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In[21]:= AB2Roots = z /. Solve[z^(2) - (1 + (3/2) * r) * z + (1/2) * r == 0, z, Complexes]
```

```
Out[21]=  $\left\{ \frac{1}{4} \left( 2 + 3r - \sqrt{4 + 4r + 9r^2} \right), \frac{1}{4} \left( 2 + 3r + \sqrt{4 + 4r + 9r^2} \right) \right\}$ 
```

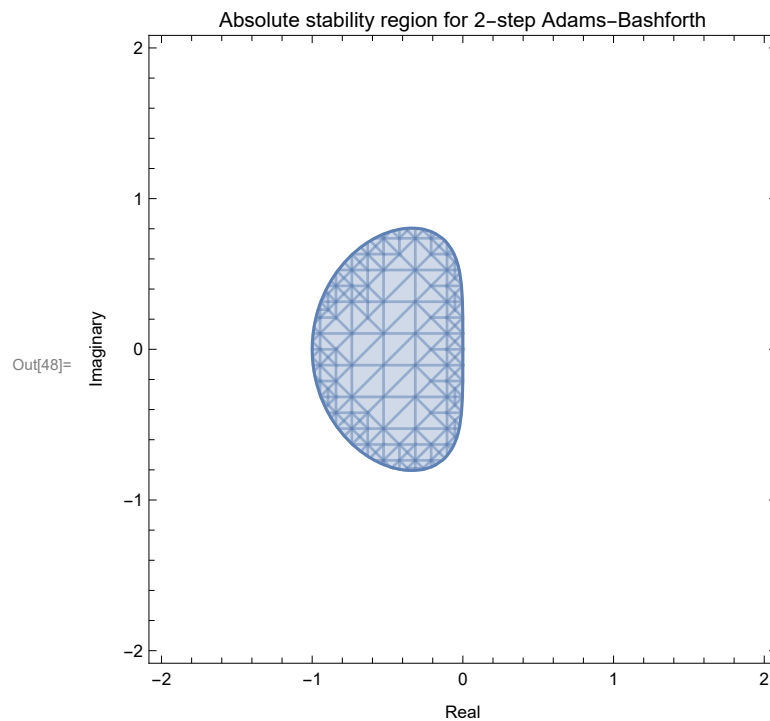
```
In[12]:= AB2Roots1 = AB2Roots[[1]]
```

```
AB2Roots2 = AB2Roots[[2]]
```

```
Out[12]=  $\frac{1}{4} \left( 2 + 3r - \sqrt{4 + 4r + 9r^2} \right)$ 
```

```
Out[13]=  $\frac{1}{4} \left( 2 + 3r + \sqrt{4 + 4r + 9r^2} \right)$ 
```

```
In[48]:= ComplexRegionPlot[{Abs[AB2Roots1] ≤ 1 && Abs[AB2Roots2] ≤ 1},
  {r, 2}, FrameLabel → {"Real", "Imaginary"},
  PlotLabel → "Absolute stability region for 2-step Adams-Bashforth"]
```



In[18]:= **AB3Roots =****z /. Solve[z^ (3) - (1 + (23 / 12) * r) * z^ (2) + (4 / 3) * r * z - (5 / 12) * r == 0, z, Complexes]**

$$\text{Out[18]} = \left\{ \frac{1}{36} (12 + 23 r) - \frac{4 \left(-1 + \frac{r}{6} - \frac{529 r^2}{144} \right)}{\left(1728 + 9288 r - 828 r^2 + 12167 r^3 + 108 \sqrt{r} \sqrt{2880 + 4308 r + 3228 r^2 + 8993 r^3} \right)^{1/3}} + \right.$$

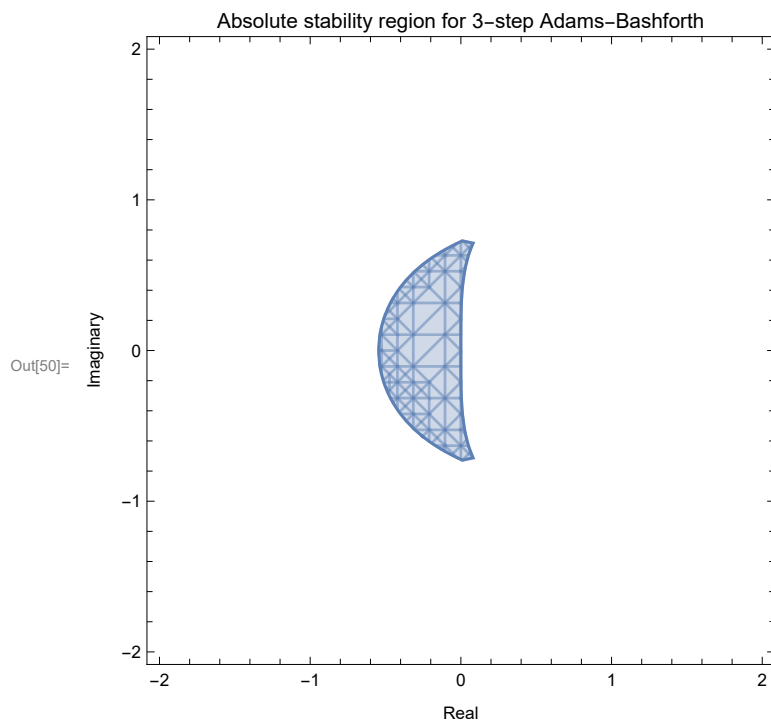
$$\frac{1}{36} \left(1728 + 9288 r - 828 r^2 + 12167 r^3 + 108 \sqrt{r} \sqrt{2880 + 4308 r + 3228 r^2 + 8993 r^3} \right)^{1/3},$$

$$\frac{1}{36} (12 + 23 r) + \frac{2 (1 + i \sqrt{3}) \left(-1 + \frac{r}{6} - \frac{529 r^2}{144} \right)}{\left(1728 + 9288 r - 828 r^2 + 12167 r^3 + 108 \sqrt{r} \sqrt{2880 + 4308 r + 3228 r^2 + 8993 r^3} \right)^{1/3}} -$$

$$\frac{1}{72} (1 - i \sqrt{3}) \left(1728 + 9288 r - 828 r^2 + 12167 r^3 + 108 \sqrt{r} \sqrt{2880 + 4308 r + 3228 r^2 + 8993 r^3} \right)^{1/3},$$

$$\frac{1}{36} (12 + 23 r) + \frac{2 (1 - i \sqrt{3}) \left(-1 + \frac{r}{6} - \frac{529 r^2}{144} \right)}{\left(1728 + 9288 r - 828 r^2 + 12167 r^3 + 108 \sqrt{r} \sqrt{2880 + 4308 r + 3228 r^2 + 8993 r^3} \right)^{1/3}} -$$

$$\left. \frac{1}{72} (1 + i \sqrt{3}) \left(1728 + 9288 r - 828 r^2 + 12167 r^3 + 108 \sqrt{r} \sqrt{2880 + 4308 r + 3228 r^2 + 8993 r^3} \right)^{1/3} \right\}$$

In[50]:= **ComplexRegionPlot[****{Abs[AB3Roots[[1]]] ≤ 1 && Abs[AB3Roots[[2]]] ≤ 1 && Abs[AB3Roots[[3]]] ≤ 1},****{r, 2}, FrameLabel → {"Real", "Imaginary"},****PlotLabel → "Absolute stability region for 3-step Adams-Bashforth"]**

In[27]:= **AM2Roots =**

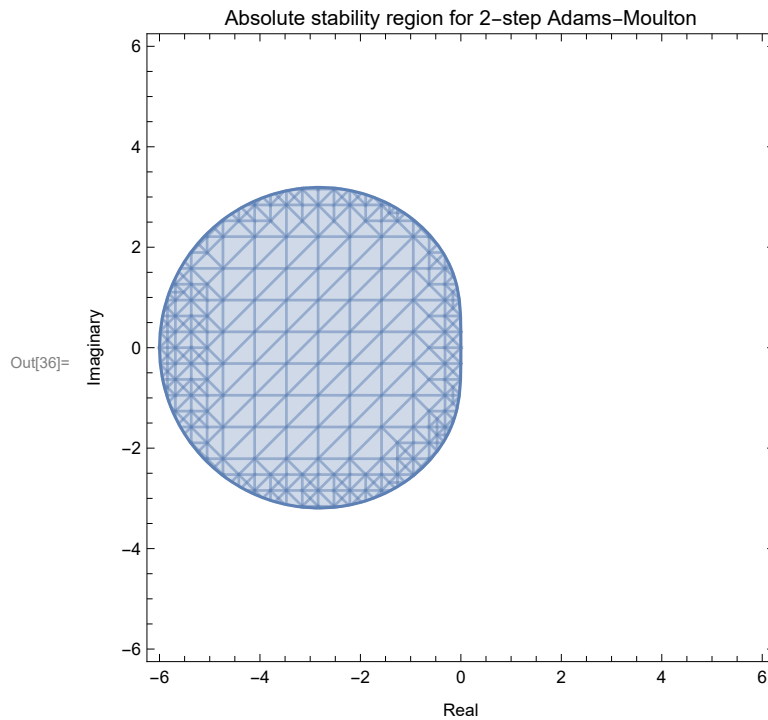
z /. Solve[(1 - (5 / 12) * r) * z^(2) - (1 + (2 / 3) * r) * z + (1 / 12) * r == 0, z, Complexes]

$$\text{Out[27]} = \left\{ \frac{-6 - 4r - \sqrt{3} \sqrt{12 + 12r + 7r^2}}{-12 + 5r}, \frac{-6 - 4r + \sqrt{3} \sqrt{12 + 12r + 7r^2}}{-12 + 5r} \right\}$$

In[36]:= **ComplexRegionPlot[Abs[AM2Roots[[1]]] ≤ 1 && Abs[AM2Roots[[2]]] ≤ 1,**

{r, 6}, FrameLabel → {"Real", "Imaginary"},

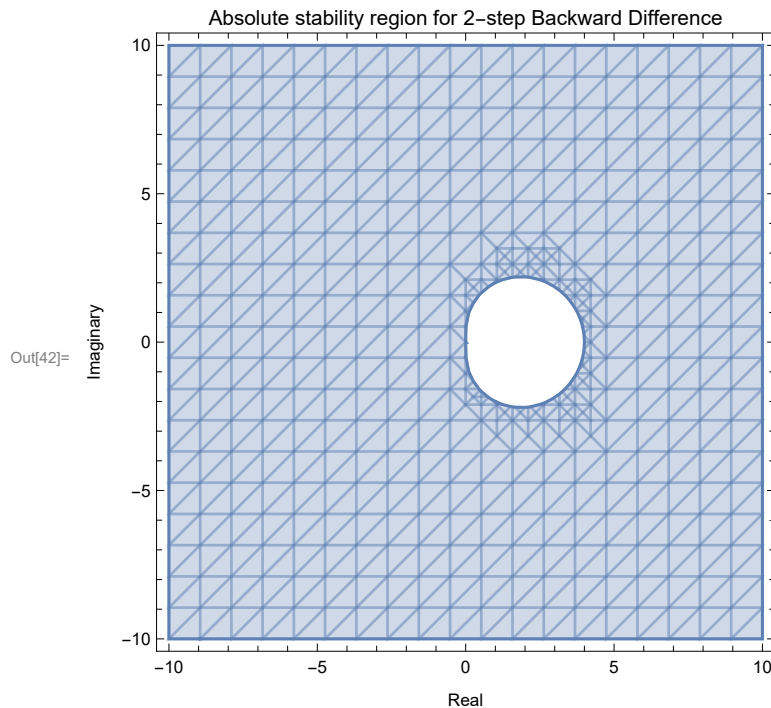
PlotLabel → "Absolute stability region for 2-step Adams-Moulton"]



In[37]:= **BDF2Roots = z /. Solve[(1 - (2 / 3) * r) * z^(2) - (4 / 3) * z + (1 / 3) == 0, z, Complexes]**

$$\text{Out[37]} = \left\{ \frac{-2 - \sqrt{1 + 2r}}{-3 + 2r}, \frac{-2 + \sqrt{1 + 2r}}{-3 + 2r} \right\}$$

```
In[42]:= ComplexRegionPlot[{Abs[BDF2Roots[[1]]] ≤ 1 && Abs[BDF2Roots[[2]]] ≤ 1},
  {r, 10}, FrameLabel → {"Real", "Imaginary"},
  PlotLabel → "Absolute stability region for 2-step Backward Difference"]
```



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In[39]:= BDF3Roots = z /. 
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```
Solve[(1 - (6 / 11) * r) * z^(3) - (18 / 11) * z^(2) + (9 / 11) * z - (2 / 11) == 0, z, Complexes]
```

Out[39]=

$$\left\{ \frac{6}{11 - 6r} - \frac{-27 - 162r}{9(11 - 6r) \left(40 + 30r + 36r^2 + \sqrt{1573 + 1914r + 864r^2 - 3672r^3 + 1296r^4} \right)^{1/3}} + \frac{\left(40 + 30r + 36r^2 + \sqrt{1573 + 1914r + 864r^2 - 3672r^3 + 1296r^4} \right)^{1/3}}{11 - 6r}, \right.$$

$$\frac{6}{11 - 6r} + \frac{(1 + i\sqrt{3})(-27 - 162r)}{18(11 - 6r) \left(40 + 30r + 36r^2 + \sqrt{1573 + 1914r + 864r^2 - 3672r^3 + 1296r^4} \right)^{1/3}} -$$

$$\frac{(1 - i\sqrt{3}) \left(40 + 30r + 36r^2 + \sqrt{1573 + 1914r + 864r^2 - 3672r^3 + 1296r^4} \right)^{1/3}}{2(11 - 6r)},$$

$$\left. \frac{6}{11 - 6r} + \frac{(1 - i\sqrt{3})(-27 - 162r)}{18(11 - 6r) \left(40 + 30r + 36r^2 + \sqrt{1573 + 1914r + 864r^2 - 3672r^3 + 1296r^4} \right)^{1/3}} - \frac{(1 + i\sqrt{3}) \left(40 + 30r + 36r^2 + \sqrt{1573 + 1914r + 864r^2 - 3672r^3 + 1296r^4} \right)^{1/3}}{2(11 - 6r)} \right\}$$

```
In[41]:= ComplexRegionPlot[
  {Abs[BDF3Roots[[1]]] ≤ 1 && Abs[BDF3Roots[[2]]] ≤ 1 && Abs[BDF3Roots[[3]]] ≤ 1},
  {r, 10}, FrameLabel → {"Real", "Imaginary"},
  PlotLabel → "Absolute stability region for 3-step Backward Difference"]
```

LessEqual: Internal precision limit \$MaxExtraPrecision = 50.` reached while evaluating

$$\frac{5}{11} - \frac{3}{11(40 + 11\sqrt{13})^{1/3}} - \frac{1}{11}(40 + 11\sqrt{13})^{1/3}.$$

