

Lesson Plan
SI Session #11
August 30, 2017

SI Leader: Eason Chang

Course: Math 18
Academic Quarter: Summer Session 2 2017
Instructor: Professor Drimbe

Topics Covered:
Dimension and Basis Col Space and Nul Space



Opener Activity:

5:05pm - 5:10pm

Talk about: Review together on what dimension and basis are for a Col Space or Null Space

Activity 1

5:10pm - 5:30pm

Matrix A with f rows and g columns: $f \times g$

- $\text{Col}(A)$ is a subspace of \mathbb{R}^f
- $\text{Nul}(A)$ is a subspace of \mathbb{R}^g
 - $\text{Nul}(A)$ is the solution to $Ax=0$, which are the x 's, x is $g \times 1$ so you need to match the x of rows of x to the # of columns of A

Practice Problem 1a:

- . Find a basis for the row space, column space, and null space of the matrix given below:

$$A = \begin{bmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{bmatrix}$$

Practice Problem 1a Solutions:

a basis for the column space of A is $\left\{ \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -5 \\ 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix} \right\}$.

If we solve $A\mathbf{x} = \mathbf{0}$, we find that x_4 is a free variable, so we set $x_4 = r$. We

obtain $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$, so $\left\{ \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \right\}$ is a basis for the nullspace of A .

Practice problem 1b:

5) a) Define column space $\text{Col}(A)$ of an $m \times n$ matrix A . Then define the null space $\text{Nul}(A)$.

b) Compute the Column space of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 0 & 1 & 1 \end{pmatrix}$. Then compute $\text{Nul}(A)$.

Practice Problem solution 1b:

$$5) a) \boxed{\text{Col}(A) = \{ A\vec{x} \mid \vec{x} \in \mathbb{R}^n \} \subset \mathbb{R}^m, \text{ if } A \text{ is } m \times n}$$

$$\boxed{\text{Nul}(A) = \{ \vec{x} \in \mathbb{R}^n \mid A\vec{x} = \vec{0} \} \subset \mathbb{R}^n}$$

$$b) \left(\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ -1 & -2 & -3 & b_2 \\ 0 & 1 & 1 & b_3 \end{array} \right) \xrightarrow{L^{-1}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ 0 & 0 & 0 & b_2 + b_1 \\ 0 & 1 & 1 & b_3 \end{array} \right) = (U|c)$$

pivots are in columns 1 and 2

free variables x_3

a Basis $\text{Col}(A) = \text{column 1 and column 2}$

$$\text{Nul}(A) = \text{Nul}(U) \begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \Rightarrow \text{Nul } A \text{ has basis } \left\{ \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

$$\boxed{\text{Col}(A) = \text{Span} \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \right\}}$$

$$x_2 = -x_3$$

x_3 free

$$x_1 = -2x_2 - 3x_3$$

$$= 2x_3 - 3x_3 = -x_3$$

$$\boxed{\text{Nul}(A) = \mathbb{R} \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}$$

Activity 2

5:30pm - 5:45pm

Practice Problem 2a:

Theorem: If A is invertible, then $\text{Nul}(A)$ is $\{0\}$.

Solution to Practice Problem 2a:

If A $m \times n$ is invertible, how many pivots does it have?

Practice Problem 2b:

(60 pts.) Suppose that A is an $n \times n$ matrix and that R is the reduced row echelon form of A . You are given R but *are not given* A . For each of the following explain why you can or cannot answer it given R but not A .

- (a) Does A^{-1} exist?
- (b) What is a basis for $\text{Nul } A$?
- (c) What is a basis for $\text{Col } A$?
- (d) What is a basis for $\text{Row } A$?
- (e) What are the eigenvalues of A ?
- (f) Does $A\mathbf{x} = \mathbf{b}$ have a solution when \mathbf{b} is the first column of the identity matrix I ?

Closure- Survey/ Feedback

5:45pm- 5:50pm

- Wrap-up:

- Please share with the group one thing you gained understanding of through the session today.

- Make a note to yourself/ write down anything you need to review/ do more practice problems on.

- Survey/ Feedback:

1. How fun was the session? (1-10)
2. How useful was the session? (1-10)
3. Would you come back? (yes or no)
4. Optional: Comments (pace of the activity), questions, concerns, suggestions, feedback on the back or wherever

Please recommend SI to your friends/ peers if you found the session useful! Thanks for coming and have a great day :)

PLANNING THE SI SESSION

SI Leader:

Session Date & Day of Week:

Course:

Course Instructor:

Warm-up/ Opening: (2-4 min.)	Content to cover:	Collaborative Learning Technique	Strategy to be used:

Please provide a **DETAILED BREAKDOWN** of warm-up activity **OR** attach corresponding document(s)

Cool-down/ Closing: (2-4 min.)	Content to cover:	Collaborative Learning Technique	Strategy to be used:

Please provide a **DETAILED BREAKDOWN** of cool-down activity **OR** attach corresponding document(s)

Workout: (44-46 min.)	Content to cover:	Collaborative Learning Technique(s)	Strategy(ies) to be used:

Please provide a **DETAILED BREAKDOWN** of workout activity **OR** attach corresponding document(s)