EC 590 HW1

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Question 1

1.

The neo-classical AHM with a missing labor market, but full land market is as follows.

$$
\begin{aligned}
& max_{x,l} u(x,l) && (1)\\
& s.t \\
& px + rA^h \leq F(L^f, A) + rA^m && (2)\\
& A = A^f + A^h && (3)\\
& E^l = L^f + l && (4)\\
& E^A = A^f + A^m && (5)\\
& && (6)
\end{aligned}
$$

There is no labor market, so labor input to the production function is just L^f, or family labor. We can show the separation hypothesis by rearranging the constraints. Plugging (3)-(5) into (2) gives us the following.

$$
\begin{aligned}
px + r(A - A^f) &\leq F(L^f, A) + r(E^A - A^f) \\
ps + rA - rA^f &\leq F(L^f, A) - rE^A - rA^f \\
px &\leq F(L, A) - rA + rE^A \\
px &\leq \Pi + rE^A && (7)
\end{aligned}
$$

$$\Pi = F(L^f, A) - rA \tag{8}$$

We can now rewrite the problem as (1) s.t (6), (7). Solving that problem gives us the following first order conditions.

$$
\begin{aligned}
& [L^f] : F_{L^f} = 0 && (9)\\
& [A] : F_A = r && (10)\\
& [x] : u_x = \lambda p && (11)\\
& [l] : u_l = 0 && (12)
\end{aligned}
$$

The separation hypothesis is confirmed by (8), and (9). The FOCs for the production function do not depend on preferences, so the household's optimal profit maximizing behavior is independent of their utility maximizing, so the household maximizes profits.

2.

Let $T = \frac{A}{1+\theta}$ whereλ is unobserved land quality, and T is the value of land we actually observe. We can easily tell that T and λ are inversely related by taking the first derivative of T wrt λ.

$$\frac{\partial T}{\partial \theta} = \frac{-A}{(a+\theta)^2} < 0$$

The derivative is negative, so there in an inverse relation. If we test the separation hypothesis using T as our independent variable, we will likely have measurement error. T will have a wider spread than A, so this is classical measurement error. In the presence of classical measurement error, coefficients are biased towards 0. So our coefficient on land will be biased towards 0, which means we are less likely to find an effect, so we will be more likely to find that the separation hypothesis fails.

3.

Now, suppose utility is a function of h which measures health of the household. h is determined by x, l, and exogenous factor η. In other words, utility is not $u(x, l, h)$, where $h = h(x, l, \eta)$. Further suppose that T from the previous part is equal to A, so $\theta = 0$. We also now have a complete labor market.

a)

First, we will keep the standard production function $F(L, A)$, and show that the household maximizes profits.

$$\begin{aligned} max_{x,l} u(x, l, h(x, l, \eta)) \quad (13) \\ s.t \\ px + wl \leq \Pi + wE^L + rE^A \quad (14) \\ \Pi = F(L, A) - wL - rA \quad (15) \\ \quad (16) \end{aligned}$$

Not the difference here is that we have a complete labor market so those terms now appear in the constraints. The FOCs that characterize this problem are as follows.

$$\begin{aligned} [L] : F_L = w \quad (17) \\ [A] : F_A = r \quad (18) \\ [x] : u_x + u_h h_l = \lambda p \quad (19) \\ [l] : u_l + u_h h_l = \lambda w \quad (20) \end{aligned}$$

Notice that again (17) and (18) the FOCs on the production inputs depend only on the relative prices of inputs, so production is again independent of consumption, so they maximize profits.

b)

Now modify the production function to depend on health $F(L, A, h)$. In this case, we'll show the separation hypothesis fails. The only thing different from the setup in part a is the modification to the production function. The FOCs are thus characterized as follows.

$$\begin{aligned} [L] : F_L L_h = w \quad (21) \\ [A] : F_A = r \quad (22) \\ [x] : u_x + \lambda F_h h_x = \lambda p \quad (23) \\ [l] : u_l + u_h h_l + \lambda F_h h_l = \lambda w \quad (24) \end{aligned}$$

(23) and (24) show the separation hypothesis fails. By rearranging (24) we get the following.

$$h_l = \frac{\lambda w - u_l}{u_h F_h}$$

This shows us that preferences and production decisions are intertwined when choosing the optimal labor, so the separation hypothesis fails.

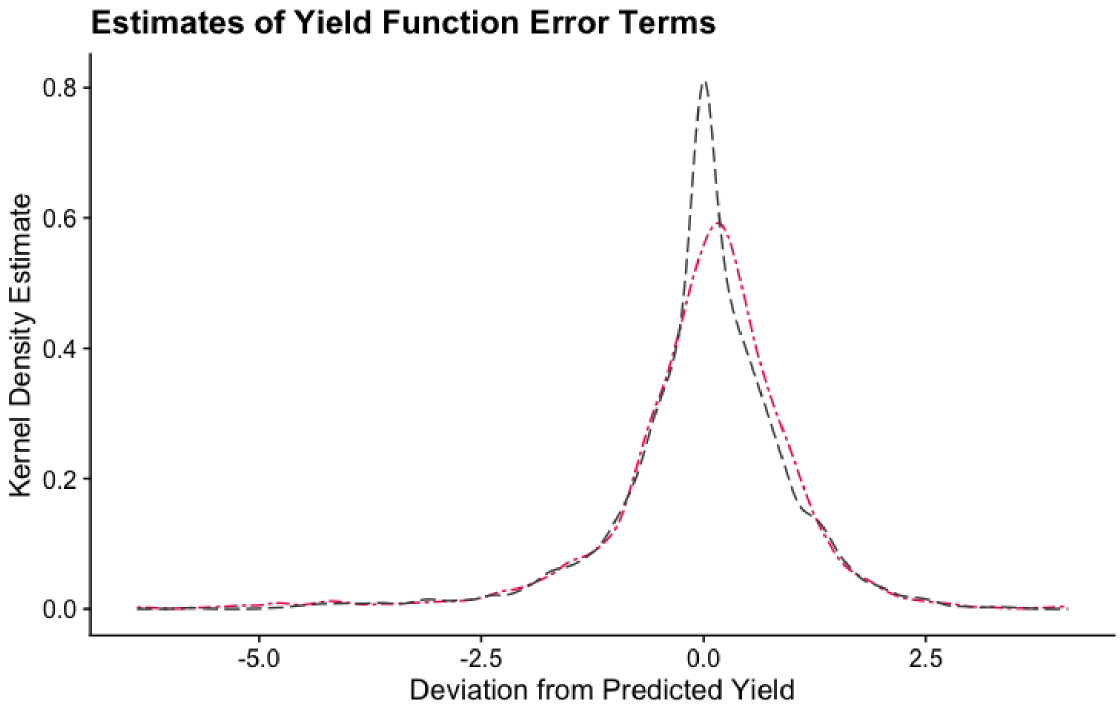

Figure 1: The black line is the error density from the regression aggregated at household level, and red is at village level.

Question 2

3.

We would expect ϵ_{vhtci} to be a diffuse distribution compared to the errors from the household-level regression. The errors represent a misallocation of resources, and the at higher levels of aggregation, we expect more inefficiencies. It is harder to optimally allocate across a whole village than across just a household.

If there were complete markets in the village, we would expect this misallocation to disappear. Optimally, the village would spread the land and labor around to produce as much as possible, and with complete markets, they could achieve that. So in the presence of complete markets, the distributions of error terms should be identical to the errors at the household level. I think we would expect it to be a degenerate distribution centered at zero deviation from expected. With complete markets, the village could insure against any variation, and always hit the optimal predicted yield.

4.

Figure 1 shows the graph of the two sets of residuals. A KS test on the two distribution yields a p-value of 2.641e-07. So we can reject the null that the two distributions are the same.

This result provides evidence that the separation hypothesis does not hold. We have shown evidence that at the village level, there is inefficiency in production, which is evidence that production is not optimized.

5.

In this section, I re-ran the regression now included a variable for the log of the size of the household. The coefficient and its relevant information is reported below. We would expect that all else equal a larger household would have a higher output because there would be more labor available for each unit of land. This is in fact what we find. A 1% increase in household size is associated with a 17% increase in the value of output. This result is highly significant.

Coefficient	Estimate	P-Value
α	0.174497	1.63e-05