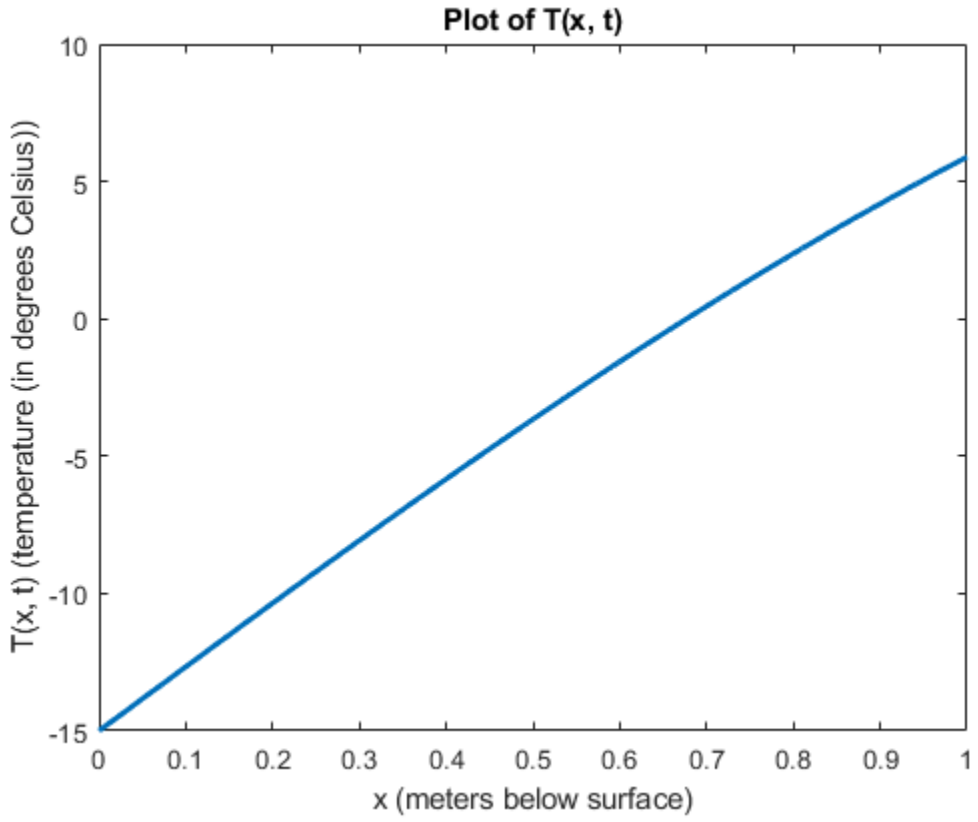
Problem 2

```
fprintf("Homework 2, Problem 2\n");
% To call our bisection implementation
addpath(genpath("C:\Users\eappe\Google Drive\1st Year\APPM
 5600\Homework 1"));
close all;

T_i = 20;
T_s = -15;
alpha = 0.138 * 1e-6;
tol = 1e-13;

% x is in meters, t is in seconds
% 60 days after the cold snap, which is 60*24*60*60 seconds
t = 60*24*60*60;
% Temperature function
T = @(x) (T_i - T_s)*erf(x./(2*sqrt(alpha*t))) + T_s;
% Derivative of temperature function
T_prime = @(x) (T_i - T_s)*(1/sqrt(pi*alpha*t))*exp(-
((x.^(2))./4*alpha*t));

a =  0;
b =  1;
x = linspace(a, b, 1000);

figure;
plot(x, T(x), "LineWidth", 2);
xlabel("x (meters below surface)");
ylabel("T(x, t) (temperature (in degrees Celsius))");
title("Plot of T(x, t)");

% Find a root of T on the interval [x_0, x_bar] via bisection
[r, iters] = bisect(a, b, T, tol);
fprintf("Root computed via bisection: %0.16f (%d iterations)\n", r,
 iters);

% Find a root of T on the interval [x_0, x_bar] via Newton's method
% Start off with an initial guess close to 0
x_0 = 0.01;
n_max = 100;
[r, iters, ~] = newton(x_0, T, T_prime, tol, n_max);
fprintf("Root computed via Newton's method: %0.16f (%d iterations, x_0
 = %0.16f)\n", ...
        r(iters), iters, x_0);

% Use Newton's method again, but make the initial guess the endpoint
 of the
% interval [a, b], i.e., x_0 = b
x_0 = b;
[r, iters, ~] = newton(x_0, T, T_prime, tol, n_max);
```

```
fprintf("Root computed via Newton's method: %0.16f (%d iterations, x_0
 = %0.16f)\n", ...
        r(iters), iters, x_0);
```

```
Homework 2, Problem 2
Root computed via bisection: 0.6769618544819309 (44 iterations)
Root computed via Newton's method: 0.6769618544819355 (14 iterations,
 x_0 = 0.0100000000000000)
Root computed via Newton's method: 0.6769618544819372 (14 iterations,
 x_0 = 1.0000000000000000)
```



Problem 4

```
fprintf("Homework 2, Problem 4\n");
close all;

p = 5;
f = @(x) ((x-1).^(p)).*exp(x);
f_prime = @(x) (p*(x-1).^(p-1)).*exp(x) + ((x-1).^(p)).*exp(x);
% f_prime = @(x) ((x-1).^(4)).*(x+4).*exp(x);

tol = 1e-15;
n_max = 150;

x_0 = 0;
[r, iters, ~] = newton(x_0, f, f_prime, tol, n_max);
```

```
fprintf("Root computed via Newton's method: %0.16f (%d iterations, x_0
 = %0.16f)\n", ...
        r(iters), iters, x_0);
fprintf("Relative error between approximate root and 1: %0.16e\n
\n", ...
        abs(r(iters) - 1));

Newton_iterates = r(1:iters);
Newton_relative_errors = abs(r(1:iters) - 1);
T1 = table(Newton_iterates, Newton_relative_errors);
disp(T1);

figure;
semilogy(1:iters, abs(r(1:iters) - 1), "LineWidth", 2);
xlabel("k");
ylabel("log(|e_{k}|)");
title("Plot of relative errors computed via Newton's method");

[r, iters, ~] = modified_newton(x_0, f, f_prime, p, tol, n_max);
fprintf("Root computed via modified Newton's method: %0.16f (%d
 iterations, x_0 = %0.16f)\n", ...
        r(iters), iters, x_0);
fprintf("Relative error between approximate root and 1: %0.16e\n
\n", ...
    abs(r(iters) - 1));

Modified_Newton_iterates = r(1:iters);
Modified_Newton_relative_errors = abs(r(1:iters) - 1);
T2 = table(Modified_Newton_iterates, Modified_Newton_relative_errors);
disp(T2);

figure;
semilogy(1:iters, abs(r(1:iters) - 1), "LineWidth", 2);
xlabel("k");
ylabel("log(|e_{k}|)");
title("Plot of relative errors computed via modified Newton's
 method");
```

```
Homework 2, Problem 4
Root computed via Newton's method: 0.9999999999999964 (149 iterations,
 x_0 = 0.0000000000000000)
Relative error between approximate root and 1: 3.5527136788005009e-15
```

Newton_iterates	Newton_relative_errors
0.00000000000000e+00	1.00000000000000e+00
2.50000000000000e-01	7.50000000000000e-01
4.26470588235294e-01	5.73529411764706e-01
5.56038694547586e-01	4.43961305452414e-01
6.53483280780737e-01	3.46516719219263e-01
7.27947224951420e-01	2.72052775048580e-01
7.85488640306659e-01	2.14511359693341e-01
8.30314023003544e-01	1.69685976996456e-01

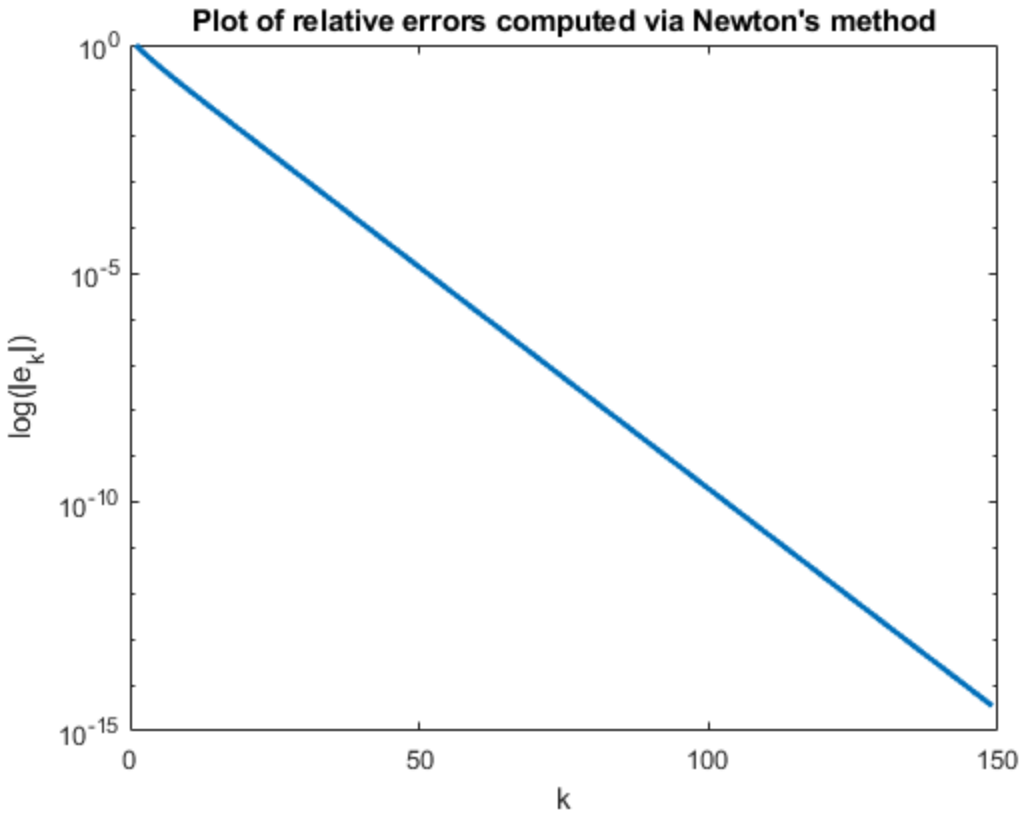
8.65443411318332e-01	1.34556588681668e-01
8.93098976763539e-01	1.06901023236461e-01
9.14946281251782e-01	8.50537187482183e-02
9.32251397906395e-01	6.77486020936053e-02
9.45987235078515e-01	5.40127649214850e-02
9.56907757585813e-01	4.30922424141867e-02
9.65601129446171e-01	3.43988705538287e-02
9.72528562733119e-01	2.74714372668814e-02
9.78053204154541e-01	2.19467958454590e-02
9.82461914737855e-01	1.75380852621445e-02
9.85981878475113e-01	1.40181215248869e-02
9.88793385188646e-01	1.12066148113539e-02
9.91039742964181e-01	8.96025703581937e-03
9.92835011585003e-01	7.16498841499735e-03
9.94270065697220e-01	5.72993430278013e-03
9.95417367350396e-01	4.58263264960446e-03
9.96334734671804e-01	3.66526532819578e-03
9.97068325498448e-01	2.93167450155218e-03
9.97655004389067e-01	2.34499561093293e-03
9.98124223574640e-01	1.87577642536019e-03
9.98499519654020e-01	1.50048034598049e-03
9.98799705807900e-01	1.20029419209955e-03
9.99039822288404e-01	9.60177711596288e-04
9.99231894715456e-01	7.68105284544296e-04
9.99385539375420e-01	6.14460624580371e-04
9.99508446604666e-01	4.91553395333688e-04
9.99606766949673e-01	3.93233050327013e-04
9.99685419745514e-01	3.14580254485830e-04
9.99748339755090e-01	2.51660244910190e-04
9.99798674337515e-01	2.01325662485430e-04
9.99838941091358e-01	1.61058908642153e-04
9.99871153910719e-01	1.28846089281431e-04
9.99896923792645e-01	1.03076207355390e-04
9.99917539459113e-01	8.24605408873413e-05
9.99934031839284e-01	6.59681607158014e-05
9.99947225645502e-01	5.27743544984638e-05
9.99957780627808e-01	4.22193721922870e-05
9.99966224573546e-01	3.37754264542189e-05
9.99972979704468e-01	2.70202955319210e-05
9.99978383792778e-01	2.16162072215198e-05
9.99982707052913e-01	1.72929470867444e-05
9.99986165654292e-01	1.38343457075640e-05
9.99988932531090e-01	1.10674689104417e-05
9.99991146029771e-01	8.85397022876155e-06
9.99992916826953e-01	7.08317304731754e-06
9.99994333463569e-01	5.66653643097048e-06
9.99995466772140e-01	4.53322786042598e-06
9.99996373418534e-01	3.62658146635386e-06
9.99997098735353e-01	2.90126464697060e-06
9.99997678988619e-01	2.32101138086804e-06
9.99998143191111e-01	1.85680888920015e-06
9.99998514553027e-01	1.48544697342601e-06
9.99998811642509e-01	1.18835749052248e-06
9.99999049314064e-01	9.50685935974249e-07

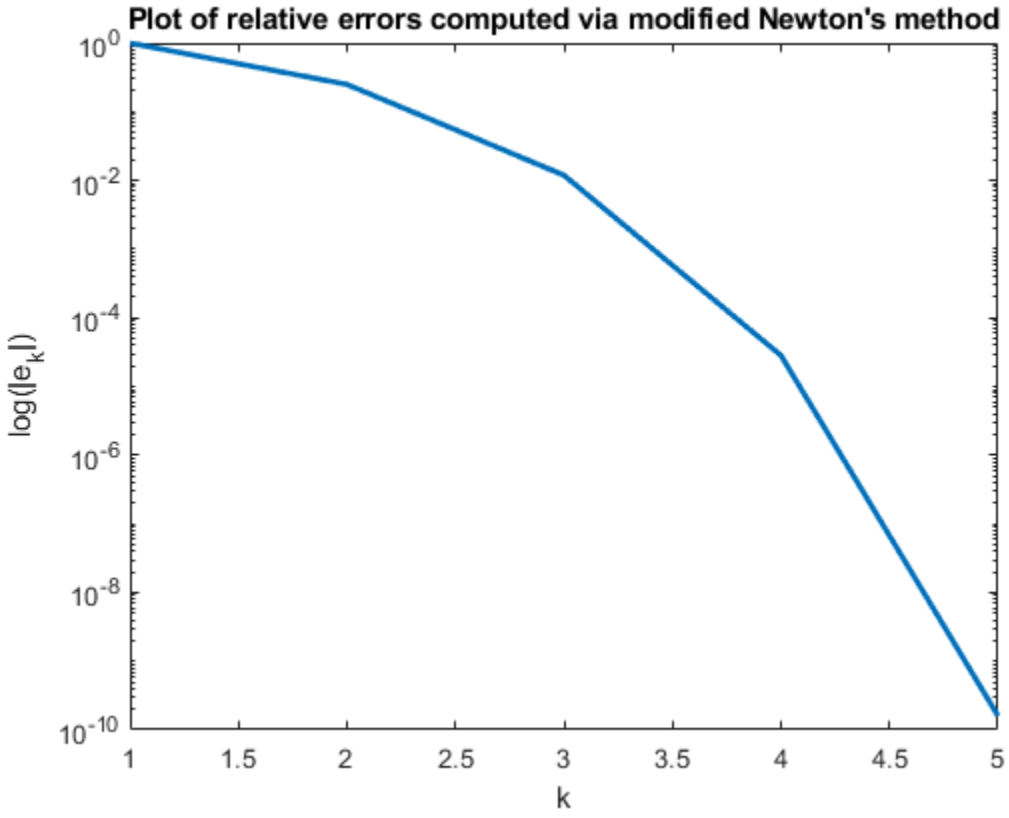
9.99999239451287e-01	7.60548712674947e-07
9.99999391561053e-01	6.08438947002909e-07
9.99999513248857e-01	4.86751142836361e-07
9.99999610599095e-01	3.89400904743376e-07
9.99999688479282e-01	3.11520717777292e-07
9.99999750783430e-01	2.49216570380462e-07
9.99999800626746e-01	1.99373253795265e-07
9.99999840501399e-01	1.59498601437491e-07
9.99999872401120e-01	1.27598880128588e-07
9.99999897920896e-01	1.02079103503350e-07
9.99999918336718e-01	8.16632823585905e-08
9.99999934669374e-01	6.53306255760100e-08
9.99999947735500e-01	5.22645002831723e-08
9.99999958188400e-01	4.18116000711066e-08
9.99999966550720e-01	3.34492800124764e-08
9.99999973240576e-01	2.67594240099811e-08
9.99999978592461e-01	2.14075391857804e-08
9.99999982873969e-01	1.71260313708288e-08
9.99999986299175e-01	1.37008250300497e-08
9.99999989039340e-01	1.09606600462442e-08
9.99999991231472e-01	8.76852801479089e-09
9.99999992985178e-01	7.01482238962825e-09
9.99999994388142e-01	5.61185786729368e-09
9.99999995510514e-01	4.48948633824386e-09
9.99999996408411e-01	3.59158902618617e-09
9.99999997126729e-01	2.87327117654002e-09
9.99999997701383e-01	2.29861696343647e-09
9.99999998161106e-01	1.83889359295364e-09
9.99999998528885e-01	1.47111489656737e-09
9.99999998823108e-01	1.17689191725390e-09
9.99999999058486e-01	9.41513533803118e-10
9.99999999246789e-01	7.53210827042494e-10
9.99999999397431e-01	6.02568661633995e-10
9.99999999517945e-01	4.82054951511657e-10
9.99999999614356e-01	3.85643961209325e-10
9.99999999691485e-01	3.08515213376381e-10
9.99999999753188e-01	2.46812126292184e-10
9.99999999802550e-01	1.97449723238208e-10
9.99999999842040e-01	1.57959756386106e-10
9.99999999873632e-01	1.26367805108885e-10
9.99999999898906e-01	1.01094244087108e-10
9.99999999919125e-01	8.08754174741466e-11
9.99999999935300e-01	6.47003561837778e-11
9.99999999948240e-01	5.17602627425617e-11
9.99999999958592e-01	4.14082101940494e-11
9.99999999966873e-01	3.31266125641605e-11
9.99999999973499e-01	2.65012456424074e-11
9.99999999978799e-01	2.12010409228469e-11
9.99999999983039e-01	1.69608771471985e-11
9.99999999986431e-01	1.35687017177588e-11
9.99999999989145e-01	1.08549835786675e-11
9.99999999991316e-01	8.68394245401305e-12
9.99999999993053e-01	6.94710955428945e-12
9.99999999994442e-01	5.55766543897107e-12

9.99999999995554e-01	4.44611014671636e-12
9.99999999996443e-01	3.55693252629408e-12
9.99999999997154e-01	2.84550161211428e-12
9.99999999997724e-01	2.27640128969142e-12
9.99999999998179e-01	1.82109882729264e-12
9.99999999998543e-01	1.45683465291313e-12
9.99999999998834e-01	1.16551213125149e-12
9.99999999999068e-01	9.32365296080206e-13
9.99999999999254e-01	7.45847827943180e-13
9.99999999999403e-01	5.96633853433559e-13
9.99999999999523e-01	4.77284878286355e-13
9.99999999999618e-01	3.81805698168591e-13
9.99999999999695e-01	3.05422354074381e-13
9.99999999999756e-01	2.44360087719997e-13
9.99999999999804e-01	1.95510274636490e-13
9.99999999999844e-01	1.56430424169685e-13
9.99999999999875e-01	1.25122134875255e-13
9.99999999999900e-01	1.00142116821189e-13
9.99999999999920e-01	8.01581023779363e-14
9.99999999999936e-01	6.41708908233340e-14
9.99999999999949e-01	5.12923037376822e-14
9.99999999999959e-01	4.10782519111308e-14
9.99999999999967e-01	3.28626015289046e-14
9.99999999999974e-01	2.63122856836162e-14
9.99999999999979e-01	2.10942374678780e-14
9.99999999999983e-01	1.68753899743024e-14
9.99999999999986e-01	1.35447209004269e-14
9.99999999999989e-01	1.08801856413265e-14
9.99999999999991e-01	8.65973959207622e-15
9.99999999999993e-01	6.88338275267597e-15
9.99999999999994e-01	5.55111512312578e-15
9.99999999999996e-01	4.44089209850063e-15
9.99999999999996e-01	3.55271367880050e-15

Root computed via modified Newton's method: 1.0000000000000000 (6 iterations, x_0 = 0.0000000000000000)
Relative error between approximate root and 1: 0.0000000000000000e+00

Modified_Newton_iterates	Modified_Newton_relative_errors
0.00000000000000e+00	1.00000000000000e+00
1.25000000000000e+00	2.50000000000000e-01
1.01190476190476e+00	1.19047619047619e-02
1.00002827734419e+00	2.82773441917517e-05
1.00000000015992e+00	1.59920743314501e-10
1.00000000000000e+00	0.00000000000000e+00


Plot of relative errors computed via Newton's method
10^{0}
10^{-5}
10^{-10}
10^{-15}
log(|e_k|)
0
50
100
150
k


Plot of relative errors computed via modified Newton's method
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
log(|e_k|)
1
1.5
2
2.5
3
3.5
4
4.5
5
k

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