Chapter 8: Comparing a Numerical Variable Across More Than Two Groups

In the previous chapter, we covered how we could compare a numerical variable across two groups using (1) a difference in two means (independent samples), and (2) the mean of the differences (paired).

ANOVA (ANalysis Of VAriance)

In this chapter, we will learn how to compare a numerical variable across more than two groups using a statistical analysis called ANOVA.

Example 8.1: IMDb Scores between Genres

Recall the IMDb Scores for movies released in 2020 from Chapter 5. The data set is comprised of the following variables collected on each movie:

Variable	Description
`Movie`	Title of the movie
`Rating`	Average IMDb user rating score from 1 to 10
`numVotes`	Number of votes from IMDb users
`Genre`	Categories the movie falls into (e.g., Action, Drama, etc.)
`2020 Gross`	Gross profit from movie viewing
`runtimeMinutes`	Length of movie (in minutes)

Below is a table summarizing the number of observations (movies) in the data set for the five most common Genres.

```
# A tibble: 5 x 2
  Genre                n
  <chr>            <int>
1 Drama               75
2 Thriller/Suspense   29
3 Documentary         26
4 Comedy              23
5 Horror              19
```

Research Question: Is there a difference in mean IMDb scores between movie genres?

The output below shows example observations from the data set.

```
head(movie_ratings, n = 10)
```

```
# A tibble: 10 x 6
   Movie             Genre          `2020 Gross` runtimeMinutes Rating numVotes
   <chr>             <fct>                 <dbl> <chr>           <dbl>    <dbl>
 1 1917              Thriller/Suspe~   157901466 "34"              5.7       23
 2 The Invisible Man Horror             64914050 "71"              7.7    29256
 3 Halloween         Horror             47274000 "91"              7.8   222169
 4 Little Women      Drama              37593127 "60"              6.5       34
 5 Just Mercy        Drama              35733621 "\\N"             7.6       14
 6 Knives Out        Drama              35244610 "\\N"             8         19
 7 Fantasy Island    Horror             26441782 "60"              6.5     6599
 8 Unhinged          Thriller/Suspe~    20831465 "79"              5.1     1548
 9 The Photograph    Drama              20578185 "32"              7         89
10 Underwater        Thriller/Suspe~    17291078 "60"              7.2       60
```

1. What is the observational unit for this study?

A movie

2. What are the variables assessed in this study? What are their roles (explanatory / response) and data types?

- Explanatory - Genre (categorical) 5 levels (Drama, Thriller, Doc, Comedy, Horror)

- Response - Rating (scale 1-10) (numeric)

3. What are the parameters of interest for this study?

μ_{Drama} : The mean ratings for all Drama movies released in 2020.

μ_{Thrill} : The mean ratings for all Thriller/Suspense movies released in 2020.

μ_{Docu} : The mean ratings for all Documentary movies released in 2020

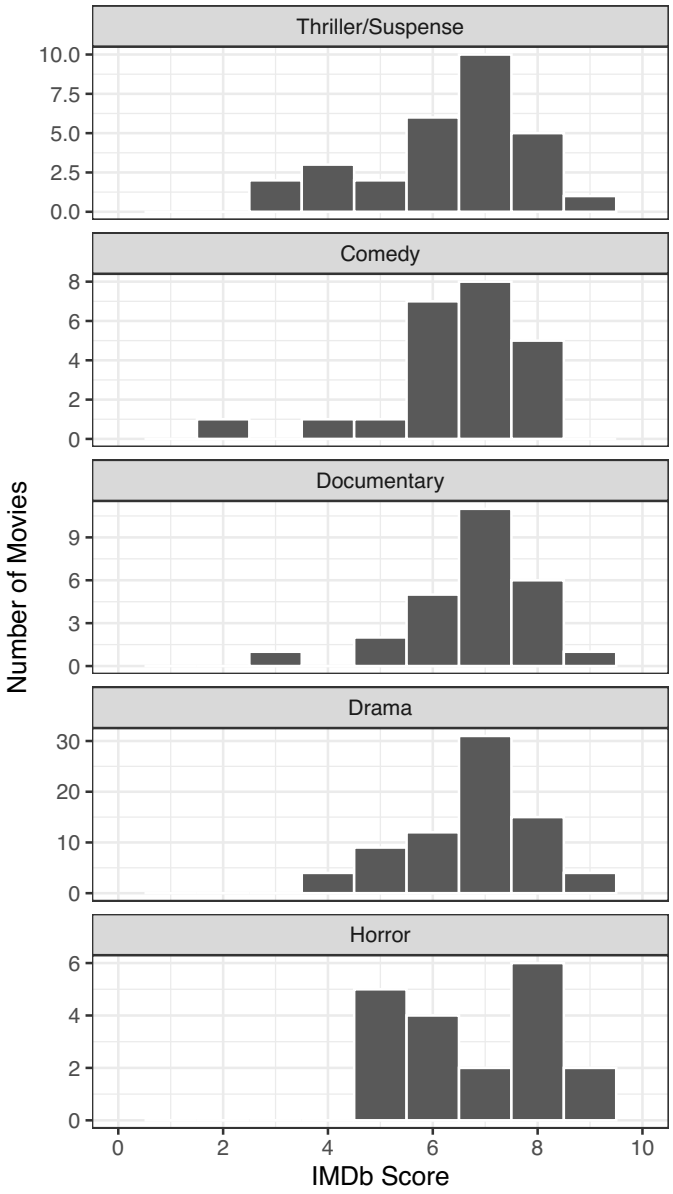
μ_{Comedy} : The mean ratings for all Comedy movies released in 2020

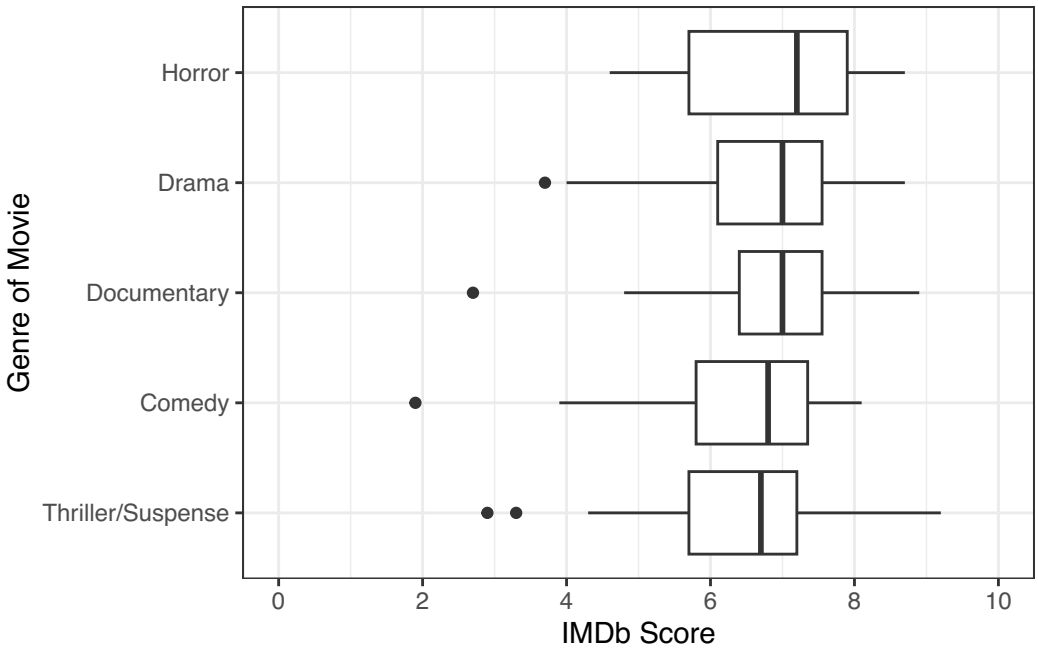
μ_{Horror} : The mean ratings for all Horror movies released in 2020.

4. Think back to last week, what were two ways we visualized one numerical variable and one categorical variable?

- Side-by-side bar plot and faceted histograms

The figures below display the IMDb Scores across the five movie genre categories.


Thriller/Suspense
Comedy
Documentary
Drama
Horror
Number of Movies
IMDb Score
0
2
4
6
8
10



Answer the following questions about the distributions of IMDb Scores across the five Genres shown above.

5. Which genre has the highest center? Horror

6. Which genre has the largest spread? Thriller/Suspense ? Horror ?

7. Which genre has the most skewed distribution? Comedy ? Drama ?

Let's obtain a more complete picture of how different these groups are with summary statistics. Our familiar friend `favstats()` can help us compare summary statistics across different genre groups.

Like before, the rating of the film is the response and the genre is the explanatory variable.

```
favstats(Rating ~ Genre, data = movie_ratings)
```

	Genre	min	Q1	median	Q3	max	mean	sd	n	missing
1	Thriller/Suspense	2.9	5.7	6.7	7.20	9.2	6.317241	1.536478	29	0
2	Comedy	1.9	5.8	6.8	7.35	8.1	6.413043	1.413025	23	0
3	Documentary	2.7	6.4	7.0	7.55	8.9	6.834615	1.203974	26	0
4	Drama	3.7	6.1	7.0	7.55	8.7	6.729333	1.148533	75	0
5	Horror	4.6	5.7	7.2	7.90	8.7	6.826316	1.370256	19	0

8. Report the observed mean rating for each genre. Use appropriate notation.

$\bar{x}_{Drama} = 6.73$ $\bar{x}_{Doc} = 6.83$

$\bar{x}_{Thrill} = 6.32$ $\bar{x}_{Horror} = 6.82$

$\bar{x}_{comedy} = 6.41$

9. Which genres have the largest difference in their mean rating?

Doc & Horror.

$\bar{x}_{Doc} - \bar{x}_{Horror} = 6.84 - 6.32 = 0.52$

10. Which genre has the largest standard deviation in ratings?

$s_{Thrill} = 1.54$

11. Which genre has the smallest standard deviation in ratings?

$s_{Drama} = 1.15$

12. How many times larger is your answer in 10 than your answer in 11 ?

$s_{Thrill} / s_{Drama} = 1.54 / 1.15 = 1.34$ times larger

Now that we have explored our data with summary statistics and visualizations, we want to use our data to draw inferences and make claims about the larger population (all movies).

14. Set up the null and alternative hypotheses.

- In words:

Null: The mean IMDb score is the same between genres for all movies released in 2020.

Alt: At least one genre has a different mean IMDb score for all movies.

- In symbols:

$H_0: \mu_{Drama} = \mu_{Thriller} = \mu_{Docu} = \mu_{comedy} = \mu_{Horror}$

$H_A: \mu_{Drama} \neq \mu_{Thriller} \neq \mu_{Docu} \neq \mu_{comedy} \neq \mu_{Horror}$

H_A: At least one μ_i differs

In order to test our research question, we could conduct a simulation similar to what we did with two categorical variables (yawn experiment) and discussed when comparing a numerical variable across two groups.

- Step 1: Write the genre and rating on 172 cards.
- Step 2: Simulate what could have happened if the null was true and rip apart the pairs
- Step 3: Generate a new data set by shuffling and randomly matching new pairs.
- Step 4: Calculate the test statistic for the new simulated data set and add it to the dot plot. ↖ this is the F-statistic

We would then repeat this process 100 or 1000 times to get an idea of what the sampling distribution of the *test statistic* looks like.

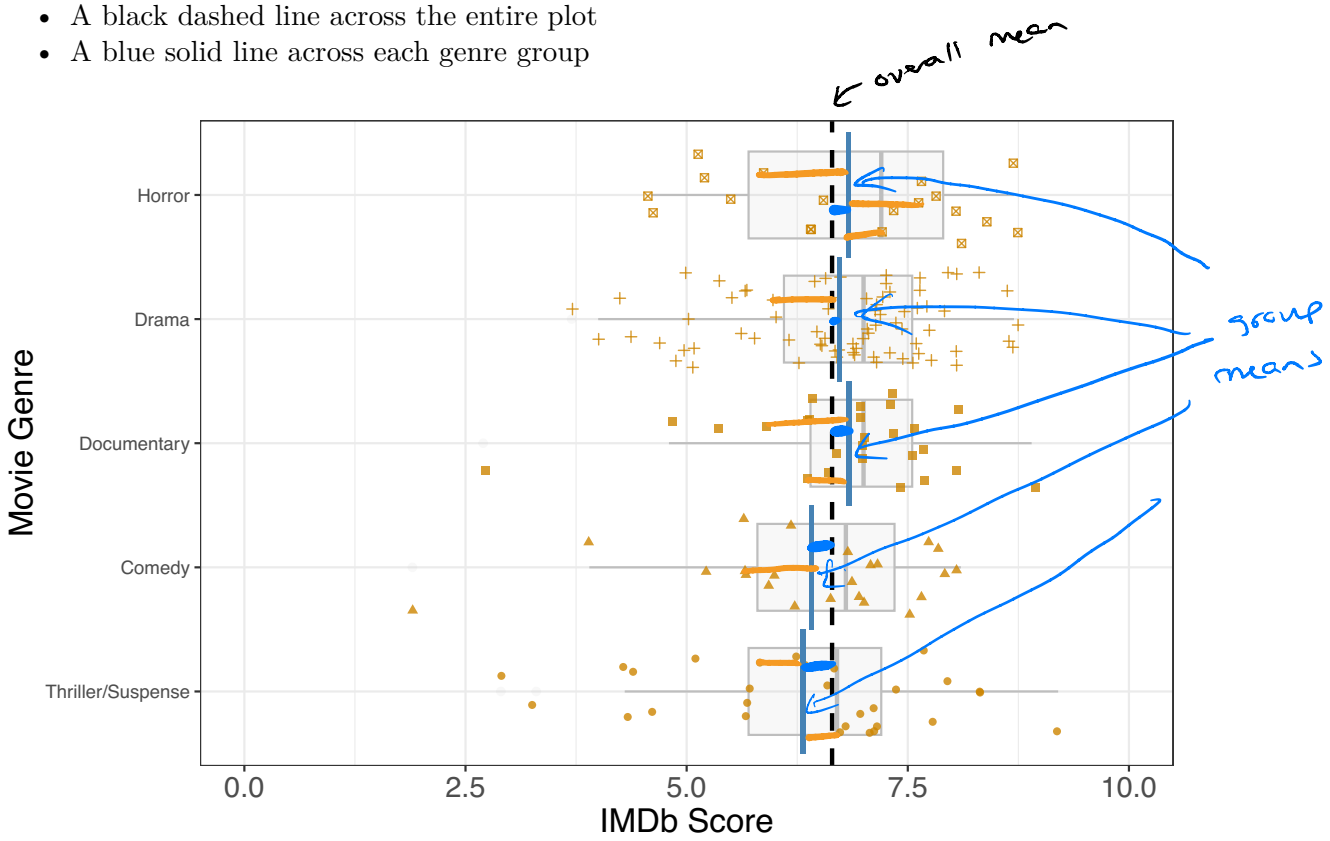
Introducing a new *test statistic*

In an ANOVA, there are more than two groups that we wish to compare how different the means are from each other. We could make every comparison of two means (Drama - Action, Horror - Documentary, Comedy - Adventure, etc.), but how would we use these numbers to summarize how different **all** of the groups are from each other?
Enter the F-statistic! An F-statistic summarizes two quantities:

- How different the means of the groups are from each other
- How different the observations in each group are from the mean of their group

To me, an F-statistic makes more sense if I visualize what these pieces mean. In the plot below, I've added three pieces,

- Orange individual points within each group (these are the movies)
- A black dashed line across the entire plot
- A blue solid line across each genre group



15. What does the black dashed line across the entire plot represent?

The overall mean Rating (no matter the genre)

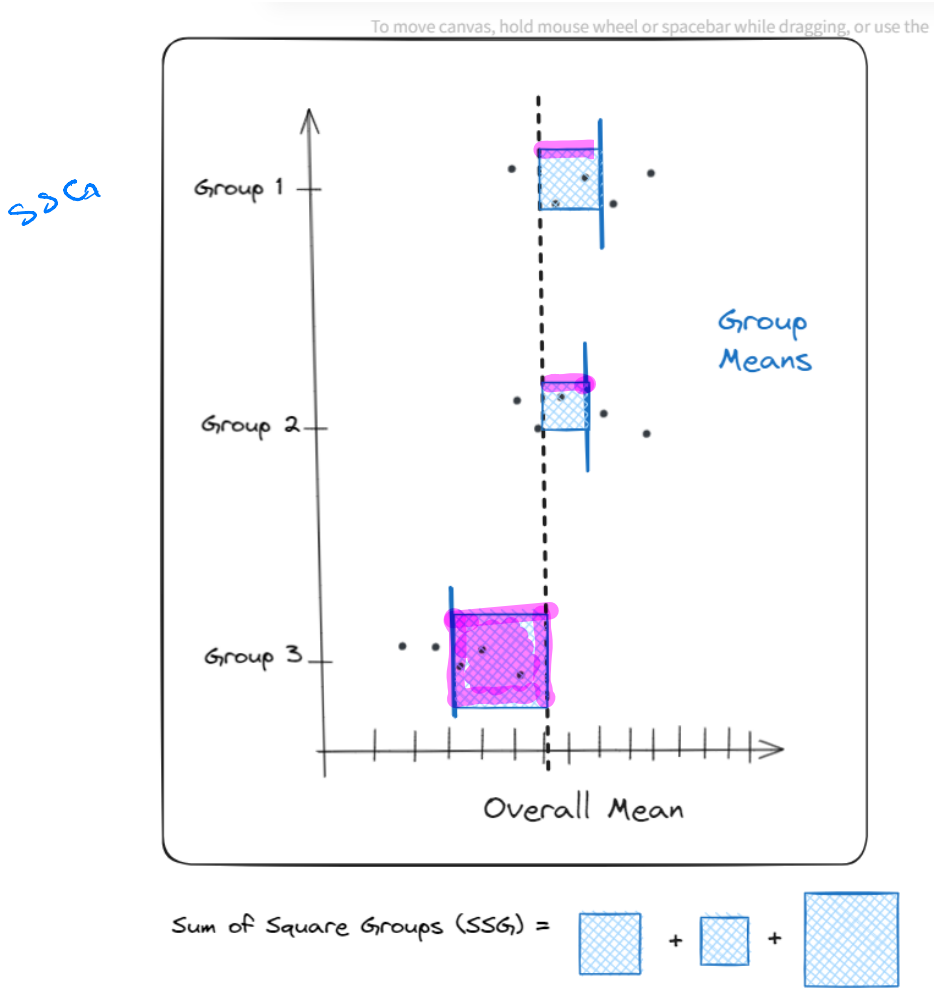
16. What do the solid blue lines across each group's boxplot represent? *Hint: The solid blue line is different from the gray solid lines.*

The mean Rating for each group (genre)

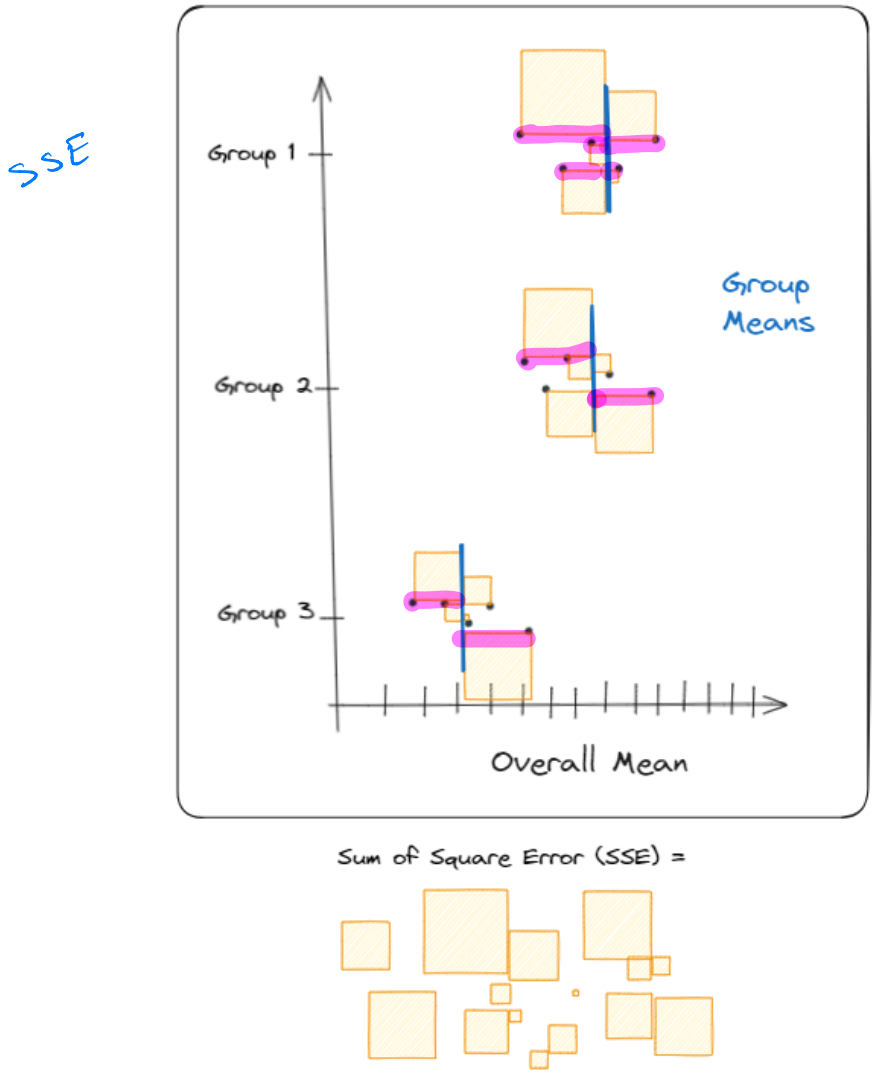
Components of an F-statistic

The two components of an F-statistic are called the *sum of squares between groups* (SSG) and the *sum of squares of the errors* (SSE). Let's break down what each of these mean.

The **SSG** compares each group's mean to the overall mean. As its name indicates, these differences are then **squared** and added together.



The **SSE** measures how far an observation is from the mean of that group. As its name indicates, these differences are **squared** and then added together.



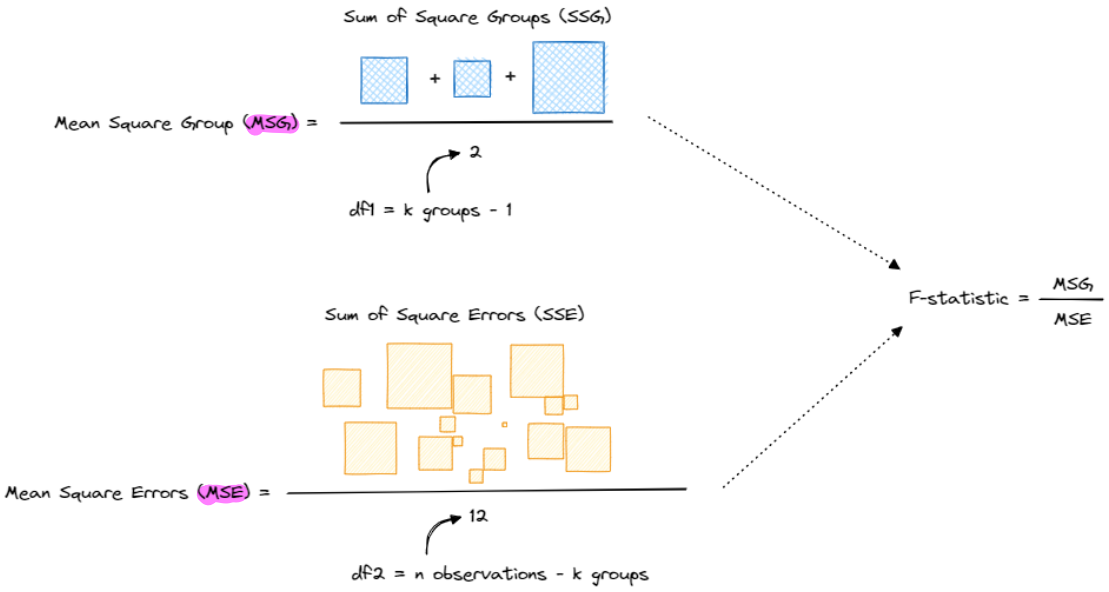
There is one final part to an F-statistic. We take each of these quantities (SSG, SSE) and divide them by their respective degrees of freedom. The degrees of freedom are calculated based on (1) the number of items available and (2) the number of statistics that need to be calculated.

For the SSG, we have k groups and we need to calculate the overall mean. So, our resulting degrees of freedom are $k - 1$.

For the SSE, we have n observations and we need to calculate k group means. So, our resulting degrees of freedom are $n - k$.

Now, putting all of these pieces together, we can obtain the magical F-statistic using the following formula:

$$\frac{\frac{SSG}{k-1}}{\frac{SSE}{n-k}} = \frac{MSG}{MSE}$$



17. Draw horizontal lines on the plot above, indicating which values are being compared when calculating the SSG.

18. Draw horizontal lines on the plot above, indicating which values are being compared with calculating the SSE.

19. How many degrees of freedom does the `Genre` variable (MSG) have?

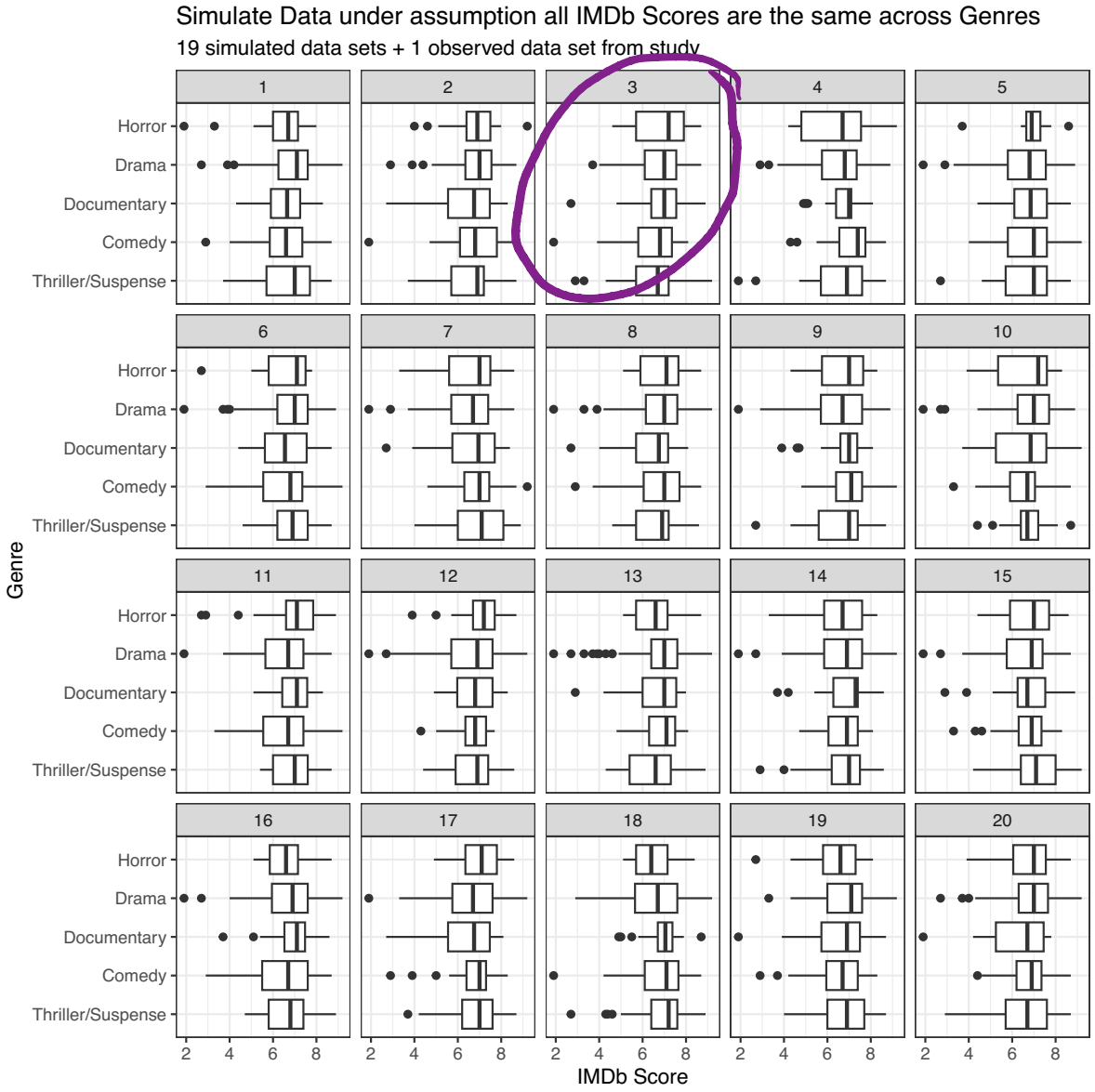
df1 = 5 - 1 = 4

20. How many degrees of freedom does the SSE for our content rating analysis have?

df2 = 172 - 5 = 167

21. Can an F-statistic be negative?

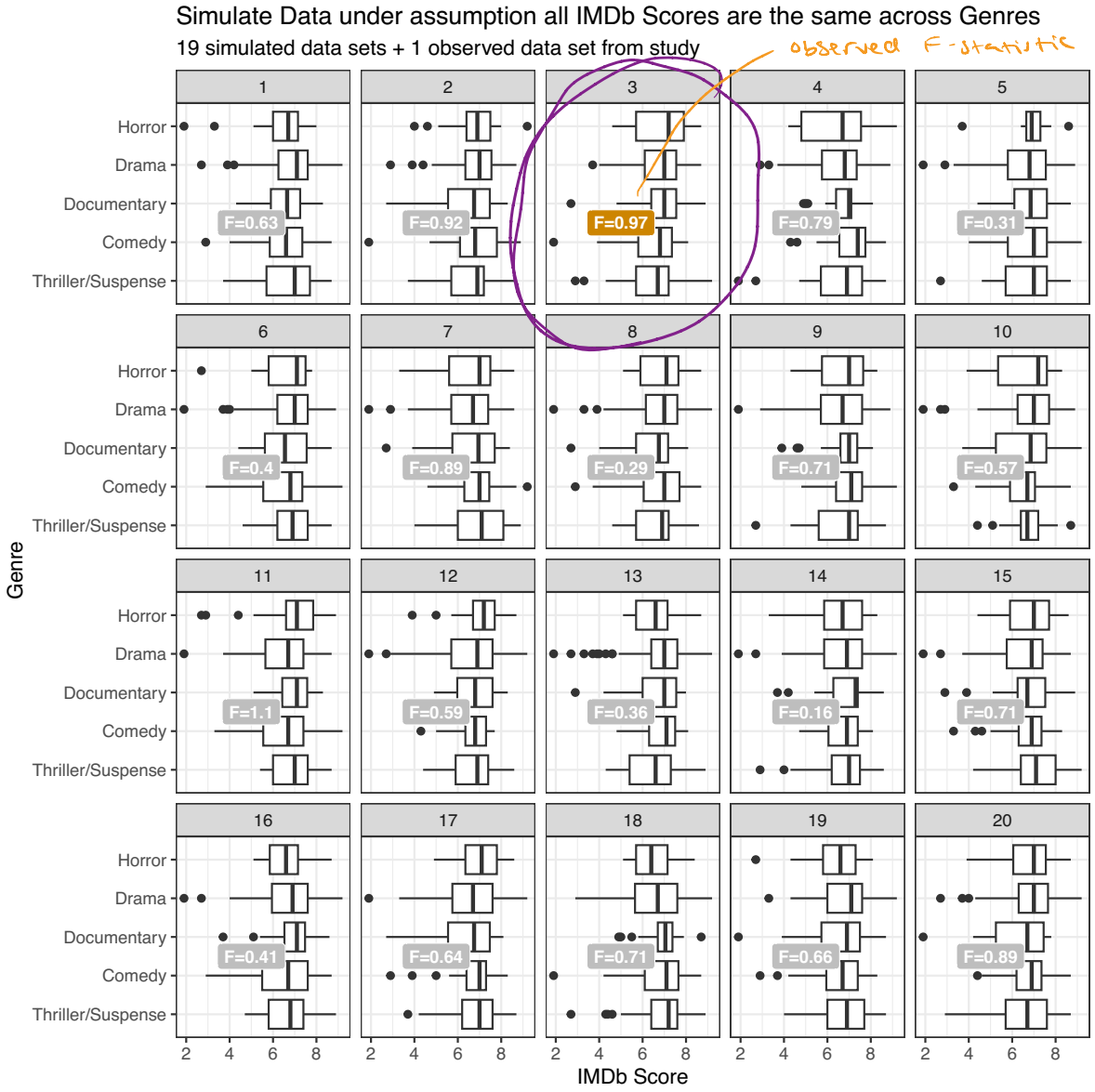
Let's use simulation to get an idea of the shape of the F-distribution.

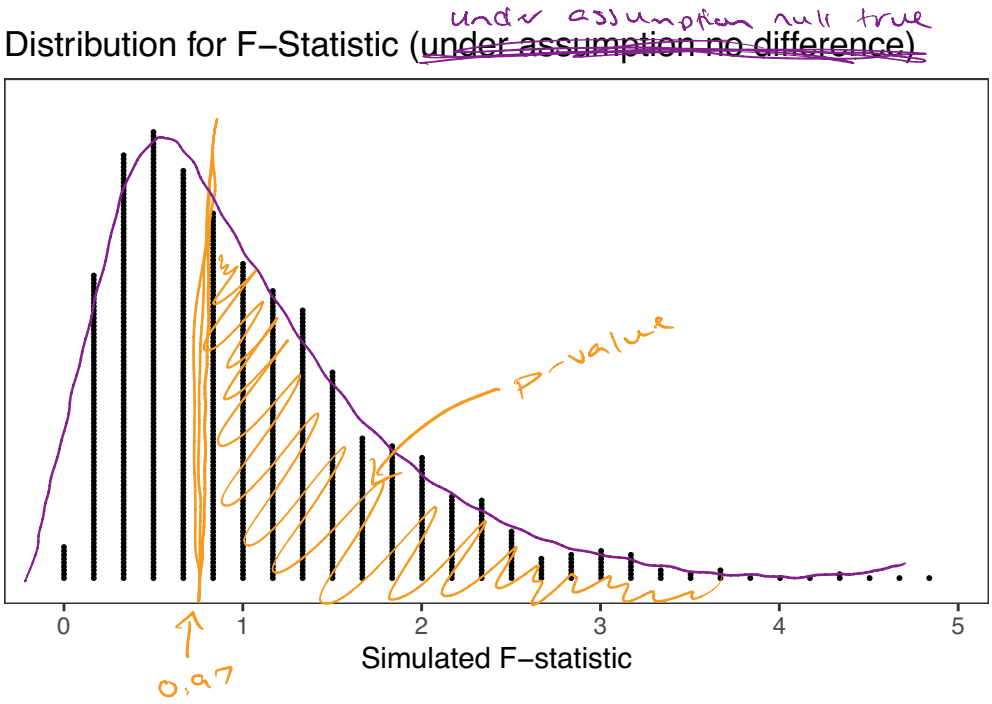


22. Which panel contains the actual data observed in the study? Was it hard to pick out? Remember what it was like trying to pick this out.

panel 3, it was pretty challenging to pick out.

We could take these and calculate the F-statistic for each panel (simulated data set) and plot this to begin creating the distribution to compare our observed test statistic to. Note, we will see how to use R to calculate the F-statistic in just a bit.





23. Take note that our observed F-statistic is 0.97, do you believe this F-statistic is likely to occur under the condition that all mean IMDb movie ratings are the same across all five genres (i.e., the null is true)?

An F-stat of 0.97 does not seem unusual if the null is true (all Genres equal)

Calculating the F-statistic by hand would be terrible! Instead, we will use `R`. The `aov()` function in `R` stands for **a**nalysis **o**f **v**ariance.

name of model so we can use later

assignment arrow

```
genre_anova <- aov(Rating ~ Genre,                    ①
    data = movie_ratings                              ②
    )

genre_anova |>
  tidy()                                              ③
```

① Save the model in an object called `genre_anova`. Provide the `aov` function a "formula" similar to `favstats()` with: `response ~ explanatory`.
② Tell `aov` what data set to use.
③ Have R output the information from the `genre_anova` object in a nice clean (`tidy`) table for you.

```
# A tibble: 2 x 6
  term          df  sumsq meansq statistic p.value
  <chr>      <dbl>  <dbl>  <dbl>     <dbl>   <dbl>
1 Genre          4   6.45   1.61     0.969   0.426
2 Residuals    167 278.     1.66    NA      NA
```

22. What is the sum of squares for `Genre` (SSG)?

SSG = 6.45

23. What is the sum of squares for the errors (SSE)?

SSE = 278

24. How was the mean squares for `Genre` (MSG) found?

$MSG = SSG/4 = 6.45/4 = 1.61$

25. How was the mean squares for the errors (MSE) found?

$MSE = SSE/167 = 278/1.66$

26. What is the resulting F-statistic?

$MSG/MSE = 1.61/1.66 = 0.969$

27. Based on the p-value associated with the F-statistic outputed above, write the conclusion in context of the problem.

We do not have evidence to conclude at least one genre has a different mean IMDb score for all movies released in 2020 ($F = 0.969$; $df_1 = 4$, $df_2 = 167$; $p = 0.426$).

Conditions for using ANOVA (F-statistic)

1. Independent observations *within* groups.
2. Independent observations *between* groups.
3. Equal variance across every group.
4. Either, all sample sizes are sufficiently large, or it reasonable to assume that the populations for each group are normally distributed.

28. Check the conditions for using ANOVA to test whether the mean IMDb score differs between genres.

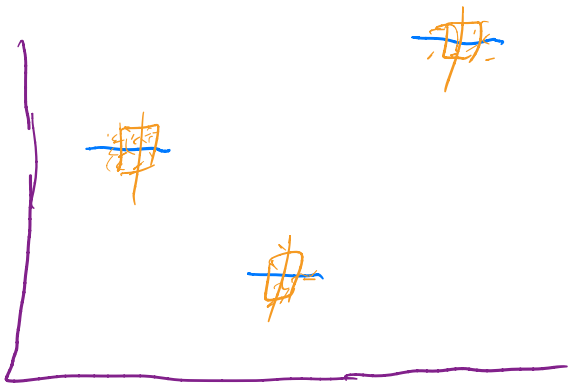
- Independent obs within: one movie's rating does not influence another movie rating of the same genre.
- Independent obs between: one movie's rating does not influence another from a diff genre.
- Equal variance: SD all right around 1.3
- Normality: Sample sizes < 30, histograms ok normal.

Alright, so we just learned about how we can analyze the differences in **many** means using ANOVA. As a refresher, with an ANOVA, we're comparing the variability *within* groups (MSE) to the variability *between* groups (MSG).

If we believe that the mean of at least one group is different from the others, ideally in a visualization we'd like to see:

- large differences in the means **between** the groups
- small amounts of variability **within** each group

29. Sketch an example of three box plots that exhibit the characteristics above.



30. Overall, do you believe any of the genres stand out as really different from the others? Recall how easy or difficult it was to pick out the data plot from all the simulated panels above.

From our movie data... no, it was hard to pick out Panel 3.

Hypothesis Testing Errors

In a hypothesis test, there are two competing hypotheses: the null and the alternative. We make a statement about which one might be true, but we might choose incorrectly. There are four possible scenarios in a hypothesis test:

	H_0 is True	H_0 is False
Reject H_0 (evidence)	Type I Error	Good Decision!
Fail to Reject H_0 (insufficient evidence)	Good Decision!	Type II Error

31. Based on the decision you reached from the ANOVA test, what type of error could you have made?

Type II error

32. With an $\alpha = 0.05$, what percent of the time would we expect to make a Type I error?

5% of the time

33. How does α relate to the probability of making a Type II error?

as $\alpha \uparrow$ the prob of Type I $\uparrow$

then the prob of Type II $\downarrow$ (not sufficient evidence more often)

Inference after ANOVA

If we had found a "significant" p-value, we could have concluded that at least one of the genres had a different mean movie rating. However, an ANOVA **does not** tell us which group(s) is(are) driving the differences.

What we could do is compare all possible combinations of two means. With five groups, that would result in 10 different hypothesis tests for a difference in means. For example:

- $\mu_{\text{Comedy}} - \mu_{\text{Documentary}}$,
- $\mu_{\text{Comedy}} - \mu_{\text{Drama}}$,
- $\mu_{\text{Horror}} - \mu_{\text{Thriller}}$,
- etc.

However, for each hypothesis test we do at an α of 0.05, we risk making a Type I error 5% of the time. In fact, we can make a mathematical equation for the probability of making a Type I Error, based on the number of tests we perform.

$$\text{Probability of Making a Type I Error} = 1 - \text{Probability of Not Making a Type I Error}$$

$$\text{Probability of Making a Type I Error} = 1 - (0.95)^{\#\text{ of tests}}$$

34. If we do 10 hypothesis tests (think of 10 pairwise comparisons between Genres), what is the probability of us making a Type I Error?

$1 - 0.95^{10} = 1 - 0.5987 = 0.4013$

Bonferonni Correction for Post-Hoc Comparisons

One solution to the problem of multiple comparisons is called the Bonferroni correction. Essentially, you take your α threshold and divide it by the number of tests you are going to perform.

$$\alpha^* = \frac{\alpha}{\text{number of pairwise comparisons}}$$

You then use this α^* value as the new threshold value for **every** pairwise comparison. If a comparison's p-value is less than α^*, then you reject H_0 (evidence to support the alternative). If a comparison's p-value is greater than α^*, then you fail to reject H_0 (insufficient evidence to support the alternative).

35. If our original α was 0.05, what value should we use for α^* with 10 pairwise comparisons?

$\alpha^* = 0.05/10 = 0.005$

Below is a table of all 10 of the pairwise comparisons (hypothesis tests) we could do when comparing the means of two genres.

```
library(emmeans)
emmeans(object = genre_anova,
        specs = ~ Genre
        ) |>
  pairs(adjust = "none")
```

contrast	estimate	SE	df	t.ratio	p.value
(Thriller/Suspense) - Comedy	-0.0958	0.360	167	-0.266	0.7905
(Thriller/Suspense) - Documentary	-0.5174	0.348	167	-1.486	0.1393
(Thriller/Suspense) - Drama	-0.4121	0.282	167	-1.461	0.1458
(Thriller/Suspense) - Horror	-0.5091	0.381	167	-1.338	0.1828
Comedy - Documentary	-0.4216	0.369	167	-1.142	0.2550
Comedy - Drama	-0.3163	0.307	167	-1.029	0.3049
Comedy - Horror	-0.4133	0.400	167	-1.034	0.3027
Documentary - Drama	0.1053	0.293	167	0.359	0.7202
Documentary - Horror	0.0083	0.389	167	0.021	0.9830
Drama - Horror	-0.0970	0.331	167	-0.293	0.7700

36. Using the α^* you found above, circle the hypothesis tests whose p-values are less than α^*.

Your α^* value should be much less than your original α of 0.05, which makes it **harder** to find evidence to support the alternative (reject the null).

Alternatively, we can ask R to do this adjustment for us by using `adjust = "bonf"` and then use our standard cut-offs.

```
emmeans(object = genre_anova,
        specs = ~ Genre
        ) |>
  pairs(adjust = "bonf")
```

```
contrast                            estimate    SE  df t.ratio p.value
(Thriller/Suspense) - Comedy         -0.0958 0.360 167 -0.266  1.0000
(Thriller/Suspense) - Documentary    -0.5174 0.348 167 -1.486  1.0000
(Thriller/Suspense) - Drama          -0.4121 0.282 167 -1.461  1.0000
(Thriller/Suspense) - Horror         -0.5091 0.381 167 -1.338  1.0000
Comedy - Documentary                 -0.4216 0.369 167 -1.142  1.0000
Comedy - Drama                       -0.3163 0.307 167 -1.029  1.0000
Comedy - Horror                      -0.4133 0.400 167 -1.034  1.0000
Documentary - Drama                   0.1053 0.293 167  0.359  1.0000
Documentary - Horror                  0.0083 0.389 167  0.021  1.0000
Drama - Horror                       -0.0970 0.331 167 -0.293  1.0000

P value adjustment: bonferroni method for 10 tests
```

Note there are multiple methods for conducting multiplicity adjustments to control your Type I error rates including `tukey`, `dunnet`, and more!