Analysis

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Question 1

In this question, I will replicate the mean return patterns for the Fama-French 25. These are the portfolios formed on size, and book-to-market value weighted. The values are given in table 1 (This table didn't round for some reason I don't know why).

Table 1:

Mean Return (% Per Month)					
	Low B/M	2	3	4	High B/M
Small	0.305390300546448	0.775708333333333	0.779899590163935	0.983272131147541	1.1271162568306
2	0.522531557377049	0.795162295081967	0.860273360655738	0.909621994535519	1.03100163934426
3	0.546485109289617	0.809150546448087	0.754173360655738	0.894013387978142	1.01733633879781
4	0.652604918032787	0.657038114754098	0.727220081967213	0.851180327868853	0.884083879781421
Big	0.594943169398907	0.555158333333333	0.589210519125683	0.531210519125683	0.70213306010929

Question 2

In this question, we will see if the Fama, French finding that market β's are approximately 1 holds up when we only include the market risk premium as a risk factor in the estimating regrssion.

a)

To do that we will run the regression shown in equation (1) below. The α's, β's, and their requisite t-statistics are shown in table 2.

$$R^e_{i,t} = \alpha_i + \beta_i RmRf_t + \epsilon_{i,t} \tag{1}$$

b)

i. The GRS test gives a test statistic of 4.3713 which has a p-value of 1.778×10^{-11}. It is highly significant. The critical values it would need to achieve for 1, 5, and 10 percent significance are reported in table 3.

The χˆ2 test gives a value of 113.307 which has a p-value of 3.294×10^{-13}. Table 4 reports reports the same critical values but for the chi-squared test as above.

Both tests strongly reject the null of the *alpha*'s being jointly 0.

ii. The mean absolute value of the α's is 0.208.

Table 2:

	Estimate					T-Statistic				
	Low B/M	2	3	4	High B/M	Low B/M	2	3	4	High B/M
Alpha										
Small	-0.5107	0.0690	0.1391	0.3825	0.5095	-2.7940	0.4311	1.070	2.8940	3.332
2	-0.2738	0.1124	0.2391	0.3190	0.3659	-2.0063	1.0082	2.318	3.0369	2.758
3	-0.2069	0.1610	0.1694	0.3134	0.3881	-1.8610	1.8716	2.005	3.2946	3.084
4	-0.0403	0.0334	0.1426	0.2740	0.2589	-0.4733	0.4900	1.804	3.0236	2.151
Big	0.0252	0.0228	0.0927	0.0176	0.1337	0.4213	0.3977	1.246	0.1811	1.002
Beta										
Small	1.4144	1.2249	1.1107	1.0412	1.0704	34.9893	34.6218	38.644	35.6202	31.651
2	1.3802	1.1834	1.0766	1.0237	1.1528	45.7287	48.0001	47.207	44.0621	39.285
3	1.3058	1.1233	1.0135	1.0063	1.0906	53.0963	59.0276	54.243	47.8262	39.189
4	1.2009	1.0809	1.0133	1.0004	1.0837	63.8357	71.7057	57.949	49.9177	40.719
Big	0.9874	0.9228	0.8606	0.8902	0.9852	74.5303	72.8998	52.297	41.3587	33.383

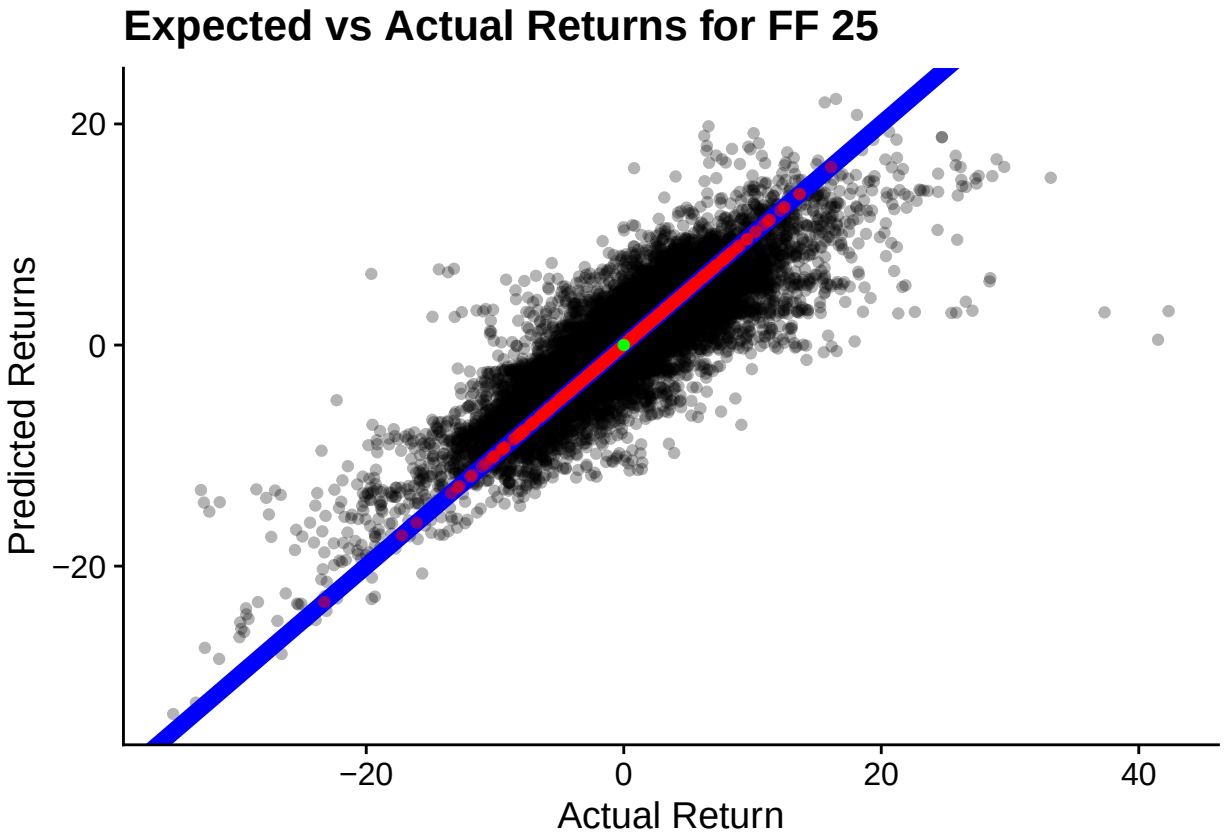
Table 3:

Critical Values		
10%	5%	1%
1.386	1.522	1.799

Table 4:

Critical Values		
10%	5%	1%
34.38	37.65	44.31

iii. Below is the plot of actual versus predicted returns. The blue line is the 45 degree line. The red points on the line are the market excess returns, and the green dot is the riks free rate.



c)

The CAPM does not work well on these portfolios. The null hypothesis of the α's being jointly 0 is strongly rejected by both tests. Individually, most of the α's also have large t-statistics. Statistically, the α's definitely matter. The mean absolute α is 0.208. This is fairly large I think. It is a fifth of the market β which seems like it matters economically.

There are predictable patterns in the mean returns we see in table 1. Generally, small firms have bigger returns that large firms, and high book to market firms have bigger returns than low book to market firms. This pattern seems to vaguely match the β's in table 2. The largest β's are the smaller firms with lower book to market ratios which were also the firms with larger mean returns.

d)

Now, I'll run the cross sectional regressions using the equation given by (2). I'll run it using only the Fama French assets, and then also include the market excess returns and the risk free asset. The results are given in table 5.

The χˆ2 test only using the ff25 is 1.6056 which has p-value 1. Including the other assets they are 2.8648 and 1. The test statistics are very small which is why the p-values are so high. Thus we fail to reject the null that the α's are jointly 0.

This result is wildly different from the time series regression. The λ's are much smaller than any of the β's. When we don't include the market and risk free rates, the λ is negative. We also don't reject the null here of all α's being 0 which we did for the time series.

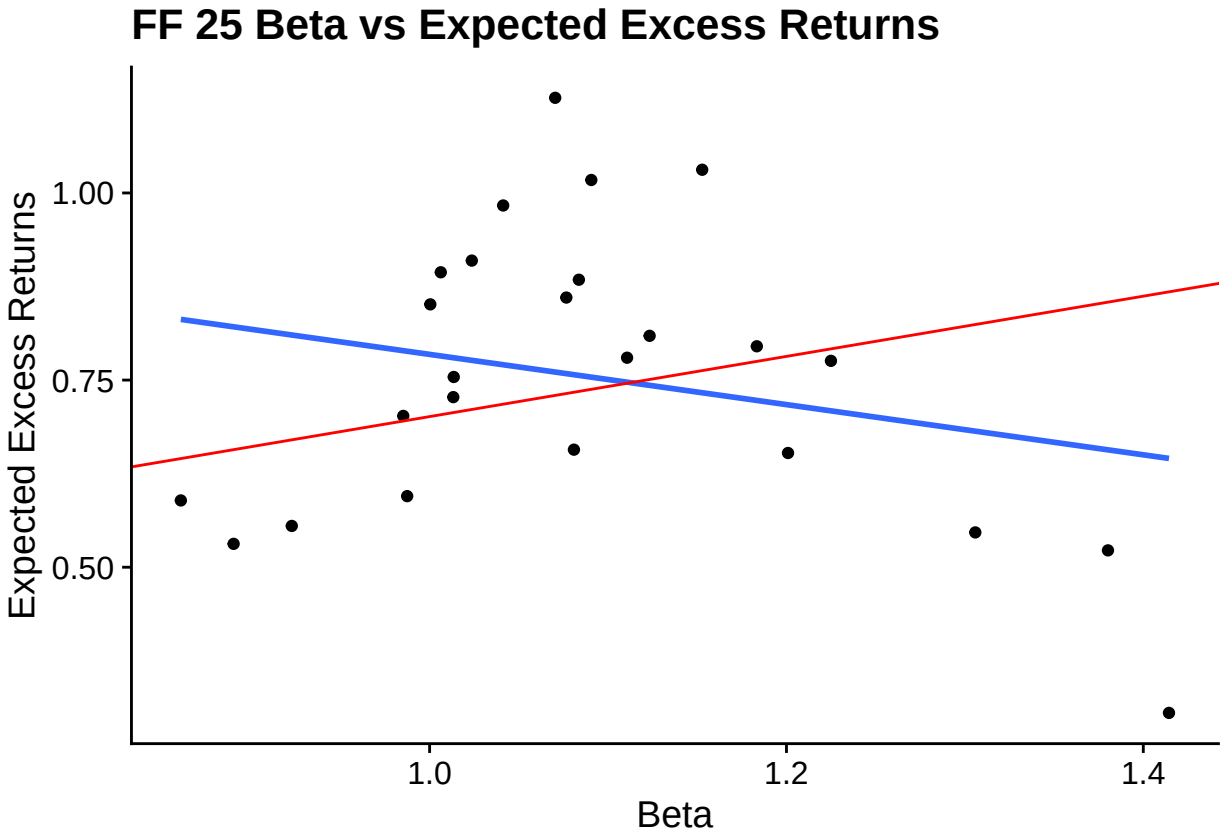
$$E[R_i^e] = \gamma + \beta_i \lambda + \alpha_i \tag{2}$$

Table 5:

	Gamma	Lambda	SE Lambda	Mean Absolute Alpha
FF 25	1.1197	-0.3354	0.2821	0.1536
FF25 + RMRF + RF	0.2991	0.4021	0.1729	0.1641

e)

i. Below is the graph showing the relationship between β's and expected excess returns for the test assets. The blue line is the relationship not including the market and risk free assets. The red line includes those. We can see they are substantially different.



ii. Below is a plot of predicted versus actual returns for the test assets. The top plot uses the regression with only the test assets. The bottom uses the regression that uses the the market and risk free assets to make the predictions. It looks like using the whole sample leads to a better fit. The bottom graph has the points more clustered around the 45 degree line.

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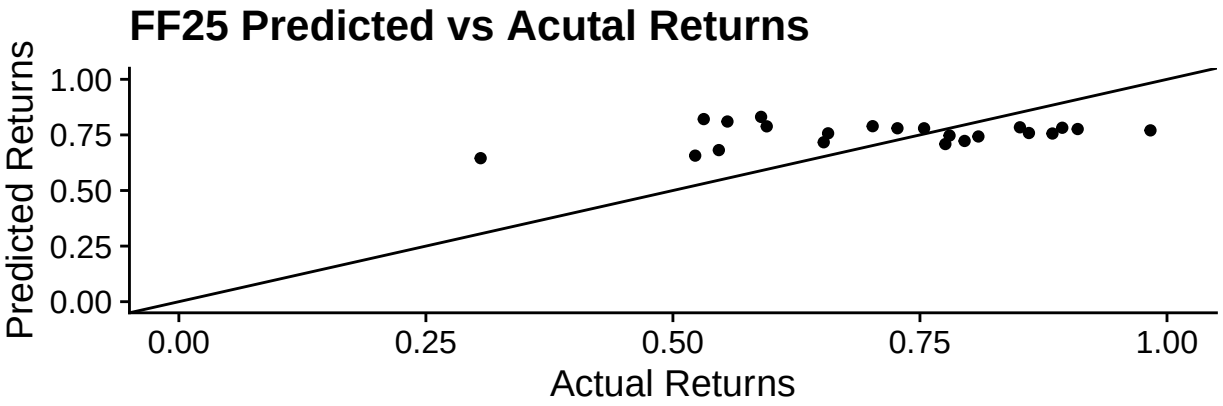

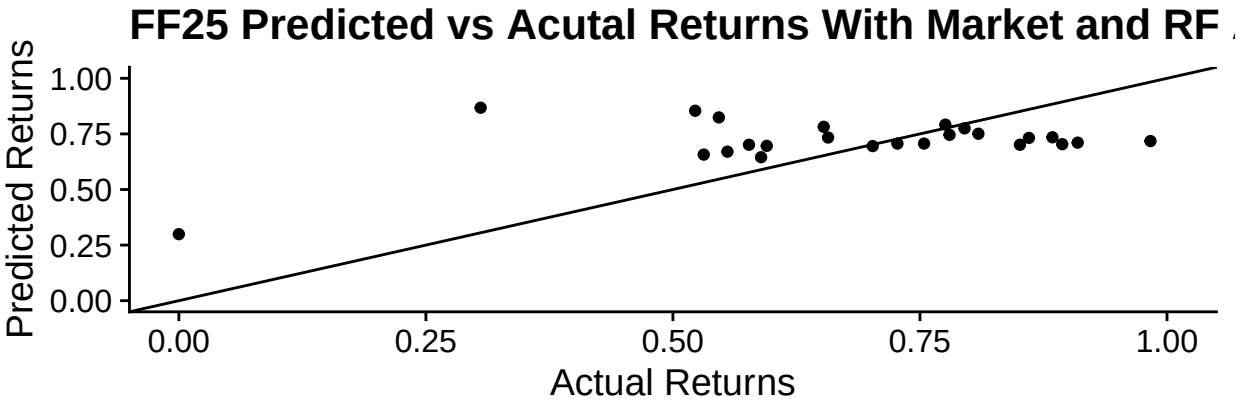

Question 3

$$R_i - R_f = a_i + b_i(R_M - R_f) + s_i SMB + h_i HML + e_i \tag{3}$$

Table 6:

Mean Return (% Per Month)					
	Low B/M	2	3	4	High B/M
a					
Small	-0.5013	-0.0200	-0.0413	0.1316	0.1731
2	-0.2097	0.0127	0.0511	0.0588	0.0042
3	-0.1109	0.0707	-0.0142	0.0521	0.0308
4	0.0792	-0.0506	-0.0293	0.0441	-0.0783
Big	0.1748	0.0075	-0.0045	-0.2236	-0.1844
b					
Small	1.0964	0.9598	0.9290	0.8883	0.9417
2	1.1288	1.0144	0.9741	0.9540	1.0820
3	1.1042	1.0235	0.9817	1.0026	1.0882
4	1.0695	1.0609	1.0369	1.0302	1.1352
Big	0.9875	0.9706	0.9451	1.0222	1.1286
s					
Small	1.4084	1.3274	1.0985	1.0813	1.1083
2	1.0233	0.9140	0.7567	0.7234	0.8893
3	0.7503	0.5897	0.4330	0.4301	0.5766
4	0.3988	0.2223	0.1663	0.2307	0.3031
Big	-0.2371	-0.1901	-0.2243	-0.2090	-0.1380
h					
Small	-0.2597	0.0112	0.2889	0.4764	0.6958
2	-0.3388	0.1084	0.3659	0.5605	0.7986
3	-0.3767	0.1381	0.4083	0.6125	0.8392
4	-0.3795	0.1829	0.4224	0.5634	0.8322
Big	-0.3521	0.0716	0.2920	0.6666	0.8558