Homework 10

STAT 984

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Exercise 8.1

Let $X_1, ..., X_n$ be a simple random sample from a Pareto distribution with density

$$f(x) = \theta c^{\theta} x^{-(\theta+1)} I\{x > c\}$$

for a known constant $c > 0$ and parameter $\theta > 0$. Derive the Wald, Rao, and likelihood ratio tests of $\theta = \theta_0$ against a two-sided alternative.

Exercise 8.2

Suppose that $\boldsymbol{X}$ is multinomial$(n, \boldsymbol{p})$, where $\boldsymbol{p} \in \mathbb{R}^k$. In order to satisfy the regularity condition that the parameter space be an open set, define $\boldsymbol{\theta} = (p_1, ..., p_{k-1})$. Suppose that we wish to test $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}^0$ against $H_1 : \boldsymbol{\theta} \neq \boldsymbol{\theta}^0$.

(a) Prove that the Wald and score tests are the same as the usual Pearson chi-square test.

(b) Derive the likelihood ratio statistic $2\Delta_n$.

Exercise 8.8

Let $X_1, ..., X_n$ be an independent sample from an exponential distribution with mean λ, and $Y_1, ..., Y_n$ be an independent sample from an exponential distribution with mean μ. Assume that X_i and Y_i are independent. We are interested in testing the hypothesis $H_0 : \lambda = \mu$ verses $H_1 : \lambda > \mu$. Consider the statistic

$$T_n = 2\sum_{i=1}^{n}(I_i - 1/2)/\sqrt{n},$$

where I_i is the indicator variable $I_i = I(X_i > Y_i)$.

(a) Derive the asymptotic distribution of T_n under the null hypothesis.

(b) Use the Lindeberg Theorem to show that, under the local alternative hypothesis $(\lambda_n, \mu_n) =)\lambda + n^{-1/2}\delta, \lambda)$, where $\delta > 0$,

$$\frac{\sum_{i=1}^{n}(I_i - \rho_n)}{\sqrt{n\rho_n(1-\rho_n)}} \xrightarrow{\mathbb{L}} N(0,1), \text{ where } \rho_n = \frac{\lambda_n}{\lambda_n + \mu_n} = \frac{\lambda + n^{-1/2}\delta}{2\lambda + n^{-1/2}\lambda}.$$

Exercise 8.9

Suppose $X_1, ...X_m$ is a simple random sample and $Y_1, ..., Y_n$ is another simple random sample independent of the X_i, with $P(X_i \leq t) = t^2$ for $t \in [0, 1]$ and $P(Y_i \leq t) = (t - \theta)^2$ for $t \in [\theta, \theta + 1]$. Assume $m/(m + n) \to \rho$ as $m, n \to \infty$ and $0 < \theta < 1$.

Find the asymptotic distribution of $\sqrt{m + n}[g(\bar{Y} - \bar{X}) - g(\theta)]$.