



DETERMINANTS

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DEFINITION OF DETERMINANT

- is a number associated with square matrix
- determines whether a square matrix is invertible

- is denoted by $|A|$, $\det A$, Δ , or

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1j} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2j} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3j} \\ \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{ij} \end{vmatrix}$$

DETERMINANT OF 1 X 1 MATRIX

Let $A = [a_{11}]$ be a 1 x 1 matrix.

The determinant of A or $|a_{11}| = a_{11}$.

Determine the determinant of each matrix.

$$1. \quad A = [2]$$

$$\det A = 2$$

$$2. \quad B = [-4]$$

$$\det B = -4$$

$$3. \quad C = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

$$\det C = \frac{2}{5}$$

DETERMINANT OF 2 X 2 MATRIX

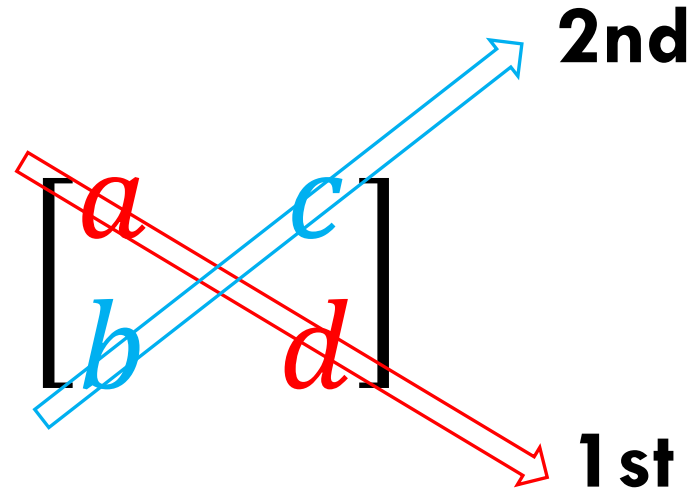
Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a 2 x 2 matrix.

The determinant of $A = a_{11}a_{22} - a_{21}a_{12}$.

ILLUSTRATION

$$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$|A| = ad - bc$$



Determine the determinant of each matrix.

$$1. \quad A = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$$

$$\det A = (3)(5) - (2)(4)$$

$$\det A = 15 - 8$$

$$\det A = 7$$

$$2. \quad B = \begin{bmatrix} -2 & 4 \\ -6 & 7 \end{bmatrix}$$

$$\det B = (-2)(7) - (-6)(4)$$

$$\det B = -14 - (-24)$$

$$\det B = -14 + 24$$

$$\det B = 10$$

Determine the determinant of each matrix.

$$3. \quad C = \begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix}$$

$$\det C = (4)(2) - (8)(1)$$

$$\det C = 8 - 8$$

$$\det C = 0$$

Note: If the determinant of a 2×2 matrix is 0, then the 2×2 matrix is noninvertible (has no inverse).

In the examples, matrices A and B are invertible, while matrix C is noninvertible.

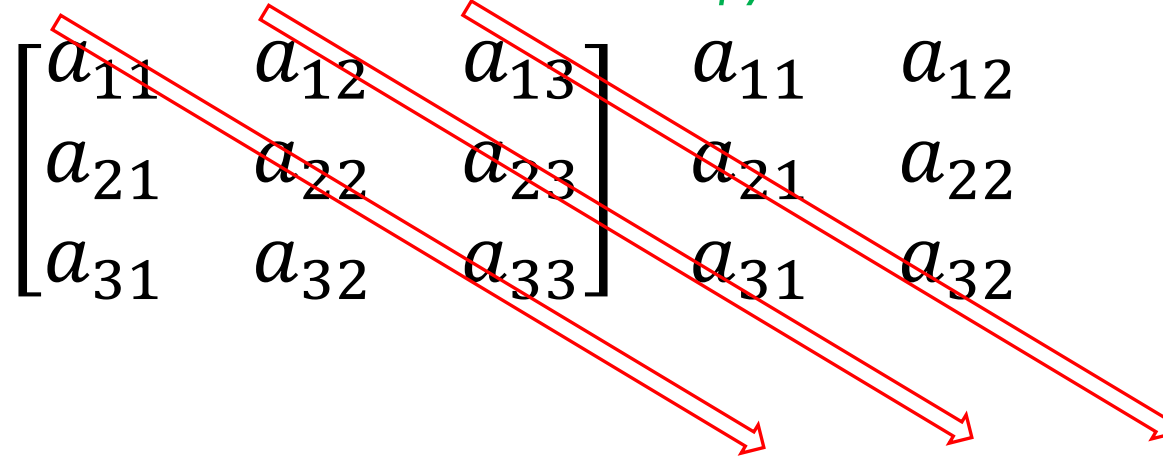
DETERMINANT OF 3 X 3 MATRIX

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a 3 x 3 matrix.

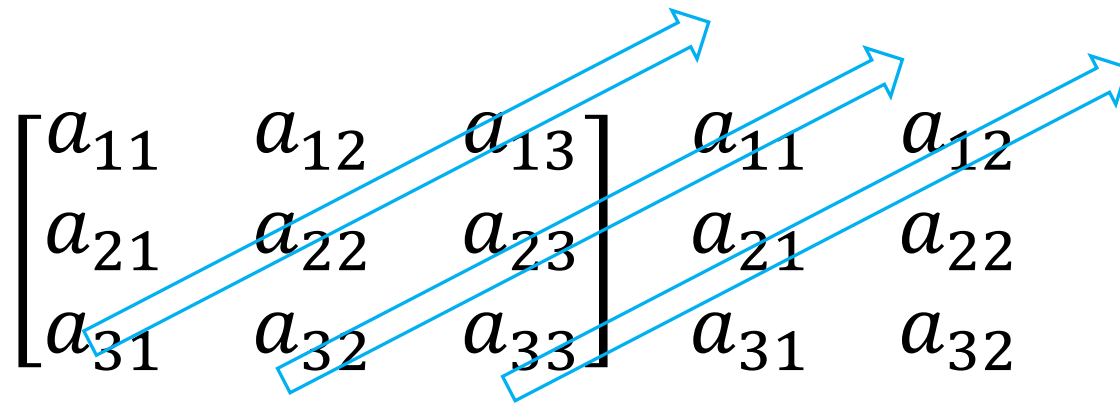
The determinant of A

$$= (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}) - (a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{33}a_{21}a_{12})$$

Copy the first two columns

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$


1st

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$


2nd

$$|A| = (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}) - (a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{33}a_{21}a_{12})$$

Determine the determinant of each matrix.

1. $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 8 \\ 5 & 6 & 1 \end{bmatrix}$

□ Copy the first two columns $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 8 \\ 5 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 5 & 6 \end{bmatrix}$

❑ Draw the downward lines first

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 8 \\ 5 & 6 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 2 \\ 5 & 6 \end{matrix}$$

❑ Get the sum of the products of the elements in each line.

$$(1 \times 2 \times 1) + (2 \times 8 \times 5) + (4 \times 3 \times 6) = 2 + 80 + 72 = 154 \text{ (Sum)}$$

❑ Draw the upward lines

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 8 \\ 5 & 6 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 2 \\ 5 & 6 \end{matrix}$$

❑ Get the sum of the products of the elements in each line.

$$(5 \times 2 \times 4) + (6 \times 8 \times 1) + (1 \times 3 \times 2) = 40 + 48 + 6 = 94$$

□ Get the difference of the sums : $\text{sum} - \text{sum}$

$$\text{Difference} = 154 - 94 = 60$$

Thus, **the determinant of A or $|A| = 60$.**

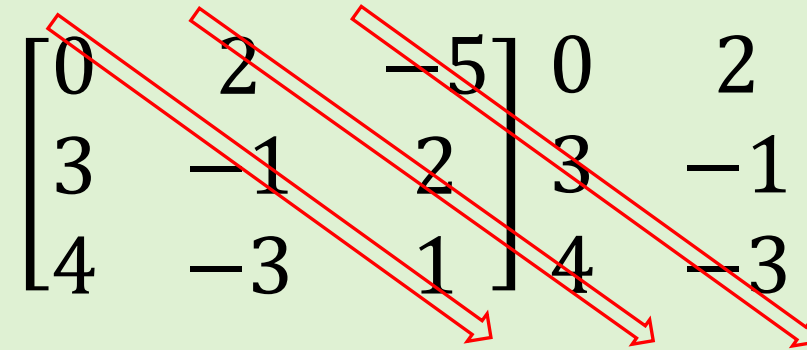
Determine the determinant of each matrix.

$$2. \quad A = \begin{bmatrix} 0 & 2 & -5 \\ 3 & -1 & 2 \\ 4 & -3 & 1 \end{bmatrix}$$

□ Copy the first two columns

$$\begin{bmatrix} 0 & 2 & -5 \\ 3 & -1 & 2 \\ 4 & -3 & 1 \end{bmatrix} \begin{array}{cc} 0 & 2 \\ 3 & -1 \\ 4 & -3 \end{array}$$

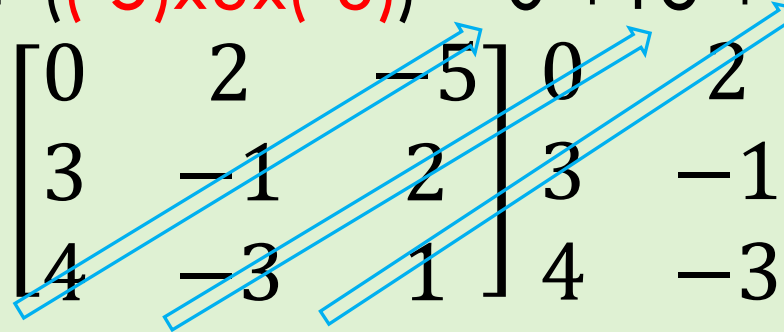
❑ Draw the downward lines first

$$\begin{bmatrix} 0 & 2 & -5 \\ 3 & -1 & 2 \\ 4 & -3 & 1 \end{bmatrix} \begin{array}{c} 0 \\ 3 \\ 4 \end{array} \begin{array}{c} 2 \\ -1 \\ -3 \end{array}$$


❑ Get the sum of the products of the elements in each line.

$$(0 \times (-1) \times 1) + (2 \times 2 \times 4) + ((-5) \times 3 \times (-3)) = 0 + 16 + 45 = 61 \text{ (Sum)}$$

❑ Draw the upward lines

$$\begin{bmatrix} 0 & 2 & -5 \\ 3 & -1 & 2 \\ 4 & -3 & 1 \end{bmatrix} \begin{array}{c} 0 \\ 3 \\ 4 \end{array} \begin{array}{c} 2 \\ -1 \\ -3 \end{array}$$


❑ Get the sum of the products of the elements in each line.

$$(4 \times (-1) \times (-5)) + ((-3) \times 2 \times 0) + (1 \times 3 \times 2) = 20 + 0 + 6 = 26$$

□ Get the difference of the sums : **sum** – **sum**

$$\text{Difference} = 61 - 26 = 35$$

Thus, **the determinant of A or $|A| = 35$.**

REFERENCES

Penney, R. (2015). *Linear algebra: ideas and applications* (4th ed.). John Wiley &

Williams, G. (2019). *Linear algebra with applications* (9th ed.). Jones & Barlett Learning.