James Ealey ‖ Stat 312

LECTURE NOTES CONT.

1 cont)

c) $H_0: \pi = .15$ $\quad H_A: \pi \neq .15$ $\quad n = 102$

$\hat{\pi} = 13/102 = .1275$ $\quad \alpha = .05$

$p = P(\hat{\pi} = .1275$ (or more extreme, both sides) | H_0 holds$) =$

$2 \cdot P(\hat{\pi} \le .1275) = 2 \cdot P\left(Z \le \frac{.1275 - .15 + \frac{.5}{102}}{\sqrt{\frac{.15(1-.15)}{102}}}\right)$

$= 2 \cdot P\left(Z \le \frac{-.017598}{\sqrt{.00125}}\right) = 2 \cdot P(Z \le -.49775) = 2(.3095)$

$= \underline{.6170} = p$

cont. for C.I. part

d) binom.test(x=13, n=102, p=.15, alternative = c("two.sided"), conf.level = .95) = $\left[\begin{array}{l}\text{p-value} = 0.6767 \\ 95\%\ CI = (.0696, .2081)\end{array}\right.$

c) part II: 95% C.I.

$\hat{\pi} = \frac{x}{n} \pm (Z_{\alpha/2})\left(\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}\right)$

95% CI =

$\frac{13}{102} = .1275 \pm (Z_{.025})\left(\sqrt{\frac{(.1275)(1-.1275)}{102}}\right)$

$= .1275 \pm (1.96)\left(\sqrt{\frac{(0.11124)}{102}}\right) = .1275 \pm (1.96)(.033025)$

$= .1275 \pm (.06473) = \boxed{(.06277, .19223)}$ $\quad \alpha = .05$, 95% confidence.

this answer assumes not asking for continuity correction for CI.

4

James Earley

312 HW3

Gamma dist:

$P(X \le x) = F(x; \alpha, \beta) = F\left(\frac{x}{\beta}; \alpha\right)$

Chi-squared (χ^2)

↳ Gamma w/ $\alpha = \nu/2$, $\beta = 2$

$\nu = df = n-1$

//NOTES — IGNORE

[Notes] $\frac{(n-1)S^2}{\sigma^2} = \frac{\Sigma(X_i - \bar{X})^2}{\sigma^2}$, $df = n-1$ → from Normal dist. pop

$\chi^2_{\alpha,\nu}$ = χ^2 critical value (α = area to right)

$P\left(\chi^2_{1-\alpha/2, n-1} < \frac{(n-1)S^2}{\sigma^2} < \chi^2_{\alpha/2, n-1}\right) = 1 - \alpha$

CI of σ^2 for normal pop:

LB: $(n-1)s^2 / \chi^2_{\alpha/2, n-1}$

UB: $(n-1)s^2 / \chi^2_{1-\alpha/2, n-1}$

↳ for σ, take sqrt of each listed above

→ to make one-sided, take corresponding side and sub. w/ α, not $\alpha/2$.

7.4: p.306, (#44)

44. $n = 9$ $s = 2.81$ (mils) $\alpha = .05$

σ^2: Lowerbound: $(9-1)(2.81)^2 / \chi^2_{(.025), 8} = 63.1688 / 17.535 = 3.6024$

Upper bound: $(9-1)(2.81)^2 / \chi^2_{(.975), 8} = 63.1688 / 2.180 = 28.9765$

→ $\boxed{95\% \text{ CI: } (3.602, 28.977) \text{ for } (\sigma^2)}$

σ: L.B: $\sqrt{(8)(2.81)^2 / \chi^2_{(.025), 8}} = (\sqrt{3.6024}) = 1.898$

U.B. $\sqrt{(8)(2.81)^2 / \chi^2_{(.975), 8}} = (\sqrt{28.9765}) = 5.383$

→ $\boxed{95\% \text{ CI: } (1.898, 5.383) \text{ for } (\sigma)}$

if $n\pi \ge 15$ and $n(1-\pi) \ge 15$, then:

$X \approx N(\mu, \sigma)$ $\begin{cases} \mu = n\pi \\ \sigma = \sqrt{n\pi(1-\pi)} \end{cases}$ i.e. $X \approx N(n\pi, \sqrt{n\pi(1-\pi)})$

$\hat{\pi} = \frac{X}{n} \approx N\left(\pi, \sqrt{\frac{\pi(1-\pi)}{n}}\right)$

//NOTES - ignore

CI for population proportion:

$\hat{\pi} = \frac{x}{n} \pm (z_{\alpha/2})\left(\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}\right)$ $n\pi \ge 15$, $n \ge 30$

James Earley

312

NOTES – IGNORE

Proportions:

Large sample tests

Null hyp: $H_0 : \pi = \pi_0$

Test statistic: $z = \dfrac{\hat{\pi} - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}}$

H_a	p value	
$H_a : p > p_0$	area to right of z	only valid if
$H_a : p < p_0$	area to left of z	$n\pi_0 \geq 15$
$H_a : p \neq p_0$	2(area to right of $\lvert z \rvert$)	$n(1-\pi_0) \geq 15$

---

p350: (#45) $H_0 : \pi = .4$ $H_A : \pi \neq .4$

45. $n = 150$ $\alpha = 0.01$ $\hat{\pi} = \frac{82}{150} = .5467$

$n \geq 30$ ✓ $n\pi_0 \geq 15? \to 60$ ✓

$n(1-\pi_0) \geq 15? \to 90$ ✓

P($\hat{\pi}_0$ extreme or more extreme than .5467 (either sides) | H_0 holds)

$= 2 \cdot P(\hat{\pi} \geq .5467) = 2 \cdot (1 - P(\hat{\pi} \leq .5467))$

$= 2 \cdot (1 - P(Z \leq \frac{.5467 - .4}{\sqrt{(.4(1-.4))/150}})) =$

$2 \cdot (1 - P(Z \leq \frac{.1467}{\sqrt{.0016}})) = 2 \cdot (1 - P(Z \leq 3.6675)) =$

$2 \cdot (1 - .999874) = 2(.000126) = .000252 = p.$

Since $p \ll \alpha = .01$, we strongly reject H_0 in favor of H_A, that there exists a statistically significant difference here between $\hat{\pi}$ and $\pi_0 = .4$. We would also reject H_0 in favor of H_A at $\alpha = .05$, because $p \ll \alpha = .05$ as well. In both cases, we conclude that the actual percentage of the population having type A blood is signif. diff. than 40%.

2

James Earley

Stat 312

8.5: p. 357 (#67)

67. $\pi_0 = 1/175 = .01333$ $\quad n = 800 \quad \hat{\pi} = 16/800 = .02$

a) $H_0: \pi = 1/175 \quad H_A: \pi \neq 1/175 \quad \alpha = 0.05$

$n \geq 30$ ✓ $\quad n\pi_0 = 10.67 \geq 10$ (book version) ✓ $\quad n(1-\pi_0) = 789 > 10$

$P(\hat{\pi} = .02$ (or more extreme, both sides) $\mid H_0$ holds$) =$

$2 \cdot P(\hat{\pi} \geq .02) = 2 \cdot (1 - P(\hat{\pi} \leq .02)) = 2\left(1 - P\left(Z \leq \frac{.02 - .01333}{\sqrt{(.01333)(1-.01333)/800}}\right)\right)$

$= 2 \cdot \left(1 - P\left(Z \leq \frac{.00667}{\sqrt{(.013152/800)}}\right)\right) = 2 \cdot (1 - P(Z \leq 1.645)) = 2 \cdot (1 - .9495)$

$= .1010 = p$

Since $p > \alpha = .05$, there is insufficient evidence to reject H_0 at the 5% signif. level, and thus we fail to reject (retain) H_0 (not statistically signif. at $\alpha = .05$).

Because we may have failed to reject H_0 "incorrectly" (bad sample) when we "should have" rejected it (if we knew ground truth), there's a chance we might have made a type II error.

b) Yes, H_0 could be rejected at $\alpha = .20$ because $p < \alpha = .20$, so we would reject H_0 in favor of H_A, that the true $\pi \neq 1/175$.

LECTURE NOTES

1. $n = 102$, $\quad H_0: \pi = .15$

a) pbinom(13, size=102, prob=.15) = $\boxed{0.3177549}$

b) $n \geq 30$? ✓ $\quad n\pi \geq 15$? $(=15.3)$ ✓ $\quad n(1-\pi) \geq 15$? $(=86.7)$ ✓

$X \sim N(n\pi, \sqrt{n\pi(1-\pi)})$ so

$Z = \frac{\hat{\pi} - \pi_0 \pm \frac{.5}{n}}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim N(0,1) \overset{so}{\Rightarrow} P\left(\hat{\pi} \leq \frac{13}{102} = .1275\right) =$

$P\left(Z \leq \frac{.1275 - .15 + \frac{.5}{102}}{\sqrt{\frac{.15(.85)}{102}}}\right) = P\left(Z \leq \frac{-.017598}{\sqrt{.00125}}\right) = P(Z \leq -.49775)$

$= \boxed{.3085}$

Continued on back

3