11403-4W11.- Fall 2013- Enrique Areyan 60/60) (23 (3.1) Verefy the rules (6.3.3) (b) w.t.s Potv = tv.Po, where v'=Po(v): Let x'=[xz] on the ene mand.

Po $t_{V}(\vec{x}) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_{1} + v_{1} \\ x_{2} \end{bmatrix} + \begin{bmatrix} v_{1} \\ v_{2} \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_{1} + v_{1} \\ x_{2} + v_{2} \end{bmatrix}$ on the one hand. = $\left[\frac{\cos(x_1+v_1)-\sin\theta(x_2+v_2)}{\sin\theta(x_1+v_1)+\cos\theta(x_2+v_2)}\right] = *$. But then, on the other ty Po = tpown Po = tpown Po = tpown [cono - sino][x] = tpown [sino cono][xz] = tpown [sino x] = tpown [sino x]

But prin = [cono - sino][vi 7 [cono - sino][vi 7] But Po(v) = [coro - sino][V] = [corovi - sinovi + corovi], so TPO(V) [Sinex+ COOX] = [COOX - Sinex] + [COOX - Sine VZ] = [Sine (X+VI) - Sine (XZ+VI)] + [Sine VI + COOX] = [Sine (X+VI) + COO(XZ+VI)] + [Sine VI + COOX] = [Sine (X+VI) + COO(XZ+VI)] + [Sine VI + COOX] = [Sine (X+VI) + COO(XZ+VI)] + [Sine VI + COOX] = [Sine (X+VI) + COO(XZ+VI)] + [Sine VI + COOX] = [Sine (X+VI) + COO(XZ+VI)] + [Sine VI + COOX] = [Sine (X+VI) + COO(XZ+VI)] + [Sine VI + COOX] = [Sine (X+VI) + COOX] = [Sine (X+VI) + COO(XZ+VI)] + [Sine (X+VI) + COOX] = [Si So (=) , we have verified the rule Poty = TviPo, whore v'=Po(v). (iii) w.t.s. rty? tur , where v'=r(v). Let x'=[xz]. On the one ha $rT_{V}(\overrightarrow{x}) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_{1} \\ x_{2} \end{bmatrix} + \begin{bmatrix} v_{1} \\ v_{1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_{1} + v_{1} \\ x_{2} + v_{2} \end{bmatrix} = \begin{bmatrix} x_{1} + v_{1} \\ -x_{2} - v_{2} \end{bmatrix}$ $t_{v'}r(\vec{x}) = t_{v'} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t_{v'} \begin{bmatrix} x_1 \\ -x_2 \end{bmatrix}$ But ty = tr(v), r(v) = [0-1][\frac{1}{12}] = [-42] = > ty' = t[\frac{1}{12}], So ty. [xi] = t[xi] [xi] - [xi] + [xi] - [xi-1] = = 500-0, we have verified the rule rty = ty'r, where v'=r(v). (iii) w.t.s. rp=Por. Let = [x2]. then, on the one hand $r_{\theta}(x) = r_{sin\theta}(x) - sin\theta \int_{x^2} x^2 = r_{sin\theta}(x) + cone(x) = r_{sin\theta}(x) - sin\theta(x)$ On the sum of the cone of the con $P_{\theta}r(\vec{x}) = P_{\theta}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = P_{\theta}\begin{bmatrix} x_1 \\ -x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ by properties of sin $= \begin{bmatrix} \cos(-\theta)X_1 + \sin(-\theta)X_2 \\ \sin(-\theta)X_1 - \cos(-\theta)X_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta)X_1 - \sin(\theta)X_2 \\ -\sin(\theta)X_1 - \cos(\theta)X_2 \end{bmatrix} = 0$ $Sin(-\theta) = -Sin(\theta)$ Cos(-0) = cos(0). So # = @, we have verified rp=Por

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M403-HWII- Fall 2013 - Enrique Areyan
(EV) W.t.S. ty In = tv+w. Let = [xz]. then,
 (t_{V}t_{W})(\overrightarrow{x}) = t_{V}(\begin{bmatrix} w_{1} \\ w_{2} \end{bmatrix} + \begin{bmatrix} x_{1} \\ x_{2} \end{bmatrix}) = t_{V}\begin{bmatrix} w_{1} + x_{1} \\ w_{2} + x_{2} \end{bmatrix} = \begin{bmatrix} v_{1} \\ w_{2} + x_{2} \end{bmatrix} + \begin{bmatrix} w_{1} + x_{1} \\ w_{2} + x_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} + x_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{1} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{1} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2} + x_{2} \\ v_{2} + w_{2} \end{bmatrix} = \begin{bmatrix} v_{1} + w_{2
                                              = t<sub>V+W</sub> [x<sub>1</sub>] = t<sub>V+W</sub> (x) => T<sub>V</sub>t<sub>W</sub> = t<sub>V+W</sub>
(V) w.t.s. Popn = Po+n. Let = [x2]. then,
 (Popn) (P) = Popn (x1) = Po ( coon - sinn ) [x1) = Po (coon x1 - sinn x2) 
= [x2] = Po [x1] = Po ( sinn coon ) [x2] = Po [sinn x1 + coon x2]
   = \begin{bmatrix} \cos\theta & -\sin\theta \end{bmatrix} \begin{bmatrix} \cos\eta x_1 & -\sin\eta x_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & (\cos\eta x_1 - \sin\eta x_2) & -\sin\theta & (\sin\eta x_1 + \cos\eta x_2) \end{bmatrix} = \begin{bmatrix} \sin\theta & (\cos\eta x_1 - \sin\eta x_2) & +\cos\theta & (\sin\eta x_1 + \cos\eta x_2) \end{bmatrix}
    = COO COSHXI-COO SINKXZ - SINO SINNXI - SINO COSHXZ
               LSin O con XI - Sin O Sin N X2 + (O) O Sin N XI + CO) O CO, N X2
   = [X1 (coso con-sing sing) - x2 (coso sing + sing cosy)]
                                                                                                                                                                By Properties of sine and come functions
                [Xi (sing cosy + cosig sinn) + Xz (cosig cosy-sing sinn)]
   = \left[ \begin{array}{c} X_1 \cos(\theta+n) - X_2 \sin(\theta+n) \\ X_1 \sin(\theta+n) + X_2 \cos(\theta+n) \end{array} \right]
   = \left[\begin{array}{cc} \cos(\theta+n) & -\sin(\theta+n) \end{array}\right] \left[\begin{array}{c} \times 1 \\ \times 2 \end{array}\right] = \left(\begin{array}{c} \theta+n \end{array}\right) \left(\begin{array}{c} \times \end{array}\right) = \left(\begin{array}{c} \theta+n \end{array}\right) \left(\begin{array}{c} \times \end{array}\right)
  (vi) w.t. 5 rr=1. Let \mathcal{P} = \begin{bmatrix} x_1 \end{bmatrix}. then,
     rr(\vec{x}) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\vec{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\vec{x} = \vec{x} = 1 rr(\vec{x}) = 1, i.e., rr = id
   Pf: Let m: X -> Axto, where A is a reflection and b is a translation, be an arbitrary.
       be an arbitrary orientation-reversing isometry. Hence m = m(mx) = m(Ax+b)

mx = Ax+b. Let us compute m^2: m^2(x) = m(mx) = m(Ax+b)
        = A(Ax+b)+b=A^2x+Ab+b=X+(Ab+b). because A is a reflection

M^2=4 translation by a vertor ALL -3 M^2-4
         m2 = translation by a vector Ab+b => m2 = tAb+b.
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M403-HWII-Fall 2013- Enrique Areyan 6.3 (3.3) Prove that a linear operator on IRZ is a reflection if and only if its eigenvalues are I and -1, and the eigenvectors with these eigenvalues Pf: (=) Let T: 122-122 be a linear operator on 122. Moreover, assume are orthogonal. T is a reflection along a line I. We want to show that : (i) eigenvalue of Tare 1 and -1 (iii) the eigenvectors with these eigenvalues are orthogonal. (i) First, note that Triges X, xxel: T(x)=x =) 1 is an eigenva of T. Moreover, let I be the only orthogonal line to I and 9 Elt, then $T(\vec{y}) = -\vec{y}' = -1$ is the other eigenvalue of T. Since Tects on 1122 it can only possibly have at most two dishert eigenvalues, so (ii) Let & be an eigenvector with eigenvalue 1. they +(a) = ax, fac ut g be an eigenvector with eigenvalue -1. then +(bg)=-by But by properties of T we must have $a\overrightarrow{v} \cdot b\overrightarrow{y} = 0$, since \overrightarrow{v} is a vector in l and g is a vector in lt, so they are orthogonal (€) Let T:112²->12² be a linear operator on 1123. Moreover, assume that its only eigenvalues are I and -1 and the eigenvectors with these eigenvalues are are orthogonal. We want to show that T is a reflection along a line! Let $V \in \mathbb{R}^2$ be an eigen vector with eigen volue Δ , i.e. T(V) = V. Since T is linear, aT(v) = av () t(av) = av, VacIR, this means that the eigenvector with eigenvalue 1 spans a line that is unaffected by Likewise, Let wells be an eigenvector with eigenvalue -1, i.e., T(w) = -1 Again, Therear implies bt(w)=-bw & T(bw)=-bw. Hence, this eigen vector span a line, which by hypothesis is orthogonal to by V. V·W=O, moreover, w is reflected along the line spanned by V. therefore, T is a reflection along the line passing through (0,0) in the direction of V.

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1 alide verter por the olide reflection	turr
(1) Determine the glide line and glide vector for the glide reflection what conditions on v and o are needed to make the glide reflection ju	ist a
rall a-tank	
Solution: Let OEIR, Ve IR? Consider the glide reflection to Por:	
(Schamatically):	
ene l'épris)	0/2
We know that for is the replection about the line at ang't the know that for is the replection about the line at ang't to a sister the change of coordinate where we trans't	late
We know that for is the replection about the line at trans' to x-axis. Consider the change of coordinate where we trans' the alide line to the x-axis:	
to x-axis. Consider the change of	
the glide line to the γ -axis: P is the position vector: $P = \begin{bmatrix} 0 \\ b \end{bmatrix}$, for some $b \in \mathbb{R}$. $P = \begin{bmatrix} 0 \\ b \end{bmatrix}$, for some $b \in \mathbb{R}$. Then $m = P \cdot t_P \cdot r \cdot t_P \cdot P \cdot e$. Now $t_P \cdot r \cdot t_P \cdot e$.	
$\vec{p} = [0]$, for some $\vec{p} = [0]$	
P Nou	u m
then m= lotp line to t	the
	Marie Land
is the glide reflection by first taking the glide line to the enigmal line x-axis, reflecting, and then moving back to the enigmal line Now, with this characterization is easy to see that the glide Now, with this characterization is easy to see that the glide Now, with this characterization is easy to see that the glide Now, with this characterization is easy to see that the glide x-axis, reflecting, and then moving back to the enigmal line of the glide line to the enigmal line is easy to see that the glide x-axis, reflecting, and then moving back to the enigmal line of the glide line to the enigmal line is easy to see that the glide line to the enigmal line is easy to see that the glide x-axis, reflecting, and then moving back to the enigmal line is easy to see that the glide line is easy to see that the glide line is easy to see that the glide x-axis, reflecting, and then moving back to the enigmal line is easy to see that the glide line is easy to see the glide line is easy to see the glide line is easy to see th	de line
1 sharacterization is easy to see I home sell	2.
1000, aith 1115 chores or	equiv
Now, with this characterization is easy to see that Now, with this characterization is easy to see that is given by $l(3) = \vec{p} + 5 lor(e_1) = \vec{p} + 5 \left[\frac{\cos(2)}{\sin(2)}\right]$, where $s \in \mathbb{N}$ is given by $l(3) = \vec{p} + 5 lor(e_1) = \vec{p} + 5 \left[\frac{\cos(2)}{\sin(2)}\right]$ to $\vec{p} = restor$ the glide vector is parallel to the glide line, i.e., a director vectorly, is a vector parallel to the glide line, i.e., a director $\vec{p} = restor$	ector o
the glide vector is parallel to valida line, i.e., a director	
lently, is a vector parallel to the girls	1 1
Lently, is a vector parallel to the glide line, i.e., where $\vec{v} = \begin{bmatrix} \cos(\frac{\theta}{2}) \end{bmatrix}$. Hence $\vec{v} = \begin{bmatrix} \cos(\frac{\theta}{2}) \end{bmatrix}$. The conditions on \vec{v} and $\vec{v} = \begin{cases} \cot(\frac{\theta}{2}) \end{bmatrix}$ and $\vec{v} = \begin{cases} \cot(\frac{\theta}{2}) \end{bmatrix}$ i.e. $\vec{v} = 0$ and $\vec{v} = 0$ i.e. $\vec{v} = 0$ and $\vec{v} = 0$ reflection are: $\vec{v} = 0$ to $\vec{v} = 0$.	0
the conditions on vanit r= Potprt-p Po ; i.e.	
JUST 1 - 1 - 1 - 1 - 10 (8)	
I have control to	1.11
(2) Prove that if L ₁ and L ₂ are lines through the origin in 1122 the composition of the reflections across the two lines is a rotal and determine the angle of rotation	tion
the composition of the reflections across the Two ones	
1 ~ 1 . 0 .	
0	
Pt: Lz Consider the reflections across zone given by for and for.	7
Consider the reflection of the sisometries and let us compose this isometries and let us compose the let us compose this isometries and let us compose the let	4
given by for and for. Let us compose this isometries and analyze the result:	
We are the second of the secon	

M403-HWII-Fall 2013- Enrique Areyan (Por)(Pnr) = Po((Pn)r) = Po((Pnr)r) By rules proved in exercise 6.3 (3.1) = (909-n)(rr) = 909-n 1 = Po-n/ therefore, the composition of the reflections across Li, Lz is a rotation with angle on, where & is the angle of the fire line wirit the x-axis and is the angle of the second line wir the x-axis. Since Lilz are arbitrary Imes through the origin, we ge the result. (3) SHOW the composition of reflections about parallel lines is a translation by a vector orthogonal to the lines reflection about line LI Sends \$ to \$' Next) Pt: Geometrically: reflection about Lz sends ₹ to R". One can see that x" is a translation of 7 by vector which is orthogonal to both Li and Lz. Algebraically, let us represent line as m= tyler lot-w, where wis a vector parallel to the u-axis to the y-axis pointing to a point on the sine on the sector line has the same angle o but possibly differs on the rector w. So let $m_z = tw_z e^{-\omega t - wz}$. Finally, let us compose those two isometing. two isometries: m10 m2 = (tw. Porlot-w.)(tw2 Porlot-wz) By rules previously proved = twilorlotwz-wilorlot-wz = tw, r lo-0 twz-w, r lo-0 t-wz = $tw_1 r tw_2 - w_1 r t - w_2$ = $tw_1 t_{\overline{z}} r r t_{-w_2}$ $7 = r(w_2 - w_1)$ = tw, tet-w2 = t= tw1-W2 showing the $= t_r(\omega_2 - \omega_1) + (\omega_1 - \omega_2) = t_{\omega_1} - \omega_2 + \omega_1 - \omega_2 = t_2(\omega_1 - \omega_2)$ result.