Analyzing the Trajectory of a Spinning Soccer Ball

Eric Anderson

Department of Physics, University of Rochester

PHY 235W: Classical Mechanics

Professor Frank Wolfs

November 25, 2020

Abstract

The trajectory of the kick of a spinning soccer ball was analyzed by starting with a theoretical derivation and running numerical simulations. The equations of motion were derived from the forces for a three-dimensional case and a two-dimensional case. By taking the assumption that the ball remains in a state of turbulent flow, or that the critical Reynolds number has been exceeded for the duration of its flight, a model was created where the drag coefficient (C_D), corresponding to the force of air resistance, and the Magnus coefficient (C_M), corresponding to force induced by the spin, are taken as constant. This model was applied to a free kick from 20 meters away from the goal using the three-dimensional equations and to a long kick using the two-dimensional equations. The physical constants chosen in the simulations were representative values of their normal range of magnitude so that a general model could be made. The simulation was iterated over various values of the C_D and C_M to obtain a wider view of the general effect of spin on the ball. The 20-meter simulations were used to map more exact trajectories and the long kick simulations were used to compare general behaviors.

Introduction

The aim of this study is to provide a succinct discussion of the general effect of spin on the kick of a soccer ball. While soccer is one of the most popular sports in the world, there is a relatively little amount of study done on the dynamics of the kicked soccer ball. Progress in this line of study is slow, possibly due to the large number of variable factors such as different types of soccer balls, variation in kicking style, and in general, the overall complexity of the problem. A past study by Bray and Kerwin (2003) used a theoretical derivation paired with a video recording experimental setup to create a general model of the path of a soccer ball. Additional studies used wind tunnels to examine the difference between a stationary and spinning ball, comparing that to visualization data of a kick caught with a camera (Asai et al., 2007). The problem gets more complex and variable when studying the small but significant differences between different soccer balls, as done by Passmore et al. (2008) as they worked to create a more standardized procedure to use a wind tunnel to measure the drag and spin coefficients associated with a particular ball.

In this study, the theoretical derivations of the equations of motion of the soccer ball were obtained using the forces associated with drag and spin. With these equations, numerical simulations were performed to demonstrate a brief overview of the effect that different spins can have. In this way, a simplified overview of the dynamics of the path of a soccer ball was created by considering the several complexities of the problem and simplifying them with general assumptions.

Theory

The forces in consideration acting on the ball are the gravitational force, the force due to air resistance, and the force due to the spin on the ball, commonly referred to as the Magnus force. Taking the spin to be at an arbitrary angle perpendicular to the initial velocity of the ball, the Magnus force will introduce a horizontal and vertical component of force. Taking only into consideration these forces and drawing upon the derivations of Bray and Kerwin (2003), the equations of motion can be obtained.

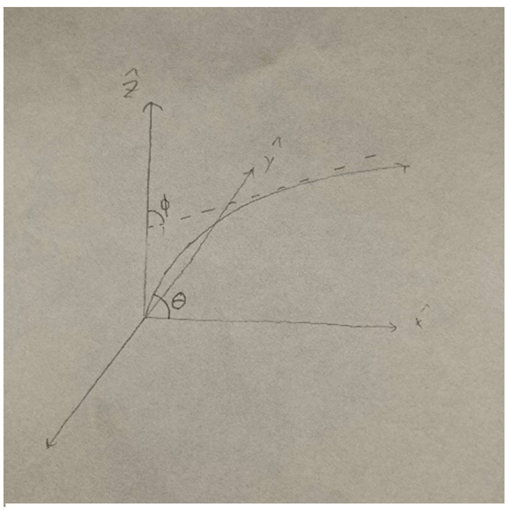

Figure 1 Schematic diagram of the kick

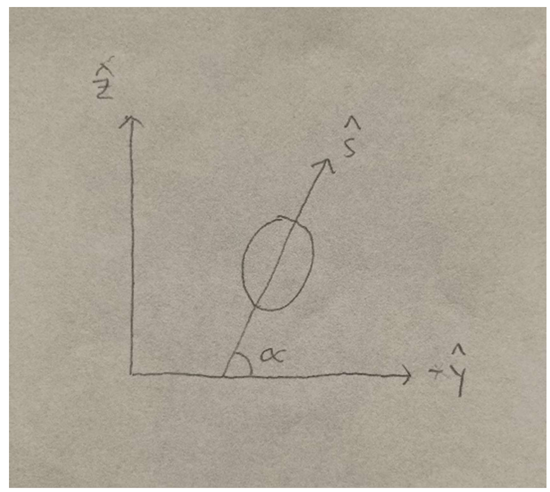

Figure 2 Schematic diagram of the spin

Figure 1 shows a ball kicked in the x̂ direction at an initial angle θ, with the angle between the tangential velocity and the z-axis ϕ at an arbitrary time. Assuming a right-footed kicker and that the direction of the spin axis will remain constant through the duration of the kick, Figure 2 shows the direction of the spin as defined by the angle α, from the perspective of the kicker. The equations for the drag force (F_D) and the Magnus force (F_M) are similar in structure and can be written as:

$$\overrightarrow{F_D} = \frac{1}{2}\rho v^2 A C_D \hat{v} \quad \overrightarrow{F_M} = \frac{1}{2}\rho v^2 A C_M \hat{s} \times \hat{v} \tag{1}$$

(where ρ is the air density, v is the velocity, A is the projected area of the ball, C_D is the drag coefficient, C_M is the Magnus coefficient, and $\hat{s}$ is the direction of the spin axis).

For simplicity, it is best to take C_D and C_M as constant. A key assumption in this is that for the duration of its flight, the ball will remain in the supercritical Reynolds number range where there is little variation in the coefficients. The number is largely dependent on velocity, thus if the ball maintains a high enough velocity, it is a reasonable assumption to take the two coefficients as constant (Asai et al., 2007).

To start the process of deriving the equations of motion, the forces can be summed:

$$\vec{F} = -m\vec{g} - \overrightarrow{F_D} + \overrightarrow{F_M} = -m\vec{g} - \frac{1}{2}\rho v^2 A C_D \hat{v} + \frac{1}{2}\rho v^2 A C_M \hat{s} \times \hat{v} \tag{2}$$

It will be convenient to define the two constants:

$$K_D = \frac{1}{2m}\rho A C_D \quad K_M = \frac{1}{2m}\rho A C_M \qquad (3), (4)$$

Then, after breaking up the equation of force into the three Cartesian components, calculating the cross-product in the Magnus force, and substituting the two new constants, the equations of motion are produced:

$$\ddot{x} = -vK_D\dot{x} - vK_M\dot{y}\sin\alpha - vK_M\dot{z}\cos\alpha \tag{5}$$

$$\ddot{y} = -vK_D\dot{y} + vK_M\dot{x}\sin\alpha \tag{6}$$

$$\ddot{z} = -g - vK_D\dot{z} + vK_M\dot{x}\cos\alpha \tag{7}$$

These are the equations of motion for a general kick of a soccer ball, assuming that the angle of the spin axis, C_D, and C_M remain constant through the kick. It will be additionally useful for study to simplify these expressions to the two-dimensional case where the angle of spin is 0 degrees. This produces the equations:

$$\ddot{x} = -vK_D\dot{x} - vK_M\dot{z} \tag{8}$$

$$\ddot{z} = -g - vK_D\dot{z} + vK_M\dot{x} \tag{9}$$

This would be the case of a kick with pure backspin and no sidespin, similar to that of a goalkeeper's goal kick.

Results

The complexity of these equations and their dependence on the velocity makes a numerical approach the viable option to examining the motion of the ball. Two analyses are done, one modeling a free kick with sidespin and backspin placed 20 meters away from the goal, and the other modeling a long kick with only backspin.

To do this, we first have to choose values for the constants. $1.2\ kg/m^3$ is used for the air density ρ; taking the average regulation size of a soccer ball, 0.11 m is used for the ball radius in calculating it's projected area; and taking the average regulation weight of a soccer ball, 0.43 kg is used for its mass. For C_D and C_M, which were assumed to remain constant through the kick, we can look at a table of calculated values from which to extrapolate numbers which will approximately map the correct motion. Using Bray and Kerwin (2003) as a guideline, who choose $C_D = 0.25$ and $C_M = 0.23$ as representative values from their calculations to explore the variation of results as C_M changes, we iterate from ($C_D = 0.25, C_M = 0.23$) to the highest C_M that they found ($C_D = 0.31, C_M = 0.29$). As both coefficients depend on the spin rate (Passmore et al., 2008), C_D was extrapolated to increase alongside the C_M at the same rate. For this simulation, a time step size (dt) of 1ms was used. This free kick is simulated as taken from 20 meters away from the goal with the ball placed in the center and the initial velocity directed straight at it.

	$\alpha = 83°$		$\alpha = 90°$		$\alpha = 62.5°$	
(C_D, C_M)	y (m)	z (m)	y (m)	z (m)	y (m)	z (m)
(0.25, 0.23)	2.49	1.53	2.51	1.23	2.25	2.35
(0.26, 0.24)	2.60	1.51	2.61	1.20	2.35	2.37
(0.27, 0.25)	2.71	1.48	2.72	1.17	2.45	2.38
(0.28, 0.26)	2.82	1.46	2.84	1.13	2.55	2.39
(0.29, 0.27)	2.93	1.43	2.95	1.09	2.65	2.40
(0.30, 0.28)	3.05	1.41	3.06	1.05	2.75	2.41
(0.31, 0.29)	3.16	1.38	3.17	1.01	2.85	2.42

Figure 3 Chart of horizontal and vertical displacements

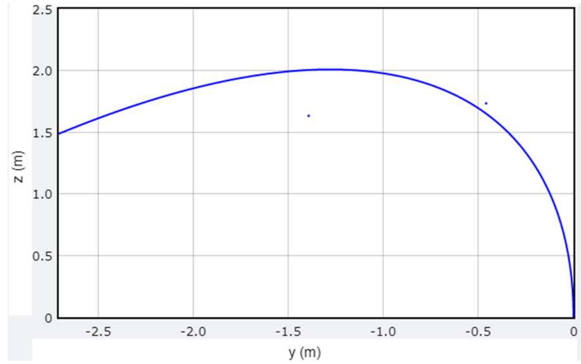

Figure 4 View of kick from player

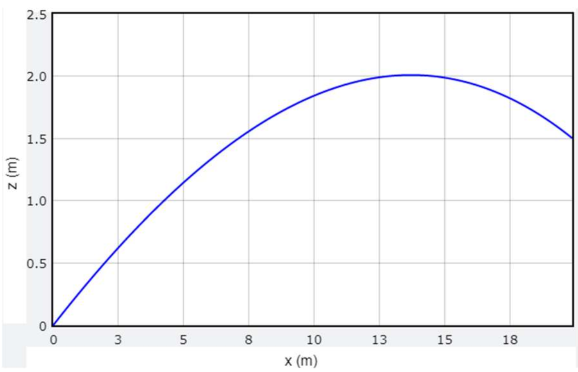

Figure 5 View of kick from side

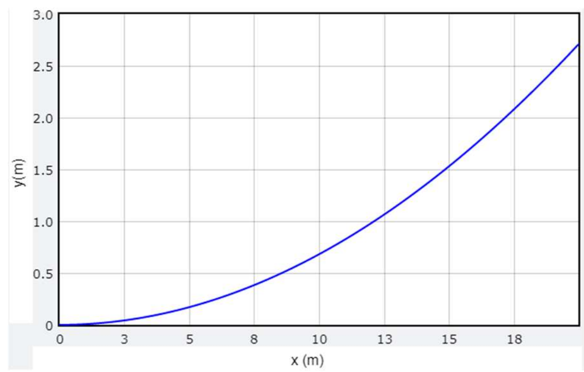

Figure 6 View of kick from above

Figures 4-6 show a visual representation of the 20-meter free kick from the three Cartesian planes at an initial velocity of 25.0 m/s, an initial angle of 15.0 degrees, and a spin axis orientation of 83.0 degrees. Using the results from Figure 3, it makes sense that as the Magnus coefficient increases the horizontal displacement of the ball due to the spin increases. Considering the size of the goal (7.32m), the ball is displaced almost to the corner of the goal, ranging from about 0.5m to 1.4m away from the post. While the spin added to the ball will produce a lifting force, its seen that the vertical position of the ball as it reaches the goal decreases with added spin for $\alpha = 83°$ and $\alpha = 90°$. The added spin increases the horizontal and total distance that the ball travels, therefore increasing the amount of time that the drag force has to take effect and for the ball to fall. In the trajectory of the ball, the increased spin rate produces a higher maximum height, but at this relatively low angle and distance, the added time of effect of the drag and gravitational forces cancel that effect out. The opposite can be seen for $\alpha = 62.5°$ where the vertical displacement increases with the increasing spin rate. Firstly, this demonstrates that the phenomenon for the first two angles is not always the case. It also shows the subtle effect that the spin can have on the kicker's placement of the ball. The crossbar lies at a height of 2.44m. For the lower spin rates, the ball will hit the bottom of the crossbar and ricochet into the goal at this spin axis orientation. For the higher spin rates, the ball will hit the crossbar and be deflected out.

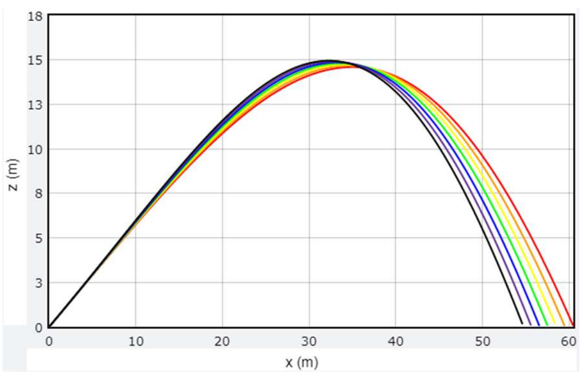

Figure 7 View of spinning kick from side, bluer colors corresponding to higher coefficients

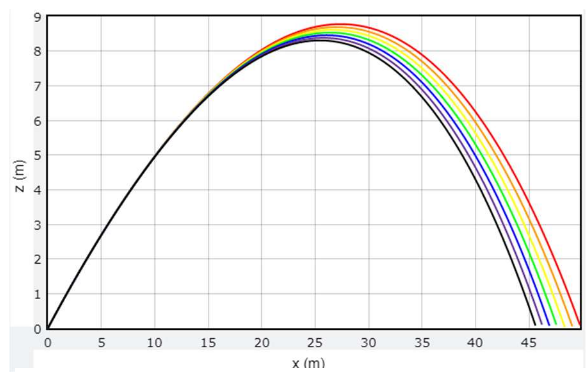

Figure 8 View of non-spinning kick from side, bluer colors corresponding to higher coefficients

	$\theta = 30.0°$		$\theta = 45.0°$	
(C_D, C_M)	x (m)	v (m/s)	x (m)	v (m/s)
(0.22, 0.20)	60.49	17.83	52.15	19.23
(0.23, 0.21)	59.50	17.63	50.88	19.01
(0.24, 0.22)	58.49	17.42	49.71	18.82
(0.25, 0.23)	57.52	17.23	48.54	18.62
(0.26, 0.24)	56.57	17.06	47.41	18.43
(0.27, 0.25)	55.66	16.90	46.33	18.25
(0.28, 0.26)	54.73	16.74	45.29	18.08

Figure 9 Chart of distance and end velocities for the spinning kick

	$\theta = 30.0°$		$\theta = 45.0°$	
(C_D, C_M)	x (m)	v (m/s)	x (m)	v (m/s)
(0.22, 0.0)	49.83	18.28	52.47	18.74
(0.23, 0.0)	49.05	18.02	51.57	18.51
(0.24, 0.0)	48.36	17.78	50.70	18.29
(0.25, 0.0)	47.62	17.54	49.87	18.07
(0.26, 0.0)	46.97	17.32	49.06	17.86
(0.27, 0.0)	46.28	17.10	48.27	17.66
(0.28, 0.0)	45.67	16.90	47.51	17.46

Figure 10 Chart of distance and end velocities for the non-spinning kick

Figure 7 shows a visual representation of long kicks in the two-dimensional plane at an initial velocity of 30.0 m/s, an initial angle of 30.0 degrees, and a spin axis angle of 90.0 degrees. This kick was simulated using a time step of 5ms. With this spin axis angle, the two-dimensional equations were used. Accounting for the increased velocity would cause a decrease in the spin rate (Asai et al., 2007), so the initial iterative values of the drag coefficient and Magnus coefficient were decreased to (0.22, 0.20) and iterated through (0.28, 0.26). In these simulations, the ball loses a significant amount of velocity. Therefore, the assumption that C_D and C_M remain constant for the duration of the kick are no longer reasonable. These simulations become better predictors of the general behavior of the ball rather than of its exact trajectory.

Figure 7 shows the kick with backspin and Figure 8 without spin for reference. This analysis shows that the maximum height of the ball increases with increasing spin rate. At these initial parameters, increasing spin rate causes a decrease in the vertical position of the ball. This occurrence is highly dependent on the initial parameters of the trajectory. For a much shallower launch angle, the ball with a higher spin rate will remain in the air much longer.

In the specific case of the goal kick that is modeled, compared to the goal kick with no backspin, the spin rate increases the distance traveled as seen by comparing Figures 9 and 10. This implies that there will be some optimal spin rate to maximize the distance traveled, which is some balance of increasing the vertical distance versus increasing the drag force and the time it has to take effect. By comparing the distance traveled at the two different angles, its shown that the optimal angle will also differ depending on the applied spin. In the case of a projectile without spin or air resistance factored in, the optimal launch angle is 45 degrees. With air resistance, Figure 10 shows that the ball kicked at 45 degrees increases the distance traveled. With backspin, the ball kicked at an angle of 30 degrees travels the furthest in comparison. This difference is caused by the added lifting force increasing the airtime of the ball and allowing for the air resistance to have a greater effect.

Discussion

There is more complexity in choosing the values of the drag and Magnus coefficients than is considered by assuming their constancy. They are both dependent on the Reynolds number and spin rate, which will both decrease during the kick of the soccer ball (Passmore et al., 2008). Taking the average range of kick speed (22m/s – 30m/s) of a professional player into account, Asai et al. (2007) determine that for this range the ball has exceeded the critical Reynolds number and has entered a region where C_D has little dependence on the number. For the case of the 20-meter free kick, where the ball will maintain a velocity in this region it is not unreasonable to assume that C_D will remain constant, but nevertheless there is still variation in its value. In the case of the goal kick, the ball will fall out of this range and will increase significantly by the end of its trajectory. Taking C_D to be constant throughout the flight of the goal kick vastly oversimplifies the problem. Assuming that the coefficient will degrade in similar fashion for each kick, it is not an unreasonable approximation to compare the simulations in terms of general behavior and not their exact trajectories.

The spin rate will decrease through the kick in each case. For the 20-meter kick, the time that the ball travels is relatively short, making the assumption that C_M is constant reasonable since there will be little variation. For the goal kick, assuming a constant C_M again vastly oversimplifies the problem; and again, assuming that the spin rates will decay in similar fashions for each kick, it is a reasonable approximation to compare the resulting trajectories in terms of general behavior.

In the actual choice of the values used for C_D and C_M, there is more added complexity. The values used are within the correct range, but in the analysis, they are approximations that are representative of an approximate kick. They are extrapolated from a chart of values, not calculated for each specific kick with different initial parameters. The linear relation between C_D and C_M that is used to account for their codependency is a simplification of their relation and requires more study.

In the case of the goal kick, the choice of coefficients is even more of an extrapolation because of the lack of data on longer kicks. While it is likely a reasonable approximation, especially due to the magnitudes of the coefficients, more study is needed on longer kicks.

Continuing further, there are a multitude of other factors that will impact the choice of coefficients. Passmore et al. (2008) demonstrated the significant impact the choice of ball – from the design, the pattern of seams, and the construction – can have on the variation in coefficients. The shoes worn by the kicker will affect the amount of friction during the kick and therefore will affect the amount of spin imparted. Weather conditions such as wind and precipitation could also affect these choices through the degradation of the coefficients in flight to the wetness of the ball impacting the friction during the kick.

However, the strengths in this analysis of the problem lie in its reduction of the possible variations to a more general formulation. By using the basic theory of the problem to obtain the equations of motion and using established ranges of values like obtained by Bray and Kerwin (2003), the kick of a soccer ball can be numerically analyzed to a reasonable approximation of reality. This approximation could then be analyzed through a large range of initial parameter values to get a more complete picture of the trajectory of the ball.

This analysis of the kick of the soccer ball tackled the problem in a similar way to Bray and Kerwin (2002) and Carre et al. (2002) by modeling the trajectory with the force-derived equations of motion. Both of those studies used high speed cameras to capture and measure the motion of a ball in reality to both apply the measured coefficients and compare their numerical results to the measured results. In this, only a small sample of numbers and variations were studied. More study is needed in the applications of these models to a wide number of different coefficients and situations, especially as studies like Asai et al. (2007) and Passmore et al. (2008) start delving into the measurement of dynamics of specific balls.

These studies all examined the kick of a soccer ball within the range of a professional soccer player kicking the ball within a short distance. This allows for the reasonable approximation that the Reynolds number will remain in the supercritical region and that C_D and C_M will remain near constant for the duration of the kick. Firstly, more study is needed into the slight degradation of these coefficients in this situation. Furthermore, more study into the dynamics of a soccer ball in other situations where the coefficient could decrease drastically during the kick, such as long kicks and non-professional kicks, is needed to more fully complete the picture. The extension of the motion of the ball in this study offers a reasonable approximation of the behavior of a long kick but does not offer a strong analysis of the exact trajectories.

Conclusions

The objective of this study was to produce a general model and simulation of the trajectory of a spinning soccer ball. For a free kick from a 20-meter distance from the goal at typical values, the drag and Magnus coefficients were iterated from (0.25, 0.23) to (0.31, 0.29).

At this range, spins ranging from purely horizontal to near horizontal could bring the ball within 0.5m of the side post. With these parameters, increasing spin caused a decrease in the vertical position when the ball reached the goal due to increased airtime. However, as the angle of the spin axis was shifted to produce more backspin, a higher spin rate increased the vertical position of the ball upon reaching the goal, demonstrating how the kicker must be sensitive to the application of spin in order to not miss the goal. Further work is needed to examine the wide variations in physical constants and coefficients.

For the case of a long kick with pure backspin in two dimensions, the assumptions of constant drag and Magnus coefficients were no longer valid to exactly simulate the trajectory of the ball but could be used to generalize the behavior. The drag and Magnus coefficients were iterated from values of (0.20, 0.22) to (0.26, 0.28). The application of spin was found to significantly increase the distance the ball traveled. However, it was found that there is an optimal combination of spin and launch angle that will maximize the distance traveled. For these values of the Magnus coefficient and initial parameters, the higher spin rates corresponded to lower distances traveled. In terms of launch angle, a 30-degree launch angle traveled further than a 45-degree launch angle. Further work is needed to study the trajectory of long kicks, especially in relation to the degradation of the drag and Magnus coefficients during the kick, in which little work has been done so far.

References

Asai, T., Seo, K., Kobayashi O. & Sakashita, R. (2007). Fundamental aerodynamics of the soccer ball. *Sports Engineering, 10,* 101-109. https://doi.org/10.1007/BF02844207

Bray, K. & Kerwin, D. (2003). Modelling the flight of a soccer ball in a direct free kick. *Journal of Sports Sciences, 21*(2), 75-85. https://doi.org/10.1080/0264041031000070994

Carre, M. J., Asia, T., Akatsuka, T. & Haake, S. J. (2002). The curve kick of a football II: Flight through the air. *Sports Engineering, 5*(4), 193-200. Retrieved from https://onlinelibrary.wiley.com/doi/abs/10.1046/j.1460-2687.2002.00109.x

Passmore, M. A, Tuplin, S., Spencer, A. & Jones, R. (2008). Experimental studies of the aerodynamics of spinning and stationary footballs. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 222,* 195-205. https://doi.org/10.1243/09544062JMES655

Code

https://www.glowscript.org/#/user/eander29/folder/MyPrograms/program/Soccer-20MeterFreeKick

https://www.glowscript.org/#/user/eander29/folder/MyPrograms/program/Soccer-LongKick