

Module 2: Completely Randomized Designs

Inference for Treatment Means

Inference for Treatment Means

ANOVA tells us whether *any* means differ.

Now we want:

- Estimates of each treatment mean (μ_i)
- Uncertainty of those treatment means (SEs + CIs)
- Interpretation in context

Example 2.1: Running Shoes

Response: Lap time (seconds)

Treatment structure:

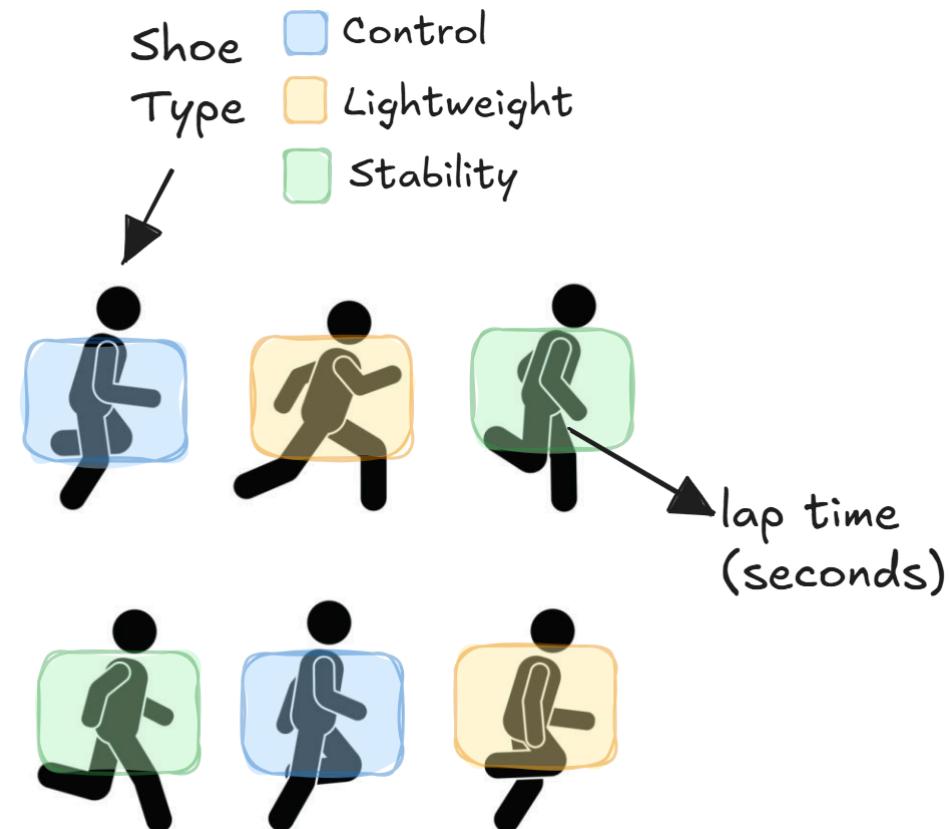
- One-way
- Factor: Shoe type
- 3 Levels: control, lightweight, and stability
- $t = 3$

Experimental structure:

- CRD
- Experimental Unit: Individual ($r = 2$)
- Measurement Unit: Individual ($N = 6$)

Goal: Determine whether shoe type affects mean lap time.

Example 2.1: Running Shoes (Blueprint)



Example 2.1: Running Shoes (Effects Model)

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \text{ with } \varepsilon_{ij} \text{ iid } \sim N(0, \sigma^2)$$

for $i = 1, 2, 3$ and $j = 1, 2$

Where:

- y_{ij} - the *observed* lap time for the j^{th} runner wearing the i^{th} shoe type.
- μ - the overall mean lap time.
- τ_i - the effect of the i^{th} shoe type.
- ε_{ij} - the experimental error associated with the j^{th} runner wearing the i^{th} shoe type.

What we Estimated

Model Reminder: $y_{ij} = \mu + \tau_i + \varepsilon_{ij}$

Analysis of Variance Table

```
Response: Lap Time (seconds)
          Df Sum Sq Mean Sq F value Pr(>F)
Shoe       2    172     86   21.5 0.01666 *
Residuals  3     12      4
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Call:

```
lm(formula = `Lap Time (seconds)` ~ Shoe, data = shoe_data)
```

Residuals:

```
 1  2  3  4  5  6
-1  1 -1  1  2 -2
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	62.0000	0.8165	75.934	5.03e-06 ***
Shoe1	-1.0000	1.1547	-0.866	0.45018
Shoe2	7.0000	1.1547	6.062	0.00901 **

Signif. codes:	0 '***'	0.001 '**'	0.01 '*'	0.05 '.' 0.1 ' ' 1
Residual standard error:	2 on 3 degrees of freedom			
Multiple R-squared:	0.9348,	Adjusted R-squared:	0.8913	
F-statistic:	21.5 on 2 and 3 DF,	p-value:	0.01666	

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	172.00000	86.0000	21.5000
Error	3	12.00000	4.0000	Prob > F
C. Total	5	184.00000		0.0167*

Expanded Estimates

Nominal factors expanded to all levels

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	62	0.816497	75.93	<.0001*
Shoe[Control]	-1	1.154701	-0.87	0.4502
Shoe[Lightweight]	7	1.154701	6.06	0.0090*
Shoe[Stability]	-6	1.154701	-5.20	0.0138*

Estimating Treatment Means $\hat{\mu}_i$ (Balanced CRD)

From the data, each treatment has a sample mean:

$$\bar{y}_i.$$

From the model, each treatment has an estimated *least squares mean*:

$$\hat{\mu}_i = \hat{\mu} + \hat{\tau}_i$$

In a **balanced** CRD (i.e., each treatment has r replications), these are the same quantity:

$$\hat{\mu}_i = \bar{y}_i.$$

Sampling Variability of $\bar{y}_{i\cdot}$.

Under the model:

$$Var(\bar{y}_{i\cdot}) = \frac{\sigma^2}{r}$$

We don't know σ^2 , so we estimate it with:

$$\hat{\sigma}^2 = MSE$$

Standard Error of a Treatment Mean $SE(\hat{\mu}_i)$

Balanced CRD (each treatment has r replications):

$$SE(\hat{\mu}_i) = \sqrt{\frac{MSE}{r}}$$

Degrees of Freedom

All inference uses the **error degrees of freedom**:

$$df_E = N - t = (r - 1)t$$

This is the same df used for the F-test in ANOVA.

Confidence Interval for a Treatment Mean

A CI for μ_i :

$$\hat{\mu}_i \pm t^*_{df_E, \alpha/2} SE(\hat{\mu}_i)$$

where t^* comes from the t distribution with $df_E = N - t = (r - 1)t$ and the $\alpha/2$ quantile associated with the confidence level.

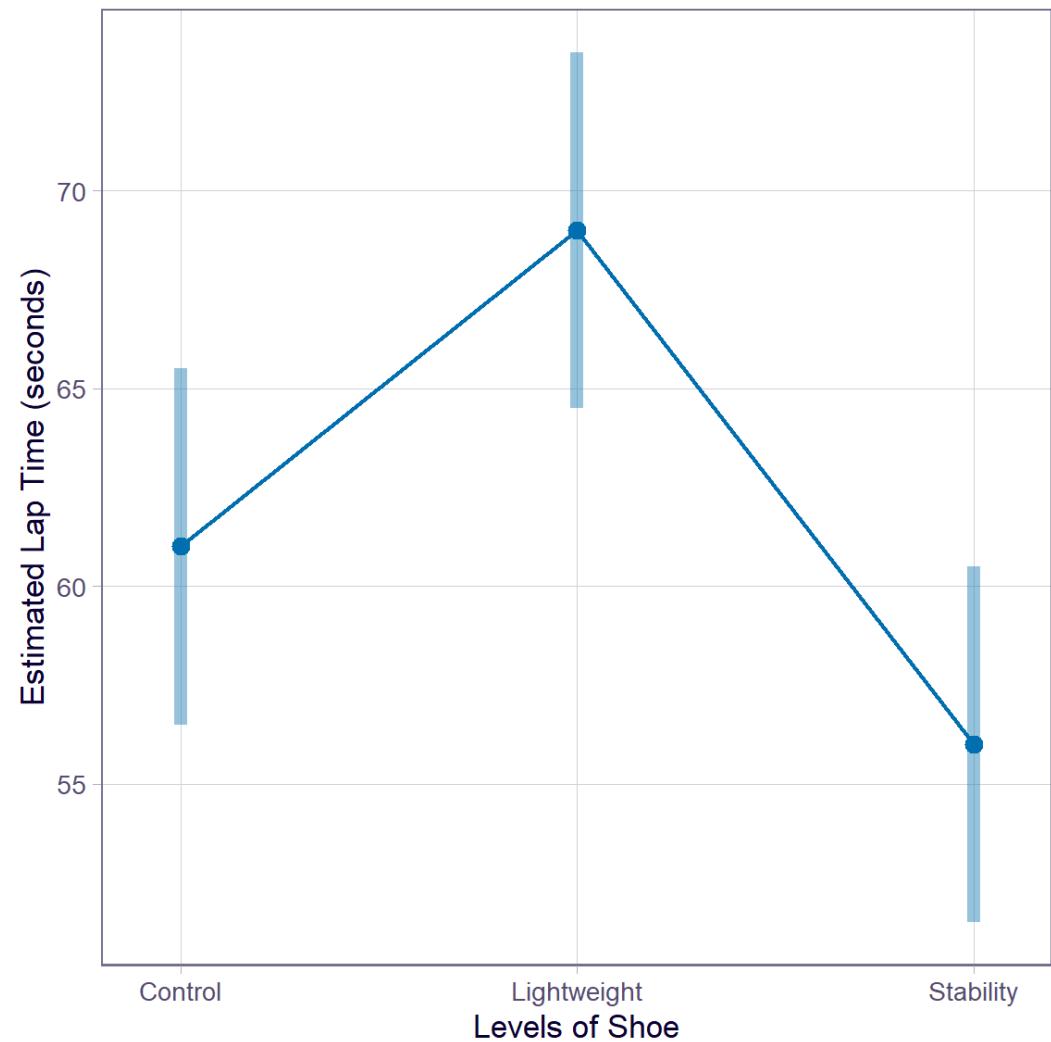
R: Estimated Means + SE + CI

```
1 library(emmeans)
2 shoe_lsmeans <- emmeans(shoe_mod, specs = ~ Shoe)
3 shoe_lsmeans
```

Shoe	emmean	SE	df	lower.CL	upper.CL
Control	61	1.41	3	56.5	65.5
Lightweight	69	1.41	3	64.5	73.5
Stability	56	1.41	3	51.5	60.5

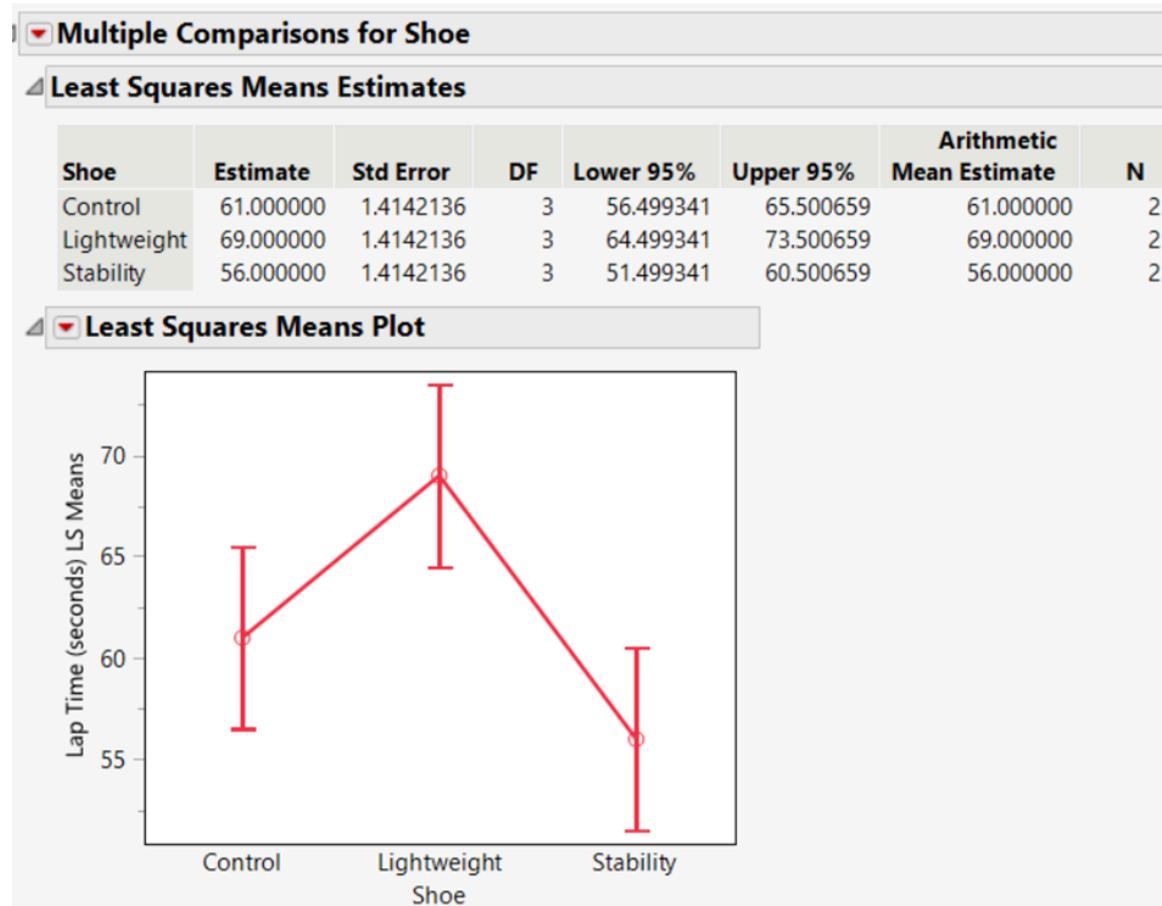
Confidence level used: 0.95

```
1 emmip(shoe_mod, ~ Shoe, CIs = T) +
2   labs(y = "Estimated Lap Time (seconds)")
```



JMP: Estimated Means + SE + CI

- ▼ Response > Multiple Comparisons > Click “Show Least Squares Means” Plot
- > OK



Interpretation of Results

The estimated mean 400m lap time for runners wearing the control shoe is 62 seconds (s.e. = 1.41).

We are 95% confident the population mean 400m lap time for all runners wearing the control shoe is between 51.5 and 60.5 seconds.

These results apply to all runner similar to those in our study under similar conditions.