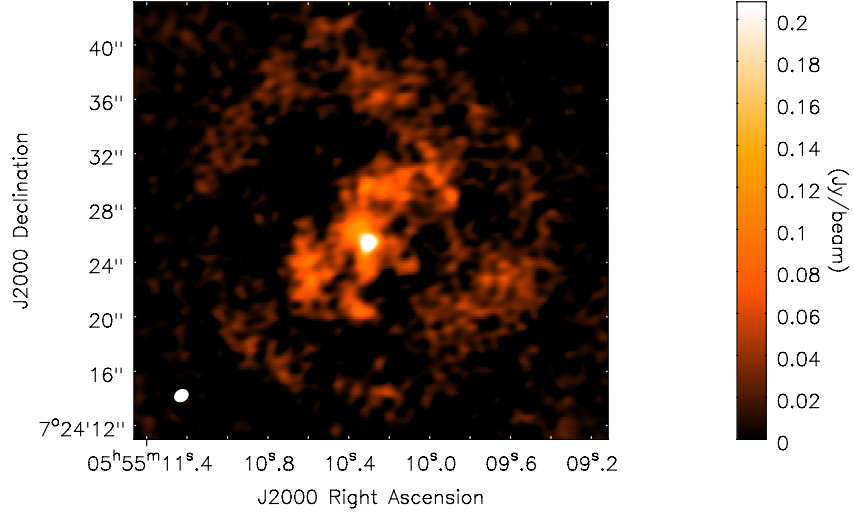


Figure 1.1: Red Giant and Asymptotic Giant Branch Stars. The left side of the figure shows the basic structure of a star on the giant branch of the HR diagram, while the right side shows a similar star after it has evolved to ascend the asymptotic giant branch. *Image Credit: Australian Telescope National Facility.*

1.3 Second Section

This section contains a basic table.

Table 1.1: Physical Properties of α Boo and α Tau.

Property	α Boo	α Tau	Reference
HD Number	124897	29139	...
Spectral Type	K2 III	K5 III	1, 2
ra (ICRS: ep=J2000)	14 ^h 15 ^m 39.672 ^s	04 ^h 35 ^m 55.239 ^s	3
dec (ICRS: ep=J2000)	+19 10 56.673	+16 30 33.489	3
pm-ra (mas yr ⁻¹)	-1093.39 ± 0.44	63.45 ± 0.84	3
pm-dec (mas yr ⁻¹)	-2000.06 ± 0.39	-188.94 ± 0.65	3
π (mas)	88.83 ± 0.54	48.94 ± 0.77	3
Distance (pc)	11.3±0.1	20.4±0.3	3
M (M_{\odot})	0.8±0.2	1.3±0.3	6, 4
θ_{UD} (mas)	21.0±0.2	20.2±0.3	5
θ_{LD} (mas)	21.0±0.2	20.2±0.3	5
L (L_{\odot})			
R (R_{\odot})	25.4±0.3	44.4±1.0	...
Log g			
T_{eff} (K)	4294±30	3970±49	5
v_{rad} (km s ⁻¹)	+5.19 ± 0.04	+54.11 ± 0.04	9
v_{esc} (km s ⁻¹)	110	106	...
v_{∞} (km s ⁻¹)	~40	~30	7, 8
T_{wind} (K)	~10,000	<10,000	7, 8
\dot{M} (M_{\odot} yr ⁻¹)	2×10^{-10}	1.6×10^{-11}	7, 8
H (H_{\odot})			...
Fe/H	-0.5±0.2	0.00±0.2	10

References.-(1);(2)Gray *et al.* (2006); (3)van Leeuwen (2007); (5)di Benedetto (1993); (6)Kallinger *et al.* (2010); (7)Drake (1985); (8)Robinson *et al.* (1998) (9)Massarotti *et al.* (2008); (10)Decin *et al.* (2003)

2

Introduction to Radio Interferometry

The poor spatial resolution provided by a single dish radio antenna can cause difficulties in obtaining accurate flux density measurements of radio astronomical sources, especially at long wavelengths. A single dish radio antenna is unable to distinguish against background radio emitters located in the primary beam, and therefore the the observed flux density can contain emission from unrelated sources. This limitation can be overcome through interferometry. An interferometer acts as a spatial filter, and can discriminate against smooth backgrounds while its higher resolution allows separation of the target from nearby confusing sources.

2.1 Radio Antenna Basics

The quality and properties of the final radio image produced from a synthesis array are partially dependent on the properties of the the individual antennas in the array. The most important such properties are discussed in the following sections and include aperture size, aperture efficiency, pointing accuracy, sidelobe level and noise temperature. We define the radio antenna as the piece of equipment which converts the electromagnetic waves emitted from the observed source into

an electric current ready to be input into to the first low noise amplifier where the signal is at the radio/sky frequency, ν_{RF} .

2.1.1 Radio Antenna Formulae

The power gain of a transmitting antenna is a measure of the antenna's capability of converting power into radio waves in a specific direction. In radio astronomy, the receiving counterpart of transmitting power gain is the effective collecting area of an antenna, $A(\nu, \theta, \phi)$, where ν is frequency and θ and ϕ are direction coordinates. An ideal radio antenna would collect all incident radiation from a distant point source and convert it to electrical power. The total spectral power P_ν , collected by it would then be a product of its geometric area and the incident spectral power per area, or flux density F_ν . By analogy then, the effective area of a real radio antenna is defined

$$A(\nu, \theta, \phi) = \frac{P_\nu}{F_\nu} = \frac{P}{I(\nu, \theta, \phi) \Delta\nu \Delta\Omega} \quad (2.1)$$

where $I(\nu, \theta, \phi)$ is the source brightness in units $\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$ that the antenna is pointing at and P is the power (in Watts) received by the antenna in bandwidth $\Delta\nu$ from element $\Delta\Omega$ of solid angle. The normalized antenna reception pattern \mathcal{A} , often referred to as the power pattern due to the duality between receiving and transmitting, is defined as

$$\mathcal{A}(\nu, \theta, \phi) = \frac{A(\nu, \theta, \phi)}{A_0} \quad (2.2)$$

where A_0 (m^2) is often referred to as the effective area of the antenna and is the response at the center of the main lobe of $A(\nu, \theta, \phi)$ [i.e. $A(\nu, 0, 0)$]. Then the beam solid angle, Ω_A , of the primary beam is

$$\Omega_A = \iint_{\text{all sky}} \mathcal{A}(\theta, \phi) d\Omega \quad (2.3)$$

and is a measure of the field of view of the antenna.

In the case of an isotropic antenna [i.e., $\mathcal{A}(\nu, \theta, \phi) = 1$], it can be shown that the product of the effective area and the primary beam solid angle is equal to the

square of the wavelength (Kraus *et al.*, 1986)

$$A_0 \Omega_A = \lambda^2. \quad (2.4)$$

Ω_A has its maximum possible value of 4π if \mathcal{A} is everywhere equal to 1. This means that the primary antenna can see the whole sky with equal sensitivity. Even though a large field of view is usually desirable in radio astronomy, Equation 2.4 ensures that for any given wavelength, when Ω_A is a maximum, the power received is a minimum and therefore the sensitivity is also at a minimum. To improve sensitivity, one could increase the collecting area of the antenna, but Equation 2.4 then ensures that the field of view must decrease. Thus, when deciding on the primary antenna size in a synthesis array, there is always a trade-off between field of view and sensitivity.

In reality, an antenna cannot radiate isotropically and will radiate preferentially in one or more directions. A Fourier transform relationship exists between the complex voltage distribution of the field, $f(u, v)$, in the aperture of the antenna and the complex far-field voltage radiation pattern, $F(l, m)$, of the antenna (Kraus *et al.*, 1986)

$$F(l, m) = \iint_{\text{aperture}} f(u, v) e^{2\pi i(ul+vm)} du dv \quad (2.5)$$

and

$$f(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(l, m) e^{-2\pi i(ul+vm)} dl dm \quad (2.6)$$

where

$$u = \sin\theta \cos\phi \quad \text{and} \quad v = \sin\theta \sin\phi \quad (2.7)$$

are the antenna coordinates and l and m are their Fourier counterparts. The form of $f(u, v)$ is determined by the manner in which the antenna feed illuminates the aperture. Therefore Equations 2.6 and 2.7 tell us that the radiation pattern in the far-field of a two-dimensional aperture is the two-dimensional Fourier transform of the aperture field illumination. For a uniformly illuminated 1-D aperture shown in Figure 2.1, the radiation pattern in the far-field is the *sinc* function. The radiation pattern in the far-field, $F(l, m)$, of such an antenna is related to the antenna power pattern, \mathcal{A} , by $\mathcal{A} = |F(l, m)|^2$. This power pattern is known

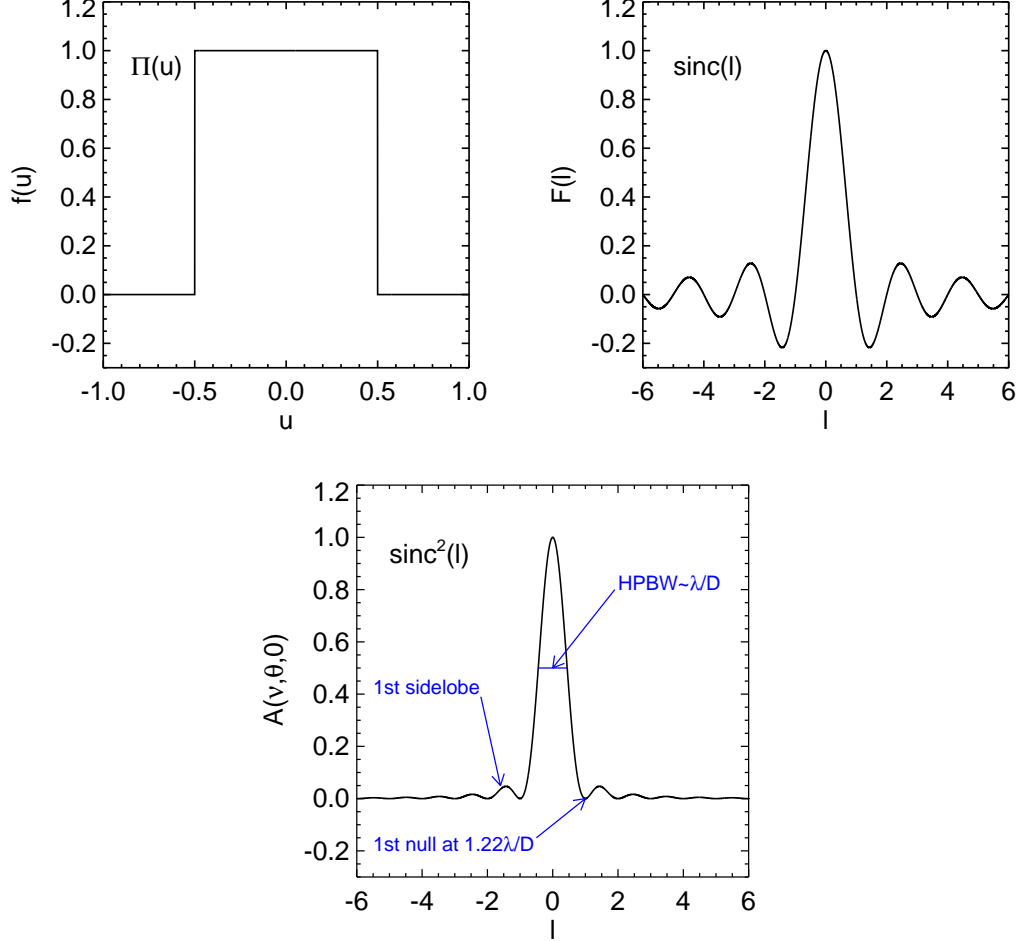


Figure 2.1: *Top Left:* A uniformly illuminated 1-D aperture $f(u)$. *Top Right:* The Fourier transform of $f(u)$ gives the antenna radiation pattern in the far-field, $F(l)$. *Bottom:* The power pattern of the antenna is given by $\mathcal{A} = |F(l)|^2$.

as the Airy pattern if the antenna is uniformly illuminated and is also shown in Figure x. The central peak of this power pattern is called the main beam while the smaller secondary peaks are called sidelobes. The antenna is maximally sensitive to radiation from the direction of the peak of the beam, but is also slightly sensitive to radiation in the direction of the side lobes. The half-power beamwidth (HPBW) of the main beam, θ_{HPBW} , is a term commonly used in the literature to

describe the field of view of an antenna/interferometer and satisfies

$$\theta_{\text{HPBW}} \propto \frac{\lambda}{D} \quad (2.8)$$

where D is the diameter of the antenna. The constant of proportionality varies slightly with the illumination taper and can be shown to be equal to ~ 0.89 for a uniformly illuminated linear aperture and ~ 1.2 for a Gaussian illuminated aperture. When the sky is scanned with a single antenna, then this HPBW is the resolution of the resulting map.

2.1.2 Antenna Structural Design

The design of the primary antenna element of an interferometric array will depend on the wavelength range to be observed. In general, wire antennas are used for wavelengths longer than ~ 1 m, while reflector antennas are typically used at shorter wavelengths. The reason why the more simple and less expensive wire antennas are not used at all wavelengths is given by Equation 2.4. For an isotropic antenna, this equation tells us that the effective area is just

$$A_0 = \frac{\lambda^2}{4\pi}. \quad (2.9)$$

Therefore, at short wavelengths a non-directional antenna such as a dipole, will have a small effective collecting area, giving it poor sensitivity for reception. Thus, wire antennas can be used at long wavelengths as they have sufficient collecting area, but cannot be used at shorter wavelengths as an impractical amount would be needed to produce useful collecting areas. Since the interferometric arrays used in this thesis use reflector antennas, the rest of this section will focus on them.

Choice of Antenna Mount. Nearly all interferometric arrays consist of antennas which have altitude over azimuth (altazimuth) mounts. These antennas lie on a horizontal azimuth track on which the antenna can turn in azimuth, and a horizontal elevation axle about which the antenna changes in zenith angle. The main advantage of such a design is simplicity and thus lower cost. Gravity always

acts on the reflector in the same plan thus reducing the problem of keeping the reflector profile accurate during the duration an observation. However, sources close to the zenith cannot be observed due to the high rate of azimuth rotation required. Also, the beam rotates with respect to the source for long duration observations which can affect the dynamic range of total intensity images of very large sources. The other type of mount occasionally used is the equatorial mount. Its polar axis is aligned parallel to the axis of rotation of the Earth and therefore only needs to rotate about the declination axis to observe a source. Its beam also doesn't have the beam rotation problem encountered by the altazimuth design and can track sources close to the zenith. Its major disadvantage and the reason for its scarce usage is the complexity of its design and resulting increased cost.

Choice of Antenna Optics. In Figure 2.2 we show the main optical systems which can be used to feed a large radio reflector. The prime focus system (e.g. used in the Giant Meter Radio Telescope) has the advantage that it can be used at long wavelengths where the use of secondary focus feeds (i.e. sub-reflectors) become impractical. However, access to, and space for, the feed and receiver is limited and sensitivity can be lost due to spillover noise from the ground. The other designs have the advantage of easier access to the feeds and receivers and less spillover noise from the ground. The off-axis Cassegrain (e.g. used in the Very Large Array) has also the advantage of increased frequency capability as many feeds can be located in a circle around the centre of the reflector and a slight rotation of the sub-reflector is all that is required to change observing frequency. The receivers and feeds in the Naysmith geometry (e.g. used in the Combined Array for Research in Millimeter wave Astronomy) are located externally to the antenna structure. Finally, the offset Cassegrain (e.g. used in the Green Bank Telescope) has no blockage and will have a circularly symmetric beam with low sidelobes but the increase complexity of its structure leads to increased costs.

2.1.3 Antenna Performance Parameters

Aperture Efficiency. The geometric collecting area of a parabolic antenna, A_{geo} ($= \pi D^2/4$), is related to the effective area (i.e. the collecting area when pointing

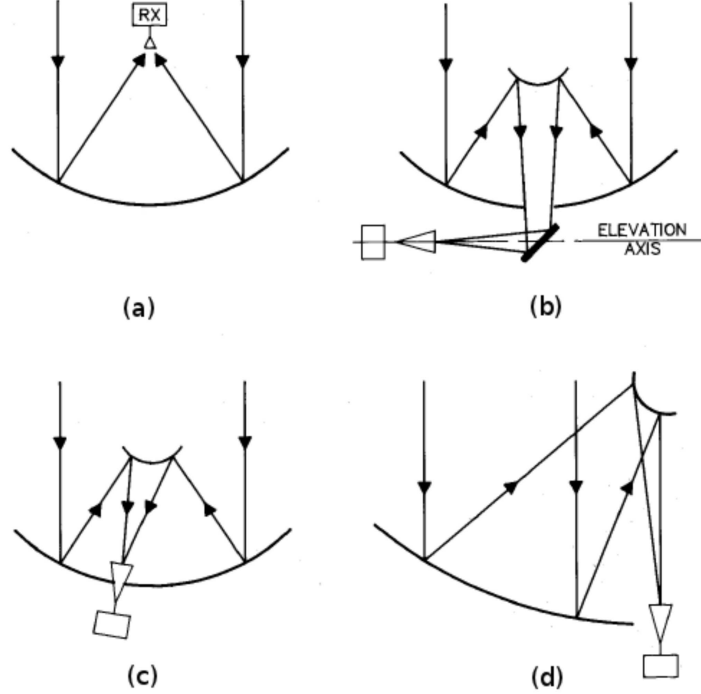


Figure 2.2: (a) Prime focus, (b) Naysmith, (c) Off-axis Cassegrain, (d) Offset Cassegrain (Figure adapted from [Taylor et al. \(1999\)](#)).

directly at a source) via the dimensionless quantity, η , known as the aperture efficiency where

$$\eta = \frac{A_0}{A_{\text{geo}}} \quad (2.10)$$

and is always less than unity. The aperture efficiency directly impacts on the sensitivity of the interferometric array and can be defined as the product of a number of different efficiency loss factors,

$$\eta = \eta_{\text{sf}} \eta_{\text{bl}} \eta_{\text{s}} \eta_{\text{t}} \eta_{\text{misc}}. \quad (2.11)$$

The surface efficiency, η_{sf} , accounts for the aperture efficiency loss as a result of reflector profile inaccuracies. Such inaccuracies result in the electric field from various parts of the aperture not adding together in phase at the feed leading to a decrease in power. The aperture blockage efficiency, η_{bl} , accounts for the fact that the subreflector (or feed) and its support structure result in a reduc-

tion in the incident radiation on the antenna. The feed spillover efficiency, η_{bl} , is best understood if the antenna is considered in transmission rather than in reception mode, and is defined as the fraction of power radiated by the feed that is intercepted by the reflector for a prime focus system, or by the subreflector for a Cassegrain system. The illumination taper efficiency, η_t , accounts for the fact that the feed pattern does not illuminate the primary reflector uniformly but illuminates the outer part of the reflector at a lower level than the inner part. Finally, the miscellaneous efficiency losses such as reflector diffraction and feed position phase errors are accounted for in η_{misc} .

Pointing Accuracy. The main lobe of an antenna's power pattern will usually not point exactly in the desired direction due to gravity deformations, wind pressure deformations, and mechanical inaccuracy. The angular offset, $\Delta\theta$, between the actual and desired pointing direction is called the pointing error. Usually, the desirable pointing error of an antenna at the highest operational frequency is $\Delta\theta < \theta_{\text{HPBW}}/20$ (Taylor *et al.*, 1999). With this specification reached, an antenna pointing at a compact source will suffer negligible intensity variations as $\mathcal{A}(\theta_{\text{HPBW}}/20) > 0.99$. However, this pointing error of only $\theta_{\text{HPBW}}/20$ will still have a substantial effect on the accuracy of the outer image. For example, a source located at the half power point will suffer a substantial fractional intensity variation of $2\mathcal{A}(\theta_{\text{HPBW}}/2 + \theta_{\text{HPBW}}/20) \simeq 0.86$. The blind pointing of a VLA antenna is only about $10''$ and can be much worse in daytime, occasionally exceeding $1'$. This means that at Q-band (45 GHz; 0.7 cm), which is the highest observing frequency on the VLA, the pointing error is only at best $\theta_{\text{HPBW}}/6$, and at worst $> \theta_{\text{HPBW}}$, meaning that the target may lie outside of the primary beam. To overcome this problem of large antenna pointing errors at high frequencies with the VLA, a technique known as referenced pointing is implemented. This technique will be discussed in detail in Chapter 3.

2.2 The Antenna Backend

2.3 Fundamentals of Radio Interferometry

The angular resolution $\Delta\theta$ of a radio antenna is the minimum angular separation which two point sources can have in order to be recognized as separate objects. The *Rayleigh criterion* is the generally accepted criterion for defining the angular resolution of a filled circular aperture of diameter D , at the observational wavelength λ and is given as

$$\Delta\theta = 1.22 \frac{\lambda}{D} \text{ rad.} \quad (2.12)$$

The Rayleigh criterion states that two objects are resolved when the first null of the diffraction pattern of one object coincides with the maximum of the diffraction pattern of the other. An immediate consequence of Equation 2.12 is that at large wavelengths, the angular resolution becomes large unless the diameter of the aperture can be increased substantially. In order to achieve modest angular resolution at radio wavelengths with a single radio antenna then, the diameter becomes impractically large. For example, in order to achieve an angular resolution of $1''$ at 6 cm a 12 km aperture would be required. Radio interferometry is a technique used in radio astronomy to overcome this problem of poor resolution at long wavelengths.

2.3.1 Young's Slits

The basic principles of interferometry can be understood through Young's double-slit experiment. If coherent radiation emitted from a distant point source propagates through two slits, an illumination pattern composed of bright and dark fringes is observed. The phenomenon is a result of the constructive and destructive interference between the secondary waves produced by the slits. The fringe separation is λ/B , where B is the projected separation of the slits and is called the baseline. The fringe contrast which is historically known as the fringe visibility, V , can be written as

$$|V| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \quad (2.13)$$

2.3 Fundamentals of Radio Interferometry

where I_{\max} and I_{\min} are the maximum and minimum intensity of the fringes, respectively. In other words, the fringe visibility is the fringe amplitude normalized by the sum of the maximum and minimum intensity.

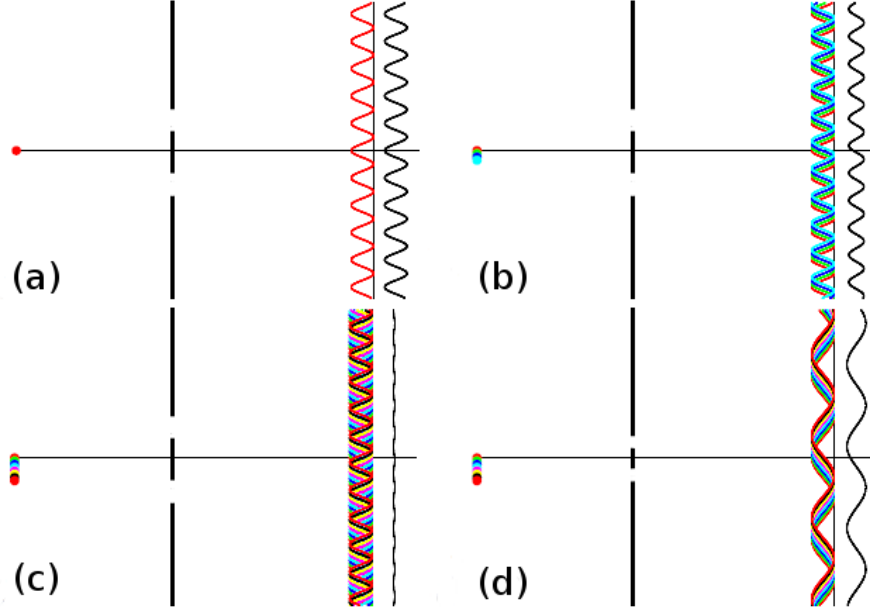


Figure 2.3: The resulting fringe pattern produced by Young’s slits under various situations. The source is shown on left left of the slits in each panel, while the separate fringe patterns (colors) along with the added fringe pattern (black) is shown on the right of the slits. (a) Point source at infinity, (Visibility = 1). Fringes are separated by an angular distance of λ/B . (b) An increase in source size results in a drop in visibility. (c) When the source size is equal to λ/B , the visibility is zero. (d) If the source size remains the same as in (c) and the slit spacing is reduced, then the fringes re-appear. Figure adapted from [Jackson \(2008\)](#).

In the simple case shown in Figure 2.3a, the angular size of the source is $\ll \lambda/B$ and the fringe visibility is 1. In interferometry, this equates to the situation in which the source size is smaller than the synthesized beam and only an upper limit of the source size can be obtained. In Figure 2.3b, the angular size of the source is now larger and can be thought of as a sequence of point sources each emitting radiation which is uncorrelated with the emission from the others. An angular shift of ϕ , called phase, in the sources position results in a shift in the corresponding fringe pattern by the same angle the other way. The total

2.3 Fundamentals of Radio Interferometry

interference intensity pattern is then just the sum of these individual patterns and the visibility is reduced. When the extension of the source equals λ/B , the fringes disappear and give a constant illumination pattern. In this case the fringe visibility is zero and the source is completely resolved as shown in Figure 2.3c. Finally, if the source size is the same as that in Figure 2.3c but the slit separation is reduced, then the fringe separation, λ/B , will again increase as shown in Figure 2.3d. This is because the source now produces much less displacement of the fringe patterns as a fraction of the fringe separation. In interferometry, this result means that extended sources can only be probed with short baselines.

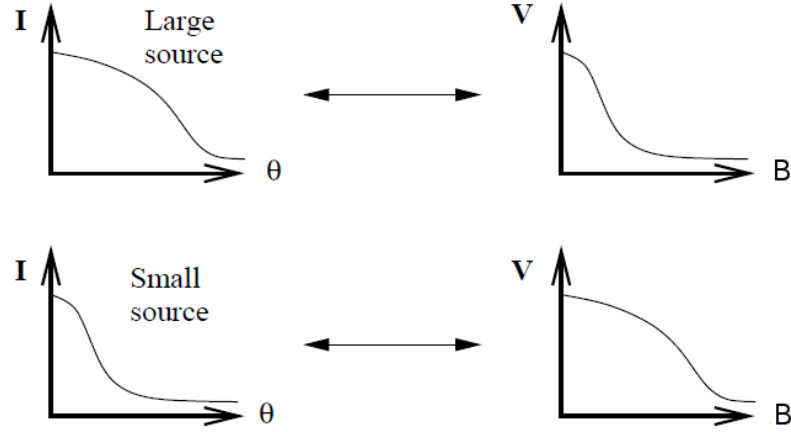


Figure 2.4: *Left column:* Intensity distribution as a function of sky angle for an extended source (top) and for a more compact source (bottom). *Right column:* The corresponding fringe visibility as a function of slit separation or baseline. Figure adapted from [Jackson \(2008\)](#).

Visibility and phase are often expressed together as the complex visibility $V = |V|e^{i\phi}$, which completely defines a pattern of interference fringes. Young’s double-slit experiment demonstrates a fundamental property of interferometry, namely that the contrast of fringes is a function of the geometry of the source. The results of the experiment are summarized in Figure 2.4. The top row shows that a large source (i.e. one whose intensity distribution extends out to a large angle on the sky) has a fringe visibility pattern which falls off quickly as projected baseline length increases. The bottom row shows that for compact sources the fringe visibility remains high out to large baselines. In the following section we

will show that the relationship between the sky brightness distribution $I(\theta)$ and the visibility $V(B)$ is a Fourier transform.

2.3.2 The Two-element Interferometer

Interferometers with N antennas can be treated as $N(N - 1)/2$ independent interferometer pairs so it is worthwhile studying the simplest case of the two-element interferometer. A simplified block diagram of the components of such an interferometer is shown in Figure 2.5. The figure shows two identical antennas separated by a baseline vector \mathbf{b} with both pointing to a distant radio source in a direction indicated by the unit vector \mathbf{s} . The plane waves from the distant radio source reach antenna 1 at a time τ_g later than they reach antenna 2. τ_g is called the geometric delay and is given by

$$\tau_g = \frac{\mathbf{b} \cdot \mathbf{s}}{c} = \frac{b \cos \theta}{c} \quad (2.14)$$

where c is the speed of light. If we assume that the interferometer only responds to a very narrow band centered on frequency $\nu = \omega/2\pi$, then the output voltages of antennas 1 and 2 at time t can be written as

$$V_1(t) = V \cos[\omega(t - \tau_g)] \quad \text{and} \quad V_2(t) = V \cos(\omega t). \quad (2.15)$$

The signals are then passed through a correlator which first multiplies these voltages to give

$$V_1(t)V_2(t) = \frac{V^2}{2} [\cos(2\omega t - \omega\tau_g) + \cos(\omega\tau_g)] \quad (2.16)$$

and then averages them over a time interval long enough such that $\Delta t \gg (2\omega)^{-1}$ to give the final output R :

$$R = \langle V_1(t)V_2(t) \rangle = \frac{V^2}{2} [\cos(\omega\tau_g)]. \quad (2.17)$$

As the Earth rotates, τ_g varies slowly with time and the resultant oscillations in the correlator output voltage represent the motion of the source. These

2.3 Fundamentals of Radio Interferometry

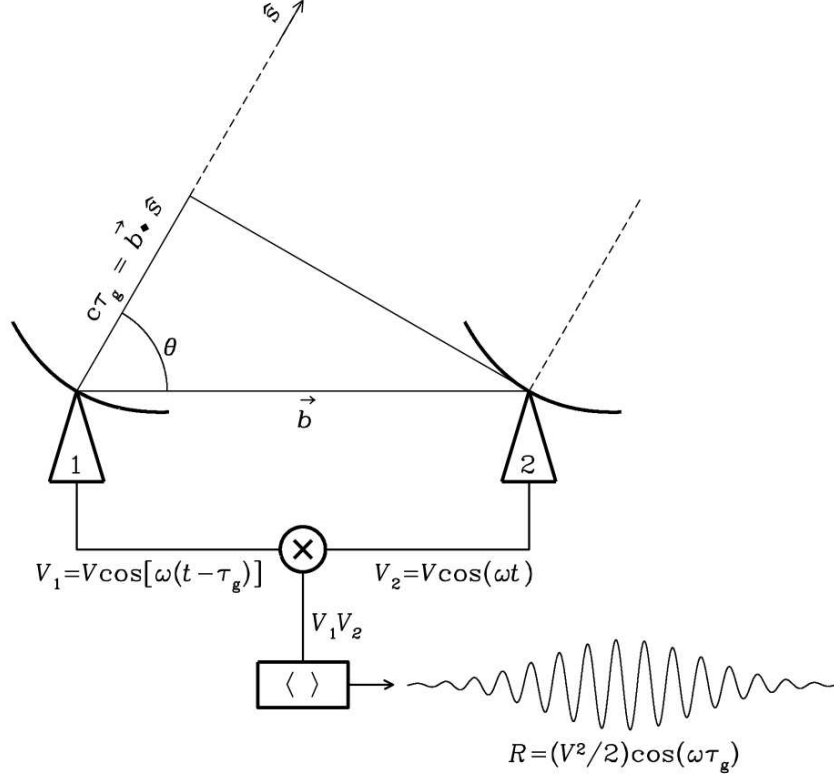


Figure 2.5: Simplified schematic diagram of a two-element interferometer. The correlator multiplies and averages the voltage outputs V_1 and V_2 of the two dishes and yields an output amplitude $V^2/2$ which is proportional to the point-source flux density F_ν .

sinusoidal oscillations are called fringes, and the fringe phase is

$$\phi = \omega \tau_g = \frac{\omega b \cos \theta}{c} \quad (2.18)$$

which changes with source direction as follows

$$\frac{d\phi}{d\theta} = \frac{\omega b \sin \theta}{c} = 2\pi \left(\frac{b \sin \theta}{\lambda} \right). \quad (2.19)$$

The fringe phase completes a full period (i.e. $\Delta\phi = 2\pi$) when an angular change $\Delta\theta = (\lambda/b \sin \theta)$ occurs. This tells us that the fringe phase is an extremely sensitive measure of source position if the projected baseline $b \sin \theta$ is many wavelengths long

2.3 Fundamentals of Radio Interferometry

and is the reason why interferometers can determine the positions of compact radio sources with exquisite accuracy.

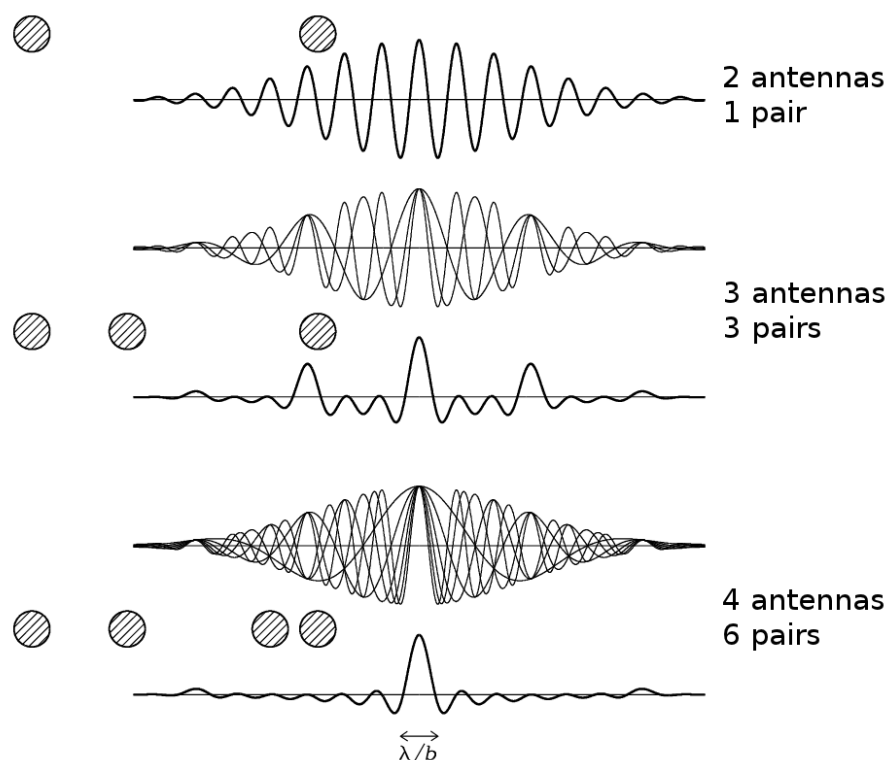


Figure 2.6: The instantaneous point source responses of an interferometer with two, three and four elements is indicated by the thick curves. The individual responses of the three pairs of two-element interferometers of the three-element interferometer and the six pairs of two-element interferometers of the four-element interferometer are plotted as thin curves. The main beam of the four-element interferometer is nearly Gaussian and has a width of $\sim \lambda/B$. This is known as the instantaneous synthesized beam of the interferometer.

If the antennas in an interferometric array are isotropic, then the point-source response of the interferometer would be a sinusoid spanning the entire sky and the interferometer would be only sensitive to one Fourier component of the sky brightness distribution having angular period $\lambda/b \sin \theta$. The response of a two-element interferometer R with non-isotropic antennas is this sinusoid multiplied by the product of the voltage patterns (i.e. defined as $f(u, v)$ in Section 3.1.1) of the individual antennas. If the antennas are identical then this product is

2.3 Fundamentals of Radio Interferometry

the power pattern of the individual antennas called the primary beam. The primary beam is usually a Gaussian that is much wider than the fringe period as $D \ll b \sin \theta$. The result is that an interferometer with directive antennas responds to a finite range of angular frequencies centered on $b \sin \theta / \lambda$. The instantaneous point source response of an interferometer is known as the synthesized beam and is the point source response obtained by averaging the outputs of all antenna pairs. The synthesized beam of an interferometer is an important quantity as it defines the maximum spatial resolution of the instrument. The synthesized beams produced by an interferometer with a various number of antennas arranged in 1-D is shown in Figure 2.7. The figure shows that the synthesized beam can be improved by acquiring more Fourier components (i.e. baselines) and rapidly approaches a Gaussian as N increases. However, sidelobes are still significant and a broad negative “bowl” exists between the main beam and the first sidelobes due to the absence of short spacings.

2.3.3 Complex Visibility

The interferometer output can be expressed in terms of the radio brightness over the sky, which is sometimes also called specific intensity and has units $\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$. If the radio brightness of a spatially incoherent extended source in the direction of unit vector \mathbf{s} is $I(\mathbf{s})$, then the response of the two-element interferometer with “cosine” correlator output near frequency $\nu = \omega/2\pi$ is obtained by treating the extended source as the sum of independent point sources:

$$R_c = \int_{\Omega} \mathcal{A}(\mathbf{s}) I_{\nu}(\mathbf{s}) \cos \left(\frac{2\pi \mathbf{b} \cdot \mathbf{s}}{\lambda} \right) d\Omega \quad (2.20)$$

where \mathcal{A} is the normalised antenna reception pattern defined in Section 3.1.1 and we call $\mathcal{A}(\mathbf{s}) I_{\nu}(\mathbf{s})$ the modified brightness distribution. However, the cosine function in the “cosine” correlator output is only sensitive to the even part of the the sky brightness distribution, which can be written as the sum of even and odd parts:

$$I(\mathbf{s}) = I_e(\mathbf{s}) + I_o(\mathbf{s}). \quad (2.21)$$

2.3 Fundamentals of Radio Interferometry

A “sine” correlator whose output is odd, is needed to detect the odd part of $I(\mathbf{s})$ and this is implemented by inserting a 90° phase delay into the signal of one of the antennas to give

$$R_s = \int_{\Omega} \mathcal{A}(\mathbf{s}) I_{\nu}(\mathbf{s}) \sin\left(\frac{2\pi \mathbf{b} \cdot \mathbf{s}}{\lambda}\right) d\Omega \quad (2.22)$$

It is convenient to write the cosines and sines as complex exponentials using the identity

$$e^{i\phi} = \cos(\phi) + i\sin(\phi) \quad (2.23)$$

and so the combination of “cosine” and “sine” correlators is called a “complex” correlator. The term *visibility* was first introduced by [Michelson \(1890\)](#) to describe the relative amplitudes of the optical fringes that he observed. The visibility is a complex quantity in radio astronomy and has dimensions of spectral power flux density ($\text{W m}^{-2} \text{Hz}^{-1}$). The complex visibility is the response of a two-element interferometer with a complex correlator to an extended source with brightness distribution $I(\mathbf{s})$ and is defined as

$$V_{\nu} = \int_{\Omega} \mathcal{A}(\mathbf{s}) I_{\nu}(\mathbf{s}) e^{-2\pi i \nu \mathbf{b} \cdot \mathbf{s} / c} d\Omega. \quad (2.24)$$

2.3.4 Coordinate Systems for Imaging

The baseline vector \mathbf{b} has coordinates (u, v, w) in three dimensions shown in Figure 2.7 where w points in the directions of interest, i.e. towards a direction s_0 that becomes the center of the synthesized image. u , v and w are measured in wavelengths (i.e. the components of \mathbf{b}/λ) and have directions towards the East, the North and the phase tracking center, respectively. An arbitrary unit vector \mathbf{s} has components (l, m, n) called direction cosines, where $n = \cos\theta = (1 - l^2 - m^2)^{1/2}$. Using these coordinates the parameters in Equation 2.24 become

$$\begin{aligned} \frac{\nu \mathbf{b} \cdot \mathbf{s}}{c} &= ul + vm + wn, \\ d\Omega &= \frac{dl dm}{n} = \frac{dl dm}{\sqrt{1 - l^2 - m^2}}. \end{aligned} \quad (2.25)$$

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Therefore Equation 2.24 can be defined in terms of the coordinate system layed out in Figurex. as

$$V(u, v, w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{A}(l, m) I_{\nu}(l, m) e^{-2\pi i[ul+vm+w(\sqrt{1-l^2-m^2})]} \frac{dl dm}{\sqrt{1-l^2-m^2}} \quad (2.26)$$

which is not a three-dimensional Fourier transform. This equation becomes a two-dimensional Fourier transform if $w = 0$ which is a good approximation for small field imaging, i.e. when $|l|$ and $|m|$ are small. In this case Equation 2.26 can be inverted to find the modified sky brightness distribution:

$$\mathcal{A}(l, m) I_{\nu}(l, m) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V(u, v) e^{2\pi i(ul+vm)} du dv. \quad (2.27)$$

Therefore Equation 2.27 demonstrates the important Fourier Transform relationship between the sky brightness distribution and the complex visibility (i.e. the interferometer response).

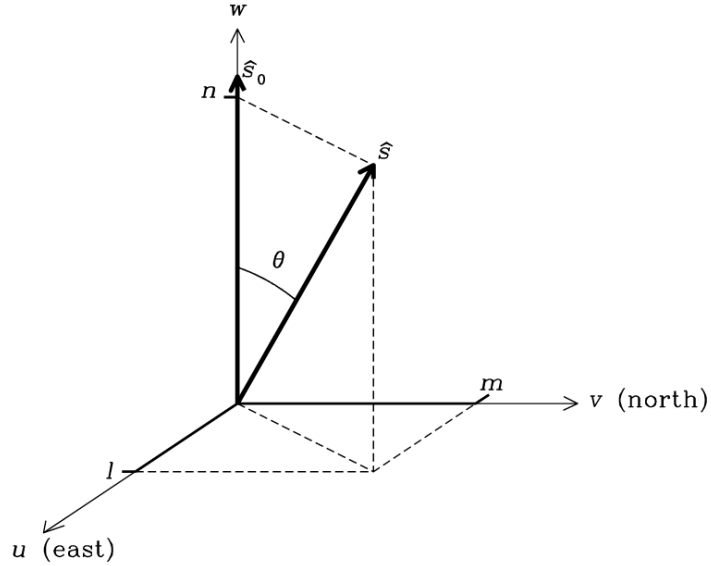


Figure 2.7: The (u, v, w) interferometric coordinate system. l , m , and n are the projections of the unit vector \mathbf{s} onto the u , v , and w axes, respectively.

2.4 Synthesis Imaging

A synthesis imaging telescope consists of a number radio elements fixed on the ground and uses the Earth’s rotation to vary the projected baseline coverage to increase the sampling of the $u - v$ plane. In this section we describe how Earth-rotation aperture synthesis is used to convert the complex visibilities outputted from the correlator to a final radio image of the observed sky.

2.4.1 Visibility Sampling

Figure 2.8 shows the overhead $u - v$ coverage of the VLA in its most extended configuration for a very short duration (i.e. snapshot) track. We define u and v as the east-west and north-south components of the projected baseline in wavelengths, respectively. As the Earth rotates, the projected baseline of each two-element pair in the array changes thus covering a different part of the $u - v$ plane. Over many hours, the $u - v$ points trace out portions of ellipses, and constructing a map over a long period of time is called Earth rotation aperture synthesis.

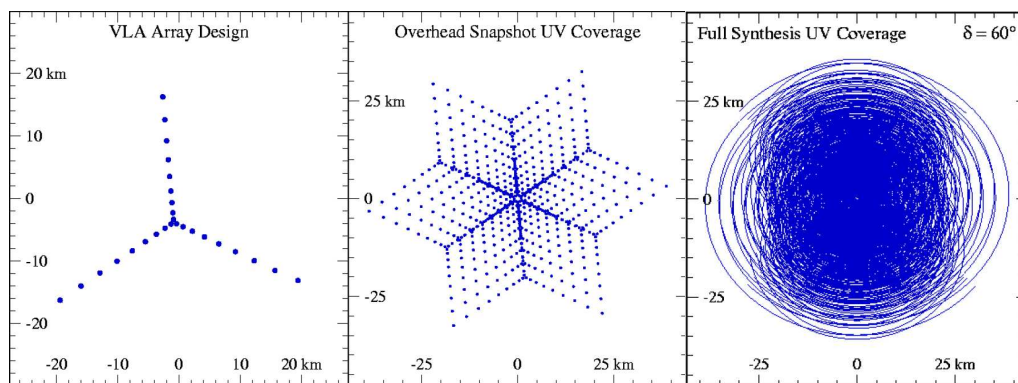


Figure 2.8: *Left:* The VLA in A-configuration is an example of an ‘Y’ shaped array design. *Middle:* The corresponding overhead snapshot $u - v$ coverage results in ‘snowflake’ pattern. *Right:* The corresponding $u - v$ coverage after a 12 hour track of a source at a declination of 60° . Note the more intense $u-v$ coverage in the direction of the three straight arms of the VLA for a snapshot track compared to the more uniform coverage over a longer duration track.

For every sky brightness distribution $I(l, m)$ there exists a continuous complex visibility function $V(u, v)$ that is its Fourier Transform. An array of antennas

will only ever measure a certain set of values of the visibility function where the measured set is called the sampling function $S(u, v)$, which is zero where no data have been taken. Then the actual data provided by the array is known as the sampled visibility function, $S(u, v)V(u, v)$. If we take the inverse Fourier transform of this function we get the *dirty image*:

$$I_\nu^D(l, m) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(u, v) V(u, v) e^{2\pi i(ul+vm)} du dv. \quad (2.28)$$

where we have used $I_\nu(l, m)$ to denote the modified sky brightness, $\mathcal{A}(l, m)I_\nu(l, m)$, as the correction for primary beam can be made at the final stage of data processing. Using the convolution theorem, the relationship between the dirty image and the desired intensity distribution $I_\nu(u, v)$ is

$$I_\nu^D(u, v) = I_\nu(u, v) * B(l, m) \quad (2.29)$$

where the asterisk implies convolution and

$$B(l, m) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(u, v) e^{2\pi i(ul+vm)} du dv \quad (2.30)$$

is the *point spread function* (PSF), or *synthesized beam*, or diffraction pattern of the aperture (i.e. the inverse Fourier Transform of the sampling function S). Equation 2.29 says that the dirty image I^D is the true intensity distribution I , convolved with the synthesized beam B .

2.4.2 Imaging (Inverting the Visibilities)

Image + FFT

Weighting

Gridding

2.4.3 Deconvolution

CLEAN +mUltiscale

MEM

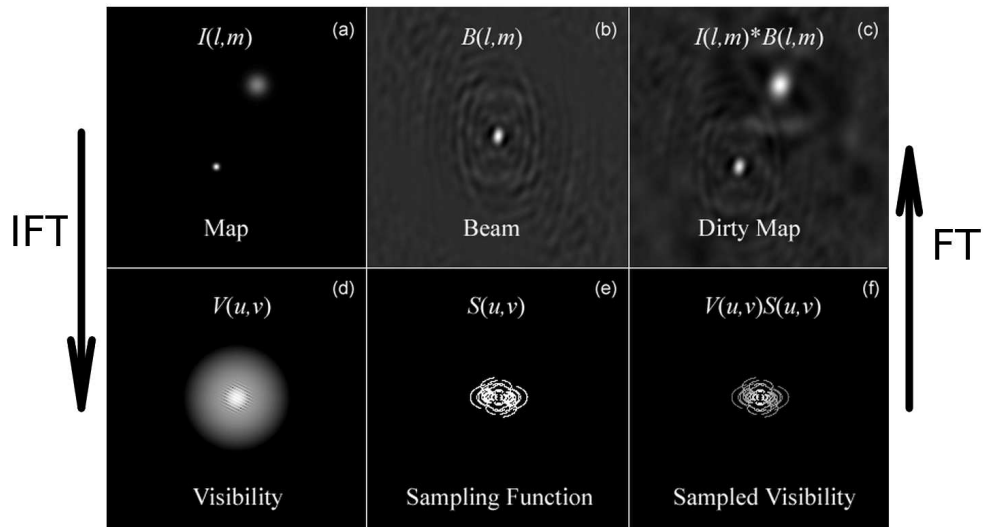


Figure 2.9

3

Instrumentation and Observations

So this Chapter has nothing really, apart from a shout out to Appendix [A](#), and maybe a few more sample references ([Harper & Brown, 2006](#); [Seaquist & Taylor, 1990](#)).

3.1 CARMA

3.1.1 The Basic Instrument

3.1.2 The CARMA Electronic System

3.1.3 Observations

3.2 VLA

3.2.1 The Basic Instrument

3.2.2 The VLA Electronic System

3.2.3 Observation Preparation

3.2.4 Observations

3.3 GMRT



A Nice Appendix

This is where the appendix would go...

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