Format for frequency comparison

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For the comparison of oscillation calculations it is convenient to have a standard format of results of adiabatic calculations, in parallel with the `fgong` format used for model comparison. I propose to base this on the ADIPLS `obs` format, but with higher precision. Specifically, the file should provide

$$l, n, \nu, E \; [, n_{\rm p}, n_{\rm g}]$$

Here l is the degree, n is the radial order, ν is the frequency in μHz and E is the mode inertia (see below). Also, the optional quantities $n_{\rm p}$ and $n_{\rm g}$ are the number of p- and g-nodes in the classification scheme, including the Takata (2006) scheme for dipolar modes (see below for details on the definition of the mode order).

For the output I suggest the fixed format
`2i8, 1p2e16.8, 2i8`
(given the very high order in the g-mode behaviour we need quite a long format for the order). The file should be ordered with the degree increasing most slowly, i.e.,

```
0 1 xxx eee ii jj
0 2 xxx eee ii jj
.....................
1 -50 xxx eee ii jj
1 -49 xxx eee ii jj
```

etc.

The mode inertia should be defined as

$$E = \frac{\int_0^{R_{\rm s}} [\xi_r^2 + l(l+1)\xi_{\rm h}^2] \rho r^2 {\rm d}r}{M[\xi_r(R_{\rm phot})^2 + l(l+1)\xi_{\rm h}(R_{\rm phot})^2]} = \frac{\int_0^{x_{\rm s}} [y_1^2 + y_2^2/l(l+1)] \, qU {\rm d}x/x}{4\pi[y_1(x_{\rm phot})^2 + y_2(x_{\rm phot})^2/l(l+1)]} \; .$$

Note in particular the normalization by the total displacement at the photosphere, $r = R_{\rm phot}$. The second equality is in terms of the ADIPLS solutions,

$$\begin{aligned}
y_1 &= \frac{\xi_r}{R} \, , \\
y_2 &= x \left(\frac{p'}{\rho} + \Phi' \right) \frac{l(l+1)}{\omega^2 r^2} = \frac{l(l+1)}{R} \xi_{\rm h} \, , \\
y_3 &= -x \frac{\Phi'}{gr} \, , \\
y_4 &= x^2 \frac{{\rm d}}{{\rm d}x} \left(\frac{y_3}{x} \right) \, ,
\end{aligned}$$

where p' and Φ' are Eulerian perturbations to pressure and gravitational potential and g is the local gravity. Also, $q = m/M$ and $U = {\rm d}\ln m / {\rm d} \ln r = 4\pi r^3 \rho / m$, where m and M are the local interior and the total mass, respectively, and ρ is density; $x = r/R_{\rm phot}$ and hence $x_{\rm phot}$ should be 1, and $R_{\rm s}$ is the radius of the outermost point in the model, with $x_{\rm s} = R_{\rm s}/R_{\rm phot}$. For $l = 0$ the terms in $\xi_{\rm h}$ and y_2 should be excluded.

I name the resulting file `fobs......`, as an extension of my normal `obs....` files.

Definition of the mode order

The 'classical' definition of the mode order is based on the so-called Eckart scheme introduced by Scuflaire (1974) and Osaki (1975). This is based on counting the number of nodes of ξ_r with a sign depending on the variation with r of the solution in a (ξ_r, ξ_h) phase diagram. We let n_p be the number of zero-crossings of ξ_r in the counter-clockwise direction and n_g the number of crossings in the clockwise direction, with increasing r; then the mode order n is obtained as

$$n = n_\mathrm{p} - n_\mathrm{g} \; .$$

It can be proved that this defines a unique and invariant labelling of the modes in the Cowling approximation, where the perturbation to the gravitational potential is neglected, or for radial modes. For nonradial modes with $l \geq 2$ it also seems to be generally valid, although no complete proof of this has been made, as far as I am aware. In these cases I suggest simply to let n_p and n_g be defined as above.

For dipolar modes when the Cowling approximation is not made the mode labelling becomes problematic for even moderately centrally condensed models, including models of the present Sun, and very much so for more evolved models. Here Takata (2006) demonstrated that a unique and invariant labelling can be defined based on the phase diagram $(\mathcal{Y}_1, \mathcal{Y}_2)$, where

$$\begin{aligned} \mathcal{Y}_1 &= \frac{1}{g}\left[\frac{\delta\Phi}{r} - \delta\left(\frac{\mathrm{d}\Phi}{\mathrm{d}r}\right)\right] \\ \mathcal{Y}_2 &= \frac{\delta p}{p} \; . \end{aligned}$$

Defining n_p and n_g in terms of zero-crossings of $\mathcal{Y}_1$, as above, the order is determined as

$$n = \begin{cases} n_\mathrm{p} - n_\mathrm{g} + 1 & \text{for } n_\mathrm{p} \geq n_\mathrm{g} \\ n_\mathrm{p} - n_\mathrm{g} & \text{for } n_\mathrm{p} < n_\mathrm{g} \; . \end{cases}$$

I recommend following this definition for $l = 1$ in the full case, despite the more complication relation between n_p, n_g and n. This is also the way that the Takata labelling is included in ADIPLS.

References:

Osaki, Y., 1975. [Nonradial oscillations of a 10 solar mass star in the main-sequence stage]. *Publ. Astron. Soc. Japan*, **27**, 237 – 258.

Scuflaire, R., 1974. [The non radial oscillations of condensed polytropes]. *Astron. Astrophys.*, **36**, 107 – 111.

Takata, M., 2006. [Analysis of adiabatic dipolar oscillations of stars]. *Publ. Astron. Soc. Japan*, **58**, 893 – 908.