ECo 602 - Analysis of Environmental Data

Conditional Probability

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Eco 602 – University of Massachusetts, Amherst – Fall 2021
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Key Concepts

- Conditional probability
- Bayes' Rule
- Differences between Frequentist and Bayesian perspectives

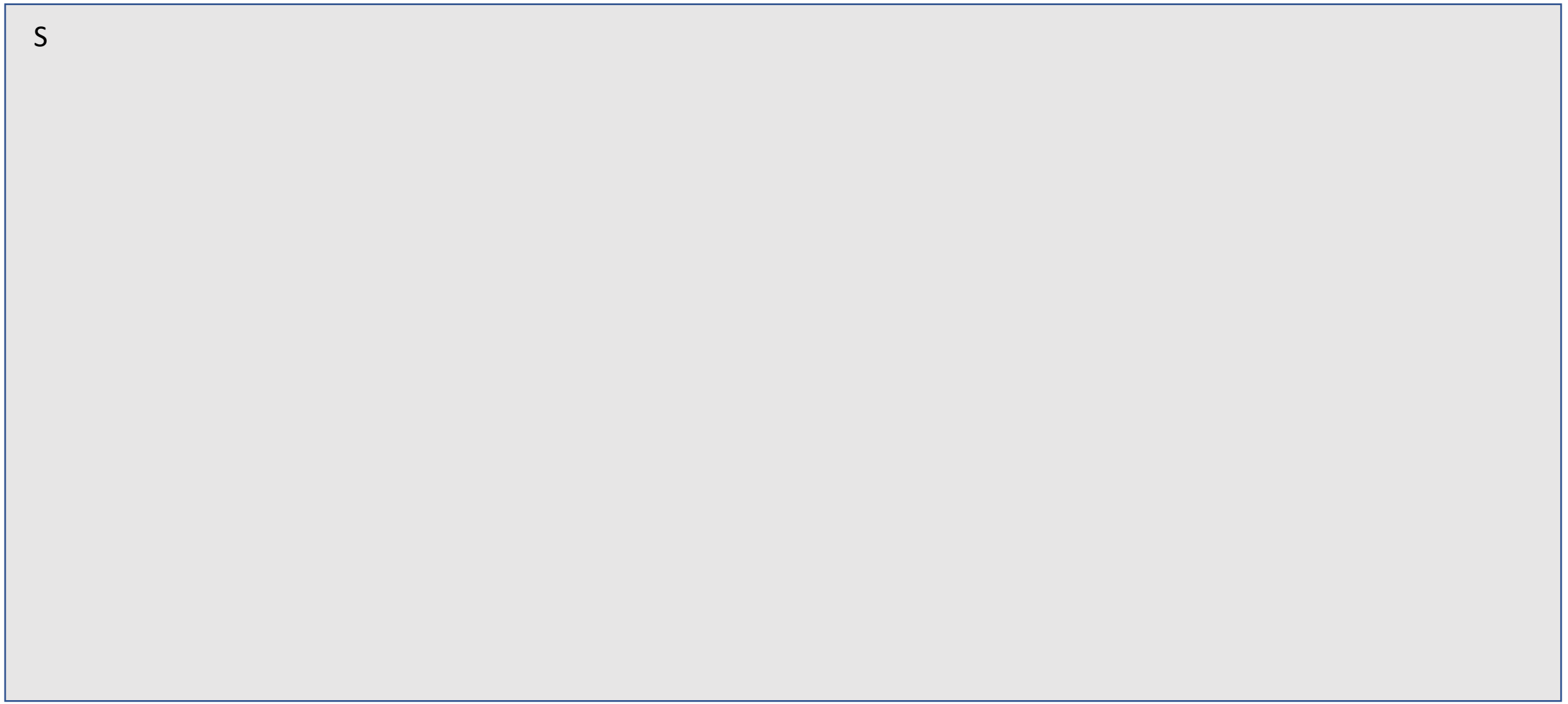
Preview of Bayesian Thinking

https://www.youtube.com/watch?v=BrK7X_XlGB8

Origin of Bayes' Theorem

https://www.youtube.com/watch?v=7GgLSnQ48os

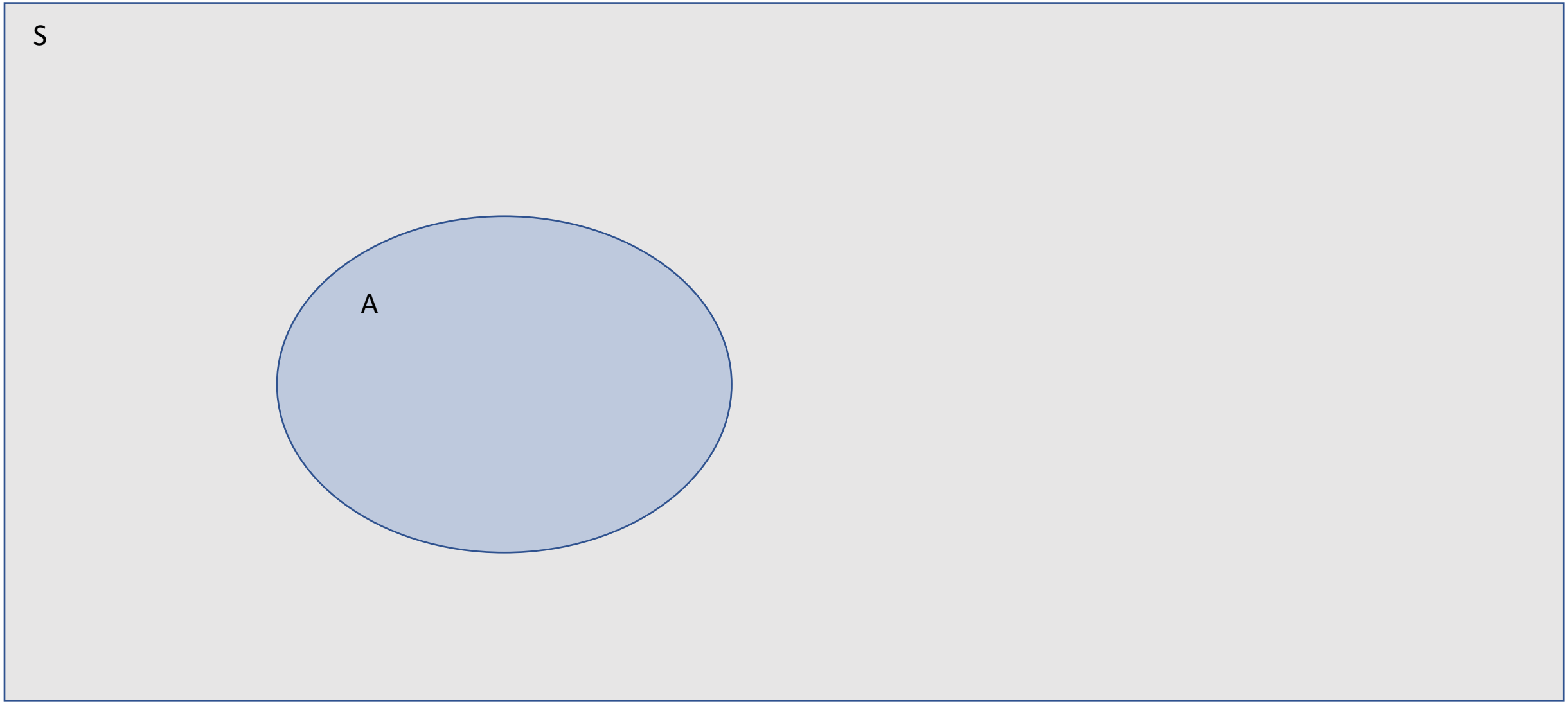
Conditional Probability: The Sample Space



Sample Space Properties

- Total area is 1.0
- The sample space contains the set of all possible events.
- $\Pr(A) = 1$

Conditional Probability: An Event

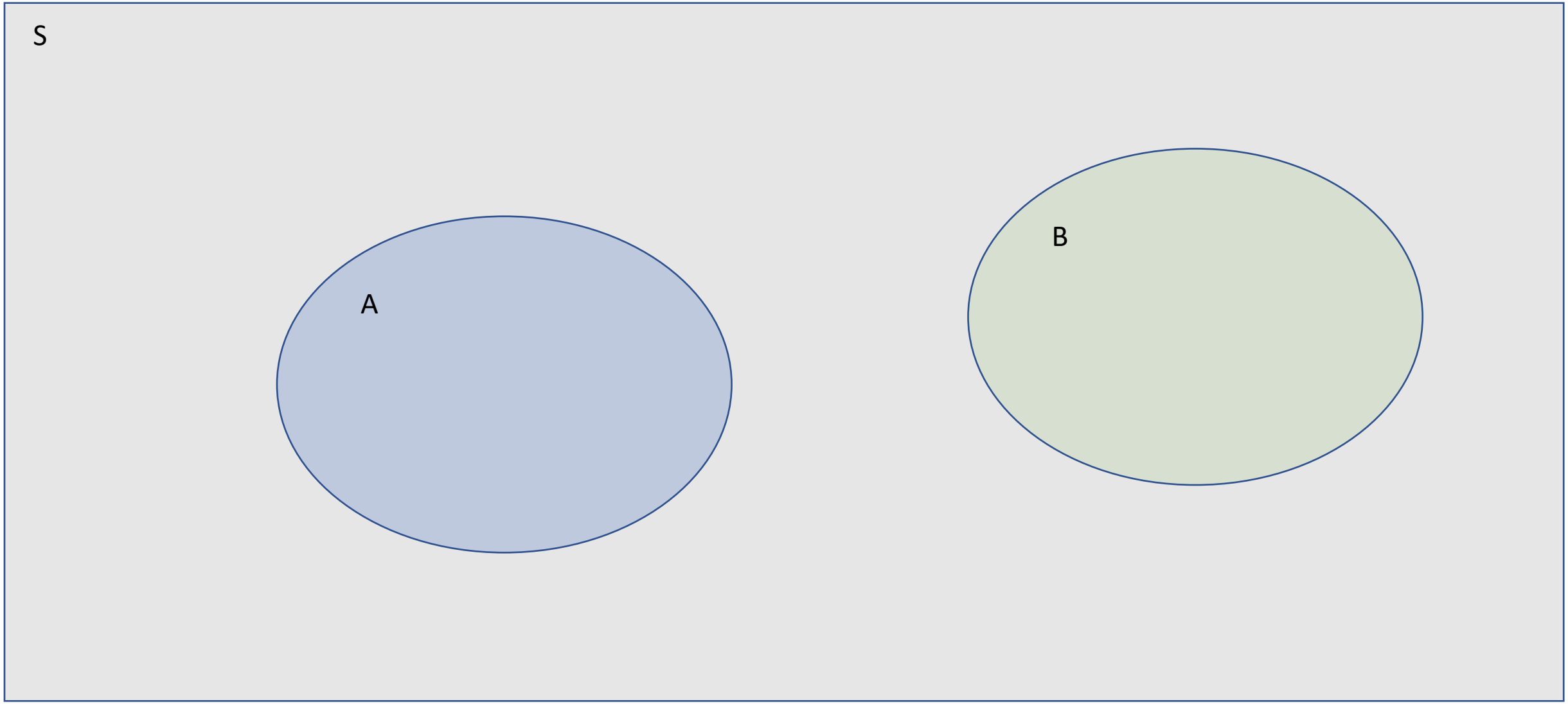


Event A Properties

- What's the probability of A?
  - We know it is equal to or less than 1.0

$$\Pr(A) = \frac{Area\ of\ A}{Area\ of\ S} = Area\ of\ A$$

Conditional Probability: Another Event



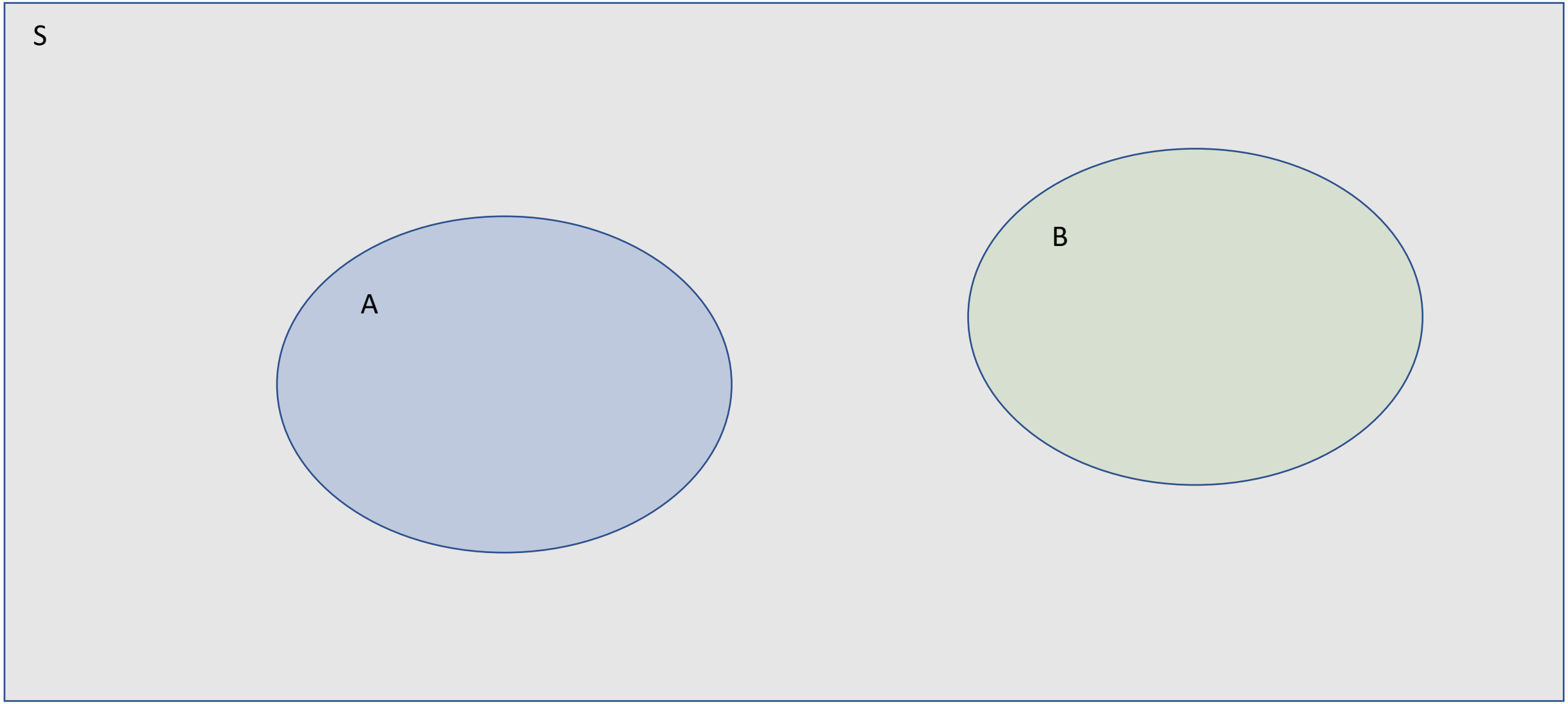
Event A Properties

- What are the event probabilities?

$$\Pr(A) = \frac{Area\ of\ A}{Area\ of\ S}$$

$$\Pr(B) = \frac{Area\ of\ B}{Area\ of\ S}$$

Exclusive Events



Exclusive Events

Some different perspectives

- If event A occurs, B cannot occur
- There is no overlap between A and B
- The conditional probability of A given B is zero.
  - $\Pr(A|B) = 0$
- The conditional probability of B given A is zero.
  - $\Pr(A|B) = 0$

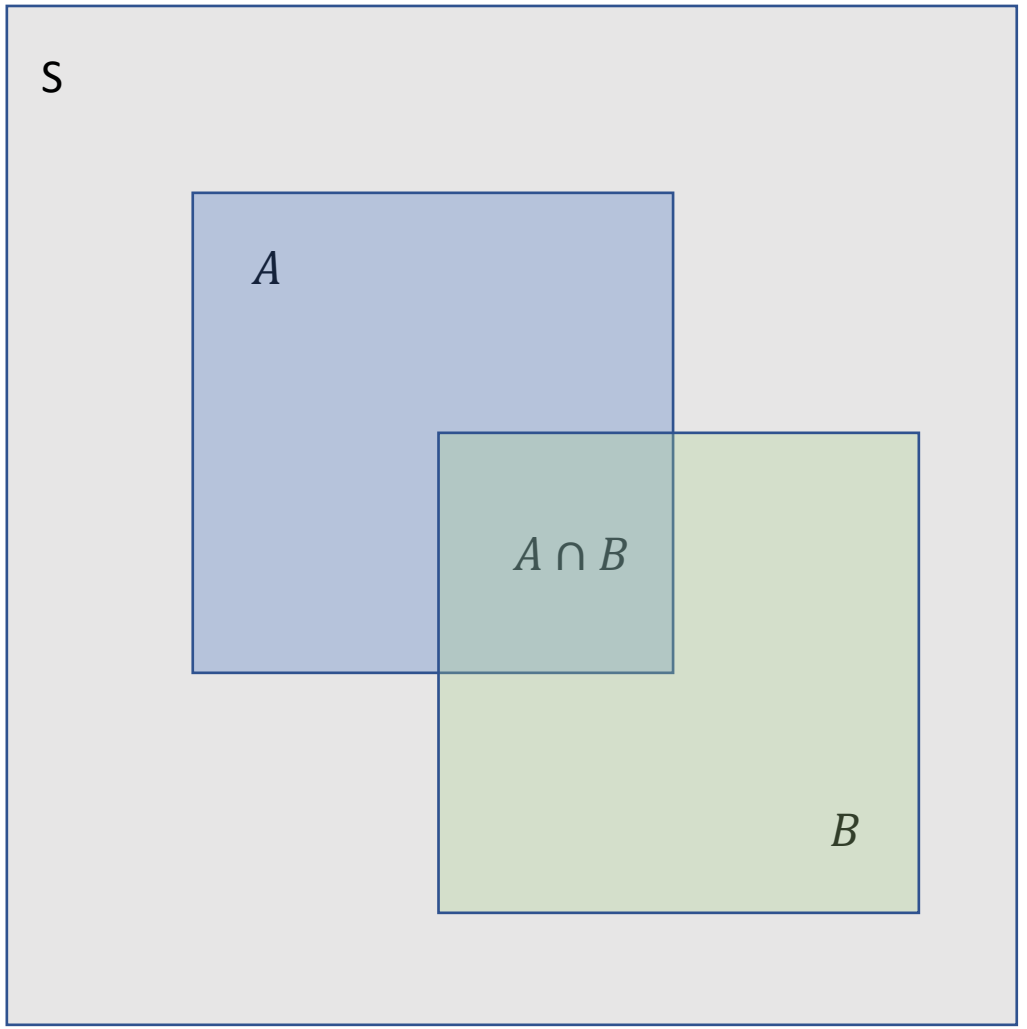
Complementary Events

Complementary Events

- Complementary events are exclusive.
  - Something can be in A or B, but not both.
  - $\Pr(A|B) = 0, \Pr(B|A) = 0$
- Complementary events fill the sample space
  - If something is not in A, then it is in B
  - $\Pr(A) + \Pr(B) = 1.0$

Overlapping Events: Conditional Probability

- We want to know



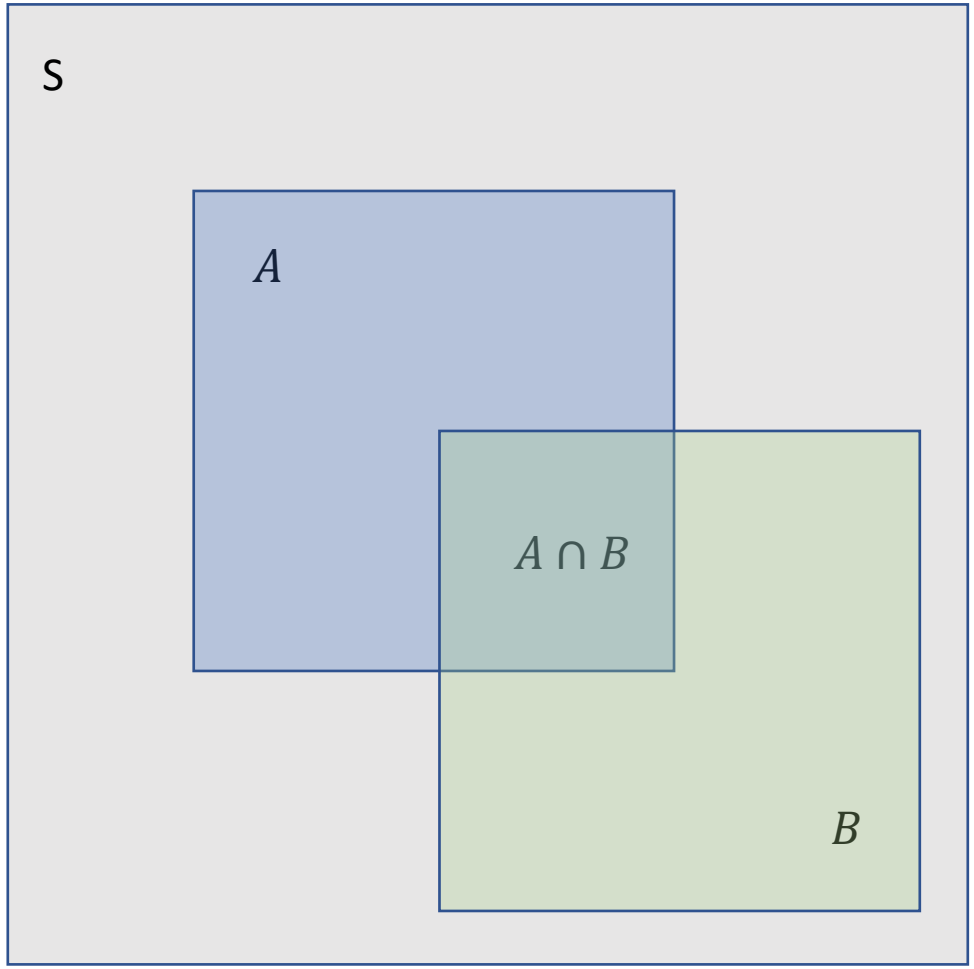
Overlapping Events: Conditional Probability

Conditional probability: We want to know how likely we are to observe an event in B if we've already observed an event in A.

In symbols:

$$\Pr(B|A)$$

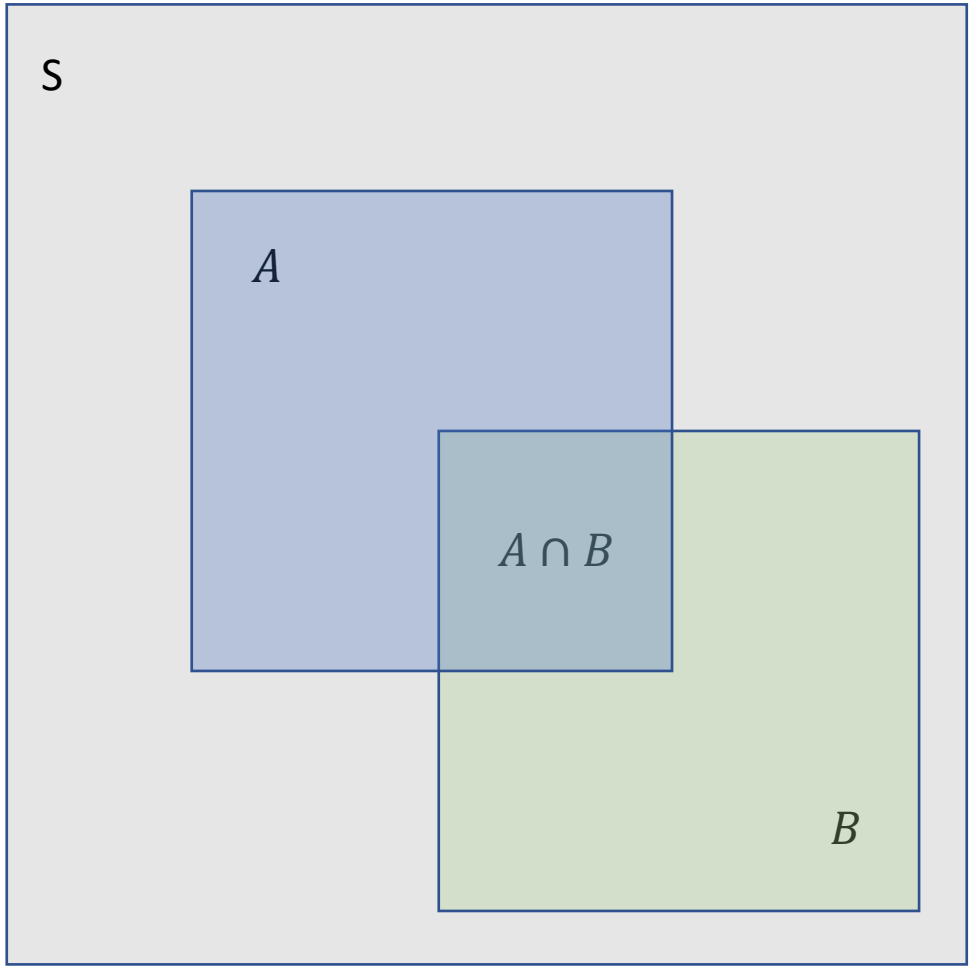
You can read this as "Probability of B given A".



Overlapping Events: Conditional Probability

What happens when we observe an event in A?

Our sample space changes…



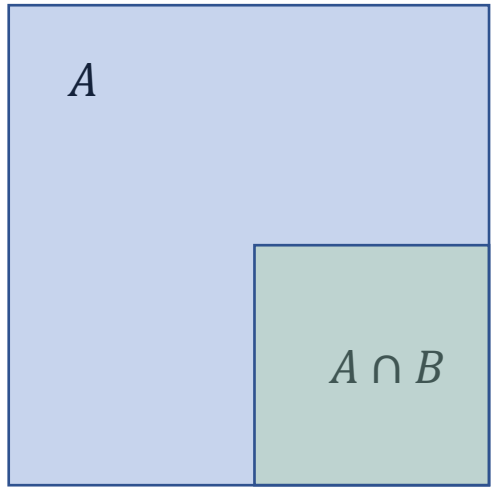
Overlapping Events: Conditional Probability

What happens when we observe an event in A?

Our sample space changes… It collapses into A.

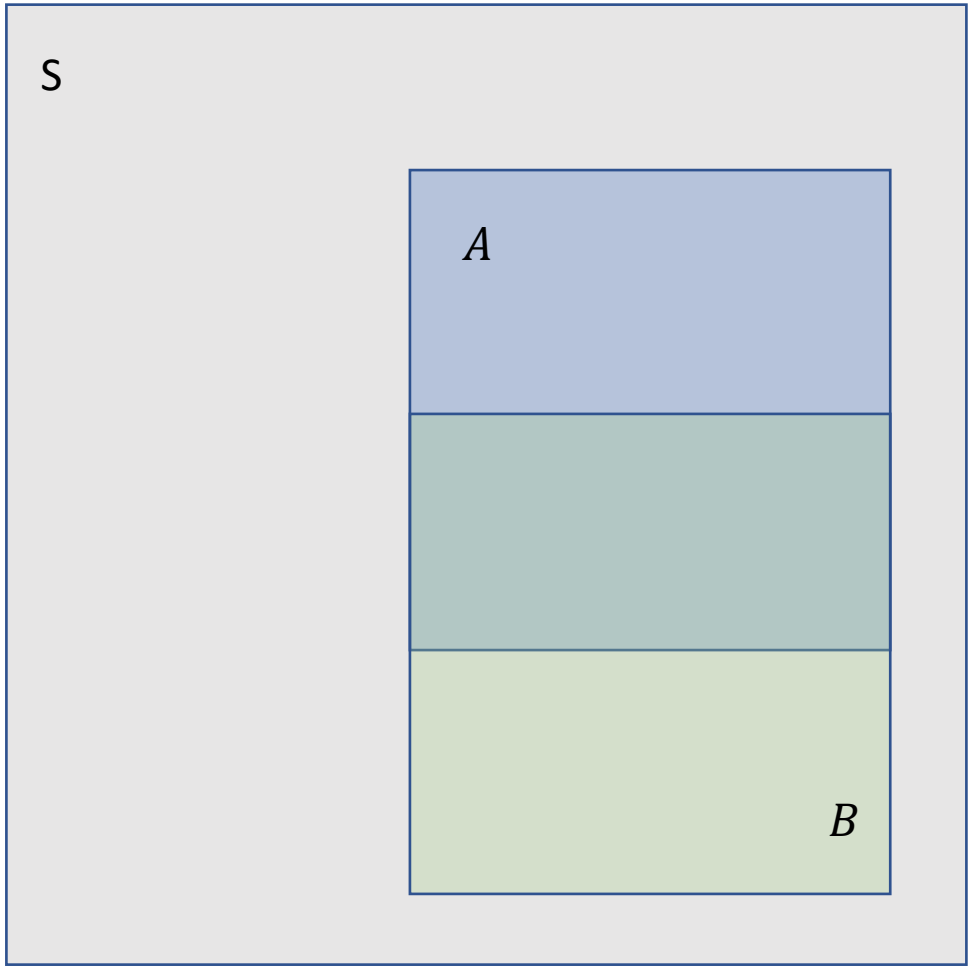
Since A contained part of B, $\Pr(B|A)$ is just:

$$\frac{\Pr(A \cap B)}{\Pr(A)}$$



Overlapping Events: Conditional Probability

Try to guess the value of $\Pr(B|A)$



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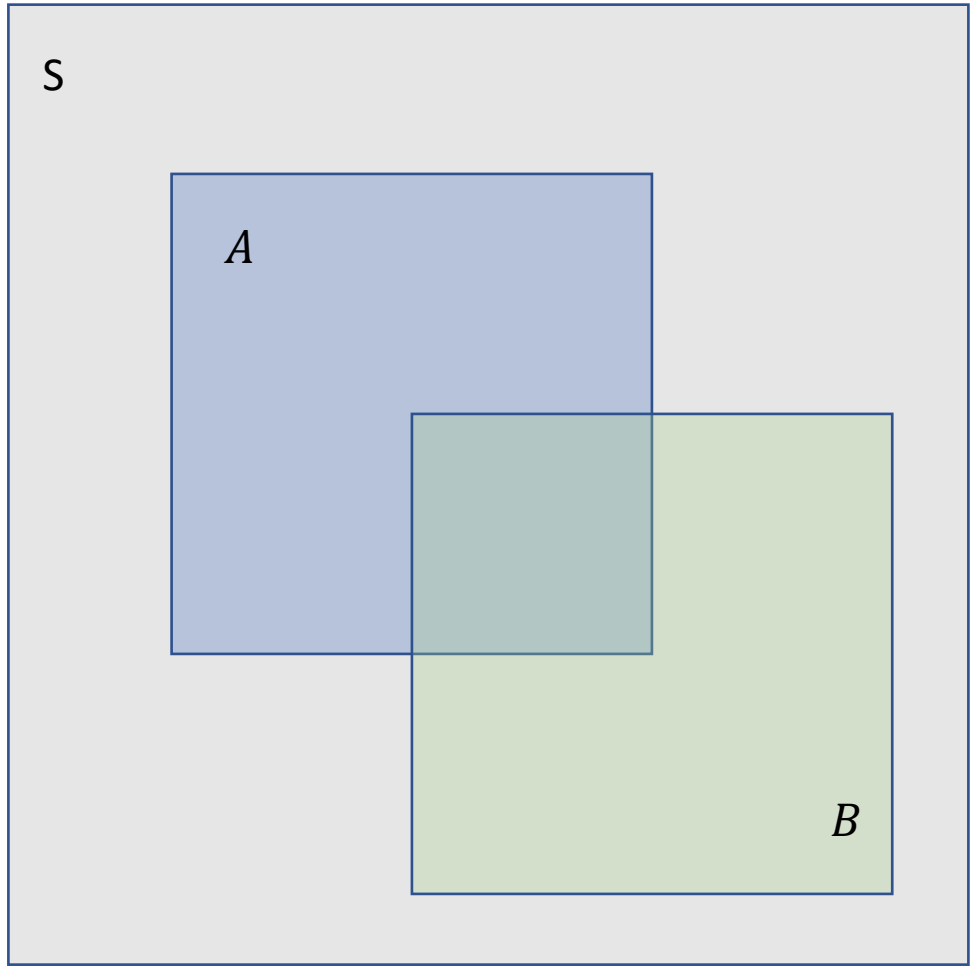
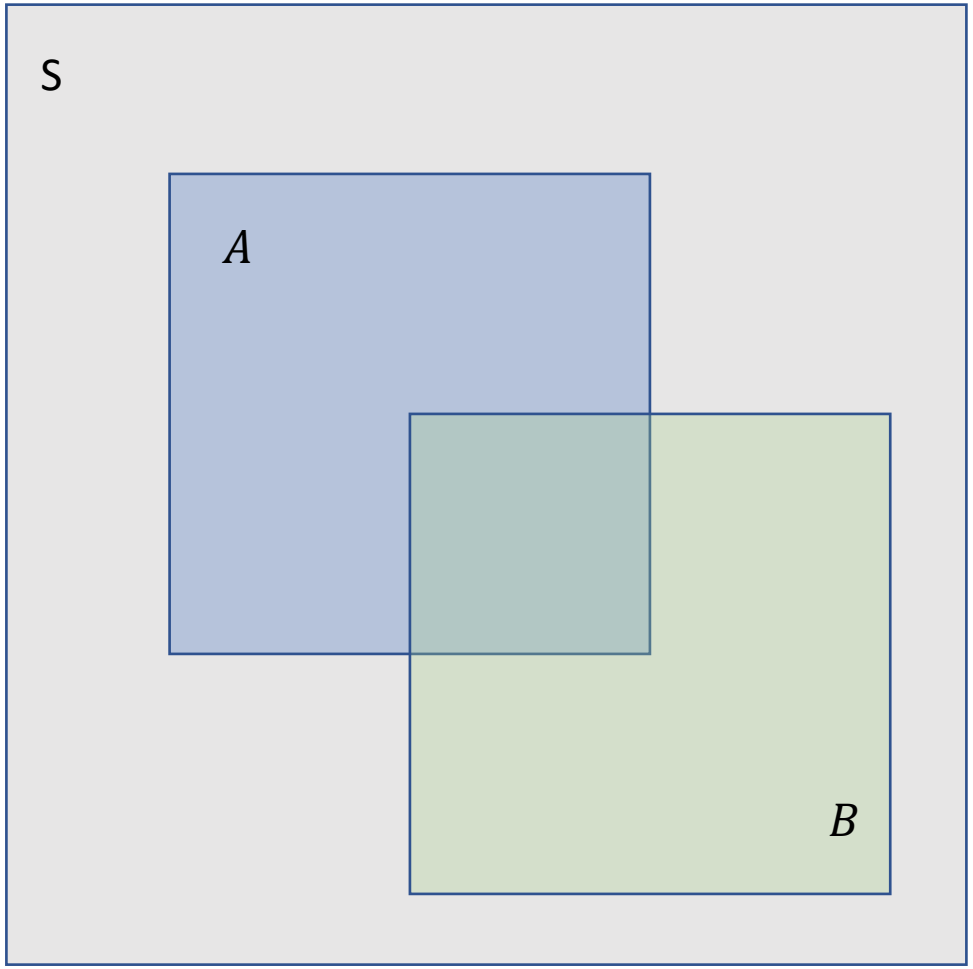


Overlapping Events: Conditional Probability

Try to guess the value of $\Pr(A|B)$



Overlapping Events: Conditional Probability

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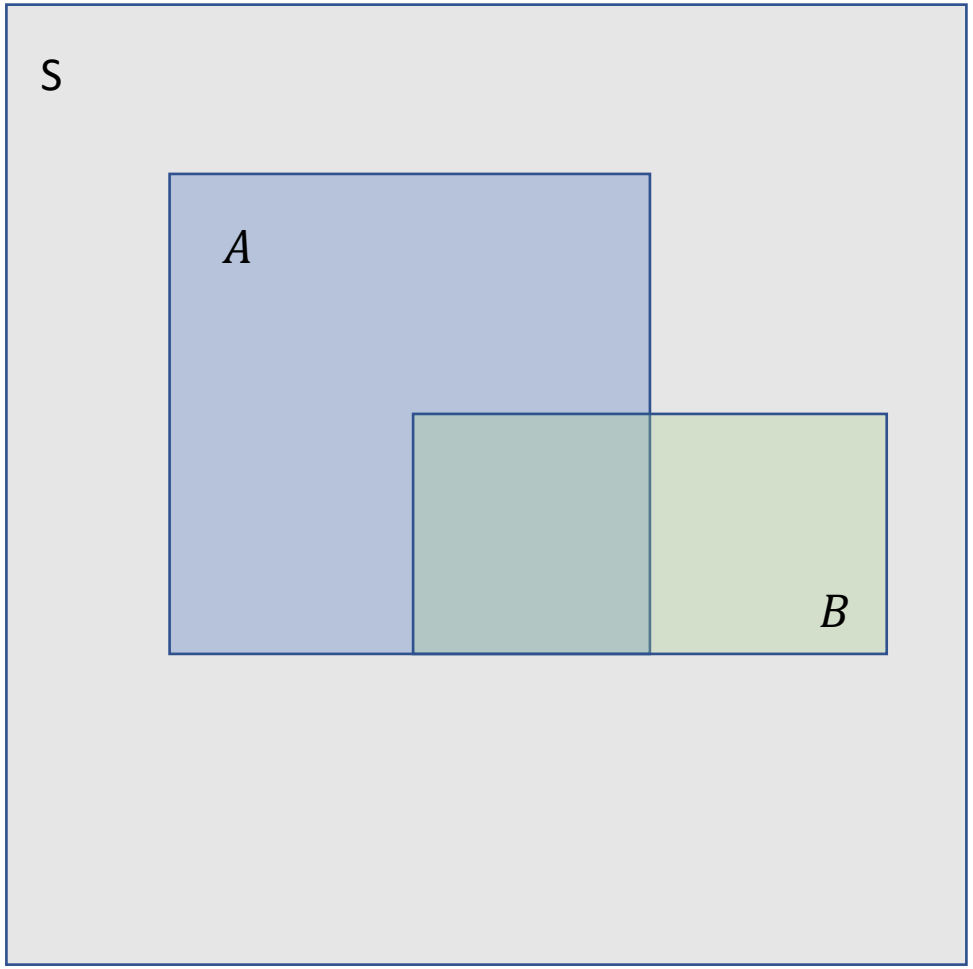


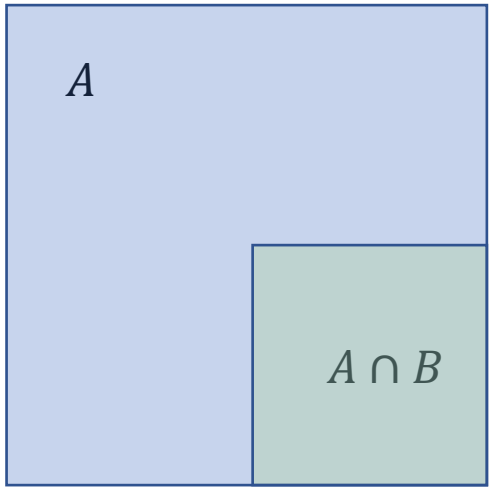
Overlapping Events: Independent Events

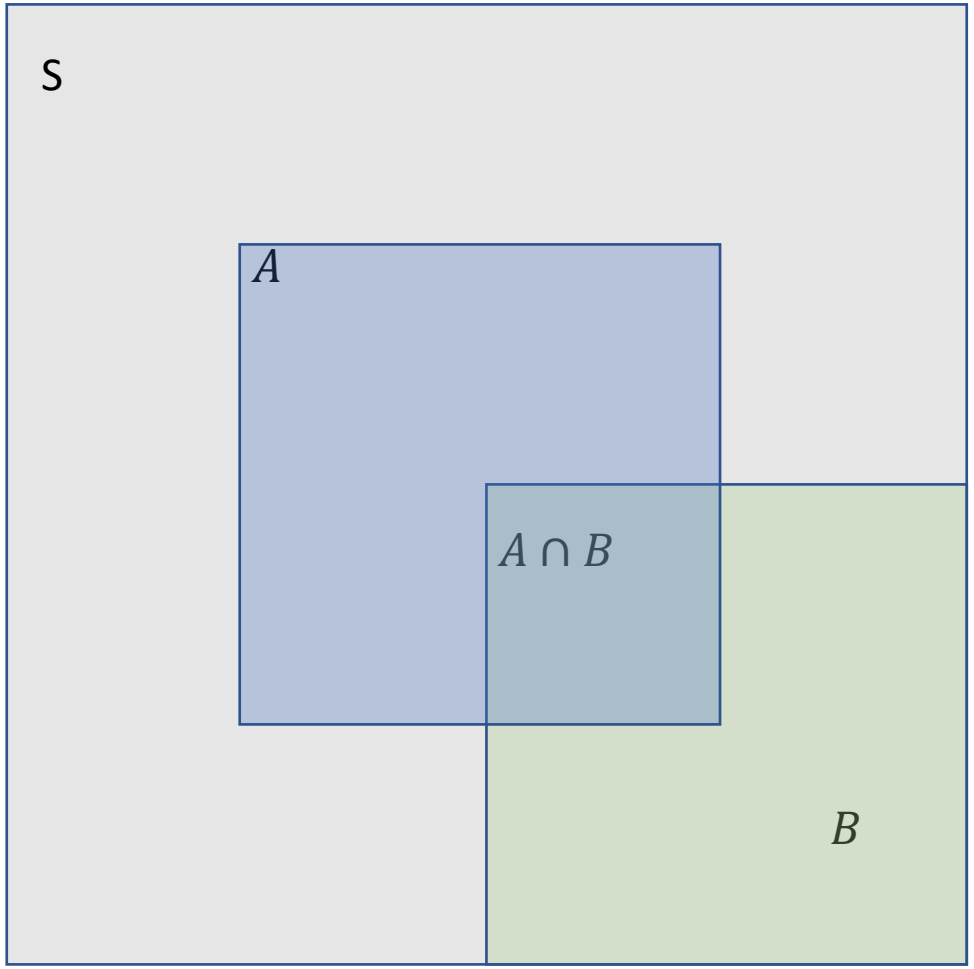
If events are independent, then:

$$\Pr(B|A) = \Pr(B)$$

In the figure, Pr(B) is 0.25

- A is ¼ of the sample space
- B is ¼ of the sample space
- $A \cap B$ is ¼ of A
- $A \cap B$ is 1/16 of the sample space





Conditional Probability: Key Points

- Sample Space
- Complementary Events
- Exclusive Events
- Overlapping Events
- To calculate a conditional probability, the sample space changes.
  - You ‘collapse’ the sample space into the conditioned event’s space