DETERMINANTS

LLOYD T. ESTRADA I HOLY ANGEL UNIVERSITY

DEFINITION OF DETERMINANT

- is a number associated with square matrix
- determines whether a square matrix is invertible
- is denoted by $|A|$, det A, Δ, or $\begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1j} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2j} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3j} \\ \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{ij} \end{vmatrix}$

DETERMINANT OF 1 X 1 MATRIX

Let A $= [a_{11}]$ be a 1 x 1 matrix.

The determinant of A or $|a_{11}| = a_{11}$.

Determine the determinant of each matrix.

1. $A = [2]$

$\det A = 2$

2. $B = [-4]$

$\det B = -4$

3. $C = \left[\frac{2}{5}\right]$

$\det C = \frac{2}{5}$

DETERMINANT OF 2 X 2 MATRIX

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a 2 x 2 matrix.

The determinant of $A = a_{11}a_{22} - a_{21}a_{12}$.

ILLUSTRATION

$$\mathrm{A} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$|\boldsymbol{A}| = ad - bc$$

2nd

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

1st

Determine the determinant of each matrix.

1. $A = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$

det A = (3)(5) – (2)(4)
det A = 15 – 8
det A = 7

2. $B = \begin{bmatrix} -2 & 4 \\ -6 & 7 \end{bmatrix}$

det B = (-2)(7) – (-6)(4)
det B = -14 – (-24)
det B = -14 + 24
det B = 10

Determine the determinant of each matrix.

3. $C = \begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix}$

det C = (4)(2) – (8)(1)
det C = 8 – 8
det C = 0

Note: If the determinant of a 2 x 2 matrix is 0, then the 2 x 2 matrix is noninvertible (has no inverse).

In the examples, matrices A and B are invertible, while matrix C is noninvertible.

DETERMINANT OF 3 X 3 MATRIX

Let A $= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a 3 x 3 matrix.

The determinant of A

$$= (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}) - (a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{ss}a_{21}a_{12})$$

Copy the first two columns

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$

1st

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$

2nd

$$|A| = (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}) - (a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{ss}a_{21}a_{12})$$

Determine the determinant of each matrix.

1. $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 8 \\ 5 & 6 & 1 \end{bmatrix}$

- Copy the first two columns $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 8 \\ 5 & 6 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 2 \\ 5 & 6 \end{matrix}$

- Draw the downward lines first $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 8 \\ 5 & 6 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 2 \\ 5 & 6 \end{matrix}$

- Get the sum of the products of the elements in each line.

  (1x2x1) + (2x8x5) + (4x3x6) = 2 + 80 + 72 = 154 (Sum)

- Draw the upward lines $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 8 \\ 5 & 6 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 2 \\ 5 & 6 \end{matrix}$

- Get the sum of the products of the elements in each line.

  (5x2x4) + (6x8x1) + (1x3x2) = 40 + 48 + 6 = 94

- Get the difference of the sums : sum – sum

Difference = 154 – 94 = 60

Thus, **the determinant of A or $|A| = 60$.**

Determine the determinant of each matrix.

2. $A = \begin{bmatrix} 0 & 2 & -5 \\ 3 & -1 & 2 \\ 4 & -3 & 1 \end{bmatrix}$

- Copy the first two columns $\begin{bmatrix} 0 & 2 & -5 \\ 3 & -1 & 2 \\ 4 & -3 & 1 \end{bmatrix} \begin{matrix} 0 & 2 \\ 3 & -1 \\ 4 & -3 \end{matrix}$

- Draw the downward lines first $\begin{bmatrix} 0 & 2 & -5 \\ 3 & -1 & 2 \\ 4 & -3 & 1 \end{bmatrix} \begin{matrix} 0 & 2 \\ 3 & -1 \\ 4 & -3 \end{matrix}$

- Get the sum of the products of the elements in each line.

  (0x(-1)x1) + (2x2x4) + ((-5)x3x(-3)) = 0 +16 + 45 = 61 (Sum)

- Draw the upward lines $\begin{bmatrix} 0 & 2 & -5 \\ 3 & -1 & 2 \\ 4 & -3 & 1 \end{bmatrix} \begin{matrix} 0 & 2 \\ 3 & -1 \\ 4 & -3 \end{matrix}$

- Get the sum of the products of the elements in each line.

  (4x(-1)x(-5)) + ((-3)x2x0) + (1x3x2) = 20 + 0 + 6 = 26

- Get the difference of the sums : sum – sum

Difference = 61 – 26 = 35

Thus, **the determinant of A or $|A| = 35$.**

REFERENCES

Penney, R. (2015). *Linear algebra: ideas and applications* (4th ed.). John Wiley &

Williams, G. (2019). *Linear algebra with applications* (9th ed.). Jones & Barlett Learning.