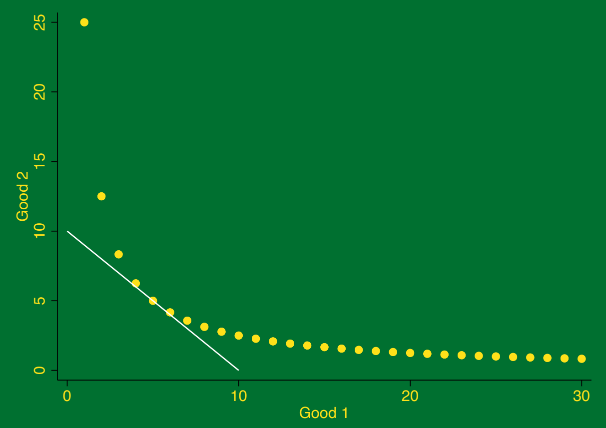
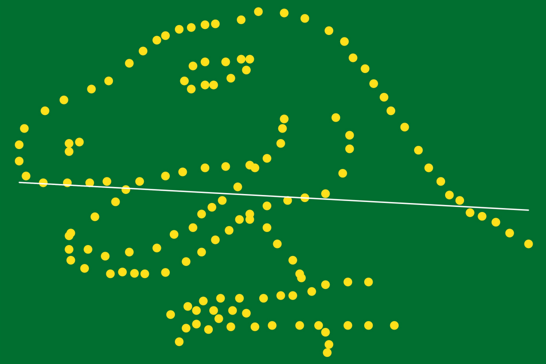
How people make choices 1: Preferences





Prof Jonathan Davis

UNIVERSITY OF OREGON

Announcements

Starting Chapter 4 Material on Consumer Behavior

- Today: Mostly Section 4.1 and 4.2 on Preferences and Indifference Curves

Budget Constraints and Preferences HW

- Due **Friday, November 12th, 5pm**

No class next Thursday (Veteran's Day)

Our Model of Consumer Theory

Our model of how consumers make choices will have two ingredients:

1. Cost (modeled with a constraint)

2. Preferences (given by a **utility function**)

Budget Constraint Recap

$$M = p_X X + p_Y Y$$

Slope is $-\frac{p_X}{p_Y}$. How much Y you have to give up to get another X.
Y intercept is $\frac{M}{p_Y}$.
X intercept is $\frac{M}{p_X}$.

How does graph change if income changes?
How does graph change if p_X changes?
How does graph change if p_Y changes?

Today: Preferences

If I google preferences, the definition that comes up first is:

a greater liking for one alternative over another

This is pretty spot on for how economists think about preferences.

Preferences Require Comparisons

The statement “I really, really like ice cream” doesn’t have much content about preferences.

But “I pick one scoop of ice cream over one slice of cake” does.

Why?

Preferences Require Comparisons

The statement “I really, really like ice cream” doesn’t have much content about preferences.

But “I pick one scoop of ice cream over one slice of cake” does.

Why?

- In the first statement, if the person really, really likes everything, then we have no idea how much they like ice cream relative to anything else.
- In the second statement, we at least know they prefer similar quantities of ice cream to similar quantities of cake.

Preferences defined by choices

Preferences are personal. There is no correct set of preferences.

So we want preferences to be flexible in our model of consumers.

We only require that preferences be logically consistent.

Assumptions about preferences

We make the following two assumptions about preferences:

1. Preferences are **complete**. This means when comparing options A and B, you can always say either:
   - You prefer A to B
   - You prefer B to A
   - You are indifferent between A and B

Assumptions about preferences

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   - You prefer A to B
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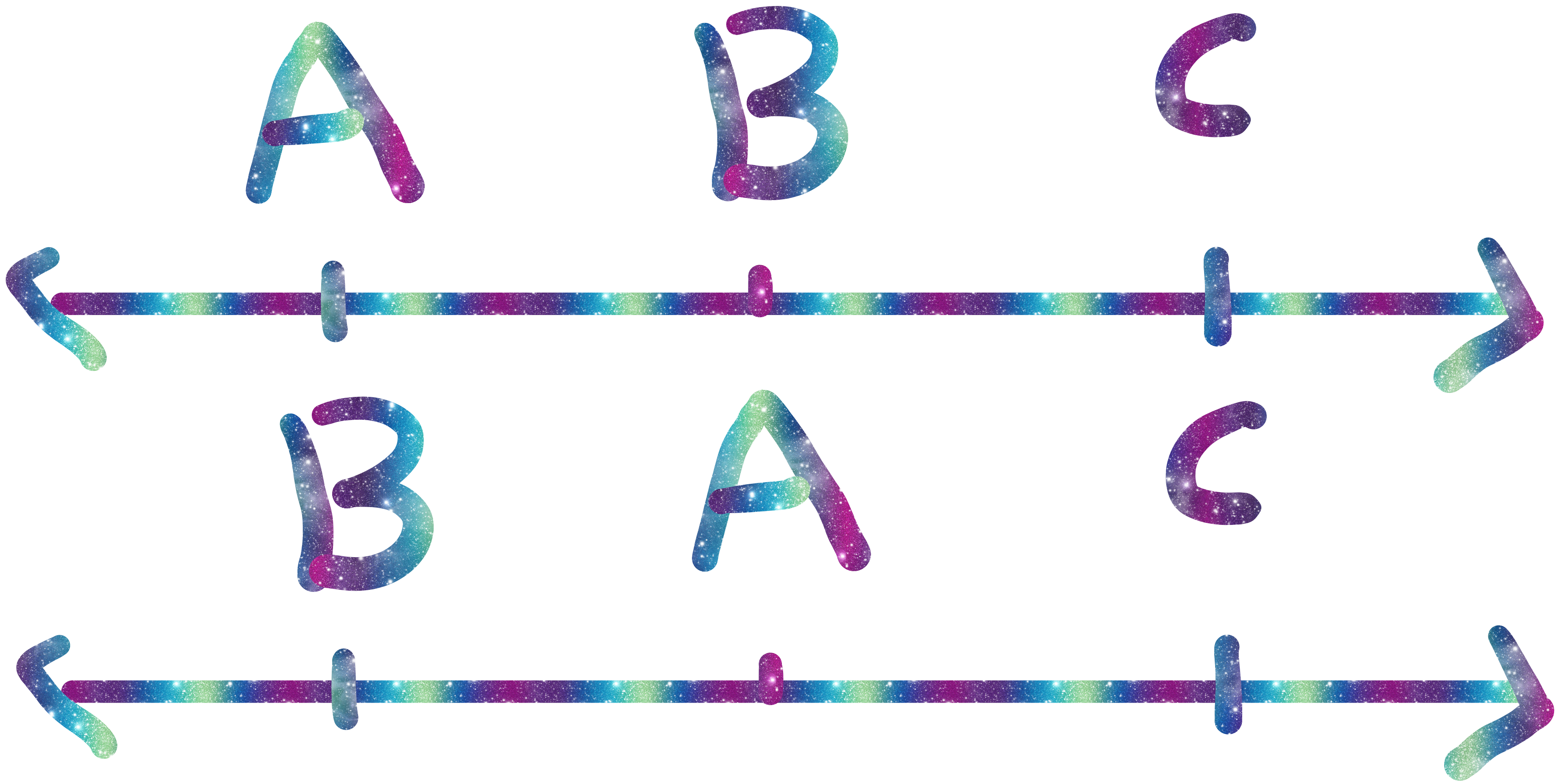
2. Preferences are **transitive**. This means when comparing options A, B, and C. If you prefer A to B and B to C, then you must prefer A to C.

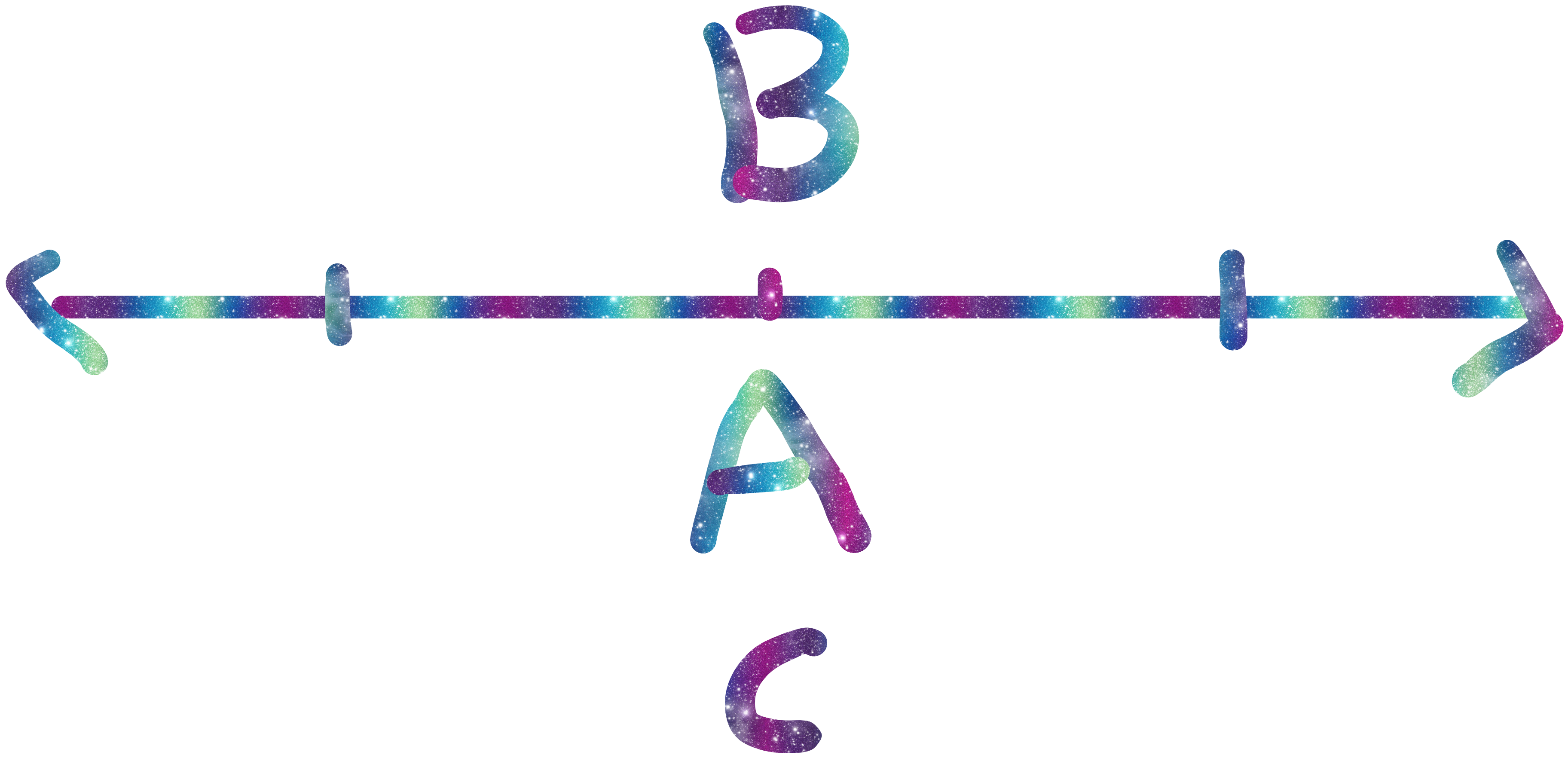
What these assumptions imply

Can place A, B, and C on a number line.

What these assumptions imply



What these assumptions imply



Preferences are ordinal

To be more concrete:

If preferences are **complete** and **transitive**, we can always place 3 bundles on the number line.

This is the sense in which we require preferences to be logically consistent.

(Try to put non-transitive preferences on the number line!)

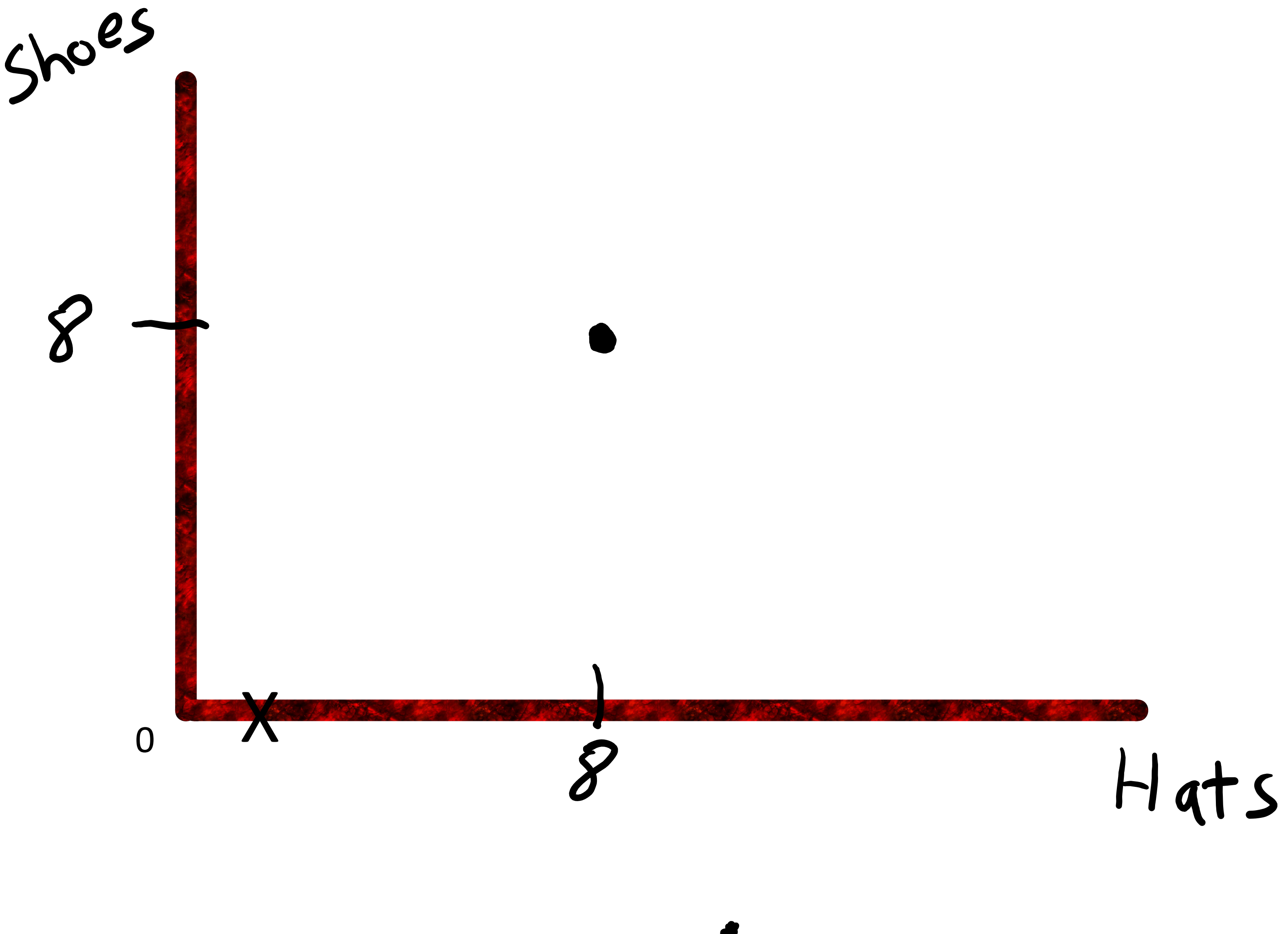
But preferences are **ordinal** which means we only know someone prefers A to B, not by how much.

How do we incorporate this information into our model of consumer choice?

iClicker: Do you prefer 8 hats and 8 pairs of shoes or 0 hats and 1 pair of shoes?

A. 8 hats and 8 pairs of shoes
B. 0 hats and 1 pair of shoes
C. Indifferent

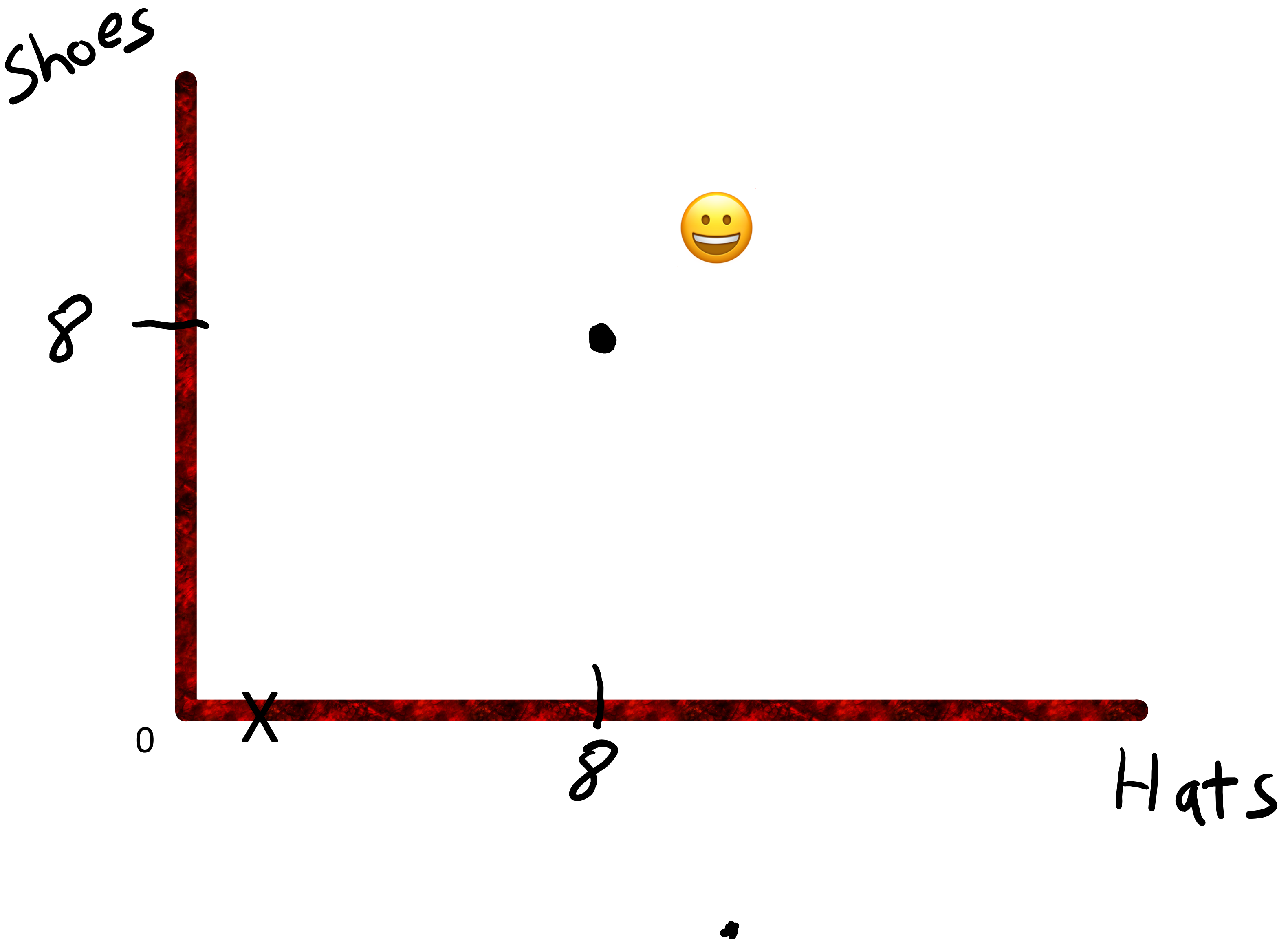
Graphing This Preference



iClicker: Do you prefer 8 hats and 8 pairs of shoes or 10 hats and 10 pair of shoes?

A. 8 hats and 8 pairs of shoes
B. 10 hats and 10 pair of shoes
C. Indifferent

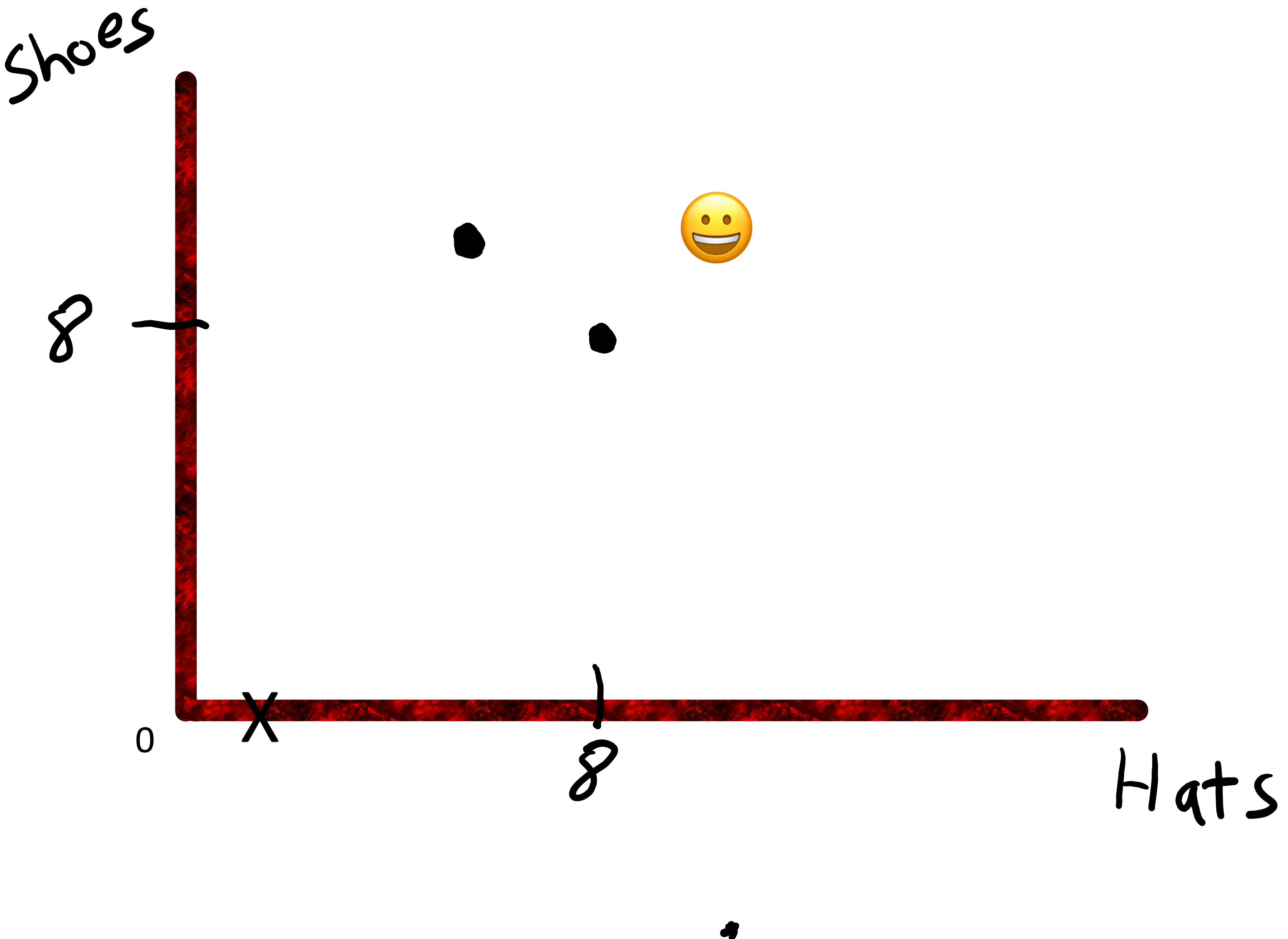
Graphing This Preference



iClicker: Do you prefer 8 hats and 8 pairs of shoes or 6 hats and 10 pair of shoes?

A. 8 hats and 8 pairs of shoes
B. 6 hats and 10 pair of shoes
C. Indifferent

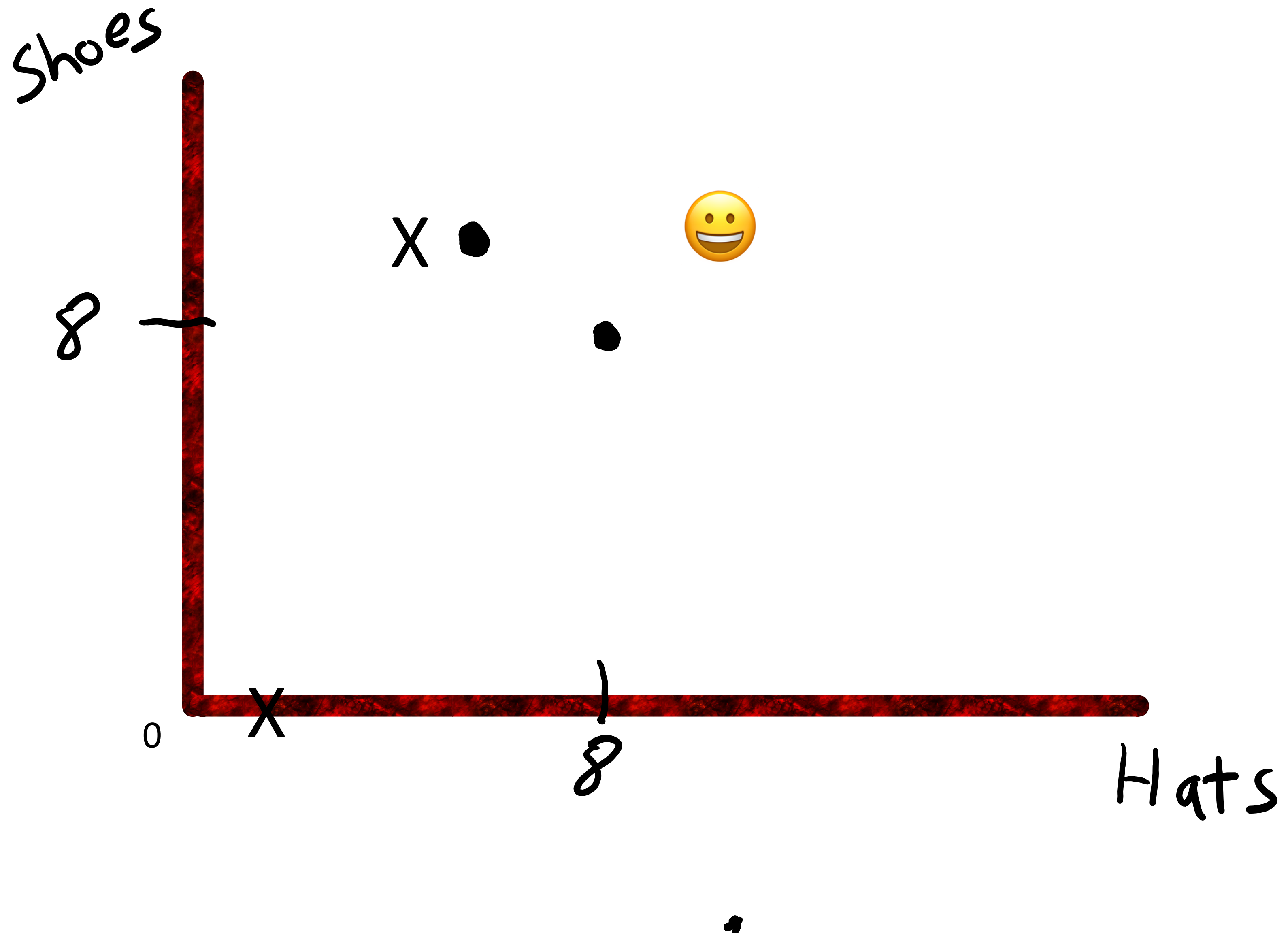
Graphing This Preference



iClicker: Do you prefer 8 hats and 8 pairs of shoes or 5 hats and 10 pair of shoes?

A. 8 hats and 8 pairs of shoes
B. 5 hats and 10 pair of shoes
C. Indifferent

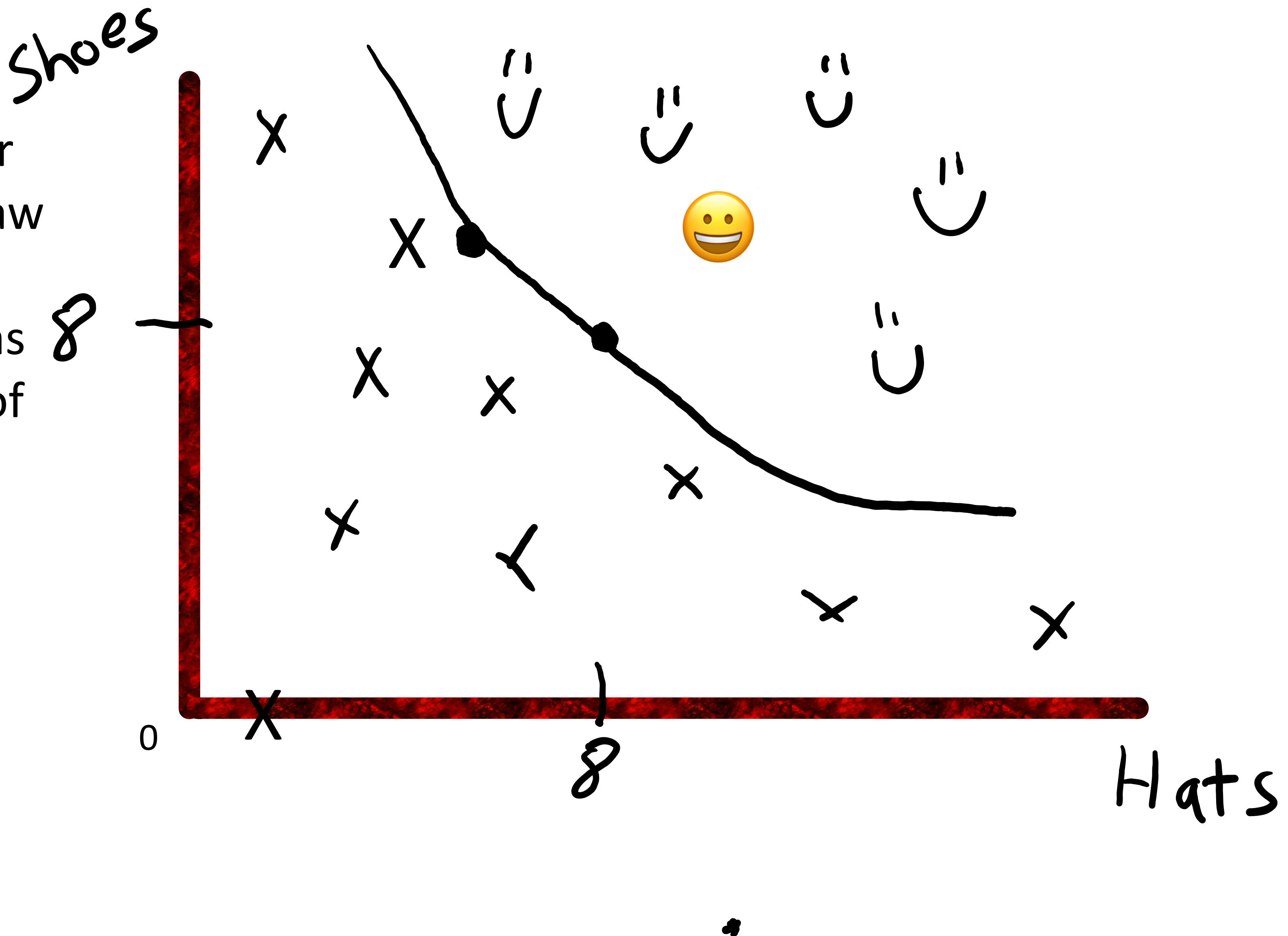
Graphing This Preference



Graphing This Preference

We could keep doing this for days until we are able to draw a curve through all the bundles you prefer exactly as much as 8 hats and 8 pairs of shoes.

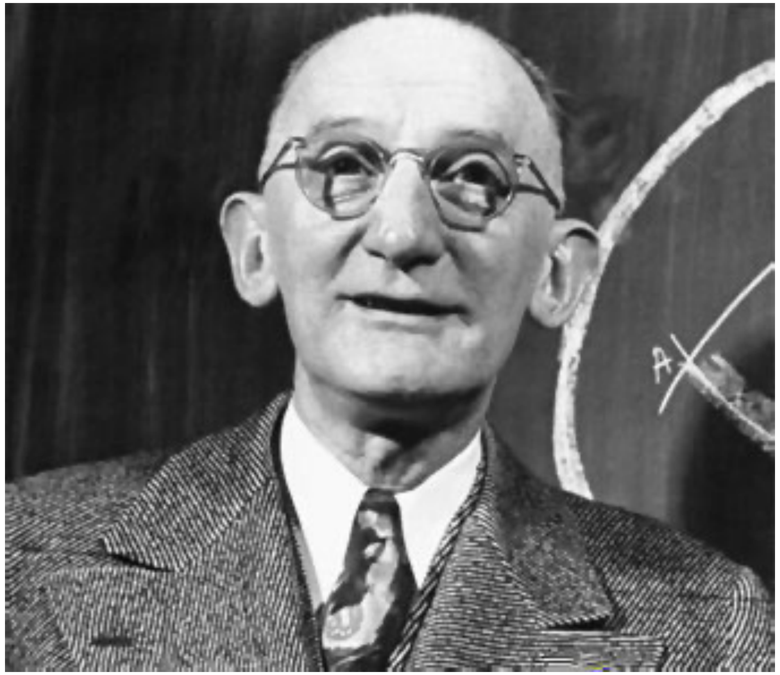
We call this curve an **indifference curve**.



Are indifference curves a good way to model preferences?

Thurston (1931)

Research Question: Can consumer preferences be adequately represented by indifference curves?



Thurston (1931): Theory

The constant method takes the following form. One of the combinations such as eight hats and eight pairs of shoes is chosen as a standard and each of the other combinations is compared directly with it. Thus in Figure 5 we should expect to find that if the subject were asked to choose (eight hats and eight pairs of shoes) or (seven hats and fourteen pairs of shoes) he might be quite willing to give up one of the hats in order to possess six additional pairs of shoes, assuming of course that the money cost to him were the same. We should therefore expect to find the point *B* marked plus because the combination at that point is preferred to that of the standard at *A*. This may be judged from Figure 5 because the point *B* lies on an indifference curve at a higher elevation of satisfaction than the point *A*.

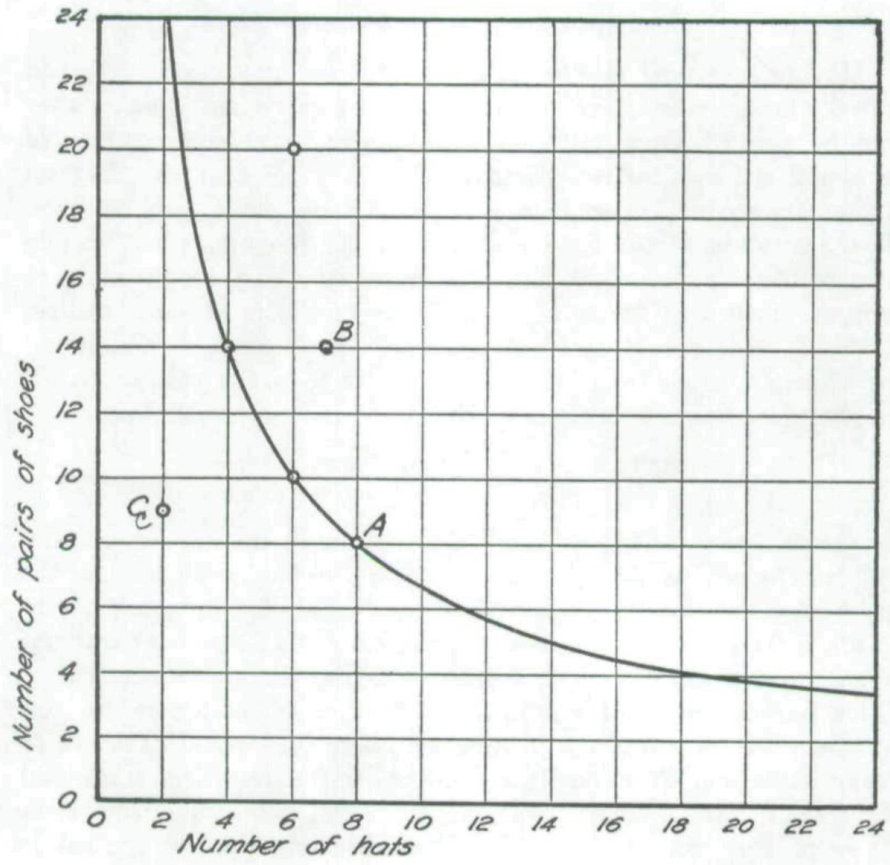


FIGURE 5

Thurston (1931): Which of the alternatives would give you more satisfaction if they cost the same?

8 hats and 8 pairs of shoes **or**
6 hats and 9 pairs of shoes

8 hats and 8 pairs of shoes **or**
4 hats and 15 pairs of shoes

8 hats and 8 pairs of shoes **or**
9 hats and 3 pairs of shoes

Etc.

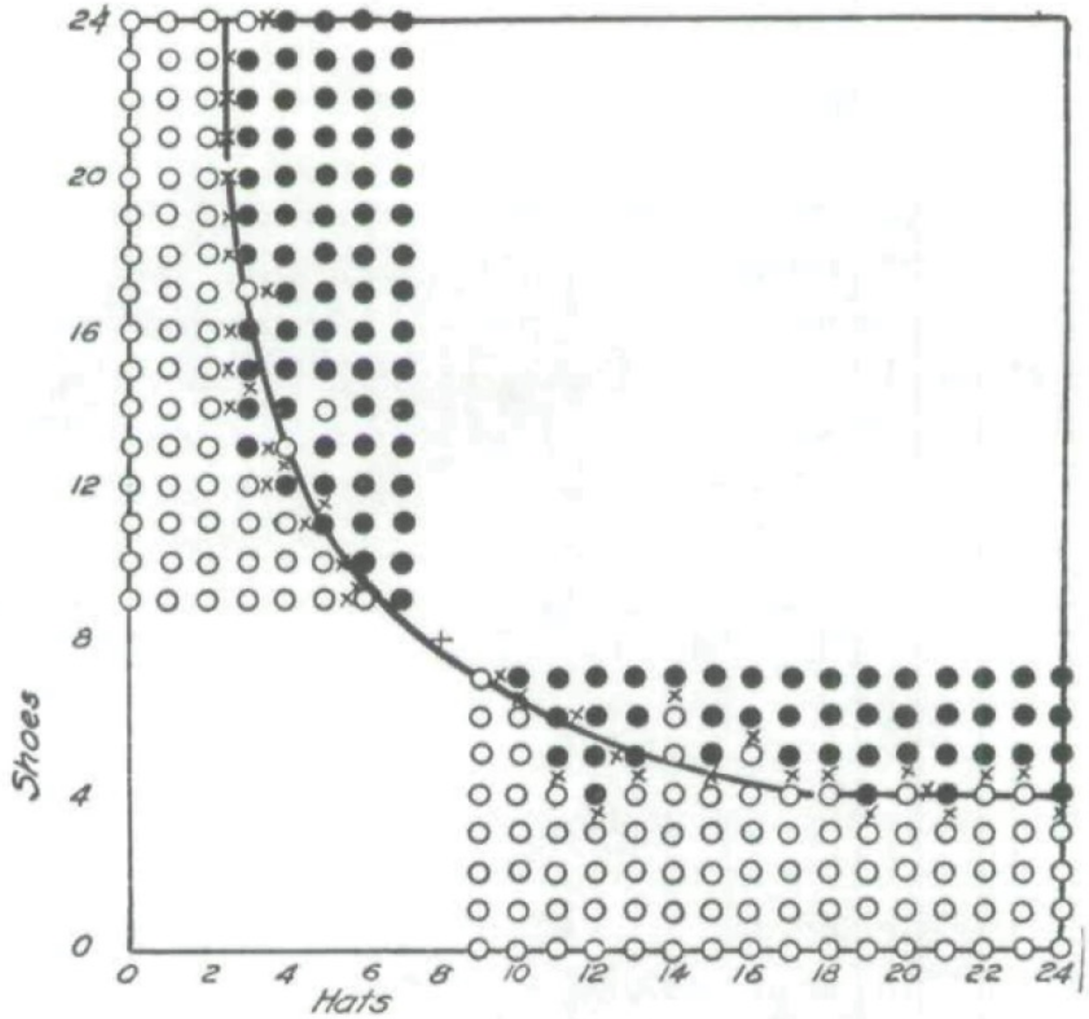


FIGURE 7

Thurston (1931)

"agreement between the predicted curves and the distributions of black and white circles is quite satisfactory." (p. 163)

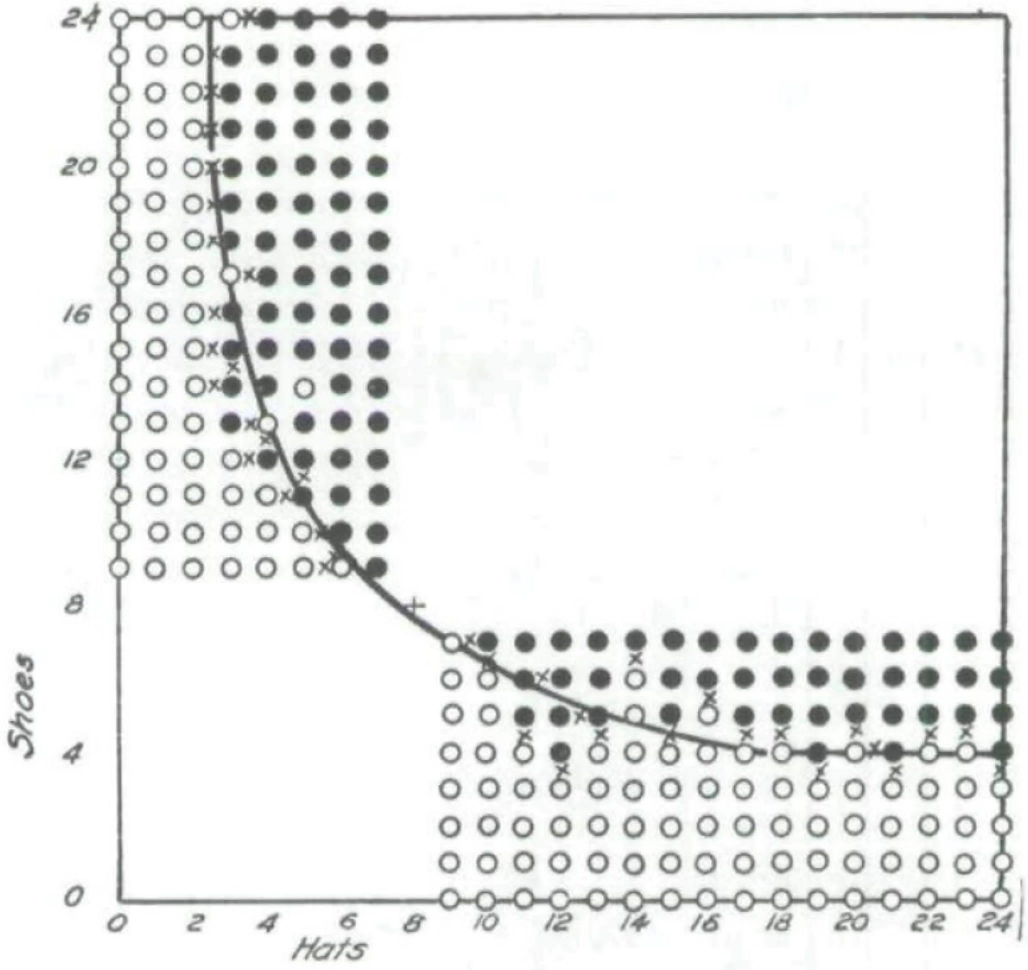


FIGURE 7

Recap of Thurston (1931)

One of first experiments in economics.

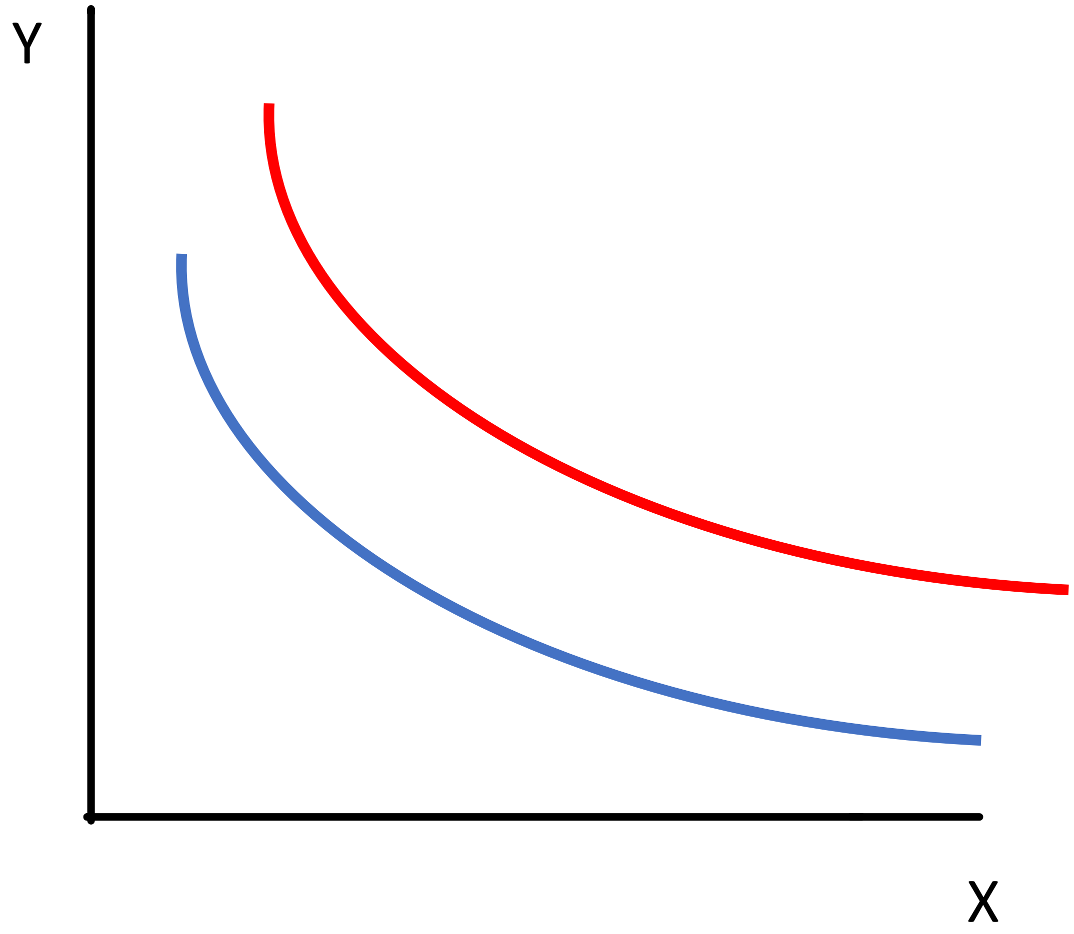
Collects new data to answer question at hand: does theory of indifference curves match reality?

Finds that theory is “ satisfactory.”

What do you think? Are you convinced by Thurston’s results?

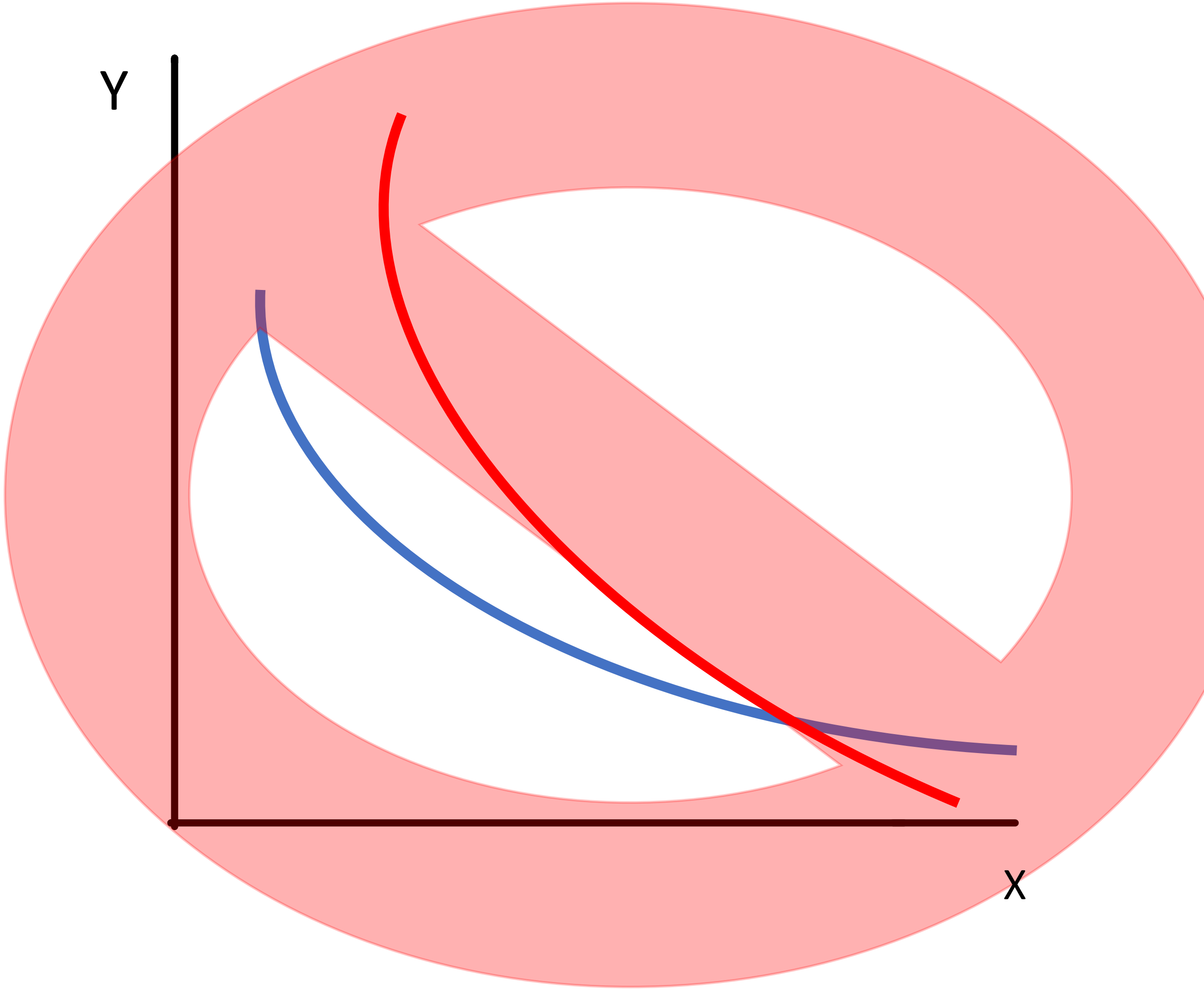
Properties of Indifference Curves

1. Curves further from the origin correspond to higher utility than curves closer to the origin.



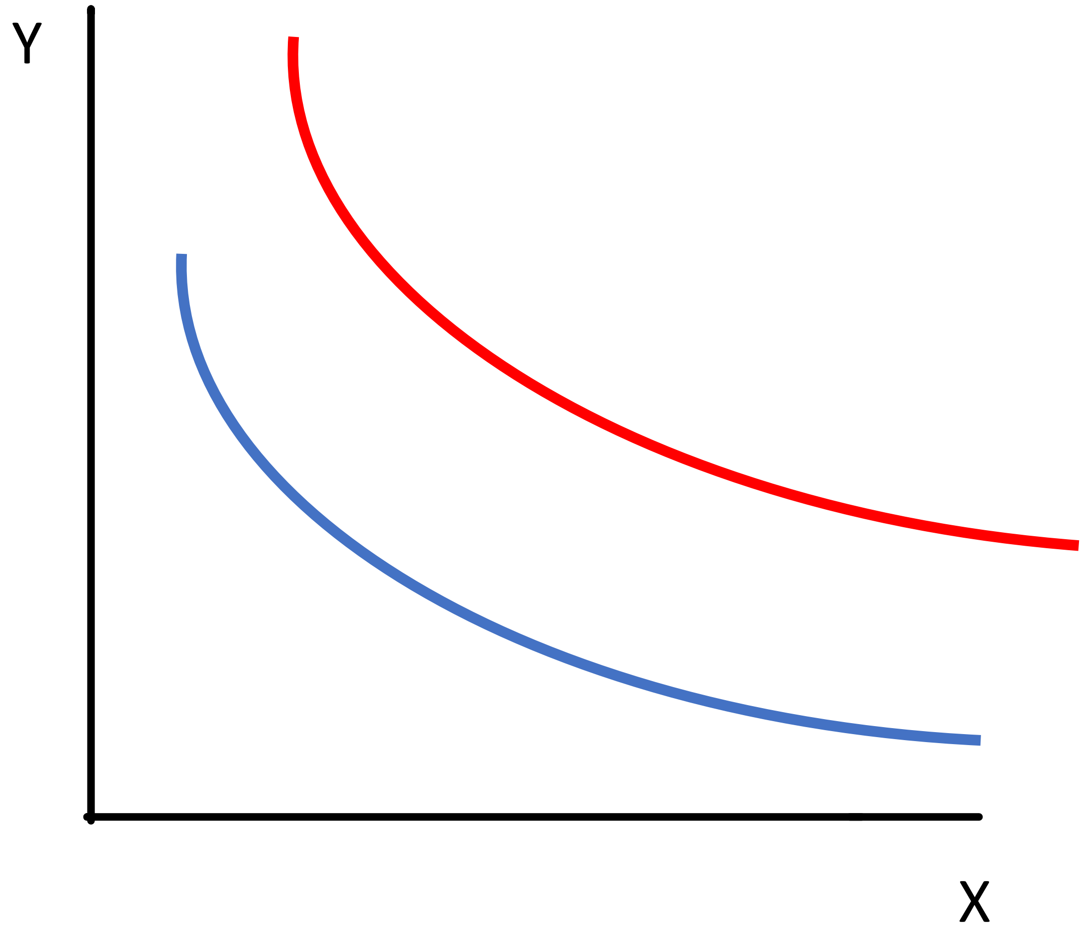
Properties of Indifference Curves

1. Curves further from the origin correspond to higher utility than curves closer to the origin.
2. Indifference curves can never cross. **Why?**



Properties of Indifference Curves

1. Curves further from the origin correspond to higher utility than curves closer to the origin.
2. Indifference curves can never cross. **Why?**
3. The slope of the indifference curve is called the **marginal rate of substitution**.



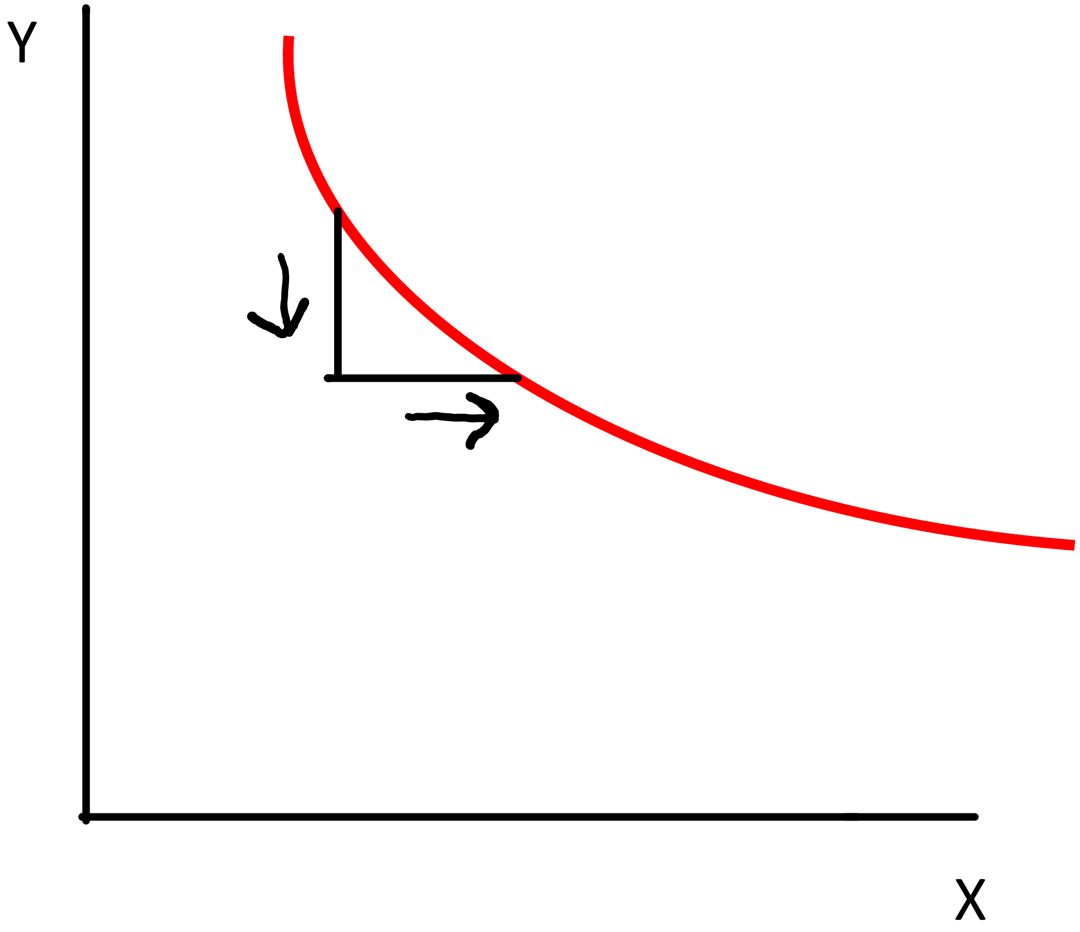
The Marginal Rate of Substitution

The Marginal Rate of Substitution is how much Y you are willing to give up in exchange for another unit of X, holding your happiness constant.

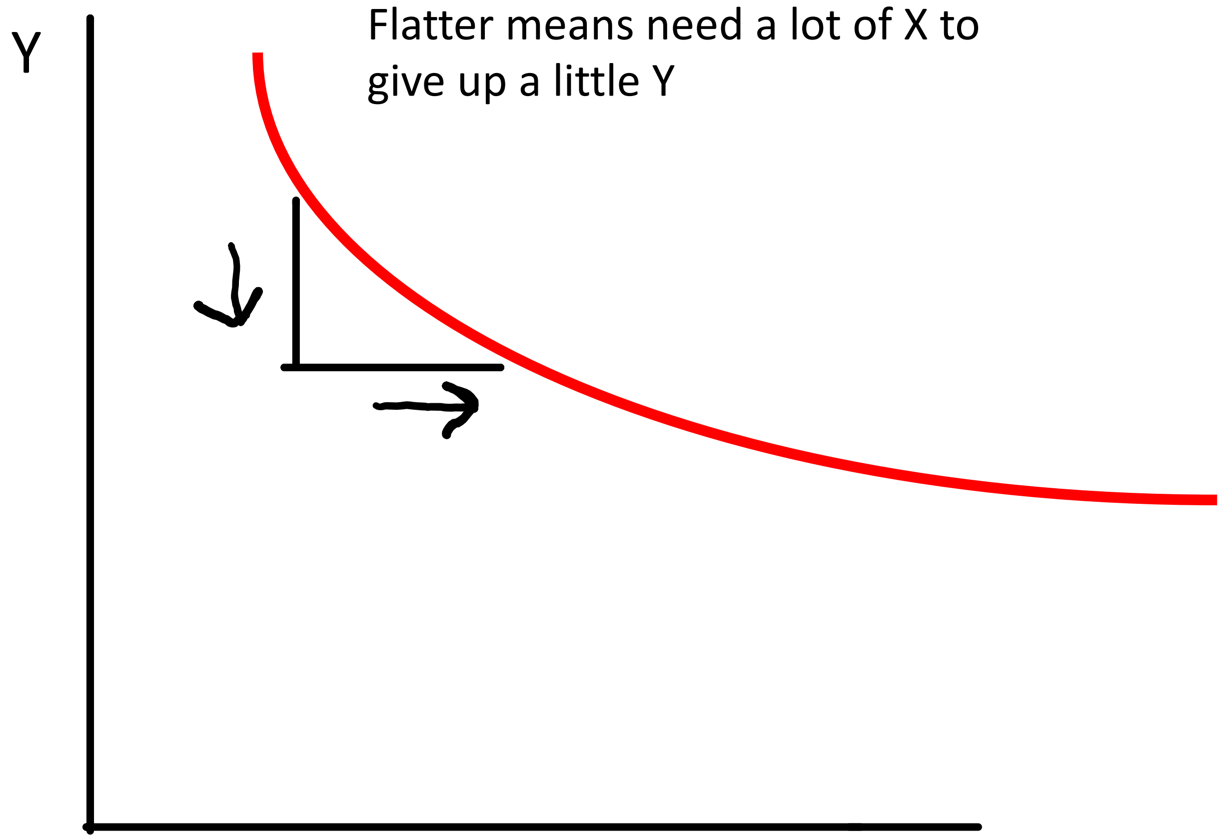
$$MRS = -\frac{\Delta Y}{\Delta X}$$

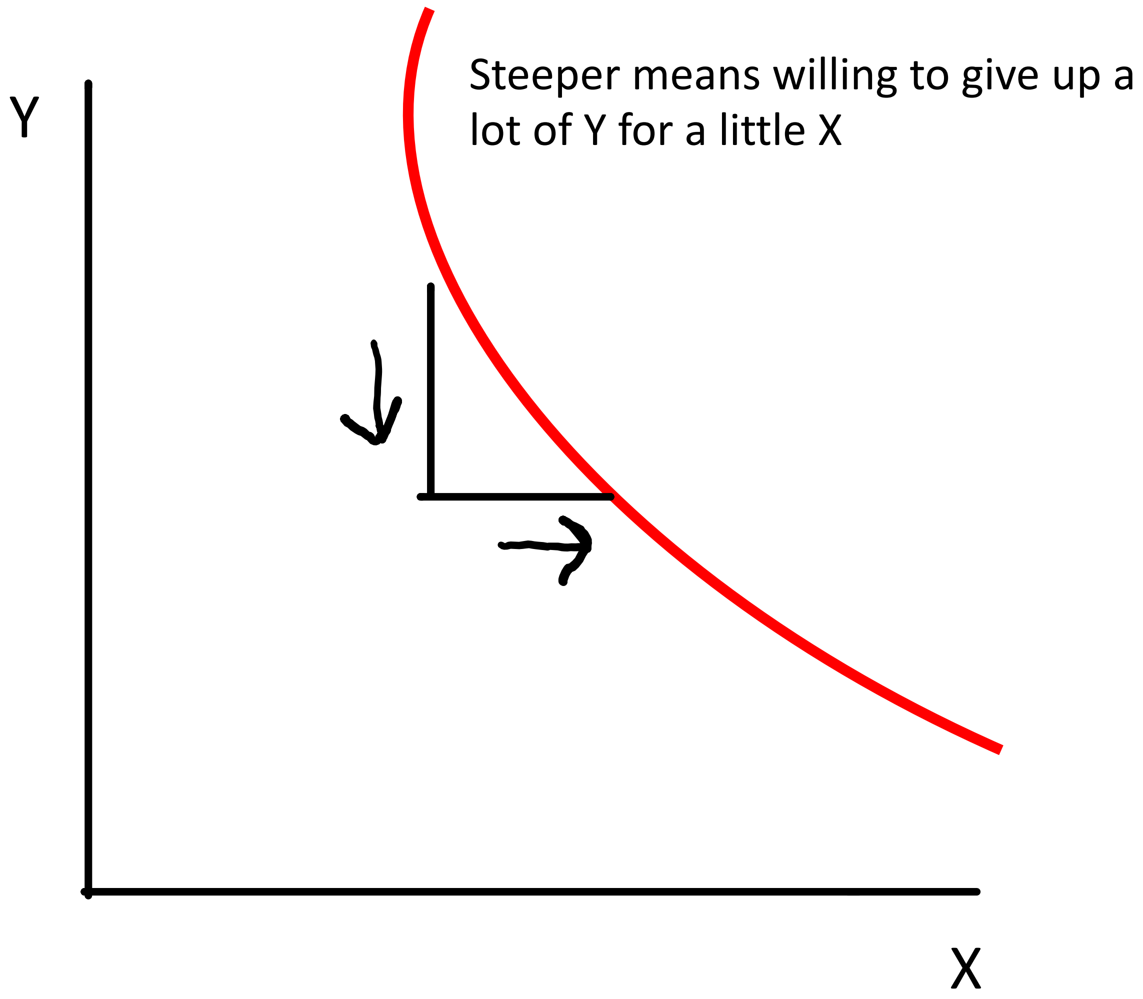
We multiply it by -1 because if you like X and Y, it will always be negative and we prefer positive numbers.

Notice that indifference curves are **non-linear**, so you can expect we will use derivatives to figure out the MRS.



The Marginal Rate of Substitution





What We Will Actually Do

Utility Functions

Rather than working directly with indifference curves, we will assume people's preferences can be represented by a **utility function**:

$$U(X, Y)$$

X is the quantity of the first good they are consuming

Y is the quantity of the second good they are consuming.

The utility function tells us how happy someone is with a particular bundle of goods.

Utility Functions are still ordinal!

If $u(C) > u(A)$ we only know C is preferred to A. Not by how much!

What are the units?

We say utility is measured in **utils** which are a made up unit.

The fact that we use a nonsense unit highlights that **the level** of utility is **irrelevant**. All we care about is comparisons across bundles.

Because the scale is irrelevant, utility contains same information if we re-scale it:

- $U(X, Y)$ represents the same preferences as $1{,}000{,}000U(X, Y) + 7$

Important Properties of Utility Functions

1. **Marginal Utility of X:** How does utility change if you get more of X?
2. **Marginal Utility of Y:** How does utility change if you get more of Y?

Important Properties of Utility Functions

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2. **Marginal Utility of Y:** How does utility change if you get more of Y?

3. **Diminish Marginal Utility of X:** How does my marginal utility from X change if I have more X? (Usually, we assume the benefit of more gets smaller when you already have a lot)
4. **Diminish Marginal Utility of Y:** How does my marginal utility from Y change if I have more Y?

Important Properties of Utility Functions

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4. **Diminishing Marginal Utility of Y:** How does my marginal utility from Y change if I have more Y?

5. **Complementarity:** How does marginal utility of X change if I have more Y?

Utility Functions

The Standard Case

Cobb-Douglas: $U(X, Y) = X^{\alpha} Y^{\beta}$

Cases that might throw you off

Perfect Substitutes: $U(X, Y) = \alpha X + \beta Y$

Perfect Complements: $U(X, Y) = \min(\alpha X, \beta Y)$

Quasi-Linear: $U(X, Y) = \alpha \log(X) + \beta Y$

Partial Derivatives

When dealing with multivariate functions, we take **partial derivatives**

The partial derivative of $f(X, Y)$ **with respect to X** is:

$$\frac{\partial f(X,Y)}{\partial X} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

Two differences from derivatives of one function:

- Write squiggly ∂ instead of d.
- Treat y like a constant.

All the derivative rules stay the same!

Quick Math Aside: Partial Derivatives

When dealing with multivariate functions, we take **partial derivatives**

The partial derivative of $f(X, Y)$ **with respect to X** is:

$$\frac{\partial f(X,Y)}{\partial X} = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$$

The partial derivative of $f(X, Y)$ **with respect to Y** is:

$$\frac{\partial f(X,Y)}{\partial Y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{h}$$

Examples

$f(X,Y)$	$\frac{\partial f}{\partial X}$	$\frac{\partial f}{\partial Y}$
XY	Y	X
XY^2	Y^2	$2XY$
$XY - X^2 - Y^2$	$Y - 2X$	$X - 2Y$
$\log(X)\, Y$	$\frac{Y}{X}$	$\log(X)$

iClicker: What is the partial derivative of $f(X, Y) = X + Y$ with respect to X?

A. 1+Y
B. 1
C. X
D. Y
E. X+Y

iClicker: What is the partial derivative of $f(X, Y) = X + Y$ with respect to X?

A. 1+Y

B. 1

C. X

D. Y

E. X+Y

Derivative of X is 1.

Treat Y like a constant. Derivative of a constant is 0.

Sum of two functions is sum of their derivatives.

So partial derivative is 1+0=1.

Cobb-Douglas Utility

This is one of most important functions in this class!

$$U(X, Y) = cX^{\alpha}Y^{\beta}$$

Examples:

$$U(X, Y) = XY$$

$$U(X, Y) = 5XY$$

$$U(X, Y) = X^{0.5}Y^{0.5}$$

$$U(X, Y) = 2X^{0.5}Y^{0.5} + 72.4$$

$$U(X, Y) = X^{0.75}Y^{0.25}$$

$$U(X, Y) = 0.75\log(X) + 0.25\log(Y)$$

Why Cobb-Douglas is the standard

Cobb-Douglas is the right model for most pairs of goods

- Consumer wants to consume some of both goods
- When bundle skewed towards one good, benefit of other good is large

Cobb-Douglas: Indifference Curve

$$U(X,Y) = XY$$

How do we graph the indifference curve?

Cobb-Douglas: Indifference Curve

$$U(X, Y) = XY$$

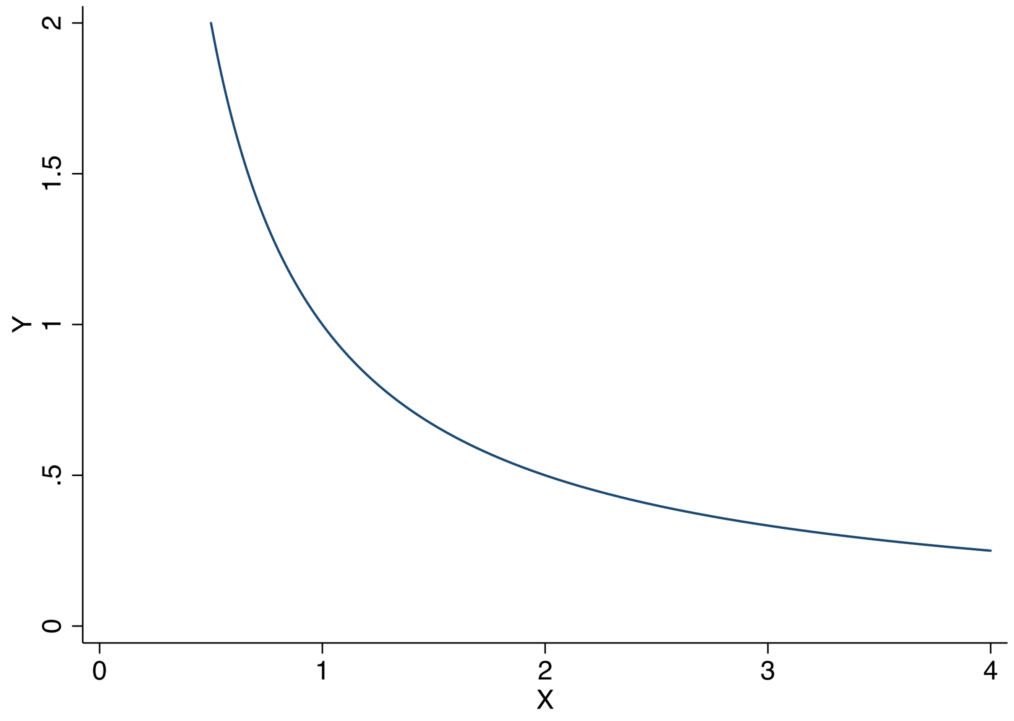
How do we graph the indifference curve?

1. Pick any level of utility, say 1.

$$1 = XY$$

2. Solve for Y as function of X:

$$Y = \frac{1}{X}$$



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$$U(X,Y) = XY$$

How do we graph the indifference curve?

1. Pick any level of utility, say 1.

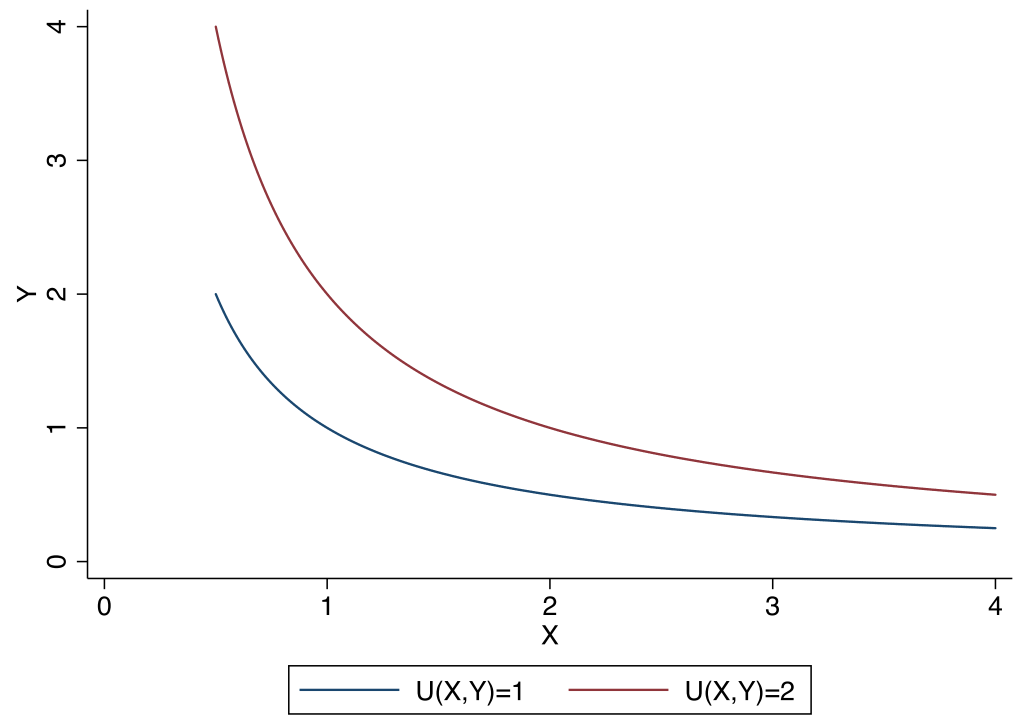
$$1 = XY$$

2. Solve for Y as function of X:

$$Y = \frac{1}{X}$$

3. Can pick a higher utility and repeat too:

$$Y = \frac{2}{X}$$



Cobb-Douglas: Marginal Utility

Marginal Utility tells us how much utility changes if we get more of one of the goods.

$$U(X, Y) = XY$$

What is the marginal utility of X?

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Marginal Utility tells us how much utility changes if we get more of one of the goods.

$$U(X, Y) = XY$$

What is the marginal utility of X?

$$\frac{\partial(XY)}{\partial x} = Y$$

Interpretation: Benefit of X is bigger when you have more of Y.

Cobb-Douglas: Marginal Utility

Marginal Utility tells us how much utility changes if we get more of one of the goods.

$$U(X, Y) = XY$$

What is the marginal utility of X?

$$MU_X = \frac{\partial(XY)}{\partial X} = Y$$

What is marginal utility of Y?

$$MU_Y = \frac{\partial(XY)}{\partial Y} = X$$

Benefit of each good is higher when have more of the other good.

iClicker: What is the marginal utility of X if $U(X,Y) = X^{0.25}Y^{0.75}$?

A. X

B. Y

C. $0.25X^{-0.75}$

D. $0.25X^{-0.75}Y^{0.75}$

E. $0.75X^{-0.75}Y^{0.75}$

iClicker: What is the marginal utility of X if $U(X,Y) = X^{0.25}Y^{0.75}$?

A. X
B. Y
C. $0.25X^{-0.75}$
D. $\mathbf{0.25X^{-0.75}Y^{0.75}}$
E. $0.75X^{-0.75}Y^{0.75}$

$$U(X,Y) = X^{0.25}Y^{0.75}$$

$$MU_X = \frac{\partial}{\partial x}(X^{0.25}Y^{0.75}) = 0.25X^{-0.75}Y^{0.75}$$

iClicker: What is the marginal utility of Y if $U(X, Y) = X^{0.5}Y^{0.5}$?

A. X

B. Y

C. $0.5X^{0.5}Y^{-0.5}$

D. $0.5X^{-0.5}Y^{-0.5}$

E. $0.5X^{0.5}Y^{0.5}$

iClicker: What is the marginal utility of Y if $U(X, Y) = X^{0.5}Y^{0.5}$?

A. X

B. Y

C. $\mathbf{0.5X^{0.5}Y^{-0.5}}$

D. $0.5X^{-0.5}Y^{-0.5}$

E. $0.5X^{0.5}Y^{0.5}$

$$U(X, Y) = X^{0.5}Y^{0.5}$$

$$MU_Y = \frac{\partial}{\partial y}(X^{0.5}Y^{0.5}) = 0.5X^{0.5}Y^{-0.5}$$

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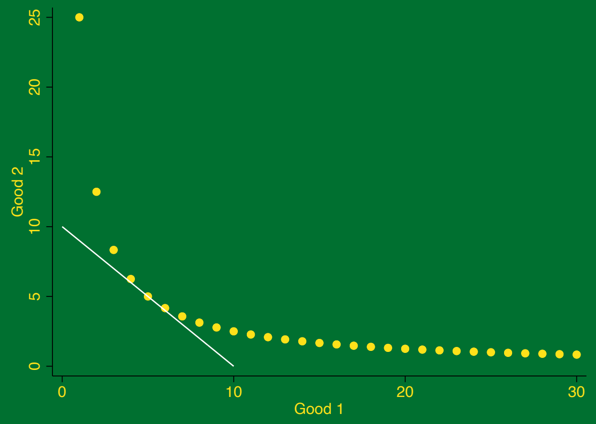

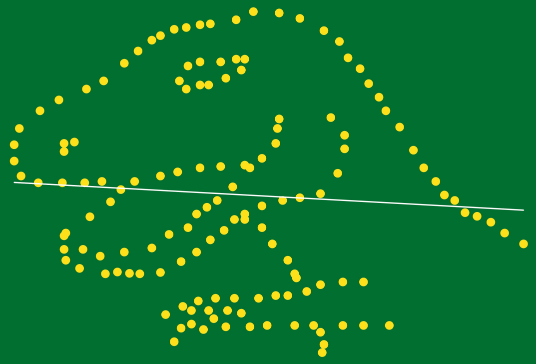

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Our Model of Consumer Theory

Our model of how consumers make choices will have two ingredients:

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2. Preferences (given by a **utility function**)

Interpreting Marginal Utility

Marginal utility tells us how much your utility increases if you get a little more X or Y given what you already have.

So if:

$$MU_Y(X,Y) = 0.5X^{0.5}Y^{-0.5}$$

Then:

- If have X=2 and Y=2, utility increases by $MU_Y = 0.5(2^{0.5}2^{-0.5}) = 0.5$
- If have X=2 and Y=10, utility increases by $MU_Y = 0.5(2^{0.5}10^{-0.5}) = 0.224$
- If have X=3 and Y=2, utility increases by $MU_Y = 0.5(3^{0.5}2^{-0.5}) = 0.612$

How does my marginal utility from Y change if I have more Y?

Let's keep working with $U(X,Y) = X^{0.5}Y^{0.5}$

So the marginal utility of Y is $MU_Y(X,Y) = 0.5X^{0.5}Y^{-0.5}$.

How does this change if I get more Y?

$$\frac{\partial MU_Y(X,Y)}{\partial Y} = 0.5X^{0.5}(-0.5Y^{-0.5-1})$$

$$\frac{\partial MU_Y(X,Y)}{\partial Y} = -0.25X^{0.5}Y^{-1.5}$$

Interpretation: This is negative. So holding amount of X constant, benefit of Y falls as we have more Y.

How does my marginal utility from Y change if I have more X?

Let's keep working with $U(X,Y) = X^{0.5}Y^{0.5}$

So the marginal utility of Y is $MU_Y(X,Y) = 0.5X^{0.5}Y^{-0.5}$.

How does this change if I get more X?

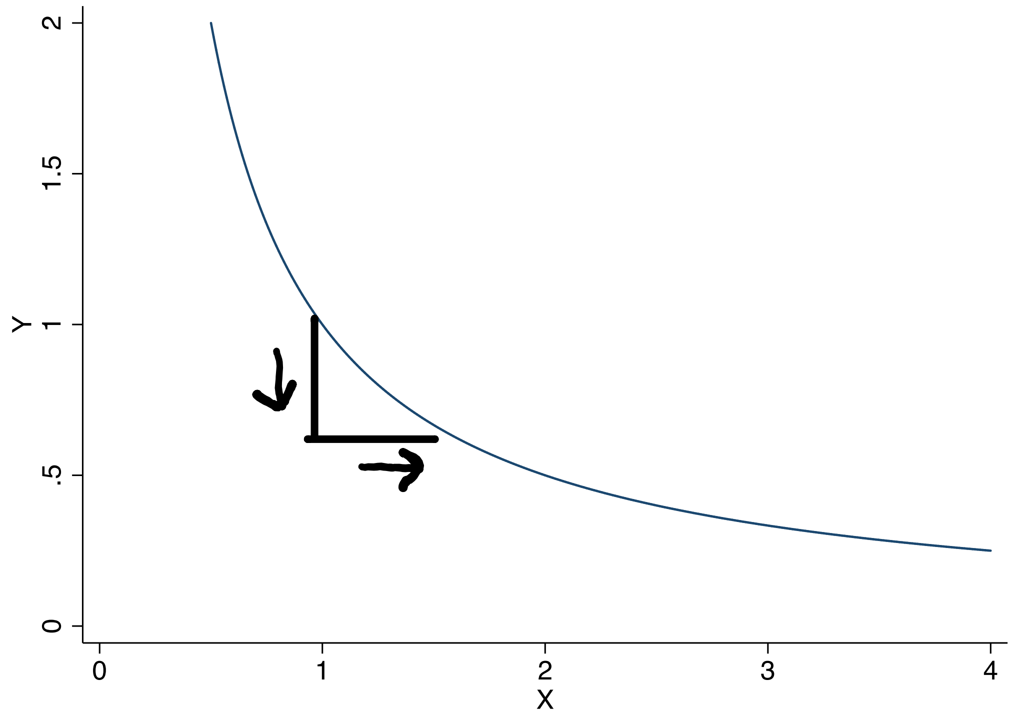
$$\frac{\partial MU_Y(X,Y)}{\partial X} = 0.5(0.5X^{0.5-1})(Y^{-0.5})$$

$$\frac{\partial MU_Y(X,Y)}{\partial X} = 0.25X^{-0.5}Y^{-0.5}$$

Interpretation: This is positive. So holding amount of Y constant, benefit of Y is bigger if we we have more X.

Cobb-Douglas: Marginal Rate of Substitution

Recall the Marginal Rate of Substitution is the slope of the indifference curve or how much Y someone is willing to give up for one more unit of X without changing their happiness.



Math Trick to Find Marginal Rate of Substitution (MRS)

If I give you a little more X, say ΔX, your utility increases by about $MU_X \times \Delta X$

Why? Marginal utility is rate of change of utility at current bundle.

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If I give you a little more X, say ΔX, your utility increases by about $MU_X \times \Delta X$

Why? Marginal utility is rate of change of utility at current bundle.

Similarly, if I take a little Y from you, say ΔY, your utility changes by $MU_Y \times \Delta Y$

Math Trick to Find Marginal Rate of Substitution (MRS)

If I give you a little more X, say ΔX, your utility increases by about $MU_X \times \Delta X$

Why? Marginal utility is rate of change of utility at current bundle.

If I take a little Y from you, say ΔY, your utility changes by $MU_Y \times \Delta Y$

If I take a little Y and give a little X so that your happiness doesn't change, it must be true that:

$$0 = MU_X \Delta X + MU_Y \Delta Y$$

Math Trick to Find Marginal Rate of Substitution (MRS)

$$0 = MU_X \Delta X + MU_Y \Delta Y$$

Recall that the MRS is defined as: $-\frac{\Delta Y}{\Delta X}$.

We just saw how to calculate MU_X and MU_Y when we know the utility function.

We have all the ingredients to find an equation for the MRS.

Math Trick to Find Marginal Rate of Substitution (MRS)

$$0 = MU_X \Delta X + MU_Y \Delta Y$$

Subtract $MU_Y \Delta Y$ from both sides:

$$-MU_Y \Delta Y = MU_X \Delta X$$

Divide both sides by ΔX:

$$-\frac{\Delta Y}{\Delta X} MU_Y = MU_X$$

Divide both sides by MU_Y:

$$-\frac{\Delta Y}{\Delta X} = \frac{MU_X}{MU_Y}$$

But $-\frac{\Delta Y}{\Delta X}$ is the **Marginal Rate of Substitution**!

The Marginal Rate of Substitution Formula

The Marginal Rate of Substitution is given by the following formula:

$$MRS = -\frac{\Delta Y}{\Delta X} = \frac{MU_X}{MU_Y}$$

Cobb-Douglas: Marginal Rate of Substitution

$$U(X,Y) = X^{0.5}Y^{0.5}$$

1. Need to calculate each of the marginal utilities.

$$MU_X = \frac{d}{dx}(X^{0.5}Y^{0.5}) = (0.5X^{0.5-1})Y^{0.5} = 0.5X^{-0.5}Y^{0.5}$$

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$$MU_Y = \frac{\partial}{\partial Y}(X^{0.5}Y^{0.5}) = X^{0.5}(0.5Y^{0.5-1}) = 0.5X^{0.5}Y^{-0.5}$$

Cobb-Douglas: Marginal Rate of Substitution

$$MU_X = 0.5X^{-0.5}Y^{0.5}$$
$$MU_Y = 0.5X^{0.5}Y^{-0.5}$$

2. Divide MU_X by MU_Y.

$$\frac{MU_X}{MU_Y} = \frac{0.5X^{-0.5}Y^{0.5}}{0.5X^{0.5}Y^{-0.5}}$$

Simplify by combining like terms and using rule $\frac{Z^a}{Z^b} = Z^{a-b}$.

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$$\frac{MU_X}{MU_Y} = \frac{\cancel{0.5}}{\cancel{0.5}}X^{-0.5-0.5}Y^{0.5-(-0.5)} = X^{-1}Y^1 = \frac{Y}{X}$$

iClicker: What is the Marginal Rate of Substitution of $U(X,Y) = XY$?

$A.$ $\frac{X^{0.5}}{Y^{0.5}}$

$B.$ $\frac{Y^{0.5}}{X^{0.5}}$

$C.$ $\frac{Y}{X}$

$D.$ $\frac{X}{Y}$

E. 1

iClicker: What is the Marginal Rate of Substitution of $U(X,Y) = XY$?

A. $\frac{X^{0.5}}{Y^{0.5}}$
B. $\frac{Y^{0.5}}{X^{0.5}}$
C. $\boldsymbol{\frac{Y}{X}}$
D. $\frac{X}{Y}$
E. 1

Cobb-Douglas: Marginal Rate of Substitution

$$U(X, Y) = XY$$

1. Need to calculate each of the marginal utilities.

$$MU_X = \frac{\partial}{\partial x}(XY) = Y$$

Cobb-Douglas: Marginal Rate of Substitution

$$U(X,Y) = XY$$

1. Need to calculate each of the marginal utilities.

$$MU_X = \frac{\partial}{\partial x}(XY) = Y$$

$$MU_Y = \frac{\partial}{\partial Y}(XY) = X$$

Cobb-Douglas: Marginal Rate of Substitution

$$MU_X = Y$$
$$MU_Y = X$$

2. Divide MU_X by MU_Y.

$$\frac{MU_X}{MU_Y} = \frac{Y}{X}$$

Interesting: Got the same MRS for both Cobb-Douglas functions we checked.

Cobb-Douglas Marginal Rate of Substitution

$$U(X, Y) = cX^{\alpha}Y^{\beta}$$

$$\frac{MU_X}{MU_Y} = \frac{\alpha}{\beta}\frac{Y}{X}$$

This is true for any Cobb-Douglas!

Remembering/Understanding this formula will save you a lot of time!

Utility Functions

The Standard Case

Cobb-Douglas: $U(X, Y) = X^{\alpha} Y^{\beta}$

Cases that might throw you off

Perfect Substitutes: $U(X, Y) = \alpha X + \beta Y$

Perfect Complements: $U(X, Y) = \min(\alpha X, \beta Y)$

Quasi-Linear: $U(X, Y) = \alpha \log(X) + \beta Y$ (Try this one on your own!)

The Two Extreme Cases: Perfect Substitutes and Perfect Complements

Perfect Substitutes

$$U(X,Y) = \alpha X + \beta Y$$

1 unit of X is identical to $\frac{\beta}{\alpha}$ units of Y

Examples

- ½ gallons of milk versus gallons of milk
- Fun sized versus full size candy bars
- 4 glasses of wine or 1 bottle of wine

iClicker: What’s the equation for the indifference curve holding utility fixed at 10 if $U(X, Y) = 3X + Y$?

A. $Y = -X$

B. $Y = 1 - X$

C. $Y = 10 - X$

D. $Y = 10 - 3X$

E. $Y = 3X$

iClicker: What's the equation for the indifference curve holding utility fixed at 10 if $U(X, Y) = 3X + Y$?

A. $Y = -X$

B. $Y = 1 - X$

C. $Y = 10 - X$

D. $\boldsymbol{Y = 10 - 3X}$

E. $Y = 3X$

$$U(X, Y) = 3X + Y$$

1. Set $U(X, Y) = 10$:

$$10 = 3X + Y$$

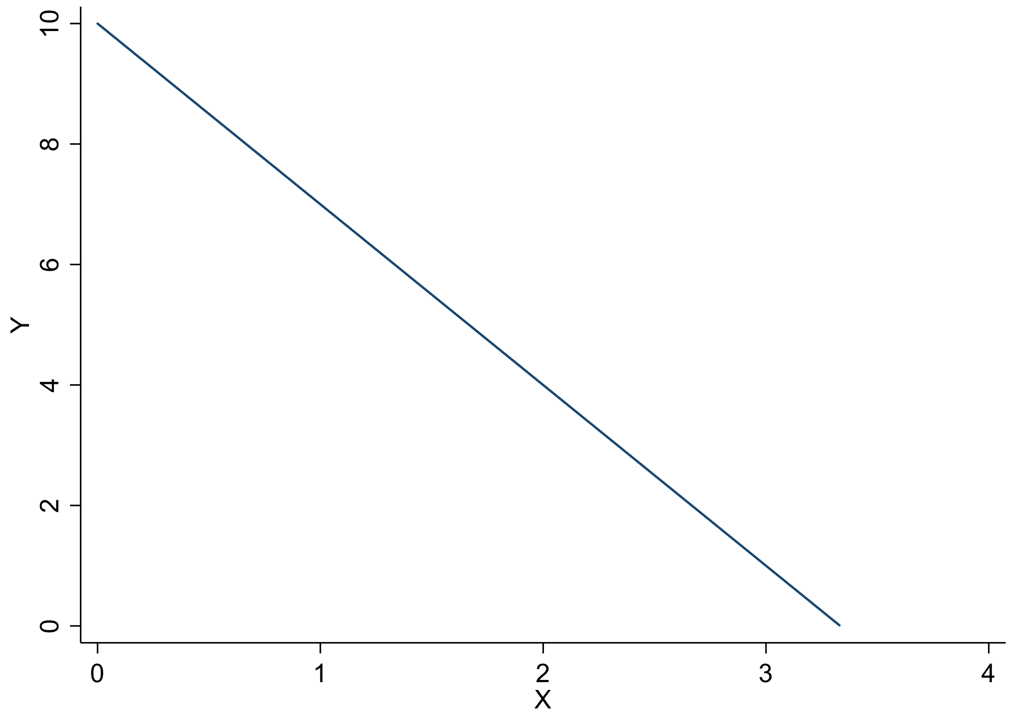
2. Solve for Y:

$$Y = 10 - 3X$$

Perfect Substitutes: Indifference Curves

$$U(X, Y) = 3X + Y$$

1. Set $U(X, Y) = 10$:
$$10 = 3X + Y$$
2. Solve for Y:
$$Y = 10 - 3X$$



Perfect Substitutes' indifference curves are linear!

Perfect Substitutes: Marginal Utilities

$$U(X,Y) = \alpha X + \beta Y$$

$$MU_X = \frac{\partial(\alpha X + \beta Y)}{\partial X} = \alpha(1X^{1-1}) + 0 = \alpha$$

$$MU_Y = \frac{\partial(\alpha X + \beta Y)}{\partial Y} = 0 + \beta(1Y^{1-1}) = \beta$$

Perfect Substitutes: Marginal Utilities

$$U(X,Y) = \alpha X + \beta Y$$

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$$MU_Y = \frac{\partial(\alpha X + \beta Y)}{\partial Y} = 0 + \beta(1Y^{1-1}) = \beta$$

Interpretation:

X gives a constant marginal utility of α

Y gives a constant marginal utility of β

iClicker: What is the marginal rate of substitution if $U(X,Y) = \alpha X + \beta Y$?

A. 1

B. α

C. β

D. $\frac{\alpha}{\beta}$

E. $\frac{\beta}{\alpha}$

iClicker: What is the marginal rate of substitution if $U(X,Y) = \alpha X + \beta Y$?

A. 1

$B.\ \alpha$

$C.\ \beta$

$\boldsymbol{D.\ \frac{\alpha}{\beta}}$

$E.\ \frac{\beta}{\alpha}$

$$MRS = -\frac{\Delta Y}{\Delta X} = \frac{MU_X}{MU_Y}$$

$$MRS = \frac{\alpha}{\beta}$$

Interpretation: Willing to give up $\frac{\alpha}{\beta}$ units of Y to get a unit of X.

Understanding Perfect Substitutes MRS

Example:

Half gallons and full gallons of milk

$$U(H, F) = H + 2F$$

$$MRS = \frac{1}{2}$$

Willing to give up ½ a full gallon of milk to get a half gallon.

Perfect Complements

$$U(X, Y) = \min(\alpha X, \beta Y)$$

Need to consume $\frac{\beta}{\alpha}$ units of Y for every unit of X.

No benefit to consuming them out of that proportion!

Examples (Not many because so extreme)

- Left shoes and right shoes

The Minimum Function

The minimum function gives the smaller of two values:

$$\min(X, Y) = \begin{cases} X \; if \; X \leq Y \\ Y \; if \; Y \leq X \end{cases}$$

Examples

- $\min(3,5) = 3$
- $\min(2,2) = 2$
- $\min(10,5) = 5$
- $\min(1,2.1) = 1$

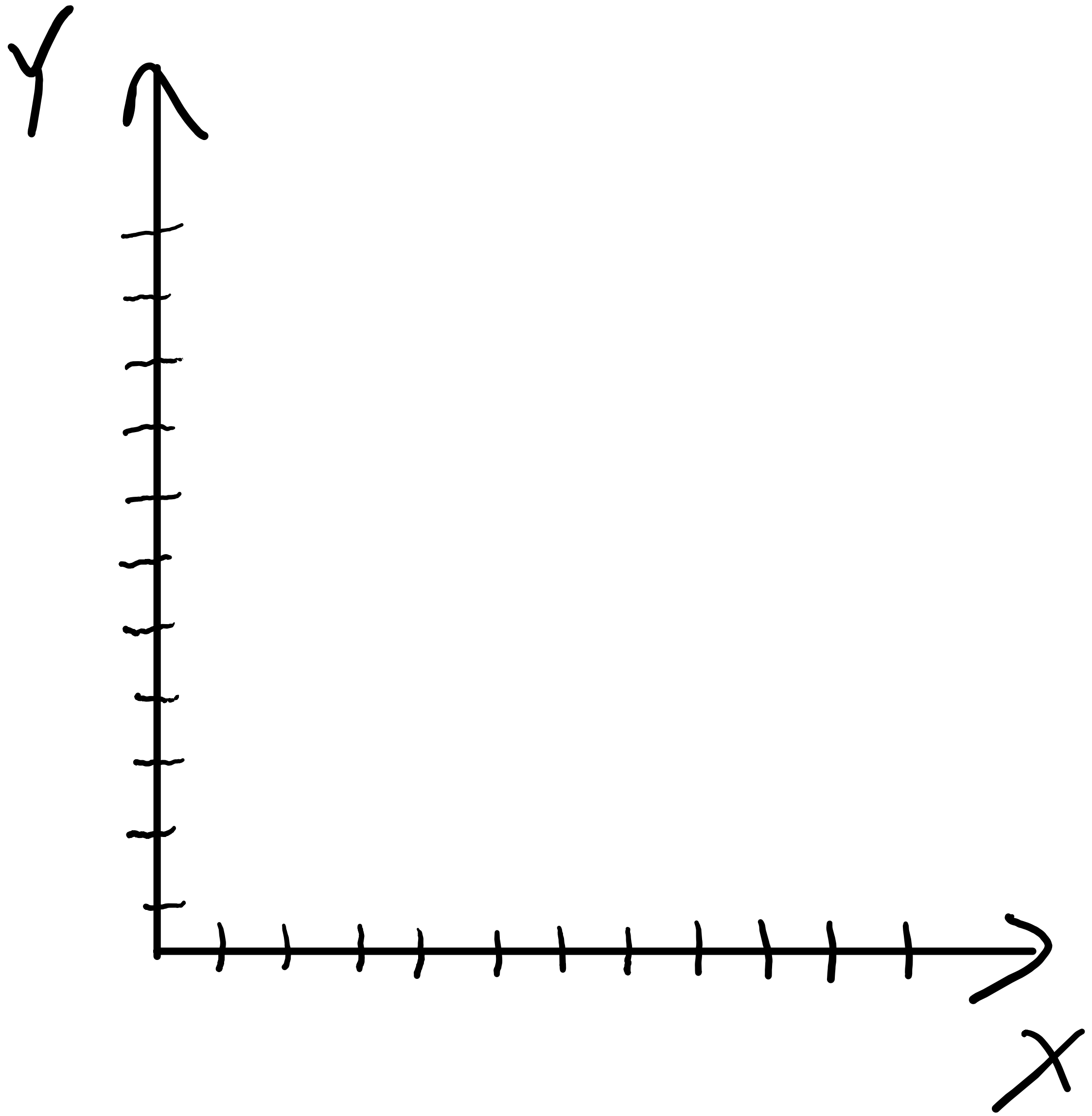
Perfect Complements: Indifference Curves

$$U(X,Y) = \min(X,Y)$$

1. Pick a utility, say 5.

$$5 = \min(X,Y)$$

2. What bundles give utility 5?



Perfect Complements: Indifference Curves

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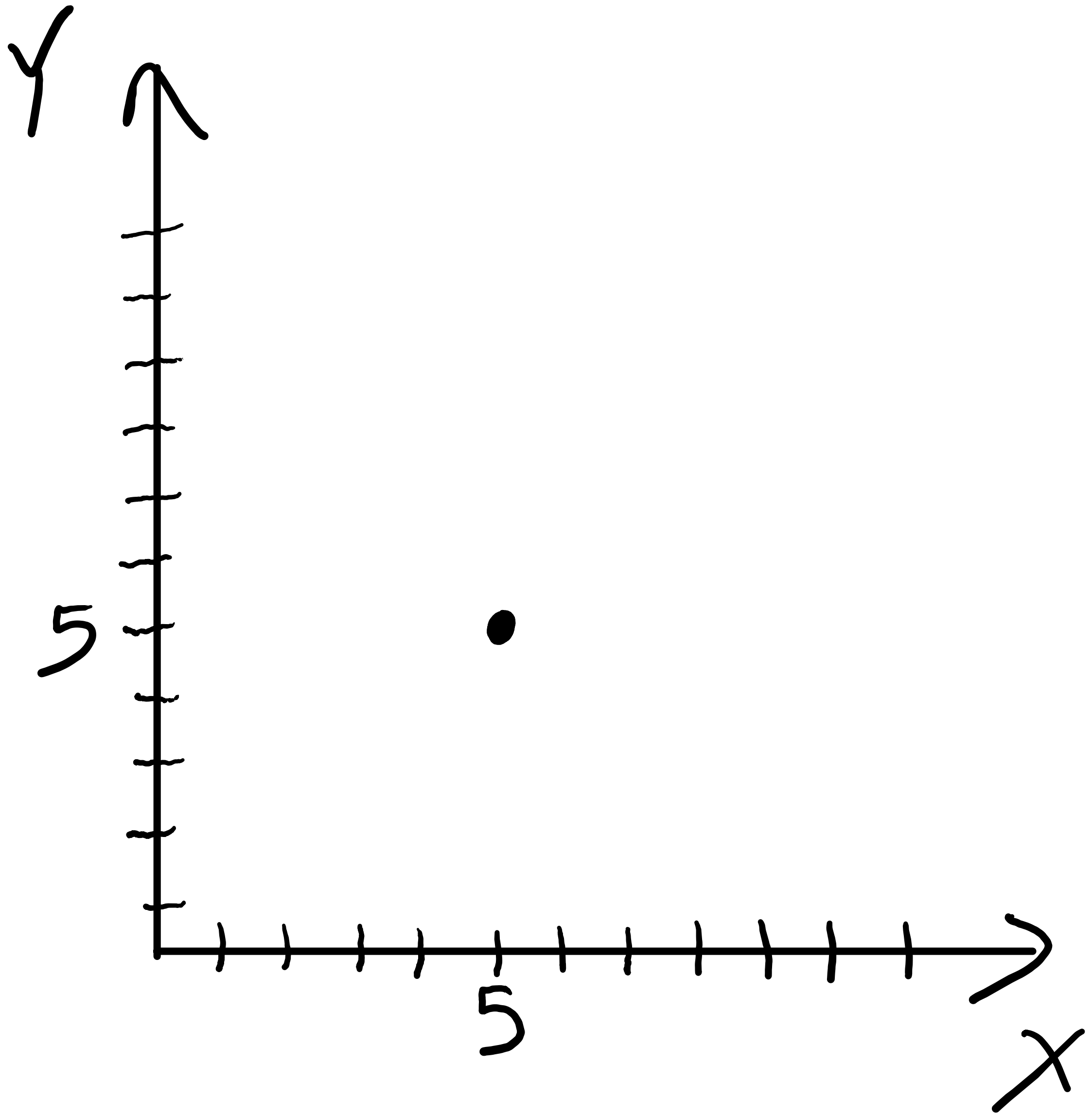
- $X = 5$ and $Y = 5$



Perfect Complements: Indifference Curves

$$U(X, Y) = \min(X, Y)$$

1. Pick a utility, say 5.

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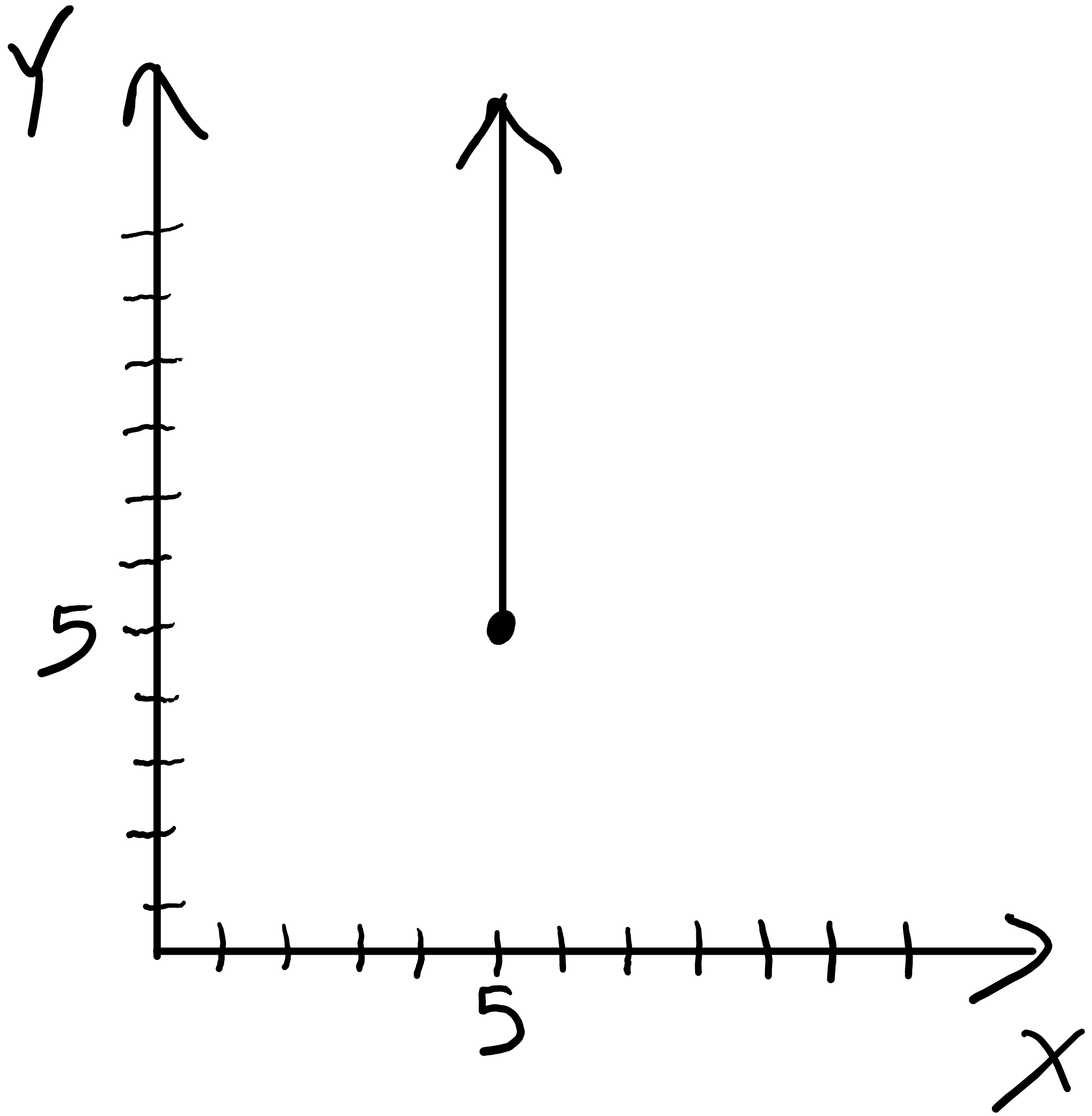
2. What bundles give utility 5?

- $X = 5$ and $Y = 5$
- $X = 5$ and $Y \geq 5$



Perfect Complements: Indifference Curves

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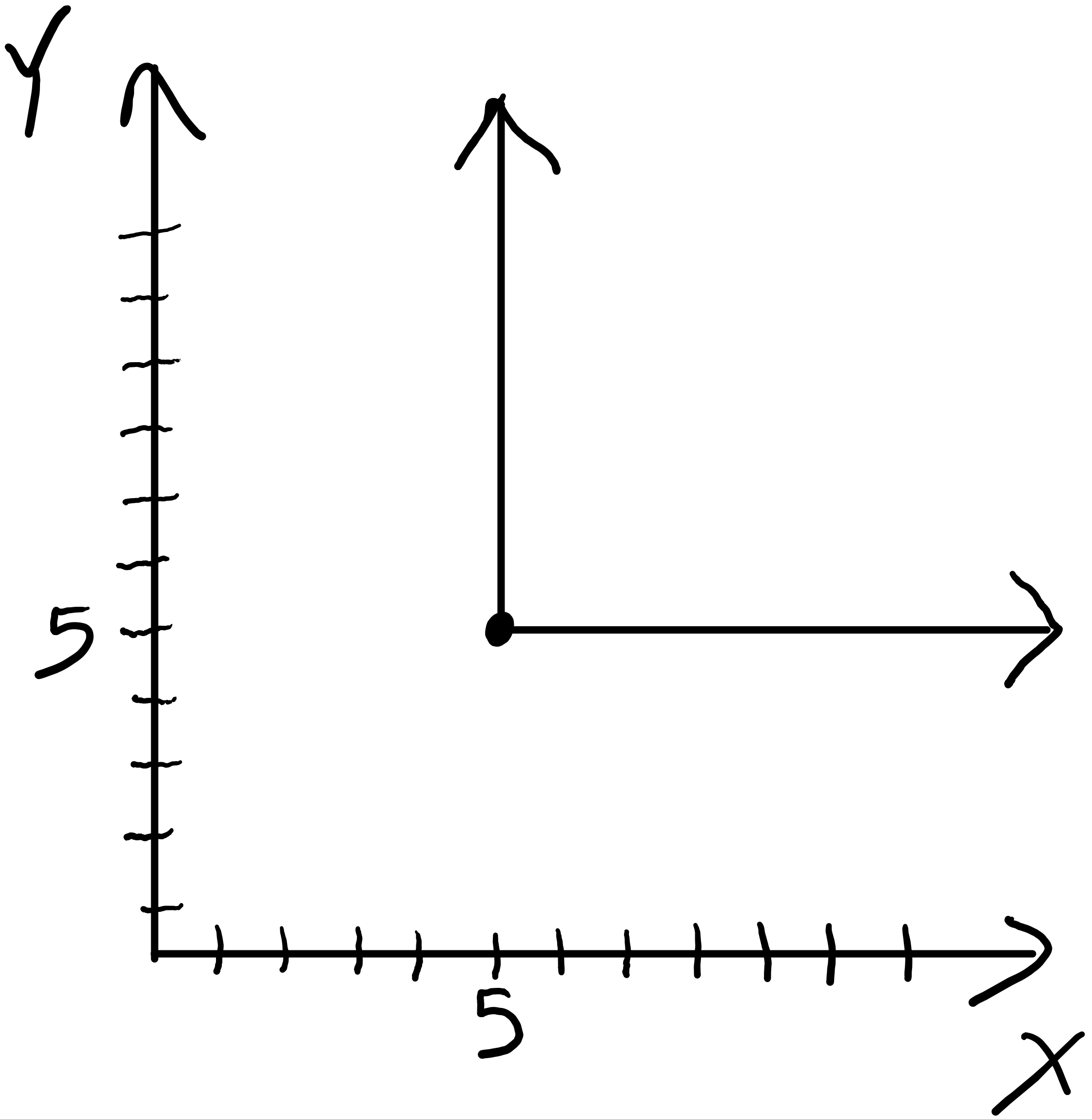
- $X = 5$ and $Y \geq 5$
- $X \geq 5$ and $Y = 5$



Perfect Complements: Indifference Curves

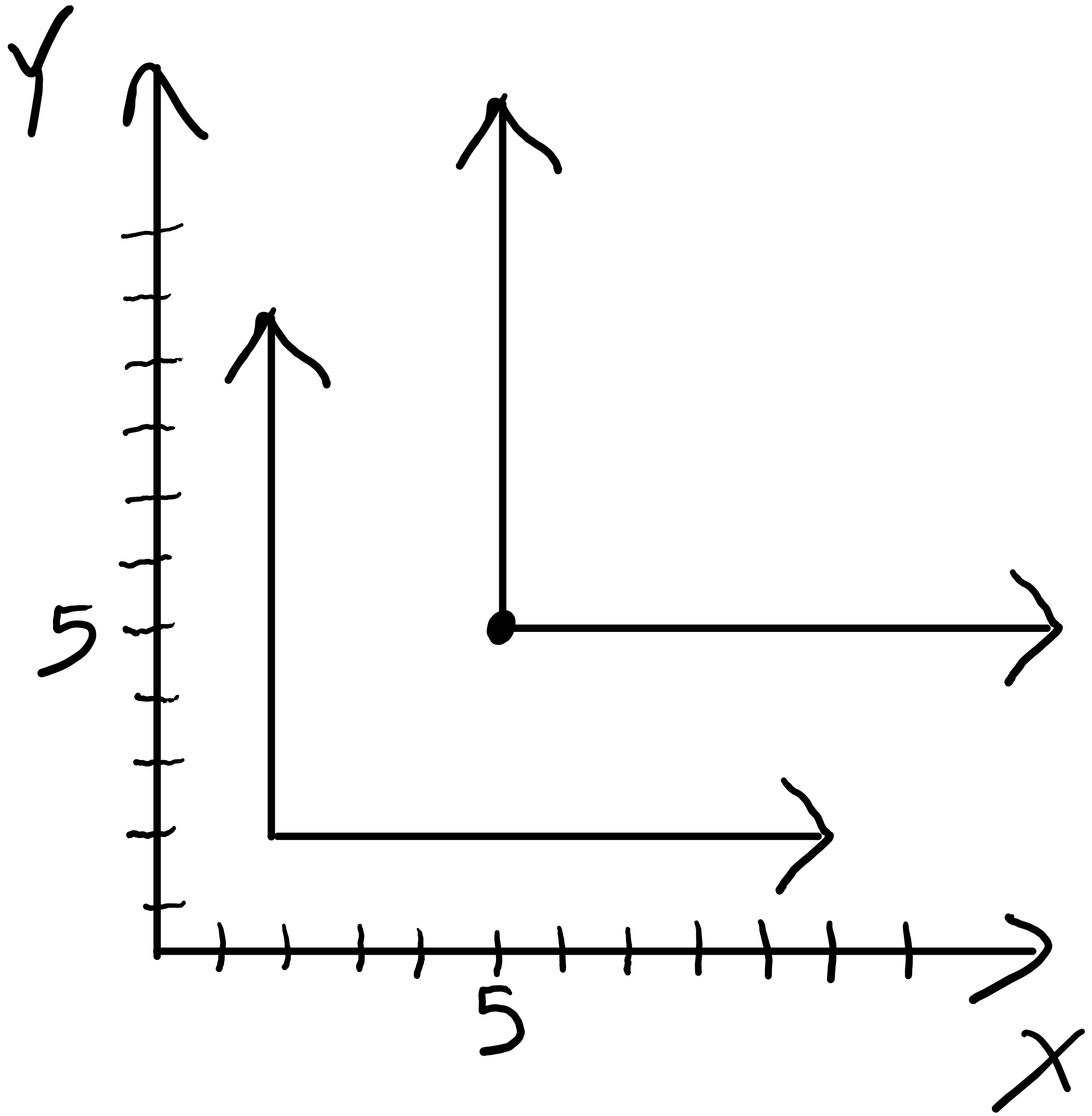
$$U(X, Y) = \min(X, Y)$$

1. Pick a utility, say 5.

$$5 = \min(X, Y)$$

2. What bundles give utility 5?

- $X = 5$ and $Y = 5$
- $X = 5$ and $Y \geq 5$
- $X \geq 5$ and $Y = 5$



Perfect Complements: Marginal Utility

How utility changes if you get another unit of X depends on Y

1. If $X < Y$, then $\min(X, Y) = X$ so another unit of X increases utility by 1.

2. If $X \geq Y$, then $\min(X, Y) = Y$ so another unit of X does not change utility.

Perfect Complements: Marginal Utility

How utility changes if you get another unit of X depends on Y

1. If $X < Y$, then $\min(X, Y) = X$ so another unit of X increases utility by 1.

2. If $X \geq Y$, then $\min(X, Y) = Y$ so another unit of X does not change utility.

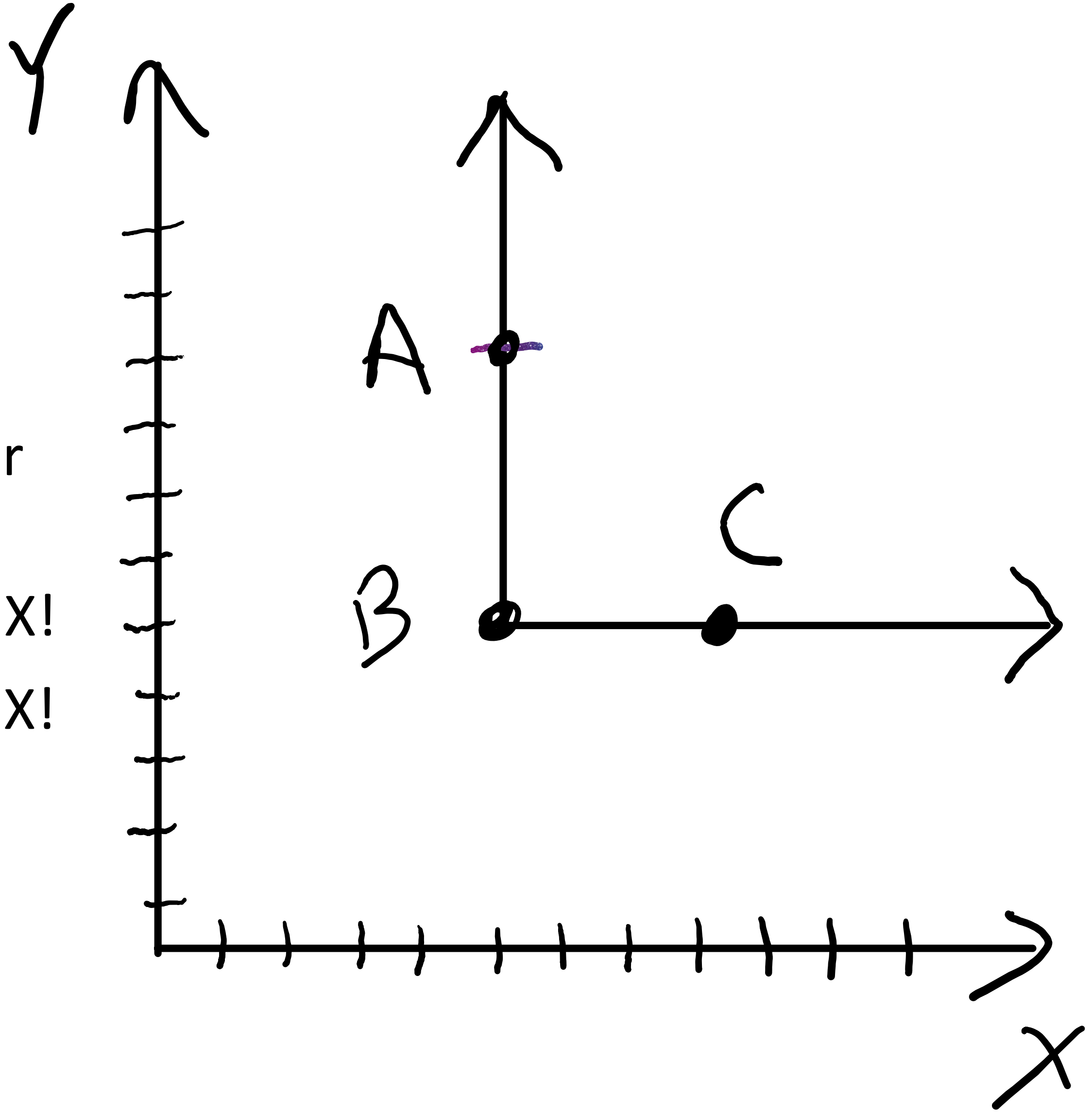
Same idea for Y: only beneficial if have more X.

Perfect Complements: Marginal Rate of Substitution

Remember the marginal rate of substitution is how much Y the agent is willing to give up for another unit of X.

This is not well defined on the curve:

- A: No worse off if give up 4 units of Y for 1 unit of X
- B: Not willing to give up any Y for more X!
- C: Not willing to give up any Y for more X!



Recap

Preferences tell us how people choose between two bundles.

Indifference Curves summarize this preference information:

- Agent exactly as happy with every bundle on the curve
- Prefers bundles on higher indifferent curves
- Does not prefer bundles on lower indifferent curves

In this class, model preferences with **utility functions**. These just summarize information in indifferent curves.

Recap

Use four main utility functions in Intermediate Micro:

- **Cobb-Douglas:** $U(X, Y) = cX^{\alpha}Y^{\beta}$
- **Perfect Substitutes:** $U(X, Y) = \alpha X + \beta Y$
- **Perfect Complements:** $U(X, Y) = \min(\alpha X, \beta Y)$
- **Quasi-Linear:** $U(X, Y) = \alpha \log(X) + \beta Y$

Important properties of each:

- Corresponding indifference curve
- Marginal Utilities
- Marginal Rate of Substitution