Transport by Lagrangian Vortices in the Eastern Pacific

Ryan Abernathey*

Columbia University, New York, New York

George Haller

ETH, Switzerland

**Corresponding author address:* Ryan Abernathey, Lamont Doherty Earth Observatory of Columbia University, 205C Oceanography, 61 Route 9W - PO Box 1000, Palisades, NY 10964-8000, US.

E-mail: rpa@ldeo.columbia.edu

ABSTRACT

1. Introduction

The mesoscale (roughly 10 - 500 km) is the most energetic scale in the ocean (Wortham and Wunsch 2014). Phenomenologically, the mesoscale comprises a disorderly jumble of waves, vortices, fronts, and filaments, and the word mesoscale frequently appears together with the word "eddy." However, a survey of the literature reveals a wide range of definitions of "eddy," which is used as both an adjective and a noun. The standard Eulerian statistical perspective defines "eddy" (an adjective) simply as a fluctuation about an Eulerian time and / or spatial mean state. The "coherent structure" perspective attempts to identify specific, discrete "eddies" (a noun) and track them through the ocean. This contribution seeks to clarify the relationship between Eulerian eddy fluxes and coherent structures.

Eulerian mesoscale eddy fluxes (i.e. statistical correlations between velocity and tracer fluctuations, a.k.a. Reynolds fluxes) play a significant role in the transport of heat, salt, momentum and other tracers through the ocean. Because climate models generally do not resolve the mesoscale, the sub-gridscale mesoscale flux must be parameterized based on the large-scale flow properties, commonly using a diffusive closure (Gent et al. 1995; Treguier et al. 1997; Visbeck et al. 1997; Vollmer and Eden 2013; Bachman and Fox-Kemper 2013). This important problem has motivated many studies of Eulerian eddy fluxes (and associated diffusivities) in observations and eddy-resolving models (e.g. Morrow et al. 1992; Stammer 1998; Roemmich and Gilson 2001; Jayne and Marotzke 2002; Volkov et al. 2008; Fox-Kemper et al. 2012; Abernathey and Marshall 2013; Klocker and Abernathey 2014; Abernathey and Wortham 2015). This work has been largely unconcerned with coherent structures, although Abernathey and Wortham (2015) did note the overlap between eddy flux spectral characteristics and the lengths scales / propagation speeds of coherent mesoscale eddies.

Many different methods have been used to identify coherent structures (CSs). These methods fall into two general categories: Eulerian[1] (based on instantaneous features of the velocity field) and Lagrangian (based on time-dependent water parcel trajectories). Early Eulerian approaches used contours of the Okubo-Weiss parameter to identify the boundaries of eddies (Isern-Fontanet et al. 2003; **?**). More recently, closed contours of the sea-surface height anomaly (SSH) field have been employed (Chelton et al. 2011, henceforth CSS11). The "eddy census" of CSS11 has been widely adopted by the community, likely due to its open publication on the web. Other recent Eulerian CS eddy census products include **?** and **?**. While these methods differ in certain details, they are all fundamentally similar in that they use the instantaneous velocity field (or streamfunction) to identify eddies at each snapshot in time, and then track these features from one snapshot to the next.

This Eulerian approach to eddy tracking has recently been challenged by researchers from the field of dynamical systems theory (see Haller 2015, for a review). The essence of the critique is that the structures identified are not frame invariant (a different observer might find different structures) and that the criteria are not objective (they depend on arbitrary parameters or thresholds). Most seriously from the perspective of transport, it has been shown that OW and SSHA eddies are not actually *materially coherent*; under Lagrangian advection, the supposed eddy boundaries become rapidly strained and filamented, implying that water does not actually remain trapped inside the coherent structure "boundary" (**?**).

Contradictions may therefore arise when such Eulerian eddy tracking methods are applied to infer material transport, as in two recent studies. Dong et al. (2014) used Eulerian eddy tracking, together with vertical structure functions of potential temperature and salinity derived statistically from ARGO profiles, to estimate the heat and salt content materially "trapped" inside the eddies.

[1] An *Eulerian method* for identifying coherent structures should not be confused with the *Eulerian* eddy flux.

By assuming no exchange with the surrounding environment for the duration of the eddy lifetime, they estimated the meridional fluxes of heat and salt on a global scale, reaching the conclusion that "...eddy heat and salt transports are mainly due to individual eddy movements." Zhang et al. (2014) used a similar method to estimate the eddy mass flux. They employed tracked Eulerian eddies together with vertical structure functions to estimate the potential vorticity field surrounding the eddies. The outermost closed potential vorticity contour was assumed to constitute an impermeable material boundary for the duration of each tracked eddy, and the eddy motion was thereby translated to a mass flux. This method estimated the westward zonal eddy mass flux in the subtropical gyre regions to be approx. 30 Sv, a surprisingly large number which is comparable to the gyre transport itself. These approaches are quite appealing because they reduce the expensive problem of observing the turbulent ocean at high spatial and temporal frequency to the more tractable one of identifying and tracking a finite number of coherent eddies. However, the work of **???** suggests that these methods strongly over-estimate the degree of material coherence in mesoscale eddies, calling into question the findings.

The goal of this study is to make a more accurate estimate of material transport due to ocean mesoscale eddies using an objective Lagrangian eddy detection method applied to surface velocity fields derived from satellite altimetry. In particular, we apply the recently introducted Rotationally Coherent Lagrangian Vortex (RCLV) definition developed by Haller et al. (2016) and further explored by Farazmand and Haller (2016). The key difference between our approach and the Eulerian methods of Dong et al. (2014) and Zhang et al. (2014) is that, by numerically advecting a dense mesh of millions of Lagrangian particles, we demonstrate (rather than assume) that our identified vortices actually remain materially coherent throughout a finite time interval. Furthermore, the full Lagrangian trajectories also allow us to estimate the more broadly-defined "eddy flux" due to the entire range of turbulent motions in the flow. By comparing this full flux with the flux trapped inside coherent vortices, we obtain an estimate of the relative importance of material transport by coherent structures to the full turbulent transport. We consider the two-dimensional surface geostrophic flow as observed by satellite altimetry, as this is the only large-scale velocity observation which resolves mesoscale structures, which limits our ability to probe subsurface transport. Nevertheless, the results strongly support the conclusion that RCLVs make only a minimal contribution to meridional eddy transport.

The paper is organized as follows. In Sec. 2, we review the RCLV definition and the concepts of Lagrangian dispersion and diffusivity. In Sec. 3, we describe the satellite data and the numerical approach to Lagrangian particle advection. Sec. 4 provides some case studies of Lagrangian vortices identified by our algorithm and summarizes their statistics. In Sec. 5, we present the eddy diffusivity and the coherent eddy diffusivity. Sec. 6 contains discussion and conclusions.

2. Theory of Lagrangian Transport and Rotationally Coherent Vortices

a. Eulerian Eddy Flux and Lagrangian Diffusivity

Consider a conserved two-dimensional scalar $c(x,y)$ advected by a two-dimensional velocity field $\boldsymbol{u}(x,y)$ where $\boldsymbol{u} = (u,v)$. The time- and zonal-mean meridional flux of the scalar across a latitude circle in a sector of the ocean is given by $\overline{vc}$. The overbar represents the time and zonal average:

$$\overline{vc} = (L_x T)^{-1} \int_{x_0}^{x_0+L_x} \int_{t_0}^{t_0+T} vc dx \tag{1}$$

where L_x is the zonal extent of the sector and T is the averaging time period. For homogeneous, statistically stationary turbulent flow, Taylor (1921) identified the relationship between this flux and the Lagrangian statistics of the flow as

$$\overline{vc} = -K_{abs} \frac{\partial \bar{c}}{\partial y} \tag{2}$$

with

$$K_{abs} = \frac{1}{2}\frac{\partial}{\partial t}\overline{Y^2} \,. \tag{3}$$

Here $\overline{Y^2}$ represents the mean squared Lagrangian displacement of water parcels from their initial position; K, the growth rate of this RMS displacement, represents the "single particle" or "absolute" diffusivity (LaCasce 2008). Regardless of whether the flow statistics are truly diffusive or not, eqs. (2) and (**??**) represent the kinematic relationship between Lagrangian displacement and Eulerian flux. K expresses the fundamental transport properties of the flow, independently of the background gradient $\partial\overline{c}/\partial y$. Note that K_{abs} is *not* a Galilean-invariant diagnostic.

From an Eulerian perspective, the "eddy" component of the flux is readily identified via a standard Eulerian Reynolds decomposition: $\overline{vc} = \overline{v}\overline{c} + \overline{v'c'}$, where the prime indicates the instantaneous deviation from the Eulerian mean. The second term $\overline{v'c'}$ is commonly termed the "eddy flux." Taylor envisioned a homogeneous, isotropic turbulent flow with no mean component, i.e. $\overline{v} = 0$. In contrast, most geophysical flows have mean flows, and the mean advection can influence K_{abs}. To remove the effects of the mean flow in the Lagrangian frame, one can instead focus on the relative diffusivity (Batchelor 1952; Bennett 1984)

$$K_{rel} = \frac{1}{2}\frac{\partial}{\partial t}\overline{\left(Y - \overline{Y}\right)^2} \,, \tag{4}$$

which represents the growth rate of the second moment of the ensemble displacement. (Relative diffusivity can equivalently be calculated from pair separation statistics [LaCasce 2008].) An additional advantage of using K_{rel} is its Galilean invariance. A detailed discussion of the relationship between K_{abs}, K_{rel}, and the mixing of a passive tracer is given by (Klocker et al. 2012). For the purposes of this study, we shall take K_{rel} to be the most relevant diagnostics of net meridional eddy diffusion. Our goal is to identify the contribution of coherent Lagrangian eddies to K_{rel}.

b. Rotationally Coherent Lagrangian Vortices

In order to partition the transport defined in (3) and (4) into a contribution from coherent Lagrangian eddies, the domain must be divided into regions inside and outside a suitably defined eddy boundary. For this boundary to be robust, it must derive from an objective, frame-invariant definition. The identification of such boundaries in unsteady turbulent flows is the subject of much recent work from the field of dynamical systems, and several possible criteria exist (for a review see Haller 2015). We emphasize again that the Eulerian eddy identification methods of CSS12 are neither objective nor frame-invariant.

One possible criteria is to define eddy boundaries as elliptic Lagrangian coherent structures which experience minimal stretching over a finite-time interval (**?**).

There is no universally accepted definition of a coherent eddy...blah blah how much to say here?

3. Satellite Data and Particle Advection

a. AVISO Surface Geostrophic Velocities

b. Advection of Lagrangian Particles

4. Identification and Statistics of Lagrangian Vortices

a. Algorithm

b. Example Vortices

c. Vortex Statistics

5. Meridional Transport by Lagrangian Vortices

6. Conclusion

Acknowledgments.

APPENDIX B

File Structure of the AMS LaTeX Package

a. AMS LaTeX files

You will be provided with a tarred, zipped LaTeX package containing 17 files. These files are

Basic style file: ametsoc.cls.

The file ametsoc.cls is the manuscript style file.

- Using `\documentclass{ametsoc}` for your .tex document will generate a PDF that follows all AMS guidelines for submission and peer review.

- Using \documentclass[twocol]{ametsoc} for your .tex document can be used to generate a PDF that closely follows the layout of an AMS journal page, including single spacing and two columns. This journal style PDF is only for the author's personal use, and any papers submitted in this style will not be accepted.

Always use \documentclass{ametsoc} when generating a PDF for submission to the AMS.

Template: template.tex, for the author to use when making his/her paper. The file provides a basic blank template with some section headings for authors to easily enter their manuscript.

Sample .tex and .pdf files: The file amspaper.tex contains the LaTeX code for the sample file. The resulting PDF can be seen in amspaper.pdf (this file).

Sample article: article formatted in draft and two-column mode.

- AMSSamp1.tex, AMSSamp1.pdf

  Formal paper done in draft mode and the resulting .pdf.

- AMSSamp2.tex, AMSSamp2.pdf

  The same paper using the [twocol] option and the resulting .pdf.

- FigOne.pdf, FigTwo.pdf, and figure01.pdf are sample figures.

Bibliography Files: ametsoc2014.bst, database2014.bib, and references.bib.

- ametsoc2014.bst is the bibliography style file.

- database2014.bib is an example of a bibliographic database file.

- references.bib should be altered with your own bibliography information.

Documention: found in AMSDocs.pdf. Additional information found in readme.txt, which contains a list of the files and how they are used.

b. Help for Authors

Questions and feedback concerning the use of the AMS LaTeX files should be directed to latex@ametsoc.org. Additional information is available on the AMS LaTeX Submission Info web page (`http://www2.ametsoc.org/ams/index.cfm/publications/authors/journal-and-bams-authors/author-resources/latex-author-info/`).

APPENDIX C

Building a PDF and Submitting Your LaTeX Manuscript Files to the AMS

a. Building your own PDF

There are a variety of different methods and programs that will create a final PDF from your LaTeX files. The easiest method is to download one of the freely available text editors/compilers such as TexWorks or TeXnicCenter. TexWorks is installed with the TeXLive distribution and provides both a text editor and the ability to compile your files into a PDF.

b. Submitting your files to the AMS for peer review

The AMS uses the Editorial Manager system for all author submissions for peer review. Editorial Manager uses the freely available TeX Live 2011 distribution. This system will automatically generate a PDF from your submitted LaTeX files and figures.

You should not upload your own PDF into the system. If the system does not build the PDF from your files correctly, refer to the AMS LaTeX FAQ page first for possible solutions. If your PDF still does not build correctly after trying the solutions on the FAQ page, email latex@ametsoc.org for help.

c. Other software

As mentioned above, there is a variety of software that can be used to edit .tex files and build a PDF. The AMS does not support LaTeX-related WYSIWYG software, such as Scientific Workplace, or WYSIWYM software, such as LyX. TeX Live (available online at `http://www.tug.org/texlive/`) is recommended for users needing an up-to-date LaTeX distribution with software that includes an editor and the ability to automatically generate a PDF.

This shows how to enter the commands for making a bibliography using BibTeX. It uses references.bib and the ametsoc2014.bst file for the style.

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LIST OF TABLES

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LIST OF FIGURES

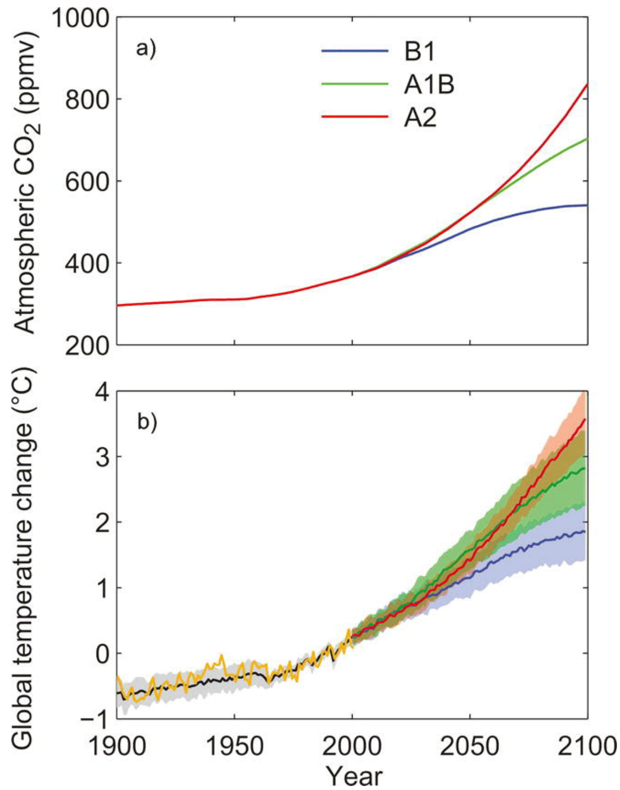

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Fig. A1. Here is the appendix figure caption.

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Fig. B1. Here is the appendix figure caption.