1413- Fall 2013 - HW7 - Enrique Areyan	
1) Prove that if t is a limit point of a bounded sequence of real number	rs
Pr. Let 1 = 2m my Pr. Suppose for a contradiction that 2 > t  an bot snitery Pm  an out snitery Pm  removes  tet 1 3 'm class that:  tet 1 3 'm class that:	nov 163
Let M= l-1. How and who proposition given in class is the	,
we have: $q = t + \eta < s = \forall k$ : $\exists m \neq k$ , $s = P_m > q = t + \eta$ . Rick such that and make.  Now, t is a limit point of $P_n$ , so all but finitely many points of $t = R_n$ . Then are going to be in any neighborhood of $t$ . But, let $0 < E < t + \eta$ ; then the neighborhood of t of radius $E$ does not contain $P_m$ for infinitely $t = R_n$ . The reighborhood of $t = R_n$ that you pick, this contradicts the fact that $t = R_n$ therefore, $t = R_n$ and $t = R_n$ .  Therefore, $t = R_n$ and $t = R_n$ .	
(2) Prove-that in 1/2, every couchy sequence converges.	
It: We prove in class the conoming of our suppose I my my for	/( <u>)</u>
therefore, it suffices to show that for area	÷
we have that limsup Pn - lim my Pn.  Note that we proved, some time ago, that cauchy implies boundeness.	
Let 4Ph3 be a Cauchy Sequence in 112. Let us prove:	

(2) lim suptipul < li

ii) Let us prove that, given 870 limasper & liming Pr + E ippose, for a contradiction, that there exists £70 s.t: S = Rom sup Pr > Rim my Pr + E = 1 + E low, since this is bounded, we have that there exists a subsequence hand of land s.t. Pro > l. Likewise, there exists a subsequence this of IPn] s.t Pn; -> S. ence, all but finitely many points of IPnil are going to be in the neighborhood I of radius E. Likewise, all but finitely many points of this are ing to be in the noighborhood of 5 of radius N. wever, this is couchy, which means that & 170: IN: 4km > N: 1Pk-Pmler for r=s-n-(l+e) and for any N, we con always And no, nj >N s.t. or Philty ; where Pa; converges to I and Pa; converges to 5 as described tore, just pick ni,n; large enough. But this contradicts the fact that . is Cauchy. Therefore limsopper & liming Pote, for only & ) liming pris liming pris (8(ii) => ly sap pn = ly my pn, call them p. By previous the onem, n > p.; since pr was an arbitrary Cauchy sequence we obtain the result Let Pn = { P1, P2,... } be a sequence of all rational numbers in [0,1]. Find (1) Ring inf Pn and (ii) Ring sup Pn tion: A theorem proved in class states that for a bounded sequence real numbers and tells: if they (a subsequence) converses to titler Dim int Pasts Pany sup Po R Ph is a bounded, non-empty set of real numbers, it has a sup, and is that 1=sup Pn and 0=ing Pn. Hence of limits Pn & limits pen & 1

Huz Est 2013- HW7- Esciola Arguan	(2)
H413-Fall 2013-HW7-Enrique Arreyan  Therefore, if we can find a subsequence of that converges to o  Likewis	) 20
If mo can try of supredicting of they that authorized	, .
Conclude that I & lim sup PN & 1 => Dimy sup PN =0 FIT I	
Who was to look and sink subsequences:	
(2) the following is a subsequence of Parthal converges to 0:	
Pay; pick any rational in Pr. s.t OK Pak 2. IMPN My "	. Pn.
	Pag 4
Pas: Pick any rational enumerated after the in April 2000 stock stock	Paris !
Pas: Pick any rational enumerated after the intent of or Having Picked Page; Pick Page on yrational enumerated after Page intent of this is a subsequence of Pa converging to 0. Therefore, 10 = em my (	Pnx
1.42 12 a 2012 still of 14 course 1. )	and the second second
Who following Subsequence of in Concerso to	
A = V D TUFN IV DICKED OF U. CAN	
then Pne is a subsequence of Pn that converges to 1.	
There is no har to be a second to the second	P > 1
(4) Given a bounded sequence IPn), define X= 1 pelle: infinitely many	. **
Prove the following:	
$(\alpha) \times (\beta) $	
(b) 5= sup x = winn	Ω >
Pt @By definition, lend is bounded it its range is bounded. Hence there exists a real number M and 9 c 112 s.t. In-21 <m *="" all="" ent="" for="" of="" pre="" property="" set.<="" td="" this=""><td>•</td></m>	•
there exists a real number M and 9 ETK 5.0. I'm this get.	
there exists a real number M and gette set.  There exists a real n	大   キ
E-W & FLIL	, , , , ,
clearly 2+M > Pn > p => 2+M is an upper bound of X. Moreover,  Since any point x < 2-M is going to be less than pn for infinitely m	M3
clearly 2+M7Pn7P => 2+M is an upper bound of X. infinitely m Since any point x < 2-M is going to be less than Pr for infinitely m	
(In fact for all), x= & M & X. So, X is not empty.	

) By part @, since X + \$ ord X is bounded above, x has a sup. et s = sup X. Claim s= em sup ipn j = l. Ps: Let us prove @ 3 > 2m, supten 5 = 0 uppose for a contradiction that 9<2. 31-E & 1+E onsider the following two properties: (1) There exists IPM s.t. PM + l. For E = 2-5, there exists No.T. Ill points Par, for nx JN; Pax > l-E. Hence, l-EEX. inco s<0 => forall K, there exist m> K s.t. Pm7, 3. Consider (=1,2,3,...) choose  $m_1,m_2,m_3,...$  s.t.  $m_1>5$ . then,  $S\in X$ . R(i) = 3 5 < l- $\epsilon$ , but  $\ell$ - $\epsilon \in X$  so  $3 \times \ell$ - $\epsilon$ ; a contradiction of but y many 1 es's Henry 57/2. (b) 5 ≤ 0m siphiphs = 2. <u>"</u>Q 2+E 5 Prose for a contradiction that 87%. onsider the following two properties: ) Let  $E = \frac{S-l}{2}70$ . Since  $l+E7l \Rightarrow there exists K5.7 <math>there$  Pn < l+E. all but finitely many points are to the cert of life. therefore, life CX. ) ALL points between like and 5 are not in X, since there are only nitely many Phis between 018 and S. This follows frama transcorp PO -> RIETX XXEX and STRIE. But Sistle Rubolx 5 < R+ & ; a contradictio, Hence 5 < R. th sal ad ssl => s=e (=> (3=emsyphing)

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3
M413-Fall 2013-HW7-Enrique Areyon
5 Compare lim sup (Pn+Pn') with lim sup hPn1 + em sup & Pn').
  (b) If em R=P then lim sup(Pn+Pn') = cim sup hPn's + cm sup hPn's
Solution
    @ claim lim sup (Pa+Pa') < lim sup hpa) + lim sup hpa's.
 An example is P_n = 0.11,0.11,...; P_n'=1,0.11,0... => P_n + P_n' = 1,1.11,1...
   lim sup (PAPA') = 1 < 2 = 1+1 = com sup 1Pas + com sup 1Pas.
Pt By definition of lim sup (Pa+Pa'), I a subsequence Part Pa'x & s.t
                     Rim (Pnt Pnx) = Rim sup (Pn+ Pn).
 there exists a subsequence of him, hence it is bounded so there exists a sub-
 sequen of IPAKS, say IPAKES that converges.
 However, I Pinke & may or may not converge, but we know it is bounded for the source of the source o
 So there exists a subsequence of IP'nke I say I P'nke I that converges.
  Since 1 Pince I converges, so does any subsequence of it, in particular h Pinceni
   So we have two convergent sequences: 2 Paren 9 and 1 Phreen]. We proved
  that the sum of convergent, real valued sequences also converges.
  Hence, & Provent Privent also converges. But 2 Provent Privent is a subsequence in
   of to Port Port I have it converges to the same value, which we assur
   is am sup (Pa+1/2). Thus, am i Parent Poren's = am sup lenter's
    But then, Rim I Parem + Parem ] = Rim / Parem ) = Rim / Parem ) = Com, sup & Part Parem )
   Finally, lay threen 9 is a subsequential finit point of that therefore,
                                   Ring & Payen ) & Ring sup & Pag - Linewise, Ring & Privery & Ring Sup & Pa
   Combining these:
     Rimsup 1Pn+Pn') = Rim 1 Pnep + Rim 1 Pnep 5 = Rim sup 1Pn 9 + Rimsup 1Pn's.
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6). By part @, we need only to prove: Em sup & Pa+Pa' & Demsib & + Emsib Pa', provided that Qm Pn = P, exists. et iPmil be a subsequence of IPmil that converges to Rimisup iPmil, i.e., Qm ( \p'n\z) = lim sup \Pn'{. ow, since we are assuming that hong converges, any subsequence of will converge to the same value, In particular April . Therefore: Rim & Part = p = lim sp ? Part. us, we can add convergent sequences and obtain a convergent sequence: " { Pox } + lim & Pox } = lim & Pox + Pox } = lim sip & Pox + lim sip & Pox } But; Charlengt Part 3 & Charles Part Part . + thus,

[ Run sup i Pa+ Pa!] > Runsup i Pa S + Rim sup i Pa's! gheter with part @, this implies that

Danis Sup Track (2) = Comsy ? (2) & Com Sup 192'S, provided that Pa-> P.