FELICITY

Finite ELement Implementation and
Computational Interface Tool for You

Class and Function Reference Guide

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Overview

This quick reference guide lists most of the functionality of **FELICITY** with *short* descriptions. More information about specific classes, class methods, or functions can be found by using MATLAB's `help` command. For example, typing the following at the MATLAB prompt

```
help MeshTriangle.Quality
```

and pressing `enter` will print the following text to the MATLAB display:

```
Quality

    This computes the quality metric of all elements in the mesh.  This uses the
    ratio of the inscribed sphere radius to circumscribed sphere radius.

    [Qual, PH] = obj.Quality(Num_Bins);

    Num_Bins = number of bins to use in histogram (optional argument).

    Qual = column vector of mesh element qualities.
    PH   = plot handle for the histogram (valid if Num_Bins > 0).
```

Therefore, if you want to know the details of how to create a class object, call a method, or execute a function, then use the MATLAB `help` command.

Mesh Classes

- `MeshInterval`: 1-D triangulations
- `MeshTriangle`: 2-D triangulations
- `MeshTetrahedron`: 3-D triangulations

Note: `MeshTriangle` and `MeshTetrahedron` are subclasses of `TriRep` (built-in MATLAB class). `MeshInterval` is a subclass of `EdgeRep` (1-D version of `TriRep`).

Methods For All Mesh Classes

- `Angles`: interior angles of each cell (element).
- `Append_Subdomain`: define sub-domain of mesh.
- `Bounding_Box`: the cartesian bounding box of the mesh vertices.
- `Create_Embedding_Data`: lower level version of `Generate_Subdomain_Embedding_Data`.
- `Create_Subdomain`: lower level version of `Append_Subdomain`.
- `Delete_Subdomain`: remove specific sub-domain from mesh.
- `Diameter`: computes the diameter of each mesh cell.
- `Generate_Subdomain_Embedding_Data`: return a struct containing embedding information for all sub-domains relative to other (given) sub-domains.
- `Geo_Dim`: return geometric dimension of mesh.
- `Get_Adjacency_Matrix`: adjacency matrix for all mesh vertices.
- `Get_Cell_Centers`: get the barycenter of all cells in the mesh.
- `Get_Facet_Info`: embedding and orientation info.
- `Get_Global_Subdomain`: return sub-domain connectivity referenced to global mesh vertex coord.
- `Get_Laplacian_Smoothing_Matrix`: output sparse matrix for "smoothing" the mesh.
- `Get_Subdomain_Cells`: return global mesh cell indices that contain given sub-domain's cells.
- `Get_Subdomain_Index`: internal use only.
- `Is_Subdomain`: indicates if the given sub-domain exists.
- `Num_Cell`: number of cells in global mesh.
- `Num_Vtx`: number of vertices in global mesh.
- `Output_Subdomain_Mesh`: return stand-alone mesh object representation of given sub-domain.
- `Plot`: default plot of global mesh.
- `Plot_Subdomain`: plot sub-domain of global mesh.
- `Quality`: compute quality metric of all mesh cells.
- `Refine`: adaptive refinement of mesh. (not implemented for 3-D meshes.)
- `Remove_Unused_Vertices`: removes all vertices not referenced by the triangulation's connectivity.
- `Reorder`: renumbers the global mesh vertices to give a tighter adjacency matrix.
- `Set_Points`: modify global vertex coordinates.
- `Top_Dim`: return topological dimension of mesh.
- `Volume`: compute volume of all cells in the mesh.
- `barycentricToCartesian`: convert point coordinates from barycentric to cartesian.
- `barycentricToReference`: convert point coordinates from barycentric to reference domain coordinates.
- `cartesianToBarycentric`: convert point coordinates from (global) cartesian to barycentric.
- `cartesianToReference`: convert point coordinates from (global) cartesian to reference domain.
- `circumcenter`: compute circumcenters of cells in the mesh.
- `edges`: return all edges in the triangulation.
- `freeBoundary`: return all "facets" referenced by only one mesh cell.
- `isConnected`: test if pair of vertices is joined by an edge.
- `nearestNeighbor`: vertex closest to a specified point.
- `neighbors`: return the mesh cell neighbor info.
- `pointLocation`: mesh cell containing specified point.
- `referenceToBarycentric`: convert point coordinates from reference domain coordinates to barycentric.
- `referenceToCartesian`: convert point coordinates from reference domain coordinates to cartesian.
- `size`: return size of triangulation connectivity.
- `vertexAttachments`: return mesh cells attached to specified vertices.

Methods For `MeshTriangle` And `MeshTetrahedron` Only

• `edgeAttachments`: return cells attached to specified edges.

• `incenter`: return incenters of specified cells.

Methods For `MeshTetrahedron` Only

• `faces`: return all triangle faces (facets) in the 3-D triangulation.

Methods For `MeshTriangle` Only

• `faceNormal`: return unit normal vectors to specified triangle cells of a surface triangulation.

• `featureEdges`: return sharp edges of a surface triangulation.

• `vertexNormal`: compute approximate normal vector at vertices of surface triangulation.

Methods For `MeshInterval` Only

• `Get_Arclength`: returns the local arc-length value at each vertex in the (space curve) mesh.

• `edgeTangent`: return unit tangent vectors to specified edge cells.

• `featurePoints`: return sharp points of a space curve mesh.

• `freeBoundary`: return all vertex indices referenced by only one mesh cell (edge).

• `vertexTangent`: compute approximate tangent vector at vertices of space curve mesh.

Note: a mesh cell is the basic unit of a mesh.

- For a 1-D mesh (MeshInterval), the cells are line segments (edges).
- For a 2-D mesh (MeshTriangle), the cells are triangles (faces).
- For a 3-D mesh (MeshTetrahedron), the cells are tetrahedra.

Finite Element Classes

- `FELSymBasisCalc`: for computing and storing symbolic expressions of basis functions.
- `FELSymFunc`: class for manipulating symbolic functions.
- `FEMatrixAccessor`: convenient access of finite element matrices by name.
- `FiniteElementSpace`: define and manipulate a finite element space.
- `GeoElementSpace`: define and manipulate a finite element space intended for higher order meshes.
- `PointSearchMesh`: for automating point searching on meshes.
- `ReferenceFiniteElement`: local finite element definition.

Note: we abbreviate finite element as **FE**, Degree-of-Freedom as **DoF**, and Degree-of-Freedom map as **DoFmap**.

Methods For `FELSymBasisCalc`

- `Compose_With_Function`: composes the basis functions (and all its derivatives) with a given function represented by `FELSymFunc`.
- `Fill_Eval`: evaluate basis function (and all its derivatives) at given points.
- `Get_Derivative`: returns a `FELSymFunc` object for the specific (multi-index) derivative you want.
- `Get_Length_Of_Multiindex`: returns the length of the "derivative multi-index" used to store basis function derivatives.
- `Get_Value`: returns basis function evaluations for a given vector component and derivative multi-index.

Methods For `FELSymFunc`

- `Compose_Function`: create a new function from composing two functions, e.g. $h(...) = f(g(...))$, where f is the current "object".
- `Differentiate`: return a function that comes from differentiating the current "object" (function).
- `Eval`: evaluates the function's symbolic expression at given points.
- `Rename_Independent_Vars`: renames the independent variables of the symbolic function.
- `input_dim`: number of independent variables the symbolic function has.
- `output_size`: number of components that the function has (can be matrix-valued).

Methods For `FEMatrixAccessor`

- `Get_Matrix`: returns sparse matrix data corresponding to a given "name" of a FE matrix.

Methods For `FiniteElementSpace`

- `Append_Fixed_Subdomain`: store mesh sub-domain where DoFs on that sub-domain are considered *fixed* (Dirichlet condition).
- `Get_DoF_Bary_Coord`: for each DoF in the FE space, returns a cell index in the mesh that contains the DoF and the associated barycentric coordinates.
- `Get_DoF_Coord`: return spatial coordinates of each DoF in the FE space.
- `Get_DoFs`: return list of DoF indices in the FE space.
- `Get_DoFs_On_Subdomain`: return list of DoF indices that are attached to a given sub-domain.
- `Get_Fixed_DoFs`: similar to `Get_DoFs`, except only returns the DoFs that are *fixed* (i.e. by some Dirichlet condition).
- `Get_Free_DoFs`: analogous to `Get_Fixed_DoFs`, except this returns the DoFs that are *free* (not fixed).
- `Get_Zero_Function`: returns a coefficient array (matrix) of all zeros representing the zero function in the FE space.
- `Set_DoFmap`: set the DoFmap, which partially defines the FE space.
- `Set_Fixed_Subdomains`: set several subdomains where the DoFs are considered *fixed*.
- `max_dof`: return largest DoF index in the FE space's DoFmap.
- `min_dof`: return smallest DoF index in the FE space's DoFmap.
- `num_dof`: return number of unique DoF indices in the FE space's DoFmap.

Methods For `GeoElementSpace`

Has same methods as `FiniteElementSpace`, in addition to:

- `Compile_MEX`: compiles a mex file for performing interpolation on a higher order mesh.
- `Get_Mapping_For_Piecewise_Linear_Mesh`: returns the FE coefficient vector representing a higher order FE function that interpolates a piecewise linear domain, i.e. the identity map on the base piecewise linear mesh.

- `Interpolate`: performs interpolation on a higher order mesh.
- `Set_mex_Dir`: sets the desired directory to store the interpolation mex file.

Methods For `PointSearchMesh`

- `Compile_MEX`: compiles mex file for actually searching the mesh.
- `Point_Search_Domain`: find closest points in a sub-domain of the mesh to the given points.
- `Setup_Search_Tree`: set up search tree for efficient searching.
- `Update_Mesh`: update mesh (and associated tree structure) for searching.

Methods For `ReferenceFiniteElement`

- `Gen_Basis_Function_Evals`: put symbolic basis functions of the reference element into a useful data structure.
- `Gen_Quadrature_Rule`: return quadrature rule on the reference element.
- `Get_Local_DoFs_On_Topological_Entity`: return info on how local DoF indices are associated with topological entities of the reference cell.
- `Get_Nodes_On_Topological_Entity`: return matrix array that specifies the local DoF indices attached to each topological entity of the reference cell; similar to `Get_Local_DoFs_On_Topological_Entity`.
- `Verify_Nodal_Kronecker_Delta_Property`: verifies Kronecker Delta property for nodal basis functions.

Note: this class is mainly used by the code generator.

Finite Elements Defined In FELICITY

Elements are defined in m-files (scripts). The m-files have the following format, where `Z` is the degree of the element, and `D` is the (topological) dimension of the reference cell on which it is defined.

- Lagrange: `lagrange_degZ_dimD.m`.
- Brezzi-Douglas-Marini: `brezzi_douglas_marini_degZ_dimD.m`.
- Raviart-Thomas: `raviart_thomas_degZ_dimD.m`.
- Nedelec-1st kind: `nedelec_1stkind_degZ_dimD.m`.

Note:

- The files above can be found in `./FELICITY/Elem_Defn/`.
- The class `FELOutputElemInfo` can be used to print information about an element.

Methods For `FELOutputElemInfo`

- `Print_Basis_Functions`: output basis function definitions in either "pretty" or LaTeX format.
- `Print_DoFs`: create a figure that graphically shows the topological layout of the Degrees-of-Freedom (DoFs) on the reference element.

Code Generation For Allocating DoFs

Procedure:

- Define an array of structs, where each struct is an element definition as in `./FELICITY/Elem_Defn/`.
- Use the command `Create_DoF_Allocator` to compile the MEX file.
- See the chapter "Automatically Generating Degree-of-Freedom Maps" in the PDF manual for more information.

Code Generation For Assembling Forms/Matrices

Procedure:

- Write an m-file defining the forms/matrices to be created. See keywords and commands below.
- Use the command `Convert_Form_Definition_to_MEX` to compile the MEX file that will assemble the matrices.
- See the chapter "Assembling Matrices" in the PDF manual for more information.

Matrix Assembly Keywords

- `Bilinear`: define a Bilinear form.
- `Coef`: define finite element coefficient function.
- `Domain`: define spatial domain.
- `Element`: define a finite element space.
- `GeoElement`: define finite element space for specifying how the geometry of the global domain is represented.
- `GeoFunc`: define geometric function for a given `Domain`.
- `Integral`: define an integral to be used for specifying `Bilinear`, `Linear`, or `Real` forms.
- `Linear`: define a Linear form.
- `Matrices`: used for collecting all forms together and outputting them to the code generation routines.
- `Real`: define a Real form, i.e. a small dense matrix.
- `Test`: define finite element test function.
- `Trial`: define finite element trial function.

Symbolic Variable Representations For `Test`, `Trial`, and `Coef`

- `div`: divergence $\nabla\cdot$.
- `ds`: 1st arc-length derivative.
- `dsds`: 2nd arc-length derivative.
- `curl`: curl operator $\nabla\times$.
- `grad`: gradient ∇.
- `hess`: hessian ∇^2.
- `val`: function value.

Symbolic Variable Representations For `GeoFunc`

- `Mesh_Size`: local mesh size.
- `X`: the identity map (position vector).
- `deriv_X`: derivatives of the local parametrization of the domain.
- `N`: normal vector.
- `T`: tangent vector.
- `Tangent_Space_Proj`: tangent space projection matrix.
- `Kappa`: signed scalar curvature (sum of the principle curvatures).
- `Kappa_Gauss`: Gaussian curvature.
- `VecKappa`: vector curvature (sum of the principle curvatures multiplied by the normal vector).
- `Shape_Op`: Shape operator.

Methods For `Matrices`

- `Append_Matrix`: append a single form (matrix).
- `Include_C_Code`: store info for including external user written C code.

Code Generation For Interpolating FE Functions

Procedure:

- Write an m-file defining the point-wise interpolations to compute. See keywords and commands below.
- Use the command `Convert_Interp_Definition_to_MEX` to compile the MEX file that will perform the interpolations.
- See the chapter "Interpolating Finite Element Data" in the PDF manual for more information.

Interpolation Keywords

- `Coef`: define finite element coefficient function.
- `Domain`: define spatial domain.
- `Element`: define a finite element space.
- `GeoElement`: define finite element space for specifying how the geometry of the global domain is represented.
- `GeoFunc`: define geometric function for a given `Domain`.
- `Interpolate`: defines an interpolation expression, which may contain FE `Coef` functions as well as `GeoFunc` functions (i.e. domain geometry).
- `Interpolations`: used for collecting all `Interpolate` objects and outputting them to the code generation routines.

Note: some of the above keywords also appear in **Matrix Assembly Keywords**.

Methods For `Interpolations`

- `Append_Interpolation`: appends a single `Interpolate` object (which contains an expression to interpolate).
- `Include_C_Code`: store info for including external user written C code.

Code Generation For Point Searching

Procedure:

- Write an m-file defining a particular point-search of a `Domain`(s). See keywords and commands below.
- Use the command `Convert_PtSearch_Definition_to_MEX` to compile the MEX file that will execute the point-search.
- See the chapter "Point Searching On A Mesh" in the PDF manual for more information.

Point-Search Keywords

- `Domain`: define spatial domain.
- `GeoElement`: define finite element space for specifying how the geometry of the global domain is represented.
- `PointSearches`: used for collecting all `Domains` (to be searched) and giving it to the code generation routines.

Note: some of the above keywords also appear in **Matrix Assembly Keywords**.

Methods For `PointSearches`

- `Append_Domain`: appends a single `Domain` object (which is a domain to be searched).
- `Include_C_Code`: store info for including external user written C code.

Note: in most situations, one will combine point-search code with interpolation code.

How To Use Hierarchical Search Trees

FELICITY contains built-in C++ code (compiled as MEX files) that implements a binary tree, *quadtree*, and *octree* for efficient nearest neighbor searching of points in space. Technical note: the search trees are implemented as "bucket trees".

Search Tree Classes

- `mexBitree`: binary tree for storing and searching points in 1-D.
- `mexQuadtree`: quadtree for storing and searching points in 2-D.
- `mexOctree`: octree for storing and searching points in 3-D.

Methods For `mexBitree`, `mexQuadtree`, And `mexOctree`

- `Check_Tree`: checks that points in the tree actually belong to their enclosing leaf cell.
- `Get_Tree_Data`: extract data about the tree, such as the enclosing cell (node) dimensions and points contained there in.
- `Plot_Tree`: plots the tree graphically.
- `Print_Tree`: print the tree info to the MATLAB display.
- `Update_Tree`: take a new set of points and update the tree to accommodate the new point coordinates.
- `delete`: destroy the tree (free its memory).
- `kNN_Search`: for a given set of points, this finds the k nearest neighbors in the tree.

Note: see the section "Hierarchical Search Trees" in the "Miscellaneous Tools" chapter of the PDF manual for more information. In addition, the Git-Hub wiki contains some tutorials.

Note: search trees can help make point-searching of meshes very fast.

How To Generate Meshes

FELICITY contains built-in routines for generating meshes and for simple mesh processing. Some are C++ codes (compiled as MEX files) and others are pure MATLAB functions.

Iso-surface Mesh Generation Classes

• `Mesher2Dmex`: Generate 2-D triangle meshes that conform to an iso-contour.

• `Mesher3Dmex`: Generate 3-D tetrahedral meshes that conform to an iso-surface.

Note: these two classes implement the **TIGER** mesh generator.

Methods For `Mesher2Dmex` And `Mesher3Dmex`

• `Get_Cut_Info`: finds all cut edges and cut points that are cut by the zero level set of a given level set function.

• `run_mex`: generate conforming mesh of the interior of a given iso-surface.

Note: see the section "Unstructured Mesh Generation: 2-D and 3-D" in the PDF manual for more information.

Simple Mesh Generator Routines

• `bcc_tetrahedral_mesh`: generate a 3-D BCC (body-centered-cubic) lattice/tetrahedral mesh of the unit cube.

• `bcc_triangle_mesh`: generate a 2-D BCC lattice/triangle mesh of the unit square.

• `Refine_Entire_Mesh`: take a given 2-D mesh and refine the mesh wherever there is a "marked" triangle.

• `regular_tetrahedral_mesh`: generate a regular tetrahedral mesh of the unit cube.

• `triangle_mesh_of_sphere`: generate a 2-D mesh of the surface of a sphere.

Mesh Smoothers

• `FEL_Mesh_Smooth`: run a Gauss-Seidel iterative ODT (Optimal-Delaunay-Triangulation) smoother multiple times on mesh vertex positions. Works for 1-D, 2-D, and 3-D simplicial meshes.

Simulation Management Classes

- `FEL_AbstractSim`: abstract class for creating specific finite element simulation subclasses. Do not use directly; use `FEL_Sim_Template` instead.
- `FEL_SaveLoad`: class for storing time-series, or indexed, simulation data and reloading it.
- `FEL_Sim_Template`: example concrete subclass of `FEL_AbstractSim`.
- `FEL_Visualize`: class for saving simulation plots and making movies.

Methods For `FEL_SaveLoad`

- `Delete_Data`: delete all files saved by the object.
- `Get_Index_String`: make simulation index string with "padded" zeros.
- `Get_Max_Index`: get largest simulation index of files saved in the object's data directory.
- `Load`: load data that was saved previously.
- `Make_FileName`: create a valid filename for the dynamic data of a given simulation index.
- `Make_FileName_Static`: create a valid filename for the static data.
- `Save`: save data with a specific file index.

Methods For `FEL_Sim_Template`

- `Assemble_Matrices`: assemble matrices on the object's mesh.
- `Build_System`: build the system matrix to be solved.
- `Define_Finite_Element_Space`: define finite element spaces for the simulation.
- `Define_Mesh`: define global mesh for the simulation.
- `Get_LU`: calls MATLAB's `lu` command; to be used with `Solve_With_LU`.
- `Initialize_Solution`: initialize solution variables (finite element coef. arrays) for the simulation.
- `Solve`: solve the system.
- `Solve_With_LU`: solve a linear system $Ax = b$ by a pre-computed LU decomp; to be used with `Get_LU`.

Methods For `FEL_Visualize`

- `Delete_Plots`: delete all plots saved by the object.
- `Make_Movie`: make a movie.
- `Save_Plot`: save figure to a file.