Lesson Plan
SI Session #4
August 15, 2017

SI Leader: Eason Chang

Course: Math 18
Academic Quarter: Summer Session 2 2017
Instructor: Professor Drimbe

Topics Covered:
Linear Dependence / Linear Independence

TEACHING + LEARNING
commons

Opener Activity:

5:05pm - 5:10pm

- Spend 1 min to review notes, and see who can recall the definitions for linear dependence, independence, and homogenous solutions.

Activity 1

5:10pm - 5:30pm

A system of linear equations is said to be **homogeneous** if it can be written in form Ax=0.

A set of vector is **linearly independent** if the vector equation has only the trivial solution

Practice Problem 1a:

$$\begin{bmatrix} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & -6 & -8 & 0 \end{bmatrix} \sim \begin{bmatrix} \textcircled{5} & 7 & 9 & 0 \\ 0 & \textcircled{2} & 4 & 0 \\ 0 & 0 & \textcircled{4} & 0 \end{bmatrix}$$

Practice Problem 1a Solutions:

Since there is no free variable, the homogenous has only the trivial solution. The vectors are linearly independent.

Practice problem 1b:

$$\left\{ \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -4 \\ 14 \end{pmatrix} \right\}$$

Practice Problem solution 1b

$$c_1 \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} + c_3 \begin{pmatrix} 4 \\ -4 \\ 14 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} c_1 + 2c_2 + 4c_3 &= 0 \\ -3c_1 + 2c_2 - 4c_3 &= 0 \\ 5c_1 + 4c_2 + 14c_3 &= 0 \end{aligned}$$

$$\left(\begin{array}{c|c|c|c} 1 & 2 & 4 & 0 \\ -3 & 2 & -4 & 0 \\ 5 & 4 & 14 & 0 \end{array}\right) \xrightarrow[-5\rho_1+\rho_3]{3\rho_1+\rho_2} \xrightarrow{(3/4)\rho_2+\rho_3} \left(\begin{array}{c|c|c|c} 1 & 2 & 4 & 0 \\ 0 & 8 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Linearly dependent

Activity 2

5:30pm - 5:45pm

Practice Problem 2a:

Find the value of *h* that will make the system *linearly dependent*.

13. $\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$

Solution to Practice Problem 2a:

To study the linear dependence of three vectors, say $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, row reduce the augmented matrix $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{0}]$:

$$\begin{bmatrix} 1 & -2 & 3 & 0 \\ 5 & -9 & h & 0 \\ -3 & 6 & -9 & 0 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & -2 & 3 & 0 \\ 0 & \textcircled{1} & h-15 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0}$ has a free variable and hence a nontrivial solution no matter what the value of h. So the vectors are linearly dependent for all values of h.

Practice Problem 2b:

14. $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$

Solution to Practice Problem 2b:

To study the linear dependence of three vectors, say $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, row reduce the augmented matrix $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{0}]$:

$$\begin{bmatrix} 1 & -5 & 1 & 0 \\ -1 & 7 & 1 & 0 \\ -3 & 8 & h & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & -7 & h+3 & 0 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & -5 & 1 & 0 \\ 0 & \textcircled{2} & 2 & 0 \\ 0 & 0 & h+10 & 0 \end{bmatrix}$$

The equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0}$ has a nontrivial solution if and only if $h + 10 = 0$ (which corresponds to x_3 being a free variable). Thus, the vectors are linearly dependent if and only if $h = -10$.

Closure- Survey/ Feedback

5:45pm- 5:50pm

- Wrap-up:
    - Please share with the group one thing you gained understanding of through the session today.
    - Make a note to yourself/ write down anything you need to review/ do more practice problems on.

- Survey/ Feedback:
    1. How fun was the session? (1-10)
    2. How useful was the session? (1-10)
    3. Would you come back? (yes or no)
    4. Optional: Comments (pace of the activity), questions, concerns, suggestions, feedback on the back or wherever

Please recommend SI to your friends/ peers if you found the session useful! Thanks for coming and have a great day :)

PLANNING THE SI SESSION

SI Leader:
Session Date & Day of Week:
Course:
Course Instructor:

	Content to cover:	**Collaborative Learning Technique**	**Strategy to be used:**
Warm-up/ Opening: **(2-4 min.)**			

Please provide a **DETAILED BREAKDOWN** of warm-up activity **OR** attach corresponding document(s)

	Content to cover:	**Collaborative Learning Technique**	**Strategy to be used:**
Cool-down/ Closing: **(2-4 min.)**			

Please provide a **DETAILED BREAKDOWN** of cool-down activity **OR** attach corresponding document(s)

Workout: **(44-46 min.)**	**Content to cover:**	**Collaborative Learning Technique(s)**	**Strategy(ies) to be used:**

Please provide a **DETAILED BREAKDOWN** of workout activity **OR** attach corresponding document(s)