

The cosmological constant in the brane world of string theory on S^1/Z_2

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Orbifold branes in string theory are investigated, and the general field equations both outside and on the branes are given explicitly for type II and heterotic string. The radion stability is studied using the Goldberger-Wise mechanism, and shown explicitly that it is stable. It is also found that the effective cosmological constant on each of the two branes can be easily lowered to its current observational value, using large extra dimensions. This is also true for type I string. Therefore, brane world of string theory provides a viable and built-in mechanism for solving the long-standing cosmological constant problem. Applying the formulas to cosmology, we obtain the generalized Friedmann equations on the branes.

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I. INTRODUCTION

One of the long-standing problems is the cosmological constant (CC) problem: its theoretical expectation values from quantum field theory exceed observational limits by 120 orders of magnitude [1]. Even if such high energies are suppressed by supersymmetry, the electroweak corrections are still 56 orders higher. This problem was further sharpened by recent observations of supernova (SN) Ia, which reveal the striking discovery that our universe has lately been in its accelerated expansion phase [2]. Cross checks from the cosmic microwave background radiation and large scale structure all confirm it [3]. In Einstein's theory of gravity, such an expansion can be achieved by a tiny positive CC, which is well consistent with all observations carried out so far [4]. Because of this remarkable fact, a large number of ambitious projects have been aimed to distinguish the CC from dynamical models [5].

Therefore, solving the CC problem now becomes more urgent than ever before. Since the problem is intimately related to quantum gravity, its solution is expected to come from quantum gravity, too. At the present, string/M-Theory is our best bet for a consistent quantum theory of gravity, so it is reasonable to ask what string/M-Theory has to say about the CC. In the string landscape [6], it is expected there are many different vacua with different local CC's [7]. Using the anthropic principle, one may select the low energy vacuum in which we can exist. However, many theorists still hope to explain the problem without invoking the existence of ourselves.

Lately, the CC problem and the late transient acceleration of the universe was studied [8] in the framework of the Horava-Witten heterotic M-Theory on S^1/Z_2 [9]. Using the Arkani-Hamed-Dimopoulos-Dvali (ADD) mechanism of large extra dimensions [10], it was shown that the

effective CC on each of the two branes can be easily lowered to its current observational value. The domination of this term is only temporary. Due to the interaction of the bulk and the brane, the universe will be in its decelerating expansion phase again, whereby all problems connected with a far future de Sitter universe [11] are resolved.

In this Letter, we shall generalize the above studies to string theory, and show that the same mechanism is also viable in all of the five versions of string theory. In addition, we also study the radion stability, using the Goldberger-Wise mechanism [12], and show explicitly that the radion is indeed stable in our current setup. Thus, brane world of string/M-Theory provides a built-in mechanism for solving the long-standing CC problem.

Before showing our above claims, we note that the CC problem was also studied in the framework of brane world in 5D spacetimes [13] and 6D supergravity [14]. However, it turned out that in the 5D case hidden fine-tunings are required [15], while in the 6D case it is still not clear whether loop corrections can be as small as required [16].

II. THE MODEL

Let us begin with the toroidal compactification of the Neveu-Schwarz/Neveu-Schwarz (NS-NS) sector of the action in $(D+d)$ dimensions, $\hat{M}_{D+d} = M_D \times \mathcal{T}_d$, where \mathcal{T}_d is a d -dimensional torus. The action takes the form [17],

$$\begin{aligned} \hat{S}_{D+d} = & -\frac{1}{2\kappa_{D+d}^2} \int_{M^{D+d}} \sqrt{|\hat{g}_{D+d}|} e^{-\hat{\Phi}} \left\{ \hat{R}_{D+d}[\hat{g}] \right. \\ & \left. + \hat{g}^{AB} (\hat{\nabla}_A \hat{\Phi}) (\hat{\nabla}_B \hat{\Phi}) - \frac{1}{12} \hat{H}^2 \right\}, \end{aligned} \quad (2.1)$$

where $\hat{\nabla}_A$ denotes the covariant derivative with respect to \hat{g}^{AB} with $A, B = 0, 1, \dots, D+d-1$; $\hat{\Phi}$ is the dilaton

field; $\hat{H} \equiv dB$ describes the NS three-form field strength of the fundamental string; and κ_{D+d}^2 is the gravitational coupling constant. It should be noted that such action is common to both type II and heterotic string [17]. For type I string, the dilaton does not couple conformally with the NS three-form. However, as to be shown below, our conclusions can be easily generalized to the latter case. In particular, our results about the CC are equally applicable to type I string.

The $(D+d)$ -dimensional spacetimes are described by the metric,

$$d\hat{s}_{D+d}^2 = \tilde{g}_{ab}(x^c) dx^a dx^b + h_{ij}(x^c) dz^i dz^j, \quad (2.2)$$

where \tilde{g}_{ab} is the metric on M_D , parametrized by the coordinates x^a with $a, b, c = 0, 1, \dots, D-1$, and h_{ij} is the metric on the compact space \mathcal{T}_d with the periodic coordinates z^i , where $i, j = D, D+1, \dots, D+d-1$. We assume that all the matter fields are functions of x^a only. This implies that the compact space \mathcal{T}_d is Ricci flat, $R_d[h] = 0$. Moreover, following [18] we also add a potential term to the total action, $\hat{S}_{D+d}^{potential} = \int_{M^{D+d}} \sqrt{|\hat{g}_{D+d}|} V_{D+d}^s$. Then, after the dimensional reduction, the D-dimensional action in the Einstein frame takes the form,

$$S_D^{(E)} = - \int_{M^D} \sqrt{|g_D|} \left\{ \frac{1}{2\kappa_D^2} R_D[g] - \mathcal{L}_D^{(E)}(\phi, \psi, B) \right\}, \quad (2.3)$$

where

$$\begin{aligned} \mathcal{L}_D^{(E)} \equiv & \frac{1}{12} \left\{ 6 \left[(\nabla\phi)^2 + (\nabla\psi)^2 - 2V_D \right] + 3e^{-\sqrt{\frac{8}{d}}\psi} \right. \\ & \times (\nabla_a B^{ij}) (\nabla^a B_{ij}) + e^{-\sqrt{\frac{8}{D-2}}\phi} H_{abc} H^{abc} \left. \right\}, \end{aligned} \quad (2.4)$$

where $B^{ij} \equiv \delta^{ik}\delta^{jl}B_{kl}$, and ∇_a denotes the covariant derivative with respect to g_{ab} , which is related to the string metric \tilde{g}_{ab} by $g_{ab} = \Omega^2 \tilde{g}_{ab}$, where $\Omega^2 = \exp(-2\tilde{\phi}/(D-2))$, $\phi = \sqrt{2/(D-2)}\tilde{\phi}$, and $\tilde{\phi} = \hat{\Phi} - (1/2)\ln|h|$. κ_D^2 is defined as $\kappa_D^2 \equiv V_0^{-1}\kappa_{D+d}^2$, with $V_0 \equiv \int d^d z$. Note that in writing down the above action, we assumed that (a) the flux B is block diagonal; and (b) the internal metric takes the form, $h_{ij} = -\exp(\sqrt{2/d}\psi)\delta_{ij}$. Then, we find that

$$V_D = V_D^0 \exp(D/\sqrt{2(D-2)}\phi + \sqrt{d/2}\psi), \text{ where } V_D^0 \equiv 2\kappa_D^2 V_0 V_{D+d}^s.$$

To study orbifold branes, we add the brane actions,

$$\begin{aligned} S_{D-1,m}^{(I)} = & - \int_{M_{D-1}^{(I)}} \sqrt{|g_{D-1}^{(I)}|} \left(\tau_{(\phi,\psi)}^{(I)} + g_k^{(I)} \right. \\ & \left. - \mathcal{L}_{D-1,m}^{(I)}(\phi, \psi, B, \chi) \right), \end{aligned} \quad (2.5)$$

where, $I = 1, 2$, and $\tau_{(\phi,\psi)}^{(I)} \equiv \epsilon_I V_{D-1}^{(I)}(\phi, \psi)$, with $V_{D-1}^{(I)}(\phi, \psi)$ denoting the potential of the scalar fields, and $\epsilon_1 = -\epsilon_2 = 1$. χ denotes collectively all matter fields, and

$g_{\kappa}^{(I)}$ are constants, as to be shown below, directly related to the $(D-1)$ -dimensional Newtonian constant $G_{D-1}^{(I)}$. Then, the field equations take the form,

$$G_{ab}^{(D)} = \kappa_D^2 T_{ab}^{(D)} + \kappa_D^2 \sum_{i=1}^2 \mathcal{T}_{ab}^{(I)} \sqrt{\left| \frac{g_{D-1}^{(I)}}{g_D} \right|} \delta(\Phi_I), \quad (2.6)$$

where $\kappa_D^2 T_{ab}^{(D)} \equiv 2\delta\mathcal{L}_D^{(E)}/\delta g^{ab} - g_{ab}\mathcal{L}_D^{(E)}$; $\mathcal{T}_{ab}^{(I)} \equiv \mathcal{T}_{\mu\nu}^{(I)} e_a^{(I,\mu)} e_b^{(I,\nu)}$; $\mathcal{T}_{\mu\nu}^{(I)} = \tau_{\mu\nu}^{(I)} + \left(\tau_{(\phi,\psi)}^{(I)} + g_k^{(I)} \right) g_{\mu\nu}^{(I)}$; $\tau_{\mu\nu}^{(I)} \equiv 2\delta\mathcal{L}_{D-1,m}^{(I)}/\delta g^{(I)\mu\nu} - g_{\mu\nu}^{(I)}\mathcal{L}_{D-1,m}^{(I)}$; and $e_{(\mu)}^{(I)a} \equiv \partial x^a/\partial\xi_{(I)}^\mu$. $\xi_{(I)}^\mu$ are the intrinsic coordinates of the I-th brane with $\mu, \nu = 0, 1, 2, \dots, D-2$; $g_{\mu\nu}^{(I)}$ is the reduced metric on the I-th brane, $g_{\mu\nu}^{(I)} \equiv e_{(\mu)}^{(I)a} e_{(\nu)}^{(I)b} g_{ab} \Big|_{M_{D-1}^{(I)}}$; $\Phi_I(x^a) = 0$ denotes the location of the I-th brane; and $\delta(x)$ the Dirac delta function.

To write down the field equations on the branes, we use the Gauss-Codacci equations [19],

$$G_{\mu\nu}^{(D-1)} = \mathcal{G}_{\mu\nu}^{(D)} + E_{\mu\nu}^{(D)} + \mathcal{F}_{\mu\nu}^{(D-1)}, \quad (2.7)$$

$$\begin{aligned} \mathcal{F}_{\mu\nu}^{(D-1)} \equiv & K_{\mu\lambda} K_\nu^\lambda - K K_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (K_{\alpha\beta} K^{\alpha\beta} - K^2), \\ \mathcal{G}_{\mu\nu}^{(D)} \equiv & \frac{(D-3)}{(D-2)(D-1)} \left\{ (D-1) G_{ab}^{(D)} e_{(\mu)}^a e_{(\nu)}^b \right. \\ & \left. - \left[(D-1) G_{ab} n^a n^b + G^{(D)} \right] g_{\mu\nu} \right\}, \end{aligned} \quad (2.8)$$

where n^a denotes the normal vector to the brane, $G^{(D)} \equiv g^{ab} G_{ab}^{(D)}$, $E_{\mu\nu}^{(D)} \equiv C_{abcd}^{(D)} n^a e_{(\mu)}^b n^c e_{(\nu)}^d$, and $C_{abcd}^{(D)}$ is the D-dimensional Weyl tensor. The extrinsic curvature $K_{\mu\nu}$ is defined as $K_{\mu\nu} \equiv e_{(\mu)}^a e_{(\nu)}^b \nabla_a n_b$.

Assuming that the branes have Z_2 symmetry, we can express the intrinsic curvatures $K_{\mu\nu}^{(I)}$ in terms of the effective energy-momentum tensor $\mathcal{T}_{\mu\nu}^{(I)}$ through the Lanczos equations [20], $[K_{\mu\nu}^{(I)}]^- - g_{\mu\nu}^{(I)} [K^{(I)}]^- = -\kappa_D^2 \mathcal{T}_{\mu\nu}^{(I)}$, where $[K_{\mu\nu}^{(I)}]^- \equiv \lim_{\Phi_I \rightarrow 0^+} K_{\mu\nu}^{(I)}^+ - \lim_{\Phi_I \rightarrow 0^-} K_{\mu\nu}^{(I)}^-$, and $[K^{(I)}]^- \equiv g^{(I)\mu\nu} [K_{\mu\nu}^{(I)}]^-$. Setting $\mathcal{S}_{\mu\nu}^{(I)} = \tau_{\mu\nu}^{(I)} + \lambda^{(I)} g_{\mu\nu}^{(I)}$, where $\lambda^{(I)}$ denotes the CC of the I-th brane, we find that $G_{\mu\nu}^{(D-1)}$ given by Eq.(2.7) can be cast in the form,

$$\begin{aligned} G_{\mu\nu}^{(D-1)} = & \mathcal{G}_{\mu\nu}^{(D)} + E_{\mu\nu}^{(D)} + \mathcal{E}_{\mu\nu}^{(D-1)} + \kappa_D^4 \pi_{\mu\nu} \\ & + \kappa_{D-1}^2 \tau_{\mu\nu} + \Lambda_{D-1} g_{\mu\nu}, \end{aligned} \quad (2.9)$$

$$\begin{aligned} \pi_{\mu\nu} \equiv & \frac{1}{8(D-2)} \left\{ 2(D-2) \tau_{\mu\lambda} \tau_\nu^\lambda - 2\tau \tau_{\mu\nu} \right. \\ & \left. - g_{\mu\nu} ((D-2)\tau^{\alpha\beta} \tau_{\alpha\beta} - \tau^2) \right\}, \end{aligned} \quad (2.10)$$

$$\begin{aligned} \mathcal{E}_{\mu\nu}^{(D-1)} \equiv & \frac{\kappa_D^4 (D-3)}{8(D-2)} \tau_{(\phi,\psi)} \\ & \times [2\tau_{\mu\nu} + (2\lambda + \tau_{(\phi,\psi)}) g_{\mu\nu}], \end{aligned} \quad (2.11)$$

$$\frac{\kappa_{D-1}^2}{\kappa_D^4} = \frac{(D-3)\lambda}{4(D-2)}, \quad \frac{\Lambda_{D-1}}{\kappa_D^4} = \frac{(D-3)\lambda^2}{8(D-2)}. \quad (2.12)$$

Note that in writing Eqs.(2.9)-(2.12), without causing any confusion, we had dropped the super indices “(I)”. In addition, the definitions of κ_{D-1} and Λ are unique, because these are the only terms that linearly proportional to the matter field $\tau_{\mu\nu}$ and the spacetime geometry $g_{\mu\nu}$. When $D = 5$ they reduce exactly to the ones defined in brane-worlds [21].

In the following, we shall restrict ourselves to the case where $D = d = 5$.

III. RADION STABILITY

In the studies of orbifold branes, an important issue is the radion stability. In this section, we shall address this problem. Let us first consider the 5-dimensional static metric with a 4-dimensional Poincaré symmetry,

$$\begin{aligned} ds_5^2 &= e^{2\sigma(y)} (\eta_{\mu\nu} dx^\mu dx^\nu - dy^2), \quad (3.1) \\ \sigma(y) &= \frac{1}{9} \ln \left(\frac{|y| + y_0}{L} \right), \\ \phi(y) &= -\sqrt{\frac{25}{54}} \ln \left(\frac{|y| + y_0}{L} \right) + \phi_0, \\ \psi(y) &= -\sqrt{\frac{5}{18}} \ln \left(\frac{|y| + y_0}{L} \right) + \psi_0, \\ B_{ij} &= 0 = B_{ab}, \end{aligned} \quad (3.2)$$

where $|y|$ is defined as that given in Fig.1 [22], L and y_0 are positive constants, and

$$\psi_0 \equiv \sqrt{\frac{2}{5}} \left(\ln \left(\frac{2}{9L^2 V_{(5)}^0} \right) - \frac{5}{\sqrt{6}} \phi_0 \right). \quad (3.3)$$

Then, it can be shown that the above solution satisfies the gravitational and matter field equations outside the branes, Eq.(2.6), for $D = d = 5$. On the other hand, to show that it also satisfies the field equations on the branes, we first note that the normal vector $n_{(I)}^a$ to the I-th brane is given simply by

$$n_{(I)}^a = -\epsilon_y^{(I)} e^{-\sigma(y_I)} \delta_y^a, \quad (3.4)$$

and that

$$\dot{t} = e^{-\sigma(y_I)}, \quad \dot{y} = 0, \quad (3.5)$$

where $y_1 = y_c > 0$ and $y_2 = 0$. Inserting the above into Eqs.(2.9)-(2.12), we find that they are satisfied for $\tau_{\mu\nu}^{(I)} = 0$, provided that the tension $\tau_{\phi,\psi}^{(I)}$ satisfies the relation,

$$\left(\tau_{(\phi,\psi)}^{(I)} + 2\rho_\Lambda^{(I)} \right)^2 = \frac{\rho_\Lambda^{(I)}}{54\pi G_4 L^2} \left(\frac{L}{y_I + y_0} \right)^{20/9}, \quad (3.6)$$

where $\rho_\Lambda^{(I)} \equiv \Lambda^{(I)}/(8\pi G_4)$ denotes the corresponding energy density of the effective cosmological constant on the

I-th brane, defined by Eq.(2.12). On the other hand, on each of the two branes, we also find that

$$\frac{\partial V_4^{(I)}(\phi, \psi)}{\partial \phi} = \sqrt{\frac{25}{54}} \frac{\epsilon_y^{(I)}}{\kappa_5^2 (y_I + y_0)}, \quad (3.7)$$

$$\frac{\partial V_4^{(I)}(\phi, \psi)}{\partial \psi} = \sqrt{\frac{5}{18}} \frac{\epsilon_y^{(I)}}{\kappa_5^2 (y_I + y_0)}. \quad (3.8)$$

For certain choices of the potentials $V_4^{(I)}(\phi, \psi)$ of the two branes, Eqs.(3.6)-(3.8) can be satisfied. For example, one may choose

$$V_4^{(I)}(\phi, \psi) = \beta_I e^{-g_I \phi} (\psi^2 - \psi_I^2)^2, \quad (3.9)$$

where β_I , g_I and ψ_I are arbitrary constants. Then, by properly choosing these parameters, Eqs.(3.6)-(3.8) can easily be satisfied.

To study the radion stability, it is found convenient to introduce the proper distance Y , defined by

$$Y = \left(\frac{9L}{10} \right) \left\{ \left(\frac{y+y_0}{L} \right)^{10/9} - \left(\frac{y_0}{L} \right)^{10/9} \right\}. \quad (3.10)$$

Then, in terms of Y , the static solution (3.1) can be written as

$$ds_5^2 = e^{-2A(Y)} \eta_{\mu\nu} dx^\mu dx^\nu - dY^2, \quad (3.11)$$

with

$$\begin{aligned} A(Y) &= -\frac{1}{10} \ln \left\{ \left(\frac{10}{9L} \right) (|Y| + Y_0) \right\}, \\ \phi(Y) &= -\sqrt{\frac{3}{8}} \ln \left\{ \left(\frac{10}{9L} \right) (|Y| + Y_0) \right\} + \phi_0, \\ \psi(Y) &= -\frac{3}{\sqrt{40}} \ln \left\{ \left(\frac{10}{9L} \right) (|Y| + Y_0) \right\} \\ &\quad + \psi_0, \end{aligned} \quad (3.12)$$

where $|Y|$ is defined also as that in Fig. 1, with

$$\begin{aligned} Y_c &\equiv \left(\frac{9L}{10} \right) \left\{ \left(\frac{y_c + y_0}{L} \right)^{10/9} - \left(\frac{y_0}{L} \right)^{10/9} \right\}, \\ Y_0 &\equiv \left(\frac{9L}{10} \right) \left(\frac{y_0}{L} \right)^{10/9}, \end{aligned} \quad (3.13)$$

and $Y_2 = 0$, $Y_1 = Y_c$.

Following [12], let us consider a massive scalar field Φ with the actions,

$$\begin{aligned} S_b &= \int d^4x \int_0^{Y_c} dY \sqrt{-g_5} ((\nabla \Phi)^2 - m^2 \Phi^2), \\ S_I &= -\alpha_I \int_{M_4^{(I)}} d^4x \sqrt{-g_4^{(I)}} (\Phi^2 - v_I^2)^2, \end{aligned} \quad (3.14)$$

where α_I and v_I are real constants. Then, it can be shown that, in the background of Eq.(3.11), the massive

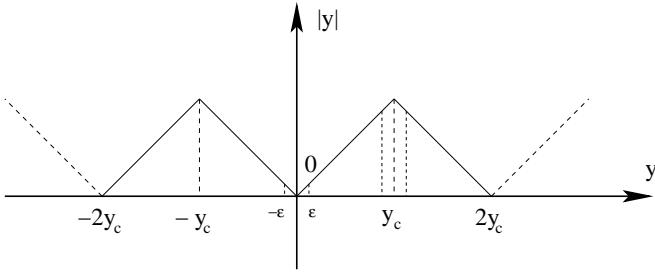


FIG. 1: The function $|y|$ appearing in the metric Eq.(5.2).

scalar field Φ satisfies the following Klein-Gordon equation

$$\Phi'' - 4A'\Phi' - m^2\Phi = \sum_{I=1}^2 2\alpha_I \Phi (\Phi^2 - v_I^2) \delta(Y - Y_I). \quad (3.15)$$

Integrating the above equation in the neighborhood of the I-th brane, we find that

$$\frac{d\Phi(Y)}{dY} \Big|_{Y_I-\epsilon}^{Y_I+\epsilon} = 2\alpha_I \Phi_I (\Phi_I^2 - v_I^2), \quad (3.16)$$

where $\Phi_I \equiv \Phi(Y_I)$. Setting

$$z \equiv m(Y + Y_0), \quad \Phi = z^\nu u(z), \quad (3.17)$$

we find that, outside of the branes, Eq.(3.15) yields,

$$\frac{d^2u}{dz^2} + \frac{1}{z} \frac{du}{dz} - \left(1 + \frac{\nu^2}{z^2}\right) u = 0, \quad (3.18)$$

where $\nu \equiv 3/10$. Eq.(3.18) is the standard modified Bessel equation [23], which has the general solution

$$u(z) = aI_\nu(z) + bK_\nu(z), \quad (3.19)$$

where $I_\nu(z)$ and $K_\nu(z)$ denote the modified Bessel functions, and a and b are the integration constants, which are uniquely determined by the boundary conditions (3.16). Since

$$\begin{aligned} \lim_{Y \rightarrow Y_c^+} \frac{d\Phi(Y)}{dY} &= - \lim_{Y \rightarrow Y_c^-} \frac{d\Phi(Y)}{dY} \equiv -\Phi'(Y_c), \\ \lim_{Y \rightarrow 0^-} \frac{d\Phi(Y)}{dY} &= - \lim_{Y \rightarrow 0^+} \frac{d\Phi(Y)}{dY} \equiv -\Phi'(0), \end{aligned} \quad (3.20)$$

we find that the conditions (3.16) can be written in the forms,

$$\Phi'(Y_c) = -\alpha_1 \Phi_1 (\Phi_1^2 - v_1^2), \quad (3.21)$$

$$\Phi'(0) = \alpha_2 \Phi_2 (\Phi_2^2 - v_2^2). \quad (3.22)$$

Inserting the above solution back to the actions (3.14), and then integrating them with respect to Y , we obtain the effective potential for the radion Y_c ,

$$\begin{aligned} V_\Phi(Y_c) &\equiv - \int_{0+\epsilon}^{Y_c-\epsilon} dY \sqrt{|g_5|} ((\nabla\Phi)^2 - m^2\Phi^2) \\ &+ \sum_{I=1}^2 \alpha_I \int_{Y_I-\epsilon}^{Y_I+\epsilon} dY \sqrt{|g_4^{(I)}|} (\Phi^2 - v_I^2)^2 \\ &\times \delta(Y - Y_I) \\ &= e^{-4A(Y)} \Phi(Y) \Phi'(Y) \Big|_0^{Y_c} \\ &+ \sum_{I=1}^2 \alpha_I (\Phi_I^2 - v_I^2)^2 e^{-4A(Y_I)}. \end{aligned} \quad (3.23)$$

In the limit that α_I 's are very large [12], Eqs.(3.21) and (3.22) show that there are solutions only when $\Phi(0) = v_2$ and $\Phi(Y_c) = v_1$, that is,

$$v_1 = z_c^\nu (aI_\nu(z_c) + bK_\nu(z_c)), \quad (3.24)$$

$$v_2 = z_0^\nu (aI_\nu(z_0) + bK_\nu(z_0)), \quad (3.25)$$

where $z_0 \equiv mY_0$ and $z_c \equiv m(Y_c + Y_0)$. Eqs.(3.24) and (3.25) have the solutions,

$$\begin{aligned} a &= \frac{1}{\Delta} (K_\nu^{(0)} z_0^\nu v_1 - K_\nu^{(c)} z_c^\nu v_2), \\ b &= \frac{1}{\Delta} (I_\nu^{(c)} z_c^\nu v_2 - I_\nu^{(0)} z_0^\nu v_1), \end{aligned} \quad (3.26)$$

where $K_\nu^{(I)} \equiv K_\nu(z_I)$, $I_\nu^{(I)} \equiv I_\nu(z_I)$, and

$$\Delta \equiv (z_0 z_c)^\nu (I_\nu^{(c)} K_\nu^{(0)} - I_\nu^{(0)} K_\nu^{(c)}). \quad (3.27)$$

A. $mY_0 \gg 1$

When $Y_0 \gg m^{-1}$, we have $z_0, z \gg 1$. Then, we find that [23],

$$\begin{aligned} I_\nu(z) &\simeq \frac{e^z}{\sqrt{2\pi z}} \simeq I'_\nu(z), \\ K_\nu(z) &\simeq \sqrt{\frac{\pi}{2z}} e^{-z} \simeq -K'_\nu(z). \end{aligned} \quad (3.28)$$

Substituting them into Eq.(3.23), we find that

$$\begin{aligned} V_\Phi(Y_c) &= \frac{1}{2} m^{3/5} \left(\frac{10}{9}\right)^{2/5} \frac{e^{-(z_0+z_c)}}{(z_0 z_c)^{3/5} \sinh(z_c - z_0)} \\ &\times \left\{ 2\nu e^{z_0+z_c} \sinh(z_c - z_0) \left(v_1^2 z_0^{3/5} - v_2^2 z_c^{3/5}\right) \right. \\ &+ (z_0 z_c)^{3/5} \left[\left(v_1 z_c^{1/5} e^{z_c} - v_2 z_0^{1/5} e^{z_0}\right)^2 \right. \\ &\left. \left. + \left(v_1 z_c^{1/5} e^{z_c} - v_2 z_0^{1/5} e^{z_0}\right)^2 \right] \right\}. \end{aligned} \quad (3.29)$$

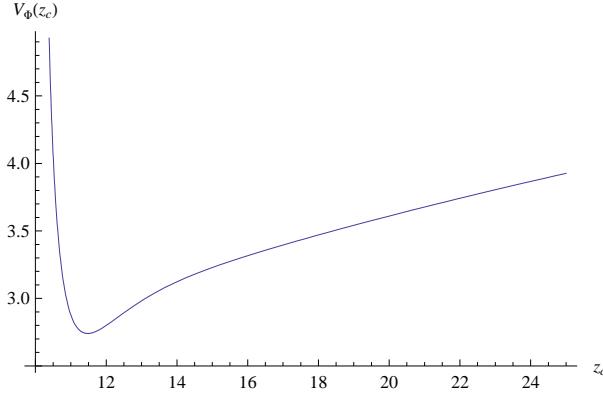


FIG. 2: The potential defined by Eq.(3.29) in the limit of large v_I and y_0 . In this particular plot, we choose $(z_0, v_1, v_2) = (10, 1.0, 0.1)$.

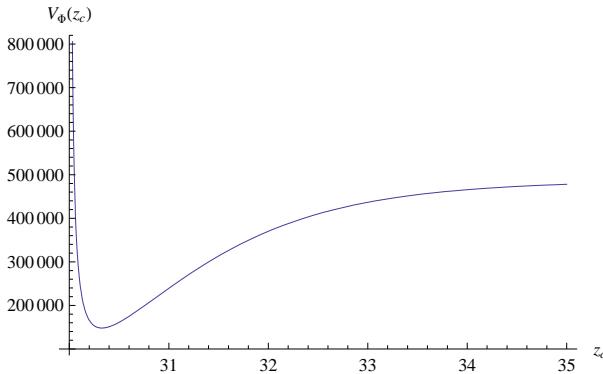


FIG. 3: The potential defined by Eq.(3.29) in the limit of large v_I and y_0 . In this particular plot, we choose $(z_0, v_1, v_2) = (30, 200, 100)$.

Then, we find that

$$V_\Phi(Y_c) = V_\Phi^{(0)} \begin{cases} \frac{(v_1 - v_2)^2 z_0^{2/5}}{\sinh(z_c - z_0)} \rightarrow \infty, & z_c \rightarrow z_0, \\ v_1^2 z_c^{2/5} \rightarrow \infty, & z_c \rightarrow \infty, \end{cases} \quad (3.30)$$

where $V_\Phi^{(0)} \equiv m^{3/5} \left(\frac{10}{9}\right)^{2/5}$. Figs. 2 and 3 show the potential for $(z_0, v_1, v_2) = (10, 1.0, 0.1)$ and $(z_0, v_1, v_2) = (30, 200, 100)$, respectively, from which we can see clearly that it has a minimum. Therefore, the radion is indeed stable in our current setup.

B. $mY_0 \ll 1$

When $mY_0 \ll 1$ and $mY_c \ll 1$, we find that [23]

$$\begin{aligned} I_\nu(z) &\simeq \frac{z^\nu}{2^\nu \Gamma(\nu + 1)}, \\ K_\nu(z) &\simeq \frac{2^{\nu-1} \Gamma(\nu)}{z^\nu}. \end{aligned} \quad (3.31)$$

Substituting them into Eq.(3.23), we obtain

$$V_\Phi(Y_c) = \frac{3}{5} m^{3/5} \left(\frac{10}{9}\right)^{2/5} \frac{(v_1 - v_2)^2}{z_c^{2\nu} - z_0^{2\nu}}. \quad (3.32)$$

Clearly, in this limit the potential has no minima, and the corresponding radion is not stable. Therefore, there exists a minimal mass for the scalar field Φ , say, m_c , only when $m > m_c$ the corresponding radion is stable.

It should be noted that, in the Randall-Sundrum setup [24], Y_c is required to be about 35 in order to solve the hierarchy problem. However, in the current setup the hierarchy problem is solved by the combination of the RS warped factor mechanism and the ADD large extra dimensions [25]. Thus, such a requirement is not needed here, which allows Y_c to have a large range of choice.

IV. THE COSMOLOGICAL CONSTANT

For $D = d = 5$, we find that

$$\kappa_5^2 = \frac{\kappa_{10}^2}{V_0} = \frac{1}{M_{10}^8 R^5}. \quad (4.1)$$

Then, from Eq.(2.12) we find

$$\rho_\Lambda \equiv \frac{\Lambda_4}{8\pi G_4} = 3 \left(\frac{R}{l_{pl}}\right)^{10} \left(\frac{M_{10}}{M_{pl}}\right)^{16} M_{pl}^{-4}, \quad (4.2)$$

where R denotes the typical size of the internal space \mathcal{T}_d , and M_{pl} and l_{pl} denote the Planck mass and length, respectively. Current observations show $\rho_\Lambda \simeq 10^{-47} \text{ GeV}^4$. If M_{10} is of the order of TeV [26], we find that Eq.(4.2) requires $R \simeq 10^{-22} \text{ m}$, which is well below the current experimental limit of the extra dimensions [27]. If $M_{10} \sim 100 \text{ TeV}$ we find that R needs to be of the order of 10^{-25} m . For $M_{10} \sim 100 \text{ eV}$, we have $R \simeq 10 \text{ microns}$. Therefore, brane world of string theory on S^1/Z_2 provides a viable mechanism to get ρ_Λ down to its current observational value. Hence, the ADD mechanism that was initially designed to solve the hierarchy problem [10] also solves the CC problem in string theory.

Although the action of Eq.(2.1) is valid only for type II and heterotic string, it is straightforward to show that our above conclusions are also true for type I string. As a matter of fact, the only difference will be in the expression of $T_{ab}^{(D)}$, while all the rest remains the same, so does Eq.(2.12), based on which our above conclusions were derived. Similarly, the addition of other matter fields, such as the Yang-Mills and Chern-Simons terms [17], in action (2.1) does not change our conclusions either.

It is remarkable to note that the same mechanism is also valid in the framework of the Horava-Witten heterotic M-Theory on S^1/Z_2 [8]. All of these strongly suggest that the above mechanism for solving the long-standing CC problem is a built-in mechanism in the brane world of string/M-Theory.

V. BRANE COSMOLOGY IN STRING THEORY

We consider spacetimes with the metric [22],

$$ds_5^2 = e^{2\sigma(t,y)}(dt^2 - dy^2) - e^{2\omega(t,y)}d\Sigma_k^2, \quad (5.1)$$

where $d\Sigma_k^2 = dr^2/(1 - kr^2) + r^2(d\theta^2 + \sin^2\theta d\phi^2)$. Assuming that the two orbifold branes are located at $y = y_I(t_I)$, we find that the reduced metric takes the form,

$$ds_5^2|_{M_4^{(I)}} = g_{\mu\nu}^{(I)}d\xi_{(I)}^\mu d\xi_{(I)}^\nu = d\tau_I^2 - a^2(\tau_I)d\Sigma_k^2, \quad (5.2)$$

where $\xi_{(I)}^\mu \equiv \{\tau_I, r, \theta, \varphi\}$, and τ_I denotes the proper time of the I-th brane, given by $d\tau_I = e^\sigma \sqrt{1 - (\dot{y}_I/t_I)^2} dt_I$, and $a(\tau_I) \equiv \exp\{\omega[t_I(\tau_I), y_I(\tau_I)]\}$, with $\dot{y}_I \equiv dy_I/d\tau_I$, etc. For the sake of simplicity and without causing any confusion, from now on we shall drop all the indices “I”. The normal vector n_a and $e_{(\mu)}^a$ are given, respectively, by $n^a = -\epsilon_y(\dot{y}\delta_t^a + \dot{t}\delta_y^a)$, $e_{(\tau)}^a = \dot{t}\delta_t^a + \dot{y}\delta_y^a$, $e_{(r)}^a = \delta_r^a$, $e_{(\theta)}^a = \delta_\theta^a$, and $e_{(\varphi)}^a = \delta_\varphi^a$, where $\epsilon_y = \pm 1$. Then, we find that, for a perfect fluid $\tau_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}$, where $u_\mu = \delta_\mu^\tau$, the field equations on the branes are given by

$$\begin{aligned} H^2 + \frac{k}{a^2} &= \frac{8\pi G}{3}(\rho + \tau_{(\phi,\psi)}) + \frac{1}{3}\Lambda + \frac{1}{3}\mathcal{G}_\tau^{(5)} + E^{(5)} \\ &\quad + \frac{2\pi G}{3\rho_\Lambda}(\rho + \tau_{(\phi,\psi)})^2, \end{aligned} \quad (5.3)$$

$$\begin{aligned} \ddot{a} &= -\frac{4\pi G}{3}(\rho + 3p - 2\tau_{(\phi,\psi)}) + \frac{1}{3}\Lambda \\ &\quad - \frac{1}{6}(\mathcal{G}_\tau^{(5)} + 3\mathcal{G}_\theta^{(5)}) - E^{(5)} - \frac{2\pi G}{3\rho_\Lambda}[\rho(2\rho + 3p) \\ &\quad + (\rho + 3p - \tau_{(\phi,\psi)})\tau_{(\phi,\psi)}], \end{aligned} \quad (5.4)$$

where $H \equiv \dot{a}/a$, and

$$\begin{aligned} E^{(5)} &\equiv \frac{1}{6}e^{-2\sigma}[\sigma_{,tt} - \omega_{,tt} - \sigma_{,yy} + \omega_{,yy} + ke^{2(\sigma-\omega)}], \\ \mathcal{G}_\tau^{(5)} &\equiv \frac{1}{3}e^{-2\sigma}[(\phi_{,t}^2 + \psi_{,t}^2) - (\phi_{,y}^2 + \psi_{,y}^2)] \\ &\quad - \frac{1}{24}\left\{5\left[(\nabla\phi)^2 + (\nabla\psi)^2\right] - 6V_5\right\}, \end{aligned}$$

$$\begin{aligned} \mathcal{G}_\theta^{(5)} &\equiv \frac{1}{24}\left\{8\left(\phi_{,n}^2 + \psi_{,n}^2\right) - 6V_5\right. \\ &\quad \left.+ 5\left[(\nabla\phi)^2 + (\nabla\psi)^2\right]\right\}, \end{aligned} \quad (5.5)$$

with $\phi_{,n} \equiv n^a\nabla_a\phi$, $\Lambda \equiv \Lambda_4$ and $G \equiv G_4$. The first two terms in the right-hand sides of Eqs.(5.3) and (5.4) also appear in the Einstein’s theory of gravity, although their origins are completely different [8]. The rest denotes the brane corrections, and the effects of which on the evolution of the universe depend on specific models to be considered.

VI. CONCLUSIONS

In this Letter, we have studied orbifold branes in string theory in (D+d)-dimensions, and obtained the general field equations both outside and on the branes for type II and heterotic string. We have investigated the radion stability, using the Goldberger-Wise mechanism [12], and shown explicitly that it is stable.

We have also shown explicitly that for $D = d = 5$ the effective cosmological constant on the branes can be easily lowered to its current observational value, using large extra dimensions. This is also true for type I string. Therefore, brane world of string theory provides a built-in mechanism for solving the long-standing cosmological constant problem.

Applying the formulas to cosmology, we have obtained the generalized Friedmann equations on each of the two branes. Investigations of their cosmological implications, including current acceleration of the universe, are under our current considerations.

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- [1] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989); and T. Padmanabhan, Phys. Rept. **380**, 235 (2003).
 - [2] A.G. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).
 - [3] A.G. Riess *et al.*, Astrophys. J. **607**, 665 (2004); P. Astier *et al.*, Astron. and Astrophys. **447**, 31 (2006); D.N. Spergel *et al.*, : Astrophys. J. Suppl. **170**, 377 (2007); W.M. Wood-Vasey *et al.*, astro-ph/0701041; T.M. Davis *et al.*, astro-ph/0701510.
 - [4] S. Sullivan, A. Cooray, and D.E. Holz, arXiv:0706.3730; A. Mantz, *et al.*, arXiv:0709.4294; and J. Dunkley, *et al.*, arXiv:0803.0586.
 - [5] A. Albrecht, *et al*, arXiv:astro-ph/0609591; J.A. Peacock, *et al*, arXiv:astro-ph/0610906.
 - [6] L. Susskind, arXiv:hep-th/0302219.
 - [7] R. Bousso and J. Polchinski, JHEP, **006**, 006 (2000).
 - [8] Y.-G. Gong, A. Wang, and Q. Wu, Phys. Lett. B**663**, 147 (2008) [arXiv:0711.1597].
 - [9] H. Horava and E. Witten, Nucl. Phys. B**460**, 506 (1996); **475**, 94 (1996).
 - [10] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B**429**, 263 (1998); Phys. Rev. D**59**, 086004 (1999); I. Antoniadis, *et al.*, Phys. Lett., B**436**, 257 (1998).
 - [11] L.M. Krauss and R.J. Scherrer, Gen. Rel. Grav. **39**, 1545

- (2007); and references therein.
- [12] W.D. Goldberger and M.B. Wise, Phys. Rev. Lett. **83**, 4922 (1999).
 - [13] N. Arkani-Hamed, *et al*, Phys. Lett. **B480**, 193 (2000); S. Kachru, M.B. Schulz, and E. Silverstein, Phys. Rev. D**62**, 045021 (2000).
 - [14] Y. Aghababaie, *et al*, Nucl. Phys. **B680**, 389 (2004); JHEP, **0309**, 037 (2003); C.P. Burgess, Ann. Phys. **313**, 283 (2004); AIP Conf. Proc. **743**, 417 (2005); C.P. Burgess , J. Matias, and F. Quevedo, Nucl.Phys. B**706**, 71 (2005).
 - [15] S. Forste, *et al*, Phys. Lett. **B481**, 360 (2000); JHEP, **0009**, 034 (2000); C. Csaki, *et al*, Nucl. Phys. **B604**, 312 (2001); J.M. Cline and H. Firouzjahi, Phys. Rev. D**65**, 043501 (2002).
 - [16] C.P. Burgess, arXiv:0708.0911.
 - [17] J.E. Lidsey, D. Wands, and E.J. Copeland, Phys. Rept. **337**, 343 (2000); M. Gasperini, *Elements of String Cosmology* (Cambridge University Press, 2007).
 - [18] T. Battefeld and S. Watson, Rev. Mod. Phys. **78**, 435 (2006).
 - [19] T. Shiromizu, K.-I. Maeda, and M. Sasaki, Phys. Rev. D**62**, 024012 (2000); A.N. Aliev and A.E. Gumrukcuoglu, Class. Quantum Grav. **21**, 5081 (2004); R.-G. Cai and L.-M. Cao, Nucl. Phys. B**785**, 135 (2007).
 - [20] C. Lanczos, Phys. Z. **23**, 539 (1922); W. Israel, Nuovo Cimento **B44**, 1 (1967); (Errata) **B48**, 463 (1967).
 - [21] V.A. Rubakov, Phys. Usp. **44**, 871 (2001); R. Maartens, Living Reviews of Relativity **7** (2004); arXiv:astro-ph/0602415 (2006); P. Brax, C. van de Bruck and A. C. Davis, Rept. Prog. Phys. **67**, 2183 (2004); V. Sahni, arXiv:astro-ph/0502032 (2005); D. Langlois, arXiv:hep-th/0509231 (2005); and A. Lue, Phys. Rept. **423**, 1 (2006).
 - [22] A. Wang, R.-G. Cai, and N.O. Santos, Nucl. Phys. B**797**, 395 (2008) [arXiv:astro-ph/0607371].
 - [23] M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions (Dover Publications, INC., New York, 1972), pp.374-8.
 - [24] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
 - [25] A. Wang and N.O. Santos, in preparation (2008).
 - [26] I. Antoniadis, J. Phys. Conf. Ser. **8**, 112 (2005); V.H.S. Kumar and P.K. Suresh, arXiv:hep-th/0606194.
 - [27] J.C. Long *et al.*, Nature (London), **421**, 922 (2003); D.J. Kapner, *et al.*, Phys. Rev. Lett. **98**, 021101 (2007).