Lesson Plan
SI Session #8
August 23, 2017

SI Leader: Eason Chang

Course: Math 18
Academic Quarter: Summer Session 2 2017
Instructor: Professor Drimbe

Topics Covered:
Transformation and Midterm Review

TEACHING + LEARNING
commons

Opener Activity:

5:05pm - 5:10pm

- Midterm Review, brainstorm on what topics and type of problems may show up on the exam and talk to each other.

Activity 1

5:10pm - 5:30pm

Practice Problem 1a:

Problem 2 (4 pts). Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ be a linearly dependent set in $\mathbb{R}^n$. Explain why the set $\{T(\mathbf{v_1}), T(\mathbf{v_2}), T(\mathbf{v_3})\}$ is linearly dependent.

Practice Problem 1a Solutions:

Solution: We want to show that there are constants k_i, not all zero, for which $k_1T(\mathbf{v_1}) + k_2T(\mathbf{v_2}) + k_3T(\mathbf{v_3}) = \mathbf{0}$.

Let c_i be numbers, not all zero, such that $c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3} = \mathbf{0}$. Then because T is linear,

$$\begin{aligned} c_1T(\mathbf{v_1}) + c_2T(\mathbf{v_2}) + c_3T(\mathbf{v_3}) &= T(c_1\mathbf{v_1}) + T(c_2\mathbf{v_2}) + T(c_3\mathbf{v_3}) \\ &= T(c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3}) \\ &= T(\mathbf{0}) \\ &= T(0\mathbf{0}) \\ &= 0T(\mathbf{0}) \\ &= \mathbf{0} \end{aligned}$$

showing that the $T(\mathbf{v_i})$s are linearly independent (with $k_i = c_i$).

Practice problem 1b:

3. (10 points) Let $n \geq 1$ and $T : \mathbb{R}^3 \to \mathbb{R}^n$ be an arbitrary linear transformation. Consider the following vectors

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

Show that $\{T(v_1), T(v_2), T(v_3)\}$ is linearly dependent.
(hint: what can you say about the vectors $\{v_1, v_2, v_3\}$?)

Practice Problem solution 1b:

$$\begin{bmatrix} 1 & -2 & 4 \\ 2 & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

can be transformed by a sequence of elementary row operations to the matrix

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Practice problem 1c: Find A-1

$$\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$$

Solution to practice problem 1c:

$$[A \quad I] = \begin{bmatrix} 1 & -2 & 1 & 1 & 0 & 0 \\ 4 & -7 & 3 & 0 & 1 & 0 \\ -2 & 6 & -4 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 2 & -2 & 2 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 10 & -2 & 1 \end{bmatrix}. \text{ The matrix } A \text{ is not invertible.}$$

Activity 2

5:30pm - 5:45pm

Midterm Review

Practice Problem 2a:

4. (10 points) Let A and B two $n \times n$ matrices with the property that A and BA^2 are invertibles. Prove that B is also an invertible matrix.

Solution to Practice Problem 2a:

Practice Problem 2b:

1. Consider the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & -3 & 1 & 1 \end{bmatrix}$$

(a) (5 points) Find the reduced echelon form of A.

Solution to Practice Problem 2b:

Divide row 1 by 2 $\left(R_1 = \frac{R_1}{2}\right)$:

$$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 & 1 \\ 1 & -3 & 1 & 1 \end{bmatrix}$$

Subtract row 1 from row 2 $(R_2 = R_2 - R_1)$:

$$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & -3 & 1 & 1 \end{bmatrix}$$

Subtract row 1 from row 3 $(R_3 = R_3 - R_1)$:

$$\begin{bmatrix} 1 & \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{9}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Make zeros in column 2 except entry at row 2, column 2 (pivot entry).

Add row 2 multiplied by 3 to row 1 $(R_1 = R_1 + (3)\,R_2)$:

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{9}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Subtract row 2 multiplied by 9 from row 3 $(R_3 = R_3 - (9)R_2)$:

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -3 & -4 \end{bmatrix}$$

Multiply row 2 by -2 $(R_2 = (-2)R_2)$:

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -3 & -4 \end{bmatrix}$$

Divide row 3 by -3 $\left(R_3 = \frac{R_3}{-3}\right)$:

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & \frac{4}{3} \end{bmatrix}$$

Subtract row 3 from row 1 $(R_1 = R_1 - R_3)$:

$$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & \frac{4}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{4}{3} \end{bmatrix}$$

Answer: $\operatorname{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & \frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 1 & \frac{4}{3} \end{bmatrix}$

Practice Problem 2c:

2. (10 points) Let $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix} \right\}$. Determine whether S is linearly independent and if it spans $\mathbb{R}^3$.

Solution to Practice Problem 2c:

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 2 \\ -3 & 4 & 2 \end{bmatrix}$$

can be transformed by a sequence of elementary row operations to the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Linear independent => spans R3

Closure- Survey/ Feedback

5:45pm- 5:50pm

- Wrap-up:
  - Please share with the group one thing you gained understanding of through the session today.
  - Make a note to yourself/ write down anything you need to review/ do more practice problems on.

- Survey/ Feedback:
  1. How fun was the session? (1-10)
  2. How useful was the session? (1-10)
  3. Would you come back? (yes or no)
  4. Optional: Comments (pace of the activity), questions, concerns, suggestions, feedback on the back or wherever

Please recommend SI to your friends/ peers if you found the session useful! Thanks for coming and have a great day :)

PLANNING THE SI SESSION

SI Leader:
Session Date & Day of Week:
Course:
Course Instructor:

Warm-up/ Opening: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of warm-up activity **OR** attach corresponding document(s)

Cool-down/ Closing: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of cool-down activity **OR** attach corresponding document(s)

Workout: **(44-46 min.)**	**Content to cover:**	**Collaborative Learning Technique(s)**	**Strategy(ies) to be used:**

Please provide a **DETAILED BREAKDOWN** of workout activity **OR** attach corresponding document(s)