

Activity 5.1: Reducing Variability with Blocking

Advertising Strategies

i Note

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A marketing team for a regional grocery chain wants to compare the impact of four advertising strategies on sales.

The four strategies are:

- (Discount) Discount-Driven Ads – highlight weekly price specials.
- (Storytelling) Brand Storytelling – focus on local farms and family ownership.
- (Digital) Digital-Only – social media and email ads only.
- (Events) Community Events – in-person promotions like pop-up tastings.

There are 16 cities interested in participating in the study. Each city will use one advertising strategy for a 6-week campaign, and overall sales volume during that period will be recorded. However, cities differ greatly in their population size and baseline sales. Larger cities naturally have higher sales, regardless of the advertising strategy.

Part A: Setup

1. How would you create blocks for this experiment?

2. Sketch a design blueprint

Clearly draw the blocks, show the experimental units, and how treatments are randomly assigned.

3. Describe the treatment structure

4. Describe the experimental structure

⚠ Part B: Incorrect Analysis

5. Run a one-way ANOVA CRD analysis (ignroing block) and record the following:

$$y_{ij} = \mu + \tau_i + \epsilon_{ij} \text{ with } \epsilon_{ij} \text{iid } \sim N(0\sigma^2)$$

for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$

- SST = _____
- SST_{Trt} = _____
- SSE = _____ → MSE = $\hat{\sigma}^2$ = _____

Part C: Treatment and Block Means

The observed sales volumes (in thousands of dollars) are shown below.

Original Sales

Advertising Strategy	Block j = 1 (Small-est)	Block j = 2	Block j = 3	Block j = 4 (Largest)	Treatment Mean Sales ($\bar{y}_{i\cdot}$)
Digital	136	153	203	200	
Discount	147	146	217	225	
Events	162	189	231	228	
Storytelling	184	208	251	257	
Block Mean Sales ($\bar{y}_{\cdot j}$)					$\bar{y}_{\cdot\cdot} =$
Block Effects ($\hat{\rho}_j = \bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot}$)					

6. Compute the treatment mean sales \bar{y}_i for each advertising strategy and record them below. Record in the table above.
7. Compute the block mean sales $\bar{y}_{\cdot j}$ for each city size block. Record in the table above.
8. Compute the overall mean sales volume $\bar{y}_{..}$. Record in the table above.

Part C: Block Effects

9. Compute the block effect for each block. Record in the table above.

$$\hat{\rho}_j = \bar{y}_{\cdot j} - \bar{y}_{..}$$

10. Use the block effects to compute SSBlk

Notice: the $(\bar{y}_{\cdot j} - \bar{y}_{..})$ inside the parentheses is the $\hat{\rho}_j$ you computed in (9).

$$SSBlk = t \sum_j (\bar{y}_{\cdot j} - \bar{y}_{..})^2 =$$

i Note

Recall, the purpose of the block design is to “remove” the variation explained by the blocking variable from the SSE, thus reducing the experimental error variation (MSE).

Part D: Block Adjusted Sales

11. Using the block effects from the table above, adjust each observed sales volume value by the associated block effect $(y_{ij} - \rho_j)$. Record in the table below.

Note, you are essentially “removing” the effect of city from the sales.

Block-adjusted Sales

Advertising Strategy	Block j = 1 (Smallest)	Block j = 2	Block j = 3	Block j = 4 (Largest)	Block-adjusted Treatment Mean Sales
Digital					
Discount					
Events					
Storytelling					

12. Calculate the block adjusted mean sales. Record in the table above.

- What do you notice about the block-adjusted mean sales compared to the original mean sales?
- Would the SST_{Trt} change when using the block-adjusted mean sales? Recall $SST_{Trt} = r \sum_i (\bar{y}_i - \bar{y}_{..})^2$.

13. What do you notice about the variability of the block-adjusted sales within each advertising strategy compared to the original variability of sales (without block-adjusted sales)?



Part E: RCBD Analysis

Consider the statistical effects model that accounts for the block effect.

$$y_{ij} = \mu + \tau_i + \rho_j + \epsilon_{ij} \text{ with } \epsilon_{ij} \text{iid} \sim N(0, \sigma^2)$$

for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$

14. Complete the skeleton ANOVA below

Source of Variation DF: Total = _____

15. Use R/JMP to fit the statistical effects model specified above. Record the following:

- SSE = _____
- MSE = $\hat{\sigma}^2$ = _____

16. How does your experimental error in (15) using the statistical effects model to account for the block effect compare to the experimental error calculated in (5) ignoring the effect of block?