Week 5 ARIMA vs Prophet

Time Series Analysis

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```
! pip install sktime
```

```
Requirement already satisfied: sktime in /usr/local/lib/python3.7/dist-package
Requirement already satisfied: numba>=0.53 in /usr/local/lib/python3.7/dist-pa
Requirement already satisfied: numpy<1.22,>=1.21.0 in /usr/local/lib/python3.7
Requirement already satisfied: scipy<1.9.0 in /usr/local/lib/python3.7/dist-pa
Requirement already satisfied: statsmodels>=0.12.1 in /usr/local/lib/python3.7
Requirement already satisfied: scikit-learn<1.2.0,>=0.24.0 in /usr/local/lib/p
Requirement already satisfied: pandas<1.5.0,>=1.1.0 in /usr/local/lib/python3.
Requirement already satisfied: deprecated>=1.2.13 in /usr/local/lib/python3.7/
Requirement already satisfied: wrapt<2,>=1.10 in /usr/local/lib/python3.7/dist
Requirement already satisfied: llvmlite<0.39,>=0.38.0rc1 in /usr/local/lib/pyt
Requirement already satisfied: setuptools in /usr/local/lib/python3.7/dist-pac
Requirement already satisfied: pytz>=2017.3 in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: python-dateutil>=2.7.3 in /usr/local/lib/python
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.7/dist-packa
Requirement already satisfied: joblib>=0.11 in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: threadpoolctl>=2.0.0 in /usr/local/lib/python3.
Requirement already satisfied: patsy>=0.5.2 in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.7/dis
Requirement already satisfied: pyparsing!=3.0.5,>=2.0.2 in /usr/local/lib/pyth
```

```
! pip install pmdarima
```

```
Requirement already satisfied: pmdarima in /usr/local/lib/python3.7/dist-packa
Requirement already satisfied: setuptools!=50.0.0,>=38.6.0 in /usr/local/lib/
Requirement already satisfied: statsmodels!=0.12.0,>=0.11 in /usr/local/lib/py
Requirement already satisfied: scikit-learn>=0.22 in /usr/local/lib/python3.7/
Requirement already satisfied: scipy>=1.3.2 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: numpy>=1.19.3 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: joblib>=0.11 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: pandas>=0.19 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: urllib3 in /usr/local/lib/python3.7/dist-packag
Requirement already satisfied: Cython!=0.29.18,>=0.29 in /usr/local/lib/python
Requirement already satisfied: python-dateutil>=2.7.3 in /usr/local/lib/python
Requirement already satisfied: pytz>=2017.3 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.7/dist-packa
Requirement already satisfied: threadpoolctl>=2.0.0 in /usr/local/lib/python3.
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.7/dis
Requirement already satisfied: patsy>=0.5.2 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: pyparsing!=3.0.5,>=2.0.2 in /usr/local/lib/pyth
```

```
from scipy import stats
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
import pandas as pd
import seaborn as sns
sns.set()

from sklearn.preprocessing import StandardScaler

from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.graphics.tsaplots import plot_pacf

from sktime.forecasting.all import ForecastingHorizon
from sktime.forecasting.all import ExponentialSmoothing
from sktime.performance_metrics.forecasting import mean_absolute_percentage_error

import warnings
warnings.filterwarnings('ignore')
```

```
/usr/local/lib/python3.7/dist-packages/sktime/utils/validation/_dependencies.p
  warnings.warn(msg)
```

1. 10 pts - Exploratory Data Analysis

1.A 5 pts Merge the data sets together, should have 735 rows of data

```
df_co2 = pd.read_csv('hw5_data_co2.csv', parse_dates = ['ds'])
df_temp = pd.read_csv('hw5_data_temp.csv', parse_dates = ['ds'])
```

```
print(df_co2.isnull().sum())
print(df_temp.isnull().sum())

print(df_co2.shape)
print(df_temp.shape)
```

```
ds     0
co2    0
dtype: int64
ds      0
temp    0
dtype: int64
(735, 2)
(1674, 2)
```

```
df_temp
```

	ds	temp
0	1880-01-01	56.40
1	1880-02-01	56.82
2	1880-03-01	56.74
3	1880-04-01	56.55
4	1880-05-01	56.85
...	...	...
1669	2019-02-01	58.37
1670	2019-03-01	58.59
1671	2019-04-01	58.50
1672	2019-05-01	58.50
1673	2019-06-01	58.50

1674 rows × 2 columns

```
df_co2
```

	ds	co2
0	1958-03-15	315.700
1	1958-04-15	317.450
2	1958-05-15	317.510
3	1958-06-15	316.685
4	1958-07-15	315.860
...	...	...
730	2019-01-15	410.920
731	2019-02-15	411.660
732	2019-03-15	412.000
733	2019-04-15	413.510
734	2019-05-15	414.830

735 rows × 2 columns

```
from datetime import datetime, timedelta
df_co2['ds'] = df_co2['ds'] - timedelta(days=14)
df = pd.merge(df_co2, df_temp, how = "inner", on = "ds")
df
```

	ds	co2	temp
0	1958-03-01	315.700	57.38
1	1958-04-01	317.450	57.29
2	1958-05-01	317.510	57.32
3	1958-06-01	316.685	57.02
4	1958-07-01	315.860	57.27
...	...	...	...
730	2019-01-01	410.920	58.29
731	2019-02-01	411.660	58.37
732	2019-03-01	412.000	58.59
733	2019-04-01	413.510	58.50
734	2019-05-01	414.830	58.50

735 rows × 3 columns

1.B. 5 pts Complete a quantitative and qualitative data exploration. Include a verbal summary of your EDA.

- Reading in the data we can observe Carbon Dioxide levels over time is our time series data topic this week. It is the only variable (besides time, of course)!
- Visually inspecting our data in a plot, we can observe a positive upward trend over time.
- There are 735 data points in our series, with a mean carbon dioxide level of 354.210673 and standard deviation of 27.922811. The minimum value in the data set is 313.2; the maximum value is 414.83. The carbon dioxide levels are recorded between a time period of March 1958 to May 2019.
- We conduct a check for null or missing values and find there are no missing values.

```
df.set_index('ds', inplace = True)
df
```

ds	co2	temp
1958-03-01	315.700	57.38
1958-04-01	317.450	57.29
1958-05-01	317.510	57.32
1958-06-01	316.685	57.02
1958-07-01	315.860	57.27
...	...	...
2019-01-01	410.920	58.29
2019-02-01	411.660	58.37
2019-03-01	412.000	58.59
2019-04-01	413.510	58.50
2019-05-01	414.830	58.50

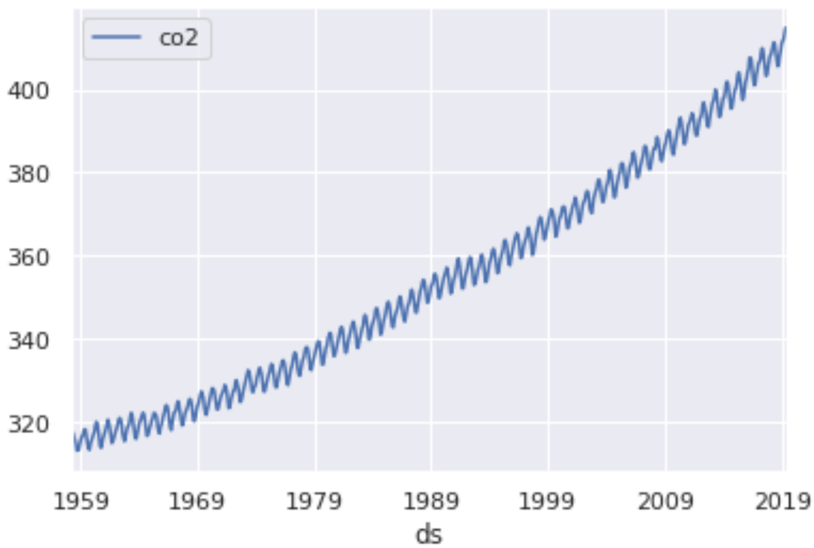
735 rows × 2 columns

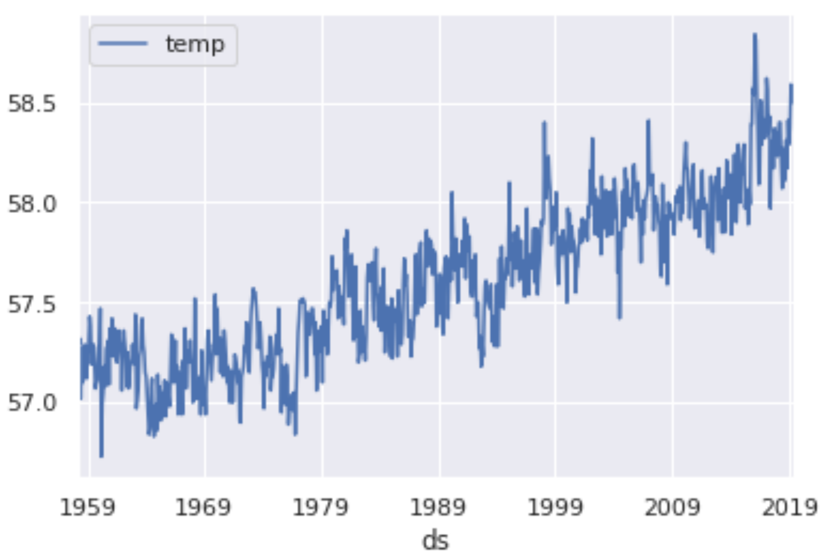
```
df.describe().T
```

	count	mean	std	min	25%	50%	75%	max
co2	735.0	354.210673	27.922811	313.20	328.785	351.34	376.515	414.83
temp	735.0	57.603293	0.397159	56.73	57.280	57.59	57.915	58.84

```
print(df[['co2']].plot())
print(df[['temp']].plot())
```

```
AxesSubplot(0.125,0.125;0.775x0.755)
AxesSubplot(0.125,0.125;0.775x0.755)
```



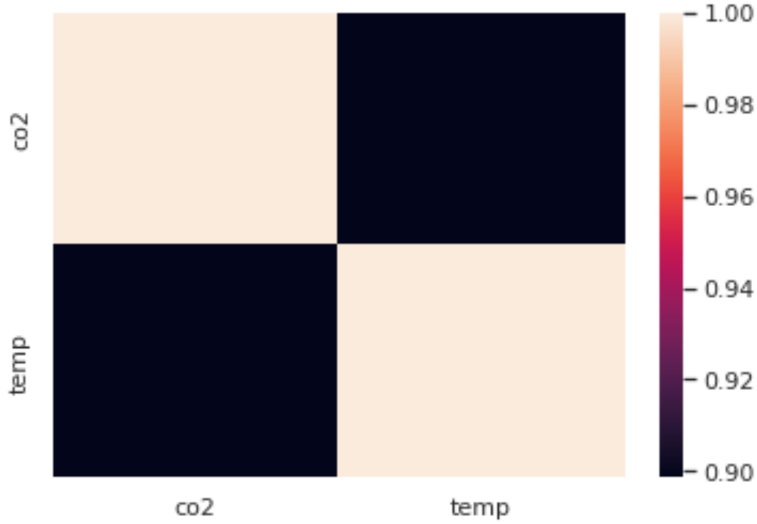



```
df.corr()
```

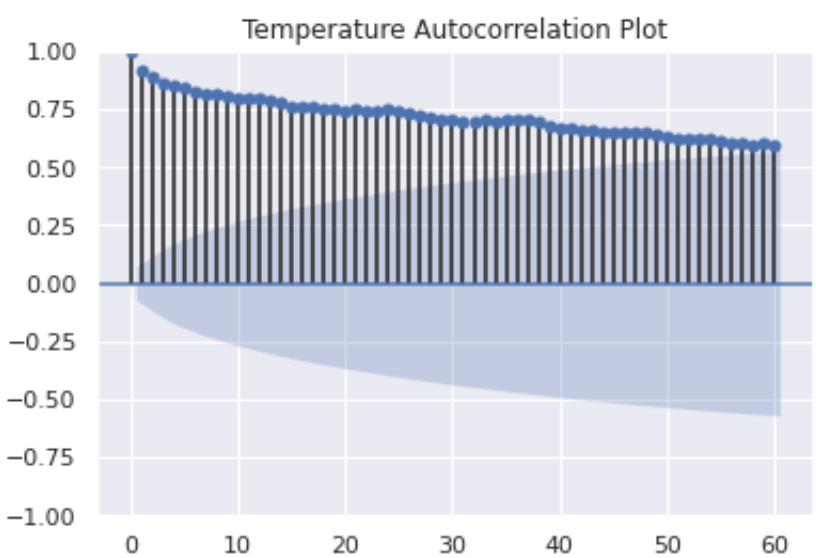
	co2	temp
co2	1.000000	0.898635
temp	0.898635	1.000000

```
sns.heatmap(df.corr())
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f8e1d22cd90>
```




```
p = plot_acf(df['temp'], lags=60, title='Temperature Autocorrelation Plot')
```



Part 2. 35 pts – ARIMA with external regressors

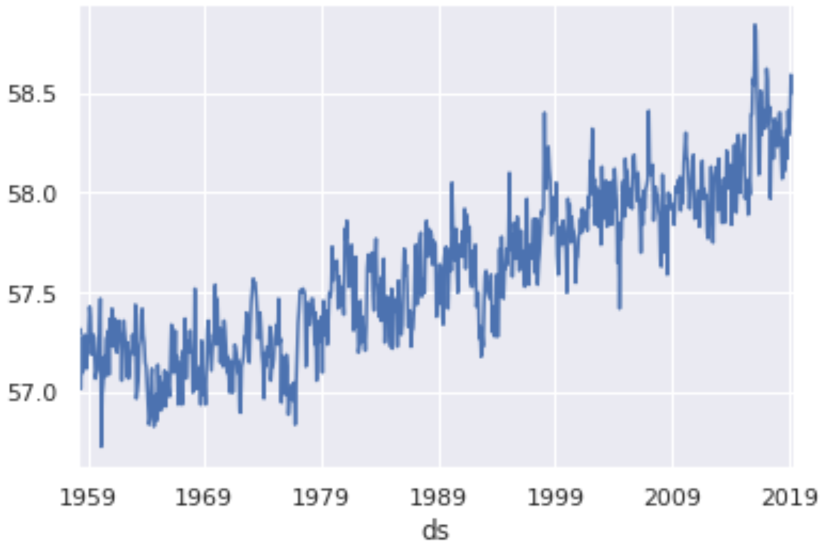
2.A. 7 pts Split Temp into train test. Determine the transformation, seasonal differencing & non seasonal differencing orders if required. Include the following: plot of differenced data, ADF results, kpss results, ACF/PACF plots.

- When initially loading the original time series data for temperature, we can immediately notice it has an upward trend and changing mean but uniform variance. Thus by definition it is not showing stationarity
- After a first seasonal differencing, the plot is observed to remain around zero for most of the time, a characteristic of stationarity. We can then confirm stationarity in the dataset by employing KPSS and ADFuller tests.
- We determine the order by differencing the data and applying ADF and KPSS tests. After the second pass, ADF test shows the p-values are less than 0.05 so the Null hypothesis can be rejected. If it is not null then we know the TS does npt possess a unit root and hence is stationary. Rejecting the Null hypothesis would mean TS is not stationary. KPSS tests also show the p-values are greater than 0.05. This means that the Null hypothesis of stationarity cannot be rejected. If it is null then we know the TS has stationary characteristics. Alternate hypothesis would mean TS is not stationary.
- The ACF plot reveal autocorrelation is present. Seasonality is observed in the PACF plot.
- Stationarity is achieved after differencing the data once each for seasonal & non-seasonal differencing orders.

```
from statsmodels.tsa.statespace import sarimax
from statsmodels.tools.eval_measures import aicc
from sktime.transformations.series import boxcox
from sktime.forecasting.arima import ARIMA
```

```
# selecting variable of intest & build uniform univariate time series
ts_temp = df.groupby(pd.PeriodIndex(df.index, freq="M"))['temp'].mean()
ts_temp.plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f8e1a4a1a50>
```




```
# take a log transformation
bctransformer = boxcox.BoxCoxTransformer()
ts_transf = bctransformer.fit_transform(ts_temp)
```

Split into Train/test ( )

```
from sktime.forecasting.all import temporal_train_test_split
y_train, y_test = temporal_train_test_split(ts_temp)
```

```
print(y_train.index.min(), y_train.index.max())
print(y_test.index.min(), y_test.index.max())
print(y_train.shape)
print(y_test.shape)
```

```
1958-03 2004-01
2004-02 2019-05
(551,)
(184,)
```

Examine Stationarity

```
from statsmodels.tsa.stattools import kpss, adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from copy import deepcopy

def adf_test(timeseries):
    print ('Dickey-Fuller Test Result:')
    dftest = adfuller(timeseries, autolag='AIC')
    dfoutput = pd.Series(dftest[0:4], index=['Test Statistic','p-value','#Lags Usec
    for key,value in dftest[4].items():
       dfoutput['Critical Value (%s)'%key] = value
    print (dfoutput)

#define KPSS
def kpss_test(timeseries, trend='c'):
    print ('KPSS Test Result:')
    kpsstest = kpss(timeseries, regression=trend)
    kpss_output = pd.Series(kpsstest[0:3], index=['Test Statistic','p-value','Lags
    for key,value in kpsstest[3].items():
        kpss_output['Critical Value (%s)'%key] = value
    print (kpss_output)


adf_test(y_train)
```

```
Dickey-Fuller Test Result:
Test Statistic                  -2.481183
p-value                          0.120120
#Lags Used                       4.000000
Number of Observations Used    546.000000
Critical Value (1%)             -3.442384
Critical Value (5%)             -2.866848
Critical Value (10%)            -2.569597
dtype: float64
```

```
adf_temp = adfuller(y_train)
ad_fuller_pval = adf_temp[1]

if ad_fuller_pval <= 0.05:
    print('stationarity from ad_fuller test: TRUE')
else:
    print('stationarity from ad_fuller test: FALSE')
```

```
stationarity from ad_fuller test: FALSE
```

```
kpss_test(y_train)
```

```
KPSS Test Result:
Test Statistic            2.970537
p-value                   0.010000
Lags Used                15.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64
```

```
kpss_temp = kpss(y_train)
kpss_pval = kpss_temp[1]

if kpss_pval >= 0.05:
    print('stationarity from kpss_fuller test: TRUE')
else:
    print('stationarity from kpss_fuller test: FALSE')
```

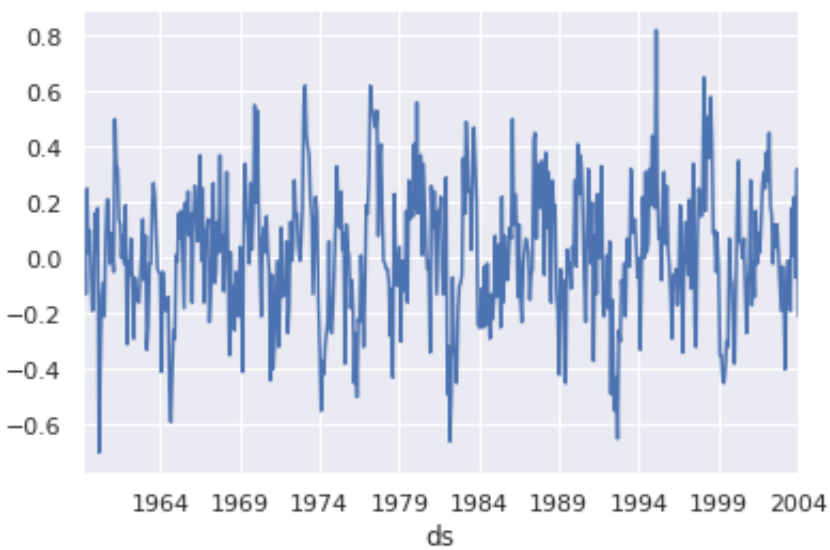
```
stationarity from kpss_fuller test: FALSE
```

No stationarity in original temp y_train... Difference the data and examine again

```
y_train_seasdiff = y_train.diff(12).dropna()
print(y_train_seasdiff)
```

```
ds
1959-03    0.05
1959-04    0.09
1959-05   -0.13
1959-06    0.25
1959-07    0.02
           ...
2003-09    0.01
2003-10    0.22
2003-11   -0.07
2003-12    0.32
2004-01   -0.21
Freq: M, Name: temp, Length: 539, dtype: float64
```

```
y_train_seasdiff.plot()
plt.show()
```



```
adf_test(y_train_seasdiff)
```

```
Dickey-Fuller Test Result:
Test Statistic                -7.250434e+00
p-value                        1.785978e-10
#Lags Used                     1.300000e+01
Number of Observations Used    5.250000e+02
Critical Value (1%)           -3.442867e+00
Critical Value (5%)           -2.867061e+00
Critical Value (10%)          -2.569710e+00
dtype: float64
```

```
adf_temp = adfuller(y_train_seasdiff)
ad_fuller_pval = adf_temp[1]

if ad_fuller_pval <= 0.05:
    print('stationarity from ad_fuller test: TRUE')
else:
    print('stationarity from ad_fuller test: FALSE')
```

```
stationarity from ad_fuller test: TRUE
```

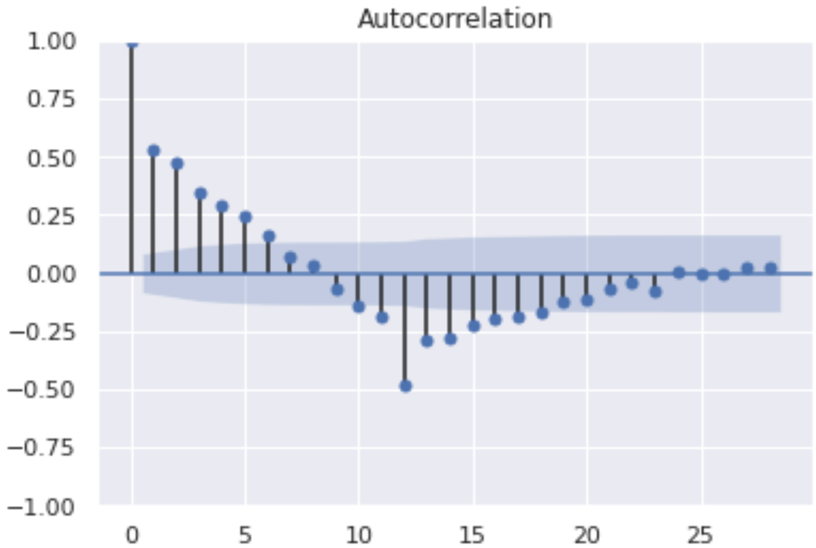
```
kpss_test(y_train_seasdiff)
```

```
KPSS Test Result:
Test Statistic            0.044985
p-value                   0.100000
Lags Used                13.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64
```

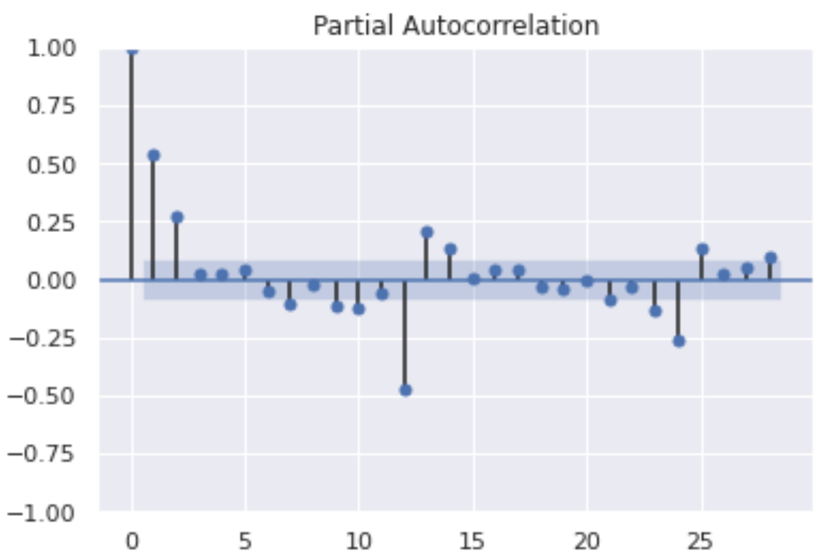
```
kpss_temp = kpss(y_train_seasdiff)
kpss_pval = kpss_temp[1]

if kpss_pval >= 0.05:
    print('stationarity from kpss_fuller test: TRUE')
else:
    print('stationarity from kpss_fuller test: FALSE')
```

```
stationarity from kpss_fuller test: TRUE
```

```
plot_acf(y_train_seasdiff)
plt.show()
```

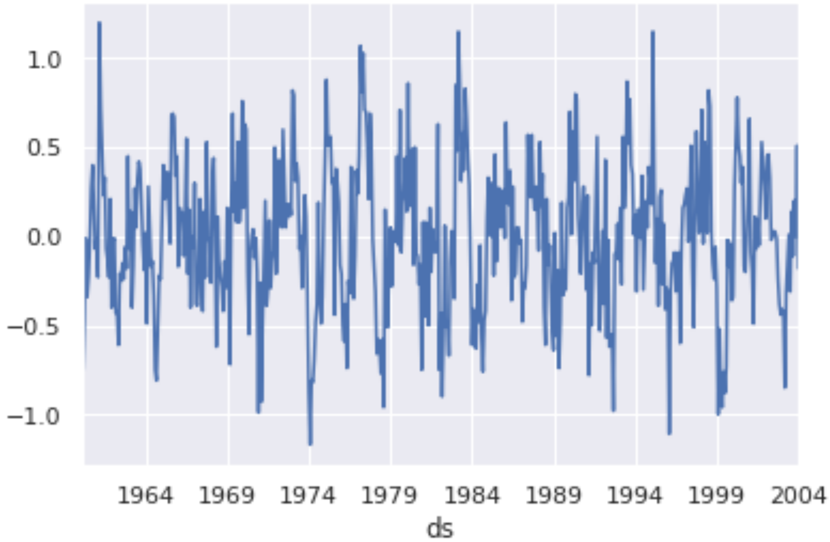



```
plot_pacf(y_train_seasdiff)
plt.show()
```




```
#y_train_nonseasdiff = np.diff(y_train_seasdiff)
y_train_nonseasdiff = y_train_seasdiff.diff(12).dropna()
```

```
y_train_nonseasdiff.plot()
plt.show()
```




```
adf_test(y_train_nonseasdiff)
```

```
Dickey-Fuller Test Result:
Test Statistic                -7.555800e+00
p-value                        3.102015e-11
#Lags Used                     1.300000e+01
Number of Observations Used    5.130000e+02
Critical Value (1%)           -3.443162e+00
Critical Value (5%)           -2.867190e+00
Critical Value (10%)          -2.569780e+00
dtype: float64
```

```
adf_temp = adfuller(y_train_nonseasdiff)
ad_fuller_pval = adf_temp[1]

if ad_fuller_pval <= 0.05:
    print('stationarity from ad_fuller test: TRUE')
else:
    print('stationarity from ad_fuller test: FALSE')
```

```
stationarity from ad_fuller test: TRUE
```

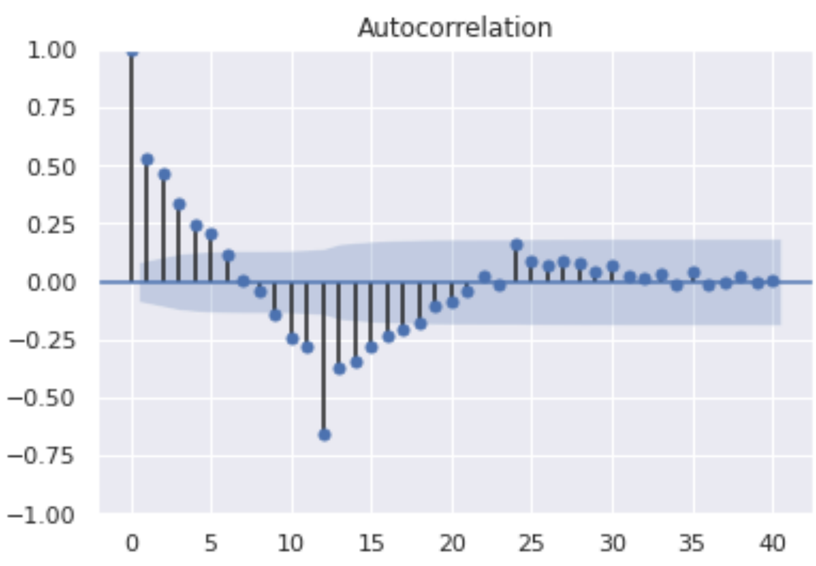
```
kpss_test(y_train_nonseasdiff)
```

```
KPSS Test Result:
Test Statistic            0.009799
p-value                   0.100000
Lags Used                13.000000
Critical Value (10%)      0.347000
Critical Value (5%)       0.463000
Critical Value (2.5%)     0.574000
Critical Value (1%)       0.739000
dtype: float64
```

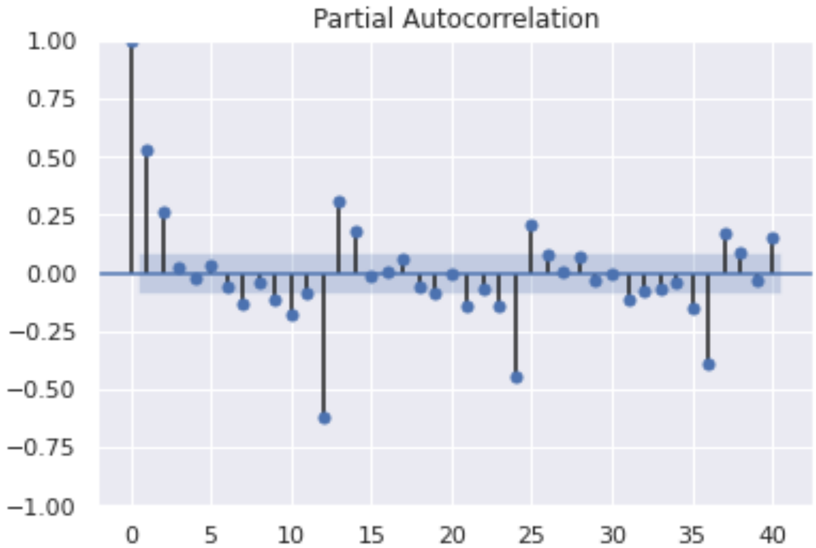
```
kpss_temp = kpss(y_train_nonseasdiff)
kpss_pval = kpss_temp[1]

if kpss_pval >= 0.05:
    print('stationarity from kpss_fuller test: TRUE')
else:
    print('stationarity from kpss_fuller test: FALSE')
```

```
stationarity from kpss_fuller test: TRUE
```

```
plot_acf(y_train_nonseasdiff, lags = 40)
plt.show()
```



```
plot_pacf(y_train_nonseasdiff, lags = 40)
plt.show()
```



2.B. 7 pts Fit a SARIMA or ARIMA model based on your examinations. Examine the residuals. Include the following: plot, histogram, ACF, Ljung Box results. Check if auto arima gives you a different order. If this is a better model, examine the residuals of this model.

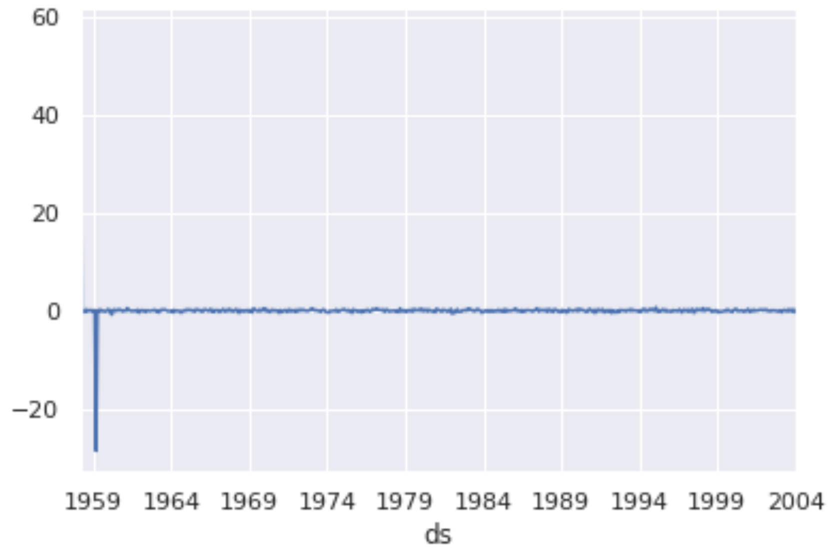
```
sarima_model = sm.tsa.statespace.SARIMAX(endog=y_train,order=(3,1,1), seasonal_orde
sarima_fit = sarima_model.fit()
print(sarima_fit.summary())
```

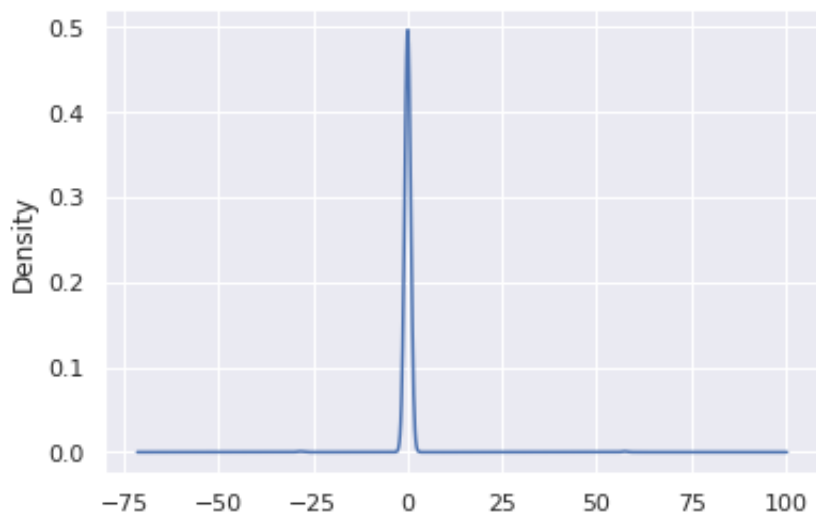
```
                                     SARIMAX Results
==============================================================================
Dep. Variable:                              temp   No. Observations:
Model:             SARIMAX(3, 1, 1)x(0, 1, [], 12)   Log Likelihood
Date:                           Sat, 14 May 2022   AIC
Time:                                   13:08:44   BIC
Sample:                               03-31-1958   HQIC
                                    - 01-31-2004
Covariance Type:                             opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.3835      0.045      8.587      0.000       0.296       0.47
ar.L2          0.2620      0.047      5.613      0.000       0.171       0.35
ar.L3          0.0273      0.047      0.585      0.558      -0.064       0.11
ma.L1         -0.9996      0.159     -6.283      0.000      -1.311      -0.68
sigma2         0.0375      0.006      6.045      0.000       0.025       0.05
==============================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):
Prob(Q):                              0.96   Prob(JB):
Heteroskedasticity (H):               1.03   Skew:
Prob(H) (two-sided):                  0.86   Kurtosis:
==============================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (comple
```

```
# Return AICc
print("AICc for order " + str(sarima_model.order) + str(sarima_model.seasonal_order
```

```
AICc for order (3, 1, 1)(0, 1, 0, 12): -224.49550706908607
```

```
# residual checks
# line plot of residuals
residuals = sarima_fit.resid
residuals.plot()
plt.show()
# density plot of residuals
residuals.plot(kind='kde')
plt.show()
# summary stats of residuals
print(residuals.describe())
```






```
count    551.000000
mean       0.055174
std        2.739116
min      -28.550001
25%       -0.120285
50%       -0.000120
75%        0.125864
max       57.380000
dtype: float64
```

```
residuals
```

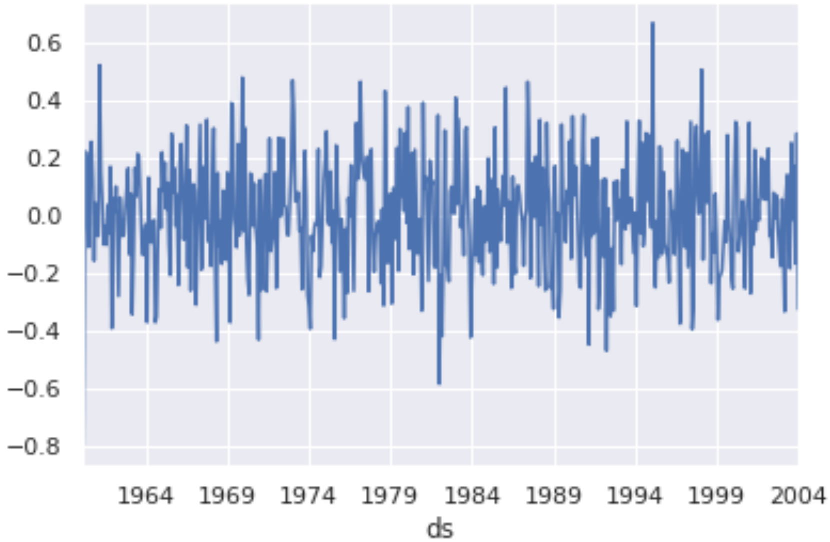
```
ds
1958-03    57.380000
1958-04    -0.089999
1958-05     0.030000
1958-06    -0.300000
1958-07     0.250000
              ...
2003-09    -0.014296
2003-10     0.168874
2003-11    -0.167513
2003-12     0.283619
2004-01    -0.326205
Freq: M, Length: 551, dtype: float64
```

```
resid = residuals[24:]
resid
```

```
ds
1960-03   -0.797779
1960-04   -0.144174
1960-05    0.220645
1960-06    0.107033
1960-07   -0.110958
             ...
2003-09   -0.014296
2003-10    0.168874
2003-11   -0.167513
2003-12    0.283619
2004-01   -0.326205
Freq: M, Length: 527, dtype: float64
```

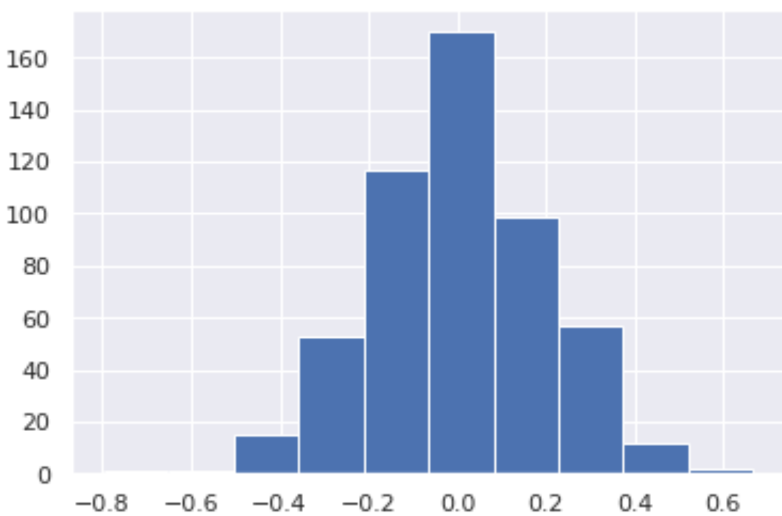
```
resid.plot()
```

<matplotlib.axes._subplots.AxesSubplot at 0x7f8e1a02fb10>

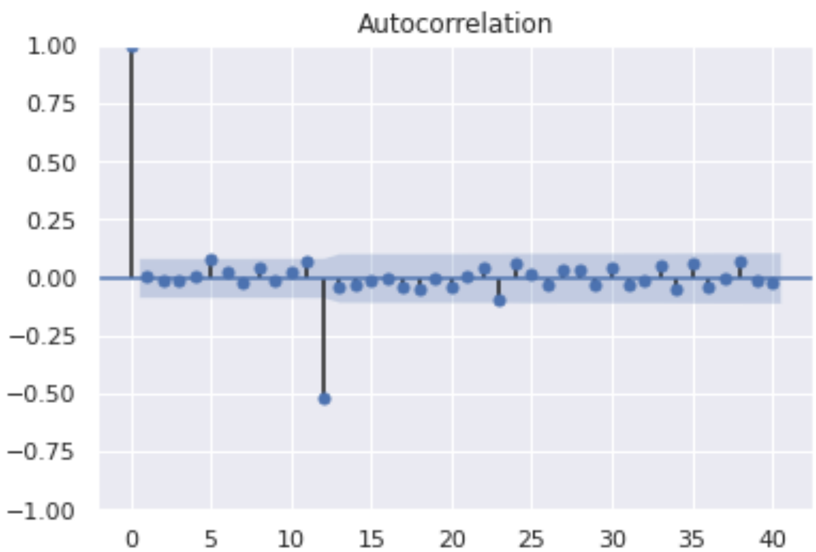



```
resid.hist()
```

<matplotlib.axes._subplots.AxesSubplot at 0x7f8e17dc8390>



```
plot_acf(resid, lags = 40)
plt.show()
```




```
sm.stats.acorr_ljungbox(resid, lags=[10], return_df=True, boxpierce=True)
```

	lb_stat	lb_pvalue	bp_stat	bp_pvalue
10	5.38012	0.864384	5.298779	0.870347

Checking against auto_arima()

```
from sktime.forecasting.arima import AutoARIMA
from pmdarima.arima import auto_arima
```

Fresh split into train and test (temperature data) for Modeling

```
y_train, y_test = temporal_train_test_split(ts_temp)
```

```
y_train.index.min(), y_train.index.max()
```

```
(Period('1958-03', 'M'), Period('2004-01', 'M'))
```

```
y_test.index.min(), y_test.index.max()
```

```
(Period('2004-02', 'M'), Period('2019-05', 'M'))
```

Model 1 (AutoARIMA)

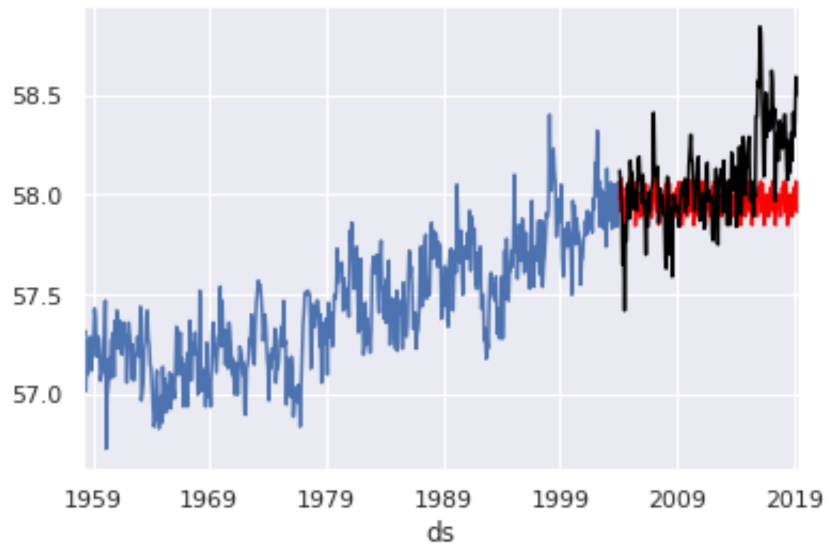
```
arima_model = AutoARIMA(D=1, sp=12)
arima_model.fit(y_train)
fh = list(range(1, 1+len(y_test)))
y_pred = arima_model.predict(fh=fh )
```

```
print(arima_model.summary())
```

```
                                      SARIMAX Results
==============================================================================
Dep. Variable:                                   y   No. Observations:
Model:             SARIMAX(1, 0, 1)x(2, 1, [], 12)   Log Likelihood
Date:                             Sat, 14 May 2022   AIC
Time:                                     13:12:23   BIC
Sample:                                          0   HQIC
                                             - 551
Covariance Type:                               opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.8809      0.033     26.698      0.000       0.816       0.94(
ma.L1         -0.4936      0.059     -8.322      0.000      -0.610      -0.37
ar.S.L12      -0.7164      0.041    -17.388      0.000      -0.797      -0.63(
ar.S.L24      -0.3194      0.040     -8.025      0.000      -0.397      -0.24
sigma2         0.0242      0.001     17.320      0.000       0.021       0.02
==============================================================================
Ljung-Box (L1) (Q):                   0.09   Jarque-Bera (JB):
Prob(Q):                              0.76   Prob(JB):
Heteroskedasticity (H):               1.11   Skew:
Prob(H) (two-sided):                  0.47   Kurtosis:
==============================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (comple
```

```
# plot
y_train.plot()
y_pred.plot(color='red')
y_test.plot(color = 'black')
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f8e17edfdd0>
```



Model 2 (auto_arima)

```
auto_arima = auto_arima(y_train, start_p=1, start_q=1,
                        max_p=10, max_q=10, m=12,
                        start_P=0, seasonal=True,
                        d=1, D=1, trace=True,
                        error_action='ignore',
                        suppress_warnings=True,
                        stepwise=True)
print(auto_arima.aic())
```

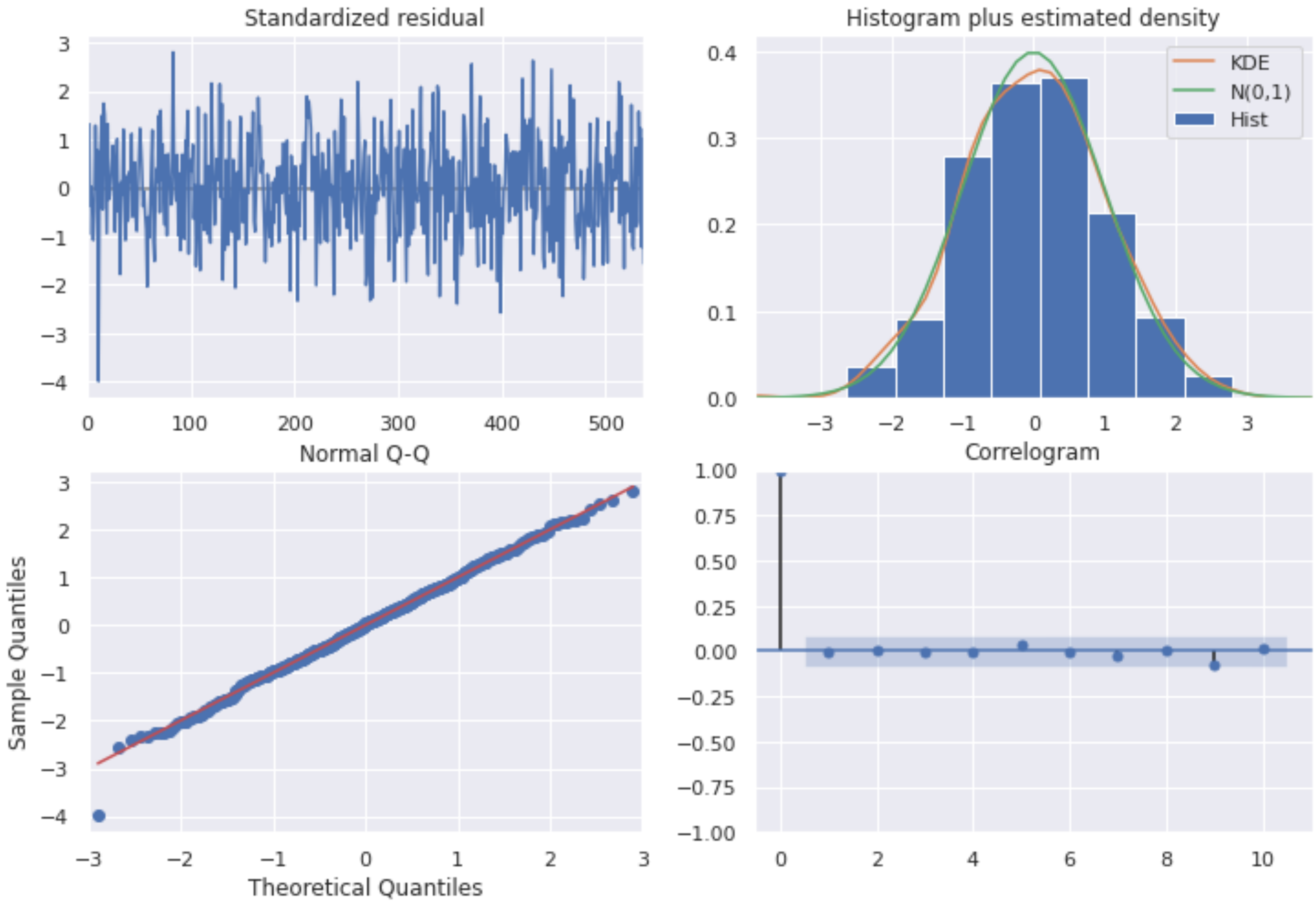
```
Performing stepwise search to minimize aic
 ARIMA(1,1,1)(0,1,1)[12]             : AIC=inf, Time=3.82 sec
 ARIMA(0,1,0)(0,1,0)[12]             : AIC=-53.968, Time=0.14 sec
 ARIMA(1,1,0)(1,1,0)[12]             : AIC=-345.175, Time=0.44 sec
 ARIMA(0,1,1)(0,1,1)[12]             : AIC=inf, Time=2.81 sec
 ARIMA(1,1,0)(0,1,0)[12]             : AIC=-167.230, Time=0.11 sec
 ARIMA(1,1,0)(2,1,0)[12]             : AIC=-395.722, Time=1.03 sec
 ARIMA(1,1,0)(2,1,1)[12]             : AIC=inf, Time=10.71 sec
 ARIMA(1,1,0)(1,1,1)[12]             : AIC=inf, Time=4.14 sec
 ARIMA(0,1,0)(2,1,0)[12]             : AIC=-284.630, Time=0.80 sec
 ARIMA(2,1,0)(2,1,0)[12]             : AIC=-405.606, Time=1.47 sec
 ARIMA(2,1,0)(1,1,0)[12]             : AIC=-353.561, Time=0.68 sec
 ARIMA(2,1,0)(2,1,1)[12]             : AIC=inf, Time=9.46 sec
 ARIMA(2,1,0)(1,1,1)[12]             : AIC=inf, Time=4.15 sec
 ARIMA(3,1,0)(2,1,0)[12]             : AIC=-424.092, Time=1.82 sec
 ARIMA(3,1,0)(1,1,0)[12]             : AIC=-367.330, Time=0.94 sec
 ARIMA(3,1,0)(2,1,1)[12]             : AIC=inf, Time=10.90 sec
 ARIMA(3,1,0)(1,1,1)[12]             : AIC=inf, Time=3.86 sec
 ARIMA(4,1,0)(2,1,0)[12]             : AIC=-432.173, Time=2.24 sec
 ARIMA(4,1,0)(1,1,0)[12]             : AIC=-376.355, Time=2.04 sec
 ARIMA(4,1,0)(2,1,1)[12]             : AIC=inf, Time=16.22 sec
 ARIMA(4,1,0)(1,1,1)[12]             : AIC=inf, Time=4.37 sec
 ARIMA(5,1,0)(2,1,0)[12]             : AIC=-431.318, Time=2.56 sec
 ARIMA(4,1,1)(2,1,0)[12]             : AIC=-431.886, Time=5.05 sec
 ARIMA(3,1,1)(2,1,0)[12]             : AIC=-433.808, Time=3.17 sec
 ARIMA(3,1,1)(1,1,0)[12]             : AIC=-376.473, Time=1.35 sec
 ARIMA(3,1,1)(2,1,1)[12]             : AIC=inf, Time=10.94 sec
 ARIMA(3,1,1)(1,1,1)[12]             : AIC=inf, Time=6.31 sec
 ARIMA(2,1,1)(2,1,0)[12]             : AIC=inf, Time=10.12 sec
 ARIMA(3,1,2)(2,1,0)[12]             : AIC=-431.817, Time=5.00 sec
 ARIMA(2,1,2)(2,1,0)[12]             : AIC=-433.194, Time=4.99 sec
 ARIMA(4,1,2)(2,1,0)[12]             : AIC=-430.006, Time=7.02 sec
 ARIMA(3,1,1)(2,1,0)[12] intercept   : AIC=-431.827, Time=12.07 sec

Best model:  ARIMA(3,1,1)(2,1,0)[12]
Total fit time: 150.765 seconds
-433.8075420485692
```

```
print(auto_arima.summary())
```

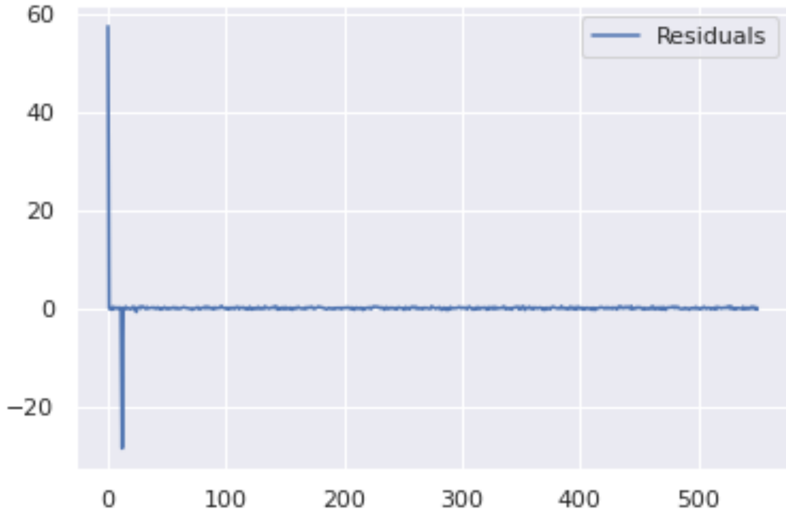
```
                                      SARIMAX Results
==========================================================================================
Dep. Variable:                                  y   No. Observations:
Model:             SARIMAX(3, 1, 1)x(2, 1, [], 12)   Log Likelihood
Date:                            Sat, 14 May 2022   AIC
Time:                                    13:14:55   BIC
Sample:                                         0   HQIC
                                            - 551
Covariance Type:                              opg
==========================================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------------------
ar.L1          0.0341      0.134      0.255      0.799      -0.228       0.29
ar.L2          0.0368      0.082      0.447      0.655      -0.124       0.19
ar.L3         -0.1145      0.060     -1.919      0.055      -0.232       0.00
ma.L1         -0.6029      0.126     -4.778      0.000      -0.850      -0.35
ar.S.L12      -0.7452      0.040    -18.648      0.000      -0.824      -0.66
ar.S.L24      -0.3332      0.040     -8.349      0.000      -0.411      -0.25
sigma2         0.0251      0.002     16.606      0.000       0.022       0.02
==========================================================================================
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):
Prob(Q):                              0.93   Prob(JB):
Heteroskedasticity (H):               1.13   Skew:
Prob(H) (two-sided):                  0.43   Kurtosis:
==========================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (comple
```

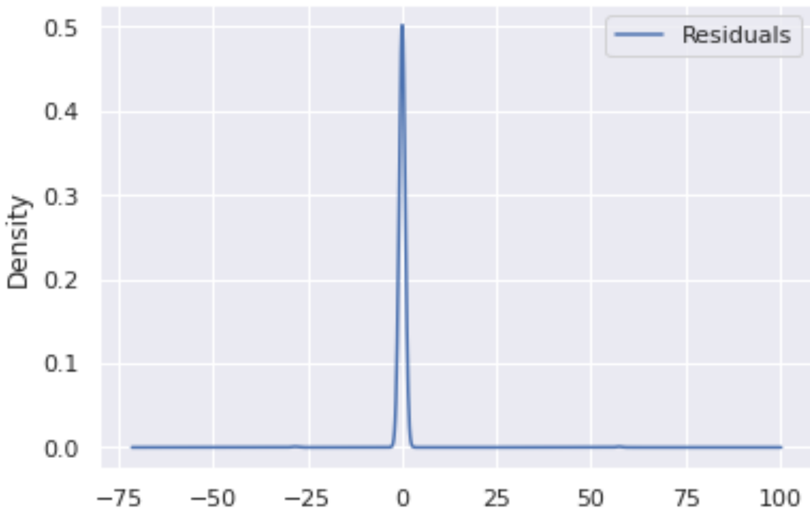
```
print(auto_arima.plot_diagnostics(figsize=(12,8)))
```

```
Figure(864x576)
```




```
# residual checks
# line plot of residuals
res = auto_arima.resid()
residuals = pd.DataFrame(res,columns=["Residuals"])
residuals.plot()
plt.show()
# density plot of residuals
residuals.plot(kind='kde')
plt.show()
# summary stats of residuals
print(residuals.describe())
```






```
       Residuals
count  551.000000
mean     0.052828
std      2.736996
min    -28.550000
25%     -0.108642
50%      0.006010
75%      0.110697
max     57.380000
```

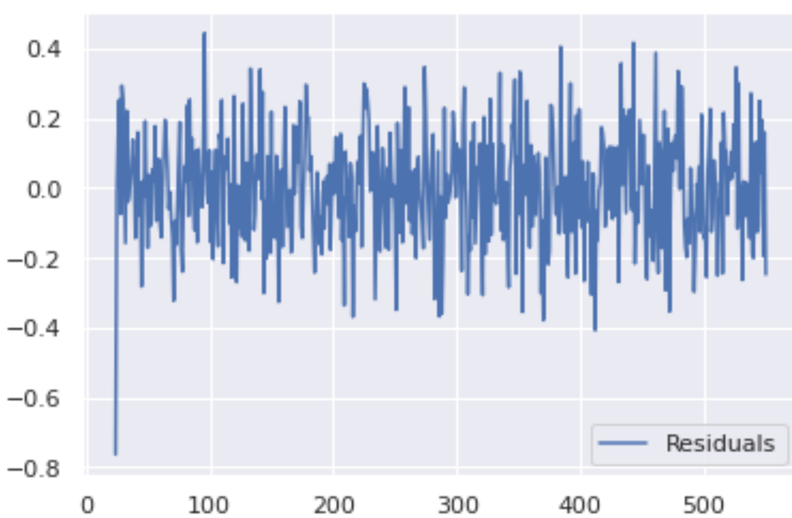
```
resid = residuals[24:]
resid
```

	Residuals
24	-0.763389
25	-0.015780
26	0.144974
27	0.252910
28	-0.074610
...	...
546	0.046744
547	0.194460
548	-0.193679
549	0.160406
550	-0.247319

527 rows × 1 columns

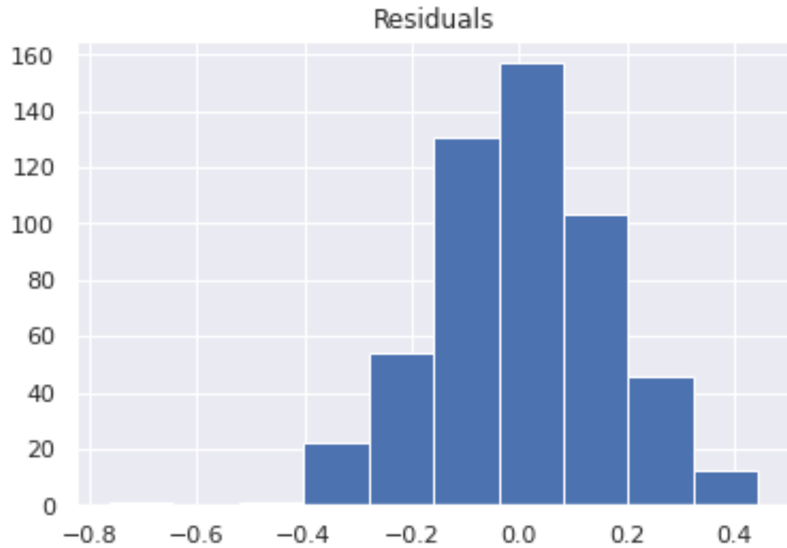
```
resid.plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f8e1d3a4090>
```

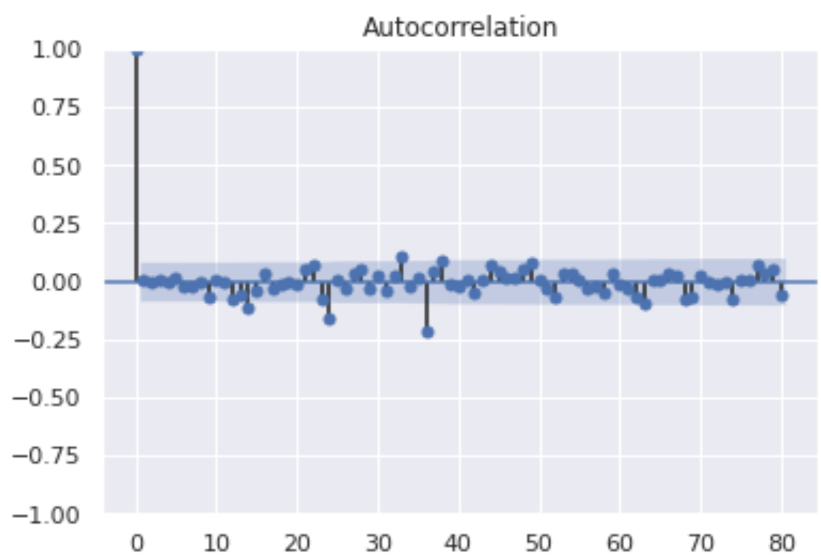


```
resid.hist()
```

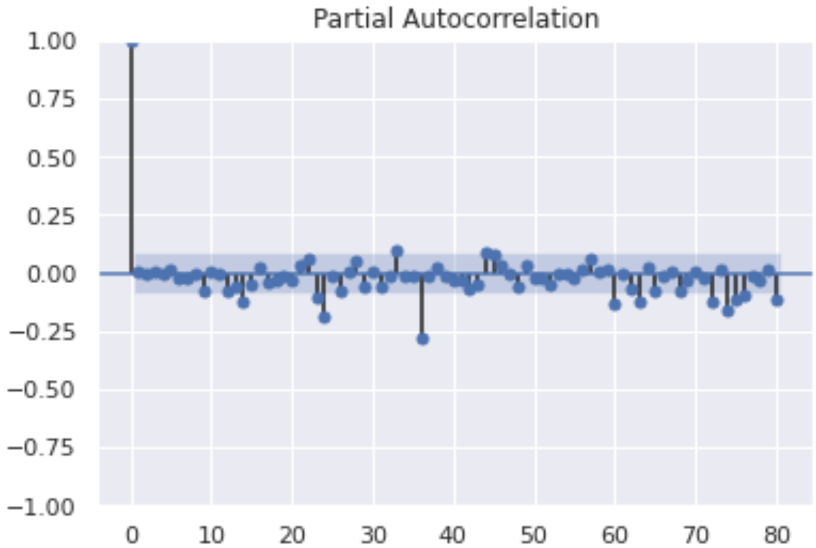
```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x7f8e1a384910>]],
      dtype=object)
```




```
plot_acf(resid, lags = 80)
plt.show()
```



```
plot_pacf(resid, lags = 80)
plt.show()
```




```
sm.stats.acorr_ljungbox(resid, lags=[10], return_df=True, boxpierce=True)
```

	lb_stat	lb_pvalue	bp_stat	bp_pvalue
10	3.571788	0.964604	3.501685	0.967041

2.C. 7 pts Using your best model, predict the test set. Include the following: MAE, MSE, MAPE, sMAPE, MASE. Can use prebuilt functions or calculate by hand.

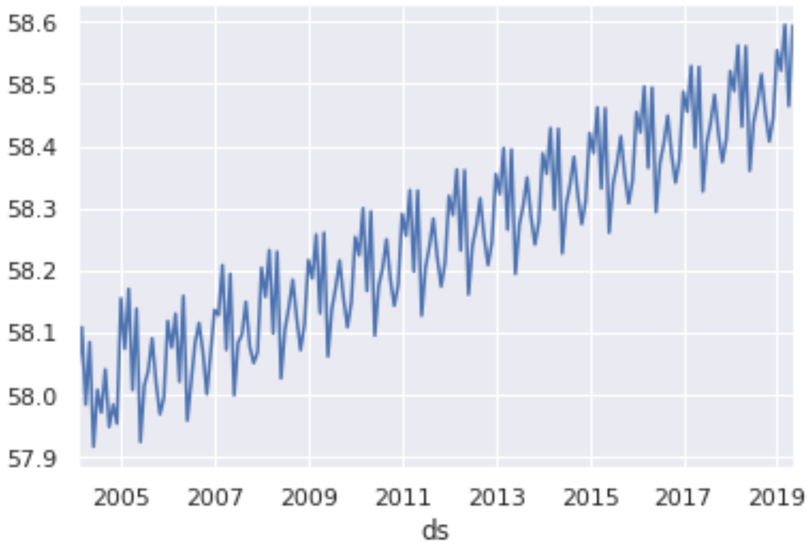
```
y_pred = pd.DataFrame(auto_arima.predict(n_periods = len(y_test)), index = y_test.i
y_pred.columns = ['pred']
y_pred
```

	pred
ds	
2004-02	58.028627
2004-03	58.108983
2004-04	57.984528
2004-05	58.084765
2004-06	57.916936
...	...
2019-01	58.553866
2019-02	58.521579
2019-03	58.595127
2019-04	58.464415
2019-05	58.593596

184 rows × 1 columns

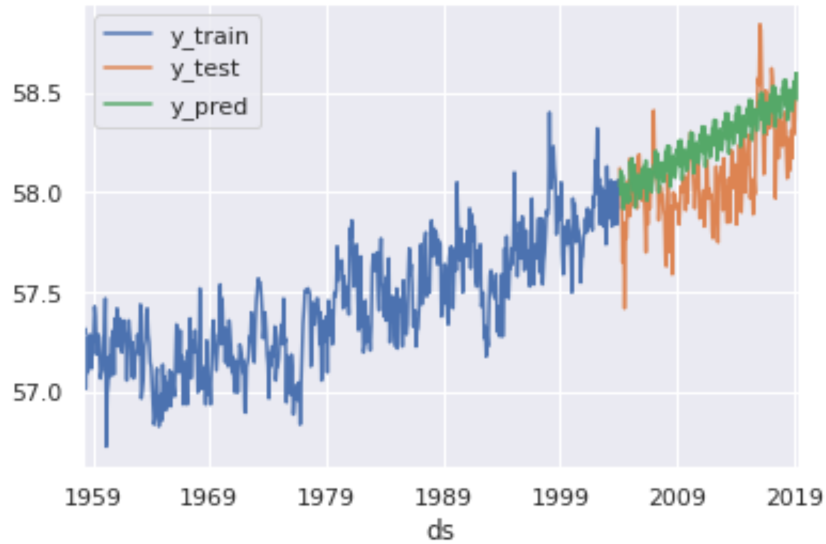
```
# selecting variable of intest & build uniform univariate time series
y_pred = y_pred.groupby(pd.PeriodIndex(y_pred.index, freq="M"))['pred'].mean()
y_pred.plot()
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x7f8e17f66ed0>
```




```
def plot_results(y_train, y_test, y_pred):
    # plot the results
    y_train.plot()
    y_test.plot()

    y_pred.plot()

    plt.legend(['y_train','y_test','y_pred'])
    plt.show()
```

```
# model prediction
plot_results(y_train, y_test, y_pred)
```




```
def smape(a, f):
    return 1/len(a) * np.sum(2 * np.abs(f-a) / (np.abs(a) + np.abs(f))*100)
```

```
#calculate SMAPE
smape(y_test, y_pred)
```

    0.34422784644276183

```
from sktime.performance_metrics.forecasting import MeanAbsoluteScaledError
mase = MeanAbsoluteScaledError()
mase(y_test, y_pred, y_train=y_train)
```

    1.5679160886858377

```
from sktime.performance_metrics.forecasting import MeanAbsoluteError
mae = MeanAbsoluteError()
mae(y_test, y_pred)
```

    0.20012310804681058

```
from sktime.performance_metrics.forecasting import MeanSquaredError
mse = MeanSquaredError()
mse(y_test, y_pred)
```

```
0.06042945368794782
```

```
from sktime.performance_metrics.forecasting import MeanAbsolutePercentageError
mape = MeanAbsolutePercentageError(symmetric=False)
mape(y_test, y_pred)
```

```
0.003450331712048208
```

2.D. 7 pts Refit your best model by adding co2 as an external regressor. Examine the residuals of this model.

[ ] ↳ 11 cells hidden

2.E. 7 pts Predict the test set. Include the following: MAE, MSE, MAPE, sMAPE, MASE. Can use prebuilt functions or calculate by hand.

[ ] ↳ 5 cells hidden

Part 3. 35 pts – Prophet with additional regressors

3.A. 10 pts Using the same train test split as part 2, fit a prophet model to your NON-differenced training set. Examine the residuals. Using the predicted values and y train, calculate the following: MAE, MSE, MAPE, sMAPE, MASE. Can use prebuilt functions or calculate by hand.

```
! pip install pystan --user
! pip install fbprophet --user
```

```
Requirement already satisfied: pystan in /usr/local/lib/python3.7/dist-package
Requirement already satisfied: Cython!=0.25.1,>=0.22 in /usr/local/lib/python3
Requirement already satisfied: numpy>=1.7 in /usr/local/lib/python3.7/dist-pac
Requirement already satisfied: fbprophet in /usr/local/lib/python3.7/dist-pacl
Requirement already satisfied: Cython>=0.22 in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: cmdstanpy==0.9.5 in /usr/local/lib/python3.7/di
Requirement already satisfied: pystan>=2.14 in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: numpy>=1.15.4 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: pandas>=1.0.4 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: matplotlib>=2.0.0 in /usr/local/lib/python3.7/c
Requirement already satisfied: LunarCalendar>=0.0.9 in /usr/local/lib/python3.
Requirement already satisfied: convertdate>=2.1.2 in /usr/local/lib/python3.7/
Requirement already satisfied: holidays>=0.10.2 in /usr/local/lib/python3.7/di
Requirement already satisfied: setuptools-git>=1.2 in /usr/local/lib/python3.7
Requirement already satisfied: python-dateutil>=2.8.0 in /usr/local/lib/pythor
Requirement already satisfied: tqdm>=4.36.1 in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: pymeeus<=1,>=0.3.13 in /usr/local/lib/python3.7
Requirement already satisfied: korean-lunar-calendar in /usr/local/lib/python3
Requirement already satisfied: hijri-converter in /usr/local/lib/python3.7/dis
Requirement already satisfied: six in /usr/local/lib/python3.7/dist-packages (
Requirement already satisfied: pytz in /usr/local/lib/python3.7/dist-packages
Requirement already satisfied: ephem>=3.7.5.3 in /usr/local/lib/python3.7/dist
Requirement already satisfied: kiwisolver>=1.0.1 in /usr/local/lib/python3.7/c
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: pyparsing!=2.0.4,!=2.1.2,!=2.1.6,>=2.0.1 in /us
Requirement already satisfied: typing-extensions in /usr/local/lib/python3.7/c
```

```
! pip install wbdata
```

```
Requirement already satisfied: wbdata in /usr/local/lib/python3.7/dist-package
Requirement already satisfied: appdirs<2.0,>=1.4 in /usr/local/lib/python3.7/c
Requirement already satisfied: decorator>=4.0 in /usr/local/lib/python3.7/dist
Requirement already satisfied: requests>=2.0 in /usr/local/lib/python3.7/dist-
Requirement already satisfied: tabulate>=0.8.5 in /usr/local/lib/python3.7/dis
Requirement already satisfied: chardet<4,>=3.0.2 in /usr/local/lib/python3.7/c
Requirement already satisfied: certifi>=2017.4.17 in /usr/local/lib/python3.7/
Requirement already satisfied: idna<3,>=2.5 in /usr/local/lib/python3.7/dist-p
Requirement already satisfied: urllib3!=1.25.0,!=1.25.1,<1.26,>=1.21.1 in /usr
```

```
from util_prophet import *
from fbprophet import Prophet
from fbprophet.plot import plot_plotly
```

```
y_train_prophet = y_train
y_train_prophet['ds'] = y_train_prophet.index
y_train_prophet.rename(columns={'temp':'y'}, inplace=True)

y_test_prophet = y_test
y_test_prophet['ds'] = y_test_prophet.index
y_test_prophet.rename(columns={'temp':'y'}, inplace=True)
```

```
y_train_prophet
```

	y	co2	ds
1958-03	57.38	315.700	1958-03
1958-04	57.29	317.450	1958-04
1958-05	57.32	317.510	1958-05
1958-06	57.02	316.685	1958-06
1958-07	57.27	315.860	1958-07
...	...	...	...
2003-09	58.02	372.980	2003-09
2003-10	58.05	373.000	2003-10
2003-11	57.84	374.350	2003-11
2003-12	58.06	375.690	2003-12
2004-01	57.92	376.790	2004-01

551 rows × 3 columns

```
y_train_prophet['ds'] = y_train_prophet['ds'].dt.strftime('%Y-%m')
print(y_train_prophet)
```

```
                y      co2       ds
1958-03  57.38  315.700  1958-03
1958-04  57.29  317.450  1958-04
1958-05  57.32  317.510  1958-05
1958-06  57.02  316.685  1958-06
1958-07  57.27  315.860  1958-07
...        ...      ...      ...
2003-09  58.02  372.980  2003-09
2003-10  58.05  373.000  2003-10
2003-11  57.84  374.350  2003-11
2003-12  58.06  375.690  2003-12
2004-01  57.92  376.790  2004-01

[551 rows x 3 columns]
```

```
model = Prophet()
model.fit(y_train_prophet)
```

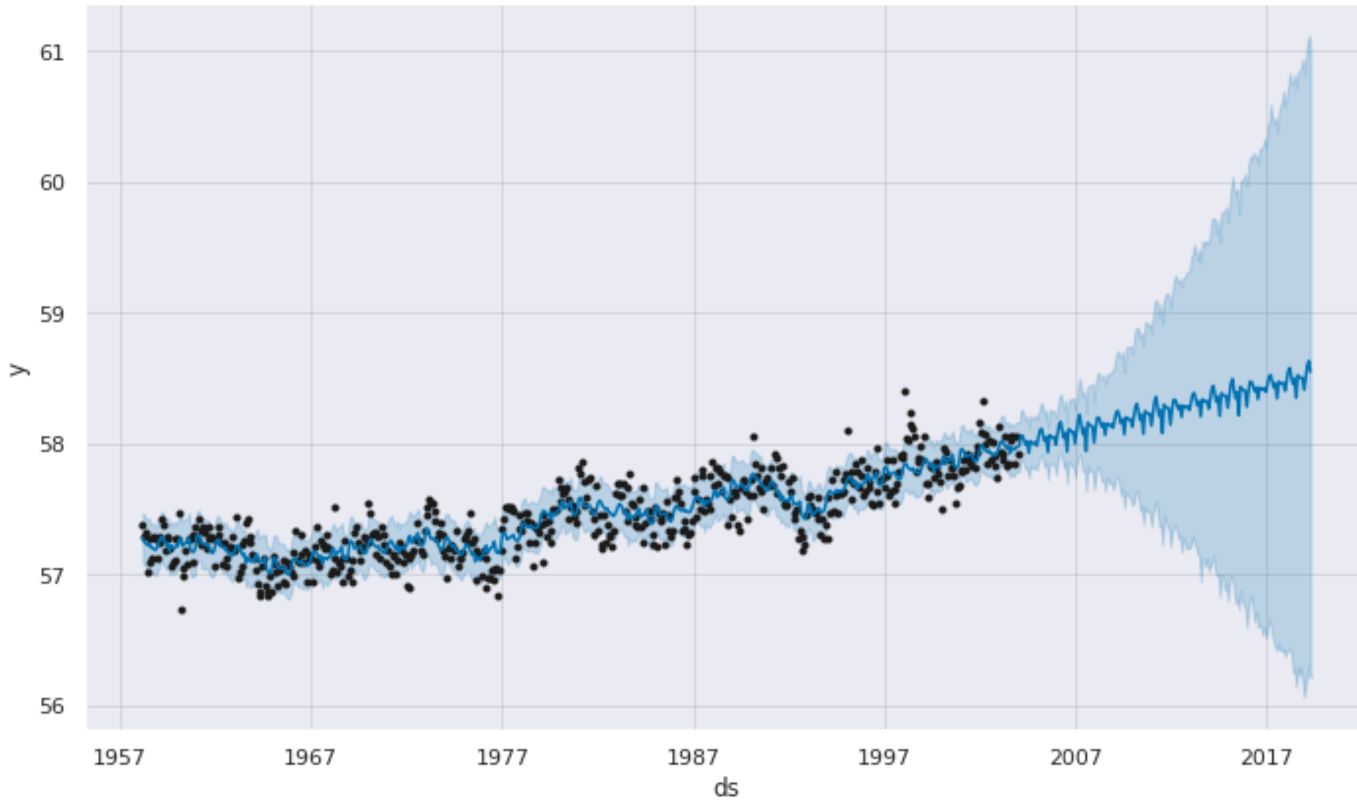
```
INFO:fbprophet:Disabling weekly seasonality. Run prophet with weekly_seasonal:
INFO:fbprophet:Disabling daily seasonality. Run prophet with daily_seasonality
<fbprophet.forecaster.Prophet at 0x7f8e15e13f90>
```

```
future_data = model.make_future_dataframe(periods=len(y_test), freq='m')
```

```
forecast_data = model.predict(future_data)
forecast_data.iloc[0:3].T
```

	0	1	2
ds	1958-03-01 00:00:00	1958-04-01 00:00:00	1958-05-01 00:00:00
trend	57.215189	57.215493	57.215787
yhat_lower	57.086823	57.07275	57.047568
yhat_upper	57.465232	57.454761	57.428443
trend_lower	57.215189	57.215493	57.215787
trend_upper	57.215189	57.215493	57.215787
additive_terms	0.062528	0.031737	0.020392
additive_terms_lower	0.062528	0.031737	0.020392
additive_terms_upper	0.062528	0.031737	0.020392
yearly	0.062528	0.031737	0.020392
yearly_lower	0.062528	0.031737	0.020392
yearly_upper	0.062528	0.031737	0.020392
multiplicative_terms	0.0	0.0	0.0
multiplicative_terms_lower	0.0	0.0	0.0
multiplicative_terms_upper	0.0	0.0	0.0
yhat	57.277717	57.24723	57.236179

```
p = model.plot(forecast_data)
```



```
forecast_data
```

	ds	trend	yhat_lower	yhat_upper	trend_lower	trend_upper	additive_
0	1958-03-01	57.215189	57.086823	57.465232	57.215189	57.215189	0.
1	1958-04-01	57.215493	57.072750	57.454761	57.215493	57.215493	0.
2	1958-05-01	57.215787	57.047568	57.428443	57.215787	57.215787	0.
3	1958-06-01	57.216091	57.034472	57.414895	57.216091	57.216091	0.
4	1958-07-01	57.216385	57.053431	57.429487	57.216385	57.216385	0.
...	...	...	...	...	...	...	
730	2018-12-31	58.493866	56.062716	60.815359	56.196467	60.926419	-0.
731	2019-01-31	58.496772	56.152361	60.945892	56.173862	60.957196	0
732	2019-02-28	58.499396	56.233727	61.058485	56.153445	60.988603	0.
733	2019-03-31	58.502302	56.303357	61.114369	56.127814	61.021028	0.
734	2019-04-30	58.505113	56.211731	61.055610	56.109263	61.035770	0.

735 rows × 16 columns

```
ind = 551
y_pred = forecast_data[ind:]['yhat']
y_true = y_test['y']
```

```
mae =  mean_absolute_error(y_pred, y_true)
mse = mean_squared_error(y_pred, y_true)
mape = mean_absolute_percentage_error(y_pred, y_true)
smape = smape(np.array(y_pred), np.array(y_true))
mase = MeanAbsoluteScaledError(sp=12)
mase_val = mase(y_pred, y_true, y_train=y_train['y'])

print('Non-differenced Prophet errors')
print(f'MAE: {mae}')
print(f'MSE: {mse}')
print(f'MAPE: {mape}')
print(f'SMAPE: {smape}')
print(f'MASE: {mase_val}')
```

```
Non-differenced Prophet errors
MAE: 0.20773835449953923
MSE: 0.0637022071638246
MAPE: 0.0035648442619457877
SMAPE: 0.35730426819346245
MASE: 1.6275797004949653
```

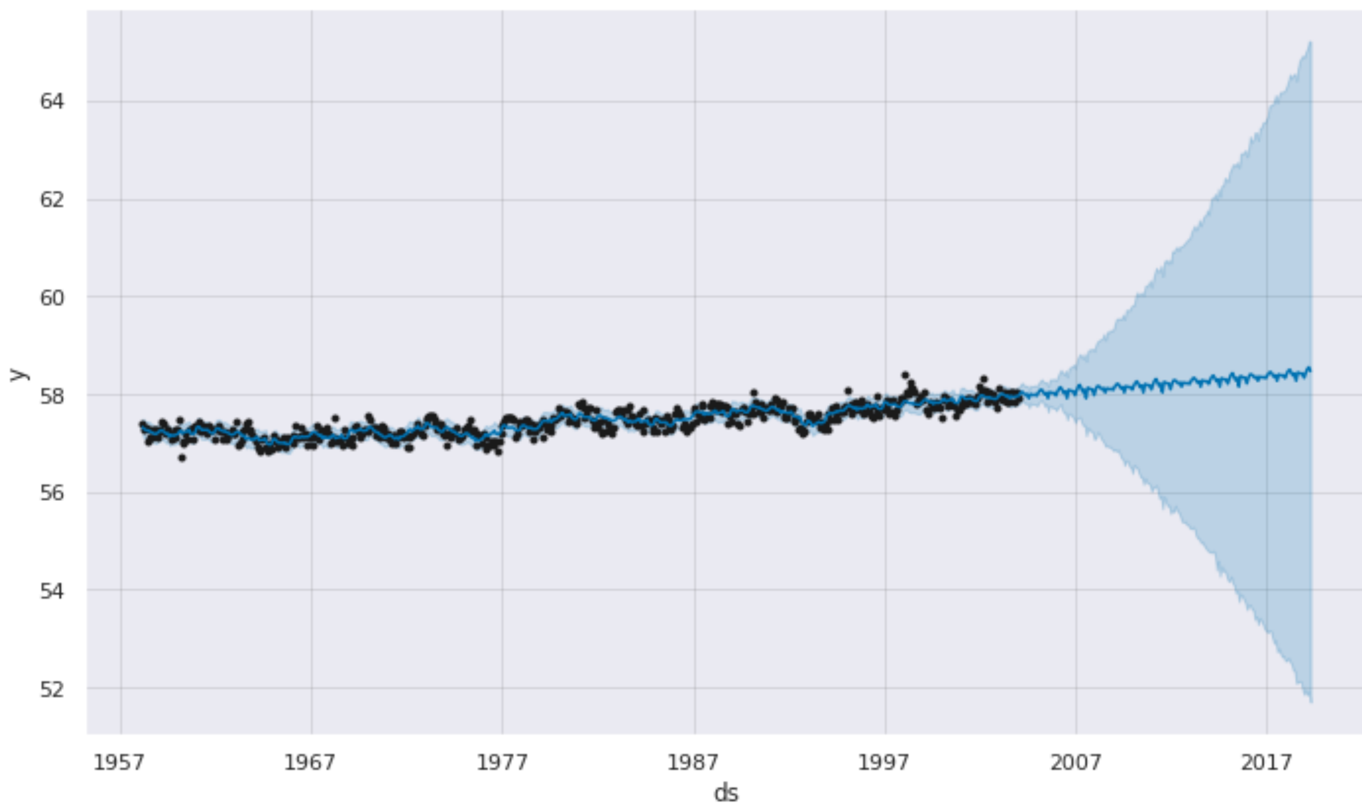
3.B. 10 pts Adjust seasonality, changepoint priors or other methods to see if you can create a better fit model. Examine the residuals. Using the predicted values and y train, calculate the following: MAE, MSE, MAPE, sMAPE, MASE. Can use prebuilt functions or calculate by hand.

```
model_seasonal = Prophet(seasonality_prior_scale=10, changepoint_prior_scale=5)
model_seasonal.fit(y_train_prophet)
future_data_s = model_seasonal.make_future_dataframe(periods=len(y_test), freq='m')
forecast_data_s = model_seasonal.predict(future_data_s)
forecast_data_s.iloc[0:3].T
```

```
INFO:fbprophet:Disabling weekly seasonality. Run prophet with weekly_seasonali
INFO:fbprophet:Disabling daily seasonality. Run prophet with daily_seasonality
```

	0	1	2
ds	1958-03-01 00:00:00	1958-04-01 00:00:00	1958-05-01 00:00:00
trend	57.258126	57.252792	57.247631
yhat_lower	57.145589	57.104178	57.089219
yhat_upper	57.499728	57.457147	57.448086
trend_lower	57.258126	57.252792	57.247631
trend_upper	57.258126	57.252792	57.247631
additive_terms	0.060569	0.030984	0.020538
additive_terms_lower	0.060569	0.030984	0.020538
additive_terms_upper	0.060569	0.030984	0.020538
yearly	0.060569	0.030984	0.020538
yearly_lower	0.060569	0.030984	0.020538
yearly_upper	0.060569	0.030984	0.020538
multiplicative_terms	0.0	0.0	0.0
multiplicative_terms_lower	0.0	0.0	0.0
multiplicative_terms_upper	0.0	0.0	0.0
yhat	57.318695	57.283777	57.268169

```
p_s = model_seasonal.plot(forecast_data_s)
```




```
ind_s = 551
y_pred_s = forecast_data_s[ind:]['yhat']
```

```
mae =  mean_absolute_error(y_pred_s, y_true)
mse = mean_squared_error(y_pred_s, y_true)
mape = mean_absolute_percentage_error(y_pred_s, y_true)
#smape = smape(np.array(y_pred_s), np.array(y_true))
mase = MeanAbsoluteScaledError(sp=12)
mase_val = mase(y_pred_s, y_true, y_train=y_train['y'])

print('Seasonal Adjusted Prophet errors')
print(f'MAE: {mae}')
print(f'MSE: {mse}')
print(f'MAPE: {mape}')
#print(f'SMAPE: {smape}')
print(f'MASE: {mase_val}')
```

```
Seasonal Adjusted Prophet errors
MAE: 0.17691574240883137
MSE: 0.04910700265611346
MAPE: 0.0030388852941335915
MASE: 1.3860919989295903
```

3.C. 10 pts Using your best prophet model, include co2 as an external regressor. Predict the test set. Include the following: MAE, MSE, MAPE, sMAPE, MASE. Can use prebuilt functions or calculate by hand.

```
y_train, y_test = temporal_train_test_split(ts)
```

```
y_train_prophet = y_train
y_train_prophet['ds'] = y_train_prophet.index
y_train_prophet.rename(columns={'temp':'y'}, inplace=True)

y_test_prophet = y_test
y_test_prophet['ds'] = y_test_prophet.index
y_test_prophet.rename(columns={'temp':'y'}, inplace=True)
```

```
y_train_prophet['ds'] = y_train_prophet['ds'].dt.strftime('%Y-%m')
```

```
y_train_prophet
```

ds	y	co2	ds
1958-03	57.38	315.700	1958-03
1958-04	57.29	317.450	1958-04
1958-05	57.32	317.510	1958-05
1958-06	57.02	316.685	1958-06
1958-07	57.27	315.860	1958-07
...	...	...	...
2003-09	58.02	372.980	2003-09
2003-10	58.05	373.000	2003-10
2003-11	57.84	374.350	2003-11
2003-12	58.06	375.690	2003-12
2004-01	57.92	376.790	2004-01

551 rows × 3 columns

```
y_test_prophet
```

	y	co2	ds
ds			
2004-02	58.12	377.37	2004-02
2004-03	58.04	378.39	2004-03
2004-04	57.96	380.50	2004-04
2004-05	57.65	380.62	2004-05
2004-06	57.85	379.55	2004-06
...	...	...	...
2019-01	58.29	410.92	2019-01
2019-02	58.37	411.66	2019-02
2019-03	58.59	412.00	2019-03
2019-04	58.50	413.51	2019-04
2019-05	58.50	414.83	2019-05

184 rows × 3 columns

```
# Python
model = Prophet()
model.fit(y_train_prophet)
```

```
INFO:fbprophet:Disabling weekly seasonality. Run prophet with weekly_seasonal:
INFO:fbprophet:Disabling daily seasonality. Run prophet with daily_seasonality
<fbprophet.forecaster.Prophet at 0x7f8e13b0efd0>
```

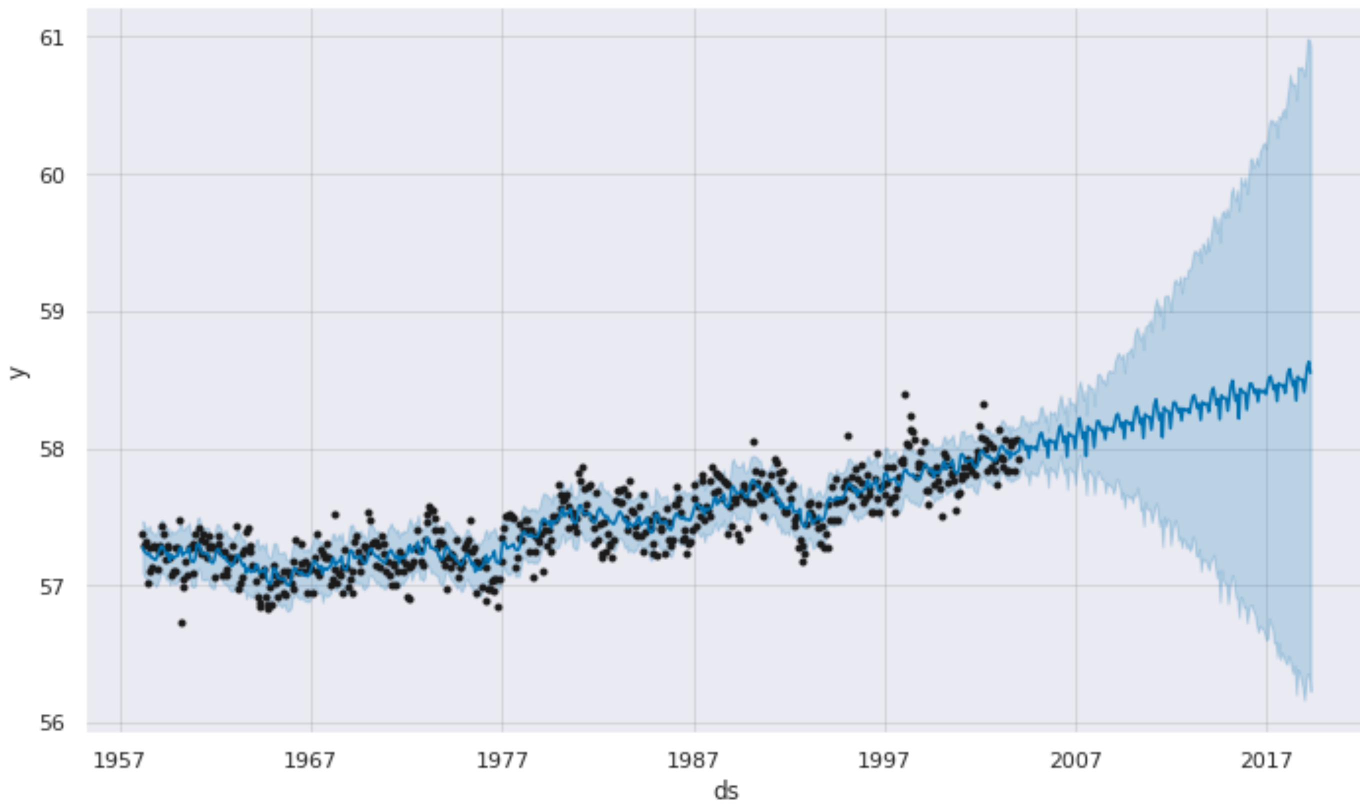
```
future_data = model.make_future_dataframe(periods=len(y_test), freq='m')
```

```
model_seasonal = Prophet(seasonality_prior_scale=10, changepoint_prior_scale=5)
model_seasonal.add_regressor('co2')
model_seasonal.fit(y_train_prophet)
future_data_s = model_seasonal.make_future_dataframe(periods=len(y_test), freq='m')
forecast_data = model.predict(future_data)
forecast_data.iloc[0:3].T
```

```
INFO:fbprophet:Disabling weekly seasonality. Run prophet with weekly_seasonali
INFO:fbprophet:Disabling daily seasonality. Run prophet with daily_seasonality
```

	0	1	2
ds	1958-03-01 00:00:00	1958-04-01 00:00:00	1958-05-01 00:00:00
trend	57.215189	57.215493	57.215787
yhat_lower	57.095134	57.061725	57.053791
yhat_upper	57.468202	57.437656	57.415308
trend_lower	57.215189	57.215493	57.215787
trend_upper	57.215189	57.215493	57.215787
additive_terms	0.062528	0.031737	0.020392
additive_terms_lower	0.062528	0.031737	0.020392
additive_terms_upper	0.062528	0.031737	0.020392
yearly	0.062528	0.031737	0.020392
yearly_lower	0.062528	0.031737	0.020392
yearly_upper	0.062528	0.031737	0.020392
multiplicative_terms	0.0	0.0	0.0
multiplicative_terms_lower	0.0	0.0	0.0
multiplicative_terms_upper	0.0	0.0	0.0
yhat	57.277717	57.24723	57.236179

```
p_r = model_seasonal.plot(forecast_data)
```



```
forecast_data
```

	ds	trend	yhat_lower	yhat_upper	trend_lower	trend_upper	additive_
0	1958-03-01	57.215189	57.095134	57.468202	57.215189	57.215189	0.
1	1958-04-01	57.215493	57.061725	57.437656	57.215493	57.215493	0.
2	1958-05-01	57.215787	57.053791	57.415308	57.215787	57.215787	0.
3	1958-06-01	57.216091	57.023707	57.407833	57.216091	57.216091	0.
4	1958-07-01	57.216385	57.027318	57.435923	57.216385	57.216385	0.
...	...	...	...	...	...	...	
730	2018-12-31	58.493866	56.163239	60.715220	56.251361	60.843148	-0.
731	2019-01-31	58.496772	56.290335	60.832557	56.242664	60.866725	0
732	2019-02-28	58.499396	56.356654	60.985098	56.236092	60.888022	0.
733	2019-03-31	58.502302	56.348506	60.981579	56.228886	60.911462	0.
734	2019-04-30	58.505113	56.228134	60.935334	56.221826	60.933383	0.

735 rows × 16 columns

```
ind_r = 551
y_pred_r = forecast_data[ind:]['yhat']

mae =  mean_absolute_error(y_pred_r, y_true)
mse = mean_squared_error(y_pred_r, y_true)
mape = mean_absolute_percentage_error(y_pred_r, y_true)
#smape = smape(np.array(y_pred_s), np.array(y_true))
mase = MeanAbsoluteScaledError(sp=12)
mase_val = mase(y_pred_r, y_true, y_train=y_train['y'])

print('Prophet Added Regressor errors')
print(f'MAE: {mae}')
print(f'MSE: {mse}')
print(f'MAPE: {mape}')
#print(f'SMAPE: {smape}')
print(f'MASE: {mase_val}')
```

```
Prophet Added Regressor errors
MAE: 0.20773835449953923
MSE: 0.0637022071638246
MAPE: 0.0035648442619457877
MASE: 1.6275797004949653
```

3.D. 5 pts Give a summary of your modeling in prophet. What improved the model, what made the fit worse. Give your reasonings why this would happen given the patterns in the data.

- For our prophet model, the seasonality used was 10 while the AutoARIMA Best model was: ARIMA(1,1,1)(0,1,1)[12]
- Comparison of various error measurements (e.g., MAE, MSE, MAPE, sMAPE, MASE) we can see very similar values, but if given the choice in a professional setting I would use Seasonal Adjusted Prophet. The errors are listed below:
- Non-differenced Prophet errors:
    - MAE: 0.20012310804681058
    - MSE: 0.06042945368794782
    - MAPE: 0.0034342826435053474
    - SMAPE: 0.34422784644276183
    - MASE: 1.5679160886858377
- Seasonal Adjusted Prophet errors
    - MAE: 0.17691574240883137
    - MSE: 0.04910700265611346
    - MAPE: 0.0030388852941335915
    - MASE: 1.3860919989295903
- Prophet Added Regressor errors:
    - MAE: 0.20773835449953923
    - MSE: 0.0637022071638246
    - MAPE: 0.0035648442619457877
    - MASE: 1.6275797004949653
- The advantage of adjusting for seasonal over the other Prophet models is that after data preprocessing step we can see what is a better seasonality and change point priors to better fit our model. In other words, we have more control by "custom fitting" for a better estimate of seasonality and change points.

Part 4. 10 pts – conclusion

4.A. 5 pts Which was the best model according to your error metrics?

The best model according to my error metrics was Seasonally adjusted prophet. We repeat our error metrics here again:

- Seasonal Adjusted Prophet errors
  - MAE: 0.17691574240883137
  - MSE: 0.04910700265611346
  - MAPE: 0.0030388852941335915
  - MASE: **1.3860919989295903**

4.B. 5 pts If you had this data for 100 locations (different pairs of time series), how would you approach modeling this data?

I would probably try the approach for Vector Auto Regression (VAR). In a VAR model, each variable is a linear function of the past values of itself and the past values of all the other variables. In our example above with temperature and Carbon dioxide, since the aim is to predict the temperature, we can simply remove the other variables (except temperature) and fit a model on the remaining univariate series. Another simple idea is to forecast values for each series individually using the techniques we already know (SARIMA, AutoARIMA, Prophet non-diff, Prophet Seasonally adjusted).

✓ 0s completed at 6:16 AM