Math 20B Lecture 4 7/10/2017

Professor Um
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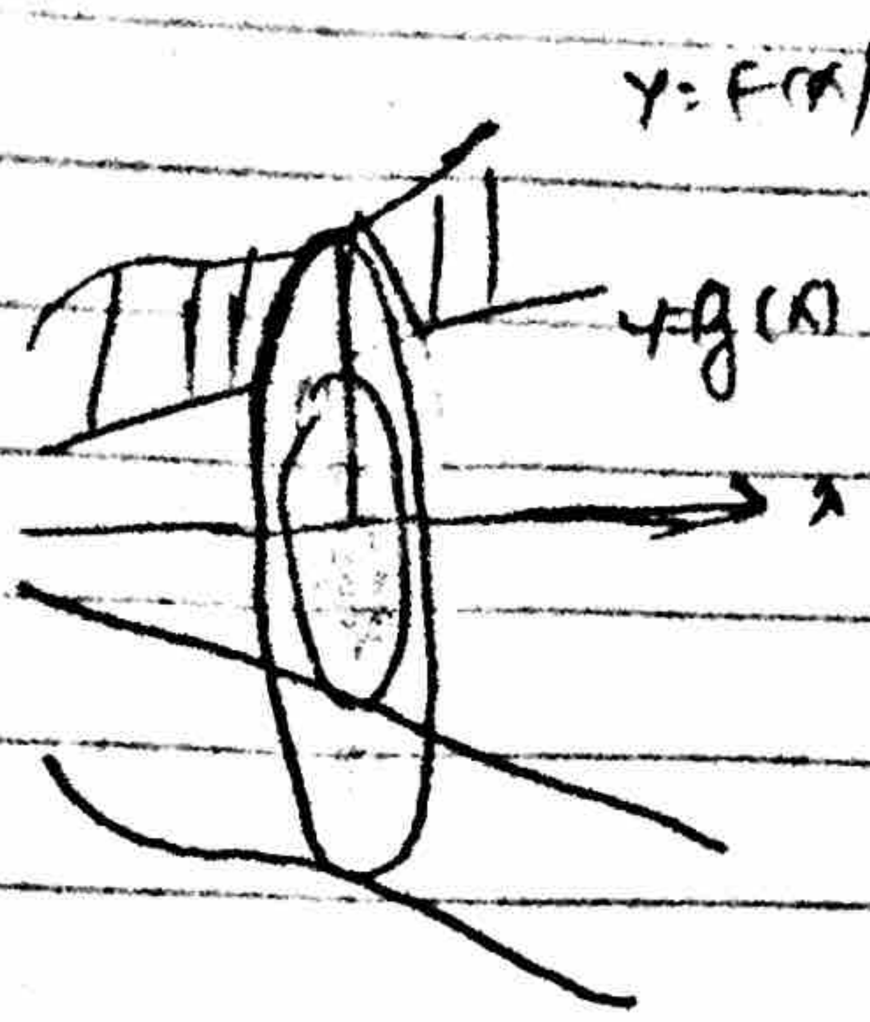
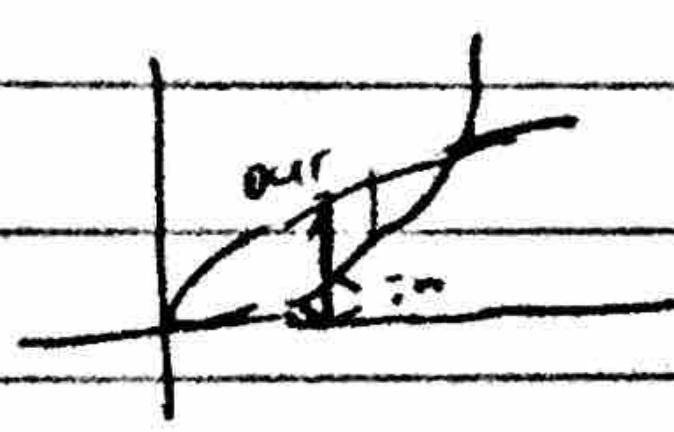
Review



$$Vol = \int_a^b \pi \left(R_{out}^2 - R_{in}^2 \right) dx$$

$R_{out} = f(x)$, $R_{in} = g(x)$

Find the vol of a solid obtained by rotating the region below about x-axis



$x^2 = \sqrt[3]{x}$

$x^6 = x$

$x^6 - x = 0$

$x(x^5 - 1) = 0$

$x = 0$ or 1

$$Vol = \int_{x=0}^{1} \pi \left(R_{out}^2 - R_{in}^2 \right) dx$$

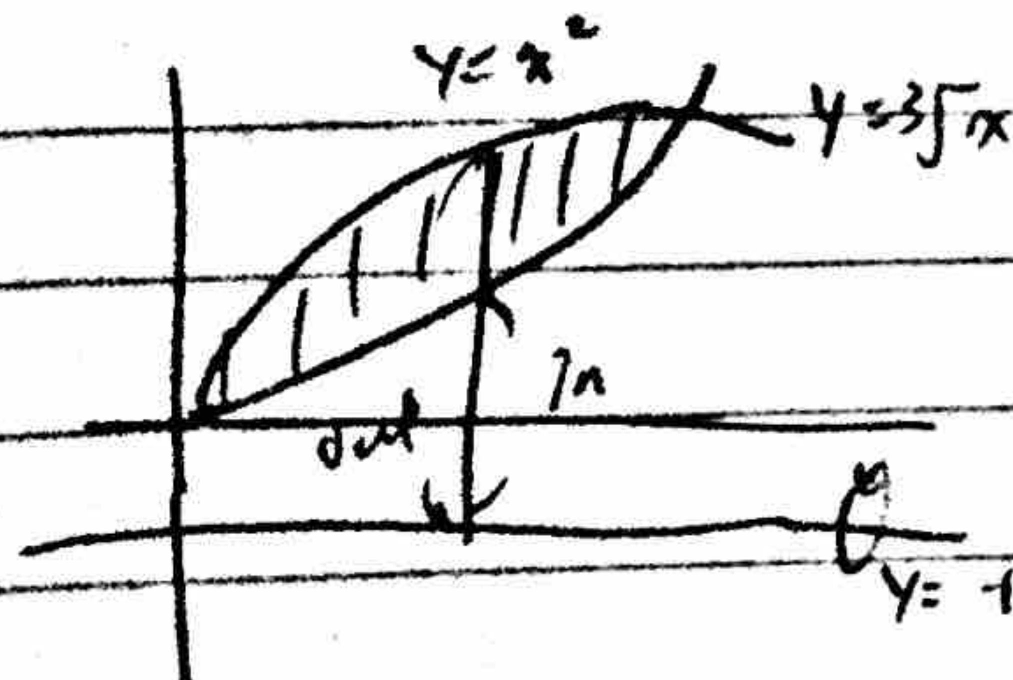
$$= \int_0^1 \pi \left((\sqrt[3]{x})^2 - (x^2)^2 \right) dx$$

$$= \pi \int_0^1 x^{\frac{2}{3}} - x^4 \, dx$$

$$= \pi \left(\frac{3}{5} x^{\frac{5}{3}} - \frac{x^5}{5} \right]_0^1$$

$$= \pi \left(\frac{3}{5} - \frac{1}{5} \right) = \frac{2}{5}\pi$$

(ii) about $y = -1$



$$Vol = \int_0^1 A(x) \, dx$$

$$A(x) = \pi \left(R_{out}^2 - R_{in}^2 \right)$$

$$= \pi \left((\sqrt[3]{x} - (-1))^2 - (x^2 + 1)^2 \right)$$

$$= \pi \left(x^{\frac{2}{3}} + 2x^{1/3} + 1 - (x^4 + 2x^2 + 1) \right)$$

$$A(x) = \pi \left(x^{2/3} + 2x^{1/3} - x^4 - 2x^2 \right)$$

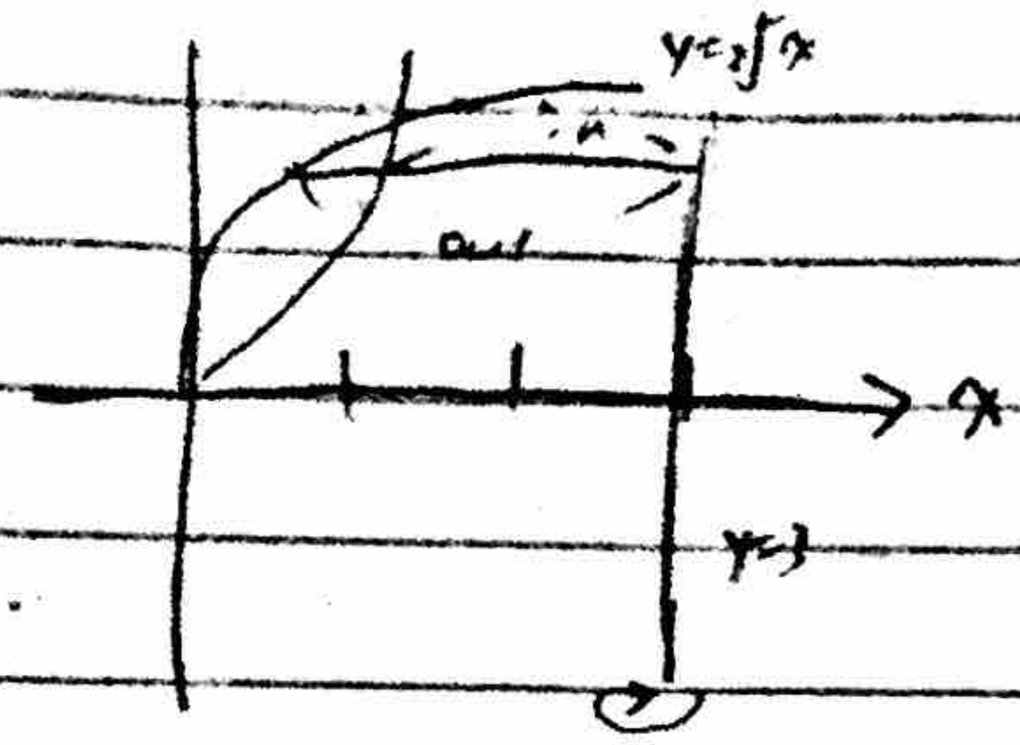
$$Vol = \int_0^1 \pi \left(x^{2/3} + 2x^{1/3} - x^4 - 2x^2 \right) dx \Rightarrow \frac{37\pi}{30}$$

(iii) about $y = 5$

$$A(x) = \pi \left(R_{out}^2 - R_{in}^2 \right)$$

$$= \pi \left((5 - x^2)^2 - (5 - \sqrt[3]{x})^2 \right)$$

(iv) about $x = 3$



$$Vol = \int_{y=0}^{1} A(y)\,dy = \text{skip} = \frac{15}{7}\pi$$

$$A(y) = \pi\left(R_{out}^2 - R_{in}^2\right)$$
$$= \pi\left(\left(3-x_L\right)^2 - \left(3-x_R\right)^2\right)$$
$$= \pi\left((3-y^3)^2 - (3-\sqrt{y})^2\right)$$

$y = \sqrt[3]{x_L} \Rightarrow y^3 = x$

$y = x_R^2 \Rightarrow \sqrt{y} = x$

$\int A(y)\,dy$ $\qquad\qquad$ $\int A(x)\,dx$

7.1 Integration By Parts

$\int x e^x\,dx = ?$

$(xe^x)' = (x)'e^x + x(e^x)'$

$\int (xe^x)'\,dx = \int e^x + xe^x\,dx$

$\Rightarrow xe^x + C = \int e^x\,dx + \int xe^x\,dx$

$\int xe^x\,dx = xe^x - \int e^x$

$= xe^x - e^x + C$

$$\int (u\cdot v)' = \int \frac{du}{dx}\cdot v + u\cdot\frac{dv}{dx}\,dx$$

$$uv = \int \frac{du}{dx}\,v\,dx + \int u\,\frac{dv}{dx}\,dx$$

$$uv = \int v\cdot du + \int u\,dv$$

Note

Suppose $\int v\cdot du$ is easier than $\int u\,dv$

$\int u\,dv = uv - \int v\,du$ $\qquad$ Int by Parts

$\ln x$ $\quad x^n$ $\qquad \sin$

$\tan^{-1}(x)$

$\sin^{-1}(x)$

$u \longleftarrow\!\!\longrightarrow dv$

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Ex Find

1) $\underbrace{\int x}_{u} \underbrace{\sin(3x)\,dx}_{dv} = \underbrace{-\frac{1}{3}x\cos(3x)}_{u\ v} + \underbrace{\int + \frac{\cos(3x)}{3}dx}_{v \cdot du}$

$= -\frac{1}{3}\cos(3x + \frac{1}{3}\int \cos(3x)\,dx$

$u = x \qquad dv \quad \sin 3x\,dx$

$du = 1 \qquad v = -\frac{1}{3}\cos 3x$

$= -\frac{1}{3}x\cos(3x) + \frac{1}{9}(\sin 3x) + C$

2) $\int \frac{\ln x}{x^2}dx = \underbrace{\int \ln x \cdot x^{2}\,dx}_{u\ dv} = \underbrace{\ln x \cdot x^{-1}}_{u\ v} + \underbrace{\int + x^{-2}\,dx}_{v \cdot du}$

$u = \ln x \qquad dv = x^{-2}$

$du = \frac{1}{x} \qquad dv = -x^{-1}$

$= -\frac{\ln x}{x} + \frac{x^{-1}}{-1} + C$

$= -\frac{\ln}{x} - \frac{1}{x} + C$

3) $\underbrace{\int e^{-x}\sin x\,dx}_{A}$ $\qquad u = \sin x \quad dv = e^{-x}dx$

$du = \cos x\,dx \quad v = -e^{-x}$

$= -e^{-x}\sin x + \int + e^{-x}\cos x\,dx$

$u = \cos x \quad dv = e^{-x}dx$

$= -e^{-x}\sin x + -e^{-x}\cos x - \int e^{-x}\sin x\,dx$ $\qquad du = -\sin x\,dx \quad v = -e^{-x}$

$= -e^{-x}\sin x - e^{-x}\cos x \underbrace{-\int e^{-x}\sin x\,dx}_{A}$

$A = -e^{-x}\sin x - e^{-x}\cos x - A$

$\frac{2A}{2} = \frac{-e^{-x}\sin x - e^{-x}\cos x + C}{2}$

$$\int_a^b u\,dv = u\cdot v\Big]_a^b - \int_a^b v\cdot du$$

why $\int_a^b (uv)' = \int_a^b \left(\frac{du}{dx} v + u\frac{dv}{dx}\right) dx$

$$uv\Big]_a^b = \int_a^b v\,du + \int_a^b u\,dv$$

Ex. $\int_1^3 \ln x \, dx$ $\qquad u = \ln x \quad dv = dx$

$\qquad\qquad du = \frac{1}{x}dx \quad v = x$

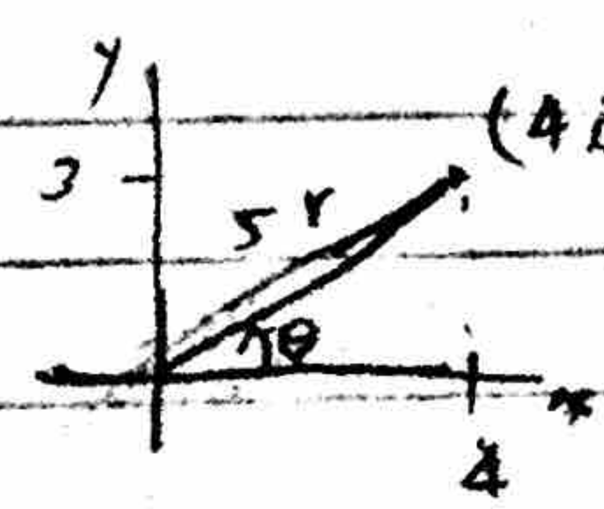
$= x\ln x\Big]_1^3 - \int_1^3 dx$

$= 3\ln 3 - 1\ln 1 - [x]_1^3$

$= 3\ln 3 - (3-1)$

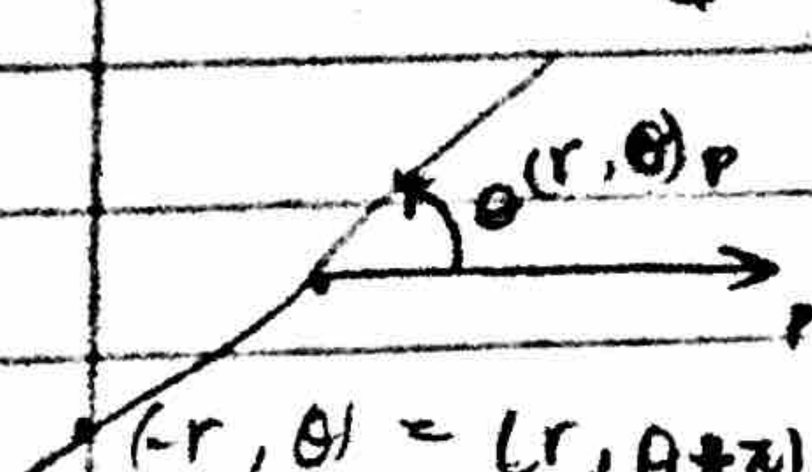
$= 3\ln - 2$

11.3 Polar Coordinates



$(4,3)_R$ rectangular coordinate

$= (r,\theta)_p$: polar coordinate



θ $\quad + \circlearrowleft$ counterclockwise

$\qquad - \circlearrowright$ clockwise

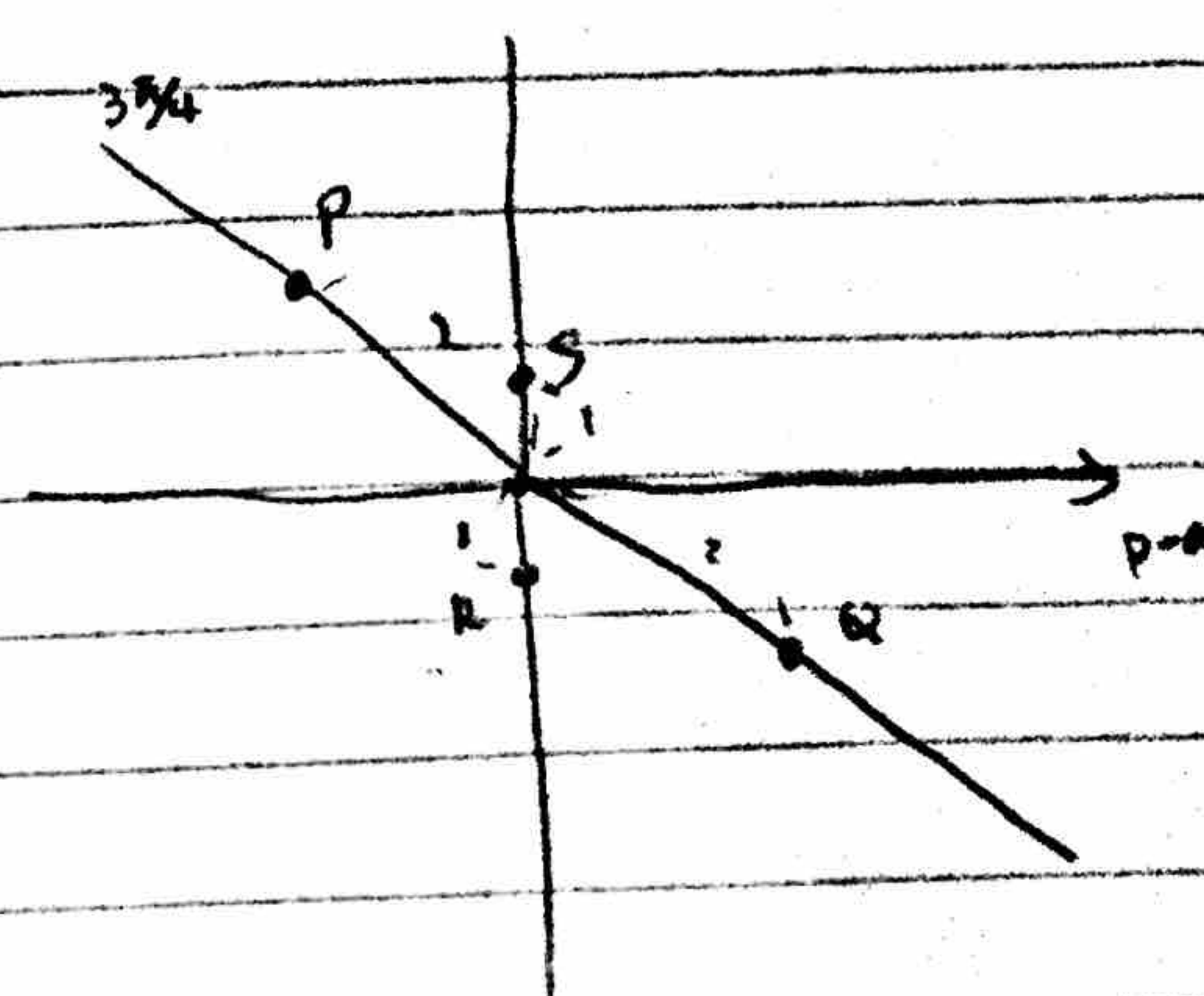
$(r,\theta) = (r, \theta + 2k\pi) \quad k\in\mathbb{Z}$

Ex. Plot $P = (2, 3\pi/4)_p$

$Q = (-2, 3\pi/4)_p$

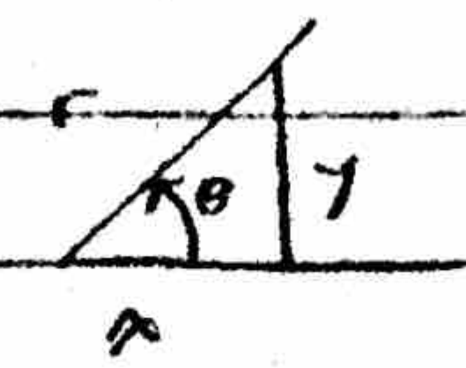
$R = (1, -\frac{\pi}{2})_p$

$S = (-1, -\frac{\pi}{2})_p$



$(x, y)_R = (r, \theta)_P$

$(r, \theta)_P \Rightarrow (x, y)_R$



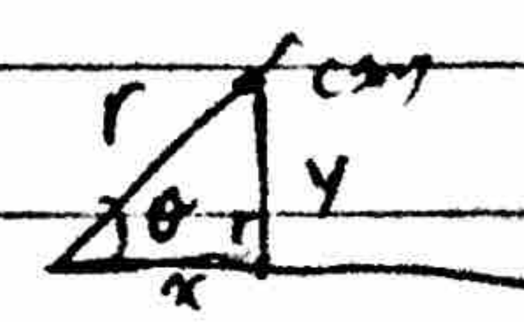
$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r} = y = r\sin\theta$

Ex. $(2, 3\pi/4)_P = (x, y)_R$

$x = 2\cos(3\pi/4) = -\sqrt{2}$
$y = 2\sin(3\pi/4) = \sqrt{2}$ $\Rightarrow (-\sqrt{2}, \sqrt{2})_R$

$(x, y)_R \Rightarrow (r, \theta)_P$



$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

Ex. $(-2, 2\sqrt{3})_R \Rightarrow (r, \theta)_P$

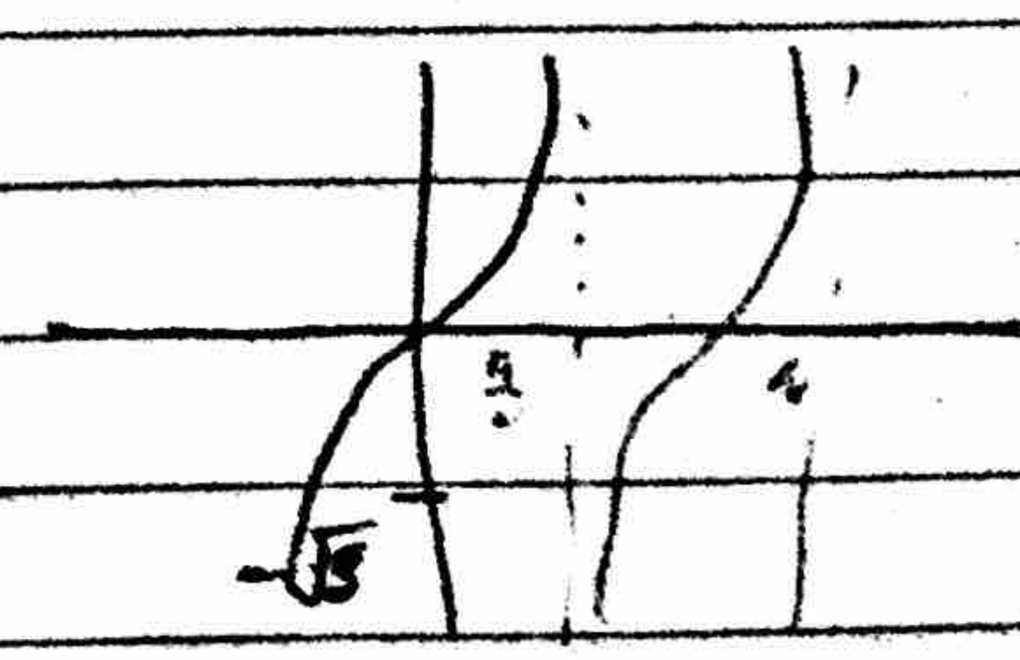
$\tan\theta = \frac{y}{x}$ $(x \neq 0)$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ $(x \neq 0)$

$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (2\sqrt{3})^2}$

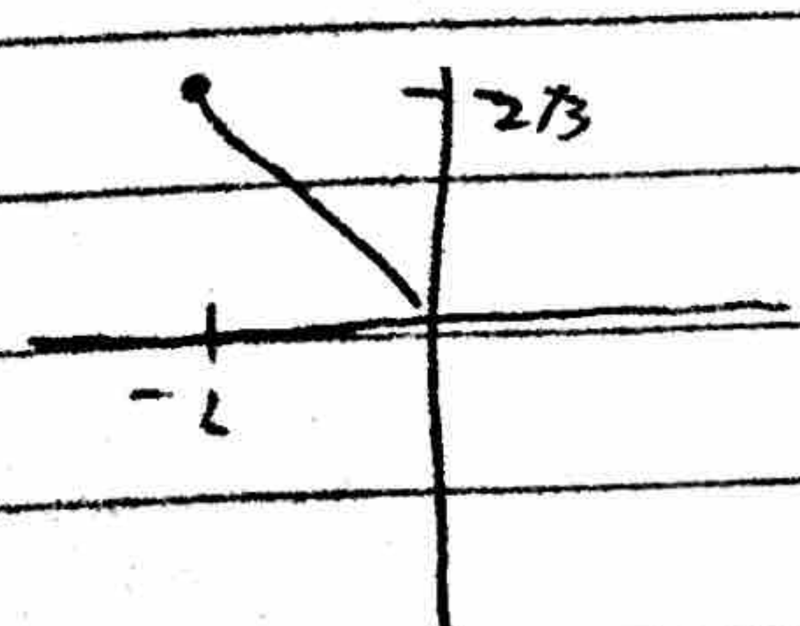
if $x = 0$, $\theta = \frac{\pi}{2}$ or $-\frac{\pi}{2}$

$= \sqrt{4 + 12} = \sqrt{16} = 4$



$\tan\theta = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\theta = \frac{2\pi}{3}$



$\left(4, \frac{2\pi}{3} + 2n\pi\right)_P$