Module 2: Completely Randomized Designs

Contrasts

From Means to Comparisons

ANOVA tells us whether *any* means differ.

Inference for means tells us *what each mean looks like.*

Contrasts let us ask specific questions about those means.

What Is a Contrast?

> ⓘ **Note**
>
> A **contrast** is a linear combination of treatment means:
>
> $$C = c_1\mu_1 + c_2\mu_2 + \cdots + c_t\mu_t$$
>
> with the constraint:
>
> $$\sum_{i=1}^{t} c_i = 0$$

What We Actually Estimate

We estimate contrasts using the estimated least squares means $\hat{\mu}_i$:

$$\hat{C} = c_1 \hat{\mu}_1 + c_2 \hat{\mu}_2 + \cdots + c_t \hat{\mu}_t$$

Example 2.1: Running Shoes

Response: Lap time (seconds)

Treatment structure:

- One-way
- Factor: Shoe type
- 3 Levels: control, lightweight, and stability
- t = 3

Experimental structure:

- CRD
- Experimental Unit: Individual (r = 2)
- Measurement Unit: Individual (N = 6)

Example 2.1: Running Shoes $(\hat{\mu}_i + \hat{\sigma}^2)$

```
1  anova(shoe_mod)
```

```
Analysis of Variance Table

Response: Lap Time (seconds)
          Df Sum Sq Mean Sq F value  Pr(>F)
Shoe       2    172      86    21.5 0.01666 *
Residuals  3     12       4
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
1  library(emmeans)
2  shoe_lsmeans <- emmeans(shoe_mod, specs = ~ Shoe)
3  shoe_lsmeans
```

```
Shoe          emmean   SE df lower.CL upper.CL
Control         61 1.41  3     56.5     65.5
Lightweight     69 1.41  3     64.5     73.5
Stability       56 1.41  3     51.5     60.5

Confidence level used: 0.95
```

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	172.00000	86.0000	21.5000
Error	3	12.00000	4.0000	**Prob > F**
C. Total	5	184.00000		0.0167*

Least Squares Means Estimates

Shoe	Estimate	Std Error	DF	Lower 95%	Upper 95%
Control	61.000000	1.4142136	3	56.499341	65.500659
Lightweight	69.000000	1.4142136	3	64.499341	73.500659
Stability	56.000000	1.4142136	3	51.499341	60.500659

Contrast 1: Lightweight vs Stability

Does the mean lap time differ between lightweight and stability shoes?

H_0 :

H_A :

Contrast:

$C =$

Coefficients: $(0, \ 1, \ -1)$

Standard Error of a Contrast (Balanced CRD)

> **ⓘ Note**
>
> For a balanced CRD with r replications per treatment:
>
> $$SE(\hat{C}) = \sqrt{MSE \sum_{i=1}^{t} \frac{c_i^2}{r}}$$

Hypothesis Test for a Contrast

$$H_0 : \mu_2 - \mu_3 = 0 \qquad \text{vs} \qquad H_A : \mu_2 - \mu_3 \neq 0$$

Test statistic:

$$t = \frac{\hat{C}}{SE(\hat{C})}$$

with $df = (r - 1)t$.

Confidence Interval for a Contrast

A CI for C:

$$\hat{C} \pm t^*_{(r-1)t,\alpha/2} \; SE(\hat{C})$$

R: Contrasts

```r
library(emmeans)
shoe_mod <- lm(`Lap Time (seconds)` ~ Shoe, data = shoe_data)
shoe_lsmeans <- emmeans(shoe_mod, ~ Shoe)
levels(shoe_data$Shoe)
```

```
[1] "Control"    "Lightweight" "Stability"
```

```r
contrast(shoe_lsmeans,
         method = list("Lightweight vs Stability" = c(0, 1, -1)),
         infer = c(TRUE, TRUE)
)
```

```
contrast                 estimate SE df lower.CL upper.CL t.ratio p.value
Lightweight vs Stability       13  2  3     6.64     19.4   6.500  0.0074

Confidence level used: 0.95
```

JMP: Contrasts

▼ Shoe > LSMeans Contrast

Interpretation: Lightweight vs Stability

At an $\alpha = 0.05$, we have evidence to conclude the mean 400m lap time for lightweight shoes is 13 seconds (s.e. = 2) slower than the mean 400m lap time for stability shoes (t = 6.5; df = 3; p = 0.0074).

We are 95% confident the population mean 400m lap time for lightweight shoes is between 6.64 and 19.4 seconds slower than for stability shoes.

These results apply to all runner similar to those in our study under similar conditions.

Contrast 2: Average of Control + Stability vs Lightweight

Is the lightweight shoe different from the average of the other two?

H_0 :

H_A :

Contrast:

$C =$

Coefficients: $(0.5, \ -1, \ 0.5)$

Contrast 2: Average of Control + Stability vs Lightweight

$\hat{C} =$

$SE(\hat{C}) =$

$t =$

R: Contrasts

```r
contrast(shoe_lsmeans,
         method = list("Control & Stability vs Lightweight" = c(0.5, -1, 0.5)),
         infer = c(TRUE, TRUE)
)
```

```
contrast                          estimate   SE df lower.CL upper.CL t.ratio
Control & Stability vs Lightweight   -10.5 1.73  3      -16    -4.99  -6.062
p.value
 0.0090

Confidence level used: 0.95
```

Note: flipping the contrasts arounds just changes the direction of the estimate and t-statistic

```r
contrast(shoe_lsmeans,
         method = list("Control & Stability vs Lightweight" = c(-0.5, 1, -0.5)),
         infer = c(TRUE, TRUE)
)
```

```
contrast                          estimate   SE df lower.CL upper.CL t.ratio
Control & Stability vs Lightweight    10.5 1.73  3     4.99       16   6.062
p.value
 0.0090

Confidence level used: 0.95
```

JMP: Contrasts

🔻 Shoe > LSMeans Contrast

Interpretation: Average of Control + Stability vs Lightweight

At an $\alpha = 0.05$, we have evidence to conclude the mean 400m lap time for control and stability shoes combined is 10.5 seconds (s.e. = 1.73) faster than the mean 400m lap time for lightweight shoes (t = -6.06; df = 3; p = 0.009).

We are 95% confident the population mean 400m lap time for control and stability shoes combined is between 4.99 and 16 seconds faster than for lightweight shoes.

These results apply to all runner similar to those in our study under similar conditions.

Preplanned vs Post Hoc Contrasts

Preplanned contrasts:

- Chosen before seeing the data

- Limited in number

- Do not require p-value multiplicity adjustment

Post hoc contrasts:

- Chosen after ANOVA

- Require multiplicity adjustment

Why Contrasts Matter

Contrasts:

- Align analysis with scientific questions

- Avoid unnecessary multiple testing

- Provide interpretable comparisons

They are often more informative than "all pairwise comparisons."