EINDHOVEN UNIVERSITY OF TECHNOLOGY

Masters Systems and Control
Control Systems Research Group
Department of Electrical Engineering

Investigation and Implementation of Direct Data-Driven Controllers on a Motion System

Internship Report

Eashwar Ravikkumar (1777033)

Supervisors:
ir. Luuk M. Spin, Dr. ir. Roland Tóth, Dr. ir. Valentina Breschi
Dr. ir. Dragan Kostić

11th November, 2023

Investigation and Implementation of Direct Data-Driven Controllers on a Motion System

Eashwar Ravikkumar (1777033)
Masters Systems and Control
Eindhoven University of Technology

***Abstract*—This report is a study of direct data-driven control strategies for input-output data assessing their efficacy and robustness on a motion system. An extended state framework is used to design data-driven controllers to ensure closed-loop stability of the system for noiseless data. It is then extended to consider noisy output data using a matrix S-procedure. For introducing performance specifications, a generalized plant framework based on behavioral theory is used. This is done using dissipativity analysis to ensure stability and performance by minimizing the l_2 gain of the system. The practical effectiveness of these control strategies are tested using a simulation model of the motion system.**

***Index Terms*—Direct data-driven control, Linear Matrix Inequalities (LMIs), Robust Control**

I. Introduction

Systems and control theory addresses the challenge of making a physical system behave according to certain desired specifications using a controller. The first step in addressing this challenge is to obtain a model of the system, either through first principles modeling or identification from data (also called system identification) [1]. The second step is to design a model-based controller using methodologies such as zero-pole placement, LQR design, lyapunov-based controller designs and feedback linearization. In particular, system identification followed by model-based control is an instance of data-driven control. Hence methods based on this combination can be classified as indirect-data driven control [2].

The model-based controller design assumes that the obtained model accurately represents the true system. But real systems may involve unmodeled dynamics, making the closed-loop system more prone to disturbances or even instability. This drives the need for robust control theory where model errors can be considered in various ways like additive and multiplicative descriptions and assumptions on priori bounds on noise or modeling errors or uncertainties [3]. However, these descriptions may not suffice for practical implementation, limiting the application of model-based robust control. Additionally, complex production processes in industries like semiconductors, chemicals, and aerospace make model development challenging and costly. Furthermore, more complex the model of the system, more is the effort or cost of designing a model-based controller. To address these limitations, direct data-driven control theory is considered, involving design of control laws directly from data without requiring an explicit mathematical model. This approach circumvents the necessity of modeling and attempts to eliminate the trade-off between informativity and complexity. Comparison between the direct and indirect data-driven control methods can be found in [4] and [5].

When working with data, an important question is how can one determine whether the obtained data contains enough information for designing control laws, i.e., is the data informative? For direct-data driven analysis, this is answered by Willems et al. in [6] and the result is popularly known as the *Fundamental Lemma*. This result has been used to perform data-driven analysis of system-theoretic properties of a *discrete-time linear time-invariant* (LTI) system in [7] and to represent its closed-loop behavior in [8]. The fundamental lemma has been utilized for data-driven control using input-output in [8] for designing data-based stabilizing controllers. It has also been used for designing optimal controllers in [8], [9], and [10]. A comprehensive survey of the existing results using the fundamental lemma have been highlighted in [12].

In these methods, the data utilized for analysis is input-output data. However, defining a state helps to easily establish stability and performance guarantees due to the extensive research available about state-space representations [14]. Hence, an alternative approach to handle the input-output data in the form of states is required. A general strategy is to create an artificial (or extended) state consisting of the shifted input-output measurements and hence consider an extended state-space representation for data-driven control. This has been proposed in [8] for noiseless data and extended in [10] for noisy data for data-driven stabilization. A preliminary aim of this work is to investigate the design of stabilizing data-driven controllers based on the extended state approach for a motion system.

The motion system considered for the direct data-driven analysis and controller implementation is a general motion stage exhibiting three-dimensional motion along the x,y and z axes. It is an example of a highly complex system where system modeling is challenging due to model inaccuracies and high cost (in terms of time, effort and/or money), thus motivating the application of direct data-driven control techniques. The motion stage consists of a moving end effector whose position is determined by a pre-defined trajectory. This motivates the primary goal of the control procedure, which is to position the end effector with the desired specifications, like tracking error and bandwidth. This objective is addressed by considering a generalized plant approach based on behavioral theory [13]. A generalized plant [14] consists of all the components of the system and control configuration except the controller. The controller and generalized plant are connected by defining an interconnection structure called a *Linear Fractional Representation* (LFR) [15]. This approach is a powerful tool which presents a systematic framework to define controllers and incorporate performance specifications. This systematic approach for ensuring system performance is investigated in this work and its applications for the motion system are explored.

The rest of this report is organized as follows. Section II deals with problem formulation for data-driven analysis and control. Section III explores data-driven stabilization for noiseless data and data in presence of measurement noise. This is carried out in an extended state framework. A generalized plant approach using behavioral theory is described in Section IV, which allows for defining performance specifications as weighting filters while considering input-output data directly. The results obtained from the implementation of these control strategies on a motion system simulator are discussed and analysed in Section V. The conclusions of this study are highlighted in Section VI.

A. Notation

The set of real numbers and non-negative integers are denoted by $\mathbb{R}$ and $\mathbb{N}$, respectively. A sequence of samples $\begin{bmatrix} z(0) & z(1) & \cdots & z(T-1) \end{bmatrix}$ of length $T \in \mathbb{N}$ such that $z(k) \in \mathbb{R}^{n_z}$ is represented as $z_{[0,T-1]}$. A column vector representation of $z_{[0,T-1]}$ is defined as $\mathrm{col}(z(0),\ z(1),\ \cdots,\ z(T-1))$. The matrix $A_L^v(z(k))$ represents $A_L^v(z(k)) = \begin{bmatrix} I_v & 0_{v\times(L-v)} \end{bmatrix}$, with I_v representing an identity matrix with of order v and $0_{v\times(L-v)}$ representing a matrix of zeros of order$(v \times (L-v))$. The notation $A^\perp \in \mathbb{R}^{m\times n}$ denotes a matrix where the columns span the null-space of A, i.e., $AA^\perp = 0_{m\times n}$. The Hankel matrix of length L associated with the sequence of samples $z_{[0,T-1]}$ is defined as

$$\mathcal{H}_L(z) = \begin{bmatrix} z(0) & z(1) & \cdots & z(T-L) \\ z(1) & z(2) & \cdots & z(T-L+1) \\ \vdots & \vdots & \ddots & \vdots \\ z(L-1) & z(L) & \cdots & z(T-1) \end{bmatrix}. \quad (1)$$

A matrix valued polynomial ring is defined as $\mathbb{R}^{n\times m}$ with ξ being an indeterminate variable. For $Z(\xi) = \Sigma_{i=0}^{l} Z_i \xi^i \in \mathbb{R}^{n\times m}$, the upper-Toeplitz matrix $\mathcal{T}_L(Z) \in \mathbb{R}^{(L-l)n\times Lm}$ of length L can be defined as

$$\mathcal{T}_L(Z) = \begin{bmatrix} Z_0 & Z_1 & \cdots & Z_l & 0 & \cdots & 0 \\ 0 & Z_0 & Z_1 & \cdots & Z_l & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & Z_0 & Z_1 & \cdots & Z_l \end{bmatrix}. \quad (2)$$

II. Problem Setting

Consider an unknown system for which only the input given to the system and the output response is known. Hence the measured input-output data is the only information known about the system, which necessitates data-driven analysis and control. In this work, the unknown system is considered as a motion system that positions an end effector according to a desired trajectory. It is emphasized that the terms *motion system* and *system* are used interchangeably. Also, the motion stage is only an example considered in this work and this framework can be extended to other examples of systems where data is available for control.

To simplify the investigation of data-driven analysis and control, a single-input-single-output (SISO) system is considered instead of the entire multiple-input-multiple-output (MIMO) system. The SISO system is designated by the X-axis channel of the motion system. The essential reason for a SISO setting is that the coupling of system dynamics which happens in a MIMO setting need not be considered. Hence, the focus of the data-driven analysis in this work is entirely on data-driven control.

The SISO system is considered as a discrete-time LTI system described by a difference equation ([8], [10])

$$\begin{aligned} y(k) + a_1 y(k-1) + \ldots + a_l y(k-l) \\ = b_0 u(k) + b_1 u(k-1) + \ldots + b_l u(k-l), \end{aligned} \quad (3)$$

with the input represented as $u(k) \in \mathbb{R}$, the output as $y(k) \in \mathbb{R}$, and l the lag (or the order) of the system [16]. The lag is generally unknown which necessitates the selection of an estimated value larger than the actual system lag. From [8], the input-output data can be used to construct an artificial (or extended) state

$$\begin{aligned} \chi(k) = \mathrm{col}(y(k-1),\ y(k-2),\ \cdots,\ y(k-L), \\ u(k-1),\ u(k-2),\ \cdots,\ u(k-L)). \end{aligned} \quad (4)$$

Suppose that $T+L$ input-output data samples are collected which are given by $u_{[0,T+L-1]}$ and $y_{[0,T+L-1]}$ respectively. Defining that variables with the subscript d are the samples collected from an experiment, these samples can be represented in data matrices as

$$\begin{aligned} U_- &= [u_d(L) \;\; u_d(L+1) \;\ldots\; u_d(T+L)], \\ Y_- &= [y_d(L) \;\; y_d(L+1) \;\ldots\; y_d(T+L)], \\ \hat{\chi}_- &= [\chi_d(L) \;\; \chi_d(L+1) \;\ldots\; \chi_d(T+L)], \end{aligned} \quad (5)$$

with the first L samples used to construct the initial conditions and the next T samples used for control.

For any data-driven control methodology, it is necessary that this collected input-output data is informative. Data informativity represents the notion of the data containing sufficient information about the system dynamics so that the system can be controlled using data-driven methods. A primary condition for data informativity is that the input fed to the system for data collection be sufficiently exciting. This is defined by the notion of persistency of excitation.

Definition 1 ([8]). *A signal $u_{[0,T-1]} \in \mathbb{R}$ is persistently exciting of order L if its Hankel matrix $\mathcal{H}_L(u)$ has full rank.*

Considering a persistently exciting input, the Fundamental Lemma is stated for input/output data gathered from a SISO system as

Lemma 1 (Fundamental Lemma, [13]). *Consider the system 3. If the input sequence $u_{[0,T+L-1]}$ is persistently exciting of order $L+l$ with $T \geq L+l$, then the input/output data obtained from the system 3 is informative if*

$$\operatorname{rank}\left(\begin{bmatrix}\mathcal{H}_L(U_-)\\ \mathcal{H}_L(Y_-)\end{bmatrix}\right) = L + l. \tag{6}$$

The lemma is satisfied for $L \geq l$, and it implies that a data-driven non-parametric representation of the system can be formulated using these Hankel matrices. Furthermore, using the extended state 4, the rank condition 6 can be reformulated. Considering that the input sequence $u_{[0,T+L-1]}$ is persistently exciting of order $2L+1$ with $T \geq 2L+1$, the input/output data obtained from the system 3 is said to be informative if the rank condition (from [8])

$$\operatorname{rank}\left(\begin{bmatrix}\mathcal{H}_1(U_-)\\ \mathcal{H}_1(\hat{\chi}_-)\end{bmatrix}\right) = 2L + 1 \tag{7}$$

is satisfied. As the lag l of the system is often not known, the height of the Hankel matrix is usually taken larger than the lag (i.e., $L > l$). This is in accordance with the Fundamental Lemma to ensure that the data represents the entire system dynamics. However, the rank condition 7 is generally not satisfied for $L > l$ and hence the data will not sufficiently represent the system. This is a primary point to be investigated, i.e, if a controller can be designed for the motion system using the extended state approach.

Considering that the collected data is informative enough (according to the Fundamental Lemma), the first step towards data-driven control is to design a stabilizing controller. A static output feedback controller is introduced as a difference equation ([8], [10])

$$\begin{aligned} y^c(k) + c_1 y^c(k-1) + \cdots + c_n y^c(k-l) &= \\ d_1 u^c(k-1) + \cdots + d_1 u^c(k-l).& \end{aligned} \tag{8}$$

This controller is connected to the system 3 through the interconnection

$$u^c(k) = y(k), \quad y^c(k) = u(k), \quad k \geq 0. \tag{9}$$

As the end effector of the motion system is to be positioned according to a desired trajectory, it is also necessary to introduce performance specifications (through weighting filters) in the controller design procedure. Instead of using the extended-state approach, the input-output data is directly used for designing a controller by using LMI-based dissipativity analysis. The analysis is done for an interconnection of data-based (system) and model-based (controller and weighting filters) components. A block-diagram representation of the control configuration is shown in Figure 1. The signal $e(k)$ represents the tracking error. W_S and W_T are weighting filters used to shape the sensitivity and complementary loop transfers, respectively. The pair of signals $(w(k), z(k))$ represent the performance channel, with $w(k)$ representing a desired reference trajectory $r(k)$ such that $w(k) = r(k)$, and $z(k)$ defined as

$$z(k) = \begin{bmatrix}\tilde{e}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix}W_S e\\ W_T y\end{bmatrix} = \begin{bmatrix}-W_S\\ W_T\end{bmatrix} y + \begin{bmatrix}W_S\\ 0\end{bmatrix} w. \tag{10}$$

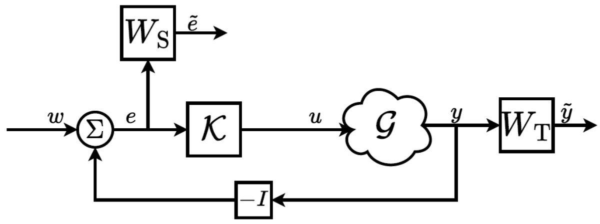

Fig. 1: Reference Tracking problem

Having defined such a representation, a generalized plant approach can be used for the interconnection of the model-based and data-based components, which facilitates this analysis.

A. Problem Statement

Consider the system 11 representing a motion system. This work explores the applicability of data-driven control techniques for ensuring stability and performance for the considered motion system. Firstly, an extended state approach is analyzed for data-driven stabilization of the system. Secondly, incorporating performance specifications into the system is explored in a data-driven methodology.

III. Data-Driven Stabilization

The stability of a system is a fundamental pre-requisite for a system to operate in a desired manner. In this section, stabilization of the system with data-driven methods is discussed.

A. System Representation

Considering a compact representation for the system 3 as

$$A(q^{-1})y(k) = B(q^{-1})u(k), \tag{11}$$

with the functions $A(q^{-1}) \in \mathbb{R}^{1\times 1}$ and $B(q^{-1}) \in \mathbb{R}^{1\times 1}$ being polynomial matrices given by

$$\begin{aligned} A(q^{-1}) &= I + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_l q^{-l},\\ B(q^{-1}) &= b_0 + b_1 q^{-1} + b_2 q^{-2} + \cdots + b_l q^{-l}, \end{aligned} \tag{12}$$

The coefficients of these polynomials are grouped together

$$\begin{aligned} \bar{A} &= \begin{bmatrix}-a_1 & -a_2 & \cdots & -a_{l-1} & -a_l\end{bmatrix},\\ \bar{B} &= \begin{bmatrix}b_1 & b_2 & \cdots & b_{l-1} & b_l\end{bmatrix},\\ B_0 &= b_0. \end{aligned} \tag{13}$$

Using the extended state 4 with $L = l$, a state space representation of the system can be created as

$$\chi(k+1) = \mathcal{A}\chi(k) + \mathcal{B}u(k), \tag{14}$$

with the coefficients

$$\mathcal{A} = \left[\begin{array}{cc|cc} \multicolumn{2}{c|}{\bar{A}} & \multicolumn{2}{c}{\bar{B}} \\ \hline I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \end{array}\right], \quad \mathcal{B} = \begin{bmatrix} B_0 \\ \hline 0 \\ I \\ 0 \end{bmatrix}. \tag{15}$$

This extended state-space system of order $2l$ is non-minimal.

For controlling the extended state-space system 14, a static output feedback controller is considered by design. The controller given by 8 can be re-written in a compact form as

$$C(q^{-1})y^c(k) = D(q^{-1})u^c(k), \tag{16}$$

with the functions $C(q^{-1}) \in \mathbb{R}^{1\times 1}$ and $D(q^{-1}) \in \mathbb{R}^{1\times 1}$ being polynomial matrices given by

$$\begin{aligned} C(q^{-1}) &= I + C_1 q^{-1} + C_2 q^{-2} + ... + C_n q^{-l}, \\ D(q^{-1}) &= D_1 q^{-1} + D_2 q^{-2} + ... + D_n q^{-l}. \end{aligned} \tag{17}$$

The coefficients of the polynomials can be grouped as

$$\bar{C} = \begin{bmatrix} -c_1 & -c_2 & \cdots & -c_{l-1} & -c_l \end{bmatrix}, \tag{18}$$
$$\bar{D} = \begin{bmatrix} d_1 & d_2 & \cdots & d_{l-1} & d_l \end{bmatrix}. \tag{19}$$

Connecting the system and the controller using the interconnection defined in 9 results in $u(k) = \begin{bmatrix} \bar{D} & \bar{C} \end{bmatrix} \chi(k)$ and the state-space representation of the closed-loop system is given by

$$\chi(k+1) = \mathcal{A}_{cl}\chi(k) = \left[\begin{array}{cc|cc} \multicolumn{2}{c|}{\bar{A}+B_0\bar{D}} & \multicolumn{2}{c}{\bar{B}+B_0\bar{C}} \\ \hline I & 0 & 0 & 0 \\ \hline \multicolumn{2}{c|}{\bar{D}} & \multicolumn{2}{c}{\bar{C}} \\ \hline 0 & 0 & I & 0 \end{array}\right] \chi(k). \tag{20}$$

Hence, the controller 8 will stabilize the system 3 if all the eigenvalues of $\mathcal{A}_{cl}$ are inside the open unit disk. The first aim of this work, which is to find a data-driven stabilizing controller, is discussed in the following subsection.

B. Stabilization for Noiseless Data

In this subsection, the data-driven control approach for a stabilizing controller described in [8] is investigated. The data considered is noiseless input-output data as defined in II. An output feedback controller 8 which stabilizes the system 11 (resulting in the closed loop representation 20) can be designed according to the below theorem.

Theorem 1 (Stabilization for noiseless data, [8]). *If the rank condition given by 7 is satisfied, then the following properties hold.*

1) *The closed-loop system 20 has an equivalent parametric representation as,*

$$\chi(k+1) = (\hat{\chi_+})\mathcal{G}_{\mathcal{K}}\chi(k) \tag{21}$$

with $\mathcal{G}_{\mathcal{K}}$ a $T \times 2l$ matrix such that

$$\begin{bmatrix} \mathcal{K} \\ I_{2l} \end{bmatrix} = \begin{bmatrix} U- \\ \hat{\chi-} \end{bmatrix} \mathcal{G}_{\mathcal{K}} \tag{22}$$

and $\mathcal{K}$ is a vector consisting of the coefficients of the controller 8 given by,

$$\begin{aligned} \mathcal{K} = [d_1 \quad d_2 \quad \cdots \quad d_{n-1} \quad d_n \\ -c_1 \quad -c_2 \quad \cdots \quad -c_{n-1} \quad -c_n] \end{aligned} \tag{23}$$

2) *Any $T \times 2l$ matrix $\mathcal{Q}$ satisfying*

$$\begin{bmatrix} \hat{\chi}_-\mathcal{Q} & \hat{\chi}_+\mathcal{Q} \\ \mathcal{Q}^\top \hat{\chi}_+^\top & \hat{\chi}_-\mathcal{Q} \end{bmatrix} > 0 \tag{24}$$

results in a controller 8 with coefficients given by

$$\mathcal{K} = U_-\mathcal{Q}(\hat{\chi}_-\mathcal{Q})^{-1} \tag{25}$$

stabilizes the system 20. Conversely, the system 11 can be stabilized by any controller 8 with its coefficients given by 25 if there exists a matrix $\mathcal{Q}$ which is a solution of 24.

This theorem is an important condition for designing a stabilizing controller for noiseless data. However as discussed in II, the extended state rank condition 7 is generally not satisfied when $L > l$. By comparison with the Fundamental Lemma 1, even when the input-output data is informative, the rank condition indicates that a controller cannot be designed without the knowledge of the exact lag l of the system. This is a significant drawback of this approach, as system identification is needed for accurately identifying the system lag, which defeats the purpose and motivation behind direct data-driven control. Additionally, as the extended state-space is non-minimal, a data-driven non-parametric system representation cannot be obtained for such cases according to the Rouché-Capelli theorem [8]. This means that Theorem 1 will not hold true.

Furthermore, noiseless data is a theoretical assumption as a real system is always subjected to noise (for example, measurement noise). However, the rank condition of the Fundamental Lemma 1 will never be satisfied in the presence of noise as the Hankel matrices will always have full rank as compared to the reduced rank requirement of the considered rank condition. This motivates the need for another method which ensures stabilization for noisy data.

C. System Representation for Noisy Data

This subsection aims to explore the stabilization of a system in the presence of noise using the notion of an extended state. Consider the system 3 to include noise dynamics and hence can be represented as an auto-regressive (AR) system

$$\begin{aligned} &y(k) + a_1 y(k-1) + ... + a_l y(k-l) \\ &\quad = b_0 u(k) + b_1 u(k-1) + ... + b_l u(k-l) + d(k), \end{aligned} \tag{26}$$

where $d(k) \in \mathbb{R}$ is a random variable representing the unknown noise.

Using the extended state representation 4, the state space representation is given by

$$\chi(k+1) = \mathcal{A}\chi(k) + \mathcal{B}u(k) + \mathcal{N}d(k), \tag{27}$$

with known matrices $\mathcal{A}$, $\mathcal{B}$ defined in 15 and

$$\mathcal{N} = \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{28}$$

The system 27 is considered to be stabilized by the output feedback controller 16. The state-space matrices $\mathcal{A}$ and $\mathcal{B}$ can be partitioned as

$$\mathcal{A} = \begin{bmatrix} \bar{A} & \bar{B} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ I_{p(l-1)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I_{m(l-1)} & 0 \end{bmatrix} = \Lambda_e + J_1,$$
$$\mathcal{B} = \begin{bmatrix} B_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I_m \\ 0 \end{bmatrix} = B_e + J_2. \tag{29}$$

Such partitioning of the state-space representation helps to separate the unknown parameter matrices Λ_e and B_e and binary matrices J_1 and J_2 of known dimensions. Since the input-output data is measured in the presence of measurement noise, noise bounds can be defined by considering an assumption on the measurement noise.

D. Assumption on Noise

Consider a noise data matrix defined as

$$N_- = [d(0) \;\; d(1) \; \dots \; d(T-1)]. \tag{30}$$

It is important to note that the noise data samples are not measured and hence the noise data matrix 30 is not known. The following assumption on the bound of the noise samples holds.

Assumption 1. *(see [9]) The noise samples collected in N_- satisfy the bound*

$$\begin{bmatrix} I \\ N_-^\top \end{bmatrix}^\top \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^\top & Q_{22} \end{bmatrix} \begin{bmatrix} I \\ N_-^\top \end{bmatrix} \geq 0, \tag{31}$$

for known matrices $Q_{11} = Q_{11}^\top$, Q_{12} and $Q_{22} = Q_{22}^\top < 0$.

An important point to be considered in Assumption 1 is that since Q_{22} is negative definite, the noise matrix N_- will satisfy the bound given by 31. Furthermore, considering a special case $Q_{11} = TH_u > 0, Q_{12} = 0$ and $Q_{22} = -I$ for some scalar $H_u > 0$, 31 can be reduced to the bounds given in 32, which implies that the energy of the noise signal is bounded in the considered finite time interval of length T.

$$N_- N_-^\top = \sum_{k=0}^{T-1} d(k)d(k)^\top \leq TH_u. \tag{32}$$

Having defined the noise bounds, it is also necessary to ensure that the (unknown) system parameter matrices are compatible with the data generated from the system by the following lemma.

Lemma 2 ([10]). *Let a noise matrix $\mathcal{Q}_e$ be defined as,*

$$\mathcal{Q}_e := \begin{bmatrix} \mathcal{N}Q_{11}\mathcal{N}^\top & \mathcal{N}Q_{12} \\ Q_{12}^\top\mathcal{N}^\top & Q_{22} \end{bmatrix}, \tag{33}$$

such that,

$$\begin{bmatrix} I & \mathcal{N}Y_- \\ 0 & -\hat{\chi}_- \\ 0 & -U_- \end{bmatrix} \mathcal{Q}_e \begin{bmatrix} I & \mathcal{N}Y_- \\ 0 & -\hat{\chi}_- \\ 0 & -U_- \end{bmatrix}^\top \geq 0. \tag{34}$$

Then, the set of all pairs of (Λ_e, B_e) 29 are compatible with the data such that 34 holds.

Having defined the assumption on the bounds of the noise samples, a data-driven controller which stabilized the system can be investigated.

E. Stabilization for Noisy Data

Using the system information and noise bounds, a controller which can stabilize the system 26 can be found according to the below definition.

Definition 2. *The data (U_-, Y_-) is informative for quadratic stabilization by the output-feedback controller 16 if there exist a $\mathcal{K}$ and $P > 0$ such that for any system matrices $\mathcal{A}$ and $\mathcal{B}$ compatible with the data,*

$$(\mathcal{A} + \mathcal{B}K)P(\mathcal{A} + \mathcal{B}K)^\top - P < 0. \tag{35}$$

Using the partitioning of matrices in 29, the stability condition 35 can be written as [10],

$$\begin{bmatrix} I \\ \Lambda_e^\top \\ B_e^\top \end{bmatrix}^\top \Pi \begin{bmatrix} I \\ \Lambda_e^\top \\ B_e^\top \end{bmatrix}, \tag{36}$$

with Π given by 37.

For solving the problem of the existence of $\mathcal{K}$ and $P > 0$ so that 36 holds true, the matrix-valued S-lemma [9] can be used.

Theorem 2 (Stabilization for Noisy data)**.** *(see [10]) The data (U_-, Y_-) are informative for quadratic stabilization by an output feedback controller 16 if there exist $L \in \mathbb{R}^{1\times 2l}, P > 0, \alpha \geq 0$ and $\beta > 0$ such that 38 holds true. For such informative data, the controller 16 can be designed with its coefficients given by $\mathcal{K} = LP^{-1}$.*

Theorem 2 and the condition 38 is sufficient for the design of stabilizing controllers for noisy output data. Additionally, if there is a matrix Z with $\bar{Z} := \mathrm{col}(I, Z)$ so that $\bar{Z}^\top \Lambda \bar{Z} > 0$, then by the generalized Slater's condition [9], the condition 38 is also a necessary condition for informativity of data for stabilization. However, the generalized Slater's condition is usually not satisfied due to the noise resulting in a singular matrix Λ. Hence, this condition is usually just a sufficient condition for stabilization.

$$\Pi = \begin{bmatrix} P-(J_1+J_2\mathcal{K})P(J_1+J_2\mathcal{K})^\top & -(J_1+J_2\mathcal{K})P & (J_1+J_2\mathcal{K})P\mathcal{K}^\top \\ -P(J_1+J_2\mathcal{K})^\top & -P & -P\mathcal{K}^\top \\ -\mathcal{K}P(J_1+J_2\mathcal{K})^\top & -\mathcal{K}P & -\mathcal{K}P\mathcal{K}^\top \end{bmatrix} \tag{37}$$

$$\begin{bmatrix} P-\beta I & -J_1P-J_2L & 0 & J_1P+J_2L \\ -PJ_1^\top - L^\top J_2^\top & -P & -L^\top & 0 \\ 0 & -L & 0 & L \\ PJ_1^\top + L^\top J_2^\top & 0 & L^\top & P \end{bmatrix} - \alpha \begin{bmatrix} I & \mathcal{N}Y_- \\ 0 & -\hat{\mathcal{X}}_- \\ 0 & -U_- \\ 0 & 0 \end{bmatrix}^\top \mathcal{Q}_e \begin{bmatrix} I & \mathcal{N}Y_- \\ 0 & -\hat{\mathcal{X}}_- \\ 0 & -U_- \\ 0 & 0 \end{bmatrix} \tag{38}$$

Having discussed stabilization (for noiseless and noisy data), it is important to also considered incorporating or improving the performance to match the desired specifications. This is discussed in the following section.

IV. Data-Driven Control for Performance

In the previous section, stabilizing controllers have been designed for both noiseless and noisy input-output data using the extended-state approach. However, there is a lack of sufficient methods to extend the analysis based on extended-state approach towards controller synthesis for performance. This motivates the necessity of a data-driven control strategy to address the problem. A solution considered in this work is to use the generalized plant approach to incorporate weighting filters using a mixed-sensitivity approach [22]. The control strategy involves defining finite-horizon dissipativity and, in particular, a finite-horizon l_2-gain. The main advantage is that the concept of a model-based generalized plant is well-defined and allows flexible interconnections to a data-driven representation. Thus, the data-driven analysis can be carried out using an already well-established control methodology.

A. Data-Driven Generalized Plant

A general representation of interconnections between data-driven and model-based representations is considered. In other words, interconnections defined as *Linear Fractional Transformations* (LFTs) are obtained by combining a model-based input-output representation with data-driven non-parametric representations provided by the Fundamental Lemma. These interconnections are represented by a *Linear Fractional Representation* (LFR). For the reference tracking problem described by 1, the corresponding LFR is shown in Figure 2.

The LFR can be explained as follows. The upper block $\mathcal{G}$ consists of all data-based components (the data-based system representation), the lower block $\mathcal{K}$ consists of all the model-based controller components, and the middle block $\mathcal{F}$ consists of the remaining control configuration and weighting filters. The blocks $\mathcal{G}$ and $\mathcal{F}$ combined constitute the upper-LFT $\mathcal{P}$ representing the generalized plant, while blocks $\mathcal{F}$ and $\mathcal{K}$ combined constitute the lower-LFT $\mathcal{M}$ representing the model-based components. The closed-loop interconnection is represented by $\mathcal{T}$. This is the advantage of using the generalized plant framework, as it assists in unifying model-based and data-driven representations into a single LFR.

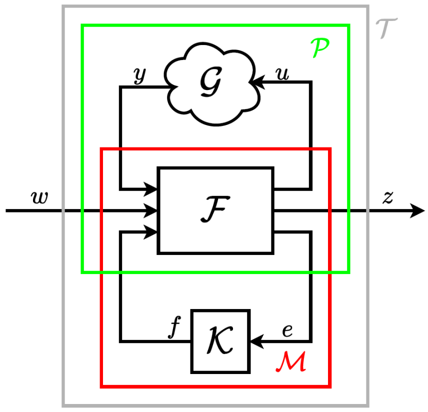

Fig. 2: LFR of the Generalized Plant

B. System Representation

Consider that the input-output behavior of the system, denoted by $\mathfrak{B}$, is defined as the collections of all possible trajectories that are admissible solutions of the system dynamics. It can be characterized as a kernel representation [13]

$$\mathcal{B} = \left\{ (u,y) \in (\mathbb{R}\times\mathbb{R})^{\mathbb{N}} | P(q) \begin{bmatrix} u \\ y \end{bmatrix} = 0 \right\}, \tag{39}$$

where $P(q^{-1}) \in \mathbb{R}^{1\times 2}$ is a polynomial matrix defined from 11 as

$$P(q^{-1}) = \Sigma_{i=0}^{l} P_i q^{-i} = [-B(q^{-1}) \quad A(q^{-1})]. \tag{40}$$

Here, l again represents the system lag. The behavioral approach is used to characterize a generalized plant representation which allows defining the desired performance.

C. Generalized Plant Interconnections

To obtain a model-based general control interconnection, consider the partitioned polynomial matrix in 40. For ease of representation, let the partitioning be defined as $P(q) = [-N(q) \ D(q)]$ with $N(q)$ and $D(q)$ implying the numerator and denominator terms respectively. Hence, $D(q)y = N(q)u$ defines an input-output representation of the system 11.

The lower model-based LFT $\mathcal{M}$ for the input-output representations can be defined by the below interconnection ([13], [20])

$$\mathcal{M} : \begin{cases} D_u^m(q)u = N_{uy}^m(q)y + N_{uw}^m(q)w, \\ D_z^m(q)z = N_{zy}^m(q)y + N_{zw}^m(q)w. \end{cases} \tag{41}$$

For the reference tracking problem, the loop transfers in 41 are

$$\begin{aligned} u &= [-K]y + [K]w, \\ z &= \begin{bmatrix} -W_S \\ W_T \end{bmatrix} y + \begin{bmatrix} W_S \\ 0 \end{bmatrix} w. \end{aligned} \tag{42}$$

Furthermore, the upper LFT representing $\mathcal{P}$ can be written in finite-horizon as $\mathcal{P}|_L$.

Lemma 3. *(see [13]) For any $w|_L \in \mathbb{R}^{Ln_w}$ and $z|L \in \mathbb{R}^{Ln_z}$, there exists a $g \in \mathbb{R}^{T-L+1}$ such that the finite-horizon plant $\mathcal{P}|_L$ can be defined as*

$$\begin{aligned} T_u^D \mathcal{H}_L(u_d)g &= T_{uy}^N \mathcal{H}_L(y_d)g + T_{uw}^N w|_L + T_{uf}^N f|_L, \\ T_z^D z|_L &= T_{zy}^N \mathcal{H}_L(y_d)g + T_{zw}^N w|_L + T_{zf}^N f|_L, \\ T_e^D e|_L &= T_{ue}^N \mathcal{H}_L(y_d)g + T_{ew}^N w|_L + T_{ef}^N f|_L. \end{aligned} \tag{43}$$

The finite-horizon plant $\mathcal{P}|_L$ is called a generalized plant if there exists a controller $\mathcal{K}$ such that the closed-loop interconnection $\mathcal{T}|_L$ is stable.

These results represents a general LFR-based interconnection framework between the data-driven and model-based representations of the system. The main motivation for defining this framework is to define the dissipativity of this representation, which is used for controller synthesis for performance.

D. Dissipativity Analysis

The notion of dissipativity can be defined for input-output representation of any dynamical system as follows.

Definition 3. *(see [21]) A system 11 with input $u : \mathbb{N} \to \mathbb{R}$, output $y : \mathbb{N} \to \mathbb{R}$ and behavior $\mathfrak{B}$ is dissipative with respect to a supply rate $\Pi \in \mathbb{S}^{2\times 2}$, if and only if*

$$\sum_{k=0}^{r} \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}^\top \Pi \begin{bmatrix} u(k) \\ y(k) \end{bmatrix} \geq 0, \forall r \in \mathbb{N}, \tag{44}$$

for all trajectories $(u, y) \in \mathfrak{B}$, with initial conditions $x_0 = 0$.

The supply rate can be defined as a quadratic supply function and is partitioned as

$$\Pi := \begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix}, \tag{45}$$

with $Q \in \mathbb{S}$, $R \in \mathbb{S}$ and $S \in \mathbb{S}^{1\times 1}$. Considering the analysis metric to be the upperbound γ on the l_2-gain gives $(Q, S, R) = (\gamma^2 I, 0, -I)$.

This classical definition of dissipativity can be extended and reformulated for finite-horizon trajectories which is the crux of the current discussion.

Definition 4. *A system 11 is said to be L-dissipative with respect to the supply rate Π if*

$$\sum_{k=0}^{L-1} \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}^\top \begin{bmatrix} I_L \otimes Q & I_L \otimes S \\ I_L \otimes S^\top & I_L \otimes R \end{bmatrix} \begin{bmatrix} u(k) \\ y(k) \end{bmatrix} \geq 0, \tag{46}$$

for all the trajectories $u_{[0,T-1]}$ and $y_{[0,T-1]}$ of the system 11.

Furthermore, the first $v \geq l + 1$ samples are considered as zero to ensure zero initial conditions. This is implemented by multiplying the Hankel matrices of the input and output signals by $V_L^v(u)$ and $V_L^v(y)$ respectively.

From Lemma 3, the finite-horizon closed-loop system $\mathcal{T}|_L$ can be defined as

$$\mathcal{T}|_L := \begin{cases} T_u^D \mathcal{H}_L(u_d)g &= T_{uy}^N \mathcal{H}_L(y_d)g + T_{uw}^N w|_L \\ T_z^D z|_L &= T_{zy}^N \mathcal{H}_L(y_d)g + T_{zw}^N w|_L. \end{cases} \tag{47}$$

This mixed representation has the advantage of unspecified finite-horizon generalized disturbance $w|_L$ and generalized performance $z|_L$, which means the dissipativity of the closed-loop system can be characterized for any arbitrary input-output pairs. The dissipativity of the closed-loop system can be verified through the below theorem.

Theorem 3 ([13]). *The closed-loop finite horizon system $\mathcal{T}|_L$ is L-dissipative with respect to the supply rate Π for*

$$(B^\perp)^\top \begin{bmatrix} 0 & 0 \\ 0 & \Pi \end{bmatrix} B^\perp \geq 0, \tag{48}$$

with

$$B = \begin{bmatrix} T_{uy}^N \mathcal{H}_L(y_d) - T_u^D \mathcal{H}_L(u_d) & T_{uw}^N & 0 \\ T_{zy}^N \mathcal{H}_L(y_d) & T_{zw}^N & -T_z^D \\ 0 & V_L^v(w) & 0 \\ 0 & 0 & V_L^v(z). \end{bmatrix} \tag{49}$$

This notion of L-dissipativity for a finite-horizon system can be used for the purpose of controller design. A controller can be obtained by minimizing the l_2-gain in the LMI 48 from Theorem 3. This results in a controller which ensures the desired performance specifications.

V. Simulation Example

In this section, the results obtained from the implementation of the control strategies illustrated previously are highlighted. Initially, the system used for generating the data is described. The various control strategies for stabilizing system in presence of noiseless and noisy data are examined. Also, performance specifications are defined through weighting filters and the data-driven control strategy is validated.

A. Simulation Environment

The X-axis channel of the motion stage is the SISO system considered for data generation along with data-driven analysis and control. The sampling frequency is $8kHz$ and hence a large number of data points can be collected within a short span of time. For the purpose of this study, the investigation of data-driven controllers is performed on a simulation environment on MATLAB/Simulink. The model consists of various coupled dynamics resulting in a nonlinear system. The input is the current along the X-axis while the output represents the position of the end effector measured using an encoder. A linearization of the nonlinear model is performed in Simulink to obtain a state-space representation of order 20. The real-time plant delays are reflected in the simulation through a delay of three samples ($0.125\ \mu sec$). This is considered in the state-space representation using `absorbDelay`, resulting in a representation of order 23. Furthermore, a balanced realization is considered to balance the energy transfers between input-state and state-output. The motivation is ensuring numerical stability and computational efficiency, since the controller design conditions are formulated as LMIs. Also, the reference trajectory of the motion system, which is considered for evaluating performance in time-domain, is shown in Figure 3.

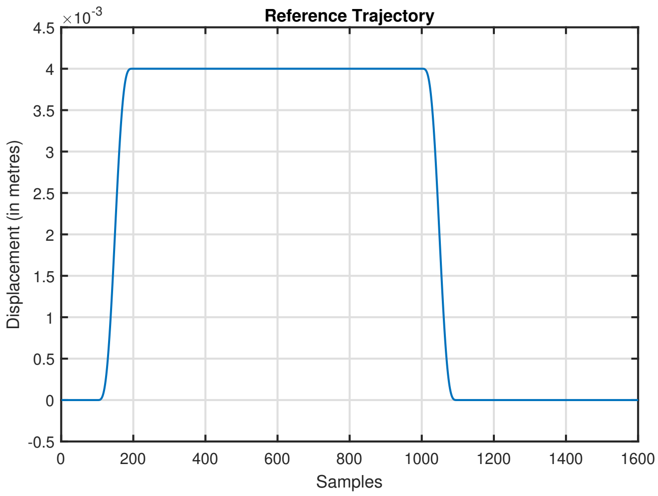

Fig. 3: Reference Trajectory

B. Model Reduction

For satisfying the rank condition of the Fundamental Lemma for the input-output data and the extended state representation, a minimal realization of the system is obtained using `minreal`. However, it is observed that the minimum realization is not an accurate representation of the system dynamics. This persists even after the tolerance is adjusted from the default value (from 1.49×10^{-8} to 10^{-6}). The difference in the dynamics are observed and highlighted through their respective Bode plots as shown in Figure 4.

To address this problem, a balanced realization is considered and model reduction is performed using balanced truncation. This results in a reduced order system of order 18 (denoted as G_x) and it is noted that the dynamics are being represented accurately by the reduced order system. In this analysis, the knowledge of the lag is considered as a known parameter (i.e. $l = 18$). The original (full-order) and reduced order systems are compared through their Bode plots as shown in Figure 5, which validates the use of the reduced order system G_x for data-driven analysis.

C. Data Collection

Having defined the data-generation system, the primary requirement for data-driven control is collecting informative data. This is done first by considering a persistently exciting input signal. There are various commonly used input signals,

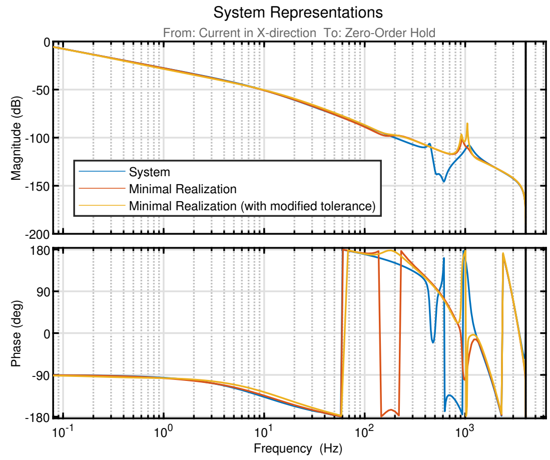

Fig. 4: System representations

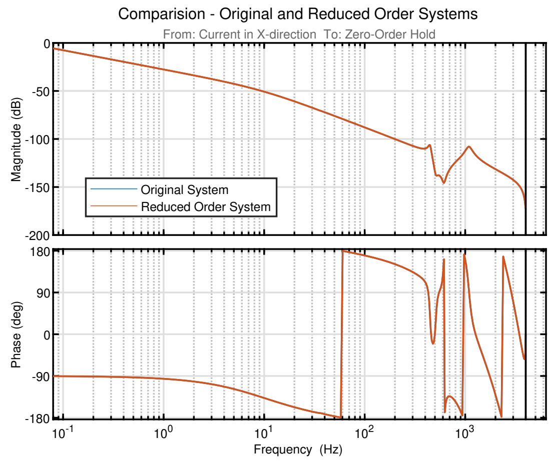

Fig. 5: Comparision - Original and Reduced Order systems

like gaussian white noise, colored noise, multisine, PRBS (Pseudo Random Binary Sequence) and sweep signals.

For simplicity, a gaussian white noise signal with zero-mean and unit standard deviation is considered as the input to the system, which ensures that the input is always persistently exciting with any desired order (of persistency of excitation). The input data samples and input power spectral density plots for 200 input data samples are shown in Figures 6 and 7. Various iterations of the experiment are performed by varying the number of collected data samples (T).

The system is excited with white noise for a period of 10 seconds and is sampled at a frequency of $8kHz$. The trailing T input-output data samples are collected. This ensures that the data is not affected by the transient response of the system. The data is normalized to ensure computational accuracy. As the normalization results in an increased gain of the system, this is adjusted by ensuring that the controller is denormalized. In the case of noisy output data, the measurement noise is

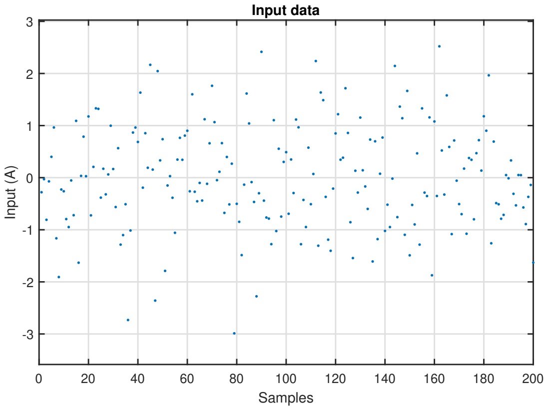

Fig. 6: Input Data Samples

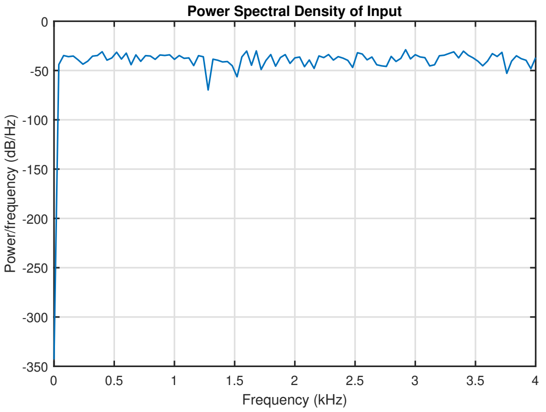

Fig. 7: Input Data - Power Spectral Density

modeled as white noise with RMS of 2×10^{-8}. This reflects the noise present in the simulator.

D. Stabilization for Noiseless Data

Consider the data-driven analysis for a stabilizing controller for noiseless data discussed in III-B. The data-generating system G_x is defined by 11 and the data is collected in the data matrices as discussed in II. An input sequence of length $T = 200$ is considered and the controller is designed based on the approach described in Theorem 1.

For the data to be informative, the input should be persistently exciting of order atleast $2l + 1$, which is ensured by using gaussian white noise with zero mean and unit standard deviation. Furthermore, the input-output data collected from the system should satisfy the rank condition 7. It is checked by varying the depth L of for the Hankel matrix $\mathcal{H}_1(\hat{\chi}_-)$ from 1 to 60, and comparing the rank of the obtained matrix with the desired rank as specified in 7. The resulting comparison plot is shown in Figure 8. The plot shows that the rank condition is only satisfied for $L \leq 14$ and it is not satisfied for $L > 15$. This implies that as the rank condition does not hold when L is taken as the lag $l = 18$, a stabilizing controller cannot be designed using the extended state approach (according to Theorem 1). Furthermore, it is not meaningful to consider $L < 18$ as the data is not a representation of the complete system dynamics.

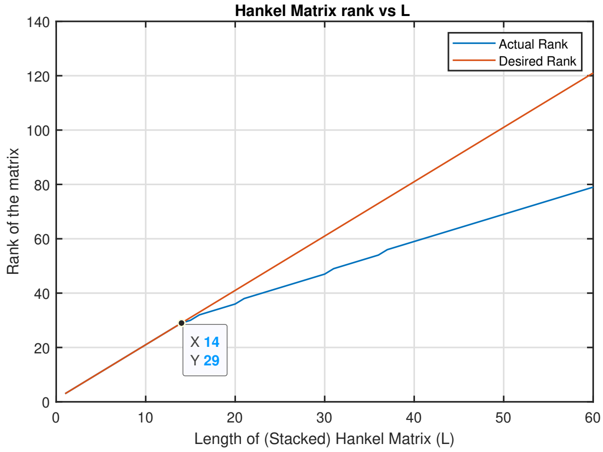

Fig. 8: Fundamental Lemma - Rank condition

These inferences are validated through simulations. It is observed that when L is considered to be the lag of G_x given by $l = 18$, a stabilizing controller is not obtained from the given input-output data. This is also verified through experiments for various values of T. The instability of the closed-loop system is observed from the infeasibility of the LMI 24. As the analysis is performed in a simulation environment with known information about the system matrices, a model-based stabilization test can also be performed. That is, the eigen values of the closed-loop state matrix $\mathcal{A}_{cl}$ (as given in 20) should lie within the unit circle. It is observed that some of the eigen values of the closed-loop system lie outside the unit circle, hence adding weight to the conclusion. The pole-zero plot of the closed-loop system for $T = 200$ and $L = 18$ is shown in Figure 9, which indicates an unstable closed-loop system. Hence, it can be concluded that this approach does not yield a stabilizing controller.

E. Stabilization for Noisy Data

In this subsection, noisy data is considered for designing a stabilizing output-feedback controller as discussed in III-E. The system G_x is again considered to be the data-generating system. However, the output data is affected by measurement noise modeled as white noise with RMS of 2×10^{-8}. The matrices Q_{11}, Q_{12} and Q_{22} in 31 are considered to be $Q_{11} = TH_u > 0, Q_{12} = 0$ and $Q_{22} = -I$ as defined in III-D. Here, T is the number of data samples collected in the experiment iteration and H_u is taken as 3×10^{-11} to satisfy 32.

An input sequence of length $T = 200$ is considered and a controller 16 is designed according to Theorem 2. It is observed that a stabilizing controller is not obtained and numerical problems are encountered while solving the LMI. To adjust for this, the tolerances of the strict inequality constraints are relaxed (from 10^{-8} to 10^{-10}). This results in the LMI becoming feasible. However the obtained controller still does not stabilize the closed loop system. This is consistently

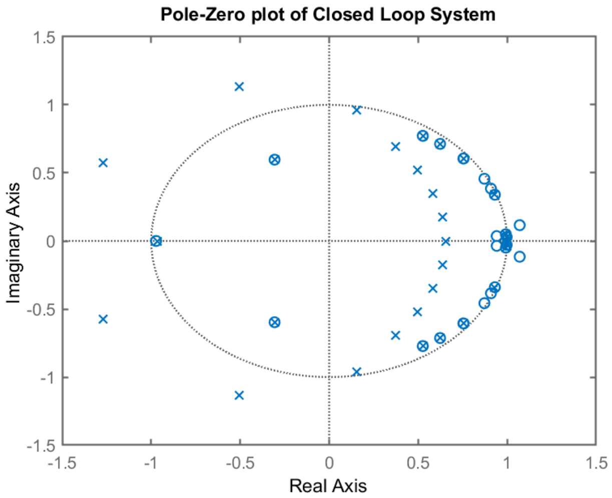

Fig. 9: Pole-Zero plot of Closed Loop system

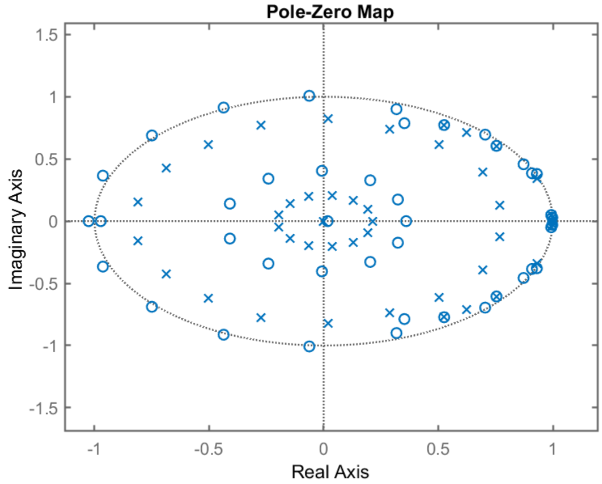

Fig. 10: Pole-Zero plot of Closed Loop system

observed even after varying the number of data samples (T) collected.

Through model-based stability tests, it is noted that the absolute value of the largest eigen values are very close to the unit circle. For the case of $T = 200$ and $l = 18$, the pole-zero plot of the closed-loop system is shown in Figure 10. It is remarked that there are zeros with a large value outside the unit circle and hence the plot is focused on the unit circle and its neighbouring region.

An added advantage of this method is the low computation time, i.e., it has a constant time which depends only l. For example, the average time taken to solve an iteration to obtain a stabilizing controller for $l = 18$ is 0.2 seconds, which increases to 1.7 seconds for $l = 28$.

A possible inference of these observations can be that the numerical errors cause the resulting instability of the closed-loop system. It can also be observed that the poles of the closed-loop system are symmetrical. These are points which can be investigated further as future work.

F. Synthesis for Performance

In this subsection, the results based on the implementation of the control strategy in IV are discussed. The input-output data is collected with white noise as input and the output data is measured without noise. This collected data is found to be informative as it satisfies the rank condition of the Fundamental Lemma 6. Also, the data is normalized and to adjust for this, the controller is denormalized, as discussed in V-C.

The desired performance specifications are a bandwidth of $W_B = 100Hz$ and a modulus margin (peak of the magnitude of the sensitivity function) $M_S < 6dB$. For incorporating these specifications, the weighting filters are designed as,

$$\begin{aligned} W_S &= \frac{0.6506q^2 - 1.2557q + 0.6121}{q^2 - 1.996q + 0.9965} \\ W_T &= \frac{25.52q - 24.15}{q + 0.9503} \end{aligned} \tag{50}$$

For the considered motion system, the controller hardware permits a controller representation of maximum sixth-order. Hence to design an implementable controller, a model-based fifth-order controller $\mathcal{K}$ is defined. This allows flexibility for an integrator to be additionally added if required. The controller is parameterized to ensure that it is stable by design

$$\mathcal{K}(q) = c\frac{(q - 0.92)(q - r_1)(q^2 - 2r_2q\cos(\theta_1) + r_2^2)}{(q - 1)(q - r_3)(q^2 - 2r_4q\cos(\theta_2) + r_4^2)}, \tag{51}$$

where c is the steady-state gain and $\{r_1, r_2, r_3, r_4, \theta_1, \theta_2\}$ are bounded parameters, such that $r_i \in [0, 0.998]\ \forall i \in \{1, 2, 3, 4\}$ and $\theta_j \in [0, \pi]\ \forall j \in \{1, 2\}$.

Assuming that the controller parameters are obtained by minimizing the finite-horizon l_2-gain of the closed-loop system, an optimization problem for finding the controller parameters can be defined as

$$\begin{aligned} &\text{minimize} \quad \gamma \\ &\text{subject to} \quad (B^\perp)^\top \begin{bmatrix} 0 & 0 \\ 0 & \Pi \end{bmatrix} B^\perp \geq 0. \end{aligned} \tag{52}$$

The optimization is done by a two-step process, parameter initialization and gradient-descent based local minima search. Firstly, good initialization conditions for the parameters of the controller $\mathcal{K}$ are defined for the optimization problem by using particle swarm optimization (PSO) [23]. These initial controller parameters are found between well-defined bounds, i.e., bounds which ensure that a stable controller is obtained by design. Then a gradient-descent based search is performed to obtain the controller parameters corresponding to the local

Algorithm 1 l_2-gain minimization algorithm

```
Set controller parameters bounds
Initialize stable = -1
while stable ≠ 1 do
    if stable = 0 then
        Increase PSO cost
    else
        Define cost as [52]
    end if
    Initialize controller parameters using PSO
    Gradient-descent based search for local minima
    Define controller using obtained parameters
    Formulate closed loop system
    Check closed loop stability
    if closed loop is stable then
        stable = 1
    else
        stable = 0
    end if
end while
```

minima. Using the obtained controller parameters, the controller [51] can be defined and closed-loop stability checked. However, it is observed that the obtained controller does not stabilize the system, which is checked using model-based tests. To address this problem, a modification is made where the additional cost is added to the cost function incase of unstable closed-loop system. This modification results in a stabilizing controller. This algorithm is shown in Algorithm 1.

Initially the number of data points and the length of the finite-horizon are considered as $T = 200$ and $L = 44$ respectively, with $v = 23$. A controller is designed as discussed in this subsection, but it does not stabilize the closed loop system. The suggested modification to the cost of PSO as described in Algorithm 1 is made. This assists in obtaining controller coefficients which stabilize the closed loop system. Furthermore, the number of data samples and length of finite-horizon were varied to compare the obtained results.

Considering $T = 500$ and $L = 50$, the pole-zero plot of the closed-loop system and the loop transfer (sensitivity, process sensitivity and open-loop) bode plots are shown in Figures 11 and 12 respectively. The modulus margin and bandwidth values can be obtained from the sensitivity function. Furthermore, the time-domain response to the reference trajectory is considered and an error profile is obtained as shown in Figure 13.

TABLE I: Analysis of Controller Performances

T	L	γ	M_S	B_w
500	50	5.4109	0.52	0.73
800	50	5.4098	0.72	1.06
800	200	57.43	2.39	3.60

To analyze the control strategy, the number of samples T and the finite-horizon L are varied and the values of l_2

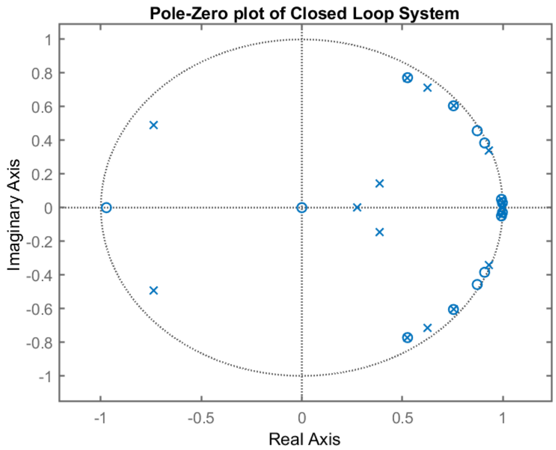

Fig. 11: Pole-Zero plot for T = 500 and L = 50

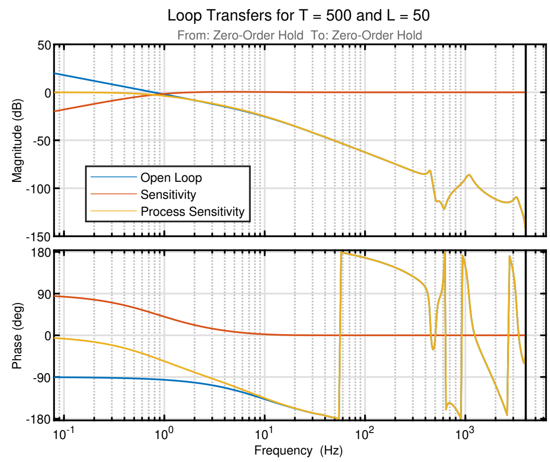

Fig. 12: Loop transfers for T = 500 and L = 50

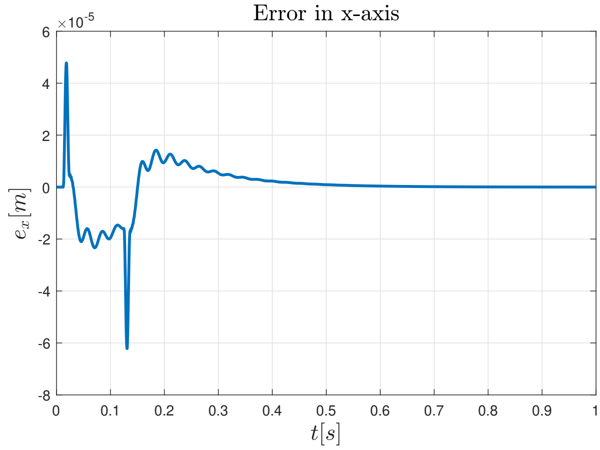

Fig. 13: Time domain response for T = 500 and L = 50

gain (γ), modulus margin (M_S) and bandwidth (B_w) are highlighted in Table I. This allows the inference that increasing L is a possible solution for improving the performance. Furthermore, the optimization process is a computationally intensive process whose complexity increases with increasing L. Hence, this limits the maximum value of L which is considered and this is a potential disadvantage of this approach. This problem can perhaps be addressed by taking advantage of the sparse structure of the various matrices used in the cost function to optimize the computation speed. It is also noted that the weighting filters are minimally tuned, and hence further tuning with stricter specifications can improve the obtained performance. The main problem with this approach is the issue of instability of the closed-loop system, which required the modification to be made to the cost function. Furthermore, when an integrator was considered in the controller formulation, the optimization algorithm was not able to find appropriate initialization conditions. This also is an interesting point for future investigation.

VI. Discussion and Conclusion

In this work, a study of data-driven control strategies and their applicability to a SISO motion stage has been conducted. Informativity conditions on the measured input-output data have been established, which are necessary to ensure that the data completely represents the system.

Data-driven stabilizing controllers have been examined in an extended state framework while considering both noiseless and noisy data obtained from persistently exciting input data. For the case of noiseless data, the rank condition on the extended state indicating data informativity does not hold, even when the collected input-output data is sufficiently informative. This is attributed to the non-minimality of the extended state-space representation. Therefore, a stabilizing controller cannot be designed for this case. Furthermore, this approach also has the additional disadvantage of failing in the case of noisy data. Hence, an alternative procedure involving definition of noise bounds and application of the matrix S-lemma is considered for stabilization. This method is also observed to not result in any stabilizing controllers. However, in contrast with the unsatisfactory rank condition in the previous case, the current method does not work due to numerical errors, which is an addressable problem for future work.

The lack of existing methods to incorporate performance specifications based on the extended state motivates a shift to use the input-output data directly for controller synthesis. To do so, an LFR is used to represent generalized interconnections between the data-driven representation (of the motion system) and model-based control configuration (consisting of the controller and weighting filters). LMI-based results have been used for dissipativity analysis and are extended for synthesizing a controller with guaranteed performance. Modifications are made to the cost function used for the dissipativity analysis which help in achieving stability. However it is observed that the desired performance specifications still need to be attained. Possible solutions include tuning the weighting filters and increasing the data points and finite-horizon length.

References

[1] L. Ljung, System Identification: Theory for the User (Prentice Hall Information and System Sciences Series). Englewood Cliffs, NJ, USA: Prentice-Hall, 1999.

[2] H. J. van Waarde, J. Eising, H. L. Trentelman, and M. K. Camlibel. Data informativity: a new perspective on data-driven analysis and control. IEEE Transactions on Automatic Control, 65(11):4753–4768, 2020

[3] Z. Hou and Z. Wang, “From model-based control to data-driven control: Survey, classification and perspective,” Inf. Sci., vol. 235, pp. 3–35, 2013.

[4] V. Krishnan and F. Pasqualetti, ”On Direct vs Indirect Data-Driven Predictive Control,” 2021 60th IEEE Conference on Decision and Control (CDC), Austin, TX, USA, 2021, pp. 736-741

[5] F. Dörfler, J. Coulson and I. Markovsky, ”Bridging Direct and Indirect Data-Driven Control Formulations via Regularizations and Relaxations,” in IEEE Transactions on Automatic Control, vol. 68, no. 2, pp. 883-897, Feb. 2023

[6] J. C. Willems, P. Rapisarda, I. Markovsky, and B. L. De Moor, “A note on persistency of excitation,” Systems & Control Letters, vol. 54, no. 4, pp. 325–329, 2005.

[7] H.J. van Waarde, J. Eising, H. L. Trentelman, and M. K. Camlibel, “Data informativity: A new perspective on data-driven analysis and control,” IEEE Trans. Autom. Control, vol. 65, no. 11, pp. 4753–4768, Nov. 2020.

[8] C. De Persis and P. Tesi, “Formulas for data-driven control: Stabilization, optimality, and robustness,” IEEE Transactions on Automatic Control, vol. 65, no. 3, pp. 909–924, 2019. Suhl, Eds. New York: Academic, 1963, pp. 271–350.

[9] H. J. van Waarde, M. K. Camlibel, and M. Mesbahi, “From noisy data to feedback controllers: Nonconservative design via a matrix S-lemma,” IEEE Trans. Autom. Control, vol. 67, no. 1, pp. 162–175, Jan. 2022,

[10] T. Steentjes, M. Lazar, and P. Van den Hof, “On data-driven control: Informativity of noisy input-output data with cross-covariance bounds,” IEEE Control Systems Letters, vol. 6, pp. 2192–2197, 2022.

[11] J. Berberich, A. Koch, C. W. Scherer, and F. Allgower, “Robust data-driven state-feedback design,” in 2020 American Control Conference (ACC), 2020, pp. 1532–1538.

[12] I. Markovsky and F. Dorfler, “Behavioral systems theory in data-driven analysis, signal processing, and control,” Annual Reviews in Control, vol. 52, pp. 42–64, 2021.

[13] L. M. Spin, C. Verhoek, W. P. M. H. Heemels, N. van de Wouw and R. Tóth, ”Unified Behavioral Data-Driven Performance Analysis: A Generalized Plant Approach,” Submitted to the 62nd IEEE Conference on Decision and Control

[14] K. Zhou and J. C. Doyle, Essentials of robust control. Prentice hall Upper Saddle River, NJ, 1998, vol. 104

[15] R. Redheffer, “On a certain linear fractional transformation,” Journal of Mathematics and Physics, vol. 39, no. 1-4, pp. 269–286, 1960

[16] J. C. Willems, “Paradigms and puzzles in the theory of dynamical systems,” IEEE Transactions on Automatic Control, vol. 36, no. 3, pp. 259–294, 1991.

[17] D. J. Hill and P. J. Moylan, “Dissipative dynamical systems: Basic input-output and state properties,” Journal of the Franklin Institute, vol. 309, no. 5, pp. 327–357, 1980.

[18] J. C. Willems, “Dissipative dynamical systems part i: General theory,” Archive for rational mechanics and analysis, vol. 45, no. 5, pp. 321–351, 1972.

[19] J. C. Willems, “Dissipative dynamical systems part ii: Linear systems with quadratic supply rates,” Archive for rational mechanics and analysis, vol. 45, pp. 352–393, 1972.

[20] M. Vidyasagar, “Control system synthesis: a factorization approach, part ii,” Synthesis lectures on control and mechatronics, vol. 2, no. 1, pp. 1–227, 2011.

[21] D. J. Hill and P. J. Moylan, “Dissipative dynamical systems: Basic input-output and state properties,” Journal of the Franklin Institute, vol. 309, no. 5, pp. 327–357, 1980.

[22] S. Skogestad and I. Postlethwaite, Multivariable feedback control: analysis and design. John Wiley & sons, 2005.

[23] J. Kennedy and R. Eberhart, ”Particle swarm optimization,” Proceedings of ICNN’95 - International Conference on Neural Networks, Perth, WA, Australia, 1995, pp. 1942-1948 vol.4