Module 2: Completely Randomized Designs

Analyzing a CRD (ANOVA)

From Design to Analysis

In a CRD, we assume:

- The systematic source of variation comes from the treatments
- All other variation is experimental error

Big idea: Does accounting for treatment reduce unexplained variability by more than we would expect by chance?

Example 2.1: Running Shoes

Response: Lap time (seconds)

Treatment structure:

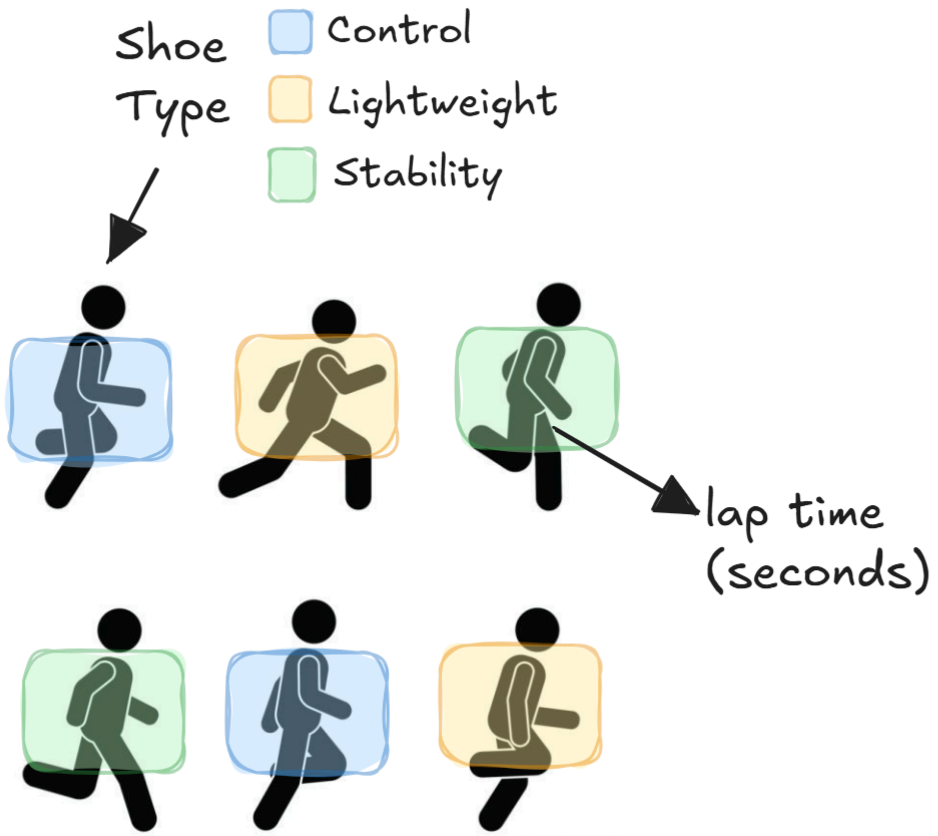
- One-way
- Factor: Shoe type
- 3 Levels: control, lightweight, and stability
- t = 3

Experimental structure:

- CRD
- Experimental Unit: Individual (r = 2)
- Measurement Unit: Individual (N = 6)

Goal: Determine whether shoe type affects mean lap time.

Example 2.1: Running Shoes (Blueprint)



Notation - y_{ij}

Note

Suppose that y_{ij} represents the response value for the j^{th} observation taken under the i^{th} treatment. In general, we have t treatments and r observations under the i^{th} treatment (number of replications).

- $\bar{y}_{..}$ – overall mean
- $\bar{y}_{i.}$ – treatment mean

Example 2.1: Running Shoes `02-shoes.csv`

Suppose the experiment was carried out, and the following lap times were recorded:

Runner	Shoe	Lap Time (seconds)
4	Control	60
6	Control	62
1	Lightweight	68
5	Lightweight	70
2	Stability	58
3	Stability	54

Example 2.1: Running Shoes

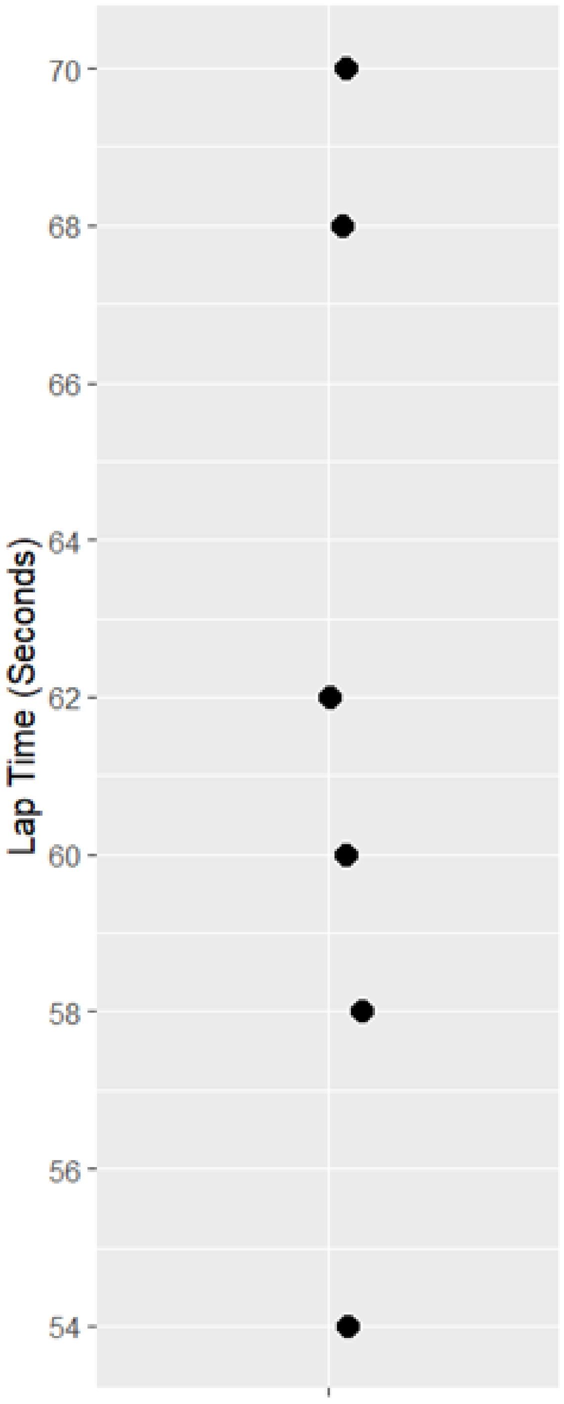
Suppose shoe has **no effect**

- All runners share the same mean lap time
- Differences are due to random variation

If we **ignore shoe type**, our best guess for any runner is the *overall average lap time*

$$\bar{y}_{..} =$$

Error = observed - overall mean

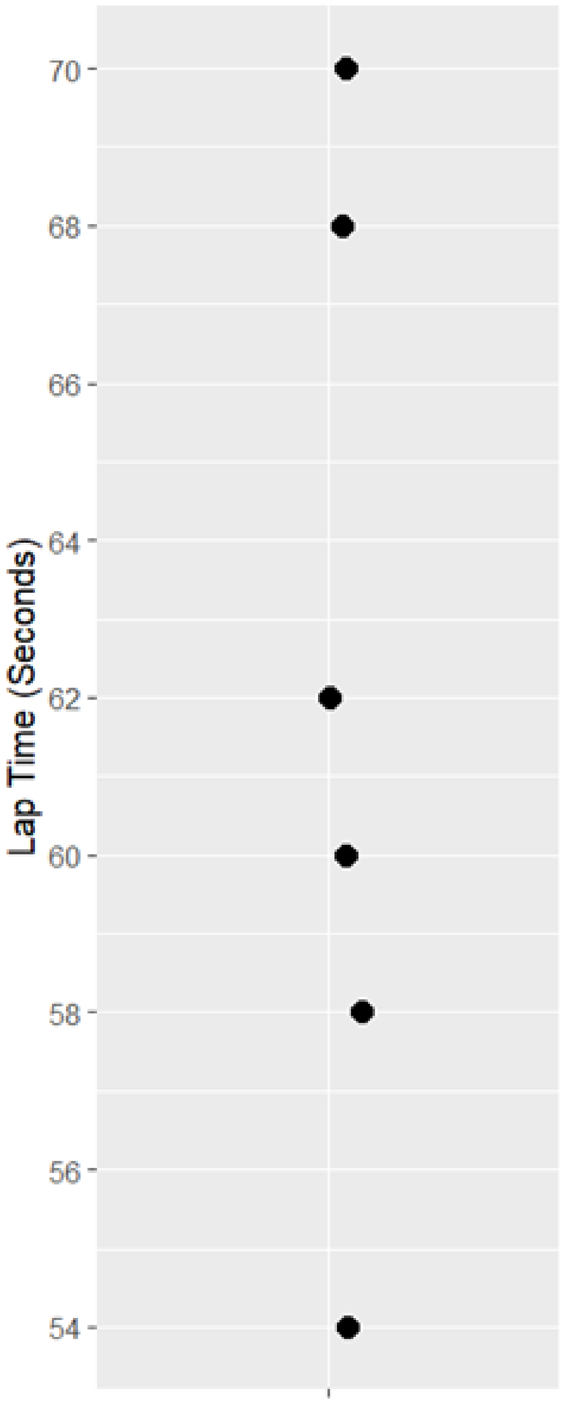


Total Sum of Squares

Total variability measures how far observations are from the overall mean.

$$SST = \sum_{i=1}^{t}\sum_{j=1}^{r}(y_{ij} - \bar{y}_{..})^2 =$$

$$(-2)^2 + (0)^2 + (6)^2 + (8)^2 + (-4)^2 + (-8)^2 =$$



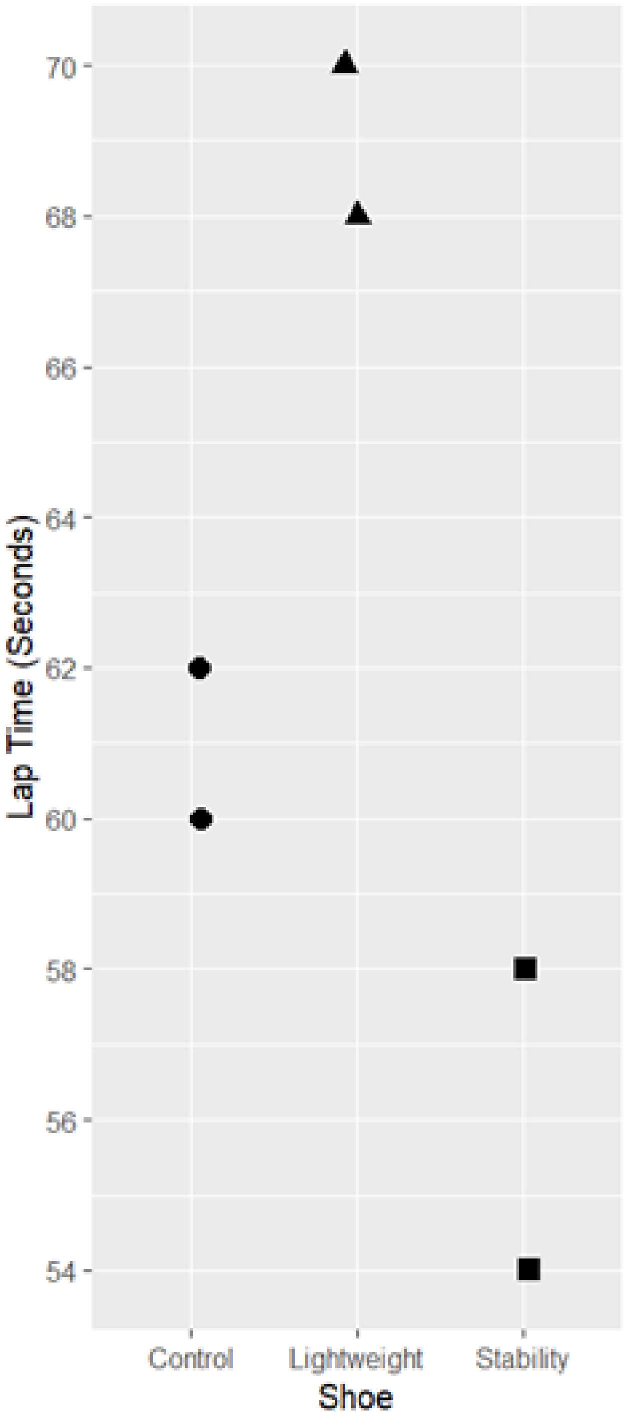
Example 2.1: Running Shoes

Now suppose shoe *does* matter. If shoe has an effect:

- Runners wearing the same shoe should have similar lap times
- Different shoes may have different mean lap times

Best guess for a runner is the mean lap time for their shoe - $\bar{y}_{i\cdot}$

Error = observed - treatment mean



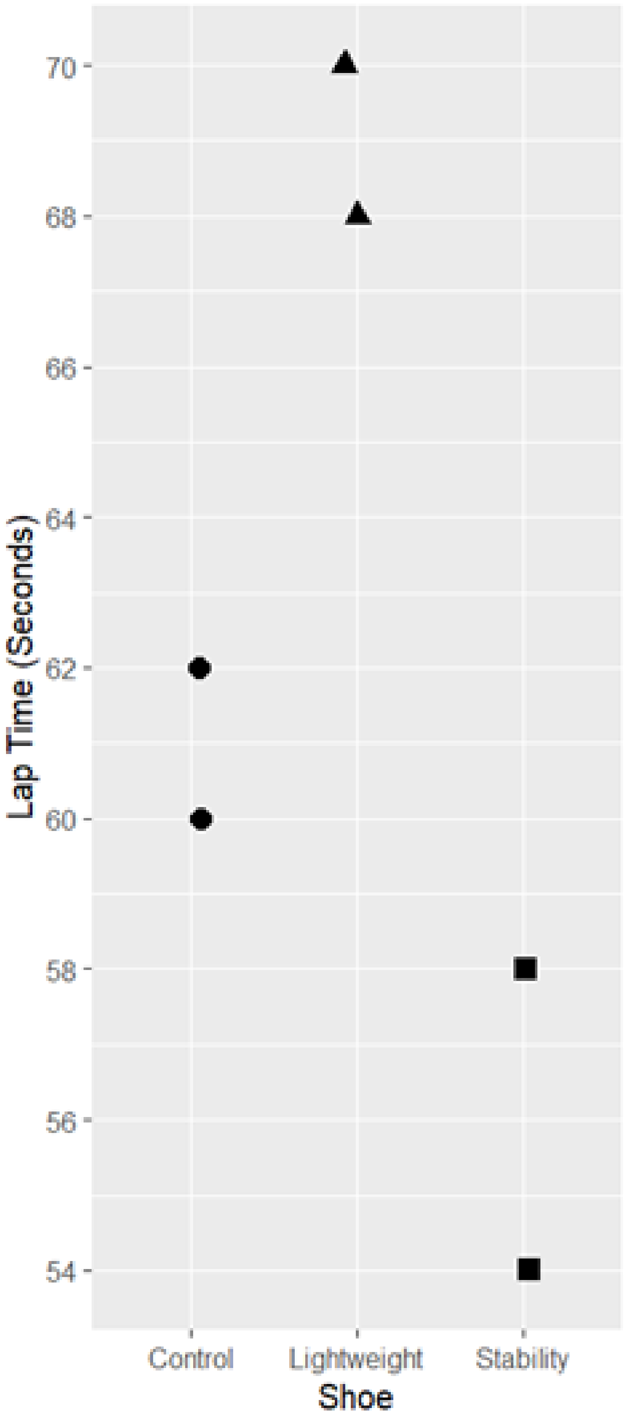
Sum of Squares Error (SSE)

This remaining variability is:

- Variation *within* shoe types
- Experimental error

$$SSE = \sum_{i=1}^{t}\sum_{j=1}^{r}(y_{ij} - \bar{y}_{i.})^2 =$$

$$(-1)^2 + (1)^2 + (-1)^2 + (1)^2(2)^2 + (-2)^2 =$$



Sum of Square Treatment (SSTrt or SSG)

What did we gain? By considering shoe total variability is reduced and the reduction is attributed to treatment.

$$SSTrt = SST - SSE =$$

Is this reduction in error big enough to claim shoe has an effect on lap time?

$$SSTrt = \sum_{i=1}^{t}\sum_{j=1}^{r}(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 =$$

$$2(61-62)^2 + 2(69-62)^2 + 2(56-62)^2 =$$

Analysis of Variance (ANOVA)

Note that $SST = SSTrt + SSE$. Thus, we have partitioned the total sums of squares into two parts:

- SSTrt: The variation between factor level means (*betweeen* treatments)
- SSE: The variation due to experimental error (*within* treatments)

ANOVA Table

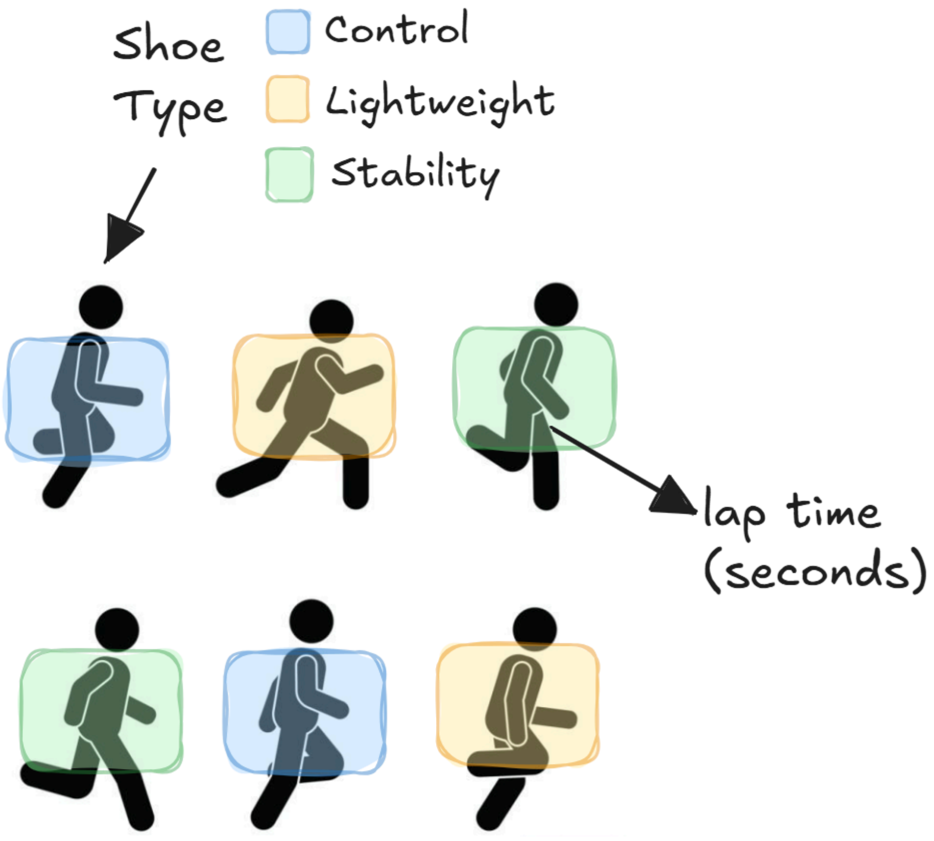
Source	**df**	**SS**	**MS**	**F**
Treatments	t − 1	SSTrt	MSTrt = SSTrt/(t−1)	MSTrt/MSE
Error	N − t	SSE	MSE = SSE/(N−t)	
Total	N − 1	SST		

- **Large F**: treatment explains substantial variability
- **F ≈ 1**: treatment explains little beyond noise

Example 1.1: Running Shoes

Source	df	SS	MS	F
Treatments		172	172 / 2 = 86	86 / 4 = 21.5
Error		12	12 / 3 = 4	
Total		184		

Skeleton ANOVA (use context!)



SV	df

What ANOVA Answers:

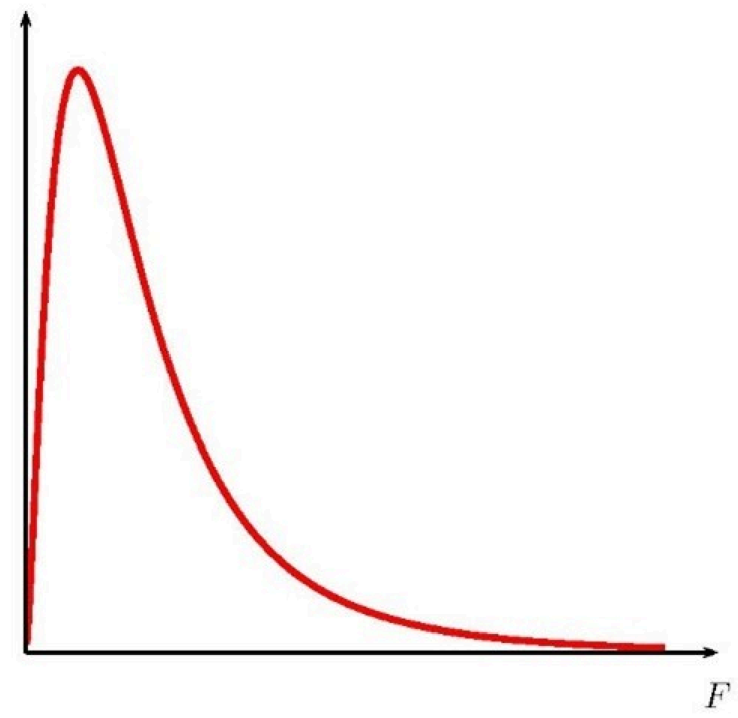
Do **any** treatment means differ?

$$H_0 : \mu_{Control} = \mu_{Lightweight} = \mu_{Stability}$$

$$H_A : \text{At least one } \mu_i \text{ differs}$$

F-distribution (assuming the null is true)

- Large F is evidence to reject H_0
- Under H_0: $F \sim F_{(t-1,\ N-t)}$



Analyzing a One-way ANOVA in R

```r
shoe_mod <- lm(`Lap Time (seconds)` ~ Shoe, data = shoe_data)
anova(shoe_mod)
```

```
Analysis of Variance Table

Response: Lap Time (seconds)
          Df Sum Sq Mean Sq F value  Pr(>F)
Shoe       2    172      86    21.5 0.01666 *
Residuals  3     12       4
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Analyzing a One-way ANOVA in JMP

Analyze > Fit Model > assign variables(Y = Lap Time, Add = Shoe) > Emphasis "Effect Leverage" > Run

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	172.00000	86.0000	21.5000
Error	3	12.00000	4.0000	Prob > F
C. Total	5	184.00000		0.0167*

Example 2.1: Running Shoes (Conclusion)

At an $\alpha = 0.05$, we have evidence to conclude there is an effect of shoe type on lap time (seconds) for all runners similar to those in our study (F = 21.5; df = 2,3; p = 0.017).

Alternative conclusion: At an $\alpha = 0.05$, we have evidence to conclude the mean lap time (seconds) differs for at least one shoe type for all runners similar to those in our study (F = 21.5; df = 2,3; p = 0.017).