Problem Set - Week 5

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```
! pip install sktime
! pip install pmdarima
```

```
Requirement already satisfied: sktime in /usr/local/lib/python3.7/dist-packages (0.11.4
Requirement already satisfied: numba>=0.53 in /usr/local/lib/python3.7/dist-packages (f
Requirement already satisfied: scikit-learn<1.2.0,>=0.24.0 in /usr/local/lib/python3.7/
Requirement already satisfied: deprecated>=1.2.13 in /usr/local/lib/python3.7/dist-pack
Requirement already satisfied: scipy<1.9.0 in /usr/local/lib/python3.7/dist-packages (f
Requirement already satisfied: statsmodels>=0.12.1 in /usr/local/lib/python3.7/dist-pac
Requirement already satisfied: pandas<1.5.0,>=1.1.0 in /usr/local/lib/python3.7/dist-pa
Requirement already satisfied: numpy<1.22,>=1.21.0 in /usr/local/lib/python3.7/dist-pac
Requirement already satisfied: wrapt<2,>=1.10 in /usr/local/lib/python3.7/dist-packages
Requirement already satisfied: setuptools in /usr/local/lib/python3.7/dist-packages (fr
Requirement already satisfied: llvmlite<0.39,>=0.38.0rc1 in /usr/local/lib/python3.7/di
Requirement already satisfied: pytz>=2017.3 in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: python-dateutil>=2.7.3 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.7/dist-packages (from
Requirement already satisfied: threadpoolctl>=2.0.0 in /usr/local/lib/python3.7/dist-pa
Requirement already satisfied: joblib>=0.11 in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.7/dist-package
Requirement already satisfied: patsy>=0.5.2 in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: pyparsing!=3.0.5,>=2.0.2 in /usr/local/lib/python3.7/dis
Requirement already satisfied: pmdarima in /usr/local/lib/python3.7/dist-packages (1.8.
Requirement already satisfied: setuptools!=50.0.0,>=38.6.0 in /usr/local/lib/python3.7/
Requirement already satisfied: pandas>=0.19 in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: statsmodels!=0.12.0,>=0.11 in /usr/local/lib/python3.7/d
Requirement already satisfied: Cython!=0.29.18,>=0.29 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: numpy>=1.19.3 in /usr/local/lib/python3.7/dist-packages
Requirement already satisfied: urllib3 in /usr/local/lib/python3.7/dist-packages (from
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Requirement already satisfied: scikit-learn>=0.22 in /usr/local/lib/python3.7/dist-pack
Requirement already satisfied: python-dateutil>=2.7.3 in /usr/local/lib/python3.7/dist-
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Requirement already satisfied: pyparsing!=3.0.5,>=2.0.2 in /usr/local/lib/python3.7/dis
```

```
from scipy import stats
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
```

```
import pandas as pd
import seaborn as sns
sns.set()

from sklearn.preprocessing import StandardScaler

from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.graphics.tsaplots import plot_pacf

from sktime.forecasting.all import ForecastingHorizon
from sktime.forecasting.all import ExponentialSmoothing
from sktime.performance_metrics.forecasting import mean_absolute_percentage_error

import warnings
warnings.filterwarnings('ignore')
```

1. EDA

1.A

```
df_co2 = pd.read_csv('hw5_data_co2.csv', parse_dates = ['ds'])
df_temp = pd.read_csv('hw5_data_temp.csv', parse_dates = ['ds'])
```

```
print(df_co2.isnull().sum())
print(df_temp.isnull().sum())

print(df_co2.shape)
print(df_temp.shape)
```

```
ds     0
co2    0
dtype: int64
ds      0
temp    0
dtype: int64
(735, 2)
(1674, 2)
```

```
from datetime import datetime, timedelta
df_co2['ds'] = df_co2['ds'] - timedelta(days=14)
df = pd.merge(df_co2, df_temp, how = "inner", on = "ds")
df
```

	ds	co2	temp
0	1958-03-01	315.700	57.38
1	1958-04-01	317.450	57.29
2	1958-05-01	317.510	57.32
3	1958-06-01	316.685	57.02
4	1958-07-01	315.860	57.27
...	...	...	...
730	2019-01-01	410.920	58.29
731	2019-02-01	411.660	58.37
732	2019-03-01	412.000	58.59

1.B.

- One dataset provides information for temperature, the other for CO2.
- There are no missing values in the dataset.
- Both variables are similar in terms of time series pattern: increasing trend over time.
- ACF plot shows autocorrelation in time for temperature

```
df.set_index('ds', inplace = True)
df
```

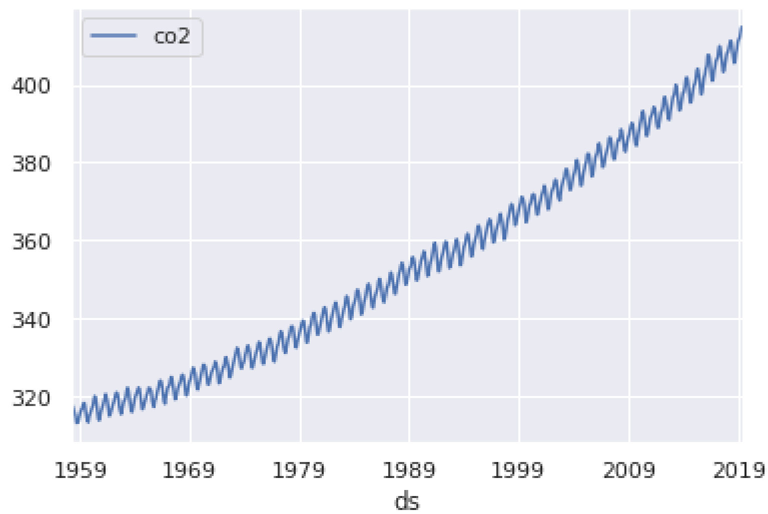
	co2	temp

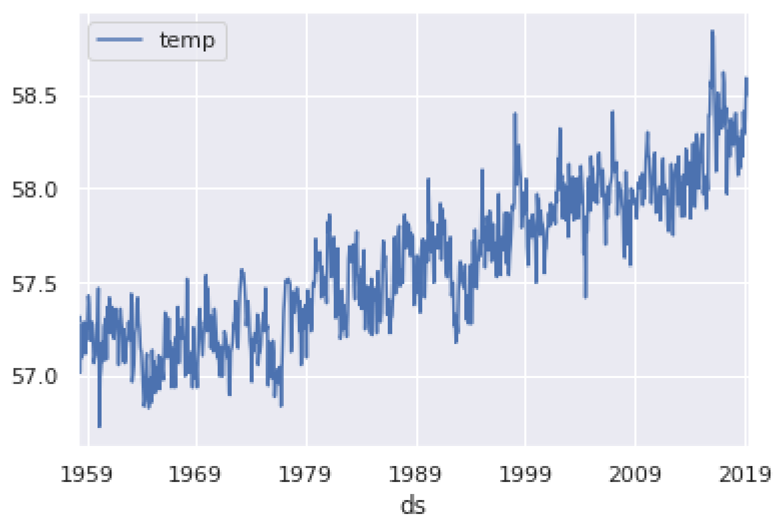
```
df.describe().T
```

	count	mean	std	min	25%	50%	75%	max
co2	735.0	354.210673	27.922811	313.20	328.785	351.34	376.515	414.83
temp	735.0	57.603293	0.397159	56.73	57.280	57.59	57.915	58.84

```
print(df[['co2']].plot())
print(df[['temp']].plot())
```

```
AxesSubplot(0.125,0.125;0.775x0.755)
AxesSubplot(0.125,0.125;0.775x0.755)
```

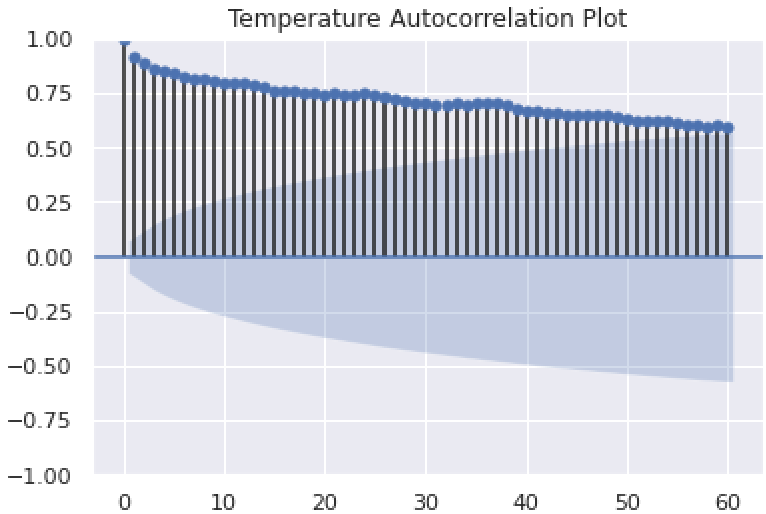





```
df.corr()
```

	co2	temp

```
p = plot_acf(df['temp'], lags=60, title='Temperature Autocorrelation Plot')
```



Part 2. ARIMA with external regressors

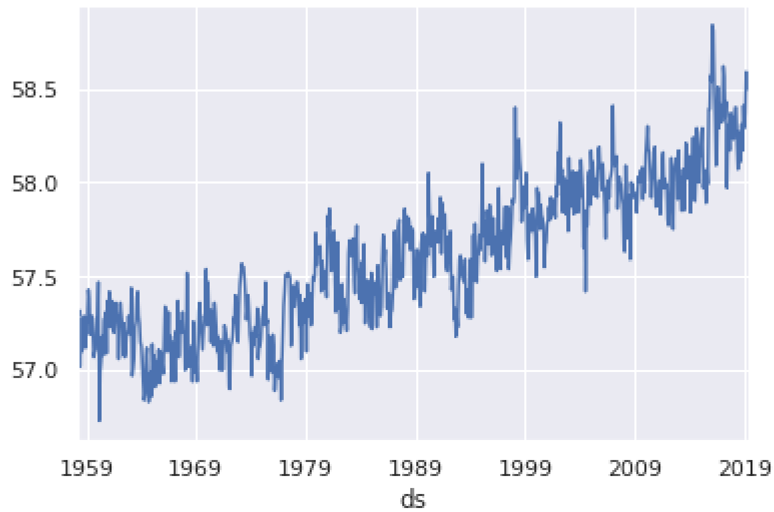
2.A.

- Our variable of interest CO2 initially shows no stationarity:
  - **ADF test** is showing that **non-stationarity can be rejected**. The p-value in the test is lower than 0.05, which means that the null hypothesis of non-stationarity can be rejected.
  - **KPSS test** is showing that **stationarity in the time series can be rejected**. The p-value is smaller than 0.05, which means that the null hypothesis of stationarity in the time series can be rejected.
- Based on ACF and PACF, we can differentiate it to make the time series stational:
  - **ADF test** is showing that **non-stationarity can be rejected**. The p-value in the test is lower than 0.05, which means that the null hypothesis of non-stationarity can be rejected.
  - **KPSS test** is showing that **stationarity in the time series cannot be rejected**. The p-value is smaller than 0.05, which means that the null hypothesis of stationarity in the time series cannot be rejected.
- See below plots and tests outputs.

```
from statsmodels.tsa.statespace import sarimax
from statsmodels.tools.eval_measures import aicc
```

```
from sktime.transformations.series import boxcox
from sktime.forecasting.arima import ARIMA


ts_temp = df.groupby(pd.PeriodIndex(df.index, freq="M"))['temp'].mean()
ts_temp.plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f12a26f0290>
```




```
bctransformer = boxcox.BoxCoxTransformer()
ts_transf = bctransformer.fit_transform(ts_temp)


from sktime.forecasting.all import temporal_train_test_split
y_train, y_test = temporal_train_test_split(ts_temp)


from statsmodels.tsa.stattools import kpss, adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from copy import deepcopy

def adf_test(timeseries):
    print ('Dickey-Fuller Test Result:')
    dftest = adfuller(timeseries, autolag='AIC')
    dfoutput = pd.Series(dftest[0:4], index=['Test Statistic','p-value','#Lags Used','Number
    for key,value in dftest[4].items():
       dfoutput['Critical Value (%s)'%key] = value
    print (dfoutput)

#define KPSS
def kpss_test(timeseries, trend='c'):
    print ('KPSS Test Result:')
    kpsstest = kpss(timeseries, regression=trend)
    kpss_output = pd.Series(kpsstest[0:3], index=['Test Statistic','p-value','Lags Used'])
    for key,value in kpsstest[3].items():
        kpss_output['Critical Value (%s)'%key] = value
    print (kpss_output)
```

```
adf_test(y_train)
```

```
Dickey-Fuller Test Result:
Test Statistic                  -2.481183
p-value                          0.120120
#Lags Used                       4.000000
Number of Observations Used    546.000000
Critical Value (1%)             -3.442384
Critical Value (5%)             -2.866848
Critical Value (10%)            -2.569597
dtype: float64
```

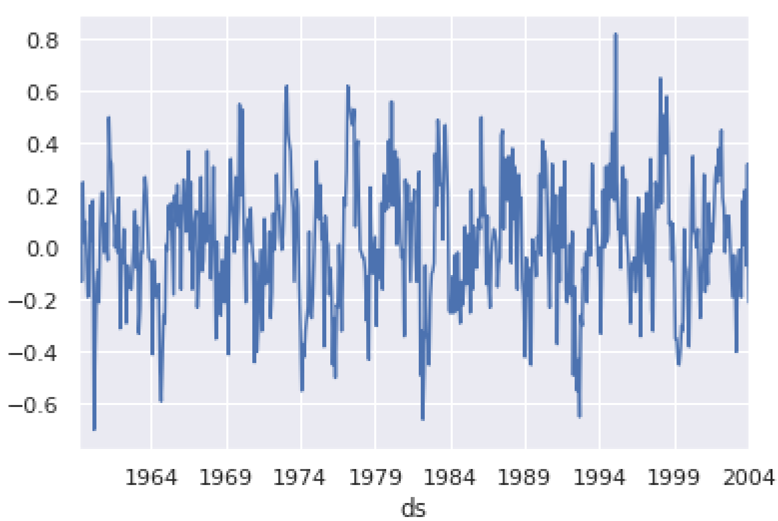
```
kpss_test(y_train)
```

```
KPSS Test Result:
Test Statistic            2.970537
p-value                   0.010000
Lags Used                15.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64
```

Differenting

```
y_train_seasdiff = y_train.diff(12).dropna()
```

```
y_train_seasdiff.plot()
plt.show()
```



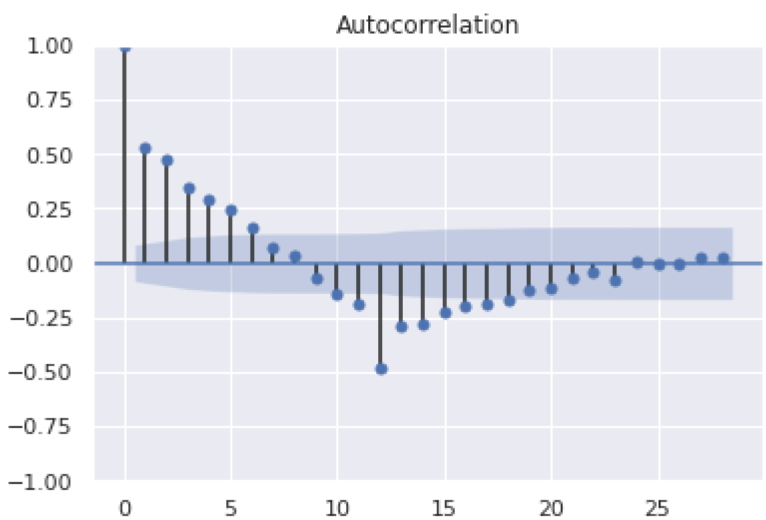

```
adf_test(y_train_seasdiff)
```

```
Dickey-Fuller Test Result:
Test Statistic                -7.250434e+00
p-value                        1.785978e-10
#Lags Used                     1.300000e+01
Number of Observations Used    5.250000e+02
Critical Value (1%)           -3.442867e+00
Critical Value (5%)           -2.867061e+00
Critical Value (10%)          -2.569710e+00
dtype: float64
```

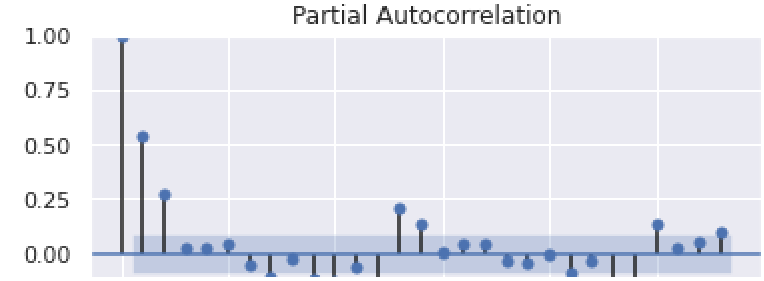
```
kpss_test(y_train_seasdiff)
```

```
KPSS Test Result:
Test Statistic            0.044985
p-value                   0.100000
Lags Used                13.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64
```

```
plot_acf(y_train_seasdiff)
plt.show()
```



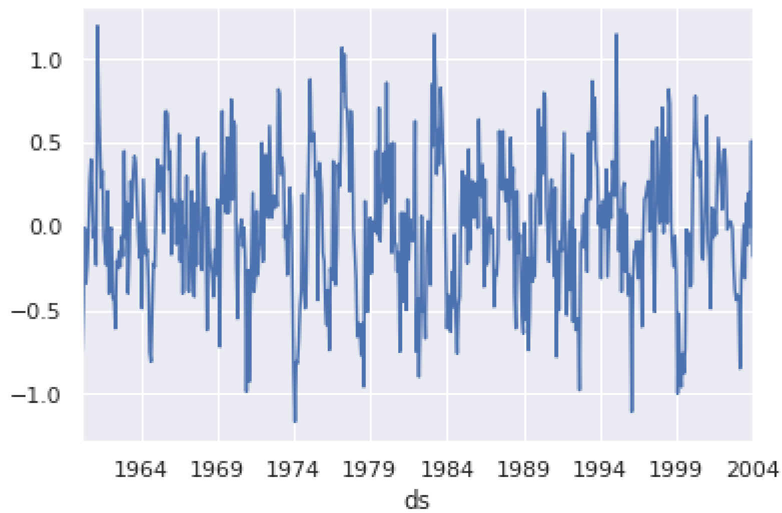

```
plot_pacf(y_train_seasdiff)
plt.show()
```




```
#y_train_nonseasdiff = np.diff(y_train_seasdiff)
y_train_nonseasdiff = y_train_seasdiff.diff(12).dropna()
```



```
y_train_nonseasdiff.plot()
plt.show()
```




```
adf_test(y_train_nonseasdiff)
```

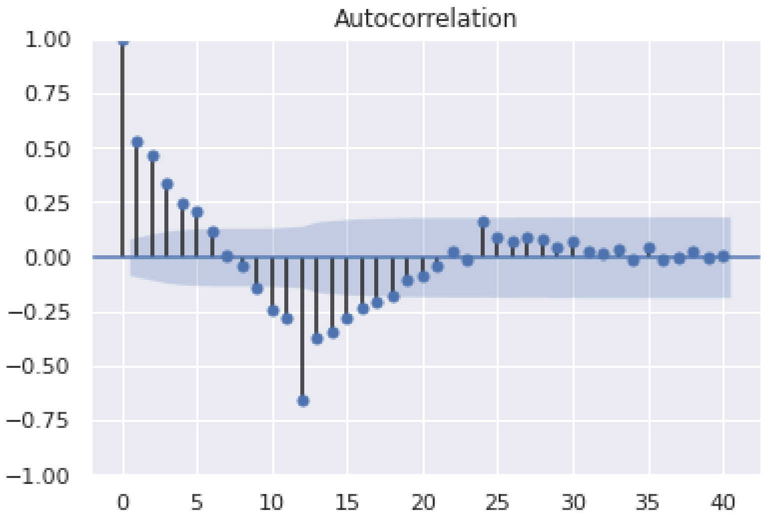
```
Dickey-Fuller Test Result:
Test Statistic                 -7.555800e+00
p-value                         3.102015e-11
#Lags Used                      1.300000e+01
Number of Observations Used     5.130000e+02
Critical Value (1%)            -3.443162e+00
Critical Value (5%)            -2.867190e+00
Critical Value (10%)           -2.569780e+00
dtype: float64
```

```
kpss_test(y_train_nonseasdiff)
```

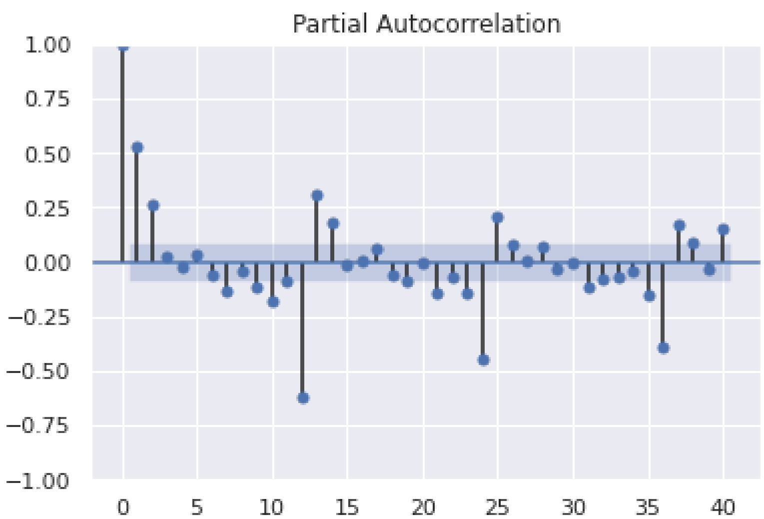
```
KPSS Test Result:
Test Statistic            0.009799
p-value                   0.100000
Lags Used                13.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
```

```
Critical Value (1%)       0.739000
dtype: float64
```

```
plot_acf(y_train_nonseasdiff, lags = 40)
plt.show()
```




```
plot_pacf(y_train_nonseasdiff, lags = 40)
plt.show()
```



2.B.

ARIMA model:

```
sarima_model = sm.tsa.statespace.SARIMAX(endog=y_train,order=(3,1,1), seasonal_order=(0, 1, 6
sarima_fit = sarima_model.fit()
```

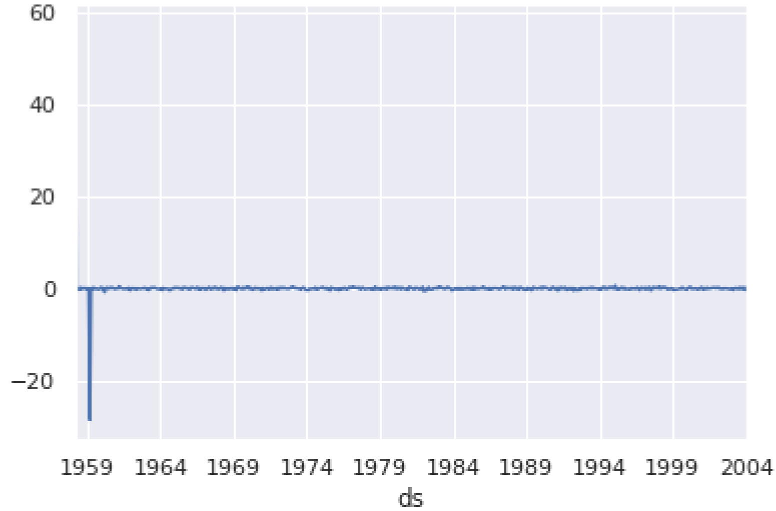
```
print(sarima_fit.summary())
```

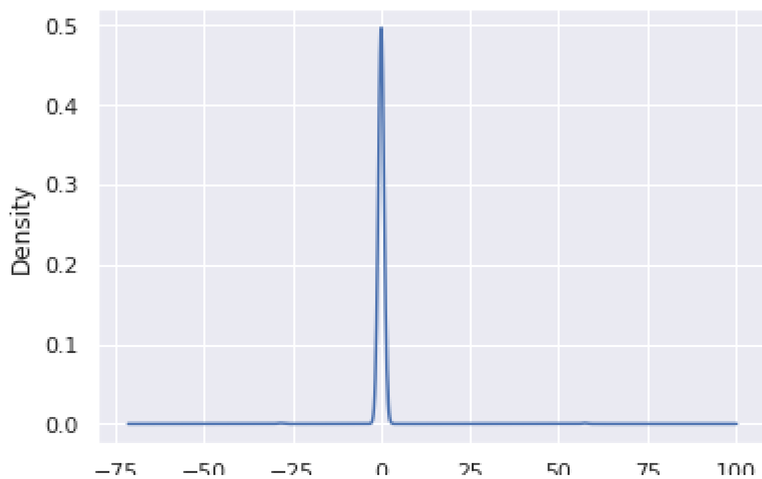
```
                                     SARIMAX Results
==========================================================================================
Dep. Variable:                                temp   No. Observations:
Model:             SARIMAX(3, 1, 1)x(0, 1, [], 12)   Log Likelihood                   117
Date:                             Sat, 14 May 2022   AIC                             -224
Time:                                     04:10:23   BIC                             -203
Sample:                                 03-31-1958   HQIC                            -216
                                      - 01-31-2004
Covariance Type:                               opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.3835      0.045      8.587      0.000       0.296       0.471
ar.L2          0.2620      0.047      5.613      0.000       0.171       0.354
ar.L3          0.0273      0.047      0.585      0.558      -0.064       0.119
ma.L1         -0.9996      0.159     -6.283      0.000      -1.311      -0.688
sigma2         0.0375      0.006      6.045      0.000       0.025       0.050
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):                 2.25
Prob(Q):                              0.96   Prob(JB):                         0.32
Heteroskedasticity (H):               1.03   Skew:                            -0.04
Prob(H) (two-sided):                  0.86   Kurtosis:                         3.31
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

```
print("AICc for order " + str(sarima_model.order) + str(sarima_model.seasonal_order) +": " +s
```

```
AICc for order (3, 1, 1)(0, 1, 0, 12): -224.49550706908607
```

```
residuals = sarima_fit.resid
residuals.plot()
plt.show()

residuals.plot(kind='kde')
plt.show()

print(residuals.describe())
```






```
residuals
```

```
ds
1958-03    57.380000
1958-04    -0.089999
1958-05     0.030000
1958-06    -0.300000
1958-07     0.250000
              ...
2003-09    -0.014296
2003-10     0.168874
2003-11    -0.167513
2003-12     0.283619
2004-01    -0.326205
Freq: M, Length: 551, dtype: float64
```

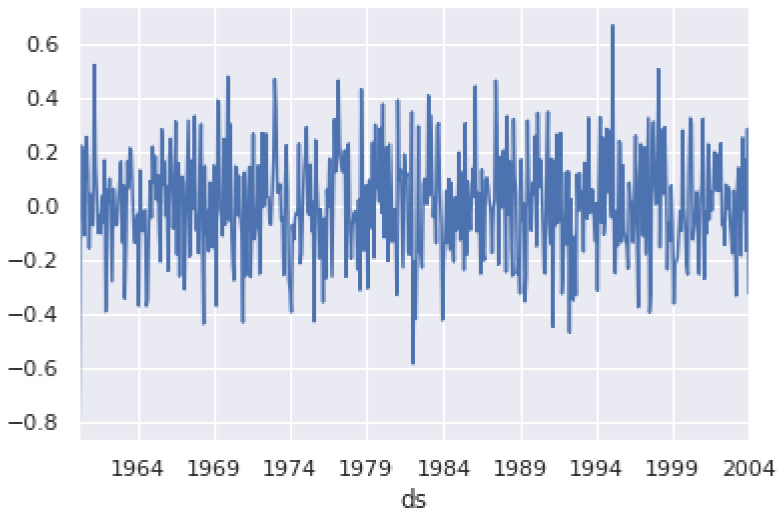
```
resid = residuals[24:]
resid
```

```
ds
1960-03   -0.797779
1960-04   -0.144174
1960-05    0.220645
1960-06    0.107033
1960-07   -0.110958
```

```
              ...
2003-09   -0.014296
2003-10    0.168874
2003-11   -0.167513
2003-12    0.283619
2004-01   -0.326205
Freq: M, Length: 527, dtype: float64
```

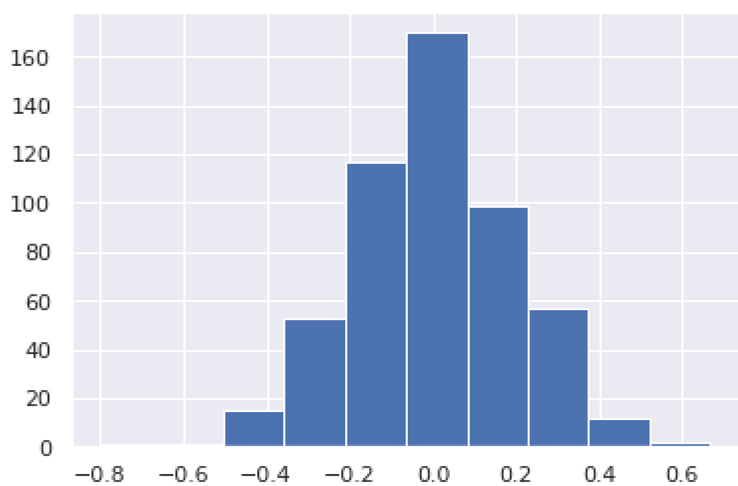
```
resid.plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f12a300d6d0>
```

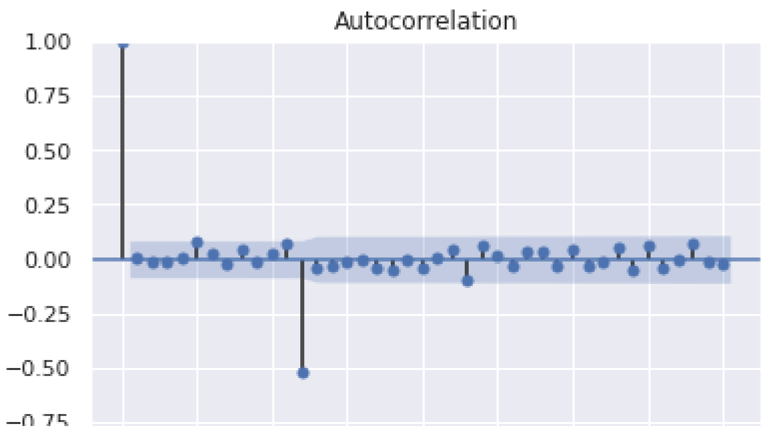



```
resid.hist()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f12a26675d0>
```




```
plot_acf(resid, lags = 40)
plt.show()
```




```
sm.stats.acorr_ljungbox(resid, lags=[10], return_df=True, boxpierce=True)
```

	lb_stat	lb_pvalue	bp_stat	bp_pvalue
10	5.38012	0.864384	5.298779	0.870347

AUTOarima model:

```
from sktime.forecasting.arima import AutoARIMA
from pmdarima.arima import auto_arima
```

```
y_train, y_test = temporal_train_test_split(ts_temp)
```

```
y_train.index.min(), y_train.index.max()
```

```
(Period('1958-03', 'M'), Period('2004-01', 'M'))
```

```
y_test.index.min(), y_test.index.max()
```

```
(Period('2004-02', 'M'), Period('2019-05', 'M'))
```

```
auto_arima = auto_arima(y_train, start_p=1, start_q=1,
                           max_p=10, max_q=10, m=12,
                           start_P=0, seasonal=True,
                           d=1, D=1, trace=True,
                           error_action='ignore',
                           suppress_warnings=True,
                           stepwise=True)
print(auto_arima.aic())
```

```
Performing stepwise search to minimize aic
 ARIMA(1,1,1)(0,1,1)[12]             : AIC=inf, Time=8.55 sec
 ARIMA(0,1,0)(0,1,0)[12]             : AIC=-53.968, Time=0.25 sec
 ARIMA(1,1,0)(1,1,0)[12]             : AIC=-345.175, Time=1.38 sec
```

```
 ARIMA(0,1,1)(0,1,1)[12]             : AIC=inf, Time=6.73 sec
 ARIMA(1,1,0)(0,1,0)[12]             : AIC=-167.230, Time=0.22 sec
 ARIMA(1,1,0)(2,1,0)[12]             : AIC=-395.722, Time=2.11 sec
 ARIMA(1,1,0)(2,1,1)[12]             : AIC=inf, Time=15.00 sec
 ARIMA(1,1,0)(1,1,1)[12]             : AIC=inf, Time=4.50 sec
 ARIMA(0,1,0)(2,1,0)[12]             : AIC=-284.630, Time=0.86 sec
 ARIMA(2,1,0)(2,1,0)[12]             : AIC=-405.606, Time=1.59 sec
 ARIMA(2,1,0)(1,1,0)[12]             : AIC=-353.561, Time=0.72 sec
 ARIMA(2,1,0)(2,1,1)[12]             : AIC=inf, Time=10.26 sec
 ARIMA(2,1,0)(1,1,1)[12]             : AIC=inf, Time=4.65 sec
 ARIMA(3,1,0)(2,1,0)[12]             : AIC=-424.092, Time=2.02 sec
 ARIMA(3,1,0)(1,1,0)[12]             : AIC=-367.330, Time=0.98 sec
 ARIMA(3,1,0)(2,1,1)[12]             : AIC=inf, Time=11.93 sec
 ARIMA(3,1,0)(1,1,1)[12]             : AIC=inf, Time=4.12 sec
 ARIMA(4,1,0)(2,1,0)[12]             : AIC=-432.173, Time=2.48 sec
 ARIMA(4,1,0)(1,1,0)[12]             : AIC=-376.355, Time=2.10 sec
 ARIMA(4,1,0)(2,1,1)[12]             : AIC=inf, Time=17.91 sec
 ARIMA(4,1,0)(1,1,1)[12]             : AIC=inf, Time=4.75 sec
 ARIMA(5,1,0)(2,1,0)[12]             : AIC=-431.318, Time=2.82 sec
 ARIMA(4,1,1)(2,1,0)[12]             : AIC=-431.886, Time=5.45 sec
 ARIMA(3,1,1)(2,1,0)[12]             : AIC=-433.808, Time=3.57 sec
 ARIMA(3,1,1)(1,1,0)[12]             : AIC=-376.473, Time=1.46 sec
 ARIMA(3,1,1)(2,1,1)[12]             : AIC=inf, Time=12.88 sec
 ARIMA(3,1,1)(1,1,1)[12]             : AIC=inf, Time=6.83 sec
 ARIMA(2,1,1)(2,1,0)[12]             : AIC=inf, Time=10.14 sec
 ARIMA(3,1,2)(2,1,0)[12]             : AIC=-431.817, Time=5.48 sec
 ARIMA(2,1,2)(2,1,0)[12]             : AIC=-433.194, Time=5.37 sec
 ARIMA(4,1,2)(2,1,0)[12]             : AIC=-430.006, Time=7.49 sec
 ARIMA(3,1,1)(2,1,0)[12] intercept   : AIC=-431.827, Time=13.19 sec

Best model:  ARIMA(3,1,1)(2,1,0)[12]
Total fit time: 177.865 seconds
-433.8075420485692
```

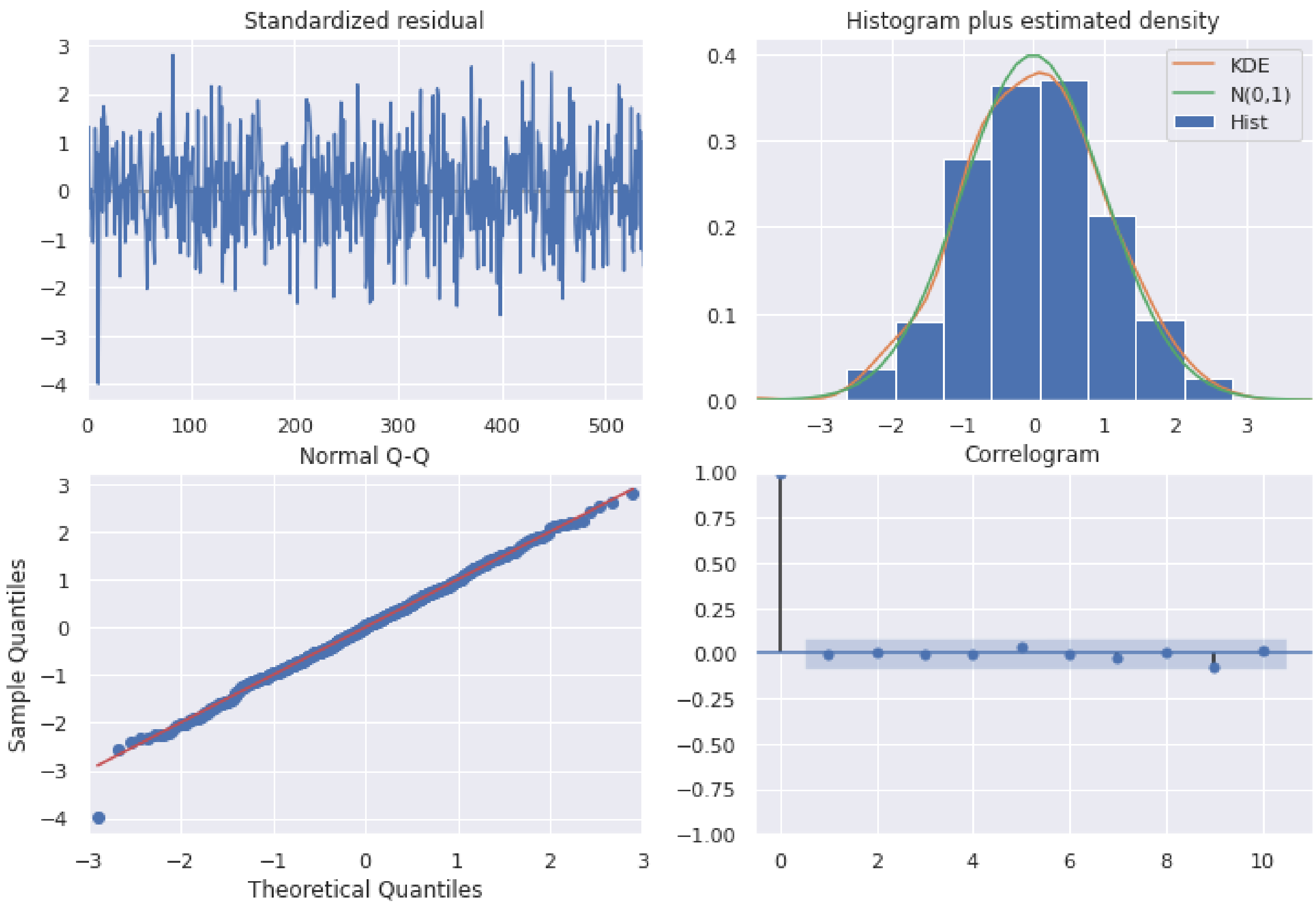
```
print(auto_arima.summary())
```

```
                                      SARIMAX Results
==========================================================================================
Dep. Variable:                                    y   No. Observations:
Model:             SARIMAX(3, 1, 1)x(2, 1, [], 12)   Log Likelihood                   223
Date:                              Sat, 14 May 2022   AIC                             -433
Time:                                      04:13:25   BIC                             -403
Sample:                                           0   HQIC                            -422
                                              - 551
Covariance Type:                                opg
=============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
-----------------------------------------------------------------------------
ar.L1          0.0341      0.134      0.255      0.799      -0.228       0.296
ar.L2          0.0368      0.082      0.447      0.655      -0.124       0.198
ar.L3         -0.1145      0.060     -1.919      0.055      -0.232       0.002
ma.L1         -0.6029      0.126     -4.778      0.000      -0.850      -0.356
ar.S.L12      -0.7452      0.040    -18.648      0.000      -0.824      -0.667
ar.S.L24      -0.3332      0.040     -8.349      0.000      -0.411      -0.255
sigma2         0.0251      0.002     16.606      0.000       0.022       0.028
```

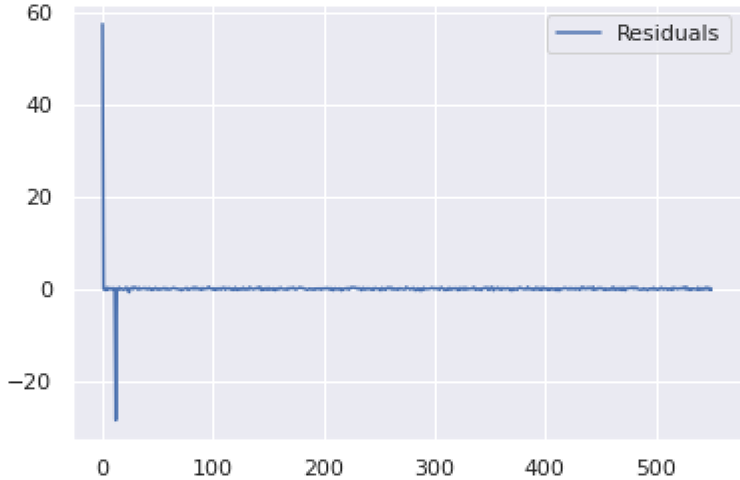
```
===================================================================================
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):                  0.65
Prob(Q):                              0.93   Prob(JB):                          0.72
Heteroskedasticity (H):               1.13   Skew:                             -0.07
Prob(H) (two-sided):                  0.43   Kurtosis:                          3.09
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

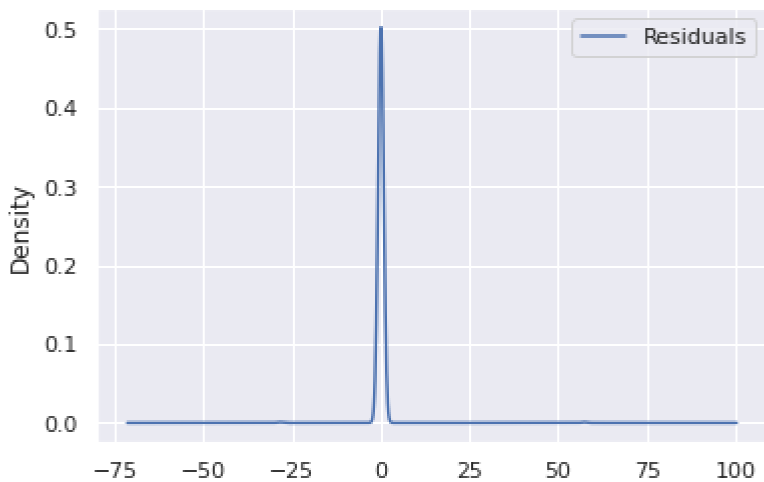
```
print(auto_arima.plot_diagnostics(figsize=(12,8)))
```

```
Figure(864x576)
```




```
res = auto_arima.resid()
residuals = pd.DataFrame(res,columns=["Residuals"])
residuals.plot()
plt.show()
residuals.plot(kind='kde')
plt.show()
print(residuals.describe())
```





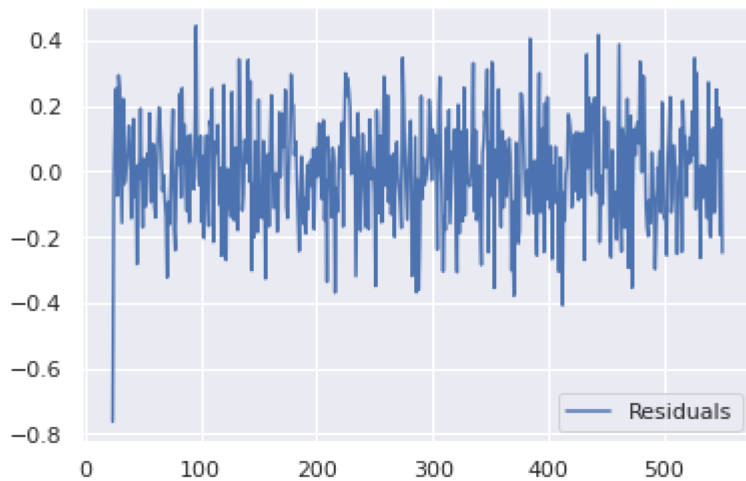

```
        Residuals
count  551.000000
mean     0.052828
std      2.736996
min    -28.550000
25%     -0.108642
50%      0.006010
75%      0.110697
```

```
resid = residuals[24:]
resid
```

	Residuals
24	-0.763389
25	-0.015780
26	0.144974
27	0.252910

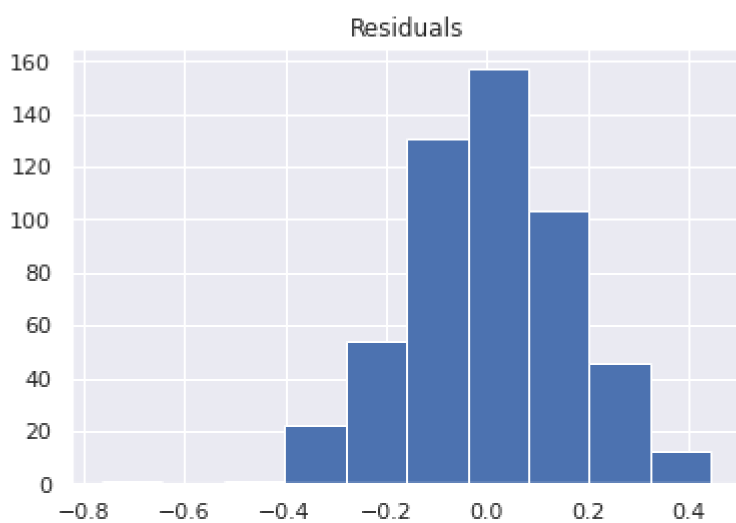
```
resid.plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f12a2513650>
```

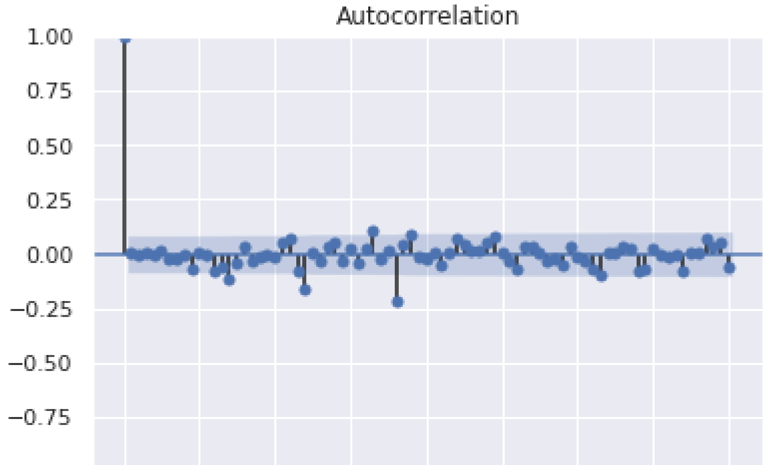



```
resid.hist()
```

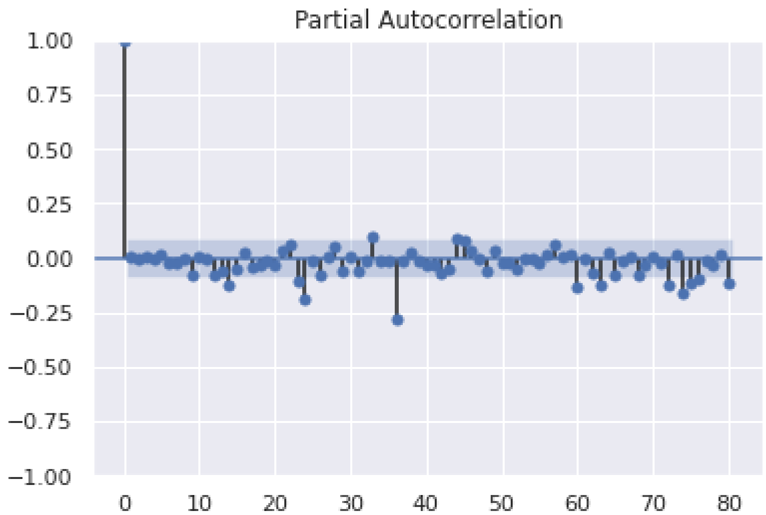
```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x7f12a2233a10>]],
      dtype=object)
```




```
plot_acf(resid, lags = 80)
plt.show()
```




```
plot_pacf(resid, lags = 80)
plt.show()
```




```
sm.stats.acorr_ljungbox(resid, lags=[10], return_df=True, boxpierce=True)
```

	lb_stat	lb_pvalue	bp_stat	bp_pvalue
10	3.571788	0.964604	3.501685	0.967041

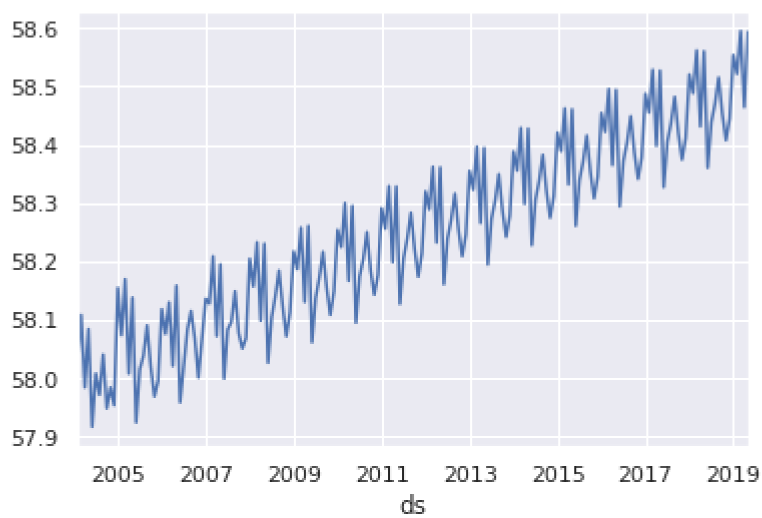
2.C.

```
y_pred = pd.DataFrame(auto_arima.predict(n_periods = len(y_test)), index = y_test.index)
y_pred.columns = ['pred']
y_pred
```

	pred
ds	
2004-02	58.028627
2004-03	58.108983
2004-04	57.984528
2004-05	58.084765
2004-06	57.916936
...	...
2019-01	58.553866
2019-02	58.521579
2019-03	58.595127
2019-04	58.464415

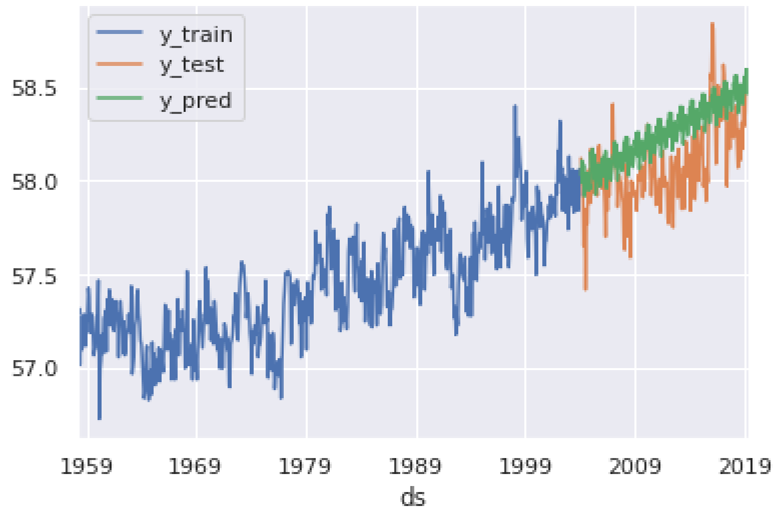
```
y_pred = y_pred.groupby(pd.PeriodIndex(y_pred.index, freq="M"))['pred'].mean()
y_pred.plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f12a209c890>
```




```
def plot_results(y_train, y_test, y_pred):
    # plot the results
    y_train.plot()
    y_test.plot()

    y_pred.plot()

    plt.legend(['y_train','y_test','y_pred'])
    plt.show()
```

```
plot_results(y_train, y_test, y_pred)
```




```
from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError
smape = MeanAbsolutePercentageError(symmetric=True)
smape(y_test, y_pred)
```

    0.003442278464427618

```
from sktime.performance_metrics.forecasting import MeanAbsoluteScaledError
mase = MeanAbsoluteScaledError()
mase(y_test, y_pred, y_train=y_train)
```

    1.5679160886858377

```
from sktime.performance_metrics.forecasting import MeanAbsoluteError
mae = MeanAbsoluteError()
mae(y_test, y_pred)
```

    0.20012310804681058

```
from sktime.performance_metrics.forecasting import MeanSquaredError
mse = MeanSquaredError()
mse(y_test, y_pred)
```

    0.06042945368794782

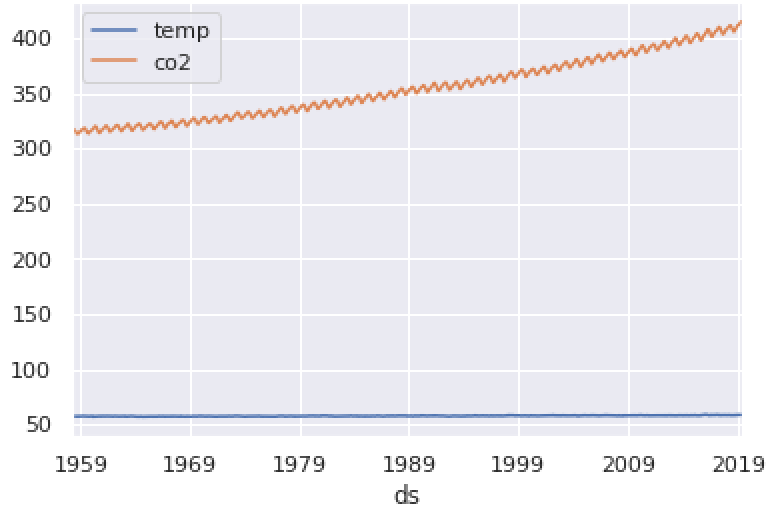
```
from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError
mape = MeanAbsolutePercentageError(symmetric=False)
mape(y_test, y_pred)
```

    0.003450331712048208

2.D.

```
ts = df.groupby(pd.PeriodIndex(df.index, freq="M"))['temp', 'co2'].mean()
ts.plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f12a1f20950>
```




```
y_train, y_test = temporal_train_test_split(ts)
```

```
from statsmodels.tsa.statespace.sarimax import SARIMAX

sarimax_model = SARIMAX(endog=y_train['temp'],
                        exog = y_train['co2'],
                        order=(3,1,1),
                        trend='c',
                        seasonal_order=(2, 1, 0, 12))
sarimax_fit = sarimax_model.fit()
print(sarimax_fit.summary())
```

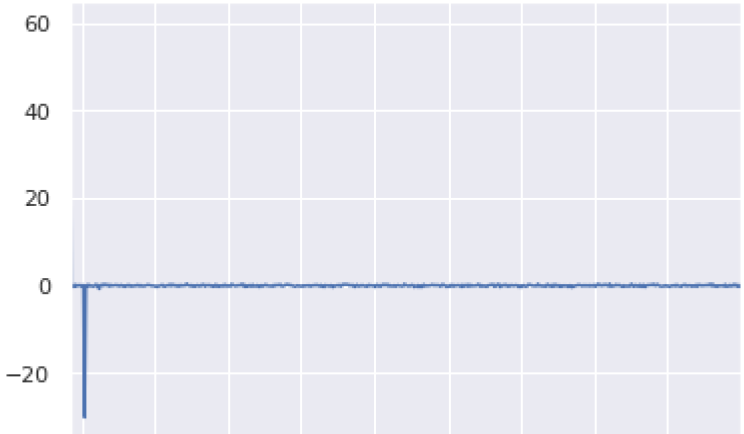
```
                                     SARIMAX Results
==========================================================================================
Dep. Variable:                                 temp   No. Observations:
Model:             SARIMAX(3, 1, 1)x(2, 1, [], 12)   Log Likelihood                   224
Date:                              Sat, 14 May 2022   AIC                             -430
Time:                                      04:13:45   BIC                             -391
Sample:                                  03-31-1958   HQIC                            -414
                                       - 01-31-2004
Covariance Type:                                opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      0.0004      0.003      0.151      0.880      -0.005       0.006
co2           -0.0097      0.022     -0.446      0.655      -0.052       0.033
ar.L1          0.0165      0.146      0.113      0.910      -0.270       0.303
ar.L2          0.0278      0.089      0.314      0.753      -0.146       0.202
ar.L3         -0.1175      0.061     -1.915      0.056      -0.238       0.003
ma.L1         -0.5834      0.140     -4.155      0.000      -0.859      -0.308
ar.S.L12      -0.7436      0.041    -18.355      0.000      -0.823      -0.664
ar.S.L24      -0.3330      0.040     -8.241      0.000      -0.412      -0.254
```

```
sigma2         0.0251      0.002     16.587      0.000       0.022       0.028
===================================================================================
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):                 0.57
Prob(Q):                              0.92   Prob(JB):                         0.75
Heteroskedasticity (H):               1.13   Skew:                            -0.07
Prob(H) (two-sided):                  0.42   Kurtosis:                         3.08
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

```
print("AICc for order " + str(sarimax_model.order) + str(sarimax_model.seasonal_order) +": "
```

```
AICc for order (3, 1, 1)(2, 1, 0, 12): -429.6824920506663
```

```
# residual checks
# line plot of residuals
residuals = sarimax_fit.resid
residuals.plot()
plt.show()
# density plot of residuals
residuals.plot(kind='kde')
plt.show()
# summary stats of residuals
print(residuals.describe())
```




```
residuals
```

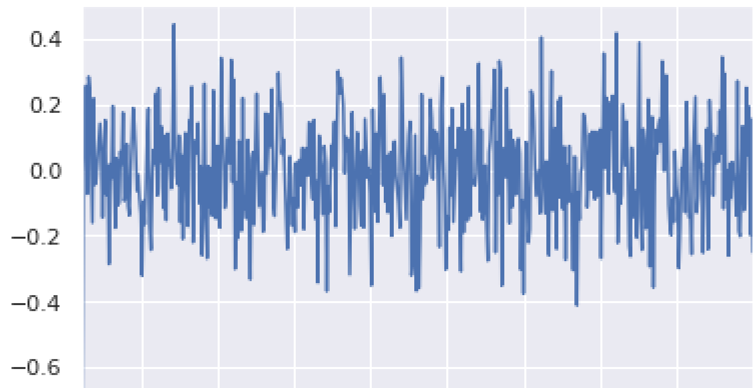
```
ds
1958-03    60.445055
1958-04    -0.073209
1958-05     0.030382
1958-06    -0.308211
1958-07     0.241789
             ...
2003-09     0.045396
2003-10     0.194082
2003-11    -0.197412
2003-12     0.155529
2004-01    -0.250193
Freq: M, Length: 551, dtype: float64
```

```
resid = residuals[24:]
resid
```

```
ds
1960-03   -0.760817
1960-04   -0.014800
1960-05    0.152592
1960-06    0.257897
1960-07   -0.072371
             ...
2003-09    0.045396
2003-10    0.194082
2003-11   -0.197412
2003-12    0.155529
2004-01   -0.250193
Freq: M, Length: 527, dtype: float64
```

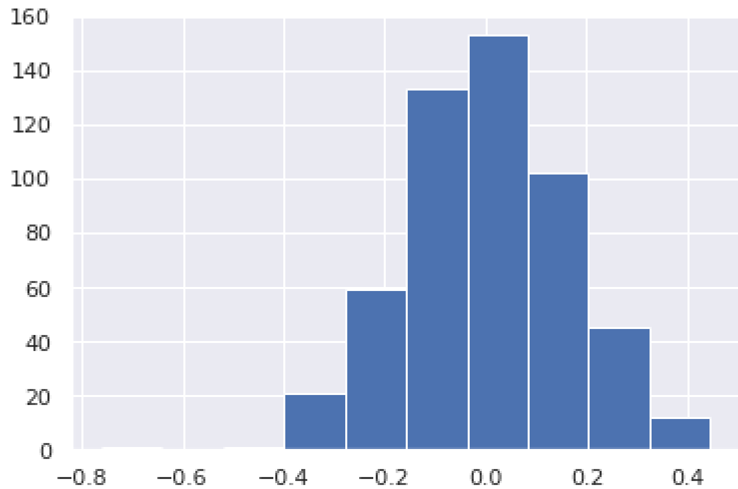
```
resid.plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f12a1da6110>
```

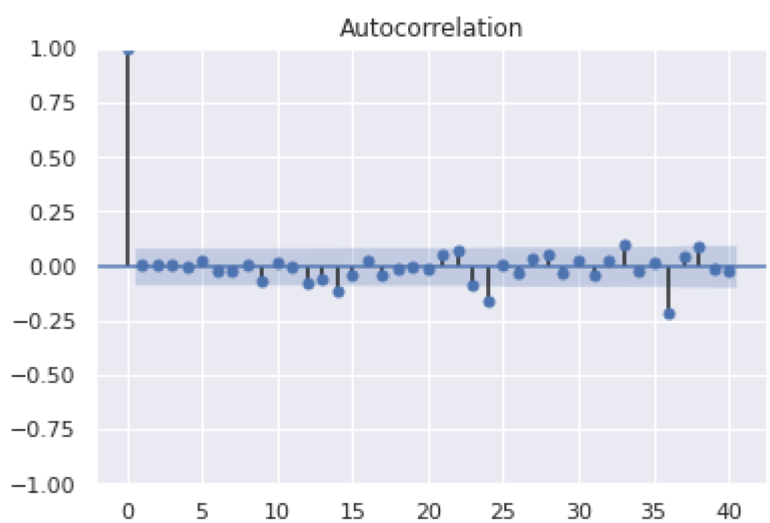



```
resid.hist()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f12a2029610>
```




```
plot_acf(resid, lags = 40)
plt.show()
```




```
sm.stats.acorr_ljungbox(resid, lags=[10], return_df=True, boxpierce=True)
```

	lb_stat	lb_pvalue	bp_stat	bp_pvalue
10	3.501347	0.967053	3.432918	0.969327

2.E.

```
y_pred = sarimax_fit.forecast(steps = len(y_test['temp']),  exog=y_test['co2'])
```

```
y_train
```

ds	temp	co2
1958-03	57.38	315.700
1958-04	57.29	317.450
1958-05	57.32	317.510
1958-06	57.02	316.685
1958-07	57.27	315.860
...	...	...
2003-09	58.02	372.980
2003-10	58.05	373.000
2003-11	57.84	374.350
2003-12	58.06	375.690
2004-01	57.92	376.790

551 rows × 2 columns

```
from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError
smape = MeanAbsolutePercentageError(symmetric=True)
smape(y_test['temp'], y_pred)
```

```
0.005245846308345825
```

```
from sktime.performance_metrics.forecasting import MeanAbsoluteError
mae = MeanAbsoluteError()
mae(y_test['temp'], y_pred)
```

```
0.3055643484278933
```

```
from sktime.performance_metrics.forecasting import MeanSquaredError
```

```
mse = MeanSquaredError()
mse(y_test['temp'], y_pred)
```

    0.1277373148574044

```
from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError
mape = MeanAbsolutePercentageError(symmetric=False)
mape(y_test['temp'], y_pred)
```

    0.005264456692487014

```
sarimax_fit.summary()
```

SARIMAX Results

Dep. Variable:	temp	**No. Observations:**	551
Model:	SARIMAX(3, 1, 1)x(2, 1, [], 12)	**Log Likelihood**	224.012
Date:	Sat, 14 May 2022	**AIC**	-430.023
Time:	04:13:47	**BIC**	-391.433
Sample:	03-31-1958	**HQIC**	-414.928
	- 01-31-2004		
Covariance Type:	opg		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
intercept	0.0004	0.003	0.151	0.880	-0.005	0.006
co2	-0.0097	0.022	-0.446	0.655	-0.052	0.033
ar.L1	0.0165	0.146	0.113	0.910	-0.270	0.303
ar.L2	0.0278	0.089	0.314	0.753	-0.146	0.202
ar.L3	-0.1175	0.061	-1.915	0.056	-0.238	0.003
ma.L1	-0.5834	0.140	-4.155	0.000	-0.859	-0.308
ar.S.L12	-0.7436	0.041	-18.355	0.000	-0.823	-0.664
ar.S.L24	-0.3330	0.040	-8.241	0.000	-0.412	-0.254
sigma2	0.0251	0.002	16.587	0.000	0.022	0.028

Ljung-Box (L1) (Q):	0.01	**Jarque-Bera (JB):**	0.57
Prob(Q):	0.92	**Prob(JB):**	0.75
Heteroskedasticity (H):	1.13	**Skew:**	-0.07
Prob(H) (two-sided):	0.42	**Kurtosis:**	3.08

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

Part 3. Prophet with additional regressors

3.A.

```
! pip install pystan --user
```

```
! pip install fbprophet --user
```

```
Requirement already satisfied: pystan in /usr/local/lib/python3.7/dist-packages (2.19.1
Requirement already satisfied: numpy>=1.7 in /usr/local/lib/python3.7/dist-packages (fr
Requirement already satisfied: Cython!=0.25.1,>=0.22 in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: fbprophet in /usr/local/lib/python3.7/dist-packages (0.7
Requirement already satisfied: Cython>=0.22 in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: cmdstanpy==0.9.5 in /usr/local/lib/python3.7/dist-packag
Requirement already satisfied: pystan>=2.14 in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: numpy>=1.15.4 in /usr/local/lib/python3.7/dist-packages
Requirement already satisfied: pandas>=1.0.4 in /usr/local/lib/python3.7/dist-packages
Requirement already satisfied: matplotlib>=2.0.0 in /usr/local/lib/python3.7/dist-packa
Requirement already satisfied: LunarCalendar>=0.0.9 in /usr/local/lib/python3.7/dist-pa
Requirement already satisfied: convertdate>=2.1.2 in /usr/local/lib/python3.7/dist-pack
Requirement already satisfied: holidays>=0.10.2 in /usr/local/lib/python3.7/dist-packag
Requirement already satisfied: setuptools-git>=1.2 in /usr/local/lib/python3.7/dist-pac
Requirement already satisfied: python-dateutil>=2.8.0 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: tqdm>=4.36.1 in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: pymeeus<=1,>=0.3.13 in /usr/local/lib/python3.7/dist-pac
Requirement already satisfied: hijri-converter in /usr/local/lib/python3.7/dist-package
Requirement already satisfied: six in /usr/local/lib/python3.7/dist-packages (from holi
Requirement already satisfied: korean-lunar-calendar in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: ephem>=3.7.5.3 in /usr/local/lib/python3.7/dist-packages
Requirement already satisfied: pytz in /usr/local/lib/python3.7/dist-packages (from Lun
Requirement already satisfied: kiwisolver>=1.0.1 in /usr/local/lib/python3.7/dist-packa
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: pyparsing!=2.0.4,!=2.1.2,!=2.1.6,>=2.0.1 in /usr/local/l
Requirement already satisfied: typing-extensions in /usr/local/lib/python3.7/dist-packa
```

```
! pip install wbdata
```

```
Requirement already satisfied: wbdata in /usr/local/lib/python3.7/dist-packages (0.3.0)
Requirement already satisfied: decorator>=4.0 in /usr/local/lib/python3.7/dist-packages
Requirement already satisfied: tabulate>=0.8.5 in /usr/local/lib/python3.7/dist-package
Requirement already satisfied: appdirs<2.0,>=1.4 in /usr/local/lib/python3.7/dist-packa
Requirement already satisfied: requests>=2.0 in /usr/local/lib/python3.7/dist-packages
Requirement already satisfied: certifi>=2017.4.17 in /usr/local/lib/python3.7/dist-pack
Requirement already satisfied: idna<3,>=2.5 in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: urllib3!=1.25.0,!=1.25.1,<1.26,>=1.21.1 in /usr/local/li
Requirement already satisfied: chardet<4,>=3.0.2 in /usr/local/lib/python3.7/dist-packa
```

```
from util_prophet import *
from fbprophet import Prophet
from fbprophet.plot import plot_plotly


y_train_prophet = y_train
y_train_prophet['ds'] = y_train_prophet.index
y_train_prophet.rename(columns={'temp':'y'}, inplace=True)

y_test_prophet = y_test
```

```
y_test_prophet['ds'] = y_test_prophet.index
y_test_prophet.rename(columns={'temp':'y'}, inplace=True)

y_train_prophet
```

ds	y	co2	ds
1958-03	57.38	315.700	1958-03
1958-04	57.29	317.450	1958-04
1958-05	57.32	317.510	1958-05
1958-06	57.02	316.685	1958-06
1958-07	57.27	315.860	1958-07
...	...	...	...
2003-09	58.02	372.980	2003-09
2003-10	58.05	373.000	2003-10
2003-11	57.84	374.350	2003-11
2003-12	58.06	375.690	2003-12
2004-01	57.92	376.790	2004-01

551 rows × 3 columns

```
y_train_prophet['ds'] = y_train_prophet['ds'].dt.strftime('%Y-%m').add('-01 00:00:00.000')
print(y_train_prophet)
```

```
             y      co2                       ds
ds
1958-03  57.38  315.700  1958-03-01 00:00:00.000
1958-04  57.29  317.450  1958-04-01 00:00:00.000
1958-05  57.32  317.510  1958-05-01 00:00:00.000
1958-06  57.02  316.685  1958-06-01 00:00:00.000
1958-07  57.27  315.860  1958-07-01 00:00:00.000
...        ...      ...                      ...
2003-09  58.02  372.980  2003-09-01 00:00:00.000
2003-10  58.05  373.000  2003-10-01 00:00:00.000
2003-11  57.84  374.350  2003-11-01 00:00:00.000
2003-12  58.06  375.690  2003-12-01 00:00:00.000
2004-01  57.92  376.790  2004-01-01 00:00:00.000

[551 rows x 3 columns]
```

```
model = Prophet()
model.fit(y_train_prophet)
```

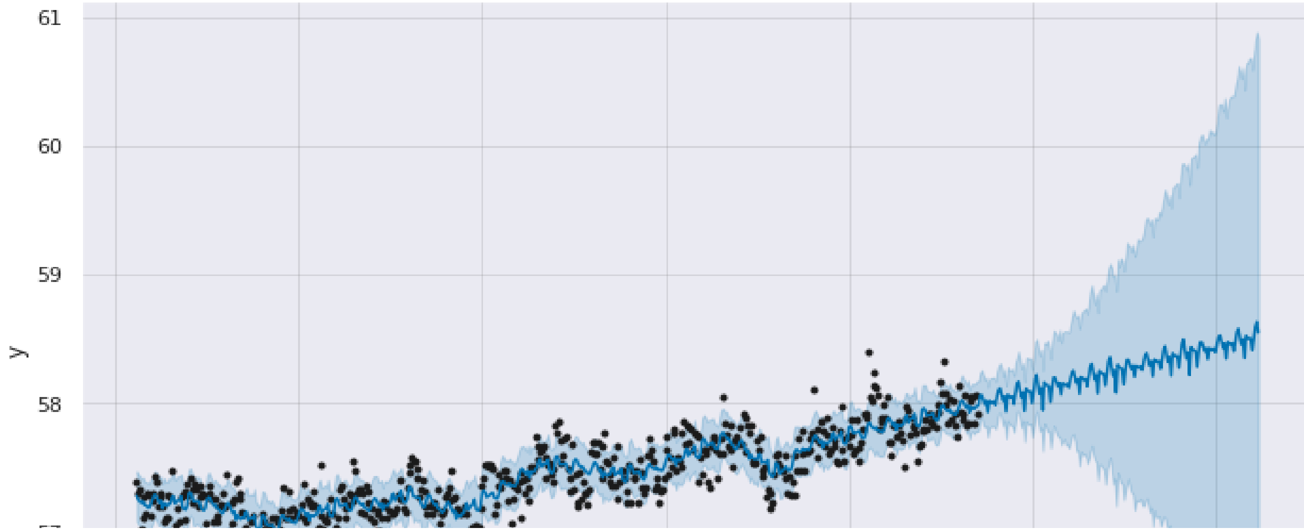
```
INFO:fbprophet:Disabling weekly seasonality. Run prophet with weekly_seasonality=True t
INFO:fbprophet:Disabling daily seasonality. Run prophet with daily_seasonality=True to
<fbprophet.forecaster.Prophet at 0x7f12a22f1a50>
```

```
future_data = model.make_future_dataframe(periods=len(y_test_prophet), freq='m')
```

```
forecast_data = model.predict(future_data)
forecast_data.iloc[0:3].T
```

	0	1	2
ds	1958-03-01 00:00:00	1958-04-01 00:00:00	1958-05-01 00:00:00
trend	57.215189	57.215493	57.215787
yhat_lower	57.079284	57.061034	57.040625
yhat_upper	57.468917	57.448108	57.428112
trend_lower	57.215189	57.215493	57.215787
trend_upper	57.215189	57.215493	57.215787
additive_terms	0.062528	0.031737	0.020392
additive_terms_lower	0.062528	0.031737	0.020392
additive_terms_upper	0.062528	0.031737	0.020392
yearly	0.062528	0.031737	0.020392
yearly_lower	0.062528	0.031737	0.020392
yearly_upper	0.062528	0.031737	0.020392
multiplicative_terms	0.0	0.0	0.0
multiplicative_terms_lower	0.0	0.0	0.0
multiplicative_terms_upper	0.0	0.0	0.0
yhat	57.277717	57.24723	57.236179

```
p = model.plot(forecast_data)
```




```
#forecast_data['ds']
```

```
0      1958-03-01
1      1958-04-01
2      1958-05-01
3      1958-06-01
4      1958-07-01
          ...
730    2018-12-31
731    2019-01-31
732    2019-02-28
733    2019-03-31
734    2019-04-30
Name: ds, Length: 735, dtype: datetime64[ns]
```

```
#forecast_data.set_index('ds', inplace = True)
```

```
---------------------------------------------------------------------------
KeyError                                  Traceback (most recent call last)
<ipython-input-421-c730bfc7766a> in <module>()
----> 1 forecast_data.set_index('ds', inplace = True)

1 frames

/usr/local/lib/python3.7/dist-packages/pandas/core/frame.py in set_index(self, keys, drop, append, inplace, verify_integrity)
   5449
   5450         if missing:
-> 5451             raise KeyError(f"None of {missing} are in the columns")
   5452
   5453         if inplace:

KeyError: "None of ['ds'] are in the columns"
```

SEARCH STACK OVERFLOW

```
forecast_data
```

	ds	trend	yhat_lower	yhat_upper	trend_lower	trend_upper	additive_terms
0	1958-03-01	57.215189	57.079284	57.468917	57.215189	57.215189	0.062528
1	1958-04-01	57.215493	57.061034	57.448108	57.215493	57.215493	0.031737
2	1958-05-01	57.215787	57.040625	57.428112	57.215787	57.215787	0.020392
3	1958-06-01	57.216091	57.033678	57.409110	57.216091	57.216091	0.007273
4	1958-07-01	57.216385	57.040420	57.419753	57.216385	57.216385	0.018871
...	...	...	...	...	...	...	...
730	2018-12-31	58.493866	55.995002	60.590794	56.058165	60.687053	-0.084658
731	2019-01-31	58.496772	56.021374	60.722300	56.024298	60.706419	0.011692
732	2019-02-28	58.499396	56.073146	60.827935	56.001594	60.726005	0.092830
733	2019-03-31	58.502302	56.163630	60.885166	55.980869	60.746802	0.127100
734	2019-04-30	58.505113	55.953883	60.832449	55.961475	60.766289	0.043005

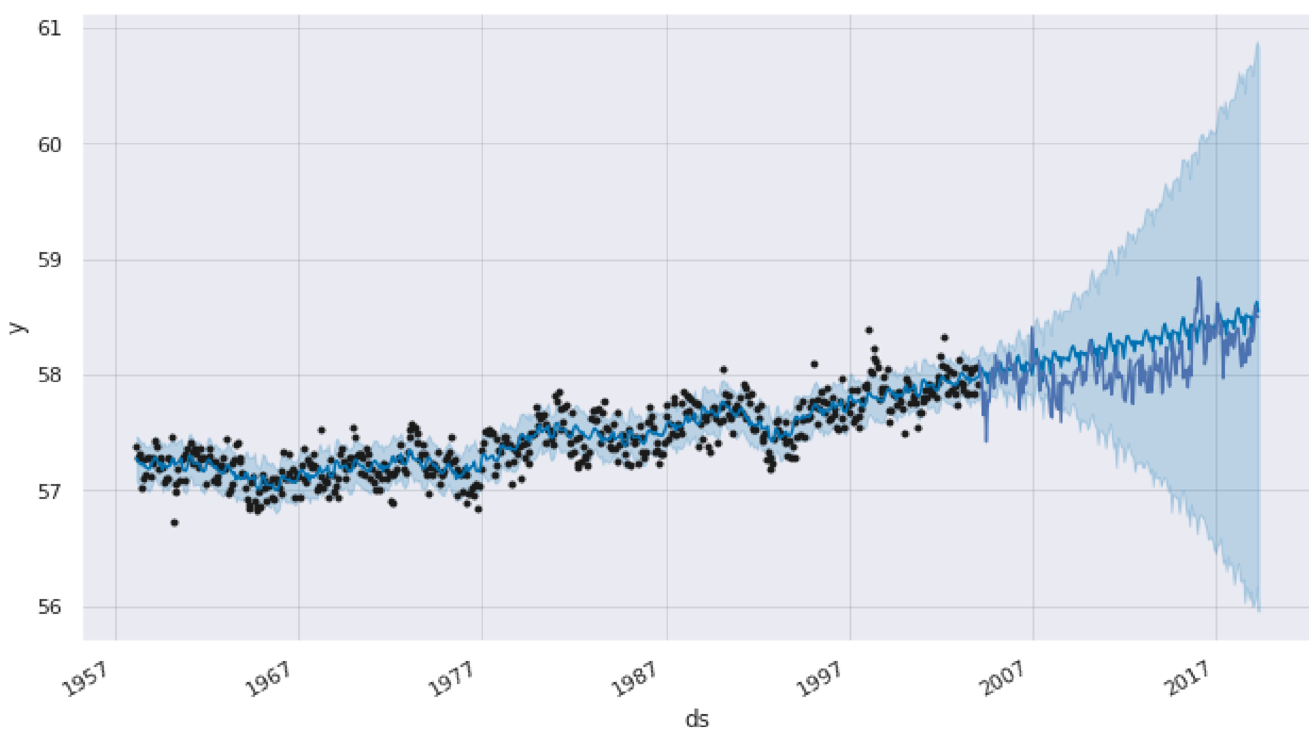
735 rows × 16 columns

```
y_test_prophet
```

	y	co2	ds
ds			
2004-02	58.12	377.37	2004-02
2004-03	58.04	378.39	2004-03
2004-04	57.96	380.50	2004-04
2004-05	57.65	380.62	2004-05

```
p = model.plot(forecast_data)
#plt.scatter(y_test_prophet['ds'],y_test_prophet['y'], color='r', marker='.')
#plt.show()
y_test_prophet['y'].plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f12a26d4590>
```




```
from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError
smape = MeanAbsolutePercentageError(symmetric=True)
smape(y_test_prophet['y'], forecast_data)
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
<ipython-input-434-e095710e47f1> in <module>()
      1 from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError
      2 smape = MeanAbsolutePercentageError(symmetric=True)
----> 3 smape(y_test_prophet['y'], forecast_data)
```

3 frames

```
/usr/local/lib/python3.7/dist-packages/sklearn/utils/validation.py in
```

3.B.

```
--> 334             % [int(l) for l in lengths]
```

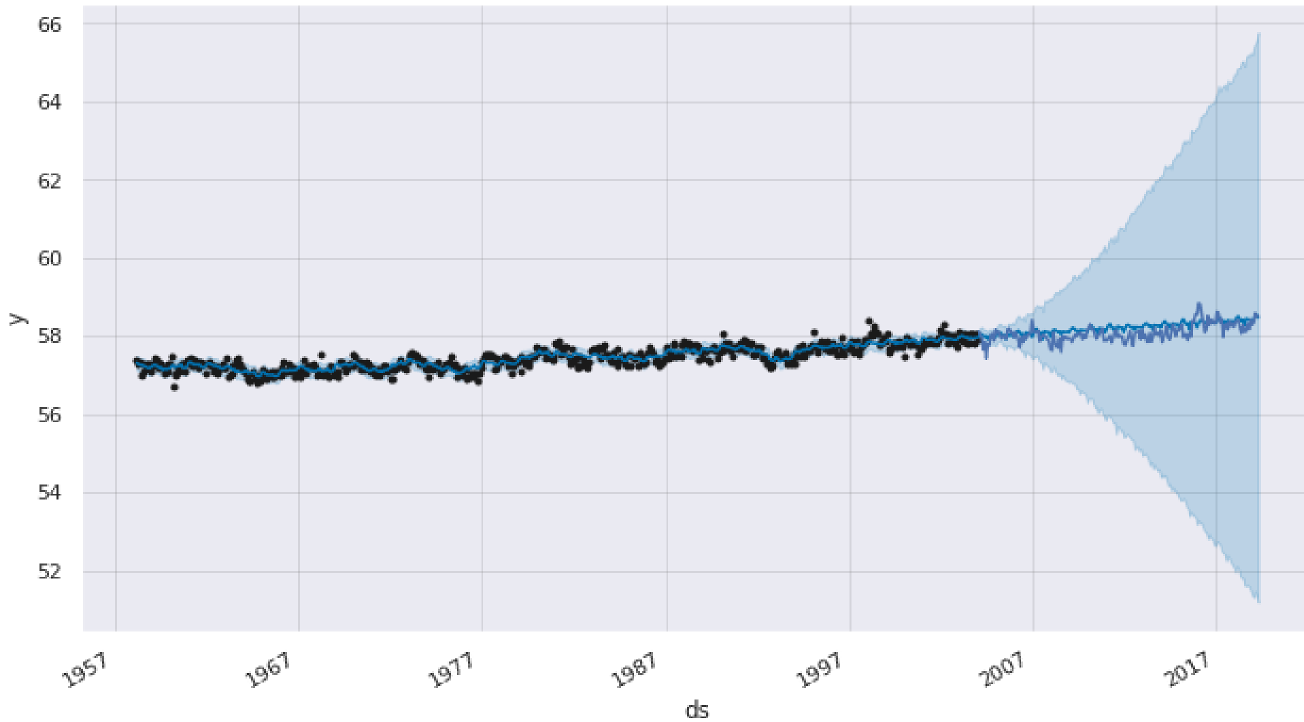
```
model_seasonal = Prophet(seasonality_prior_scale=10, changepoint_prior_scale=5)
model_seasonal.fit(y_train_prophet)
future_data_s = model_seasonal.make_future_dataframe(periods=len(y_test), freq='m')
forecast_data_s = model_seasonal.predict(future_data_s)
forecast_data_s.iloc[0:3].T
```

```
INFO:fbprophet:Disabling weekly seasonality. Run prophet with weekly_seasonality=True t
INFO:fbprophet:Disabling daily seasonality. Run prophet with daily_seasonality=True to
```

	0	1	2
ds	1958-03-01 00:00:00	1958-04-01 00:00:00	1958-05-01 00:00:00
trend	57.258126	57.252792	57.247631
yhat_lower	57.12644	57.106863	57.086879
yhat_upper	57.495843	57.45507	57.445642
trend_lower	57.258126	57.252792	57.247631
trend_upper	57.258126	57.252792	57.247631
additive_terms	0.060569	0.030984	0.020538
additive_terms_lower	0.060569	0.030984	0.020538
additive_terms_upper	0.060569	0.030984	0.020538
yearly	0.060569	0.030984	0.020538
yearly_lower	0.060569	0.030984	0.020538
yearly_upper	0.060569	0.030984	0.020538
multiplicative_terms	0.0	0.0	0.0
multiplicative_terms_lower	0.0	0.0	0.0
multiplicative_terms_upper	0.0	0.0	0.0
yhat	57.318695	57.283777	57.268169

```
p_s = model_seasonal.plot(forecast_data_s)
y_test_prophet['y'].plot()
```

<matplotlib.axes._subplots.AxesSubplot at 0x7f12a330a710>




```
ind_s = forecast_data_s[forecast_data_s['ds'] == y_test.index[0]].index[0]
y_pred_s = forecast_data_s[ind:]['yhat']
```

```
---------------------------------------------------------------------------
IndexError                                Traceback (most recent call last)
<ipython-input-438-0f1a2f49f211> in <module>()
----> 1 ind_s = forecast_data_s[forecast_data_s['ds'] == y_test.index[0]].index[0]
      2 y_pred_s = forecast_data_s[ind:]['yhat']

/usr/local/lib/python3.7/dist-packages/pandas/core/indexes/base.py in
__getitem__(self, key)
   4602         if is_scalar(key):
   4603             key = com.cast_scalar_indexer(key, warn_float=True)
-> 4604             return getitem(key)
   4605 
   4606         if isinstance(key, slice):

IndexError: index 0 is out of bounds for axis 0 with size 0
```

SEARCH STACK OVERFLOW

```
from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError
smape = MeanAbsolutePercentageError(symmetric=True)
smape(y_test_prophet['y'], forecast_data)
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
<ipython-input-447-e095710e47f1> in <module>()
      1 from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError
      2 smape = MeanAbsolutePercentageError(symmetric=True)
----> 3 smape(y_test_prophet['y'], forecast_data)
```

3 frames

```
/usr/local/lib/python3.7/dist-packages/sklearn/utils/validation.py in
check_consistent_length(*arrays)
    332         raise ValueError(
    333             "Found input variables with inconsistent numbers of samples: %r"
--> 334             % [int(l) for l in lengths]
    335         )
    336

ValueError: Found input variables with inconsistent numbers of samples: [184, 735]
```

3.C.

```
y_train_prophet
```

ds	y	co2	ds
1958-03	57.38	315.700	1958-03-01 00:00:00.000
1958-04	57.29	317.450	1958-04-01 00:00:00.000
1958-05	57.32	317.510	1958-05-01 00:00:00.000
1958-06	57.02	316.685	1958-06-01 00:00:00.000
1958-07	57.27	315.860	1958-07-01 00:00:00.000
...	...	...	...
2003-09	58.02	372.980	2003-09-01 00:00:00.000
2003-10	58.05	373.000	2003-10-01 00:00:00.000
2003-11	57.84	374.350	2003-11-01 00:00:00.000
2003-12	58.06	375.690	2003-12-01 00:00:00.000
2004-01	57.92	376.790	2004-01-01 00:00:00.000

551 rows × 3 columns

```
y_test_prophet
```

	y	co2	ds
ds			
2004-02	58.12	377.37	2004-02
2004-03	58.04	378.39	2004-03
2004-04	57.96	380.50	2004-04
2004-05	57.65	380.62	2004-05
2004-06	57.85	379.55	2004-06
...	...	...	...
2019-01	58.29	410.92	2019-01
2019-02	58.37	411.66	2019-02
2019-03	58.59	412.00	2019-03
2019-04	58.50	413.51	2019-04
2019-05	58.50	414.83	2019-05

```
def co2(ds):
    date = (pd.to_datetime(ds)).date()

    if y_train_prohet[date:].empty:
        return y_test_prohet[date:]['co2'].values[0]
    else:
        return (y_train_prophet[date:]['co2']).values[0]

    return 0


mr = Prophet(seasonality_prior_scale=10, changepoint_prior_scale=5)
mr.add_regressor('co2')
mr.fit(y_train_prophet)
future_r = mr.make_future_dataframe(periods=len(y_test_prophet), freq='m')
future_r['co2'] = future_r['ds'].apply(co2)
forecast_r = mr.predict(future_r)
```

```
INFO:fbprophet:Disabling weekly seasonality. Run prophet with weekly_seasonality=True t
INFO:fbprophet:Disabling daily seasonality. Run prophet with daily_seasonality=True to
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-443-e882fa91fb62> in <module>()
      3 mr.fit(y_train_prophet)
      4 future_r = mr.make_future_dataframe(periods=len(y_test_prophet), freq='m')
----> 5 future_r['co2'] = future_r['ds'].apply(co2)
      6 forecast_r = mr.predict(future_r)
```

```
forecast_r
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-444-30089bd039fc> in <module>()
----> 1 forecast_r

NameError: name 'forecast_r' is not defined
```

SEARCH STACK OVERFLOW

```
ind_r = forecast_r[forecast_r['ds'] == y_test_prophet.index[0]].index[0]
y_pred_r = forecast_r[ind:]['yhat']

mae =  mean_absolute_error(y_pred_r, y_true)
mse = mean_squared_error(y_pred_r, y_true)
mape = mean_absolute_percentage_error(y_pred_r, y_true)
#smape = smape(np.array(y_pred_s), np.array(y_true))
mase = MeanAbsoluteScaledError(sp=12)
mase_val = mase(y_pred_r, y_true, y_train=y_train['y'])

print('Seasonal Arima errors')
print(f'MAE: {mae}')
print(f'MSE: {mse}')
print(f'MAPE: {mape}')
#print(f'SMAPE: {smape}')
print(f'MASE: {mase_val}')
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-445-d7457e43d4c4> in <module>()
----> 1 ind_r = forecast_r[forecast_r['ds'] == y_test_prophet.index[0]].index[0]
      2 y_pred_r = forecast_r[ind:]['yhat']
      3 
      4 mae =  mean_absolute_error(y_pred_r, y_true)
      5 mse = mean_squared_error(y_pred_r, y_true)

NameError: name 'forecast_r' is not defined
```

SEARCH STACK OVERFLOW

3.D.

Prophet model is the one with a better predictive power in contrast with previous models. Also, by adjusting per seasonality and changes points, and adding a external regressors we have the possibility of increasing the forecast performance of the fitted model.

Part 4. Conclusion

4.A.

According to performance metrics (MAE, MSE, MAPE, sMAPE, MASE), Prophet model is the one with a better predictive power. Also, we can increase the forecast power by adjusting per seasonality and changes points, and using external regressor it is possible to increase the forecast power of the fitted model.

```
### 4.B.

If I had this data for 100 locations (differen
I would apply multiple time series modeling us
would take advantage of the options to fitt a
performance, as by adjusting per seasonality a
adding external regressors.
```

4.B.

If I had this data for 100 locations (different pairs of time series), I would apply multiple time series modeling using prophet. Also, I would take advantage of the options to fitt a model with a better performance, as by adjusting per seasonality and changes points, and adding external regressors.

0 s completado a las 23:38