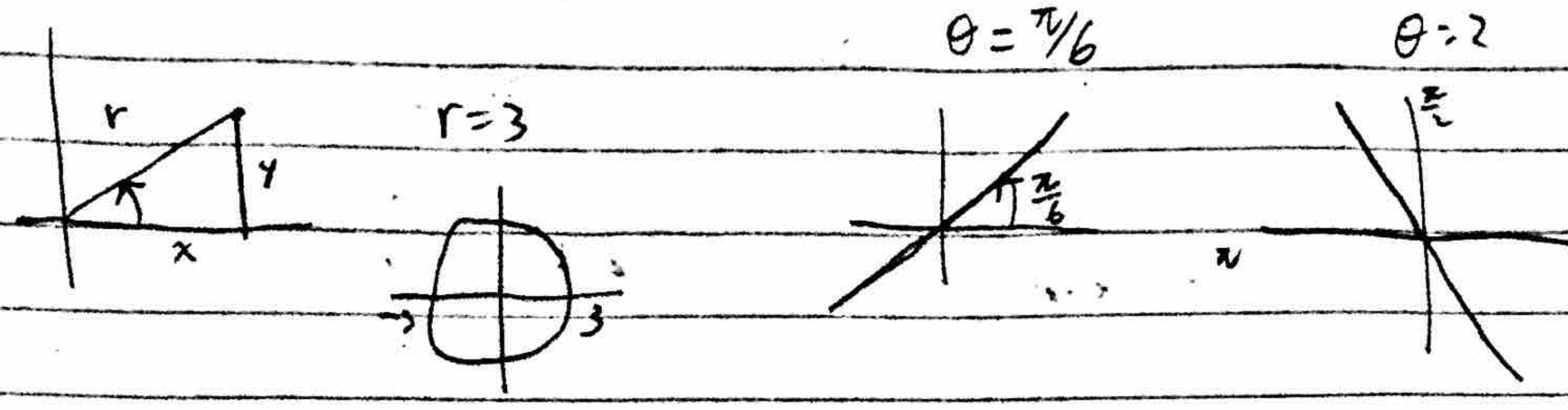
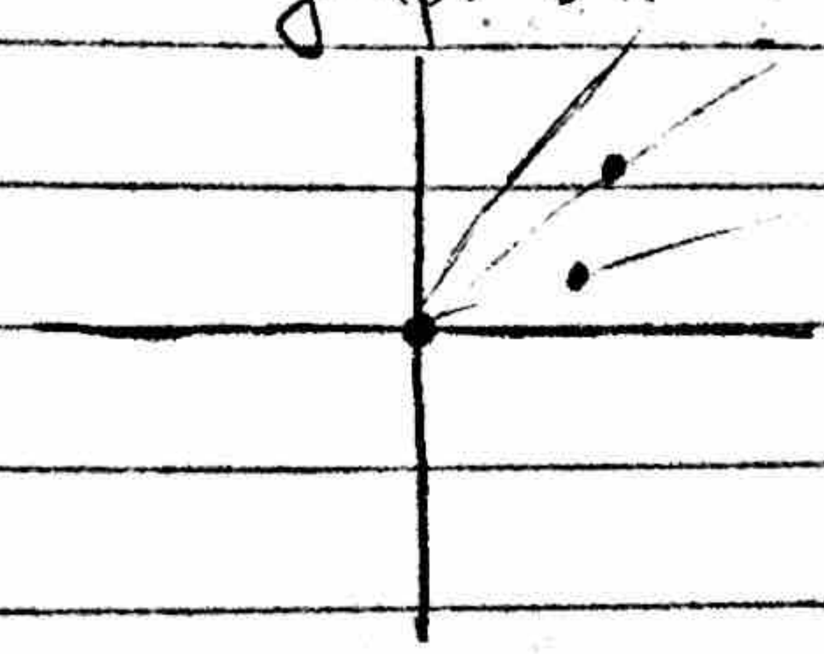
Professor Um
SSL: Eason

Math 20B Lecture 5 7/11

$(r, \theta)_P = (x, y)_R = (x, y)$



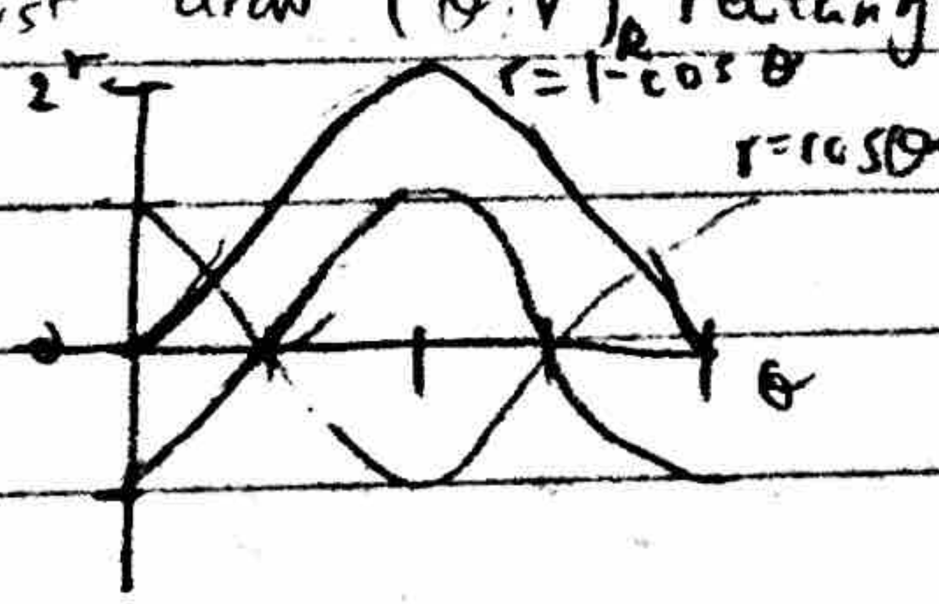
The graph of $r = 1 - \cos\theta$

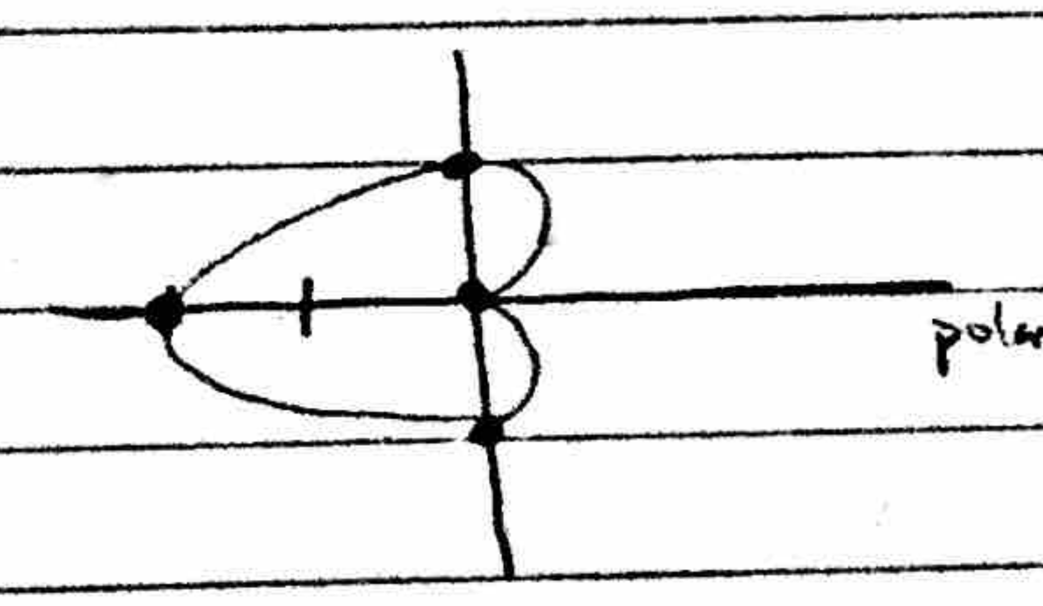


very hard to tell from table

θ	r
0	0
$\frac{\pi}{6}$	$1 - \frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$1 - \frac{1}{\sqrt{2}}$

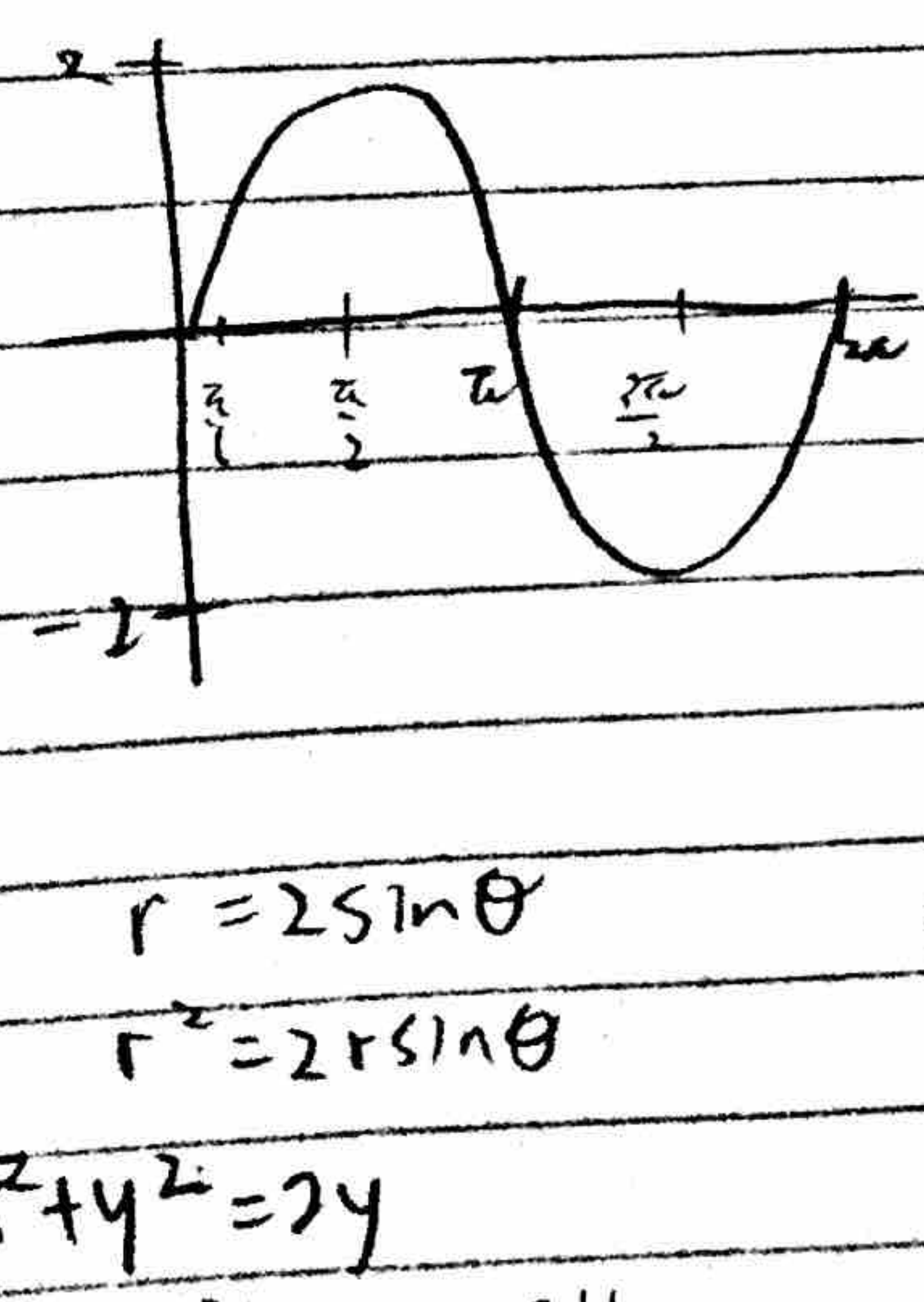
First draw $(\theta, r)_R$ rectangular coordinates

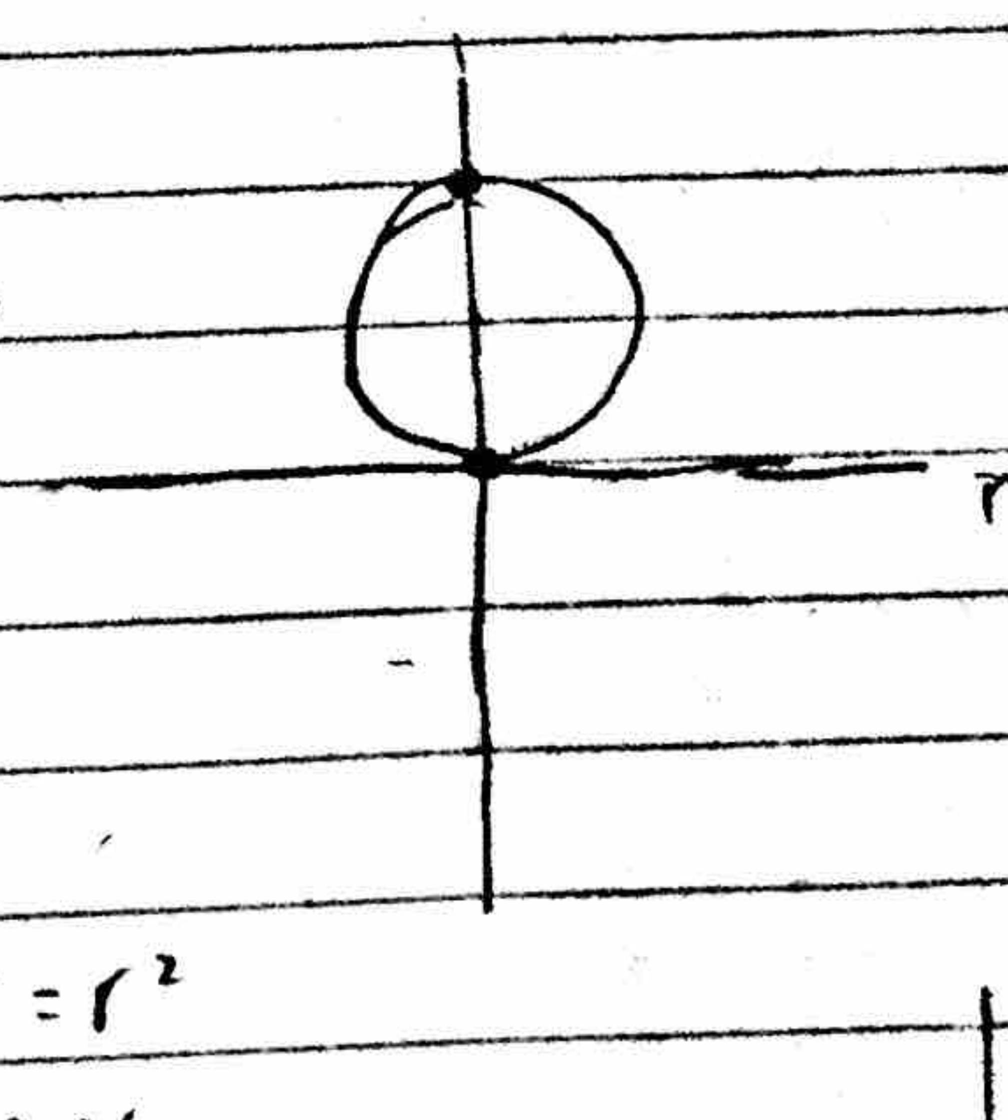




$r = 1 - \cos\theta$ is called cardio

(i) The graph of $r = 2\sin\theta$





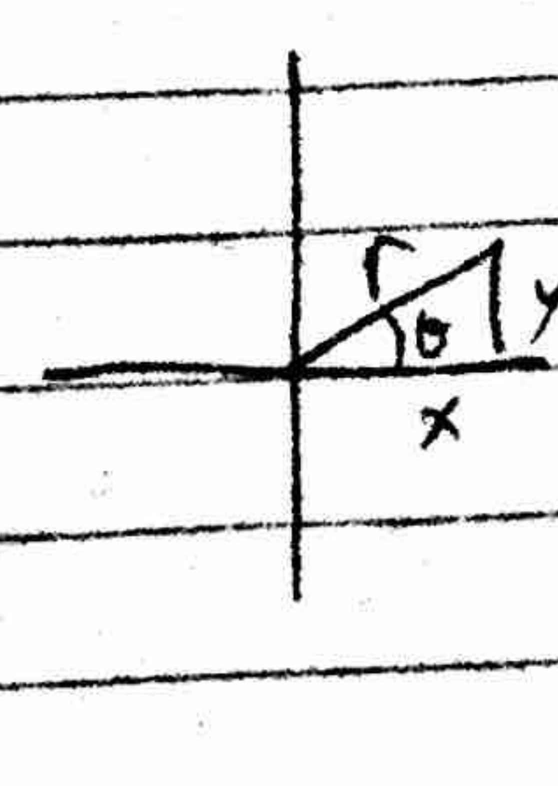
$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

$r = 2\sin\theta$ $\qquad$ $x^2 + y^2 = r^2$

$r^2 = 2r\sin\theta$ $\qquad$ $x = r\cos\theta$

$x^2 + y^2 = 2y$ $\qquad$ $y = r\sin\theta$



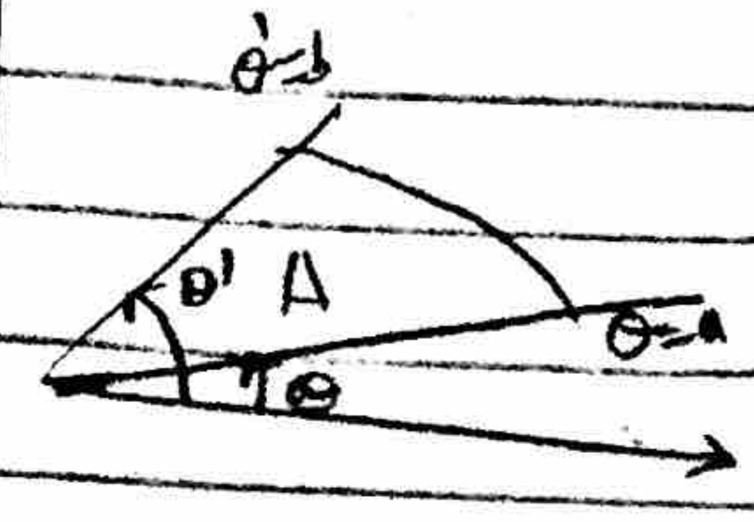
$x^2 + y^2 - 2y + 1 = 0 + 1$

$x^2 + (y-1)^2 = 1^2$

circle centered at $(0,1)$ with radius 1.

11.4 Area in Polar Coordinates (skip length)

$r = f(\theta)$

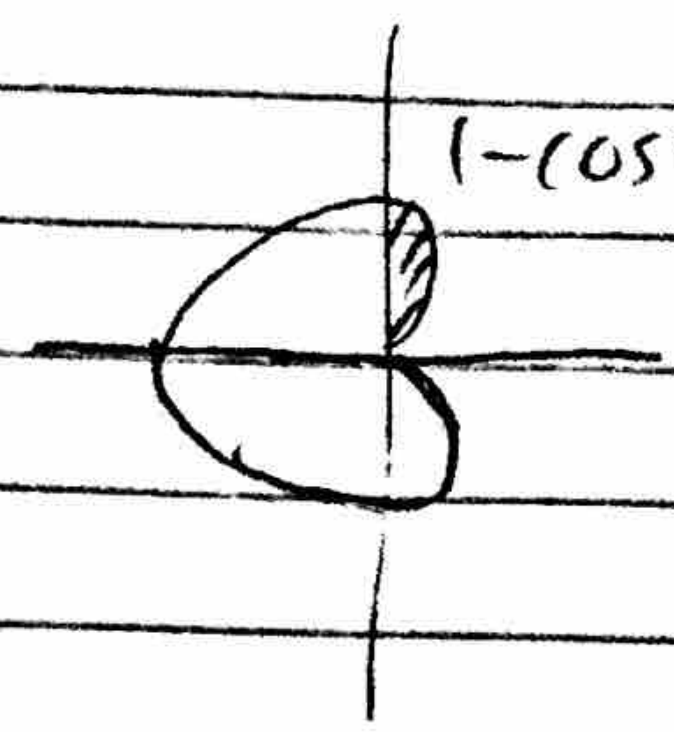


$$\frac{A_1}{\pi(f(\theta_1))^2} = \frac{\Delta\theta}{2\pi} \rightarrow A_1 = \frac{\pi}{2\pi}(f(\theta_1))^2\,\Delta\theta$$

$$= \frac{1}{2}\left[f(\theta_1)\right]^2\Delta\theta$$

$$\text{Area} = \lim_{N\to\infty} \frac{1}{2}(f(\theta_1))^2\Delta\theta + \frac{1}{2}(f(\theta_2))^2\Delta\theta + \dots \frac{1}{2}(f(\theta_N))^2\Delta\theta$$

$$\stackrel{\text{def}}{=} \int_{\theta=a}^{b} \frac{1}{2}(f(\theta))^2\,d\theta$$



$$\text{Area} \int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 1 - 2\cos\theta + \cos^2\theta\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} 1 - 2\cos\theta + \frac{1+\cos(2\theta)}{2}\,d\theta$$

Trig ID

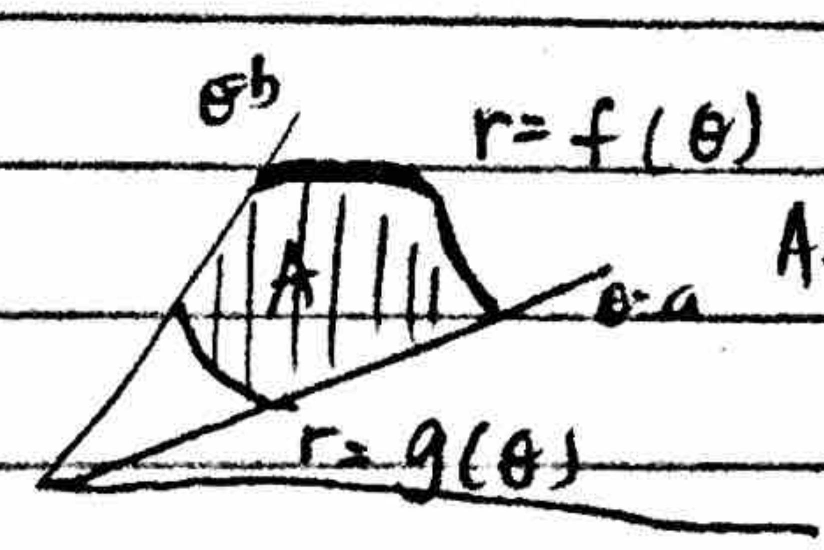
$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = \frac{1+\cos(2x)}{2}$

$\sin^2 x = \frac{1-\cos(2x)}{2}$

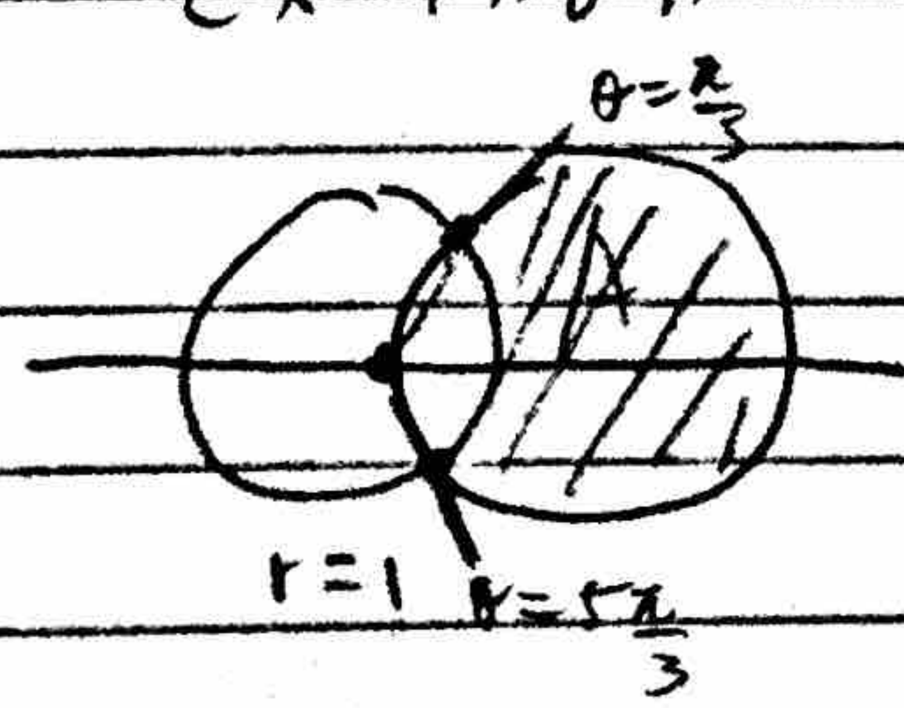
$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}\left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{2}\frac{\sin(2\theta)}{2}\right]_0^{\frac{\pi}{2}} = \frac{3\pi}{8} - 1$$



$$A = \int_a^b \frac{1}{2}(f(\theta))^2\,d\theta - \int_a^b \frac{1}{2}((g\theta))^2\,d\theta$$

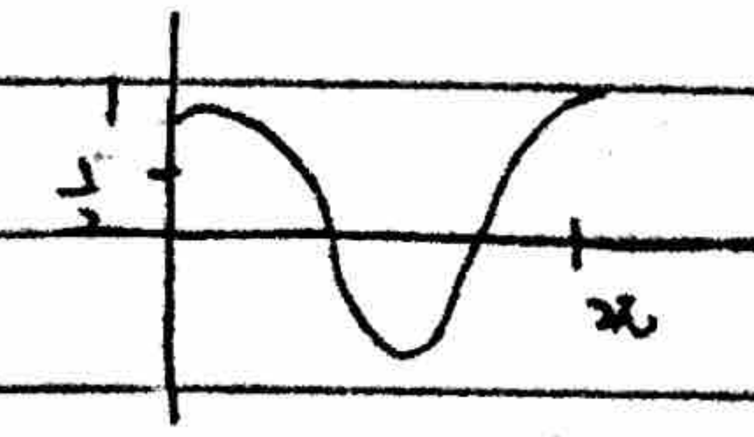
Ex Find A

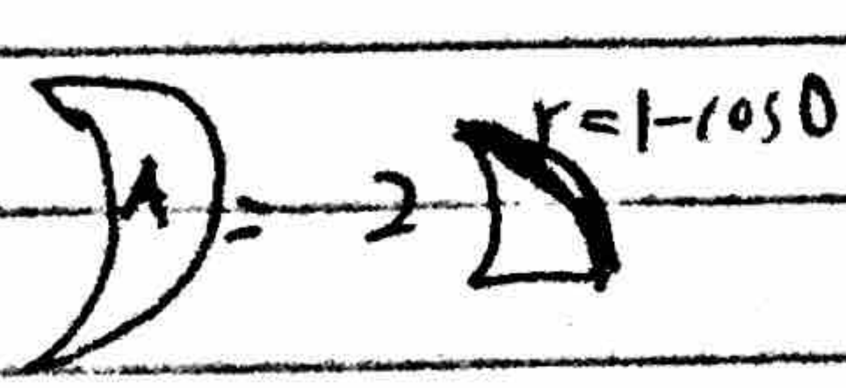


$1 = 2\cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$





$$2\left\{\int_0^{\frac{\pi}{3}} \frac{1}{2}(1-\cos\theta)^2\,d\theta - \int_0^{\frac{\pi}{3}} \frac{1}{2}\cdot 1^2\,d\theta\right\}$$

7.2 Trignometric Integrals

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos^2 x = \frac{1 + \cos 2x}{2}$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\int \sin^4 x \cdot \cos^3 x \, dx$ $\quad u = \sin x$, $du = \cos x \, dx$

$= \int \sin^4 x \cos^2 x \cdot \cos x \, dx$

$= \sin^4 x (1 - \sin^2 x) \, du$

$= \int u^4 (1 - u^2) \, du$

$= \int u^4 - u^6 \, du$

$= \frac{u^5}{5} - \frac{u^7}{7} + C$

$= \frac{(\sin x)^5}{5} - \frac{(\sin x)^7}{7} + C$

$\int \sin^{any} x \cos^{odd} x \, dx$

$u = \sin x$
$du = \cos x \, dx$

$\int \sin^{odd} x \cdot \cos^{any} x \, dx$

$u = \cos$
$du = -\sin x \, dx$