DESIGN OF A ROBUST GUIDANCE STRATEGY FOR TARGET CAPTURE

A thesis submitted in partial fulfilment of the requirements for the award of the degree of

B.Tech.

in

Instrumentation and Control Engineering

By

Eashwar Ravikkumar (110118026)

Completed at the

Distributed Control and Decision Laboratory, IIT Kanpur, India

INSTRUMENTATION AND CONTROL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

TIRUCHIRAPALLI-620015

MAY 2022

BONAFIDE CERTIFICATE

This is to certify that the project titled **DESIGN OF A ROBUST GUIDANCE STRATEGY FOR TARGET CAPTURE** is a bonafide record of the work done by

Eashwar Ravikkumar (110118026)

in partial fulfilment of the requirements for the award of the degree of **Bachelor of Technology in Instrumentation and Control Engineering of the NATIONAL INSTITUTE OF TECHNOLOGY, TIRUCHIRAPPALLI**, during the year **2021-2022.**

Dr. K. Srinivasan	**Dr. Twinkle Tripathy**	**Dr. K. Dhanalakshmi**
Internal Guide	External Guide	Head of Department
Associate Professor	Associate Professor	Professor
NIT Trichy	IIT Kanpur	NIT Trichy

ABSTRACT

The aim of this project is to design a suitable guidance law which will ensure target capture. Guidance refers to the process of guiding a pursuer towards a target. In this report, a classical guidance law - deviated pure pursuit guidance law, is implemented by defining an appropriate geometrical rule, which is a rule defined from mathematics allowing us to guide any pursuer. This ensures control of the motion of a pursuer towards a stationary target and guarantee target capture. Additionally, an additional constraint – impact time, has been introduced, improving the target capture mechanism by allowing the target interception at a desired time. A robust control strategy – the Sliding Mode Controller (SMC) has been studied and designed to implement the theoretical framework while rejecting external disturbances. Simulation is essential as it is carried out before physical deployment of systems to improve cost and system efficiency, and the validation of the framework is performed using MATLAB.

Keywords: Target capture; Guidance; Geometrical Rule; Sliding Mode Controller; MATLAB

ACKNOWLEDGEMENTS

I wish to express my sincere thanks to my research guides, Dr. K. Srinivasan and Dr. Twinkle Tripathy for providing me with the expertise and knowledge for the implementation of the project work and for having faith in our aptitude through this entire period. I would also like to extend my gratitude to the Project Review Committee consisting of Mr. Goldin R Bennet, Dr. C. Geetha, and Dr. N. Sivakumaran for ensuring a seamless progression of the project. Finally, I wish to thank the Head of Department, Dr. K. Dhanalakshmi for her support, without which the project would never see completion.

TABLE OF CONTENTS

ABSTRACT 3

ACKNOWLEDGEMENTS 4

LIST OF FIGURES 7

LIST OF TABLES 8

CHAPTER 1 – INTRODUCTION 9

1.1 Guidance 9
1.2 Robust Controller 10
1.3 Sliding Mode Controller 10
1.4 Necessity of Sliding Mode Controller 11

CHAPTER 2 – LITERATURE REVIEW 13

CHAPTER 3 – GUIDANCE METHODOLOGY 15

3.1 System Kinematics 15
3.2 Conditions for Guaranteed Target Capture 16
3.3 Guidance Law Design 16
3.4 Geometrical Rule for Target Capture 18
3.5 Constraints on Geometrical Rule for Desired Impact Time 18
3.6 Target Capture with Desired Impact Time 19

CHAPTER 4 – DESIGN METHODOLOGY OF A SLIDING MODE CONTROLLER 20

4.1 Sliding Surface 20
4.2 Controller Design 21
4.2.1 Equivalent Controller 21
4.2.2 Uncertainty Controller 21
4.2.3 Final Controller Parameter 21
4.3 Convergence to Sliding Surface 21

CHAPTER 5 – INTEGRATING THE SMC WITH THE GUIDANCE FRAMEWORK 23

5.1 Modified System Kinematics ... 23
5.2 Design of Modified SMC ... 23

CHAPTER 6 – SIMULATION RESULTS AND DISCUSSIONS ... 25
6.1 Target Capture ... 25
6.2 Target Capture with Desired Impact Time ... 28
6.3 Sliding Mode Controller (SMC) Design ... 31
6.4 Integration of SMC with Guidance Framework ... 32

CHAPTER 7 – CONCLUSIONS ... 35

ABBREVIATIONS ... 36

BIBLIOGRAPHY ... 37

LIST OF FIGURES

Figure No.	Figure Name	Page No.
1.1	Block diagram of guidance system	9
1.2	State-space representation of SMC	11
3.1	Engagement geometry of pursuer and target	15
3.2	$V_R - V_\theta$ engagement plot	17
6.1	Target capture trajectory of pursuer	25
6.2	Deviation angle variation during target capture	26
6.3	Velocity variation during target capture	26
6.4	Target capture trajectory with varying LOS angles	27
6.5	Target capture trajectory with varying Deviation angles	27
6.6	Target capture trajectory with desired impact time	28
6.7	Deviation angle variation for desired impact time	29
6.8	Velocity variation for desired impact time	29
6.9	Target capture trajectory with varying LOS angles for desired impact time	30
6.10	Target capture trajectory with varying Deviation angles for desired impact time	30
6.11	Target capture trajectory with varying desired impact times	31
6.12	Design of Sliding Mode Controller	34
6.13	Trajectory with and without disturbance	33
6.14	Deviation angle variation during SMC controller target capture	33

LIST OF TABLES

Table No.	Table Name	Page No.
6.1	Sliding Mode controller parameters	31

CHAPTER 1 – INTRODUCTION

1.1 Guidance

Guidance is the process of guiding the path of an object (called pursuer) towards a given point (called target), which can either be stationary or moving. Moving targets can further be classified as non-maneuvering (constant velocity) or maneuvering (accelerating) targets.

We have three main types of guidance:

- Autonomous guidance – pursuer doesn't require external aid for guidance
- Non-autonomous guidance – external communication is required
- Homing – pursuer tracks target by the energy emitted by the latter.

There are three levels of guidance:

- Geometrical rule – a rule derived from geometry, which helps in guidance
- Guidance law – implementation of the geometric rule by having a closed guidance loop
- Control – improve response by using a body control feedback loop.

Figure 1.1 shows the block diagram of the guidance system.

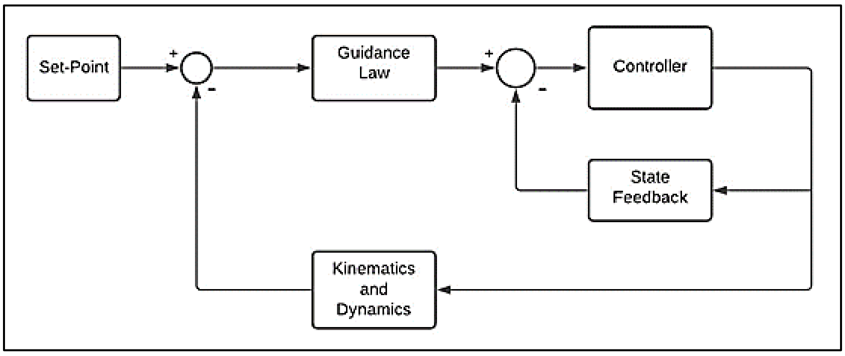


Figure 1.1 - Block Diagram of Guidance System

A classical guidance law which has been the primary motivation for the designing process is the Deviated Pure Pursuit (DPP) rule. DPP states that the pursuer P would direct itself at a target T so that its velocity vector V_P is incident at a lead or deviation angle with R.

1.2 Robust Controller

A robust controller is a feedback controller with unknown parameters and unknown model dynamics that provides strong steady-state tracking and disturbance rejection.

A controller has two primary functions: guiding the servo system's response to achieve the desired behaviour and maintaining that behaviour during operation despite system irregularities.

The second criteria – 'robustness to uncertainty' is the one to which we are giving importance here. It is crucial to the system's reliability. Most control systems are designed and built based on an idealised and simplified representation (i.e., the model) of the real system. It must be resistant to model flaws, such as differences between the model and the real system, physical parameter excesses, and external disruptions, to function successfully.

The fundamental benefit of robust control approaches is that they can develop control laws that meet both of the following conditions. Given a specification of intended behaviour and frequency estimates of the size of uncertainty, the robust control theory assesses feasibility, generates an appropriate control law, and guarantees the control law's range of validity (strength).

1.3 Sliding Mode Controller

Sliding mode control (SMC) is a nonlinear robust control approach with excellent accuracy, robustness, and simplicity of tuning and implementation. In practical scenarios, SMC provides for the control of nonlinear processes that are exposed to external disturbances and large model uncertainties.

The SMC has been widely used in numerous areas because of its effectiveness and ease of implementation in practice. SMC systems are built to drive system states onto a specific state-space surface known as a sliding surface. When the sliding surface is reached, sliding mode control keeps the states in the immediate neighbourhood of the sliding surface.

The sliding mode control is a two-part controller. The first step includes designing a sliding surface to ensure that the sliding motion meets the design specifications. The second step is to choose a control law that would make the switching surface attractive to the system state.

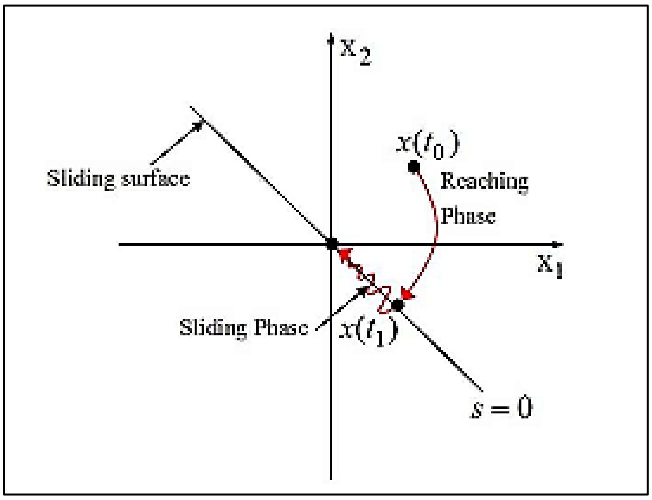


Figure 1.2 - State-space representation of SMC

The state-space representation of the SMC is shown in Figure 1.2. The nominal response in the state-space is given by σ=*0*, where σ is the sliding surface or manifold. In other words, the state X=0 is an asymptotically stable equilibrium point. The first phase is called the Reaching Phase, where the controller brings the system from any random state on the state-space to the sliding surface. In the Sliding Phase, the controller ensures that the state remains on the manifold and slides to the nominal state.

One observation which we can make is that this is a discontinuous control law, in the sense that the continuity is disturbed when the state-shift happens to the Sliding Phase.

Sliding mode control has two major advantages.

1. The first is that the sliding function can be used to customize the system's dynamic behaviour - effectively, the sliding function is a measure of intended performance.
2. Second, the closed-loop response becomes completely unaffected by a certain class of uncertainty known as matched uncertainty. It is defined by the uncertainty present in the input channels. This principle applies to bounded model parameter uncertainties, disturbance, and non-linearity.

1.4 Necessity of Sliding Mode Controller

As discussed in the previous sub-section, a sliding mode controller (SMC) helps to reduce the effect of system uncertainties entering through the input channel. By mitigating the effect of uncertainties on the system performance, the SMC increases the tolerance and makes the system robust.

Considering our system, the motion is controlled by the angular acceleration a_P of the pursuer. Even a small variation in the value of a_P (which could be due to wind disturbances, variation in generated thrust, hardware failure, etc.) might cause a cascading effect on the system and affect the overall stability. The SMC ensures that the pursuer will not experience an increased deviation from its trajectory and will ensure the capture of the target. There are certain disadvantages of the SMC (like chattering effect), but it is still quite effective in handling the uncertainties observed in the system, which has allowed it to be prevalently used in systems like pursuer guidance and capture.

CHAPTER 2 – LITERATURE REVIEW

Pursuer guidance is an evolving research area, with many applications ranging from missile systems to autonomous tracking and navigation. It has been considered for missile guidance in various literature.

15Ghose has discussed the basics of missile guidance in [1]. Several control approaches and guidance laws have been investigated in the literature for impact time guidance. The classical guidance laws, which incorporate the geometrical rules of pure pursuit (PP) and deviated pure pursuit (DPP), can be used to achieve a pre-specific impact angle [2]. The pursuer's velocity vector is directed towards the target in PP resulting in a tail-chase scenario, whereas it maintains a constant lead angle from the line of sight (LOS) vector in DPP. Shima has highlighted the importance and the practical application of various classical guidance laws in [3] and [4].

The development of optimal control and estimation theory resulted in modern guidance laws that can incorporate terminal constraints. Chen and Wang [5] have proposed a biased PNG guidance law for impact time and angle control . A PNG term with the navigation constant N=4 and a bias term relating to impact angle inaccuracy make up the guidance law. The given notional trajectory's related time-to-go estimate is calculated. Using a feedback controller, the time-to-go error is made to asymptotically converge to zero.

Trajectory shaping of the pursuit curves is a relatively new research area, where the focus is on shaping the curve to ensure desired target capture parameters such as impact angle, impact time and lateral acceleration. Geometrical rules derived from mathematical curves are popularly used for trajectory shaping. Tripathy and Shima used trajectory shaping guidance for a general curved trajectory towards the target, with a higher convergence rate than pure pursuit in [6]. They have considered the Archimedean spiral as the geometrical rule to control the curvature of the trajectory, ensuring target capture at a desired intercept angle.

Nonlinear control strategies, like sliding mode control, have been applied for allowing the pursuer to achieve a desired impact time. The basic principles of sliding mode control have been explained and highlighted in [8]. Sarah Spurgeon has expanded and has gone into detail in [9]. A sliding mode control-based guidance law to control terminal constraints like impact angle while ensuring target capture have been proposed by Kumar, Rao and Ghose in [10].

CHAPTER 3 – GUIDANCE METHODOLOGY

3.1 System Kinematics

The primary objective of this project is to create guidance laws that will guide a pursuer to intercept a stationary target. A stationary target, which is designated by T, can be a beacon, a structure, or any other landmark. The pursuer P is assumed to be a point-mass object having nonlinear unicycle kinematics given by –

$$\dot{x_P} = V_P \cos(\alpha) \tag{1}$$

$$\dot{y_P} = V_P \sin(\alpha) \tag{2}$$

$$\dot{\alpha} = \frac{a_P}{V_P} \tag{3}$$

where, $x_P\ (t)$ and $y_P\ (t)$ are the position coordinates of the pursuer at any time t and $\alpha(t)$ is its heading angle. It has a constant linear speed V_P, while a_P is its angular acceleration. The target is assumed to be a point-mass object with coordinates (x_T, y_T). The planar engagement geometry between the pursuer P and the target T is depicted in Figure 3.1.

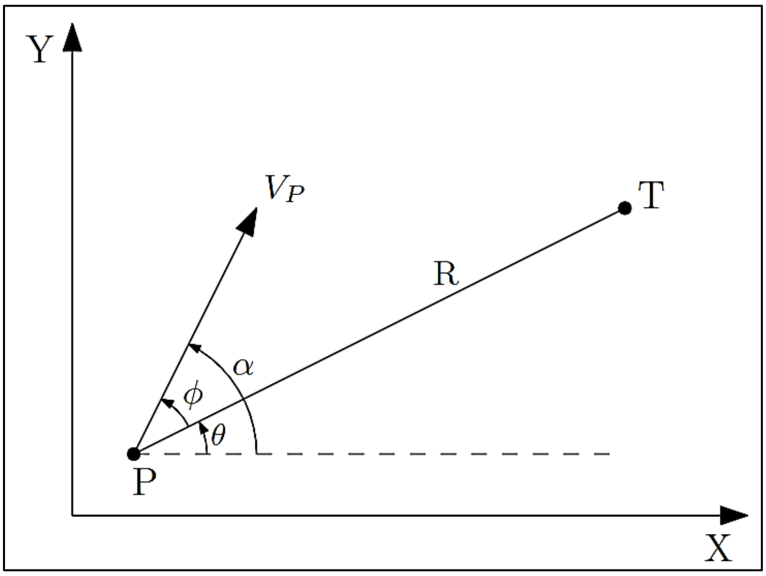


Figure 3.1- Engagement geometry of pursuer and target

The line-of-sight (LOS) distance between the pursuer and the target at time t is given by R and the LOS angle by θ. The relative kinematics between them is nonlinear in nature and is governed by the parameters R and θ as shown below,

$$\dot{R} = -V_P \cos(\alpha - \theta) = -V_P \cos(\phi) \tag{4}$$

$$R\dot{\theta} = -V_P \sin(\alpha - \theta) = -V_P \sin(\phi) \tag{5}$$

The parameter ϕ known as the lead angle expresses the kinematics more concisely. It is defined as,

$$\phi = \alpha - \theta \tag{6}$$

Having defined the relative motion kinematics, the guidance law can be constructed based on these criteria to achieve target capture while imposing engagement (terminal) restrictions such as impact time. The proposed design method has the advantage of providing an integrated environment for ensuring target capture while implementing the constraints as well.

3.2 Conditions for Guaranteed Target Capture

This sub-section discusses the conditions for guaranteed target capture, based on the relative motion of the pursuer. Guaranteed capture happens when the distance between the pursuer and the target is zero. Mathematically, it can be denoted as,

$$R(t_f) = 0 \tag{7}$$

where t_f is the time of interception, that is, the time at which the capture happens successfully.

For implementing this, the equations of motion are re-defined and two velocity components associated with the LOS have been introduced as –

$$V_R = \dot{R} \tag{8}$$

$$V_\theta = R\dot{\theta} \tag{9}$$

where, V_R and V_θ are the two components of the relative velocity between the pursuer and the target. V_R is the tangential velocity component along the LOS, and V_θ is the perpendicular velocity component normal to the LOS.

The objective in the next sub-section is to analyse the target capture problem in the (V_θ, V_R) plane. The motivation stems from the interesting properties associated with these velocity components.

3.3 Guidance Law Design

The relative motion kinematics is obtained directly from Equations (4)-(5) as given below,

$$V_R = -V_P \cos(\phi) \tag{10}$$

$$V_\theta = -V_P \sin(\phi) \tag{11}$$

Squaring and summing both sides of the above equations, we get $\mathrm{V_R^2 + V_\theta^2 = V_P^2}$. This is the locus of a circle in the $(\mathrm{V_\theta, V_R})$ space centered at the Origin (0,0) having a radius equal to V_P as shown in Figure 3.2.

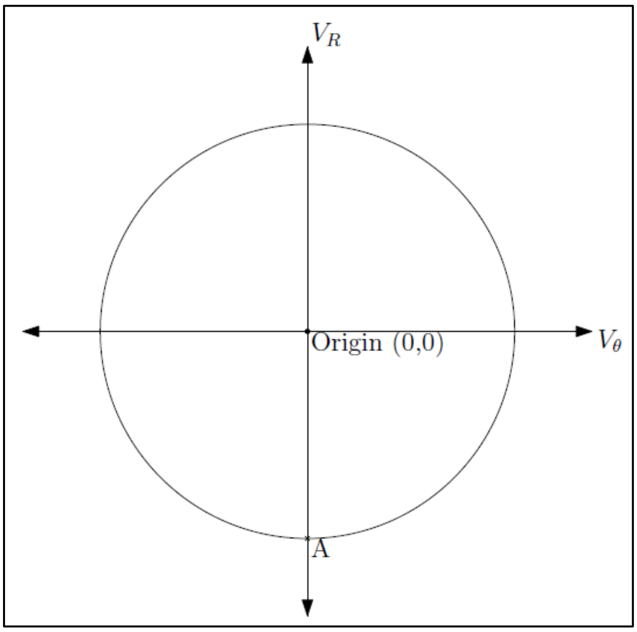


Figure 3.2 -V_θ, V_R engagement plot

This circle can be mathematically defined as –

$$C = \{(V_\theta, V_R) \big| V_\theta^2 + V_R^2 = V_P^2\} \tag{12}$$

It shows us that the point (V_θ, V_R) will only move on this circle as the interception proceeds. The graphical representation helps us to better understand and design the required guidance law to drive the (V_θ, V_R) values to specific values corresponding to target capture conditions.

As discussed, for achieving target capture, the value of R should reduce to 0. This means we need a negative $\dot{R}$ which means a negative V_R. This implies that the point (V_θ, V_R) should move towards the lower half of the plane, if it is not already in that region, and should remain there.

By observation, if the direction of the motion of the point (V_θ, V_R) is always towards point A, we will achieve guaranteed target capture. In other words, the required condition evolves to,

$$\dot{V_R} < 0 \tag{13}$$

The direction of motion of the points can be found from the derivatives of the two velocity components.

$$\dot{V}_R = V_P \sin(\phi)\, \dot{\phi} = -V_\theta \dot{\phi} \tag{14}$$

$$\dot{V}_\theta = -V_P \cos(\phi)\, \dot{\phi} = V_R \dot{\phi} \tag{15}$$

This is the primary contribution and novelty of this project work, as a novel guidance law can be implemented that guarantees target capture.

3.4 Geometrical Rule for Target Capture

Geometric rules can be defined as the mathematical rules derived from geometry which help us in the guidance of the pursuer. Based on my understanding, I have designed a geometric rule which places constraints on $\dot{\phi}$ (and equivalently a_P) resulting in a family of guidance laws.

Consider the following law –

$$a_P = -\frac{V_R \sin\phi}{R} - \frac{AV_P}{\sin\phi} \tag{16}$$

where A is the guidance law parameter and $\dot{\phi}$ is given by,

$$\dot{\phi} = -\left(\frac{A}{\sin\phi}\right) \tag{17}$$

From Equation (9), we know that $\dot{V}_R < 0$ is required, which translates to $\dot{V}_R = -V_P A$. Since $V_P > 0$, the guidance law will lead to target capture for every A > 0.

3.5 Constraints on Geometrical Rule for Desired Impact Time

In this subsection, I have used the results discussed previously to enforce a desired impact time, which would be always greater than the minimum time of capture given by the pursuer intercepting the target head-on in a straight line. Consider the proposed guidance law given by Equation (16) with $A > 0$. As discussed above, this guarantees target capture.

To impose a desired impact time t_d, we have A as a free parameter which is only required to be positive. To find the relation between A and t_d, we integrate $\dot{V}_R$ using the initial conditions –

$$V_R\,(t=0) = V_{R0} = V_P\, cos\phi_0\,; \quad R\,(t=0) = R_0$$

This gives,

$$R(t) = -AV_P\,\frac{t_d^2}{2} - V_P\cos(\phi_0)\, t_d + R_0 \tag{18}$$

From Equation (7), we can substitute $R(t_f) = 0$ for target interception, and algebraically reduce the Equation (18) to obtain the value of the free parameter A as,

$$A = 2\frac{(-V_P \cos(\phi_0)\, t_d + R_0)}{V_P t_d^2} \tag{19}$$

3.6 Target Capture with Desired Impact Time

The essential condition for target capture is $A > 0$. I have imposed this condition to get the constraint on the impact time as –

$$t_d < \frac{R_0}{V_P \cos(\phi_0)} \tag{20}$$

The above equation shows as that any desired time can be imposed using the guidance law given by Equation (16). It is understood that for a positive time of impact, the magnitude of $\cos\phi_0$ should be positive. Also, by changing the initial conditions like position, distance and lead angle, we can obtain the desired impact time.

CHAPTER 4 – DESIGN METHODOLOGY OF A SLIDING MODE CONTROLLER

4.1 Sliding Surface

The selection of the sliding surface is essentially the most important step as it decides the application of the controller. Considering the framework, uncertainty would be introduced through the input of angular acceleration. Hence, the LOS angle θ is chosen as the error to be reduced and have appropriately selected the sliding surface.

The error and its derivative are shown below,

$$e = \theta - \theta_d \tag{21}$$

$$\dot{e} = \dot{\theta} = -\frac{V_P \sin(\phi)}{R} = \frac{V_\theta}{R} \tag{22}$$

The sliding surface often depends upon the tracking error and a certain number of its derivatives. The function σ should be selected that its vanishing $\sigma = 0$ will give rise to a stable differential equation which will tend to zero eventually. Hence, the most common way to choose the sliding manifold is to take a linear combination of its derivatives.

The error signal is of degree 2, in the sense that it has the input (to be controlled) a_P in its second derivative. Therefore, the sliding manifold is selected as shown below, where τ is the time constant.

$$\sigma = \tau\dot{e} + e \tag{23}$$

$$\dot{\sigma} = \tau\ddot{e} + \dot{e} \tag{24}$$

$$\dot{\sigma} = \tau\left(\frac{\dot{V}_\theta R - V_\theta \dot{R}}{R^2}\right) + \dot{\theta}$$

$$\dot{\sigma} = \tau\left(-\frac{a_P \cos(\phi)}{R} - \frac{2V_R V_\theta}{R^2}\right) + \frac{V_\theta}{R}$$

$$\dot{\sigma} = \tau\left(-\frac{a_P \cos(\phi)}{R} - \frac{V_P^2 \sin(2\phi)}{R^2}\right) - \frac{V_P \sin(\phi)}{R} \tag{25}$$

This is the required expression for the Sliding surface and its derivative. I have now used this to design the SMC, which has two parts – equivalent and uncertainty controller.

4.2 Controller Design

The sliding mode controller a_P has two parts – equivalent controller denoted a_{Peu} and the uncertainty controller denoted by a_{Puc}. Hence, here we will get the controller as –

$$a_P = a_{Peq} + a_{Puc} \tag{26}$$

4.2.1 Equivalent Controller

In the absence of modelling errors and target manoeuvre commands, the equivalent controller is designed to keep the system on the sliding surface by imposing $\sigma=0$. Finding the derivative of sigma, and nulling the uncertainty part gives us the equivalent controller as,

$$a_{Peq} = -\frac{V_P \tan(\phi)}{\tau} - \frac{2V_P^2 \sin(\phi)}{R} \tag{27}$$

4.2.2 Uncertainty Controller

The system may not be in sliding mode at first, or it may deviate from the sliding surface due to modelling mistakes or target motions. In the face of these uncertainties, the uncertainty controller is designed to push the system to the sliding surface in finite time. It is represented mathematically by the signum function, and hence is discontinuous, as mentioned earlier.

$$a_{Pun} = \beta sgn(\sigma) \tag{28}$$

4.2.3 Final Controller Parameter

The final controller is obtained from Equation (26) as shown below,

$$a_P = -\frac{V_P \tan(\phi)}{\tau} - \frac{2V_P^2 \sin(\phi)}{R} + \beta sgn(\sigma) \tag{29}$$

4.3 Convergence to Sliding Surface

The analysis of the convergence of the state to the sliding surface σ can be done using a candidate Lyapunov function, as shown.

$$L = \frac{1}{2}\sigma^2 \tag{30}$$

The time derivative of this Lyapunov function gives us the reaching condition, solving which gives us the required constrain or condition on the initial parameters for achieving finite time to the sliding manifold.

$$\dot{L} = \sigma\dot{\sigma} \tag{31}$$

$$\dot{L} = \sigma\tau\left(-\frac{a_P\cos(\phi)}{R} - \frac{V_P^2\sin(2\phi)}{R^2}\right) - \sigma\frac{V_P\sin(\phi)}{R}$$

$$\dot{L} = \sigma\frac{\tau\cos(\phi)}{R}\left(-\frac{V_P\tan(\phi)}{\tau} - \frac{2V_P^2\sin(\phi)}{R} + \beta sgn(\sigma)\right) - \sigma\frac{V_P^2\sin(2\phi)}{R^2} - \sigma\frac{V_P\sin(\phi)}{R}$$

$$\dot{L} = -\sigma\tau\left(\frac{\beta\cos(\phi)\,sgn(\sigma)}{R}\right) \leq \frac{= |\sigma|\tau\beta\cos(\phi)}{R} \qquad (32)$$

which gives us the condition on the uncertainty parameter β as,

$$\beta\cos(\phi) > 0 \qquad (33)$$

The value of β which satisfies this condition will ensure the negative definiteness of the Lyapunov function, and by extension, ensure that the sliding manifold is reached in finite-time.

CHAPTER 5 – INTEGRATING THE SMC WITH THE GUIDANCE FRAMEWORK

5.1 Modified System Kinematics

In our system, we consider that the noise can enter through the input channel of angular acceleration. To implement the SMC, we need to introduce certain modifications in the existing kinematics of the system.

$$\dot{x_P} = V_P \cos(\alpha) \tag{34}$$

$$\dot{y_P} = V_P \sin(\alpha) \tag{35}$$

$$\dot{\alpha} = \frac{a_P + w}{V_P} \tag{36}$$

where, *w* is the external disturbance entering the system, which belongs to the class of matched uncertainties.

5.2 Design of Modified SMC

The sliding surface is still considered as $\sigma = \tau\dot{e} + e$. The relationship regarding the angular acceleration changes and hence the Equation (29) will become

$$a_P = a_{Peq} + a_{Puc} + w \tag{37}$$

We get the updated sliding surface as

$$\dot{\sigma} = \tau\left(-\frac{a_P\cos(\phi)}{R} - \frac{V_P^2 \sin(2\phi)}{R^2}\right) - \frac{V_P\sin(\phi)}{R}$$

$$\dot{\sigma} = \frac{\tau\cos(\phi)}{R}\left(-\frac{V_P\tan(\phi)}{\tau} - \frac{2V_P^2\sin(\phi)}{R} + \beta sgn(\sigma) + w\right) - \frac{V_P^2\sin(2\phi)}{R^2} - \frac{V_P\sin(\phi)}{R}$$

$$\dot{\sigma} = \tau\cos(\phi)\left(\frac{\beta sgn(\sigma) + w}{R}\right) \tag{38}$$

The new Lagrangian is

$$\dot{L} = \sigma\dot{\sigma} = \sigma\tau\left(-\frac{a_P\cos(\phi)}{R} - \frac{V_P^2\sin(2\phi)}{R^2}\right) - \sigma\frac{V_P\sin(\phi)}{R} \tag{39}$$

$$\dot{L} = \sigma\frac{\tau\cos(\phi)}{R}\left(-\frac{V_P\tan(\phi)}{\tau} - \frac{2V_P^2\sin(\phi)}{R} + \beta sgn(\sigma) + w\right) - \sigma\frac{V_P^2\sin(2\phi)}{R^2} - \sigma\frac{V_P\sin(\phi)}{R}$$

$$\dot{L} = -\sigma\tau\cos(\phi)\left(\frac{\beta sgn(\sigma) + w}{R}\right) \leq \frac{= |\sigma|\tau(\beta + w)\cos(\phi)}{R} \tag{40}$$

which gives us the new condition on the uncertainty parameter β as,

$$\beta\cos(\phi) > w\cos(\phi) \tag{41}$$

Depending on the value of the deviation angle, we can determine the value of the uncertainty parameter. Furthermore, using Equations (19) and (20) guarantees target interception in a desired time, as discussed in earlier sections.

CHAPTER 6 – SIMULATION RESULTS AND DISCUSSIONS

This section contains the numerical simulations that were used to validate the proposed geometrical rule. Throughout this phase, the pursuer maintains a constant speed. The target is at the origin and immobile. In a noise-free situation, target capture has been examined using the designed guidance law, and then considered different impact times. A circle, a plus, and an asterisk, respectively, depict the pursuer's initial and final positions, as well as the target.

6.1 Target Capture

In this subsection, the guidance rule proposed in Equation (11) is considered for target capture. Three types of simulations are discussed. Firstly, consider the target capture for a specific set of initial conditions. The second one has different LoS angle with the same deviation angle. The third simulation considers different deviation angles with the same LoS angle.

In the first case, consider the scenario given by : $V_P = 20$, $R_0 = 370$, $A = 0.025$, $\phi_0 = 5\pi/12$ and $\theta_0 = \pi$. The engagement trajectory of the pursuer in Figure 6.1 shows that the target is being captured successfully. Also, the variations of ϕ and V_R with time are shown in Figure 6.2 and Figure 6.3 respectively.

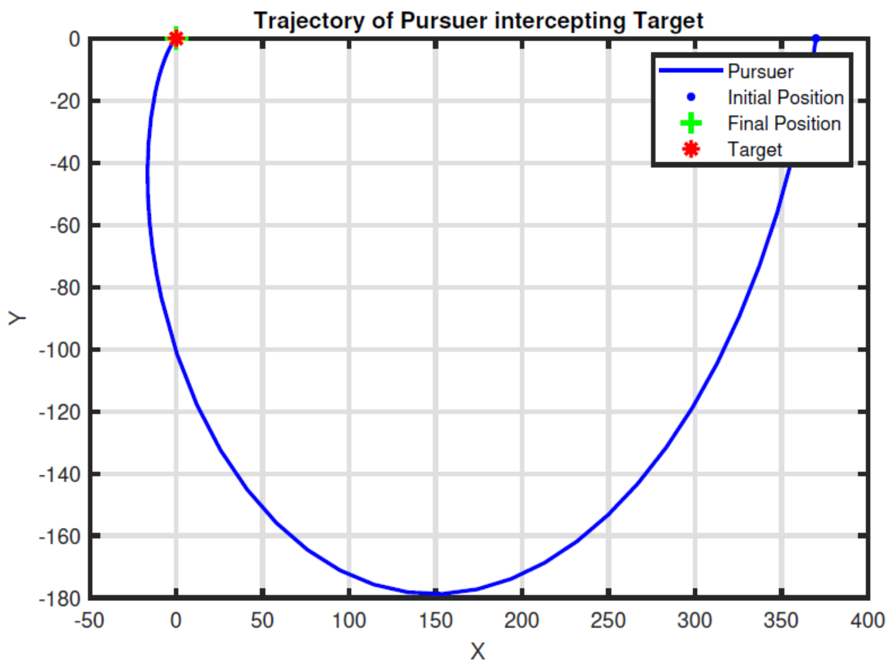


Figure 6.1 - Target capture trajectory of pursuer

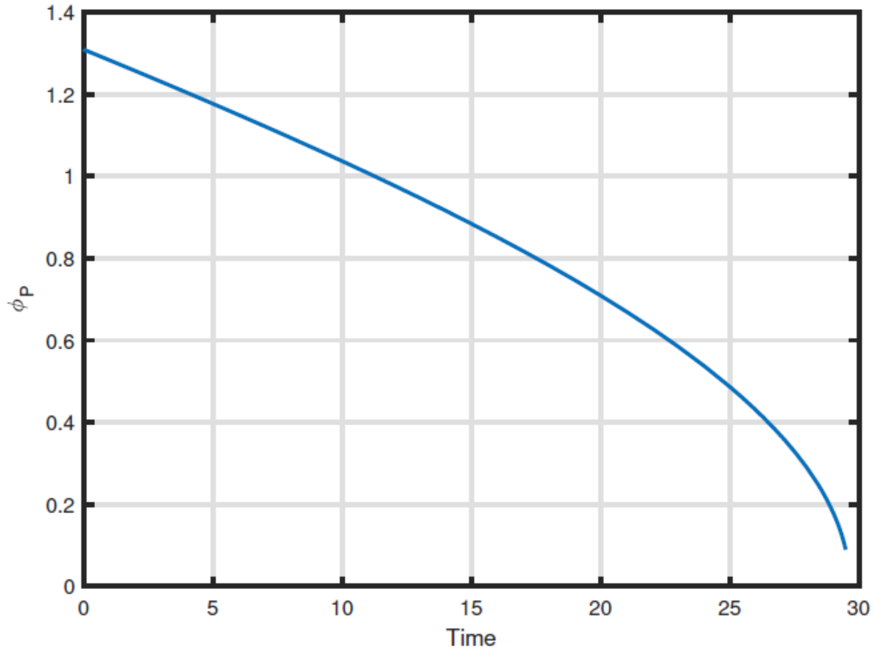


Figure 6.2 - Deviation angle variation during target capture

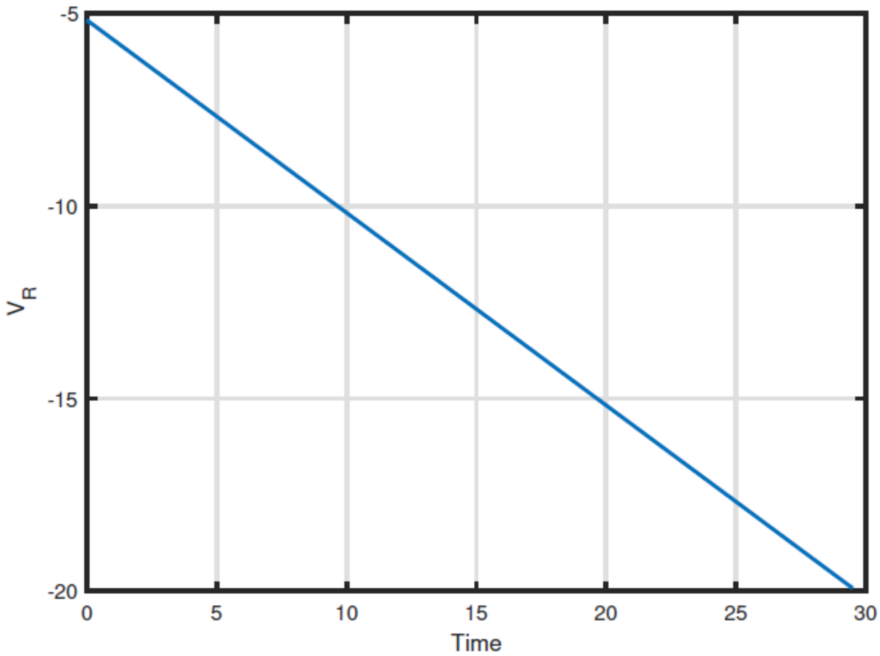


Figure 6.3 - Velocity variation during target capture

Now, consider the second case where there are different initial line-of-sight angles of the pursuer, while the initial deviation angle is $\phi_0 = 4\pi/9$. The trajectories are shown in Figure 6.4, where the initial angles are given by $\theta_0 \in \{\pi/4 , 3\pi/4 , 5\pi/4 , 7\pi/4\}$.

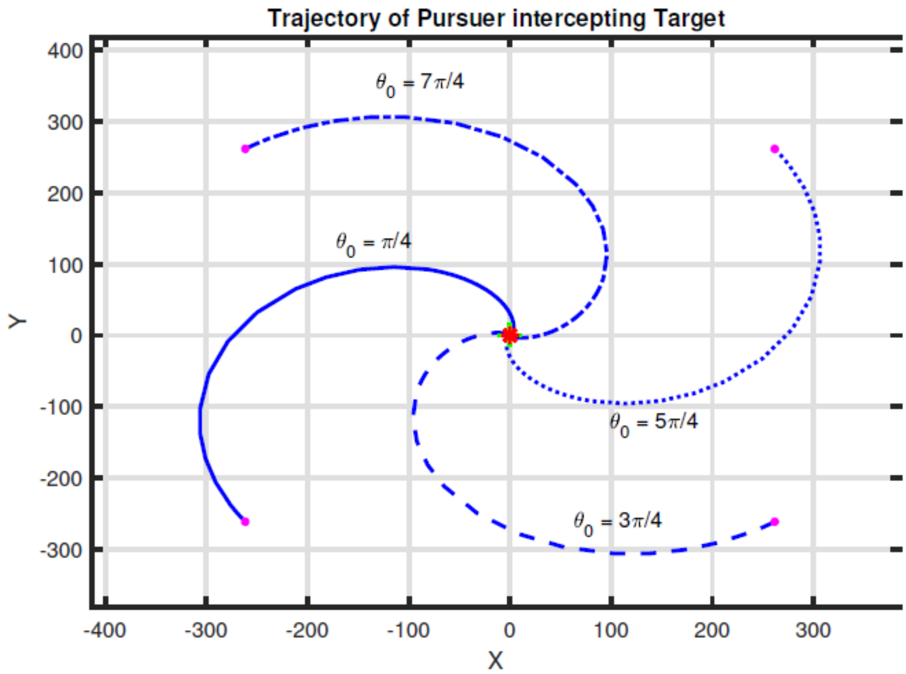


Figure 6.4 - Target capture trajectory with varying LOS angles

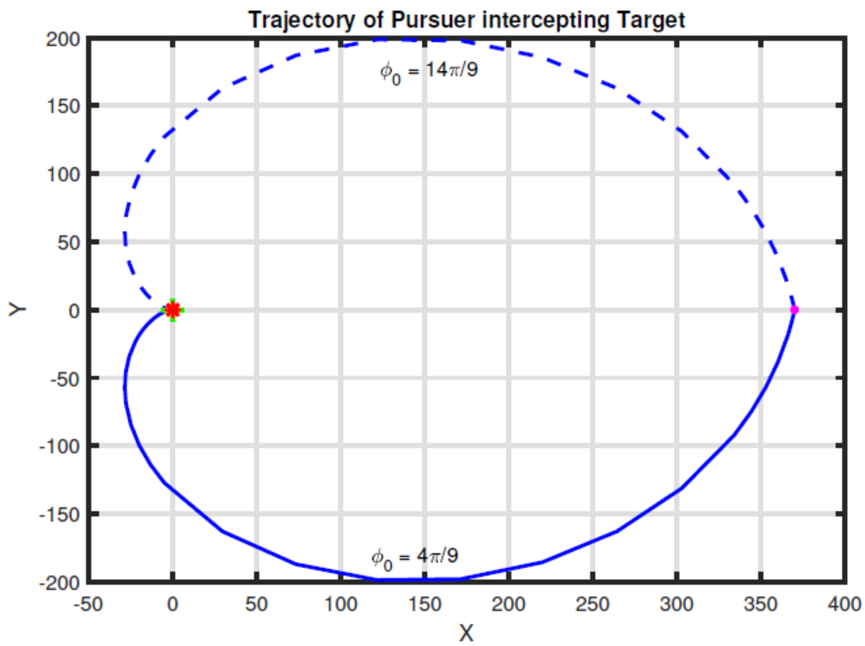


Figure 6.5 - Target capture trajectory with varying Deviation angles

Consider the final case; there are different initial deviation angles of the pursuer, while the initial LoS angle is θ_0=π. The trajectories are shown in Figure 6.5, where the initial angles are given by $\phi_0 \in \{4\pi/9, 14\pi/4\}$.

The results discussed in this sub-section are the proof-of-concept that a geometrical rule is used to design a DPP guidance law which guarantees target capture constrained by the condition $\dot{V}_R < 0$.

6.2 Target Capture with Desired Impact Time

In this subsection, the guidance rule proposed in Equation (11) is considered for target capture with the constraint on the free parameter A given by Equation (14). Also, the parameters should be satisfying the condition given by Equation (15).

Four types of simulations are discussed. Firstly, we consider the target capture for a specific set of initial conditions. The second one has different LoS angle with the same deviation angle. The third simulation considers different deviation angles with the same LoS angle. Lastly, the fourth one showcases the variations in the trajectory by changing the desired impact time.

In the first case, consider the scenario given by : $V_P = 25$, $R_0 = 325$, $t_d = 30$, $\phi_0 = \pi/2$ and $\theta_0 = 9\pi/10$. The trajectory of the pursuer capturing the target in the desired impact time is shown by Figure 6.6. The variations of ϕ and V_R with time are shown in Figure 6.7 and Figure 6.8 respectively.

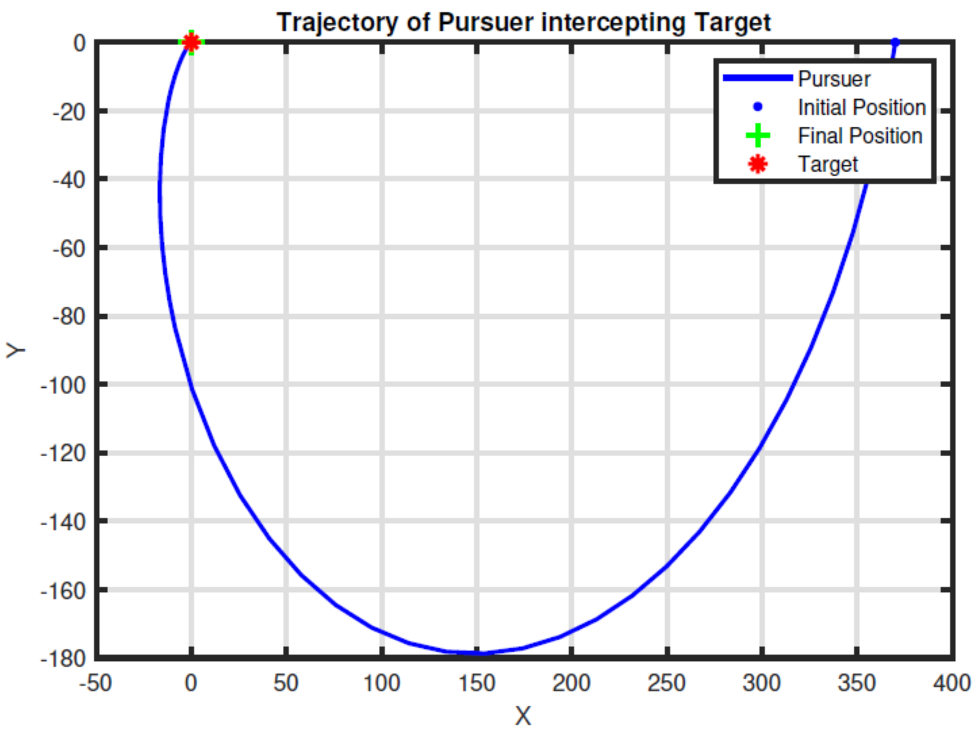


Figure 6.6 - Target capture trajectory with desired impact time

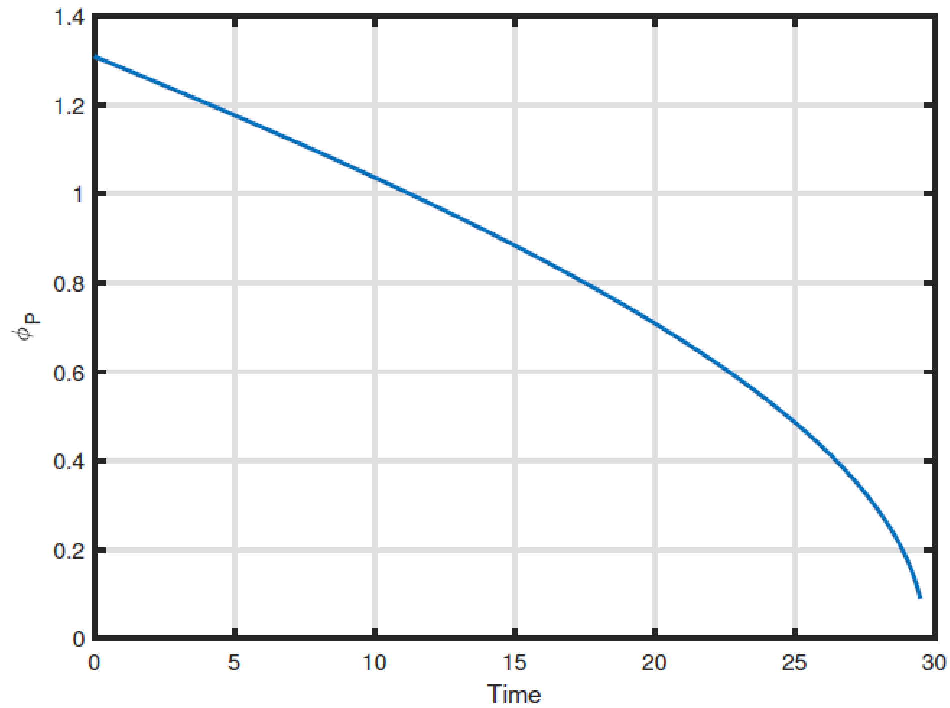


Figure 6.7 - Deviation angle variation for desired impact time

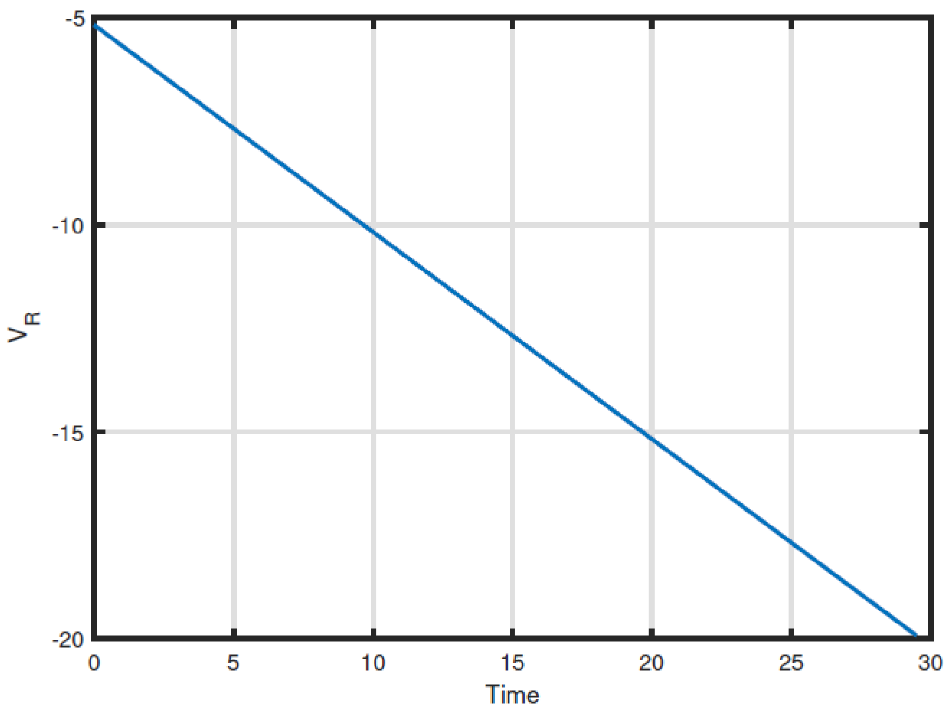


Figure 6.8 - Velocity variation for desired impact time

Now, we consider the second case where we have different initial line-of-sight angles of the pursuer, while the initial deviation angle is $\phi_0 = 5\pi/12$. The desired time remains constant at 30 seconds. The trajectory is shown in Figure 6.9, where the initial angles are given by $\theta_0 \in \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$.

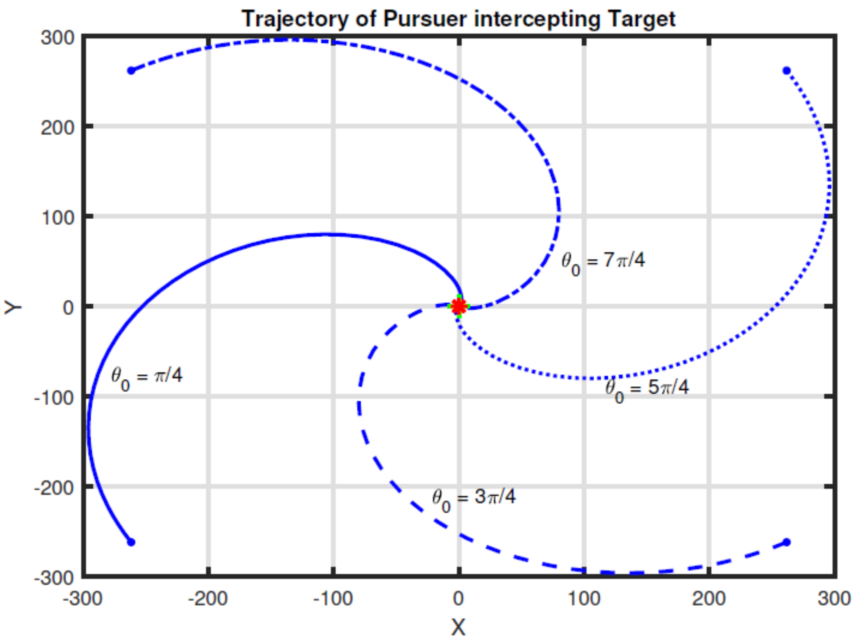


Figure 6.9 - Target capture trajectory with varying LOS angles for desired impact time

Considering the third case, we have different initial deviation angles of the pursuer, while the initial LoS angle is $\theta_0 = \pi$, with no change in the impact time. The trajectories are shown in Figure 6.10, where the initial angles are given by $\phi_0 \in \{7\pi/18, 5\pi/3\}$.

For the final case, we vary the impact time given by $t_d \in \{30, 40, 60\}$ while keeping other parameters same as in the first case. The trajectories are shown in Figure 6.11.

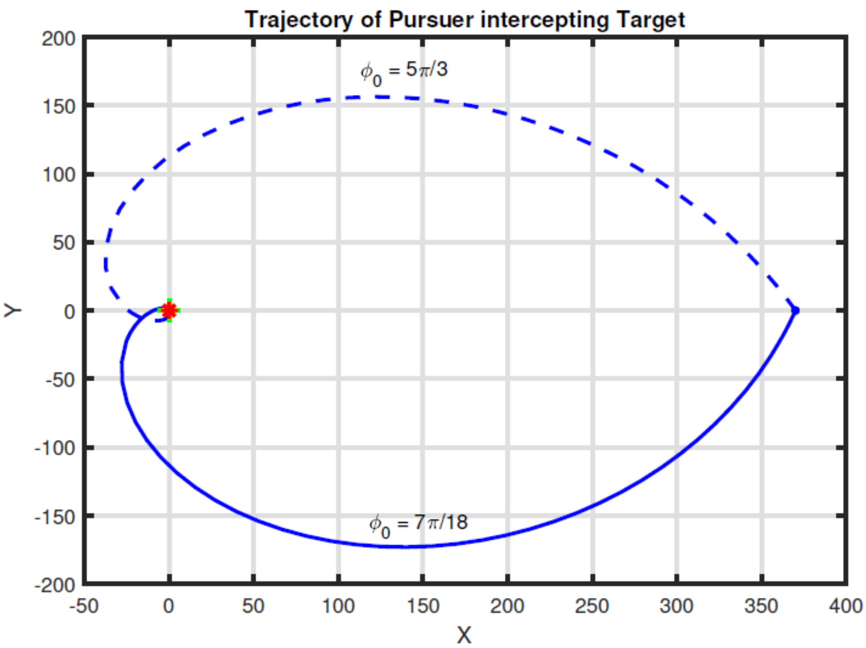


Figure 6.10 - Target capture trajectory with varying Deviation angles for desired impact time

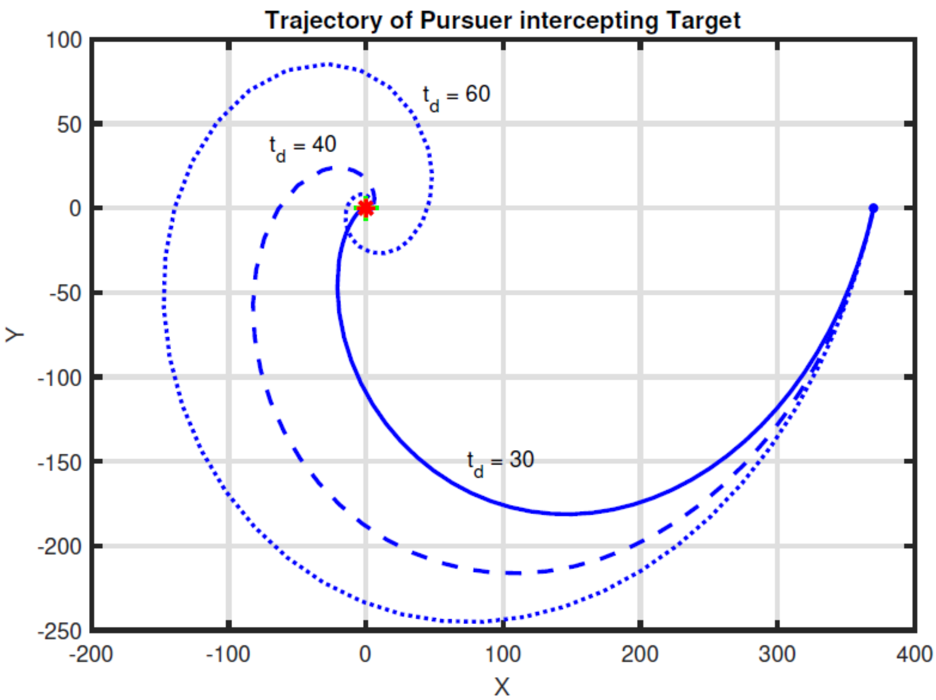


Figure 6.11 - Target capture trajectory with varying desired impact times

The results discussed in this sub-section show that a desired impact time can be imposed by having additional constraints on the existing framework itself.

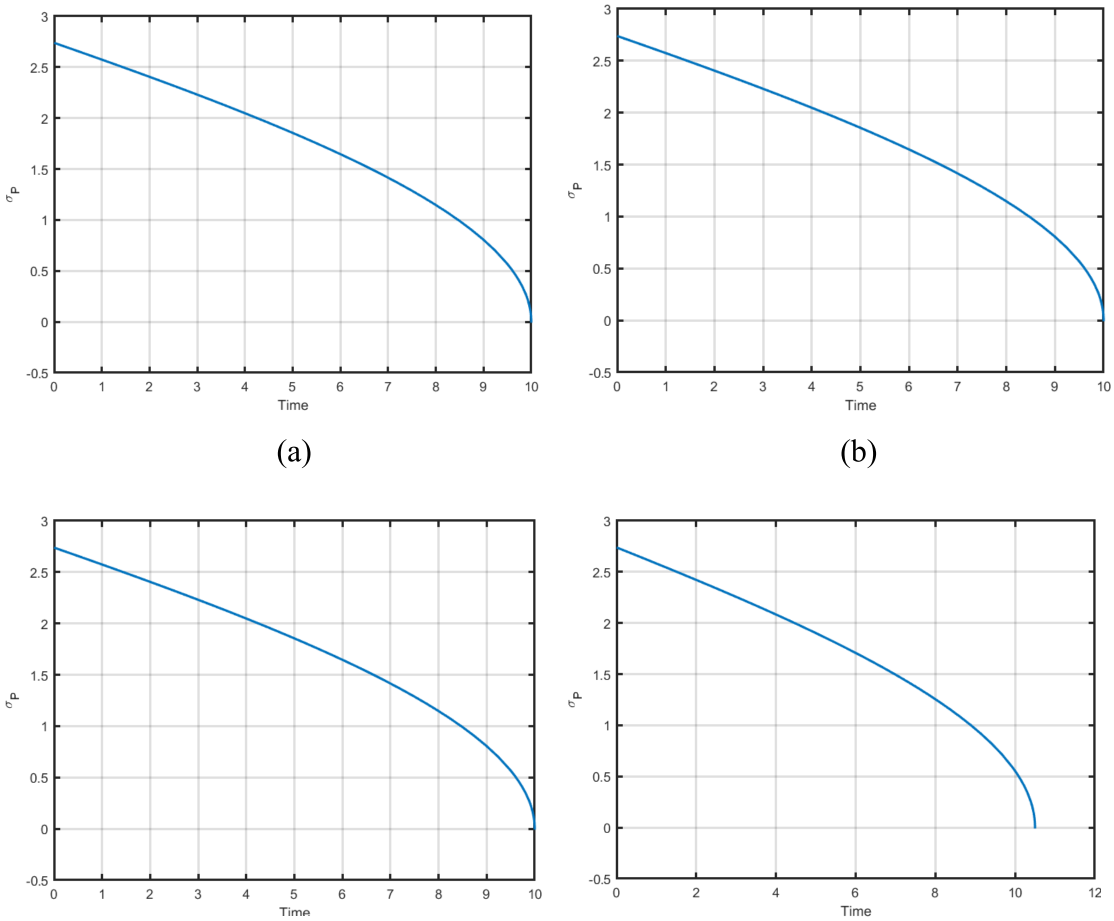
6.3 Sliding Mode Controller (SMC) Design

This sub-section consists of the numerical simulations that have been used to validate the proposed sliding mode controller (SMC).

The main aim is to ensure that the sliding manifold is reached in a finite time. In other words, σ should become zero in a finite time. The below table summarizes the various iterations of the controller and the following diagrams show the validation of the designed sliding mode controller. We can also study the effects of time constant and uncertainty constant.

Table 6.1 - Sliding Mode controller parameters

S.No.	Time Constant	Uncertainty Constant	Figure
1	0.001	1	Figure 6.12 (a)
2	0.001	100	Figure 6.12 (b)
3	0.001	10000	Figure 6.12 (c)
4	0.01	100	Figure 6.12 (d)



(a) (b)

(c) (d)

Figure 6.12 - Design of Sliding Mode Controller

6.4 Integration of SMC with Guidance Framework

In this subsection, we are implementing the controller design in the existing framework which guarantees target capture along with implementation of a desired impact time. This is done in accordance with the modifications discussed in Section 3.

The desired impact time is taken as 50 seconds while the initial distance is 750 metres. The uncertainty parameter β is taken as 0.01, with ϕ_0 as -80 degrees. The magnitude of the external disturbance w is considered as 0.1 and is a constant bias.

The trajectory of the pursuer with and without external disturbances is depicted in Figure 6.13. The modifications in the lead angle are depicted in Figure 6.14.

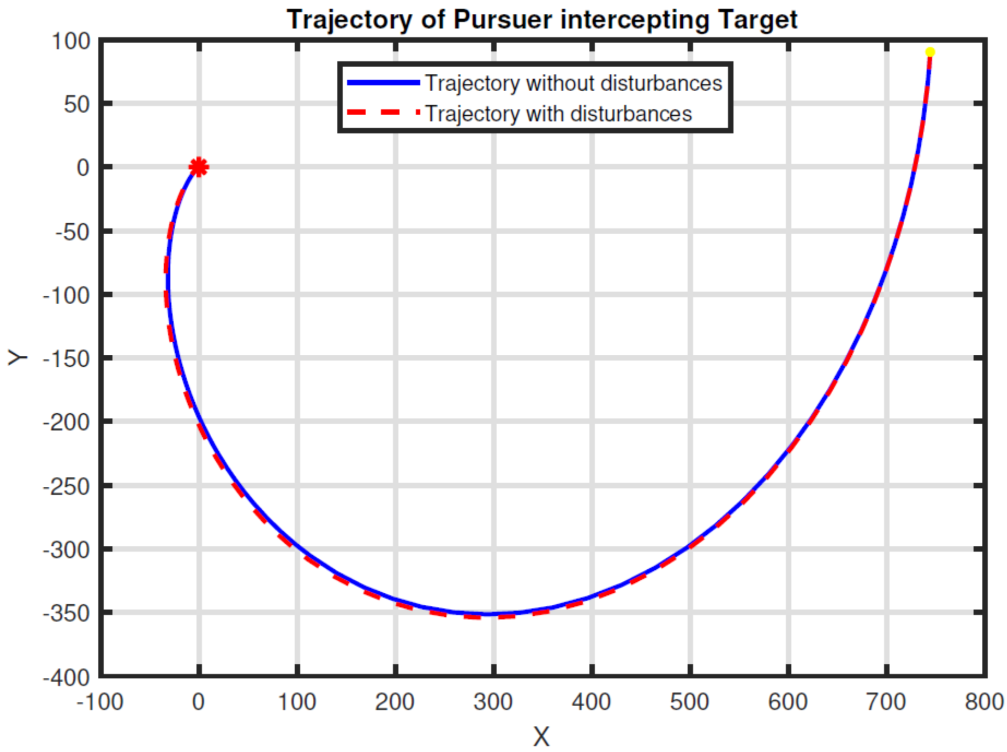


Figure 6.13 - Trajectory with and without disturbance

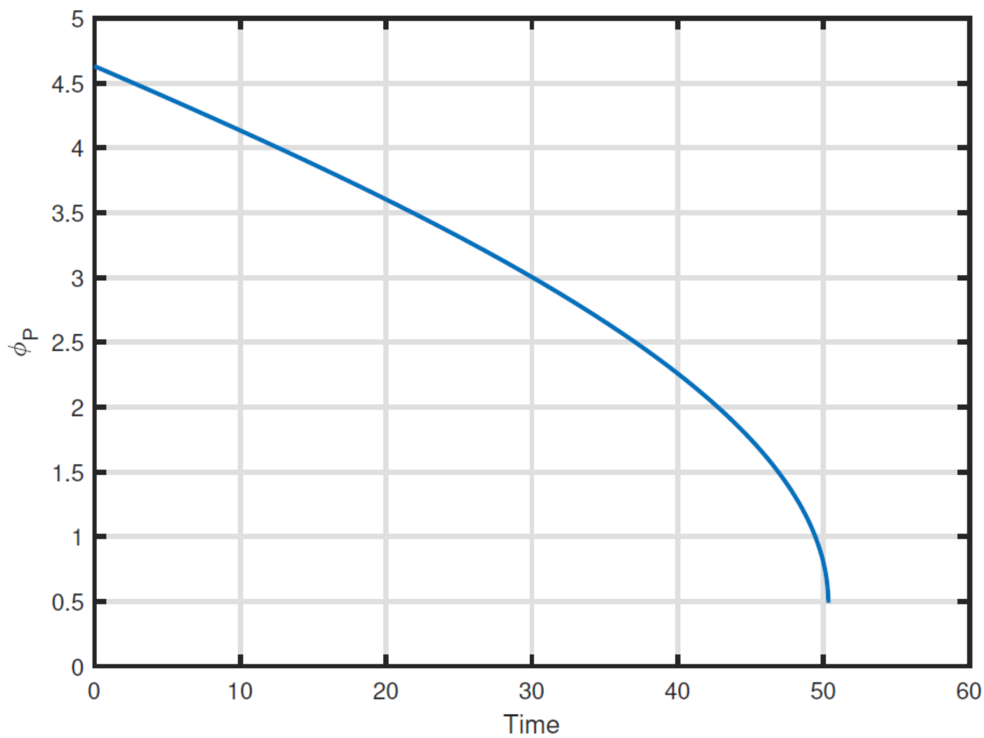


Figure 6.14 - Deviation angle variation during SMC controller target capture

The results discussed in this sub-section establish the validity of the SMC designed for the existing kinematic framework of guidance and target capture with desired impact time. The simulations also show the controller’s resistance to uncertainty and disturbances.

An interesting observation is that the existence of external disturbances lengthens the time it takes for the pursuer to catch the target compared to the intended impact time. This is expected as the system is experiencing disturbances over and above the existing system parameters.

CHAPTER 7 – CONCLUSIONS

Target capture is a very important concept as we have various use cases for it. Consider a missile which has been launched from a launch pad. By implementing this guidance law, it is possible to increase the accuracy of it striking the target in a realistic scenario. Furthermore, the control of impact time also allows the missile to selectively strike the target as-and-when the situation demands of it. Extending the applications, considering a warehouse where objects must be fetched from the shelves and carried to a certain location. In such a case, the importance of time-to-capture rises even further, as precision and punctuality of delivery would be assured. This is the physical relevance of my work.

Having established the validity of theoretical framework through simulations, the design of a robust controller was studied. In a real-world scenario, the system must be able to reject noise and uncertainty, and this is where a robust controller, such as the sliding-mode controller comes into play. The controller design, as explained, is simple and intuitive. It also allows flexibility of modification of the sliding surface/manifold according to the performance specifications. The SMC has been integrated with the guidance framework with an optimized sliding surface for the guidance law to improve the performance by ensuring target capture at a desired impact time.

ABBREVIATIONS

MATLAB	–	MATrix LABoratory
SMC	–	Sliding Mode Controller
LOS	–	Line of Sight
DPP	–	Deviated Pure Pursuit

BIBLIOGRAPHY

1. Debasish Ghose – Guidance of Missiles (IISc, NPTEL)
2. Missile Guidance and Pursuit: Kinematics, Dynamics and Control 1st edition by NA Shneydor
3. Shima.T, "Head Pursuit Guidance", Journal of Guidance, Control, and Dynamics
4. Shima.T, "Deviated Pure Pursuit", Journal of Guidance, Control, and Dynamics
5. Chen, X., and Wang, J., "Optimal control-based guidance law to control both impact time and impact angle," Aerospace Science and Technology
6. Tripathy, T., and Shima, T., "Archimedean Spiral-Based Intercept Angle Guidance," Journal of Guidance, Control, and Dynamics
7. Ratnoo, A., Hayoun, S. Y., Granot, A., & Shima, T. (2015). Path following using trajectory shaping guidance. *Journal of Guidance, Control, and Dynamics*
8. "A Quick Introduction to Sliding Mode Control and its Applications", Manual by Electrical and Electronics Department, University of Cagliari
9. "Sliding Mode Control: A tutorial" by Surah Spurgeon, School of Engineering and Digital Arts, University of Kent
10. Kumar, S.K., Rao, S., & Ghose, D., "Nonsingular Terminal Sliding Mode Guidance with Impact Angle Constraints