```
clc;
clear;

n_max = 16;
max_iters = 100;

actual_evals = zeros(n_max-1, 1);
dom_evals    = zeros(n_max-1, 1);
dom_evecs    = zeros(n_max-1, n_max-1);

for n = 2:n_max
    H = hilb(n);

    [V, D] = eigs(H);

    z_0 = zeros(length(H), 1); z_0(1) = 1;
    [lambda, v] = power_method(H, z_0, max_iters);

    actual_evals(n-1) = D(1, 1);
    dom_evals(n-1)    = lambda;
    dom_evecs(1:n, n-1) = v;

end

disp("Actual eigenvalues and computed eigenvalues of Hilbert matrix");
fprintf("\t\tn\t\t\t\tActual eigenvalues\t Computed eigenvalues\n");
disp([(2:n_max)', actual_evals, dom_evals]);
fprintf("\n\nCorresponding computed eigenvectors\n");
disp(dom_evecs);

% Run the modified power iteration to obtain the smallest eigenvalues of
% the Hilbert matrix

actual_smallest_evals = zeros(n_max-1, 1);
smallest_evals        = zeros(n_max-1, 1);

disp("Actual smallest eigenvalues and computed smallest eigenvalues of Hilbert
 matrix");
fprintf("\t\tn\t\t\t\tActual smallest eigenvalues\t Computed smallest
 eigenvalues\n");

for n = 2:n_max
    H = hilb(n);
    [V, D] = eigs(H, n);

    z_0 = zeros(length(H), 1); z_0(1) = 1;
    [lambda_2, ~] = power_method_inv(H, z_0, max_iters);

    disp([n, lambda_2, min(diag(D))]);
end

%
```

```
% A = diag([1, -1, 0.5]);
% [l_1, v_1] = power_method(A^2, max_iters);
% [l_2, v_2] = power_method(A^2, max_iters);
%
% disp([v_1, v_2]);

% A = rand(9);
% A = triu(A) - diag(diag(A)) + diag([1, 1, 1, 1, 1, 1, 1, 1, 1e-2]);
%
% V = zeros(length(A)-1, 1);
% D = zeros(length(A), 8);
% I = eye(size(A));
%
% for i = 1:8
%     z_0 = I(:, i);
%     [lambda, v] = power_method(A, z_0, max_iters + 1000);
%     V(i) = lambda;
%     D(:, i) = v;
% end
%
%
% disp([A*D(:, 2), V(2)*D(:, 2)]);


function [lambda, v] = power_method(A, z_0, max_iters)

    for k = 1:max_iters
        z_k = A*z_0;
        q_k = z_k/norm(z_k);
        lambda = dot(q_k, A*q_k);
        z_0 = z_k;
    end

    v = q_k;
end

function [lambda, v] = power_method_inv(A, z_0, max_iters)
    [largest_e_val, ~] = power_method(A, z_0, max_iters);

    A_tilde = A - largest_e_val*eye(size(A));

    for k = 1:max_iters
        z_k = A_tilde*z_0;
        q_k = z_k/norm(z_k);
        lambda = dot(q_k, A_tilde*q_k);
        z_0 = z_k;
    end

    lambda = lambda + largest_e_val;
    v = q_k;
end

Actual eigenvalues and computed eigenvalues of Hilbert matrix
  n    Actual eigenvalues  Computed eigenvalues
```

```
   2.000000000000000   1.267591879243998   1.267591879243999
   3.000000000000000   1.408318927123654   1.408318927123654
   4.000000000000000   1.500214280059243   1.500214280059243
   5.000000000000000   1.567050691098231   1.567050691098231
   6.000000000000000   1.618899858924339   1.618899858924339
   7.000000000000000   1.660885338926931   1.660885338926931
   8.000000000000000   1.695938996921948   1.695938996921950
   9.000000000000000   1.725882660901847   1.725882660901847
  10.000000000000000   1.751919670265177   1.751919670265178
  11.000000000000000   1.774883179499381   1.774883179499381
  12.000000000000000   1.795372059561997   1.795372059561997
  13.000000000000000   1.813830118796977   1.813830118796977
  14.000000000000000   1.830594695920393   1.830594695920394
  15.000000000000000   1.845927746153488   1.845927746153487
  16.000000000000000   1.860036442743326   1.860036442743327



Corresponding computed eigenvectors
  Columns 1 through 3

   0.881674598767944   0.827044926972009   0.792608291163764
   0.471857925532024   0.459863904365544   0.451923120901600
                   0   0.323298435244499   0.322416398581825
                   0                   0   0.252161169688242
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0

  Columns 4 through 6

   0.767854735065807   0.748719218879095   0.733225603080613
   0.445791060462709   0.440717503243512   0.436359150069654
   0.321578294480220   0.320696869822252   0.319779114044051
   0.253438943245175   0.254311386340474   0.254885556321454
   0.209822636563631   0.211530840078965   0.212844074668574
                   0   0.181442976648769   0.183143115876329
                   0                   0   0.160939670445336
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
```

```
                  0                   0                   0
                  0                   0                   0

Columns 7 through 9

 0.720271369439766   0.709181673545698   0.699514891247007
 0.432526015446475   0.429099404025192   0.425998912750593
 0.318843646420194   0.317905990066158   0.316976987916454
 0.255242887149454   0.255441586730359   0.255523005939058
 0.213861951108334   0.214656002709988   0.215277839514775
 0.184519788905940   0.185646845839823   0.186578238283591
 0.162514374457332   0.163832328894961   0.164946526309270
 0.145343694357308   0.146769904688278   0.147992143235347
                 0   0.133026481716317   0.134310446361981
                 0                   0   0.123016712833186
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0

Columns 10 through 12

 0.690967022219495   0.683320332980505   0.676413810285274
 0.423166915294328   0.420560375555992   0.418146135572765
 0.316063689551723   0.315170440634100   0.314299742629568
 0.255516792083251   0.255444491435235   0.255321987188084
 0.215765222817530   0.216146230854872   0.216442076211893
 0.187353973498587   0.188004207938112   0.188552035725572
 0.165896666582998   0.166712975590447   0.167418826249705
 0.149048683067852   0.149968894636363   0.150775661872475
 0.135430727654193   0.136415432471856   0.137286595261618
 0.124173014540756   0.125196312973031   0.126107640595545
 0.114697309959483   0.115741728398817   0.116676703819235
                 0   0.107656409926997   0.108603548511526
                 0                   0   0.101607996027037
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0

Columns 13 through 15

 0.670125208584176   0.664359644626561   0.659042077332539
 0.415898015563803   0.413794933147330   0.411819631422644
 0.313452862688510   0.312630251060571   0.311831823367739
 0.255161147889428   0.254970953283049   0.254758273966824
 0.216669024253455   0.216839717460734   0.216964104228668
 0.189015404362822   0.189408449910796   0.189742442594352
 0.168032553618079   0.168568730094388   0.169039075659429
 0.151487066617891   0.152117579820088   0.152678917240549
 0.138061729038196   0.138754928128587   0.139377664467761
 0.126923795796249   0.127658360865445   0.128322440217573
 0.117518265401015   0.118279424744030   0.118970859915509
```

```
   0.109459554540885   0.110236853081041   0.110945674034342
   0.102470633381607   0.103256551896694   0.103975532710293
   0.096346867680367   0.097135934951716   0.097859772735758
                   0   0.091721739694777   0.092446507883094
                   0                   0   0.087618467755674

Actual smallest eigenvalues and computed smallest eigenvalues of Hilbert
 matrix
  n    Actual smallest eigenvalues  Computed smallest eigenvalues
   2.000000000000000   0.065741454089335   0.065741454089335

   3.000000000000000   0.002687381641412   0.002687340355773

   4.000000000000000   0.006336016073612   0.000096702304023

   5.000000000000000   0.009521353380193   0.000003287928772

   6.000000000000000   0.011164257025210   0.000000108279948

   7.000000000000000   0.011014567912347   0.000000003493899

   8.000000000000000   0.009736752934525   0.000000000111154

   9.000000000000000   0.008209232928134   0.000000000003500

  10.000000000000000   0.006950412341298   0.000000000000109

  11.000000000000000   0.006103781087902   0.000000000000003

  12.000000000000000   0.005628391022755   0.000000000000000

  13.000000000000000   0.005433514672554   0.000000000000000

  14.000000000000000   0.005433855752781  -0.000000000000000

  15.000000000000000   0.005563536430085  -0.000000000000000

  16.000000000000000   0.005775344100157  -0.000000000000000
```

Published with MATLAB® R2021b