

# Module 4: Factorial Treatment Structure

Higher-Order Interactions and Analysis Strategy

## Example 4.4: Exercise Performance

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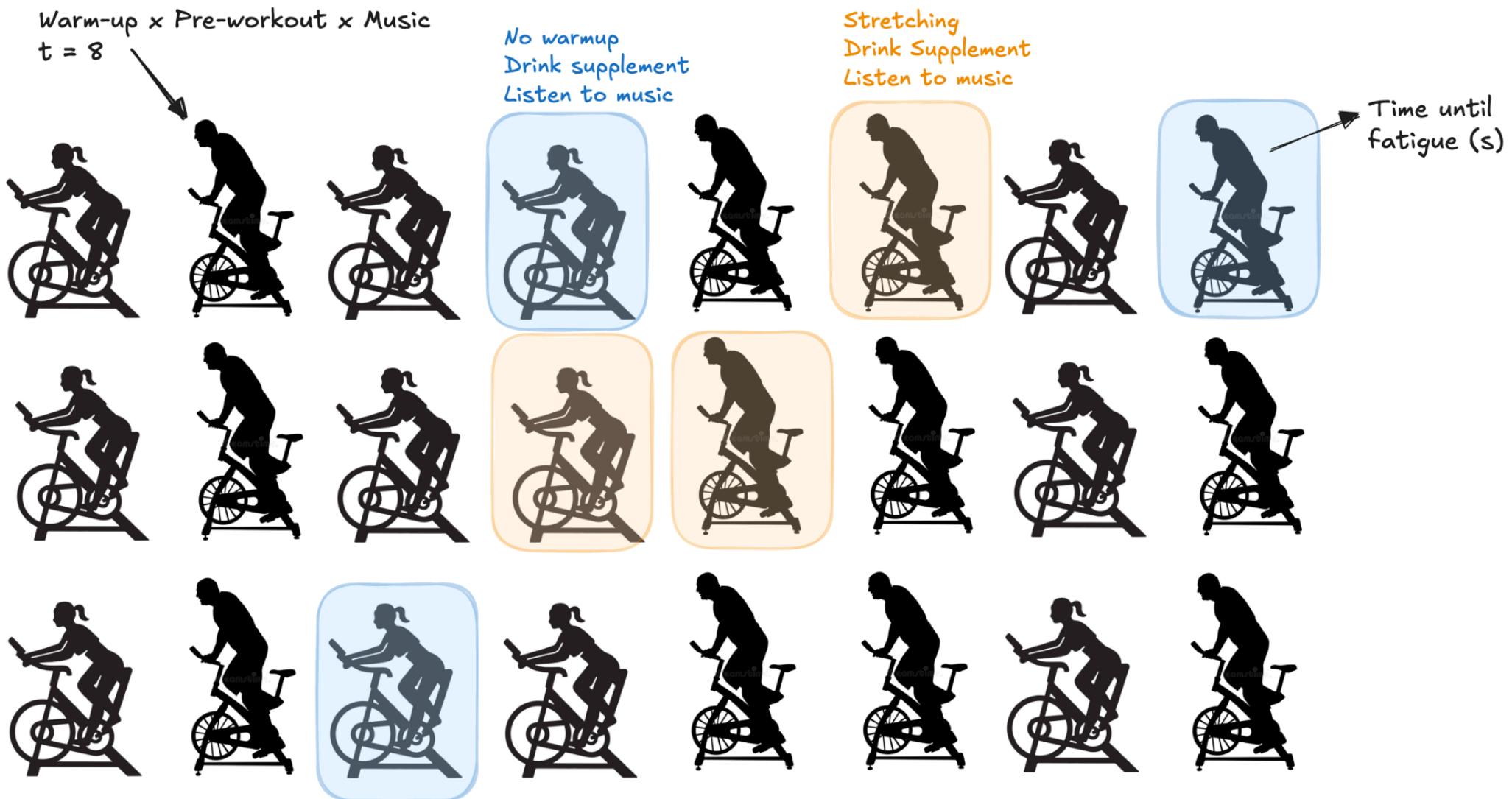
The effects of warm-up type (no warm-up vs. dynamic stretching), pre-workout supplement (none vs. pre-workout drink), and music type (no music vs. upbeat music) on exercise tolerance were studied in a small-scale experiment involving 24 adults aged 25-35. Exercise tolerance was measured as the number of minutes until the subject reached fatigue while performing on a stationary bicycle. Each participant was randomly assigned to a warm-up type – pre-workout supplement – music type condition before completing the exercise tolerance stress test. The data can be found in [stress\\_test.csv](#).

# Example 4.4: The Data

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Subject	Warmup	Preworkout	Music	Tolerance
1	Dynamic stretching	Pre-workout drink	Upbeat music	29.2
2	No warm-up	Pre-workout drink	No music	14.8
3	Dynamic stretching	Pre-workout drink	Upbeat music	24.1
4	No warm-up	None	No music	6.1
5	Dynamic stretching	None	Upbeat music	14.6
6	Dynamic stretching	Pre-workout drink	No music	17.6

# Study Blueprint



# Study Structure

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## Treatment Structure

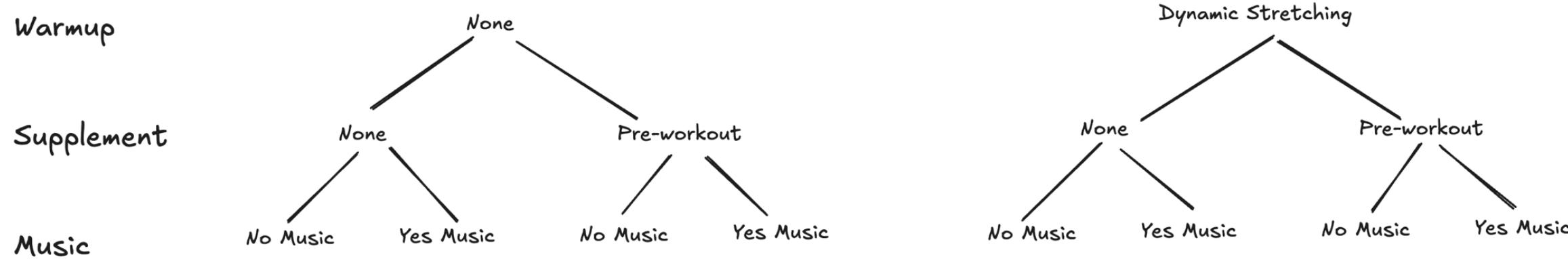
A 2x2x2 full factorial between warm-up type (no warm-up vs. dynamic stretching), pre-workout supplement (none vs. pre-workout supplement), and music type (no music vs. upbeat music) for a total of  $t = 8$  treatments.

## Experimental Structure

Warm-up type, pre-workout supplement, and music type combinations are randomly assigned to subjects (e.u.) in a CRD with  $r = 3$ . The time until reaching fatigue on a stationary bicycle (seconds) is recorded for each subject (m.u.) for a total  $N = 24$  subjects.

# Adding a Third Factor (2x2x2)

With three factors  $\rightarrow t = 8$  treatments

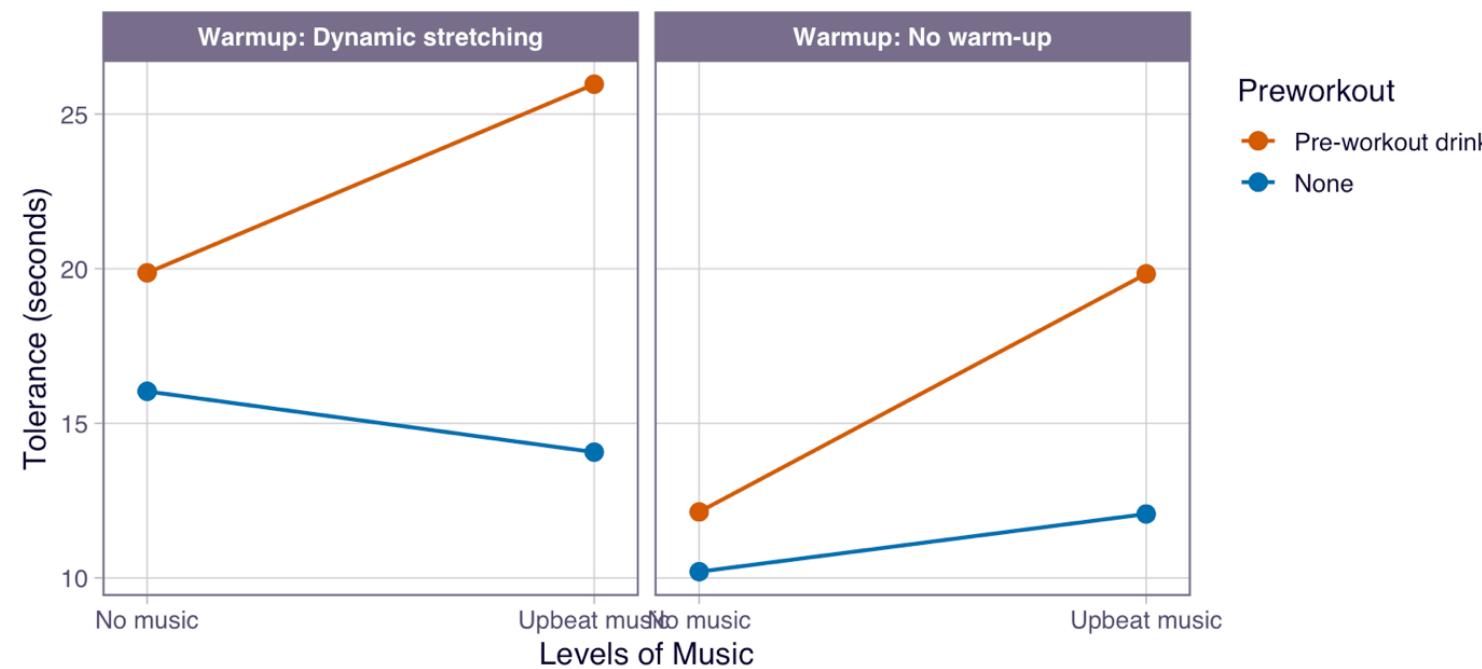


- main effects: Warm-up, Pre-workout, Music
- two-way interactions: Warm-up x Pre-workout, Warm-up x Music, Pre-workout x Music
- three-way interaction: Warm-up x Pre-workout x Music

# What is a three-way interaction?

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- A three-way interaction exists when the way two factors interact depends on the level of a third factor.
- AKA: a two-way interaction is not consistent across levels of a third factor.



# **3-way Treatment Effects Model**

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$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \alpha\beta_{ij} + \alpha\gamma_{ik} + \beta\gamma_{jk} + \alpha\beta\gamma_{ijk} + \epsilon_{ijkl} \text{ with } \epsilon_{ijkl} \text{ iid } \sim N(0, \sigma^2)$$

for  $i = 1, 2; j = 1, 2; k = 1, 2; l = 1, 2, 3$

where... continued on next slide!

# 3-way Treatment Effects Model

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- $y_{ijkl}$ : is the exercise tolerance (number of minutes until the subject reached fatigue) for the  $l^{th}$  individual receiving the  $i^{th}$  warm-up type,  $j^{th}$  pre-workout supplement, and  $k^{th}$  music type.
- $\alpha_i$ : the effect of the  $i^{th}$  level of warm-up type.
- $\beta_j$ : the effect of the  $j^{th}$  level of pre-workout supplement.
- $\gamma_k$ : the effect of the  $k^{th}$  music type.
- $\alpha\beta_{ij}$ : the interaction effect between the  $i^{th}$  level of warm-up type and  $j^{th}$  level of pre-workout supplement.
- $\alpha\gamma_{ik}$ : the interaction effect between the  $i^{th}$  level of warm-up type and  $k^{th}$  level of music type.
- $\beta\gamma_{jk}$ : the interaction effect between the  $j^{th}$  level of pre-workout supplement and  $k^{th}$  level of music type.
- $\alpha\beta\gamma_{ijk}$ : the interaction effect between the  $i^{th}$  level of warm-up type,  $j^{th}$  level of pre-workout supplement, and  $k^{th}$  level of music type.
- $\epsilon_{ijkl}$ : the experimental error associated with the  $l^{th}$  individual receiving the  $i^{th}$  warm-up type,  $j^{th}$  pre-workout supplement, and  $k^{th}$  music type.

# 3-way ANOVA Table

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SV	DF	SS	MS = SS/DF	F
A	a-1	SSA	MSA	MSA/MSE
B	b-1	SSB	MSB	MSB/MSE
C	c-a	SSC	MSC	MSC/MSE
AB	(a-1)(b-1)	SSAB	MSAB	MSAB/MSE
AC	(a-1)(c-1)	SSAC	MSAC	MSAC/MSE
BC	(b-1)(c-1)	SSBC	MSBC	MSBC/MSE
ABC	(a-1)(b-1)(c-1)	SSABC	MSABC	MSABC/MSE
Error	(r-1)(abc)	SSE	MSE	
<i>Total</i>	N-1			

# Example 4.4: 3-way Skeleton ANOVA

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Source of Variation	DF
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# R: Fit 3-way Analysis

```
1 options(contrasts = c("contr.sum", "contr.poly"))
2 # same as Tolerance ~ Warmup + Preworkout + Music + Warmup:Preworkout + Warmup:Music + Preworkout:Music + Warmup:Preworkout:Music
3 stress_mod <- lm(Tolerance ~ Warmup*Preworkout*Music, data = stress_data)
```

This is your roadmap!

Analysis of Variance Table

```
Response: Tolerance
          Df  Sum Sq Mean Sq F value    Pr(>F)
Warmup       1 176.584 176.584 18.9155 0.0004971 ***
Preworkout   1 242.570 242.570 25.9839 0.0001076 ***
Music        1  70.384  70.384  7.5394 0.0143574 *
Warmup:Preworkout  1 13.650 13.650  1.4622 0.2441432
Warmup:Music   1 11.070 11.070  1.1859 0.2922989
Preworkout:Music 1 72.454 72.454  7.7612 0.0132205 *
Warmup:Preworkout:Music 1  1.870  1.870  0.2004 0.6604336
Residuals    16 149.367  9.335
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

These are your parameter estimates!

```
Call:
lm(formula = Tolerance ~ Warmup * Preworkout * Music, data = stress_data)

Residuals:
    Min      1Q  Median      3Q     Max
-4.100  -1.842 -0.950  2.217  4.367

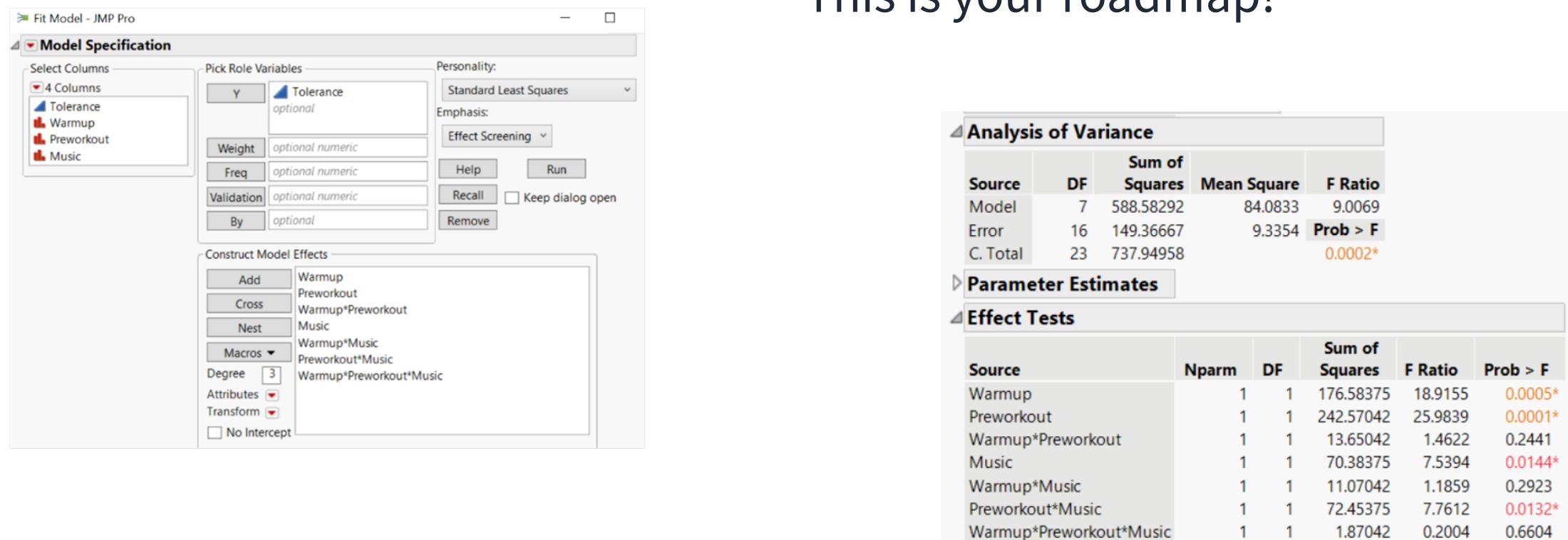
Coefficients:
                                         Estimate Std. Error t value Pr(>|t|)    
(Intercept)                         16.2708   0.6237 26.088 1.54e-14 ***
Warmup1                               2.7125   0.6237  4.349 0.000497 ***
Preworkout1                          -3.1792   0.6237 -5.097 0.000108 ***
Music1                                -1.7125   0.6237 -2.746 0.014357 *  
Warmup1:Preworkout1                  -0.7542   0.6237 -1.209 0.244143
Warmup1:Music1                        0.6792   0.6237  1.089 0.292299
Preworkout1:Music1                   1.7375   0.6237  2.786 0.013221 *  
Warmup1:Preworkout1:Music1           0.2792   0.6237  0.448 0.660434
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.055 on 16 degrees of freedom
Multiple R-squared:  0.7976,    Adjusted R-squared:  0.709 
F-statistic: 9.007 on 7 and 16 DF,  p-value: 0.0001525
```

# JMP: Fit 3-way Analysis

► Analyze > Fit Model > Assign Y = Response + Highlight both treatment factors and click Macros > Full Factorial

This is your roadmap!



The image shows the JMP Fit Model dialog and the resulting Analysis of Variance (ANOVA) table.

**Fit Model - JMP Pro**

**Model Specification**

- Select Columns:** 4 Columns (Tolerance, Warmup, Preworkout, Music) are selected.
- Pick Role Variables:** Y is assigned as the Response.
- Personality:** Standard Least Squares.
- Emphasis:** Effect Screening.
- Construct Model Effects:** Macros are selected, and the Degree is set to 3. The effects listed are Warmup, Preworkout, Warmup\*Preworkout, Music, Warmup\*Music, Preworkout\*Music, and Warmup\*Preworkout\*Music.

**Analysis of Variance**

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Model	7	588.58292	84.0833	9.0069	
Error	16	149.36667	9.3354	Prob > F	0.0002*
C. Total	23	737.94958			

**Parameter Estimates**

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Warmup	1	1	176.58375	18.9155	0.0005*
Preworkout	1	1	242.57042	25.9839	0.0001*
Warmup*Preworkout	1	1	13.65042	1.4622	0.2441
Music	1	1	70.38375	7.5394	0.0144*
Warmup*Music	1	1	11.07042	1.1859	0.2923
Preworkout*Music	1	1	72.45375	7.7612	0.0132*
Warmup*Preworkout*Music	1	1	1.87042	0.2004	0.6604

**Effect Tests**

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Warmup	1	1	176.58375	18.9155	0.0005*
Preworkout	1	1	242.57042	25.9839	0.0001*
Warmup*Preworkout	1	1	13.65042	1.4622	0.2441
Music	1	1	70.38375	7.5394	0.0144*
Warmup*Music	1	1	11.07042	1.1859	0.2923
Preworkout*Music	1	1	72.45375	7.7612	0.0132*
Warmup*Preworkout*Music	1	1	1.87042	0.2004	0.6604

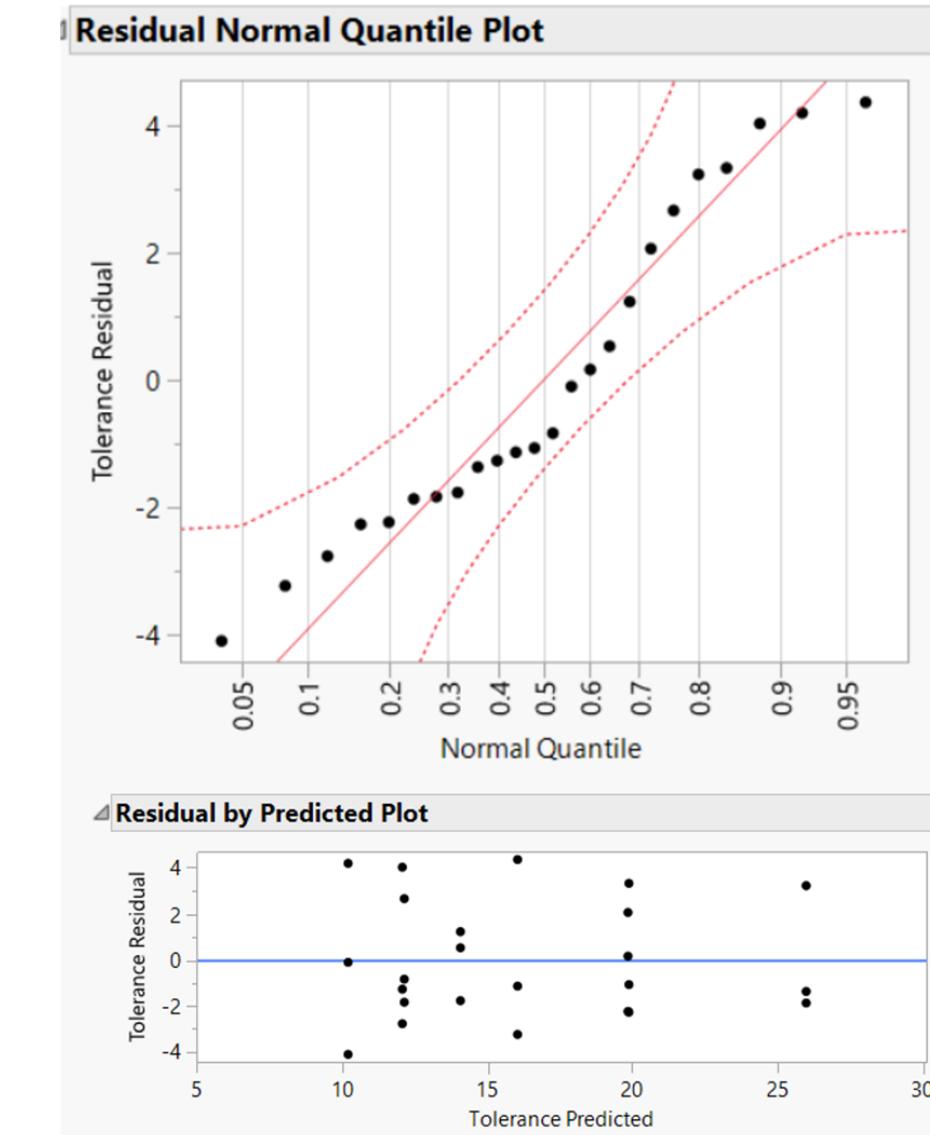
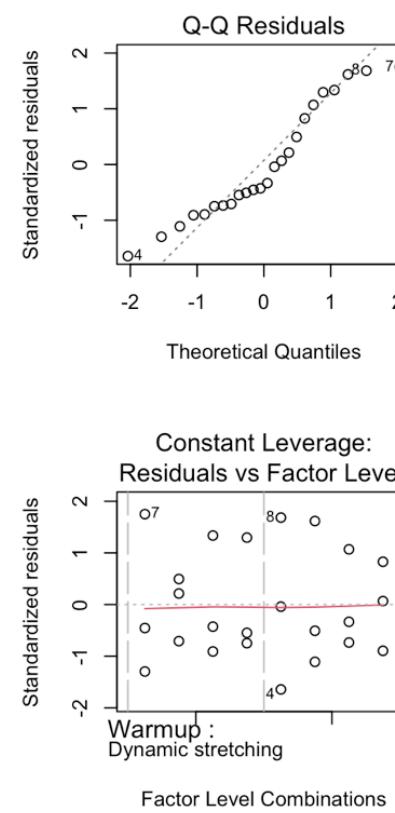
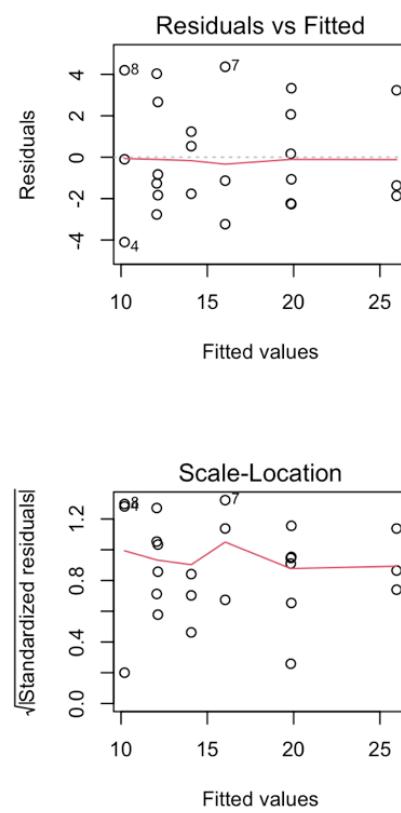
# JMP: Fit 3-way Analysis

## ▼ Response > Estimates > Expanded Estimates

Expanded Estimates				
Nominal factors expanded to all levels				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	16.270833	0.623679	26.09	<.0001*
Warmup[Dynamic stretching]	2.7125	0.623679	4.35	0.0005*
Warmup[No warm-up]	-2.7125	0.623679	-4.35	0.0005*
Preworkout[None]	-3.179167	0.623679	-5.10	0.0001*
Preworkout[Pre-workout drink]	3.1791667	0.623679	5.10	0.0001*
Warmup[Dynamic stretching]*Preworkout[None]	-0.754167	0.623679	-1.21	0.2441
Warmup[Dynamic stretching]*Preworkout[Pre-workout drink]	0.7541667	0.623679	1.21	0.2441
Warmup[No warm-up]*Preworkout[None]	0.7541667	0.623679	1.21	0.2441
Warmup[No warm-up]*Preworkout[Pre-workout drink]	-0.754167	0.623679	-1.21	0.2441
Music[No music]	-1.7125	0.623679	-2.75	0.0144*
Music[Upbeat music]	1.7125	0.623679	2.75	0.0144*
Warmup[Dynamic stretching]*Music[No music]	0.6791667	0.623679	1.09	0.2923
Warmup[Dynamic stretching]*Music[Upbeat music]	-0.679167	0.623679	-1.09	0.2923
Warmup[No warm-up]*Music[No music]	-0.679167	0.623679	-1.09	0.2923
Warmup[No warm-up]*Music[Upbeat music]	0.6791667	0.623679	1.09	0.2923
Preworkout[None]*Music[No music]	1.7375	0.623679	2.79	0.0132*
Preworkout[None]*Music[Upbeat music]	-1.7375	0.623679	-2.79	0.0132*
Preworkout[Pre-workout drink]*Music[No music]	-1.7375	0.623679	-2.79	0.0132*
Preworkout[Pre-workout drink]*Music[Upbeat music]	1.7375	0.623679	2.79	0.0132*
Warmup[Dynamic stretching]*Preworkout[None]*Music[No music]	0.2791667	0.623679	0.45	0.6604
Warmup[Dynamic stretching]*Preworkout[None]*Music[Upbeat music]	-0.279167	0.623679	-0.45	0.6604
Warmup[Dynamic stretching]*Preworkout[Pre-workout drink]*Music[No music]	-0.279167	0.623679	-0.45	0.6604
Warmup[Dynamic stretching]*Preworkout[Pre-workout drink]*Music[Upbeat music]	0.2791667	0.623679	0.45	0.6604
Warmup[No warm-up]*Preworkout[None]*Music[No music]	-0.279167	0.623679	-0.45	0.6604
Warmup[No warm-up]*Preworkout[None]*Music[Upbeat music]	0.2791667	0.623679	0.45	0.6604
Warmup[No warm-up]*Preworkout[Pre-workout drink]*Music[No music]	0.2791667	0.623679	0.45	0.6604
Warmup[No warm-up]*Preworkout[Pre-workout drink]*Music[Upbeat music]	-0.279167	0.623679	-0.45	0.6604

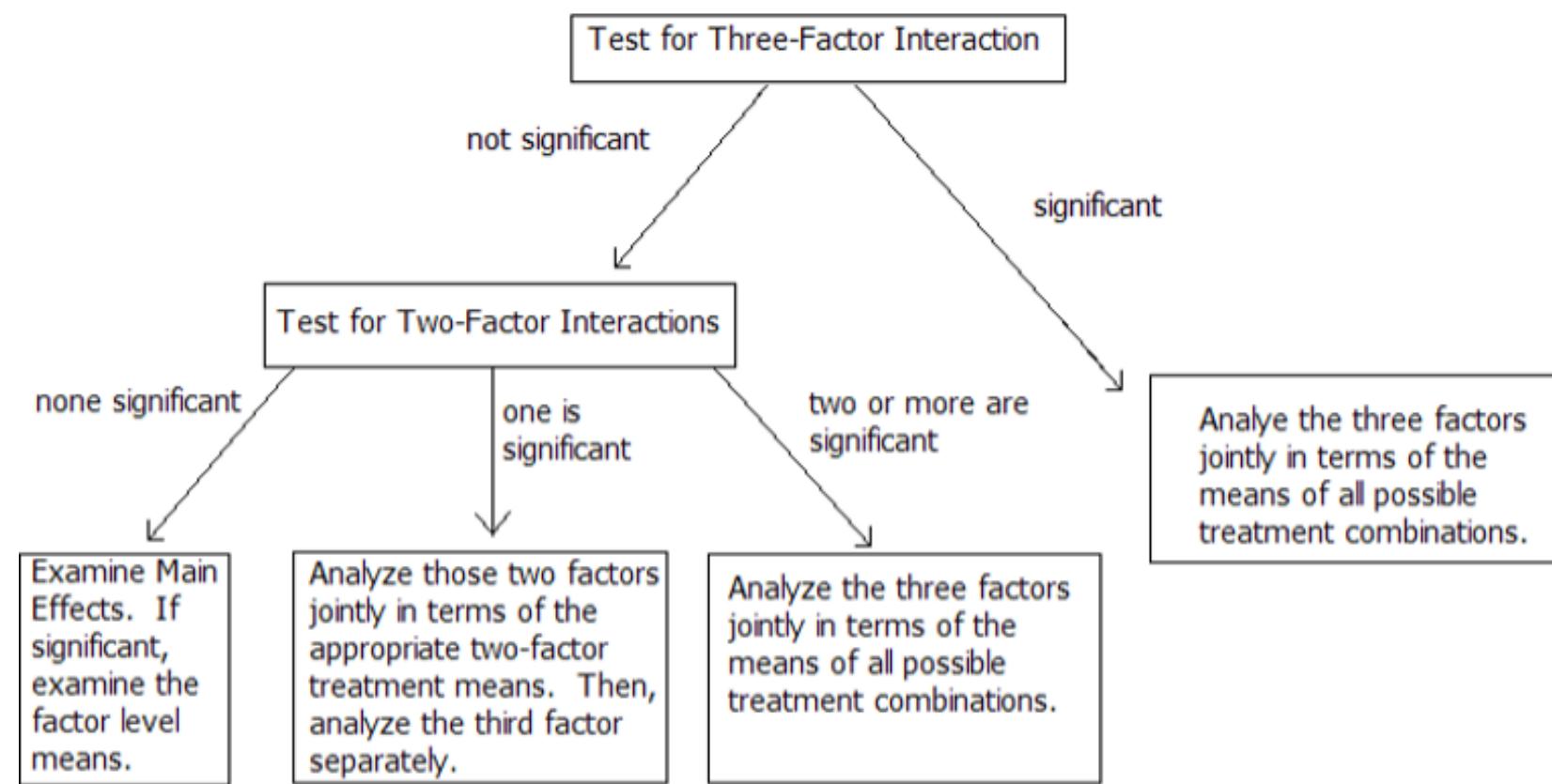
# Check Model Assumptions: $\epsilon_{ijkl} \text{iid} \sim N(0, \sigma^2)$

```
1 par(mfrow = c(2,2))
2 plot(stress_mod)
```



# 3-way Decision Flowchart

---



# What are we testing with the F-tests?

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## 3-way Interaction

$H_0 : \text{All } \alpha\beta\gamma_{ijk} = 0$  vs  $H_A : \text{At least one } \alpha\beta\gamma_{ijk} \neq 0$

## Main Effects

$H_0 : \text{All } \alpha_i = 0$  vs  $H_A : \text{At least one } \alpha_i \neq 0$

## 2-way Interactions

$H_0 : \text{All } \beta_j = 0$  vs  $H_A : \text{At least one } \beta_j \neq 0$

$H_0 : \text{All } \alpha\beta_{ij} = 0$  vs  $H_A : \text{At least one } \alpha\beta_{ij} \neq 0$

$H_0 : \text{All } \gamma_k = 0$  vs  $H_A : \text{At least one } \gamma_k \neq 0$

$H_0 : \text{All } \alpha\gamma_{ik} = 0$  vs  $H_A : \text{At least one } \alpha\gamma_{ik} \neq 0$

$H_0 : \text{All } \beta\gamma_{jk} = 0$  vs  $H_A : \text{At least one } \beta\gamma_{jk} \neq 0$

# Let's go back to our ANOVA Table (i.e., Roadmap)

What should we test first? Then what?

Analysis of Variance Table						
Response: Tolerance						
	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
Warmup	1	176.584	176.584	18.9155	0.0004971	***
Preworkout	1	242.570	242.570	25.9839	0.0001076	***
Music	1	70.384	70.384	7.5394	0.0143574	*
Warmup:Preworkout	1	13.650	13.650	1.4622	0.2441432	
Warmup:Music	1	11.070	11.070	1.1859	0.2922989	
Preworkout:Music	1	72.454	72.454	7.7612	0.0132205	*
Warmup:Preworkout:Music	1	1.870	1.870	0.2004	0.6604336	
Residuals	16	149.367	9.335			

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Model	7	588.58292	84.0833	9.0069	
Error	16	149.36667	9.3354	Prob > F	
C. Total	23	737.94958		0.0002*	

Parameter Estimates					

Effect Tests					
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Warmup		1	176.58375	18.9155	0.0005*
Preworkout		1	242.57042	25.9839	0.0001*
Warmup*Preworkout		1	13.65042	1.4622	0.2441
Music		1	70.38375	7.5394	0.0144*
Warmup*Music		1	11.07042	1.1859	0.2923
Preworkout*Music		1	72.45375	7.7612	0.0132*
Warmup*Preworkout*Music		1	1.87042	0.2004	0.6604

# What should we “dig into”?

---

## 1. 3-way interaction:

- At an  $\alpha = 0.05$  we do not have enough evidence to conclude there is a significant 3-way interaction between warmup, preworkout, and music on the tolerance ( $F = 0.2$ ;  $df = 1,16$ ;  $p = 0.6604$ ).

# What should we “dig into”?

---

## 2. 2-way interactions:

- We have enough evidence to conclude there is a significant 2-way interaction between preworkout and music on the tolerance ( $F = 7.76$ ;  $df = 1,16$ ;  $p = 0.0132$ ).
- We do not have enough evidence to conclude there is a significant 2-way interaction between warmup and music on the tolerance ( $F = 1.19$ ;  $df = 1,16$ ;  $p = 0.2922$ ).
- We do not have enough evidence to conclude there is a significant 2-way interaction between warmup and preworkout on the tolerance ( $F = 1.46$ ;  $df = 1,16$ ;  $p = 0.2441$ ).

# What should we “dig into”?

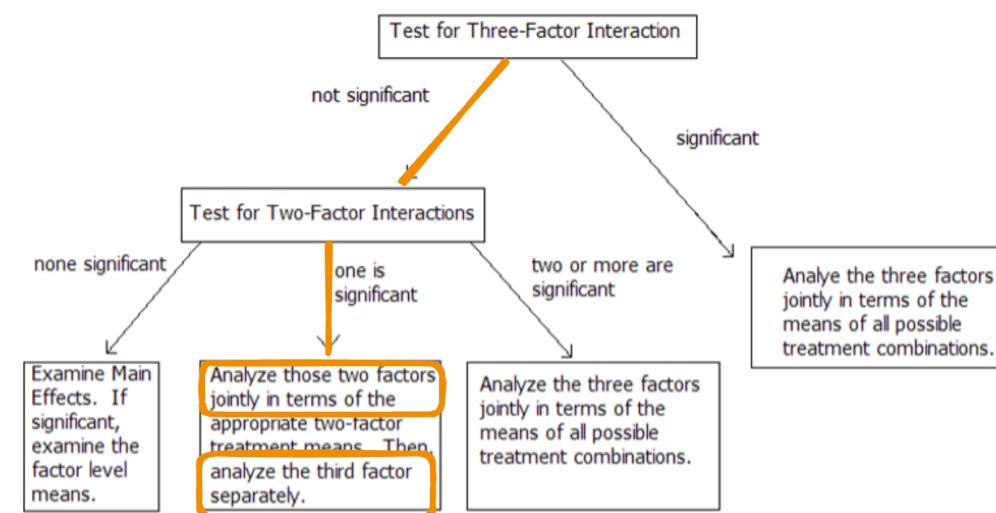
---

## 3. Main effects:

- Music.. don’t care.
- Preworkout.. don’t care
- **We have enough evidence to conclude there is a significant main effect of warmup on the tolerance ( $F = 18.91$ ;  $df = 1,16$ ;  $p = 0.0005$ ).**

# What should we “dig into”?

---

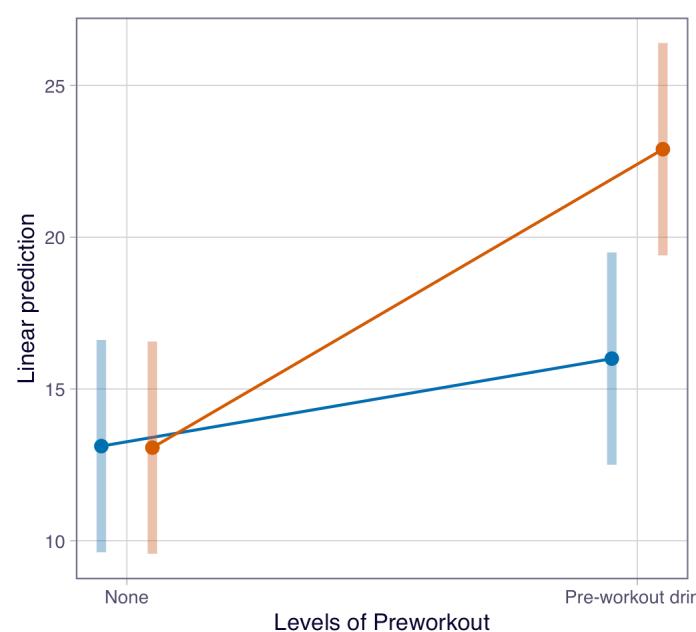


- 2-way interaction between preworkout x music (averaged over warm-up)
- Main effect of warm-up (separately)

# R: 2-way Interaction Effect Preworkout x Music

*Averaged over workout levels: none/stretching*

```
1 library(emmeans)
2 library(multcomp)
3 emmip(stress_mod, Music ~ Preworkout, CIs = TRUE, adjust = "tukey")
```



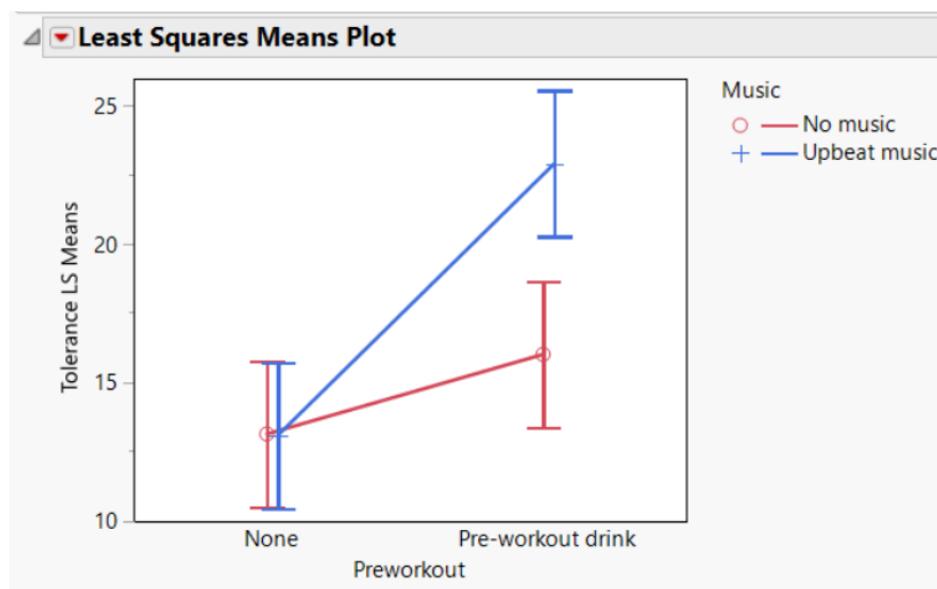
```
1 emmeans(stress_mod, specs = ~ Preworkout*Music) |>
2 cld(Letters = LETTERS, decreasing = T, adjust = "tukey")
```

Preworkout	Music	emmmean	SE	df	lower.CL	upper.CL	group
Pre-workout drink	Upbeat music	22.9	1.25	16	19.40	26.4	A
Pre-workout drink	No music	16.0	1.25	16	12.50	19.5	B
None	No music	13.1	1.25	16	9.62	16.6	B
None	Upbeat music	13.1	1.25	16	9.57	16.6	B

Results are averaged over the levels of: Warmup  
Confidence level used: 0.95  
Conf-level adjustment: sidak method for 4 estimates  
P value adjustment: tukey method for comparing a family of 4 estimates  
significance level used: alpha = 0.05  
NOTE: If two or more means share the same grouping symbol,  
then we cannot show them to be different.  
But we also did not show them to be the same.

# JMP: 2-way Interaction Effect Preworkout x Music

Scroll over to  Preworkout\*Music



Level	Least Sq Mean	Std Error
Pre-workout drink,Upbeat music	A 22.900	1.2474
Pre-workout drink,No music	B 16.000	1.2474
None,No music	B 13.117	1.2474
None,Upbeat music	B 13.067	1.2474

Levels not connected by same letter are significantly different.

# Pairwise Comparisons: Preworkout x Music (avg over Workout)

```
1 emmeans(stress_mod, specs = ~ Preworkout*Music) |>
2   pairs(adjust = "tukey", infer = c(T,T))
```

contrast	estimate	SE	df	lower.CL	upper.CL	t.ratio	p.value
None No music - (Pre-workout drink No music)	-2.88	1.76	16	-7.93	2.16	-1.635	0.3883
None No music - None Upbeat music	0.05	1.76	16	-5.00	5.10	0.028	1.0000
None No music - (Pre-workout drink Upbeat music)	-9.78	1.76	16	-14.83	-4.74	-5.546	0.0002
(Pre-workout drink No music) - None Upbeat music	2.93	1.76	16	-2.11	7.98	1.663	0.3740
(Pre-workout drink No music) - (Pre-workout drink Upbeat music)	-6.90	1.76	16	-11.95	-1.85	-3.911	0.0061
None Upbeat music - (Pre-workout drink Upbeat music)	-9.83	1.76	16	-14.88	-4.79	-5.574	0.0002

Results are averaged over the levels of: Warmup

Confidence level used: 0.95

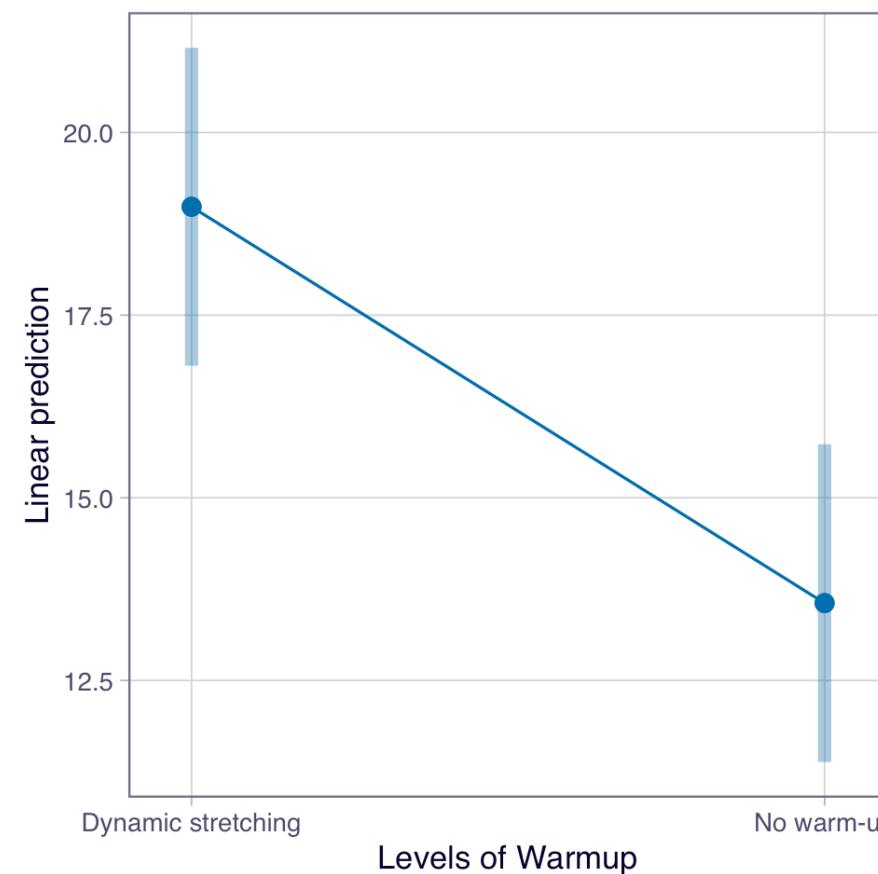
Conf-level adjustment: tukey method for comparing a family of 4 estimates

P value adjustment: tukey method for comparing a family of 4 estimates

Level	- Level	Difference	Std Err Dif	Lower CL	Upper CL	p-Value
Pre-workout drink,Upbeat music	None,Upbeat music	9.833333	1.764031	4.78640	14.88026	0.0002*
Pre-workout drink,Upbeat music	None,No music	9.783333	1.764031	4.73640	14.83026	0.0002*
Pre-workout drink,Upbeat music	Pre-workout drink,No music	6.900000	1.764031	1.85307	11.94693	0.0061*
Pre-workout drink,No music	None,Upbeat music	2.933333	1.764031	-2.11360	7.98026	0.3740
Pre-workout drink,No music	None,No music	2.883333	1.764031	-2.16360	7.93026	0.3883
None,No music	None,Upbeat music	0.050000	1.764031	-4.99693	5.09693	1.0000

# R: Main effect of Warmup

```
1 library(emmeans)
2 library(multcomp)
3 emmip(stress_mod, ~ Warmup, CIs = TRUE, adjust = "tukey")
```



```
1 emmeans(stress_mod, specs = ~ Warmup) |>
2   cld(Letters = LETTERS, decreasing = T, adjust = "tukey")
```

Warmup	emmean	SE	df	lower.CL	upper.CL	group
Dynamic stretching	19.0	0.882	16	16.8	21.2	A
No warm-up	13.6	0.882	16	11.4	15.7	B

Results are averaged over the levels of: Preworkout, Music  
 Confidence level used: 0.95  
 Conf-level adjustment: sidak method for 2 estimates  
 significance level used: alpha = 0.05  
 NOTE: If two or more means share the same grouping symbol,  
 then we cannot show them to be different.  
 But we also did not show them to be the same.

```
1 emmeans(stress_mod, specs = ~ Warmup) |>
2   pairs(adjust = "tukey")
```

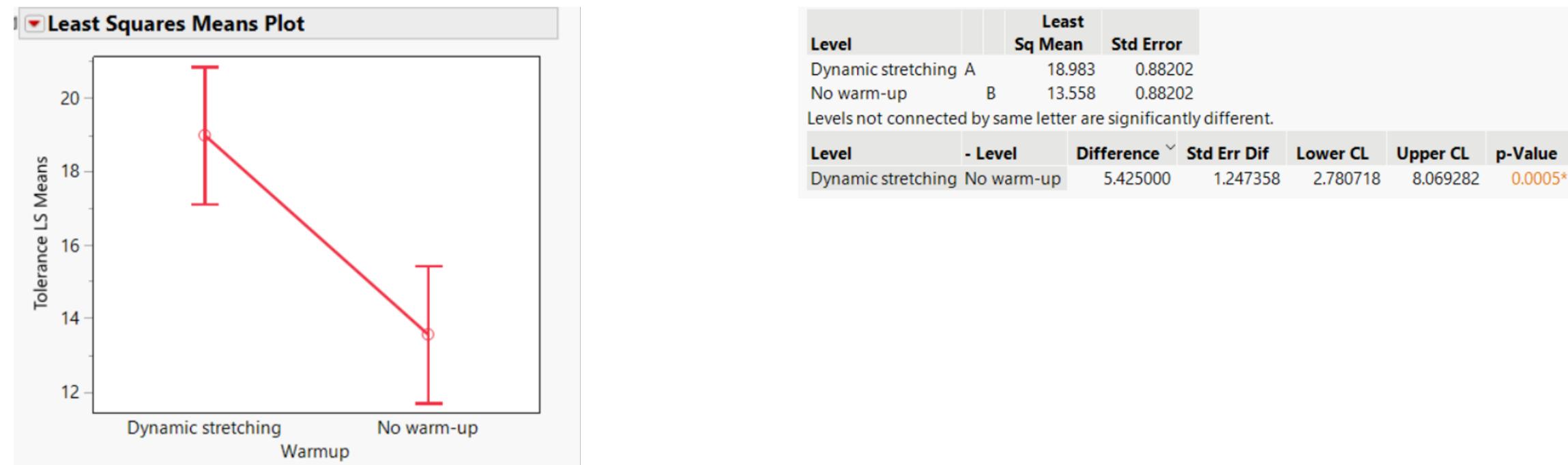
contrast	estimate	SE	df	t.ratio	p.value
Dynamic stretching - (No warm-up)	5.42	1.25	16	4.349	0.0005

Results are averaged over the levels of: Preworkout, Music

# JMP: Main effect of Warmup

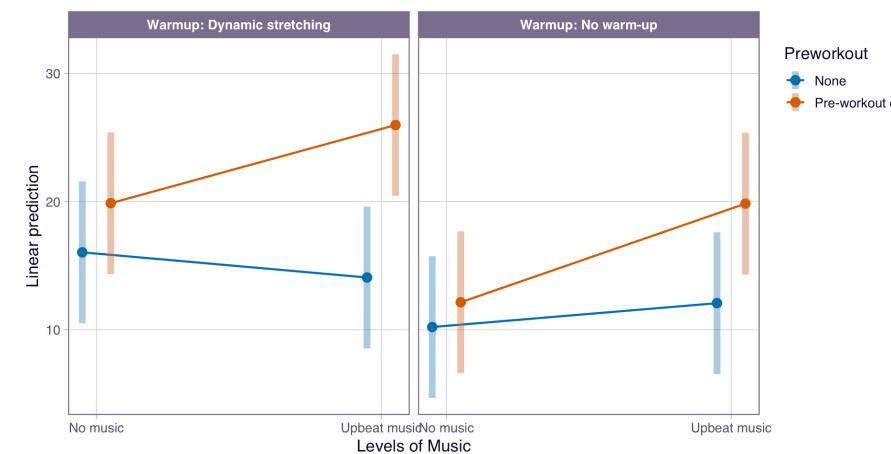
---

Scroll over to ❤️ Warmup



# R: What if there had been a 3-way interaction?

```
1 emmip(stress_mod, Preworkout ~ Music | Warmup,
2   CIs = TRUE,
3   adjust = "tukey")
```



```
1 emmeans(stress_mod, ~ Preworkout:Music:Warmup) |>
2   cld(Letters = LETTERS, decreasing = T, adjust = "tukey")
```

Preworkout	Music	Warmup	emmean	SE	df	lower.CL	upper.CL	group
Pre-workout drink	Upbeat music	Dynamic stretching	26.0	1.76	16	20.44	31.5	A
Pre-workout drink	No music	Dynamic stretching	19.9	1.76	16	14.34	25.4	AB
Pre-workout drink	Upbeat music	No warm-up	19.8	1.76	16	14.30	25.4	AB
None	No music	Dynamic stretching	16.0	1.76	16	10.50	21.6	BC
None	Upbeat music	Dynamic stretching	14.1	1.76	16	8.54	19.6	BC
Pre-workout drink	No music	No warm-up	12.1	1.76	16	6.60	17.7	BC
None	Upbeat music	No warm-up	12.1	1.76	16	6.54	17.6	BC
None	No music	No warm-up	10.2	1.76	16	4.67	15.7	C

Confidence level used: 0.95

Conf-level adjustment: sidak method for 8 estimates

P value adjustment: tukey method for comparing a family of 8 estimates

significance level used: alpha = 0.05

NOTE: If two or more means share the same grouping symbol,  
then we cannot show them to be different.  
But we also did not show them to be the same.

# JMP: *What if there had been a 3-way interaction?*

Scroll over to ♥ Warmup\*Preworkout\*Music

