CAAM 520, Spring 2020 - Problem Set 4

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NOTE: Change `PETSC_DIR` in `makefile` to match your PETSc installation directory!

1. *Generalized Jacobi method with PETSc*

   (a) See code.

   (b) See code.

   (c) The following are the number of iterations each test took, differentiated by the linear system solver:

      i. **Jacobi method**

         A. `nasa2146` with $\omega = 0.25$ and a tolerance of 10^{-6}: 6541 iterations

         B. `plbuckle.bin` with $\omega = 0.6$ and a tolerance of 10^{-6}: 21778 iterations

         C. `bcsstk08.bin` with $\omega = 0.7$ and a tolerance of 10^{-6}: 9108 iterations

      ii. **Conjugate gradient**

         A. 1 process

         - `nasa2146` with $\omega = 0.25$ and a tolerance of 10^{-6}: 11 iterations
         - `plbuckle.bin` with $\omega = 0.6$ and a tolerance of 10^{-6}: 3 iterations
         - `bcsstk08.bin` with $\omega = 0.7$ and a tolerance of 10^{-6}: 24 iterations

         B. 2 processes

         - `nasa2146` with $\omega = 0.25$ and a tolerance of 10^{-6}: 40 iterations
         - `plbuckle.bin` with $\omega = 0.6$ and a tolerance of 10^{-6}: 70 iterations
         - `bcsstk08.bin` with $\omega = 0.7$ and a tolerance of 10^{-6}: 58 iterations

         C. 3 processes

         - `nasa2146` with $\omega = 0.25$ and a tolerance of 10^{-6}: 48 iterations
         - `plbuckle.bin` with $\omega = 0.6$ and a tolerance of 10^{-6}: 77 iterations
         - `bcsstk08.bin` with $\omega = 0.7$ and a tolerance of 10^{-6}: 102 iterations

         D. 4 processes

         - `nasa2146` with $\omega = 0.25$ and a tolerance of 10^{-6}: 55 iterations
         - `plbuckle.bin` with $\omega = 0.6$ and a tolerance of 10^{-6}: 92 iterations
         - `bcsstk08.bin` with $\omega = 0.7$ and a tolerance of 10^{-6}: 105 iterations

      In running the above numerical tests, the Jacobi iteration took the same number of iterations regardless of the number of processes used, whereas the conjugate gradient method took more iterations as the number of processes increased. However, the conjugate gradient method vastly outperformed the Jacobi iteration in all four cases, which was to be expected as the Jacobi iteration is typically very slow.

2. *Inverse iteration*

(a) See code.

(b) The following are the results of running `inverse_iteration` on `plbuckle.bin` for various values of μ and a tolerance of 10^{-3}:

i. $\mu = 0.0$:
   A. $\lambda \approx 1.634155$
   B. 4 iterations

ii. $\mu = 2.5$:
   A. $\lambda \approx 1.634155$
   B. 3 iterations

iii. $\mu = 5$:
   A. $\lambda \approx 6.415979$
   B. 17 iterations

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iv. $\mu = 7.5$:
   A. $\lambda \approx 6.415979$
   B. 9 iterations

,

v. $\mu = 10$:
   A. $\lambda \approx 6.415979$
   B. 38 iterations

We see that the first two choices of μ lead to the same eigenvalue approximation of 1.634155, but the choice of $\mu = 2.5$ results in one fewer iteration than the choice of $\mu = 0$. Similarly, the last three choices of μ lead to the same eigenvalue approximation of 6.415979, but the choice of $\mu = 7.5$ takes the fewest number of iterations to converge, the choice of $\mu = 5$ takes the second fewest number of iterations to converge, and the choice of $\mu = 10$ takes the most number of iterations to converge.

It appears that the inverse iteration takes fewer iterations to converge the closer μ is to the eigenvalue we want to approximate, and this is evident from the above numerical results. In the case of the first eigenvalue approximation, $\mu = 2.5$ is closer to the approximate eigenvalue 1.634155 than $\mu = 0$ is to said eigenvalue. The number of iterations grow correspondingly.

In the case of the second eigenvalue approximation, $\mu = 7.5$ is closest to the approximate eigenvalue 6.415979, $\mu = 5$ is the second closest to the said eigenvalue, and $\mu = 10$ is the furthest from the same eigenvalue. The number of iterations grows correspondingly.