Math 20B Lecture 7 7/13/2019

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Ex. $\int \frac{18}{(x+3)(x^2+9)}\,dx$

$\frac{A}{x+3} + \frac{Bx+C}{x^2+9}$

$= \int \frac{1}{x+3}\,dx - \int \frac{x-3}{x^2+9}$

$= \ln|x+3| - \int \frac{x}{x^2+9}\,dx + 3\int \frac{1}{x^2+9}\,dx$

$= \ln|x+3| - \frac{1}{2}\ln|x^2+9| + \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$

$x^2 - a^2 = (x+a)(x-a)$

Partial Fraction work

$18 = A(x^2+9) + (Bx+C)(x+3)$

$x=-3$ $\quad 18 = A(9+9) \Rightarrow A = 1$

$18 = A(x^2+9 + (Bx+C)(x+3))$

$18 = Ax^2 + 9A + Bx^2 + 3Bx + Cx + 3C$

$18 = \underline{(A+B)}x^2 + \underline{(3B+C)}x + \underline{9A+3C}$

$A+B=0$ $\quad 9A+3C=18$ $\Rightarrow$ $B=-1$, $C=3$

$3B+C=0$

2nd term
U-sub

$u = x^2+9$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+9| + C$

3rd term

$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C$

$\int \frac{1}{x^2+9}\,dx = \frac{1}{9}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx$ $\quad u = \frac{x}{3},\ du = \frac{1}{3}dx$

$= \frac{1}{9}\int \frac{1}{u^2+1}\,3\,du = \frac{3}{9}\tan^{-1}(u) + C$

$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C$

$\int \frac{1}{x^2+a} = \frac{1}{\sqrt{a}}\tan^{-1}\left(\frac{x}{\sqrt{a}}\right) + C$

iii) $\int \frac{18}{(x+3)(x^2-9)} dx$

$= \int \frac{18}{(x+3)(x+3)(x-3)} dx$

$= \int \frac{18}{(x+3)^2(x-3)}$

$= \frac{Ax+B}{(x+3)^2} + \frac{C}{x-3}$

$= \frac{A}{x+3} + \frac{B^* - 3A}{(x+3)^2} + \frac{C}{x-3}$

$= \int \frac{-1/2}{x+3} + \frac{-3}{(x+3)^2} + \frac{1/2}{x-3} dx$

$\frac{18}{(x+3)^2(x-3)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-3}$

$= -\frac{1}{2}\ln|x+3| + \frac{1}{2}\ln|x-3| - 3\int \frac{1}{(x+3)^2} dx$

$18 = A(x+3)(x-3) + B(x-3) + C(x+3)^2$

$= -\frac{1}{2}\ln|x+3| + \frac{1}{2}\ln|x-3| + \frac{3}{x+3} + C$

$A = -\frac{1}{2}$

$B = -3$

$C = \frac{1}{2}$

$u = x+3$

$du = dx$

$\int \frac{1}{u^2} du = -\frac{1}{u} = -\frac{1}{x+3}$

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How to set up A B C

(power lower than 3)

$\frac{P(x)}{(x+3)(2x-11)x} = \frac{A}{x+3} + \frac{B}{2x-11} + \frac{C}{x}$

$\frac{P(x)}{(x+3)(x^2+10)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+10}$

$\frac{P(x)}{(x+1)^3(2x-5)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{2x-5}$

$\frac{1}{x^3(2x-5)} = \frac{Ax^2+Bx+C}{x^3} + \frac{D}{2x-5} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{2x-5}$

$\frac{P(x)}{(x-1)(x^2+2x+5)^3} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+5} + \frac{Dx+E}{(x^2+2x+5)^2} + \frac{Fx+G}{(x^2+2x+5)^3}$

$(\sqrt{\square})' = \frac{1}{2}\square^{-\frac{1}{2}}\square'$

not a rational function

Ex $\int \frac{\sqrt{x+4}}{x}\,dx$ $\qquad u=\sqrt{x+4} \qquad u^2 = x+4$

$\int \frac{u}{x}\, 2\sqrt{x+4}\,du$ $\qquad du = \frac{1}{2}\frac{1}{\sqrt{x+4}}\,dx \qquad x = u^2-4$

$2\sqrt{x+4}\,du = dx$

$\int \frac{u}{u^2-4}\,2u\,du$

$\int \frac{2u^2}{(u+2)(u-2)}\,du$ ← now it's a rational function

$$\begin{array}{r} 2 \\ u^2-4\,\overline{)\,2u^2} \\ -\,2u^2-8 \\ \hline 8 \end{array}$$

long div

$= \int 2 + \frac{8}{(u-2)(u+2)}$

$= 2(u + \ln|u-2| - \ln|u+2|) + C$

$= 2\sqrt{x+4} + 2\ln|\sqrt{x+4}-2| - 2\ln|\sqrt{x+4}-2| + C$

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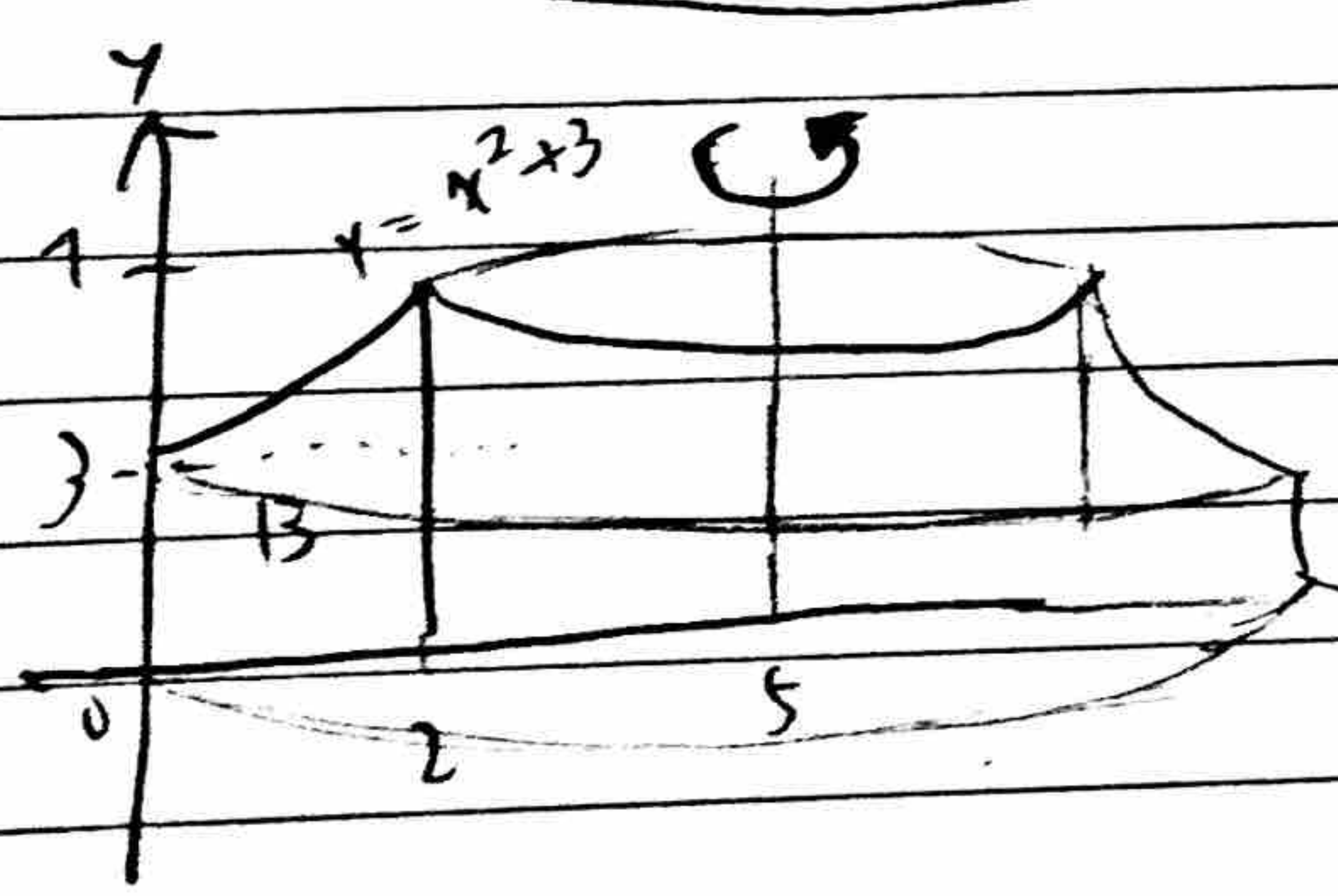



$y = x^2+3$

$x^2 = y-3$

$x = \sqrt{y-3}$

$\pi(25 - 10\sqrt{y-3} + y - 3 - 9$

If $y>3$ $\quad A(y) = \pi(R_{out}^2 - R_{in}^2)$

$= \pi((5-x)^2 - (3)^2)$

$= \pi(25 - 10x + x^2 - 9)$

$\text{Vol} = \int_0^7 A(y)\,dy$

$= \int_0^3 A(y)\,dy + \int_3^7 A(y)\,dy$

If $0 \le y \le 3$ $\quad A(y) = \pi(R_{out}^2 - R_{in}^2)$

$= \pi(5^2 - 3^2)$

$= 16\pi$

Fall 2012

#1 Area enclosed by $y = x^2 + 3x - 2$, $y = 3x + 2$

#2 (i) ~~$\int \tan x \, dx$~~

(ii) $\int \arctan x \, dx$ $\qquad u = \tan^{-1} x \quad dv = 1dx$

$= x \cdot \tan^{-1} x - \int \frac{x}{1+x^2} dx \qquad du = \frac{1}{1+x^2} \qquad v = x$

$= \quad " \quad - \frac{1}{2} \int \frac{1}{a} da$

$a = x^2 + 1$

$= x \cdot \tan^{-1} x - \frac{1}{2} \ln|x^2+1| + C \qquad \frac{da}{2} = \frac{\cancel{2}x \, dx}{\cancel{2}}$

$x^{\frac{1}{2}}$

#3 $\int_4^9 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$    $u = \sqrt{x}$

$-2du = \frac{1}{\sqrt{x}}\,dx$

$\int_4^9 \frac{\sin u}{} (-2\,du)$

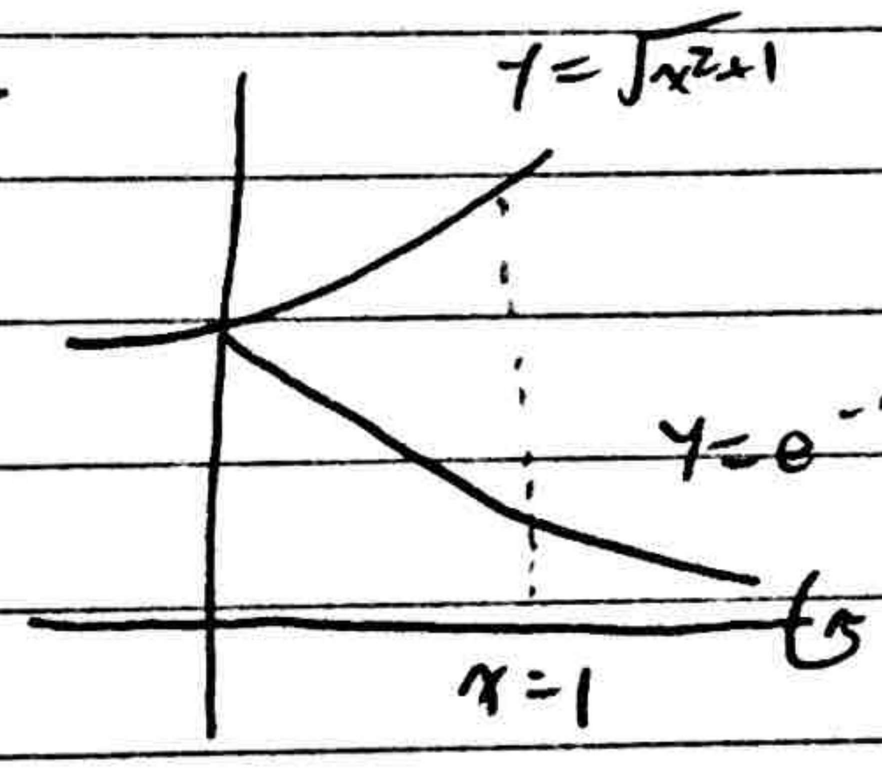
$-2\int_4^9 \sin u\,du$

$-2\left[\cos u\right]_4^9$

$-2\left[\cos\sqrt{x}\right]_4^9$

$= -2\left[\cos 3 - \cos 2\right]$

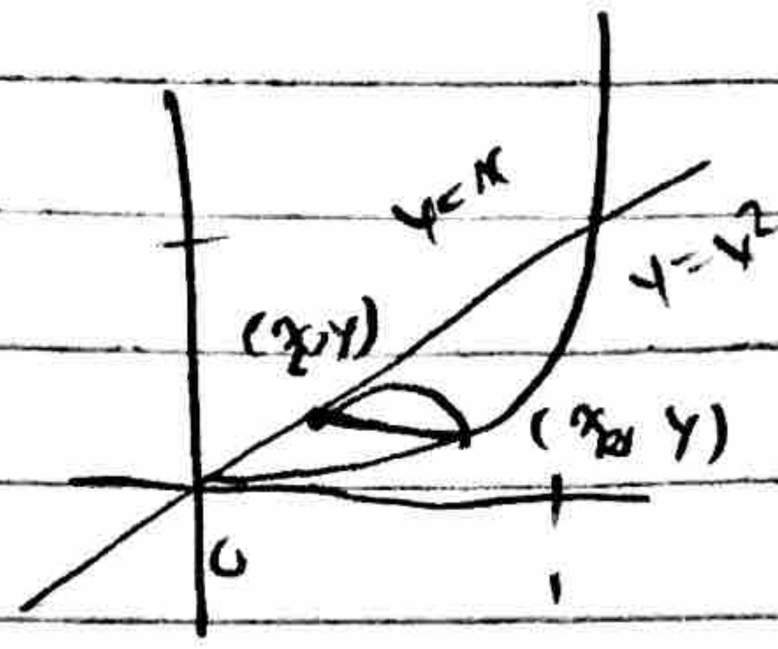
#4



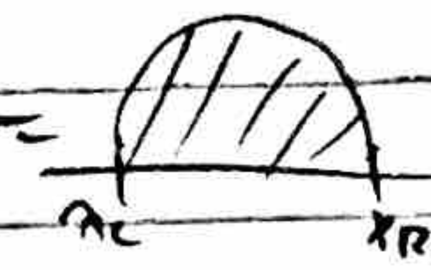
$y = x_R^2$

$y = x_L$

#5




cross section = semi circle

$A(y) =$ 


Find the volume

$x^2 = x$

$x^2 - x = 0$

$x(x+1) = 0$

$x = 0$ or $1$

$\frac{1}{2}\pi\left(\frac{1}{2}(x_R - x_L)\right)^2$

$= \frac{\pi}{8}(x_R - x_L)^2$

$= \frac{\pi}{8}(\sqrt{y} - y)^2$

$$\text{Vol} = \int_0^1 A(y)\,dy$$

$$= \int_0^1 \frac{\pi}{8}(\sqrt{y} - y)^2\,dy$$