

Module 5: Randomized Complete Block Designs (RCBD)

Factorial Treatment Design in a RCBD

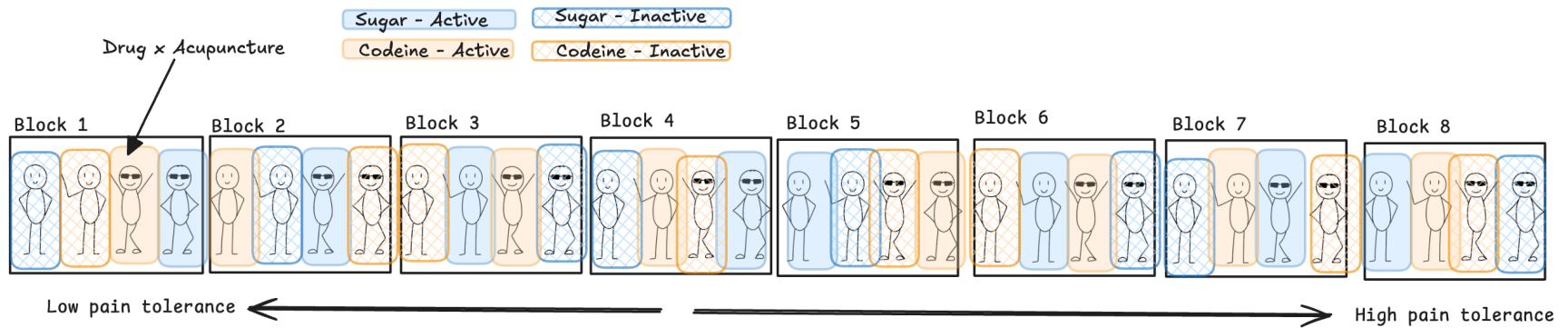
Example 5.2: Dental

An anesthesiologist made a comparative study of the effects of acupuncture and codeine on postoperative dental pain in male subjects. The four treatment combinations were as follows:

- Placebo treatment (a sugar pill and two inactive acupuncture points)
- Codeine treatment (a codeine pill and two inactive acupuncture points)
- Acupuncture only (a sugar pill and two active acupuncture points)
- Codeine and acupuncture (a codeine pill and two active acupuncture points)

Thirty-two subjects were grouped into 8 blocks of size 4 according to an initial evaluation of their pain tolerance. The subjects in each block were then randomly assigned to the four treatment combinations. The pain relief scores were obtained for all subjects two hours after dental treatment (data were collected on a double-blind basis). The higher the pain relief score the more effective the treatment.

Example 5.2 Blueprint



2. Why do you think *pain tolerance* was used as a blocking variable?

Example 5.2: Study Structure

Treatment Structure

A 2×2 full factorial between Drug (sugar/codeine) and Acupuncture (active/inactive) for a total of $t = 4$ treatment combinations.

Experimental Structure

Drug and acupuncture treatment combinations were randomly assigned to subjects (e.u.) in a RCBD with $r = 8$ replications (blocks) where pain tolerance is the blocking factor (block size = 4). We measured the pain relief scores on each subject (m.u.) for a total $N = 32$ subjects.

Statistical Effects Model

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \rho_k + \epsilon_{ijk} \text{ with } \epsilon_{ijk} \text{ iid } \sim N(0, \sigma^2)$$

for $i = 1, 2; j = 1, 2; k = 1, 2, \dots, 8$

Where:

- y_{ijk} is the pain relief score from the subject in the k^{th} pain tolerance block receiving the i^{th} level of drug and the j^{th} level acupuncture
- μ is the overall mean pain relief score
- α_i is the effect of the i^{th} level of drug
- β_j is the effect of the j^{th} level of acupuncture
- $\alpha\beta_{ij}$ is the interaction effect between the i^{th} level of drug and the j^{th} level of acupuncture
- ρ_k is the effect of the k th block
- ϵ_{ijk} is the experimental error associated with the subject from the k^{th} pain tolerance block receiving the i^{th} level of drug and the j^{th} level of acupuncture

ANOVA Table

Source of Variation	DF	SS	MS	F
Blocks	$r-1$	SSB _{ik}	MSB _{ik}	MSB _{ik} /MSE
A	$a-1$	SSA	MSA	MSA/MSE
B	$b-1$	SSB	MSB	MSB/MSE
AxB	$(a-1)(b-1)$	SSAB	MSAB	MSAB/MSE
Block x AB → error	$(r-1)(ab-1)$	SSE	MSE	
Total (N = rab)	N-1			

Skeleton ANOVA

Source of Variation	DF
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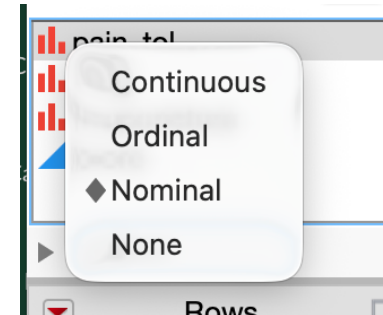
Example 5.2: The Data

Warning

Ensure “block” is a factor/character!

```
1 library(tidyverse)
2 dental_data <- read_csv("data/05_dental_data.csv") |>
3   mutate(pain_tol = as.factor(pain_tol))
4 head(dental_data, n = 8)
```

```
# A tibble: 8 × 4
  pain_tol drug  acupuncture score
  <fct>    <chr>   <chr>         <dbl>
1 1      sugar    active         0.6
2 1    codeine inactive         0.5
3 1      sugar    inactive         0
4 1    codeine active         1.2
5 2    codeine inactive         0.6
6 2      sugar    active         0.7
7 2      sugar    inactive         0.3
8 2    codeine active         1.3
```



R: Analysis

```
1 options(contrasts = c("contr.sum", "contr.poly"))
2 # score ~ drug*acupuncture + pain_tol
3 dental_mod <- lm(score ~ pain_tol + drug + acupuncture + drug:acupuncture,
4                  data = dental_data)
5 anova(dental_mod)
```

Analysis of Variance Table

Response: score

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
pain_tol	7	5.5988	0.7998	55.2963	4.126e-12 ***
drug	1	2.3113	2.3113	159.7901	2.773e-11 ***
acupuncture	1	3.3800	3.3800	233.6790	7.465e-13 ***
drug:acupuncture	1	0.0450	0.0450	3.1111	0.0923 .
Residuals	21	0.3038	0.0145		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
1 summary(dental_mod)
```

Call:

```
lm(formula = score ~ pain_tol + drug + acupuncture + drug:acupuncture,
    data = dental_data)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.18125	-0.06250	0.00000	0.04688	0.24375

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.15625	0.02126	54.385	< 2e-16 ***
pain_tol1	-0.58125	0.05625	-10.333	1.09e-09 ***
pain_tol2	-0.43125	0.05625	-7.667	1.62e-07 ***
pain_tol3	-0.25625	0.05625	-4.556	0.000172 ***
pain_tol4	-0.28125	0.05625	-5.000	5.99e-05 ***
pain_tol5	0.09375	0.05625	1.667	0.110427
pain_tol6	0.39375	0.05625	7.000	6.53e-07 ***
pain_tol7	0.49375	0.05625	8.778	1.80e-08 ***
drug1	0.26875	0.02126	12.641	2.77e-11 ***
acupuncture1	0.32500	0.02126	15.287	7.47e-13 ***
drug1:acupuncture1	0.03750	0.02126	1.764	0.092304 .

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1203 on 21 degrees of freedom

JMP: Analysis

▼ Analyze > Fit Model

Fit Model

Model Specification

Select Columns

4 Columns

- pain_tol
- drug
- acupuncture
- score

Pick Role Variables

Y: score (optional)

Weight: optional numeric

Freq: optional numeric

Validation: optional numeric

By: optional

Construct Model Effects

Add

Cross

Nest

Macros

Degree: 2

Attributes

Transform

No Intercept

Personality: Standard Least Squares

Emphasis: Effect Leverage

Recall

Keep dialog open

Remove

Run

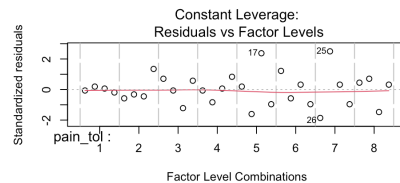
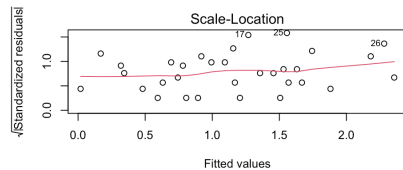
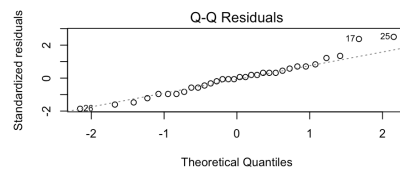
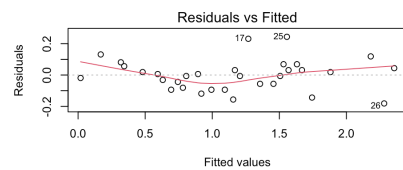
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Ratio	
Model	10	11.335000	1.13350	78.3654	
Error	21	0.303750	0.01446	Prob > F	
C. Total	31	11.638750		<.0001*	

Parameter Estimates					
Term	Estimate	Std Error	t Ratio	Prob> t	
Intercept	1.15625	0.021261	54.38	<.0001*	
pain_tol[1]	-0.58125	0.05625	-10.33	<.0001*	
pain_tol[2]	-0.43125	0.05625	-7.67	<.0001*	
pain_tol[3]	-0.25625	0.05625	-4.56	0.0002*	
pain_tol[4]	-0.28125	0.05625	-5.00	<.0001*	
pain_tol[5]	0.09375	0.05625	1.67	0.1104	
pain_tol[6]	0.39375	0.05625	7.00	<.0001*	
pain_tol[7]	0.49375	0.05625	8.78	<.0001*	
drug[codeine]	0.26875	0.021261	12.64	<.0001*	
acupuncture[active]	0.325	0.021261	15.29	<.0001*	
drug[codeine]*acupuncture[active]	0.0375	0.021261	1.76	0.0923	

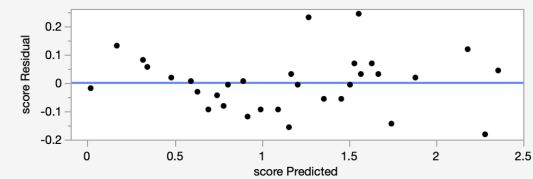
Effect Tests					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
pain_tol	7	7	5.5987500	55.2963	<.0001*
drug	1	1	2.3112500	159.7901	<.0001*
acupuncture	1	1	3.3800000	233.6790	<.0001*
drug*acupuncture	1	1	0.0450000	3.1111	0.0923

Check Model Assumptions $\epsilon_{ijk} \text{ iid } \sim N(0, \sigma^2)$

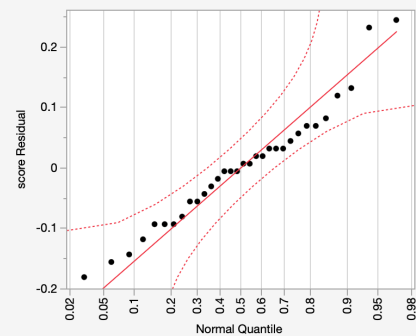
```
1 par(mfrow = c(2,2))
2 plot(dental_mod)
```



Residual by Predicted Plot



Residual Normal Quantile Plot



Factorial flow chart

Where should we proceed with our analysis?

Analysis of Variance Table

Response: score

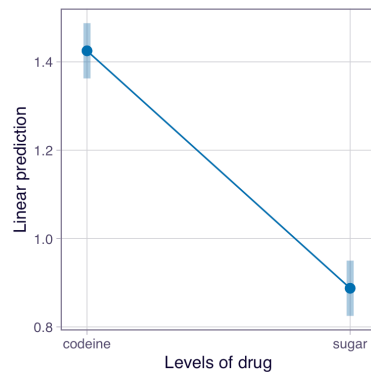
	Df	Sum Sq	Mean Sq	F value	Pr(>F)
pain_tol	7	5.5988	0.7998	55.2963	4.126e-12 ***
drug	1	2.3113	2.3113	159.7901	2.773e-11 ***
acupuncture	1	3.3800	3.3800	233.6790	7.465e-13 ***
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Effect Tests

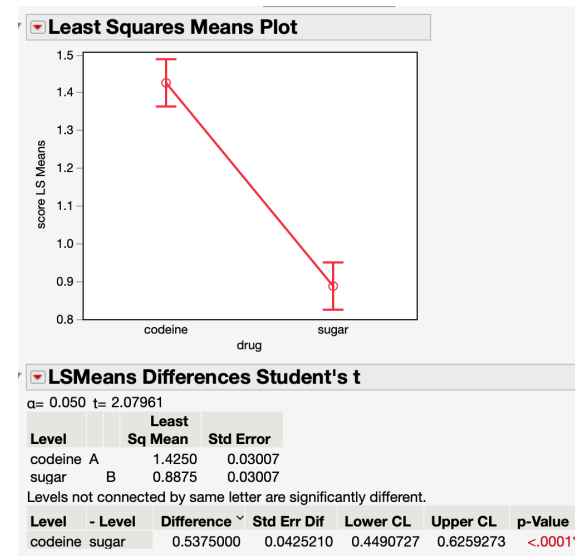
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
pain_tol	7	7	5.5987500	55.2963	<.0001*
drug	1	1	2.3112500	159.7901	<.0001*
acupuncture	1	1	3.3800000	233.6790	<.0001*
drug*acupuncture	1	1	0.0450000	3.1111	0.0923

Main Effect of Drug



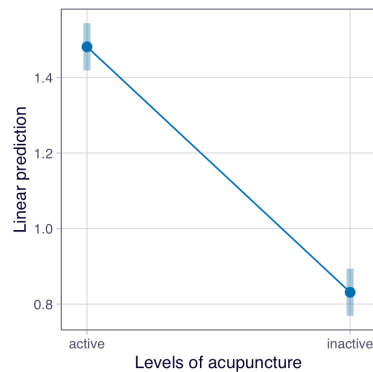
```
contrast      estimate    SE df lower.CL upper.CL t.ratio p.value
codeine - sugar    0.537 0.0425 21    0.449    0.626  12.641 <0.0001
```

Results are averaged over the levels of: pain_tol, acupuncture
Confidence level used: 0.95



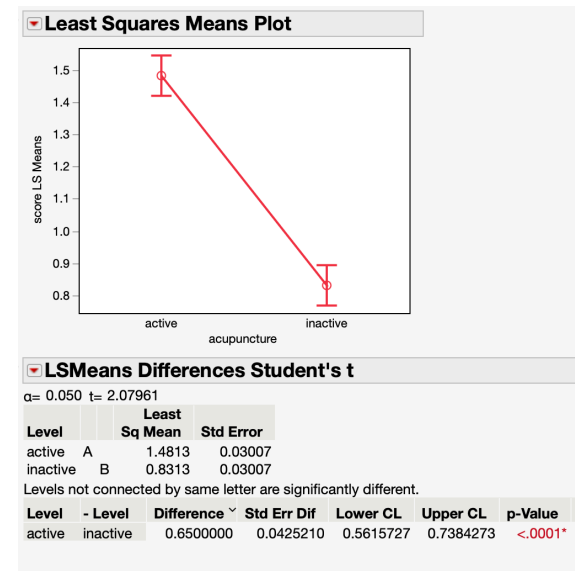
Codeine results in the highest mean pain relief at 1.42 (s.e. 0.03). This is, on average, 0.53 points more relief than the sugar pill ($t = 12.64$; $df = 21$; $p < 0.0001$).

Main Effect of Acupuncture



contrast	estimate	SE	df	lower.CL	upper.CL	t.ratio	p.value
active - inactive	0.65	0.0425	21	0.562	0.738	15.287	<0.0001

Results are averaged over the levels of: pain_tol, drug
Confidence level used: 0.95



Active acupuncture results in the highest mean pain relief at 1.48 (s.e. = 0.03). This is, on average, 0.65 points higher than inactive acupuncture ($t = 15.29$; $df = 21$; $p < 0.0001$).