HOLY ANGEL UNIVERSITY
School of Arts and Sciences

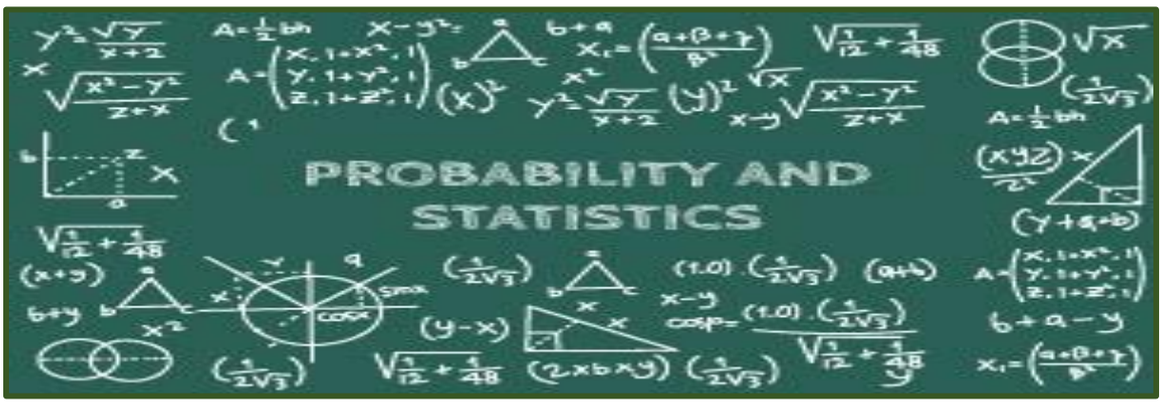

Image Source: https://alison.com/course/strand-1-leaving-certificate-ordinary-level-probability-and-statistics-revised

Lesson 2: Permutation & Combination

Permutations that involve ordering *r* of *n* distinct objects obey the formula:

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

where *n* is the number of objects taken *r* at a time.

The factorial symbol ! in the formula is defined as:

n! = n (n - 1) (n - 2) (n - 3)…(1)

Example:

6! = 6 x 5 x 4 x 3 x 2 x 1 = 720

3! = 3 x 2 x 1 = 6

Note that by definition 0! = 1

Read the material: https://documents.uow.edu.au/content/groups/public/@web/@eis/@maas/documents/mm/uow168693.pdf

Watch the video: https://www.youtube.com/watch?v=uNS1QvDzCVw

Example 1: A debating team consists of 4 speakers.

a) In how many ways can all 4 speakers be arranged in a row for a photo?

b) How many ways can the captain and vice-captain be chosen?

Solution:

a. 4 x 3 x 2 x 1 = 4! or ${}_4P_4$

b. 4 x 3 = 12 or ${}_4P_2$

Example 2: There are 7 horses in a race. a) In how many different orders can the horses finish? b) How many trifectas (1st, 2nd and 3rd) are possible?

Solution:

a. 7x6x5x4x3x2x1 = 7! or ${}_7P_7$

b. 7x6x5 = 210 or ${}_7P_3$

Example 3: In how many ways can 5 boys and 4 girls be arranged on a bench if a) there are no restrictions? b) boys and girls alternate?

Solution

a. 9! or ${}_9P_9$
b. A boy will be on each end

BGBGBGBGB = 5 .4. 4.3.3.2. 2. 1. 1 = = 5! x 4! or ${}_5P_5$ x ${}_4P_4$

The number of distinct permutations of *n* objects where n1 of the objects are identical, n2 of the objects are identical… nr of the objects are identical is found by the formula:

$$\frac{n!}{n_1!\cdot n_2!\cdot n_3!\cdots n_r!}$$

Example 4: How many different arrangements of the word PARRAMATTA are possible?

Solution:

10 letters but note repetition (4 A's, 2 R's, 2 T's)

No. of arrangements = 10! / 4! 2! 2! = 37 800 ways

Example 5: In how many ways can the letters in the word "TALLAHASSEE" be arranged?

Solution:

Of the 11 letters, 3 are A's, 2 are L's, 2 are S's and 2 are E's.

No. of arrangement = 11! / 3!2!2!2! = 831,000 ways

End of discussion. Should you have questions regarding the discussion, please contact your instructor during his/her consultation hours. Otherwise, you may proceed with **Activity No. 3**

A **combination of *n* elements taken *r* at a time** is a subset of the collection of elements where order is *not* important.

Using the letters, A, B, C, and D, find all the possible combinations using two of the letters.

{AB}, {AC}, {AD}, {BC}, {BD}, {CD}

There are six different combinations using 2 of the 4 letters.

This is the same as {BA}

The formula for the number of **combinations of *n* elements taken *r* at a time** is

$$_nC_r = \frac{n!}{(n-r)!r!}.$$

where *n* is the number of objects taken *r* at a time.

Example 1: How many ways can a basketball team of 5 players be chosen from 12 players?

Solution:

Since n = 12; r = 5, then the answer is $_{12}C_5$ or 792 ways.

Example 2: A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible if a) there are no restrictions? b) one person must be chosen on the committee? c) one woman must be excluded from the committee?

Solution:

a. n = 10, r = 5 so the answer is ${}_{10}C_5$ or 252 ways
b. 1 x ${}_{9}C_4$ = 126 ways
c. ${}_{9}C_5$ = 125 ways

Example 3: In a hand of poker, 5 cards are dealt from a regular pack of 52 cards. a. In how many of these hands are there all Hearts? b. all the same color? c. 3 Aces and two Kings?

Solution:

a. n = 13 (hearts) r = 5, therefore the answer is ${}_{13}C_5$ or 1,287 ways
b. Red or Black = ${}_{26}C_5$ + ${}_{26}C_5$ = 131, 560 ways
c. ${}_{4}C_3$ x ${}_{4}C_2$ = 4 x 6 = 24 ways

End of discussion. Should you have questions regarding the discussion, please contact your instructor during his/her consultation hours. Otherwise, you may proceed with **Activity No. 4**