Requirements for achieving Strong Response Time Fairness

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Definition 1. *An ordering process O is strongly fair if it satisfies the following conditions,*

$$C1: \textit{If } A_i(k) < A_i(l), \textit{ then, } O(i,k) < O(i,l).$$
$$C2: \textit{If } f_i(k) = f_j(l) \wedge rt_i(k) < rt_j(l), \textit{ then, } O(i,k) < O(j,l).$$

Condition $C1$ states that a MP is always better-off submitting the trade order as early as possible. $C2$ states that trade orders generated based on the same market data point should be ordered based on the response time of the MPs.

Theorem 1. *The necessary and sufficient conditions on the delivery processes for strongly fair ordering are given by,*

$$D_i(x+1) - D_i(x) = D_j(x+1) - D_j(x), \qquad \forall i, j, x.$$

The theorem states that for strong fairness the inter-delivery times should be the same across all MPs. In other words, the delivery clocks at all RBs (at any given delivery clock time) must advance at the same rate.

Proof. Necessary: To prove that the above condition is necessary we will show that if this condition is not met then no ordering process exists which is strongly fair for arbitrary trade orders.

Consider the following scenario (Fig. 1) where the above condition is not met. Let $D_i(x+1) - D_i(x) = c1$, $D_j(x+1) - D_j(x) = c2$. Without loss of generality we assume $c1 < c2$. Consider hypothetical trades (i,k) and (j,l) s.t. $A_i(k) = D_i(x+1) + c3$ and $A_j(l) = D_j(x+1) + c4$. Further, we can pick $A_i(k)$ and $A_j(l)$ s.t. $c3 > c4$ and $c1 + c3 < c2 + c4$. Now we consider two scenarios for how these trades were generated. These two scenarios are indistinguishable at the OB.
Case 1: $f_i(k) = f_j(l) = x + 1$. Here,

$$rt_i(k) = c3, rt_j(l) = c4. \tag{1}$$

Since $c3 > c4$, a strongly fair ordering (condition $C2$) must satisfy, $O(i,k) > O(j,l)$.
Case 2: $f_i(k) = f_j(l) = x$. Here,

$$rt_i(k) = c1 + c3, rt_j(l) = c2 + c4. \tag{2}$$

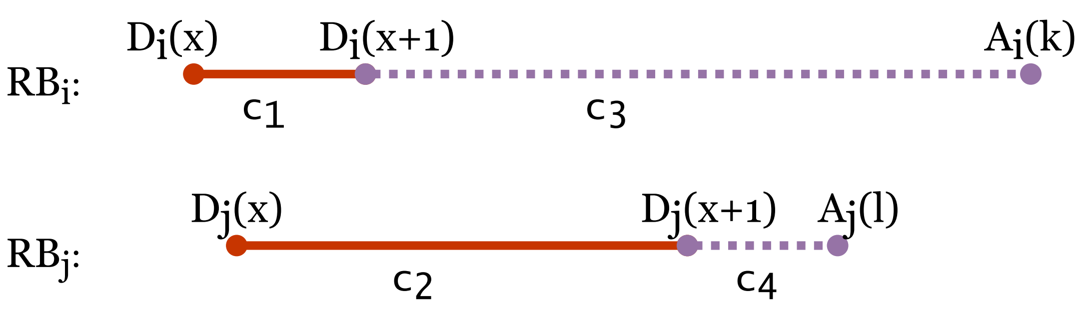

Figure 1: **Proof of Theorem 1.**

In this case, since $c1+c3 < c2+c4$, a strongly fair ordering must instead satisfy the opposite, $O(i, k) < O(j, l)$. A contradiction! Thus, no ordering process can be strongly fair in both these scenarios.

Sufficient: We will now show that if the inter-delivery times are same across MPs, then a strongly fair ordering exists.

Assuming same inter-delivery times, DBO trivially satisfies $C1$. DBO also satisfies $C2$, i.e., if $f_i(k) = f_j(l) \wedge rt_i(k) < rt_j(l)$, then,

$$DC_i(A_i(k)) < DC_j(A_j(l)). \tag{3}$$

Intuitively, this is because market data $f_i(k)(= f_j(l))$ is delivered to each MP at the same delivery clock time (by definition). Further, delivery clocks advance at the same rate across all MPs. When measured from the delivery of $f_i(k)(= f_j(l))$, delivery clock of RB_j in duration $rt_j(l)$ will advance more than delivery clock of RB_i in duration $rt_i(k)$. Therefore, DBO is strongly fair.[1] $\square$

[1] Note that, DBO is not the only ordering process that can achieve strong fairness. Other ordering processes that also order trades from MPs based on when they received the market data can also achieve strong fairness.