

Final Homework

Due: Noon Wednesday, December 18, 2019

1. Modify the ESS function to also estimate a 95% HPD interval. Include your function in the printed version of the homework.
2. Consider the following model:

$$\begin{aligned}y_i|\kappa_i &\stackrel{\text{ind}}{\sim} \text{exponential}(\kappa_i) \\ \kappa_i &= \prod_j \theta_j^{x_{ij}} \\ \theta_j &\stackrel{\text{ind}}{\sim} \text{gamma}(\alpha_j, \lambda_j)\end{aligned}$$

where x_{ij} are known covariates, and α_j and λ_j are known hyperparameters. In some cases the y_i are censored at time t_i so the data are the pairs (t_i, w_i) where $w_i = 1$ if t_i is an uncensored time and $w_i = 0$ if t_i is an uncensored time yielding

$$f_{\mathbf{t}, \mathbf{w}|\boldsymbol{\kappa}}(\mathbf{t}, \mathbf{w}|\boldsymbol{\kappa}) = \prod_i \kappa_i^{w_i} e^{-t_i \kappa_i}.$$

Note: Consistent with the book we are using the parameterization where κ_i and λ_j are rate parameters as opposed to scale parameters.

- (a) Derive the score function and Hessian matrix necessary to compute the MLE estimates of $\boldsymbol{\theta}$ using the Newton-Raphson algorithm. Note: This will not involve the prior distributions, $\text{gamma}(\alpha_j, \lambda_j)$.
- (b) Write a function to compute MLE estimates of $\boldsymbol{\theta}$ along with their approximate standard errors given \mathbf{t} , \mathbf{w} , and \mathbf{x} . Include your function in the printed version of the homework.
- (c) Derive the conditional distributions necessary to implement the Gibbs sampler for $\boldsymbol{\theta}$.
- (d) Write a function that to implement your Gibbs sampler given \mathbf{t} , \mathbf{w} , \mathbf{x} , $\boldsymbol{\alpha}$, and $\boldsymbol{\lambda}$. Include your function in the printed version of the homework.
- (e) Using the data and model described in problem 7.4.
 - i. Run your MLE function to obtain maximum likelihood estimates and approximate standard errors of $\boldsymbol{\theta}$.
 - ii. Run and evaluate the performance of your Gibbs sampler using a single chain.
 - iii. Using the `doParallel` and `foreach` packages run multiple chains of your Gibbs sampler and evaluate the performance of your Gibbs sampler.
 - iv. Compute summary statistics of the estimated joint posterior distribution of the $\boldsymbol{\theta}$ along with the mean remission times for control and treated patients, including marginal means, standard deviations, and 95% probability intervals.

3. Using the data and change point model for problem 7.6. For the prior assume $\lambda_i \sim \text{Gamma}(\gamma_1, \alpha)$ for $i = 1, 2$ and $\alpha \sim \text{Gamma}(\gamma_2, \gamma_3)$ where γ_1 , γ_2 , and γ_3 are known hyperparameters.
- (a) Derive the conditional distributions necessary to implement a change point model Gibbs sampler.
 - (b) Write a function that to implement a change point model Gibbs sampler given \mathbf{X} and the three gamma distribution hyperparameters. Include your function in the printed version of the homework.
 - (c) Run your Gibbs sample using the coal mine data and compute summary statistics of the estimated joint posterior distribution of the θ , λ_1 , and λ_2 , including marginal means, standard deviations, and 95% probability intervals.