Problem Set - Week 6

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Part 1. EDA

A

In [1]:
```
! pip install sktime
```

```
Requirement already satisfied: sktime in c:\users\guill\anaconda3\lib\site-pack
ages (0.11.2)
Requirement already satisfied: pandas<1.5.0,>=1.1.0 in c:\users\guill\anaconda3
\lib\site-packages (from sktime) (1.4.2)
Requirement already satisfied: numba>=0.53 in c:\users\guill\anaconda3\lib\site
-packages (from sktime) (0.53.1)
Requirement already satisfied: deprecated>=1.2.13 in c:\users\guill\anaconda3\l
ib\site-packages (from sktime) (1.2.13)
Requirement already satisfied: scipy<1.8.0 in c:\users\guill\anaconda3\lib\site
-packages (from sktime) (1.6.2)
Requirement already satisfied: scikit-learn>=0.24.0 in c:\users\guill\anaconda3
\lib\site-packages (from sktime) (0.24.1)
Requirement already satisfied: numpy<1.22,>=1.21.0 in c:\users\guill\anaconda3
\lib\site-packages (from sktime) (1.21.6)
Requirement already satisfied: statsmodels>=0.12.1 in c:\users\guill\anaconda3
\lib\site-packages (from sktime) (0.12.2)
Requirement already satisfied: wrapt<2,>=1.10 in c:\users\guill\anaconda3\lib\s
ite-packages (from deprecated>=1.2.13->sktime) (1.12.1)
Requirement already satisfied: setuptools in c:\users\guill\anaconda3\lib\site-
packages (from numba>=0.53->sktime) (52.0.0.post20210125)
Requirement already satisfied: llvmlite<0.37,>=0.36.0rc1 in c:\users\guill\anac
onda3\lib\site-packages (from numba>=0.53->sktime) (0.36.0)
Requirement already satisfied: pytz>=2020.1 in c:\users\guill\anaconda3\lib\sit
e-packages (from pandas<1.5.0,>=1.1.0->sktime) (2021.1)
Requirement already satisfied: python-dateutil>=2.8.1 in c:\users\guill\anacond
a3\lib\site-packages (from pandas<1.5.0,>=1.1.0->sktime) (2.8.1)
Requirement already satisfied: six>=1.5 in c:\users\guill\anaconda3\lib\site-pa
ckages (from python-dateutil>=2.8.1->pandas<1.5.0,>=1.1.0->sktime) (1.15.0)
Requirement already satisfied: threadpoolctl>=2.0.0 in c:\users\guill\anaconda3
\lib\site-packages (from scikit-learn>=0.24.0->sktime) (2.1.0)
Requirement already satisfied: joblib>=0.11 in c:\users\guill\anaconda3\lib\sit
e-packages (from scikit-learn>=0.24.0->sktime) (1.0.1)
Requirement already satisfied: patsy>=0.5 in c:\users\guill\anaconda3\lib\site-
packages (from statsmodels>=0.12.1->sktime) (0.5.1)

WARNING: Ignoring invalid distribution -atplotlib (c:\users\guill\anaconda3\lib
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WARNING: Ignoring invalid distribution -atplotlib (c:\users\guill\anaconda3\lib
\site-packages)
WARNING: You are using pip version 22.0.4; however, version 22.1 is available.
You should consider upgrading via the 'C:\Users\guill\anaconda3\python.exe -m p
ip install --upgrade pip' command.
```

In [2]:
```
! pip install pmdarima
```

```
Requirement already satisfied: pmdarima in c:\users\guill\anaconda3\lib\site-pa
ckages (1.8.5)
Requirement already satisfied: Cython!=0.29.18,>=0.29 in c:\users\guill\anacond
a3\lib\site-packages (from pmdarima) (0.29.23)
Requirement already satisfied: scipy>=1.3.2 in c:\users\guill\anaconda3\lib\sit
e-packages (from pmdarima) (1.6.2)
Requirement already satisfied: numpy>=1.19.3 in c:\users\guill\anaconda3\lib\si
te-packages (from pmdarima) (1.21.6)
Requirement already satisfied: setuptools!=50.0.0,>=38.6.0 in c:\users\guill\an
aconda3\lib\site-packages (from pmdarima) (52.0.0.post20210125)
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Requirement already satisfied: joblib>=0.11 in c:\users\guill\anaconda3\lib\sit
e-packages (from pmdarima) (1.0.1)
Requirement already satisfied: urllib3 in c:\users\guill\anaconda3\lib\site-pac
kages (from pmdarima) (1.26.4)
Requirement already satisfied: statsmodels!=0.12.0,>=0.11 in c:\users\guill\ana
conda3\lib\site-packages (from pmdarima) (0.12.2)
Requirement already satisfied: scikit-learn>=0.22 in c:\users\guill\anaconda3\l
ib\site-packages (from pmdarima) (0.24.1)
Requirement already satisfied: python-dateutil>=2.8.1 in c:\users\guill\anacond
a3\lib\site-packages (from pandas>=0.19->pmdarima) (2.8.1)
Requirement already satisfied: pytz>=2020.1 in c:\users\guill\anaconda3\lib\sit
e-packages (from pandas>=0.19->pmdarima) (2021.1)
Requirement already satisfied: six>=1.5 in c:\users\guill\anaconda3\lib\site-pa
ckages (from python-dateutil>=2.8.1->pandas>=0.19->pmdarima) (1.15.0)
Requirement already satisfied: threadpoolctl>=2.0.0 in c:\users\guill\anaconda3
\lib\site-packages (from scikit-learn>=0.22->pmdarima) (2.1.0)
Requirement already satisfied: patsy>=0.5 in c:\users\guill\anaconda3\lib\site-
packages (from statsmodels!=0.12.0,>=0.11->pmdarima) (0.5.1)


WARNING: Ignoring invalid distribution -atplotlib (c:\users\guill\anaconda3\lib
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WARNING: Ignoring invalid distribution -atplotlib (c:\users\guill\anaconda3\lib
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WARNING: Ignoring invalid distribution -atplotlib (c:\users\guill\anaconda3\lib
\site-packages)
WARNING: You are using pip version 22.0.4; however, version 22.1 is available.
You should consider upgrading via the 'C:\Users\guill\anaconda3\python.exe -m p
ip install --upgrade pip' command.
```

In [3]:
```
import warnings
warnings.filterwarnings('ignore')
import pandas as pd
from statsmodels.tsa.stattools import kpss, adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from copy import deepcopy
import matplotlib.pyplot as plt
import numpy as np
from sktime.forecasting.all import temporal_train_test_split
from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError, M
from sklearn.metrics import mean_squared_error
import seaborn as sns
import numpy as np
import statsmodels.api as sm
```

In [4]:
```
df = pd.read_csv("hw6_data_var.csv", parse_dates=["Unnamed: 0"]).rename(columns={
df.set_index('ds', inplace=True)
df
```

Out[4]:

ds	PCE	AHE	PCEPI
2006-03-01	9122.1	20.04	88.473
2006-04-01	9174.8	20.17	88.850
2006-05-01	9215.1	20.13	89.070
2006-06-01	9240.8	20.22	89.285
2006-07-01	9322.6	20.30	89.601
...	...	...	...
2019-08-01	14650.9	28.16	110.115
2019-09-01	14673.2	28.15	110.167
2019-10-01	14728.5	28.24	110.377
2019-11-01	14752.8	28.33	110.461
2019-12-01	14796.3	28.36	110.750

166 rows × 3 columns

In [5]:
```
df.describe().T
```

Out[5]:

	count	mean	std	min	25%	50%	75%	max
PCE	166.0	11507.872289	1630.579647	9122.100	10009.0000	11204.100	12789.90000	14796.30
AHE	166.0	23.938193	2.235573	20.040	22.2375	23.765	25.67500	28.36
PCEPI	166.0	100.006169	6.028324	88.473	95.1450	100.861	104.18625	110.75

In [6]:
```
df.isnull().sum()
```

Out[6]:
```
PCE      0
AHE      0
PCEPI    0
dtype: int64
```

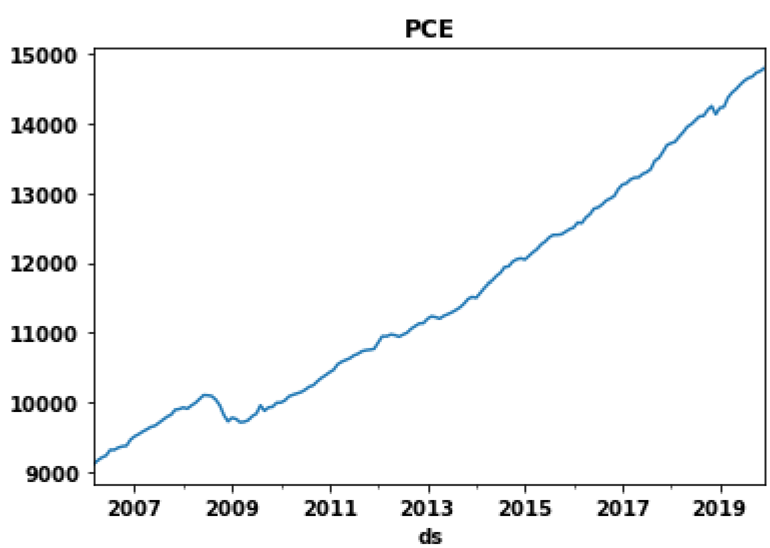
In [7]:
```
print('Start date:', df.index.min())
print('End date:', df.index.max())
df.agg(
    {
        "PCE": ["min", "max", "median", "mean", "std"],
        "AHE": ["min", "max", "median", "mean", "std"],
        "PCEPI": ["min", "max", "median", "mean", "std"],
    }
)
```

```
Start date: 2006-03-01 00:00:00
End date: 2019-12-01 00:00:00
```

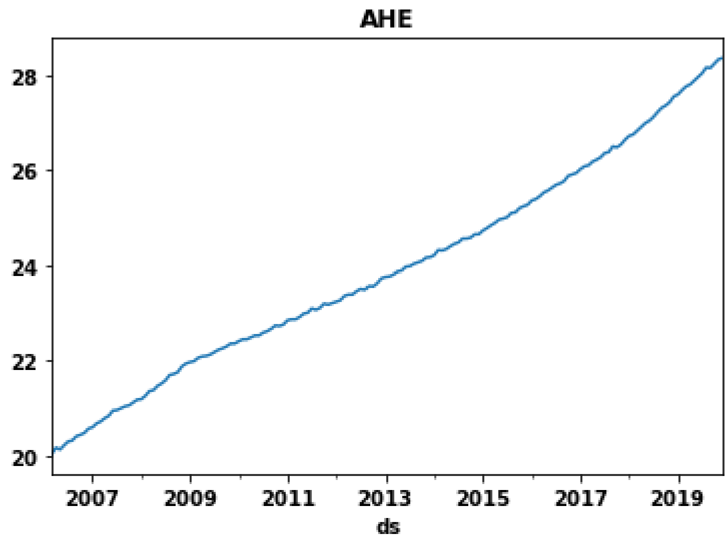
Out[7]:

	PCE	AHE	PCEPI
min	9122.100000	20.040000	88.473000
max	14796.300000	28.360000	110.750000
median	11204.100000	23.765000	100.861000
mean	11507.872289	23.938193	100.006169
std	1630.579647	2.235573	6.028324

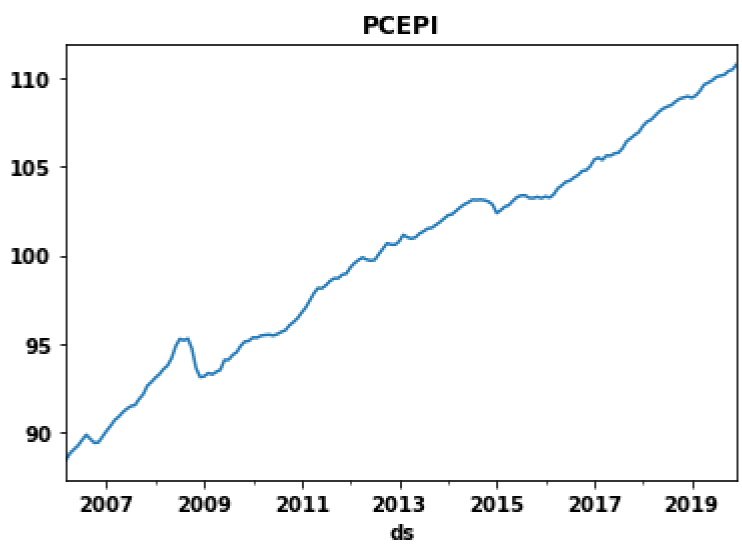
In [8]:
```
df['PCE'].dropna().plot()
plt.title('PCE')
plt.show()
```



In [9]:
```
df['AHE'].plot()
plt.title('AHE')
plt.show()
```



In [10]:
```
df['PCEPI'].plot()
plt.title('PCEPI')
plt.show()
```



In [11]:
```
df.corr()
```

Out[11]:

	PCE	AHE	PCEPI
PCE	1.000000	0.988292	0.973768
AHE	0.988292	1.000000	0.988329
PCEPI	0.973768	0.988329	1.000000

In [12]:
```
from statsmodels.tsa.stattools import kpss, adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from copy import deepcopy

def adf_test(timeseries):
    print ('Dickey-Fuller Test Result:')
    dftest = adfuller(timeseries, autolag='AIC')
    dfoutput = pd.Series(dftest[0:4], index=['Test Statistic','p-value','#Lags Us
    for key,value in dftest[4].items():
       dfoutput['Critical Value (%s)'%key] = value
    print (dfoutput)

#define KPSS
def kpss_test(timeseries, trend='c'):
    print ('KPSS Test Result:')
    kpsstest = kpss(timeseries, regression=trend)
    kpss_output = pd.Series(kpsstest[0:3], index=['Test Statistic','p-value','Lag
    for key,value in kpsstest[3].items():
        kpss_output['Critical Value (%s)'%key] = value
    print (kpss_output)
```

In [13]:
```
adf_test(df['PCE'])
```

```
Dickey-Fuller Test Result:
Test Statistic                   2.189889
p-value                          0.998874
#Lags Used                       2.000000
Number of Observations Used    163.000000
Critical Value (1%)             -3.471119
Critical Value (5%)             -2.879441
Critical Value (10%)            -2.576314
dtype: float64
```

In [14]:
```
adf_test(df['PCEPI'])
```

```
Dickey-Fuller Test Result:
Test Statistic                  -0.434152
p-value                          0.904239
#Lags Used                       1.000000
Number of Observations Used    164.000000
Critical Value (1%)             -3.470866
Critical Value (5%)             -2.879330
Critical Value (10%)            -2.576255
dtype: float64
```

In [15]:
```
adf_test(df['AHE'])
```

```
Dickey-Fuller Test Result:
Test Statistic                  1.680032
p-value                         0.998079
#Lags Used                      9.000000
Number of Observations Used   156.000000
Critical Value (1%)            -3.472979
Critical Value (5%)            -2.880252
Critical Value (10%)           -2.576747
dtype: float64
```

In [16]:
```
kpss_test(df['PCE'])
```

```
KPSS Test Result:
Test Statistic           1.190019
p-value                  0.010000
Lags Used               14.000000
Critical Value (10%)     0.347000
Critical Value (5%)      0.463000
Critical Value (2.5%)    0.574000
Critical Value (1%)      0.739000
dtype: float64

C:\Users\guill\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:1906: I
nterpolationWarning: The test statistic is outside of the range of p-values ava
ilable in the
look-up table. The actual p-value is smaller than the p-value returned.

  warnings.warn(
```

In [17]:
```
kpss_test(df['AHE'])
```

```
KPSS Test Result:
Test Statistic           1.207634
p-value                  0.010000
Lags Used               14.000000
Critical Value (10%)     0.347000
Critical Value (5%)      0.463000
Critical Value (2.5%)    0.574000
Critical Value (1%)      0.739000
dtype: float64

C:\Users\guill\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:1906: I
nterpolationWarning: The test statistic is outside of the range of p-values ava
ilable in the
look-up table. The actual p-value is smaller than the p-value returned.

  warnings.warn(
```

In [18]:
```
kpss_test(df['PCEPI'])
```

```
KPSS Test Result:
Test Statistic             1.206769
p-value                    0.010000
Lags Used                 14.000000
Critical Value (10%)       0.347000
Critical Value (5%)        0.463000
Critical Value (2.5%)      0.574000
Critical Value (1%)        0.739000
dtype: float64

C:\Users\guill\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:1906: I
nterpolationWarning: The test statistic is outside of the range of p-values ava
ilable in the
look-up table. The actual p-value is smaller than the p-value returned.

  warnings.warn(
```

B.

- The dataset contains three variables:
  - PCE: Personal consumptions expenditures is the primary measure of consumer spending on goods and services in the US economy. This accounts for 2/3 of domestic spending and this is the primary engine that drives future economic growth https://www.bea.gov/resources/methodologies/nipa-handbook/pdf/chapter-05.pdf (https://www.bea.gov/resources/methodologies/nipa-handbook/pdf/chapter-05.pdf) (Links to an external site.)
  - AHE: Average hourly earnings is reported in dollars per hour and is reported monthly
  - PCEPI: Personal consumptions expenditures price index is a measure of the average increase in prices for all domestic personal consumption. A major inflationary measure in the United States
- There are no missing values in the dataset.
- Correlation is present in the relationship between the three variables (higher than 0.97 in all cases).
- The three variables are similar in terms of time series pattern: increasing trend over time.
- ADF and KPSS test show that the time series is stationary for the three variables:
  - **ADF test** is showing, for the three of them, that **non-stationarity cannot be rejected**. The p-value in the test is greater than 0.05 for the three of them, which means that the null hypothesis of non-stationarity cannot be rejected.
  - **KPSS test** is showing that **stationarity in the time series can be rejected**. The p-value is smaller than 0.05, which means that the null hypothesis of stationarity in the time series can be rejected.

Part 2 – Granger Causality

A.

The EDA shows that these two variables are strongly correlated, which intuitively make sense. We don't know the direction of the causal relationship between them, but it is expected that higher the average hourly earnings that greater the personal consumption expenditures and the other way around.

B.

In [19]:
```
df['PCE_diff'] = df['PCE'].diff()
df['AHE_diff'] = df['AHE'].diff()
df['PCEPI_diff'] = df['PCEPI'].diff()
```

C.

In [20]:
```
data = df[['PCE_diff', 'AHE_diff']].dropna()
data
```

Out[20]:

	PCE_diff	AHE_diff
ds		
2006-04-01	52.7	0.13
2006-05-01	40.3	-0.04
2006-06-01	25.7	0.09
2006-07-01	81.8	0.08
2006-08-01	-0.8	0.02
...	...	...
2019-08-01	39.0	0.12
2019-09-01	22.3	-0.01
2019-10-01	55.3	0.09
2019-11-01	24.3	0.09
2019-12-01	43.5	0.03

165 rows × 2 columns

In [21]:
```
alpha=0.05
```

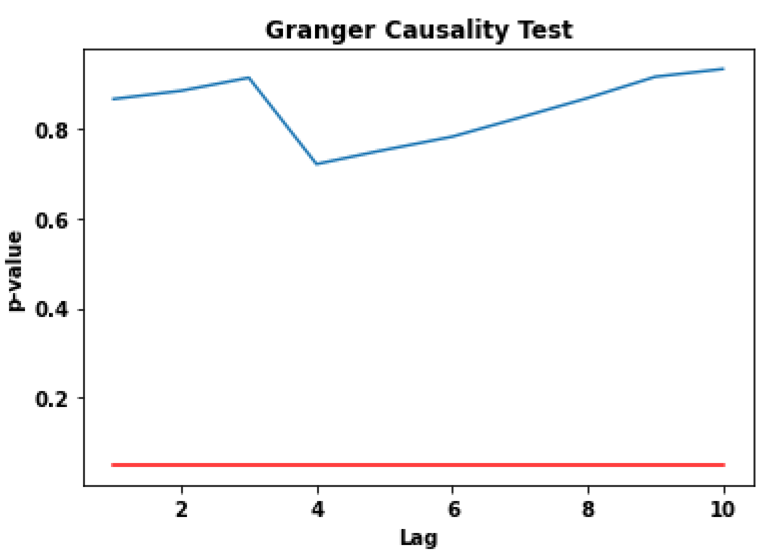
In [22]:
```
from statsmodels.tsa.stattools import grangercausalitytests, q_stat
granger_results = grangercausalitytests(data, maxlag=10, verbose=True)
```

```
Granger Causality
number of lags (no zero) 1
ssr based F test:         F=0.0285  , p=0.8662  , df_denom=161, df_num=1
ssr based chi2 test:   chi2=0.0290  , p=0.8648  , df=1
likelihood ratio test: chi2=0.0290  , p=0.8648  , df=1
parameter F test:         F=0.0285  , p=0.8662  , df_denom=161, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:         F=0.1227  , p=0.8846  , df_denom=158, df_num=2
ssr based chi2 test:   chi2=0.2531  , p=0.8811  , df=2
likelihood ratio test: chi2=0.2529  , p=0.8812  , df=2
parameter F test:         F=0.1227  , p=0.8846  , df_denom=158, df_num=2

Granger Causality
number of lags (no zero) 3
ssr based F test:         F=0.1740  , p=0.9138  , df_denom=155, df_num=3
ssr based chi2 test:   chi2=0.5457  , p=0.9087  , df=3
likelihood ratio test: chi2=0.5448  , p=0.9089  , df=3
parameter F test:         F=0.1740  , p=0.9138  , df_denom=155, df_num=3

Granger Causality
number of lags (no zero) 4
ssr based F test:         F=0.5205  , p=0.7208  , df_denom=152, df_num=4
ssr based chi2 test:   chi2=2.2052  , p=0.6981  , df=4
likelihood ratio test: chi2=2.1903  , p=0.7008  , df=4
parameter F test:         F=0.5205  , p=0.7208  , df_denom=152, df_num=4

Granger Causality
number of lags (no zero) 5
ssr based F test:         F=0.5312  , p=0.7524  , df_denom=149, df_num=5
ssr based chi2 test:   chi2=2.8518  , p=0.7228  , df=5
likelihood ratio test: chi2=2.8267  , p=0.7267  , df=5
parameter F test:         F=0.5312  , p=0.7524  , df_denom=149, df_num=5

Granger Causality
number of lags (no zero) 6
ssr based F test:         F=0.5335  , p=0.7821  , df_denom=146, df_num=6
ssr based chi2 test:   chi2=3.4861  , p=0.7458  , df=6
likelihood ratio test: chi2=3.4484  , p=0.7508  , df=6
parameter F test:         F=0.5335  , p=0.7821  , df_denom=146, df_num=6

Granger Causality
number of lags (no zero) 7
ssr based F test:         F=0.5116  , p=0.8247  , df_denom=143, df_num=7
ssr based chi2 test:   chi2=3.9568  , p=0.7847  , df=7
likelihood ratio test: chi2=3.9081  , p=0.7903  , df=7
parameter F test:         F=0.5116  , p=0.8247  , df_denom=143, df_num=7

Granger Causality
number of lags (no zero) 8
ssr based F test:         F=0.4807  , p=0.8683  , df_denom=140, df_num=8
ssr based chi2 test:   chi2=4.3122  , p=0.8279  , df=8
```

```
likelihood ratio test: chi2=4.2540  , p=0.8335  , df=8
parameter F test:         F=0.4807  , p=0.8683  , df_denom=140, df_num=8

Granger Causality
number of lags (no zero) 9
ssr based F test:         F=0.4320  , p=0.9159  , df_denom=137, df_num=9
ssr based chi2 test:   chi2=4.4271  , p=0.8811  , df=9
likelihood ratio test: chi2=4.3654  , p=0.8858  , df=9
parameter F test:         F=0.4320  , p=0.9159  , df_denom=137, df_num=9

Granger Causality
number of lags (no zero) 10
ssr based F test:         F=0.4233  , p=0.9333  , df_denom=134, df_num=10
ssr based chi2 test:   chi2=4.8967  , p=0.8980  , df=10
likelihood ratio test: chi2=4.8209  , p=0.9028  , df=10
parameter F test:         F=0.4233  , p=0.9333  , df_denom=134, df_num=10
```

In [23]:

```python
res_list = []
for lag, v in granger_results.items():
    res = {}
    res['Lag'] = lag
    for test, stats in v[0].items():
        res[test] = stats[1]

    res_list.append(res)

pvals = pd.DataFrame(res_list)
pvals.set_index('Lag', inplace=True)
pvals_graph = pvals['params_ftest']

pvals_graph.plot(title='Granger Causality Test')
alpha_ser = pd.Series([alpha]*len(pvals_graph), index=pvals_graph.index)
alpha_ser.plot(color='red')
plt.ylabel('p-value')
plt.show()
```



D.

In [24]:
```
data = df[['AHE_diff', 'PCE_diff']].dropna()
data
```

Out[24]:

ds	AHE_diff	PCE_diff
2006-04-01	0.13	52.7
2006-05-01	-0.04	40.3
2006-06-01	0.09	25.7
2006-07-01	0.08	81.8
2006-08-01	0.02	-0.8
...	...	...
2019-08-01	0.12	39.0
2019-09-01	-0.01	22.3
2019-10-01	0.09	55.3
2019-11-01	0.09	24.3
2019-12-01	0.03	43.5

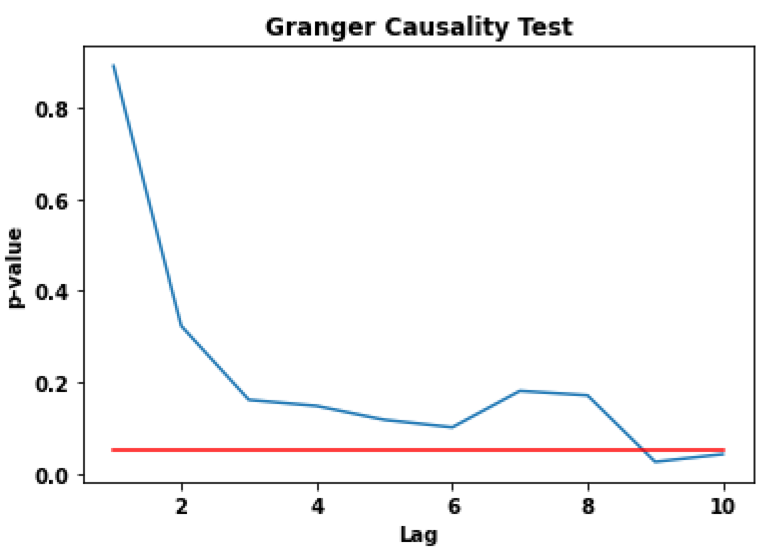
165 rows × 2 columns

In [25]:
```
granger_results = grangercausalitytests(data, maxlag=10, verbose=True)
```

```
Granger Causality
number of lags (no zero) 1
ssr based F test:         F=0.0186  , p=0.8917  , df_denom=161, df_num=1
ssr based chi2 test:   chi2=0.0189  , p=0.8906  , df=1
likelihood ratio test: chi2=0.0189  , p=0.8906  , df=1
parameter F test:         F=0.0186  , p=0.8917  , df_denom=161, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:         F=1.1373  , p=0.3233  , df_denom=158, df_num=2
ssr based chi2 test:   chi2=2.3466  , p=0.3094  , df=2
likelihood ratio test: chi2=2.3298  , p=0.3119  , df=2
parameter F test:         F=1.1373  , p=0.3233  , df_denom=158, df_num=2

Granger Causality
number of lags (no zero) 3
ssr based F test:         F=1.7396  , p=0.1612  , df_denom=155, df_num=3
ssr based chi2 test:   chi2=5.4546  , p=0.1414  , df=3
likelihood ratio test: chi2=5.3648  , p=0.1470  , df=3
parameter F test:         F=1.7396  , p=0.1612  , df_denom=155, df_num=3

Granger Causality
number of lags (no zero) 4
ssr based F test:         F=1.7223  , p=0.1479  , df_denom=152, df_num=4
ssr based chi2 test:   chi2=7.2970  , p=0.1210  , df=4
likelihood ratio test: chi2=7.1365  , p=0.1288  , df=4
parameter F test:         F=1.7223  , p=0.1479  , df_denom=152, df_num=4

Granger Causality
number of lags (no zero) 5
ssr based F test:         F=1.7941  , p=0.1174  , df_denom=149, df_num=5
ssr based chi2 test:   chi2=9.6329  , p=0.0863  , df=5
likelihood ratio test: chi2=9.3540  , p=0.0957  , df=5
parameter F test:         F=1.7941  , p=0.1174  , df_denom=149, df_num=5

Granger Causality
number of lags (no zero) 6
ssr based F test:         F=1.8099  , p=0.1010  , df_denom=146, df_num=6
ssr based chi2 test:   chi2=11.8261 , p=0.0660  , df=6
likelihood ratio test: chi2=11.4069 , p=0.0766  , df=6
parameter F test:         F=1.8099  , p=0.1010  , df_denom=146, df_num=6

Granger Causality
number of lags (no zero) 7
ssr based F test:         F=1.4748  , p=0.1808  , df_denom=143, df_num=7
ssr based chi2 test:   chi2=11.4064 , p=0.1218  , df=7
likelihood ratio test: chi2=11.0134 , p=0.1380  , df=7
parameter F test:         F=1.4748  , p=0.1808  , df_denom=143, df_num=7

Granger Causality
number of lags (no zero) 8
ssr based F test:         F=1.4762  , p=0.1711  , df_denom=140, df_num=8
ssr based chi2 test:   chi2=13.2438 , p=0.1037  , df=8
likelihood ratio test: chi2=12.7147 , p=0.1220  , df=8
```

```
parameter F test:         F=1.4762  , p=0.1711  , df_denom=140, df_num=8

Granger Causality
number of lags (no zero) 9
ssr based F test:         F=2.2035  , p=0.0253  , df_denom=137, df_num=9
ssr based chi2 test:   chi2=22.5822 , p=0.0072  , df=9
likelihood ratio test: chi2=21.0901 , p=0.0123  , df=9
parameter F test:         F=2.2035  , p=0.0253  , df_denom=137, df_num=9

Granger Causality
number of lags (no zero) 10
ssr based F test:         F=1.9624  , p=0.0422  , df_denom=134, df_num=10
ssr based chi2 test:   chi2=22.6999 , p=0.0119  , df=10
likelihood ratio test: chi2=21.1840 , p=0.0198  , df=10
parameter F test:         F=1.9624  , p=0.0422  , df_denom=134, df_num=10
```

In [26]:
```
res_list = []
for lag, v in granger_results.items():
    res = {}
    res['Lag'] = lag
    for test, stats in v[0].items():
        res[test] = stats[1]

    res_list.append(res)

pvals = pd.DataFrame(res_list)
pvals.set_index('Lag', inplace=True)
pvals_graph = pvals['params_ftest']

pvals_graph.plot(title='Granger Causality Test')
alpha_ser = pd.Series([alpha]*len(pvals_graph), index=pvals_graph.index)
alpha_ser.plot(color='red')
plt.ylabel('p-value')
plt.show()
```



E

- Interpretation for the null hypothesis of Granger Test:

- The null hypothesis in the test is that the lagged values of "x" (the explanatory variable) do not explain the variation in "y" (the explained variable).
- First, where x = AHE and y = PCE, we are seeing that the pvalues are greater than 0.05 (until the 10th lag), which means that we cannot reject the null hypothesis of no causal relationship.
- Second, where x = PCE and y = AHE, we are seeing that the pvalues are decreasing with number of laggs until reaching a point in which it gets statistical significant (less than 0.05) by 9th lag, which means that we can reject the null hypothesis of no causal relationship.

Part 3 - VARMA modeling

A.

In [27]:
```
adf_test(df['PCE_diff'].dropna())
```

```
Dickey-Fuller Test Result:
Test Statistic                -6.605709e+00
p-value                        6.564988e-09
#Lags Used                     1.000000e+00
Number of Observations Used    1.630000e+02
Critical Value (1%)           -3.471119e+00
Critical Value (5%)           -2.879441e+00
Critical Value (10%)          -2.576314e+00
dtype: float64
```

In [28]:
```
adf_test(df['PCEPI_diff'].dropna())
```

```
Dickey-Fuller Test Result:
Test Statistic                -7.507425e+00
p-value                        4.099451e-11
#Lags Used                     0.000000e+00
Number of Observations Used    1.640000e+02
Critical Value (1%)           -3.470866e+00
Critical Value (5%)           -2.879330e+00
Critical Value (10%)          -2.576255e+00
dtype: float64
```

In [29]:
```
adf_test(df['AHE_diff'].dropna())
```

```
Dickey-Fuller Test Result:
Test Statistic                 -1.012276
p-value                         0.748761
#Lags Used                     14.000000
Number of Observations Used   150.000000
Critical Value (1%)            -3.474715
Critical Value (5%)            -2.881009
Critical Value (10%)           -2.577151
dtype: float64
```

In [30]:
```
kpss_test(df['PCE_diff'].dropna())
```

```
KPSS Test Result:
Test Statistic            0.487339
p-value                   0.044518
Lags Used                14.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64
```

In [31]:
```
kpss_test(df['AHE_diff'].dropna())
```

```
KPSS Test Result:
Test Statistic            0.456203
p-value                   0.052930
Lags Used                14.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64
```

In [32]:
```
kpss_test(df['PCEPI_diff'].dropna())
```

```
KPSS Test Result:
Test Statistic            0.094052
p-value                   0.100000
Lags Used                14.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64

C:\Users\guill\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:1910: I
nterpolationWarning: The test statistic is outside of the range of p-values ava
ilable in the
look-up table. The actual p-value is greater than the p-value returned.

  warnings.warn(
```

Adding differences to get AHE and stationary

- Based on ADF, we need to differentiate for AHC
- Based on KPSS, we need to differentiate for AHC and PCE

In [33]:
```
df['AHE_diff2'] = df['AHE_diff'].diff().diff()
```

In [34]:
```
adf_test(df['AHE_diff2'].dropna())
```

```
Dickey-Fuller Test Result:
Test Statistic                -7.001846e+00
p-value                        7.287822e-10
#Lags Used                     1.400000e+01
Number of Observations Used    1.480000e+02
Critical Value (1%)           -3.475325e+00
Critical Value (5%)           -2.881275e+00
Critical Value (10%)          -2.577293e+00
dtype: float64
```

In [35]:
```
kpss_test(df['AHE_diff2'].dropna())
```

```
KPSS Test Result:
Test Statistic            0.189893
p-value                   0.100000
Lags Used                14.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64

C:\Users\guill\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:1910: I
nterpolationWarning: The test statistic is outside of the range of p-values ava
ilable in the
look-up table. The actual p-value is greater than the p-value returned.

  warnings.warn(
```

In [36]:
```
df['PCE_diff2'] = df['PCE_diff'].diff().diff()
```

In [37]:
```
adf_test(df['PCE_diff2'].dropna())
```

```
Dickey-Fuller Test Result:
Test Statistic                -7.301904e+00
p-value                        1.331855e-10
#Lags Used                     1.200000e+01
Number of Observations Used    1.500000e+02
Critical Value (1%)           -3.474715e+00
Critical Value (5%)           -2.881009e+00
Critical Value (10%)          -2.577151e+00
dtype: float64
```

In [38]:
```
kpss_test(df['PCE_diff2'].dropna())
```

```
KPSS Test Result:
Test Statistic            0.043249
p-value                   0.100000
Lags Used                14.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64

C:\Users\guill\anaconda3\lib\site-packages\statsmodels\tsa\stattools.py:1910: I
nterpolationWarning: The test statistic is outside of the range of p-values ava
ilable in the
look-up table. The actual p-value is greater than the p-value returned.

  warnings.warn(
```

We can identify then that the best order is order (p, q) is (3, 0) and it is for VAR model:

In [39]:
```
from statsmodels.tsa.statespace.varmax import VARMAX
from statsmodels.tsa.vector_ar.var_model import VAR
from math import sqrt
import warnings
warnings.filterwarnings('ignore')
```

In [40]:
```
varma_df = df[['PCE','AHE', 'PCEPI']].dropna()
train = varma_df[:-13]
test = varma_df[-13:]
var_df = df[['PCE_diff2','AHE_diff2', 'PCEPI_diff']].dropna()
train_var = var_df[:-12]
test_var = var_df[-12:]
prange = range(0,4)
qrange = range(1,4)
max_aic = np.inf
best_order = None

print('Running search of VARMA')

for p in prange:
    for q in qrange:
        order = (p, q)
        varma_model = VARMAX(train, order=order).fit(disp=False)
        current_aic = varma_model.aic
        print('\t Order is', order, 'with AIC of', current_aic)
        if current_aic < max_aic:
            max_aic = current_aic
            best_order = order

print('Best VARMA order is', best_order, 'with AIC of', max_aic)

max_aic_var = np.inf
best_order_var = None
print('Running search of VAR')
for p in prange:
    order = (p, 0)
    var_model = VAR(train).fit(p)
    current_aic = var_model.aic
    print('\t Order is', order, 'with AIC of', current_aic)
    if current_aic < max_aic_var:
        max_aic_var = current_aic
        best_order_var = order

print('Best VAR order is', best_order_var, 'with AIC of', max_aic_var)
```

```
Running search of VARMA
         Order is (0, 1) with AIC of 2979.350418228753
         Order is (0, 2) with AIC of 2819.0934531058083
         Order is (0, 3) with AIC of 2742.530256952685
         Order is (1, 1) with AIC of 4134.1471965117835
         Order is (1, 2) with AIC of 3299.7086685803542
         Order is (1, 3) with AIC of 2957.048379272
         Order is (2, 1) with AIC of 6651.8173717998125
         Order is (2, 2) with AIC of 5300.240365204367
         Order is (2, 3) with AIC of 5432.601010813803
         Order is (3, 1) with AIC of 7410.405155283706
         Order is (3, 2) with AIC of 3900.275660444901
         Order is (3, 3) with AIC of 4071.669154725519
Best VARMA order is (0, 3) with AIC of 2742.530256952685
Running search of VAR
         Order is (0, 0) with AIC of 12.243589209130015
         Order is (1, 0) with AIC of -3.318112474025692
```

```
         Order is (2, 0) with AIC of -3.8363653797902275
         Order is (3, 0) with AIC of -3.958260862334076
Best VAR order is (3, 0) with AIC of -3.958260862334076
```

B.

In [41]:
```
var_model = VAR(train_var).fit(3)
var_model.summary()
```

Out[41]:
```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Fri, 20, May, 2022
Time:                     20:47:21
--------------------------------------------------------------------
No. of Equations:         3.00000    BIC:                   -1.41681
Nobs:                     148.000    HQIC:                  -1.77751
Log likelihood:          -450.207    FPE:                   0.132161
AIC:                     -2.02435    Det(Omega_mle):        0.108622
--------------------------------------------------------------------
Results for equation PCE_diff2
================================================================================
=
                   coefficient       std. error           t-stat            pro
b
--------------------------------------------------------------------------------
-
const                14.465303         4.946489            2.924           0.00
3
L1.PCE_diff2         -1.270115         0.083301          -15.247           0.00
0
L1.AHE_diff2        -66.941721        80.310653           -0.834           0.40
5
L1.PCEPI_diff       -39.330608        22.087009           -1.781           0.07
5
L2.PCE_diff2         -1.012974         0.114003           -8.885           0.00
0
L2.AHE_diff2        -32.465107       105.804078           -0.307           0.75
9
L2.PCEPI_diff       -58.019701        25.585973           -2.268           0.02
3
L3.PCE_diff2         -0.474896         0.082490           -5.757           0.00
0
L3.AHE_diff2         25.237543        76.963877            0.328           0.74
3
L3.PCEPI_diff       -19.350999        22.765489           -0.850           0.39
5
================================================================================
=

Results for equation AHE_diff2
================================================================================
=
                   coefficient       std. error           t-stat            pro
b
--------------------------------------------------------------------------------
-
const                -0.001290         0.004694           -0.275           0.78
4
L1.PCE_diff2         -0.000095         0.000079           -1.205           0.22
8
L1.AHE_diff2         -1.470929         0.076214          -19.300           0.00
```

```
0
L1.PCEPI_diff        -0.021927         0.020960           -1.046           0.29
5
L2.PCE_diff2         -0.000025         0.000108           -0.231           0.81
7
L2.AHE_diff2         -1.237347         0.100408          -12.323           0.00
0
L2.PCEPI_diff         0.020959         0.024281            0.863           0.38
8
L3.PCE_diff2          0.000054         0.000078            0.694           0.48
8
L3.AHE_diff2         -0.451943         0.073038           -6.188           0.00
0
L3.PCEPI_diff         0.011063         0.021604            0.512           0.60
9
================================================================================
=

Results for equation PCEPI_diff
================================================================================
=
                   coefficient       std. error           t-stat            pro
b
--------------------------------------------------------------------------------
-
const                 0.069651         0.019752            3.526           0.00
0
L1.PCE_diff2          0.000475         0.000333            1.428           0.15
3
L1.AHE_diff2          0.173416         0.320686            0.541           0.58
9
L1.PCEPI_diff         0.495448         0.088195            5.618           0.00
0
L2.PCE_diff2          0.000467         0.000455            1.026           0.30
5
L2.AHE_diff2          0.188986         0.422484            0.447           0.65
5
L2.PCEPI_diff         0.025005         0.102167            0.245           0.80
7
L3.PCE_diff2          0.000080         0.000329            0.243           0.80
8
L3.AHE_diff2          0.142347         0.307322            0.463           0.64
3
L3.PCEPI_diff        -0.065241         0.090904           -0.718           0.47
3
================================================================================
=

Correlation matrix of residuals
              PCE_diff2  AHE_diff2  PCEPI_diff
PCE_diff2      1.000000  -0.100657    0.301401
AHE_diff2     -0.100657   1.000000   -0.100679
PCEPI_diff     0.301401  -0.100679    1.000000
```

C.

We are expecting the errors in the matrix to be 1 in the diagonal and 0 in the rest of the matrix, which means perfect correlation within the same variables - same series - but not with other variables. However, what we are observing in the matrix is that perfect correlation holds but not the second part: even when lower, there exist correlation across series, which is slighty greater between PCE and PCEPI.

After testing for the statistical significance of residuals correlation, we found that we reject the null hypothesis of no correlation (the p-value of the test is smaller than 0.05 for a null hypothesis of no correlation).

In [42]:
```
var_model.test_whiteness(signif =0.05).summary()
```

Out[42]:

Portmanteau-test for residual autocorrelation. H_0: residual autocorrelation up to lag 10 is zero. Conclusion: reject H_0 at 5% significance level.

Test statistic	Critical value	p-value	df
130.0	82.53	0.000	63

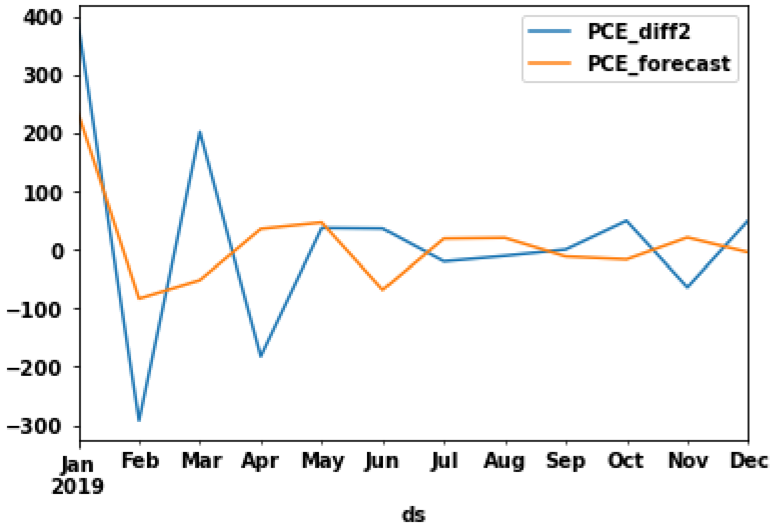
D

In [43]:
```
yhat = var_model.forecast(var_model.endog, steps=12)
idx=test_var.index
df_preds = pd.DataFrame(yhat).set_index(idx)
df_preds.rename(columns={0: 'PCE_forecast', 1: 'AHE_forecast', 2: 'PCEPI_forecast
```

True values vs Forecast for PCE

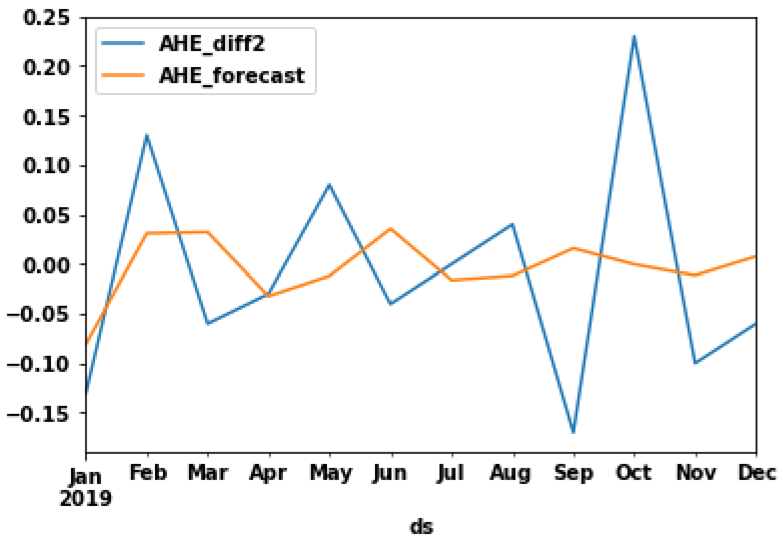
In [44]:
```
df_results = pd.concat([test_var, df_preds], axis=1)
df_results[['PCE_diff2', 'PCE_forecast']].plot()
```

Out[44]: <AxesSubplot:xlabel='ds'>



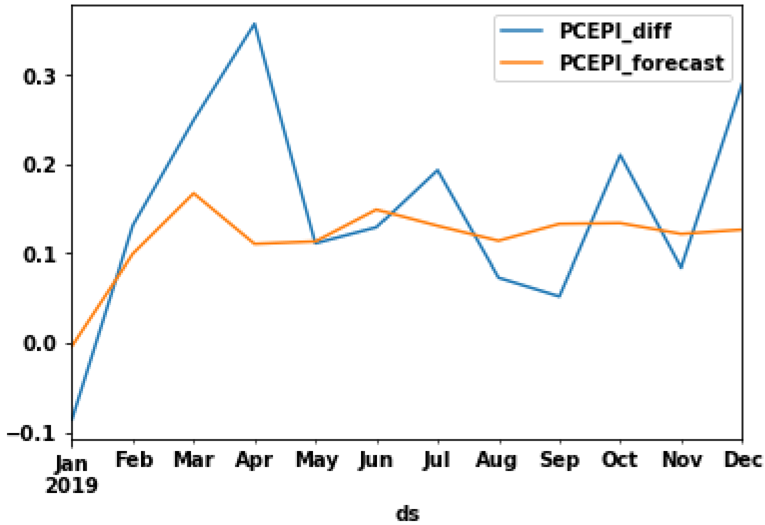
True values vs Forecast for AHE

In [45]:
```
df_results[['AHE_diff2', 'AHE_forecast']].plot()
plt.show()
```



True values vs Forecast for PCEPI

In [46]:
```
df_results[['PCEPI_diff', 'PCEPI_forecast']].plot()
plt.show()
```

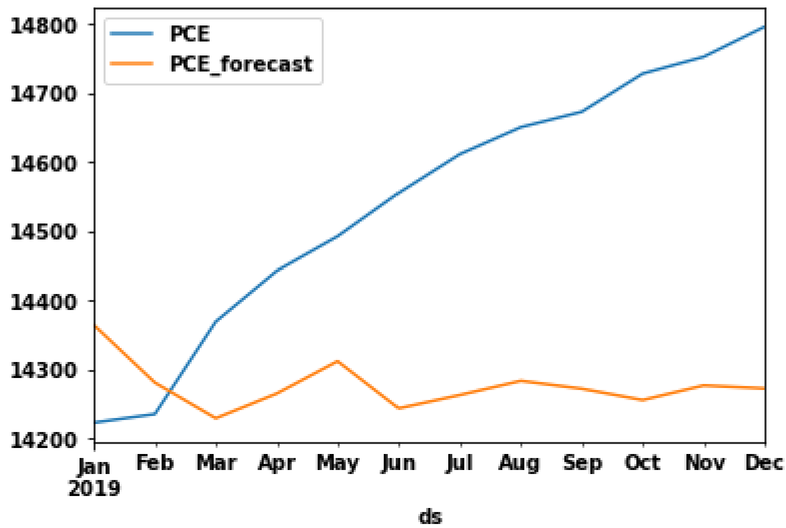


Getting cumulative sums:

In [47]:
```
df_preds.rename(columns={'PCE_forecast':'PCE',
                         'AHE_forecast': 'AHE',
                         'PCEPI_forecast': 'PCEPI'}, inplace=True)

forecast_results1 = pd.concat([df[-13:-12][['PCE', 'PCEPI']], df_preds[['PCE', 'P
                    .cumsum()\
                    .drop([0])\
                    .set_index(idx)

forecast_results1.columns = ['PCE_forecast', 'PCEPI_forecast']
```

True values vs Forecast for PCE:

In [48]:
```
forecast_pce = pd.concat([df[-12:]['PCE'],forecast_results1[['PCE_forecast']]], a
forecast_pce.plot()
plt.show()
```



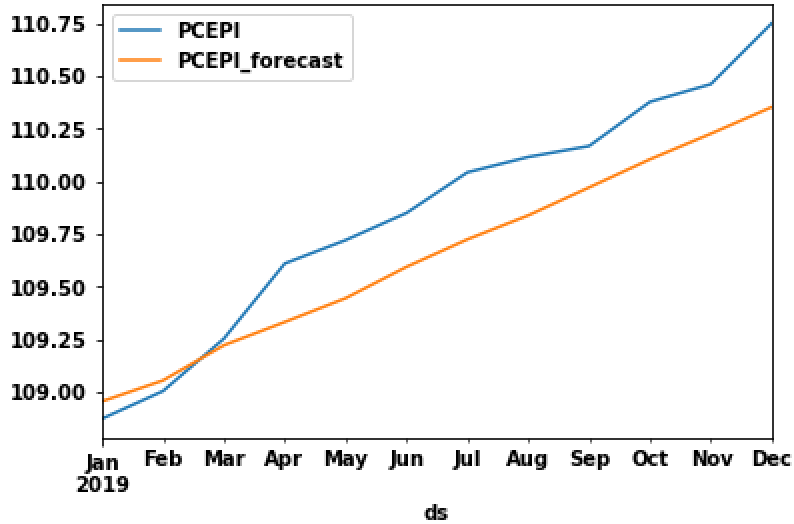
RMSE for PCE:

In [49]:
```
str(sqrt(mean_squared_error(df[-12:]['PCE'],forecast_results1['PCE_forecast'])))
```

Out[49]: '334.26314689466614'

True values vs Forecast for PCEPI:

In [50]:
```
forecast_pcepi = pd.concat([df[-12:]['PCEPI'],forecast_results1[['PCEPI_forecast'
forecast_pcepi.plot()
plt.show()
```



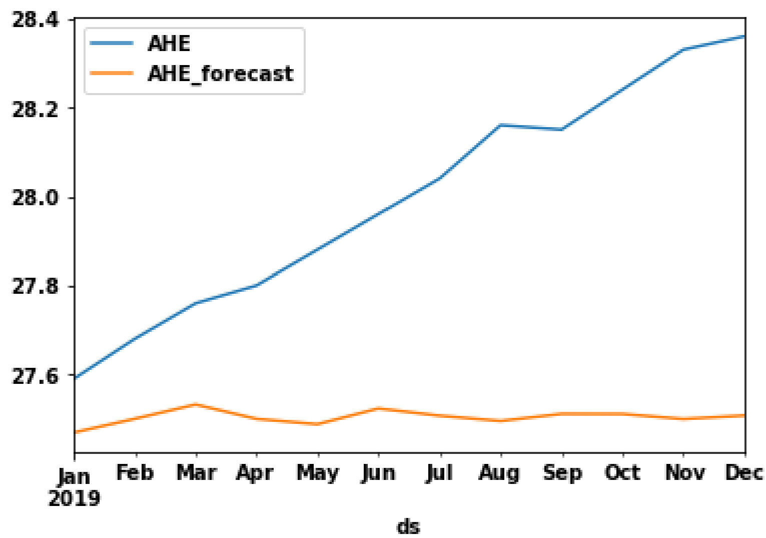
RMSE for PCEPI:

In [52]:
```
str(sqrt(mean_squared_error(df[-12:]['PCEPI'],forecast_results1['PCEPI_forecast']
```

Out[52]: '0.24742881664174932'

True values vs Forecast for AHE:

In [54]:
```
# AHE second difference - double cumsum
forecast_results2 = pd.concat([df[-13:-12][['AHE']],df_preds[['AHE']]], axis=0).r
                    .cumsum()\
                    .drop([0])\
                    .set_index(idx)

forecast_results2.columns=['AHE_forecast']

forecast_ahe = pd.concat([df[-12:]['AHE'],forecast_results2[['AHE_forecast']]], a
forecast_ahe.plot()
plt.show()
```



RMSE for AHE:

In [55]:
```
str(sqrt(mean_squared_error(df[-12:]['AHE'],forecast_results2['AHE_forecast'])))
```

Out[55]: '0.5487657637033092'

E.

The advantage of the VAR model is that we can estimate the series simultanously in comparison to the procedure we follow by applying ARIMA or Prophet models. Another difference is that we can add with VARMA lags for regressor while in the case of ARIMA of Phophet models we can add exogenous regressors.