The Subsidy Immunity Formula (SIF)

An Optimal Stopping Framework for Retail Mean Reversion Strategies

Technical Whitepaper

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Abstract

Retail investors frequently suffer from the Disposition Effect, holding losing positions due to a lack of explicit opportunity cost quantification. This paper formalizes the **Subsidy Immunity Formula (SIF)**, derived from the *Principle of Indifference*. By equating the value of immediate sale against the expected value of waiting, and algebraically isolating the volatility time horizon from the *Reflection Principle*, we define two novel metrics: **Intrinsic Duration (D_i)** and **Dynamic Convexity (C)**. The interaction of these terms determines the **Strategic Shielding Time (SST)**, identifying the critical "Log-Moneyness" barrier (δ_{crit}) where a position becomes mathematically insolvent.

1 Introduction

The core dilemma in managing a losing position is distinguishing between a temporary drawdown and an inefficient use of capital. We approach this through **Revenue Management**: capital is perishable inventory. Holding a losing stock "subsidizes" the position using the risk-free rate (r). We aim to find the Time-Based Efficiency limit.

2 Model Definitions

The asset price S_t follows a Geometric Brownian Motion (GBM). We define:

- S_t: Current Spot Price (x).
- K: Target Price (T).
- δ: Log-Moneyness, $\delta = \ln(K/S_t)$.
- σ: Annualized volatility.
- τ: Volatility time horizon.

3 Mathematical Derivation

The SIF model resolves the conflict between the *Funding Time* (linear) and the *Volatility Time* (quadratic).

3.1 Phase 1: The Indifference Principle

In Revenue Management, we establish a "Protection Level" where the investor is indifferent between realizing a guaranteed loss now or waiting for an uncertain recovery. We define two value functions:

1. Value of Immediate Sale (V_{now}): If we sell the asset at current price S_t today and invest the recovered capital at the risk-free rate r for a duration t, the future value is:

$$V_{now}(t) = S_t \cdot e^{rt} \tag{1}$$

2. Value of Waiting (V_{wait}): The expected value of holding the position until it hits the target K. This is the target value adjusted by the probability of success P:

$$V_{wait} = K \cdot P \tag{2}$$

3. The Equilibrium Condition: To find the maximum rational holding time, we equate the certainty of the bank deposit with the expectation of the trade:

$$V_{wait} = V_{now}(t) \implies K \cdot P = S_t \cdot e^{r \cdot t} \tag{3}$$

4. Solving for Funding Time (t_{fund}): Isolating the time variable t purely from the funding perspective (assuming $P \to 1$ for the intrinsic component), we obtain the linear funding limit:

$$t_{fund} = \frac{\ln(K/S_t)}{r} = \frac{\delta}{r} \tag{4}$$

3.2 Phase 2: The Statistical Constraint

We cannot simply assume $P = 1$. Under the **Reflection Principle** for Brownian Motion, the probability of hitting barrier K depends on the observation time τ:

$$P = 2 \cdot \Phi(-z) \quad \text{where} \quad z = \frac{\delta}{\sigma\sqrt{\tau}} \tag{5}$$

Substituting this into the equilibrium logic, we equate the probability of success to the inverse of the moneyness ratio (S_t/K) required to break even:

$$2 \cdot \Phi\left(-\frac{\delta}{\sigma\sqrt{\tau}}\right) = \frac{S_t}{K} \tag{6}$$

3.3 Phase 3: Algebraic Isolation of τ

Equation 6 defines τ implicitly. To obtain a closed-form solution, we isolate τ (Volatility Latency) in sequential steps:

Step 1: Isolate the CDF Φ:

$$\Phi\left(-\frac{\delta}{\sigma\sqrt{\tau}}\right) = \frac{S_t}{2K} \tag{7}$$

Step 2: Apply the Inverse Normal CDF (Φ^{-1}):

$$-\frac{\delta}{\sigma\sqrt{\tau}} = \Phi^{-1}\left(\frac{S_t}{2K}\right) \tag{8}$$

Step 3: Solve for τ_{vol}: Squaring both sides removes the negative sign and the root, yielding the volatility time penalty:

$$\tau_{vol} = \left[\frac{\delta}{\sigma \cdot \Phi^{-1}\left(\frac{S_t}{2K}\right)}\right]^2 \tag{9}$$

We define the denominator term as the **Exigence Coefficient (EC)**, which captures the exponential decay of probability as S_t drops ($S_t/K \approx e^{-\delta}$):

$$\mathrm{EC}(\delta) = \left|\Phi^{-1}\left(\frac{1}{2e^{\delta}}\right)\right| \tag{10}$$

3.4 Phase 4: Synthesis and Definitions

The **Strategic Shielding Time (SST)** is the net strategic advantage: $SST = t_{fund} - \tau_{vol}$. Substituting the terms, we formally define the components of the SIF model:

Definition 1: Intrinsic Duration (D_i) Represents the linear sensitivity of the funding horizon to displacement.

$$D_i = \frac{1}{r} \tag{11}$$

Definition 2: Shielding Convexity ($C(\delta)$) Represents the quadratic penalty imposed by volatility and market exigence. Unlike bond convexity, this is dynamic:

$$C(\delta) = \frac{1}{[\sigma \cdot \mathrm{EC}(\delta)]^2} \tag{12}$$

The SIF Master Equation: Combining definitions, we obtain the canonical form:

$$\mathrm{SST} = D_i \cdot \delta - C(\delta) \cdot \delta^2 \tag{13}$$

4 Critical Liquidation Analysis

The position becomes insolvent when the cost of time outweighs the probability of recovery, i.e., $SST \leq 0$.

4.1 Critical Log-Moneyness (δ_{crit})

Solving $(D_i \cdot \delta) - (C(\delta) \cdot \delta^2) = 0$:

$$D_i = C(\delta) \cdot \delta \implies \frac{1}{r} = \frac{\delta}{(\sigma \cdot \mathrm{EC})^2} \tag{14}$$

Rearranging for δ, we find the structural limit:

$$\delta_{crit} = \frac{(\sigma \cdot \mathrm{EC}(\delta_{crit}))^2}{r} \tag{15}$$

Interpretation: The maximum sustainable distance is proportional to the variance (σ^2) scaled by exigence, and inversely proportional to the cost of capital (r).

5 The Greeks of Shielding

To manage the strategy dynamically, we derive the sensitivities.

5.1 Shielding Vega (ν_{sif})

Measures sensitivity to volatility.

$$\nu_{sif} = \frac{\partial \mathrm{SST}}{\partial \sigma} = \frac{2\delta^2}{\sigma^3 \cdot \mathrm{EC}^2} > 0 \tag{16}$$

Higher volatility reduces the convexity penalty $C(\delta)$, extending the shielding time.

5.2 Shielding Rho (ρ_{sif})

Measures sensitivity to interest rates.

$$\rho_{sif} = \frac{\partial \mathrm{SST}}{\partial r} = -\frac{\delta}{r^2} < 0 \tag{17}$$

Higher rates reduce D_i, accelerating the collapse of the shield.

6 Numerical Illustration

We simulate the SST collapse for two distinct risk profiles. The **Blue curve** represents a standard asset ($r = 5\%, \sigma = 20\%$), while the **Red curve** illustrates a high-volatility asset ($r = 5\%, \sigma = 50\%$).

Consistent with our derivative analysis ($\nu_{sif} > 0$), the graph confirms that higher volatility reduces the convexity penalty $C(\delta)$, effectively extending the strategic shielding time before insolvency.

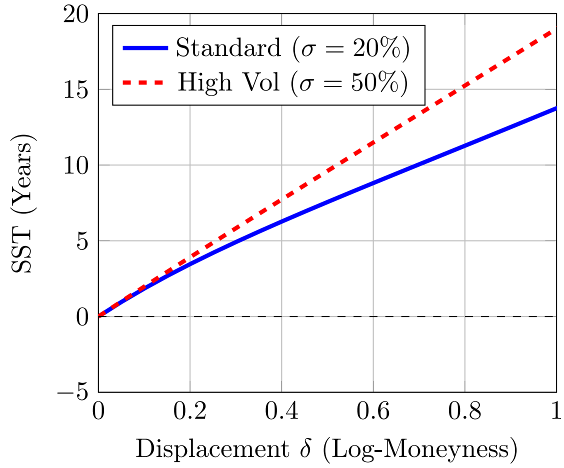

Figure 1: SST Comparison: Volatility buys time.

7 Sensitivity Analysis

To validate the model's behavior, we perform a comparative statics analysis, isolating the effects of Intrinsic Duration and Shielding Convexity.

7.1 Impact of Rates (Duration Effect)

We vary the risk-free rate r while holding volatility constant ($\sigma = 20\%$).

- **Low Rates** ($r = 2\%$)**:** The cost of capital is negligible, extending the Intrinsic Duration ($D_i = 50$). The SST remains positive for large displacements.

- **High Rates** ($r = 10\%$)**:** The opportunity cost is punitive ($D_i = 10$). The SST collapses rapidly, forcing early liquidation.

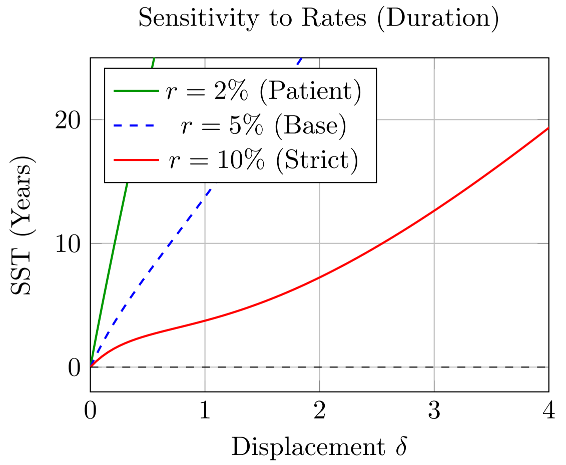

Figure 2: Higher rates accelerate shield collapse.

7.2 Impact of Volatility (Convexity Effect)

We vary volatility σ while holding rates constant ($r = 5\%$). This illustrates the "Option Value" of waiting.

- **Low Volatility** ($\sigma = 10\%$)**:** Recovery is statistically unlikely. The Convexity penalty is massive ($C \uparrow$), killing the position immediately.

- **High Volatility** ($\sigma = 40\%$)**:** The distribution tails are fat. The penalty shrinks ($C \downarrow$), granting the investor "survival time".

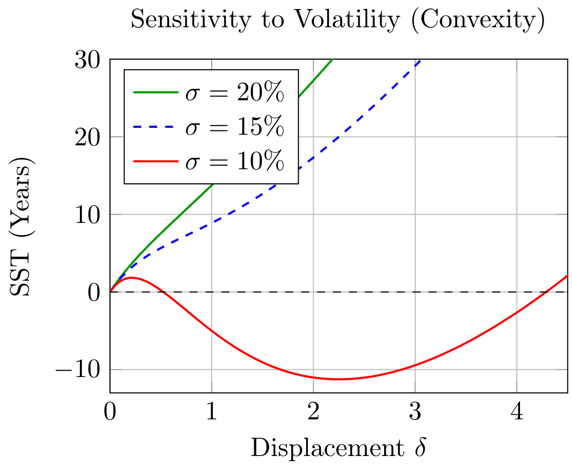

Figure 3: Volatility buys time (reduces Convexity).

8 Scenario Analysis: The Greeks in Action

To bridge theory and practice, we analyze the SIF model's behavior under two common market regimes, illustrating the practical implications of ρ_{sif} and ν_{sif}.

8.1 The "Hawk" Regime (Rho Risk)

Consider a mean-reversion strategy in a rising rate environment, such as the 2022 tightening cycle. As the central bank raises the risk-free rate r from 0% to 5%:

- **Impact:** The Intrinsic Duration ($D_i = 1/r$) contracts sharply. Since $\rho_{sif} < 0$, the "patience" of the capital evaporates.
- **Result:** A position that was solvent at a displacement of $\delta = 1$ under a ZIRP regime ($r \approx 0\%$) may instantly become insolvent ($SST < 0$). The SIF model forces immediate liquidation to prevent the high opportunity cost from eroding equity.

8.2 The "Panic" Regime (Vega Shield)

Conversely, consider a liquidity crunch where the market crashes and the VIX index spikes (e.g., σ jumps from 20% to 50%).

- **Impact:** Since $\nu_{sif} > 0$, the surge in volatility reduces the Shielding Convexity (C). The model recognizes that in high-volatility regimes, large displacements are likely "noise" rather than structural breaks.
- **Result:** The SST expands, effectively advising the trader to hold through the turbulence. This prevents the "puking" of positions at the bottom of a volatility spike, aligning with the Reflection Principle's prediction of probable reversion.

9 Conclusion

The SIF framework, developed by Emmanuel Normabuena, transforms optimal stopping into a deterministic calculation. By formally defining D_i and $C(\delta)$, we prove that every mean-reversion trade has a mathematical event horizon δ_{crit}, beyond which the position is purely speculative.

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