Abstracts (Chronological Order)

A decomposition theorem of surface vector fields and spectral structure of the Neumann-Poincaré operator in elasticity

Hyeonbae Kang (Inha University)

We prove that the space of vector fields on the boundary of a bounded domain in three dimensions is decomposed into three subspaces: elements of the first one extend to the inside the domain as divergence-free and rotation-free vector fields, the second one to the outside as divergence-free and rotation-free vector fields, and the third one to both the inside and the outside as divergence-free harmonic vector fields. We apply this decomposition theorem to investigate spectral properties of the Neumann-Poincaré operator in elasticity, whose cubic polynomial is known to be compact. We prove that each linear factor of the cubic polynomial is compact on each subspace of decomposition separately and those subspaces characterize eigenspaces of the Neumann-Poincaré operator. This talk is based on a joint work with S. Fukushima and Y.-G. Ji.

Perturbation of fields by perfect conductors with interface resistance

Shota Fukushima (Gumma University)

Transmission problem consists of an underlying partial differential equation (PDE) and transmission conditions on interfaces. Typical transmission condition is the ideal interface condition, where the continuity of the solution and its flux across the interfaces is assumed. On the other hand, in practice, it often occurs that the interfaces have a resistance, which causes a jump of the solution to PDE across the interface. This jump is controlled by the interface resistance. In this talk, we show that the solution to the Laplace equation with interface resistance exhibits distinct behavior from the ideal interface case, whereas it converges to the solution to the Laplace equation with ideal interface as the interface resistance tends to zero. This talk is based on joint work with Yong-Gwan Ji (KIAS), Hyeonbae Kang (Inha Univ.) and Xiaofei Li (Zhejiang Univ. of Tech.).

Neumann-Poincaré operators and Laplace screen problem

Yong-Gwan Ji (Korea Institute for Advanced Study)

Dirichlet and Neumann boundary value problems for closed surfaces (or curves) are well understood both analytically and numerically. However, when the boundary consists of only a portion of a surface (or a curve), the corresponding problems, referred to as Laplace screen problem problems, are more delicate, both analytically and numerically. Such problems typically arise in the modeling of very thin structures. In this talk, we investigate the spectral properties of the Neumann–Poincaré operator on the confocal family of oblate spheroids and ellipses, which degenerate to a disk and an interval, respectively, as the confocal parameter tends to zero. Using these results, we investigate the Laplace screen problem where the boundary is either a disk or an interval.

Dislocations with corners in an elastic body with applications to fault detection

Qingle Meng (City University of Hong Kong)

In this talk, we pay attention to an elastic dislocation problem, which is motivated by applications in the geophysical and seismological communities. In our model, the displacement satisfies the Lamé system in a bounded domain with a mixed homogeneous boundary condition. We also allow the occurrence of discontinuities in both the displacement and traction fields on the fault curve/surface. By the variational approach, we first prove the well-posedness of the direct dislocation problem in a rather general setting with the Lamé parameters being real-valued L^∞ functions and satisfying the strong convexity condition. Next, by considering that the Lamé parameters are constant and the fault curve/surface possesses certain corner singularities, we establish a local characterization of the jump vectors at the corner points over the dislocation curve/surface. In our study, the dislocation is geometrically rather general and may be open or closed. We establish the unique results for the inverse problem of determining the dislocation curve/surface and the jump vectors for both cases. This work is joint work with Huaian Diao (Jilin University) and Hongyu Liu (City University of Hong Kong).

Fractional Borg-Levinson problem

Tuhin Ghosh (Harish-Chandra Research Institute)

In this talk, we will explore how Gel'fand spectral data can be used to determine the perturbation potential associated with the fractional Laplacian on a smooth bounded domain.

Lipschitz stability of inverse problems for the time-fractional advection-diffusion equation and its relation to the linear Boltzmann equation

Manabu Machida (Kindai University)

We will consider the advection-diffusion equation perturbed by a time-fractional term and investigate inverse problems of determining coefficients of this time-fractional advection-diffusion equation.
In the first part, I will show that the time-fractional advection-diffusion equation appears in the asymptotic limit of the linear Boltzmann equation in which the waiting time is taken into account. In the second part, I will prove the Lipschitz stability in determining coefficients of the fractional diffusion equation.

Recovery of spatially varying order of time derivation in subdiffusion

Jiho Hong (Chinese University of Hong Kong)

We demonstrate some uniqueness and stability theorems for the inverse problem to determine the variable order of time derivation for the time-fractional subdiffusion. The variable order, i.e. the slowness of diffusion, is a function defined on the spatial domain in various dimensions. We introduce some extra information, e.g. monotonicity, and prove that this allows the stable determination of the variable order from the boundary measurement at just one excitation and at one point. The proofs are based on the asymptotic behavior of the Laplace transform of the boundary data at certain frequencies. This is a joint work with Prof. Dr. Jin and Prof. Dr. Kian.

On inverse problems in coupled biological models

Yuhan Li (City University of Hong Kong)

This report addresses the unique identifiability of unknown parameters in coupled biological systems from boundary measurements. Our primary focus is on Lotka-Volterra predator-prey models, where the non-negativity of solutions—representing population densities—poses a significant challenge for inverse problems. We overcome this by employing the high-order variation method, which guarantees solution positivity. A key innovation in our work is the choice of a more general base solution, enabling the simultaneous recovery of all interaction terms, including those related to prey attack, crowding, carrying capacity, and other critical factors. We illustrate these results through applications to the hydra-effects.
Building on this foundation, we then tackle a far more complex and unexplored problem: the unique identification of parameters in a fully coupled nonlinear system of mixed parabolic-elliptic-elliptic type, which models attraction-repulsion chemotaxis. The mixed-type nature of this system introduces profound theoretical obstacles not present in purely parabolic models. To overcome this, we develop a novel approach that strategically construct special CGO form as well as the combination of fundamental theorem of calculus and inverse Fourier transform.

Inverse stability for hyperbolic equations

Shiqi Ma (Jilin University)

Coefficient recovery of hyperbolic equations using boundary measurement is one of the very classical inverse problems. The inverse stability of coefficient recovery has been under extensive study. Bukhgeim and Klibanov had shown some preliminary results in early 80s. Later on, Japanese mathematician Masahiro Yamamoto and others have established Lipschitz stability for this problem using Carleman estimates. However, current results usually require that the initial data of the equation possess some positiveness condition and requires a time-reflection operation. These conditions limit the application of such results to other cases such as the inverse scattering problem for the wave equation using fixed-angle data. Currently, the single incident wave inverse scattering problem remains an open challenge. In this talk we provide a review of the aforementioned areas and presents the latest research findings.

SVD-based spectral approaches for efficient imaging and learning

Soomin Jeon (Dong-A University)

Singular Value Decomposition (SVD) remains a cornerstone of applied mathematics, especially in problems of data compression and efficient representation. In this talk, I will highlight two applications of SVD that address pressing challenges in medical imaging and machine learning. The first involves medical image compression, where we introduce an adaptive rank selection method to produce compact yet clinically useful image representations. The second examines active learning from the perspective of Neural Tangent Kernels (NTKs), using spectral properties of NTK matrices to identify the most informative training batches. Across both settings, SVD provides a unifying framework for maximizing information content while reducing computational cost. These studies illustrate how spectral methods can drive efficiency and impact across diverse domains.

Deep variational EIT via coupled neural potentials and stream functions

Kwancheol Shin (Ewha Womans University)

We propose a variational deep learning framework for solving the inverse conductivity problem in Electrical Impedance Tomography (EIT). The governing equation is reformulated as a system of first-order partial differential equations by introducing both the electrical potential and a streamline function representing the divergence-free current density. This leads to two coupled convex minimization problems: one corresponding to a Dirichlet problem for the potential, and the other to a Neumann problem for the current density. By combining these into a single variational functional, we train neural networks for the potential and the streamline function in an unsupervised manner, using boundary measurements only. The conductivity distribution is subsequently recovered explicitly from the learned fields through a closed-form update formula.

Quantum state tomography under the logarithmic loss

Yen-Huan Li (National Taiwan University)

Quantum state tomography refers to the task of estimating the quantum state of an unknown quantum system from measurement outcomes. It is essential both for verifying the correctness of quantum computing units and for extracting information from quantum computations. This talk focuses on quantum state tomography under the logarithmic loss. In the batch setting, this corresponds to maximum-likelihood quantum state tomography; in the online setting, it gives rise to a non-commutative generalization of online portfolio selection, a classical open problem in online learning theory.

The logarithmic loss introduces significant analytical challenges: It is not globally Lipschitz, nor does it have a Lipschitz gradient, so standard arguments in optimization and learning theory do not directly apply. In this talk, I will present our recent results on batch and online algorithms for quantum state tomography under the logarithmic loss, along with some surprising applications and open problems.

Guide star in the speckle, mathematical framework for speed of sound reconstruction in medical ultrasound imaging

Pierre Millien (Institut Langevin)

We present a mathematical model and analysis for a new experimental method [Bureau and al., arXiv:2409.13901, 2024] for effective sound velocity estimation in medical ultrasound imaging. Using recent results on stochastic homogenization of the Helmholtz equation, we provide a representation formula for the field scattered by a random multi-scale medium (whose acoustic behavior is similar to a biological tissue) in the time harmonic regime. We then prove that statistical moments of the imaging function can be accessed from data collected with only one realization of the medium. We show that it is possible to locally extract the point spread function from an image constituted only of speckle and build an estimator for the effective sound velocity in the micro-structured medium.

Analytical formulae for flow resistance of a periodic shrouded-fin heat sink: exact and asymptotic solutions

Hiroyuki Miyoshi (The University of Tokyo)

Analytical formulae are developed for the friction factor times Reynolds number (fRe) for fully-developed flow through a 5-sided duct. The value fRe is relevant to the flow resistance of longitudinal-fin heat sinks with or without clearance between the tips of the fins and a shroud as per the classic problem numerically resolved by Sparrow *et al*. [ASME J. Heat Transfer, 100, 1978]. Simple formulae for fRe in the limit of vanishing fin thickness are derived via matched asymptotic expansions in two settings: (i) when the fin spacing to fin height is small, and (ii) when the fin clearance to fin height is small. Numerical calculations show that the asymptotic formulae remain good approximants for fRe even outside the asymptotic parameter range for which they were derived.
This work was carried out in collaboration with T. Kirk at University of Southampton, M. Hodes at Tufts University, and D. G. Crowdy at Imperial College London, and has recently been published in Journal of Fluid Mechanics, 991 A2, 2024.

Time-domain direct sampling method for inverse scattering problems with a single incident source

Xianchao Wang (Harbin Institute of Technology)

In this talk, we introduce the inverse medium scattering problem of reconstructing unknown objects from time-dependent boundary measurements. A novel time-domain direct sampling method is developed for determining the locations of unknown scatterers by using only a single incident source. Notably, our method imposes no restrictions on the waveform of the incident wave. Based on the Fourier--Laplace transform, we first establish the connection between the frequency-domain and the time-domain direct sampling method. Furthermore, we elucidate the mathematical mechanism of the imaging functional through the properties of modified Bessel functions. Theoretical justifications and stability analyses are provided to demonstrate the effectiveness of the proposed method. Finally, several numerical experiments are presented to illustrate the feasibility of our approach.

Reconstructing cell differentiation dynamics via optimal transport

Toshiaki Yachimura (Tohoku University)

Optimal transport (OT) theory provides a variational framework to define distances and optimal matchings between probability measures. Recent theoretical and computational advances in entropic optimal transport have made it feasible to apply OT to large-scale data, leading to a wide range of applications such as image processing, machine learning, and single-cell biology.

In this talk, I will introduce the entropic Gaussian mixture optimal transport (EGOT) and its applications to single-cell biology. In particular, I will present scEGOT, a comprehensive framework developed based on this theory for the reconstruction of cell differentiation dynamics. Unlike conventional trajectory inference methods that only construct cell-state graphs, scEGOT is capable of estimating gene expression velocities, dynamic trajectories (animations), Waddington-like potential landscapes representing differentiation plasticity, and gene regulatory networks that shape these dynamics, all within a unified and computationally efficient framework.

As a case study, I will report results obtained by applying scEGOT to time-series scRNA-seq data on the induction of human primordial germ cell-like cells (hPGCLCs) from iPS cells. The method successfully identified novel marker genes associated with PGC progenitors and their induction processes. If time permits, I will also briefly discuss my recent work on mixture Wasserstein spline interpolation.
