EC428/528: Problem Set 3
Due by class time on Monday, May 23rd

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In this homework, we will practice working with the intertemporal utility maximization model from class. We will also think about a couple applications of behavioral economics.

Ground Rules

- May work in groups of up to 5 (e-mail me if you need help finding a group).
- Only submit one assignment per group.
- Homework can be typed or (legibly) handwritten.
- Homework can be submitted in class or via Canvas.
- I will only answer e-mailed questions if sent by Tuesday, May 3rd.

1 Time Preferences (75 points)

In class, we solved a two-period savings model where a consumer allocates income across two periods. We assumed the consumer's intertemporal utility function was given by: $U(c_1, c_2) = log(c_1) + \delta log(c_2)$ and that their intertemporal budget constraint was $M_1 + \frac{M_2}{1+r} = c_1 + \frac{c_2}{1+r}$.

Along the way to solving that problem, we found that consumers should select their consumption in each period so that:

$$u'(c_1) = \delta(1+r)u'(c_2),$$

where δ is the exponential discount rate and r is the interest rate.

In this problem, we will extend this problem from two to three periods. We will solve it with exponential discounting and quasi-hyperbolic discounting.

1.1. Assume the consumer's "flow" utility is given by $log(c_t)$ in each period and that the consumer has an exponential discount rate δ. What is the intertemporal utility function with three periods (instead of two)? (10 points)

1.2. Assume the consumer receives income M_1 in period 1, M_2 in period 2, and M_3 in period 3 and that the interest rate is still r. What is the intertemporal budget constraint with three periods (instead of two)? (10 points)

1.3. We normally solve utility maximization problems where the consumer only chooses two things. We use two equations to solve for these two unknowns: (1) the marginal rate of substitution equals the price ratio and (2) the budget constraint. When solving for more than two things, we can replace the MRS=price ratio equation with the requirement that the marginal utility per dollar for each good must be the same. If it is helpful, you can write for each good, g, $MU_g/p_g = \Lambda$ where Λ is sometimes called the "Lagrange Multiplier." With only two goods, this gives you exactly the same information as MRS=price ratio, but with three goods it gives you more equations.

Use this to extend the condition that $u'(c_1) = \delta(1+r)u'(c_2)$ to include utility in the third period. (5 points)

1.4. Solve for consumption in each period assuming $u(c_t) = log(c_t)$ in every period, $\delta = 0.95$, $r = 0.05$, and $M_1 = M_2 = M_3 = 100$. (25 points)

1.5. Now, assume the consumer has quasi-hyperbolic time preferences with additional parameter $\beta = 0.9$. Solve for consumption in each period assuming $u(c_t) = log(c_t)$ in every period, $\delta = 0.95$, $r = 0.05$, and $M_1 = M_2 = M_3 = 100$. (25 points)

2 Applications (25 points total - 12.5 points each)

Comment on the following statements. Are they consistent with the standard economic model? Explain why or why not.

2.1. A couple near retirement age have all of their savings ($\tilde{\$}$200,000) in safe bonds but buys a lottery ticket every week.

2.2. Most people who save for retirement save exactly the percentage that their employer matches or the maximum the government allows them to contribute to tax protected plans.