Math 20B

Lecture 2 7/5/2017

Professor Um

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$(x^2+1)' = 2x$

$\int 2x \, dx = x^2 + C$

5.7 Subsititution Method

Consider $(e^{x^2})' \xrightarrow{\text{chain rule}} e^{x^2} \cdot 2x$ ⊛

$\int e^{x^2} 2x \, dx = e^{x^2} + c$

What if we dont know ⊛ ?

$\int e^{x^2} \underline{(2x) \, dx}$ Let $u = x^2$

$\frac{du}{dx} = 2x \rightarrow \underline{du = 2x \, dx}$

$= \int e^u \, du$ simpler integral

$= e^u + C$

$= e^{x^2} + C$

In general $\int \underline{f(g(x))} \cdot \underline{g'(x) \, dx}$ $u = g(x)$

~~$du = g'(x)\,dx$~~

$\int f(u) \quad du$

☆ u-subsititution ☆

Ex (i) $\int 2x \cos(x^2+1) \, dx$ set $u = x^2+1$

$du = 2x \, dx$

$= \int \cos u \, du$

$= \sin u + C$

$= \sin(x^2+1) + C$

(ii) $\int \frac{1}{3\sqrt{3x+5}} 3\,dx$ $\quad$ $u = 3x+3$

$du = 3\,dx$

$= \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$

$= \frac{1}{3}\int u^{-\frac{1}{2}}\,du$

$= \frac{1}{3} 2\sqrt{u} + C = \frac{2}{3}\sqrt{u} + C = \frac{2}{3}\sqrt{3x+5} + C$

(iii) $\int x^5 \sqrt{x^3+1}\,dx$ $\quad$ Set $u = x^3+1$

$du = 3x^2\,dx$

$= \frac{1}{3}\int x^3 \sqrt{u}\, \underline{3x^2\,dx}$

$= \frac{1}{3}\int x^3 \sqrt{u}\,du$

$= \frac{1}{3}\int (u-1)\sqrt{u}\,du$

$= \frac{1}{3}\int u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du$

$= \frac{1}{3}\left(\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right) + C$

$= \frac{2}{15}u^{\frac{5}{2}} - \frac{2}{9}u^{\frac{3}{2}} + C$

$= \frac{2}{15}(x^3+1)^{\frac{5}{2}} - \frac{2}{9}(x^3+1)^{\frac{3}{2}} + C$

Change of Variable Formula for Definite Integral

$$\int_a^b f(g(x))\cdot g'(x)\,dx \quad \int_{g(a)}^{g(b)} f(u)\,du$$

Set $u = g(x)$

If $x = a$, then $u = g(a)$

If $x = b$, $u = g(b)$

Ex. $\int_1^e \frac{(\ln x)^2}{x}\,dx = \int_{x=1}^{x=e} (\ln x)^2 \frac{1}{x}\,dx$

Set. $u = \ln x$

$du = \frac{1}{x}\,dx$

If $x=1$, $u = \ln 1 = 0$

If $x=e$, $u = \ln e = 1$.

$= \int_{u=0}^{u=1} u^2\,du$

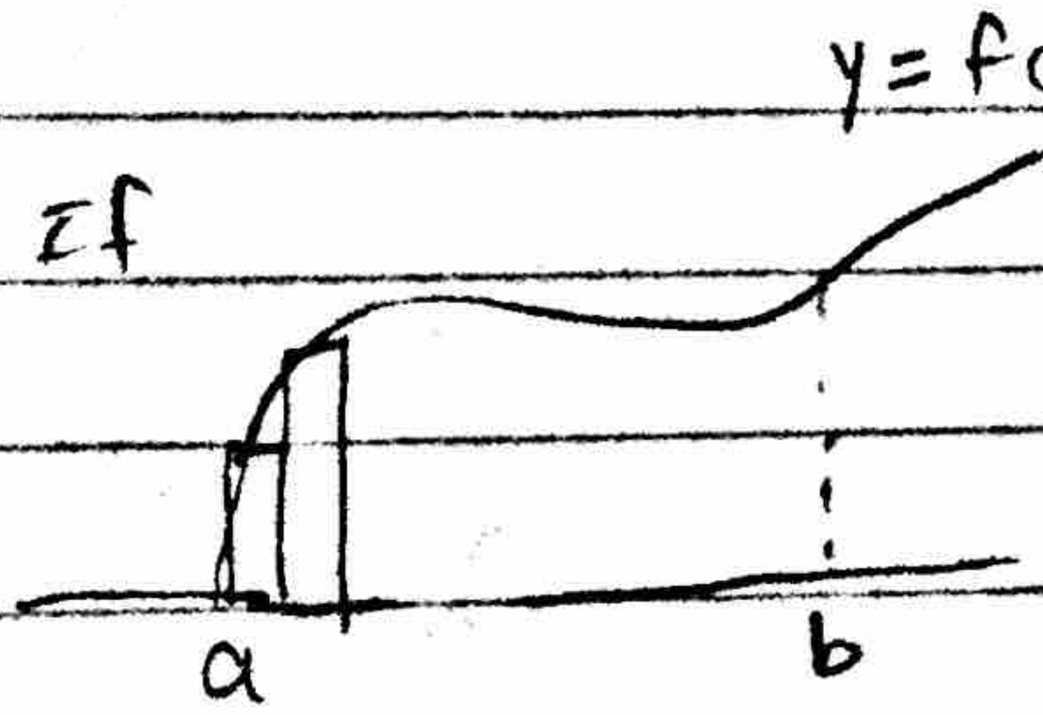
$= \frac{u^3}{3} + C \Big]_{u=0}^{u=1}$

$= \left(\frac{1}{3} + C\right) - (0 + C) = \frac{1}{3}$

6. Application of the Integral

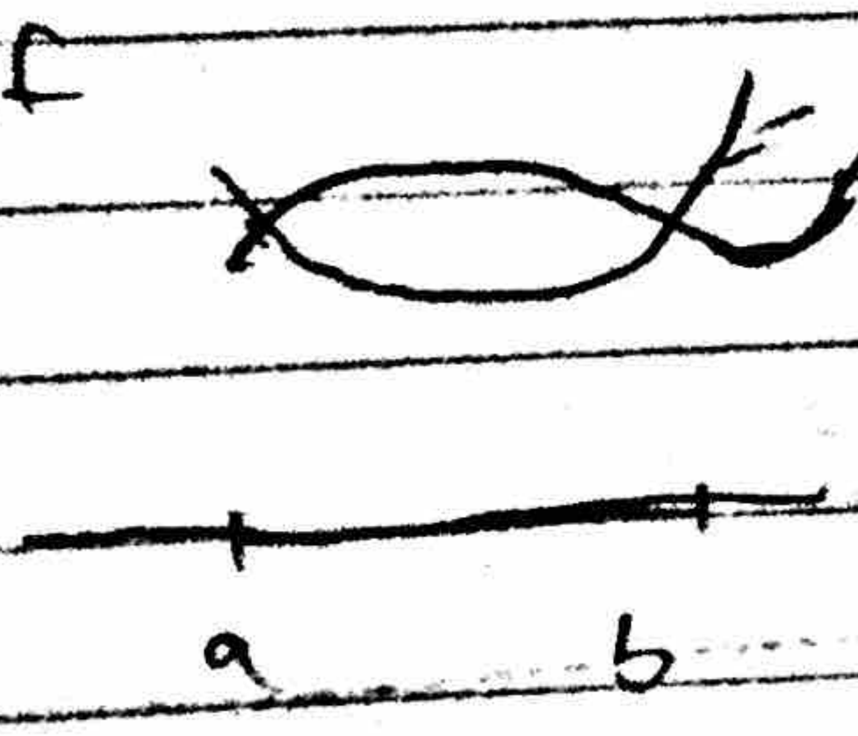
6.1 Area Between Curves

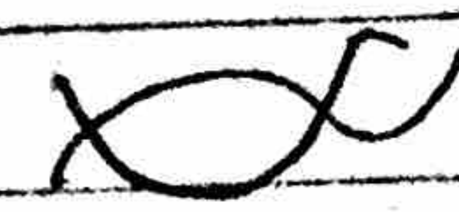
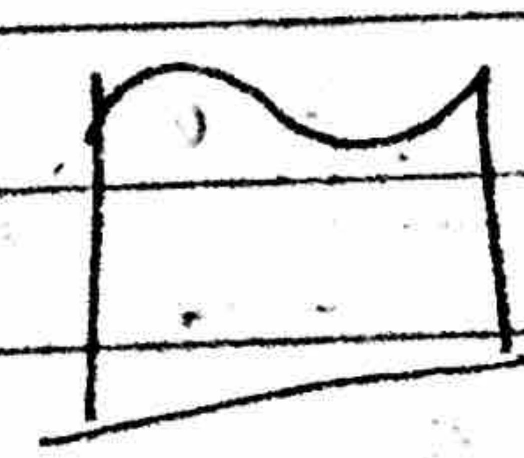
If



then Area $= \int_a^b f(x)\,dx$

If



Area  =  

$= \int_a^b f(x)\,dx \qquad \int_a^b g(x)\,dx$

$= \int_a^b f(x) - g(x)\,dx$ if a vertical line intersects f and g only once

Ex. Find the area of the region enclosed by
$f(x) = x^3 - 10x$, $g(x) = 6x$

To find intersection point:

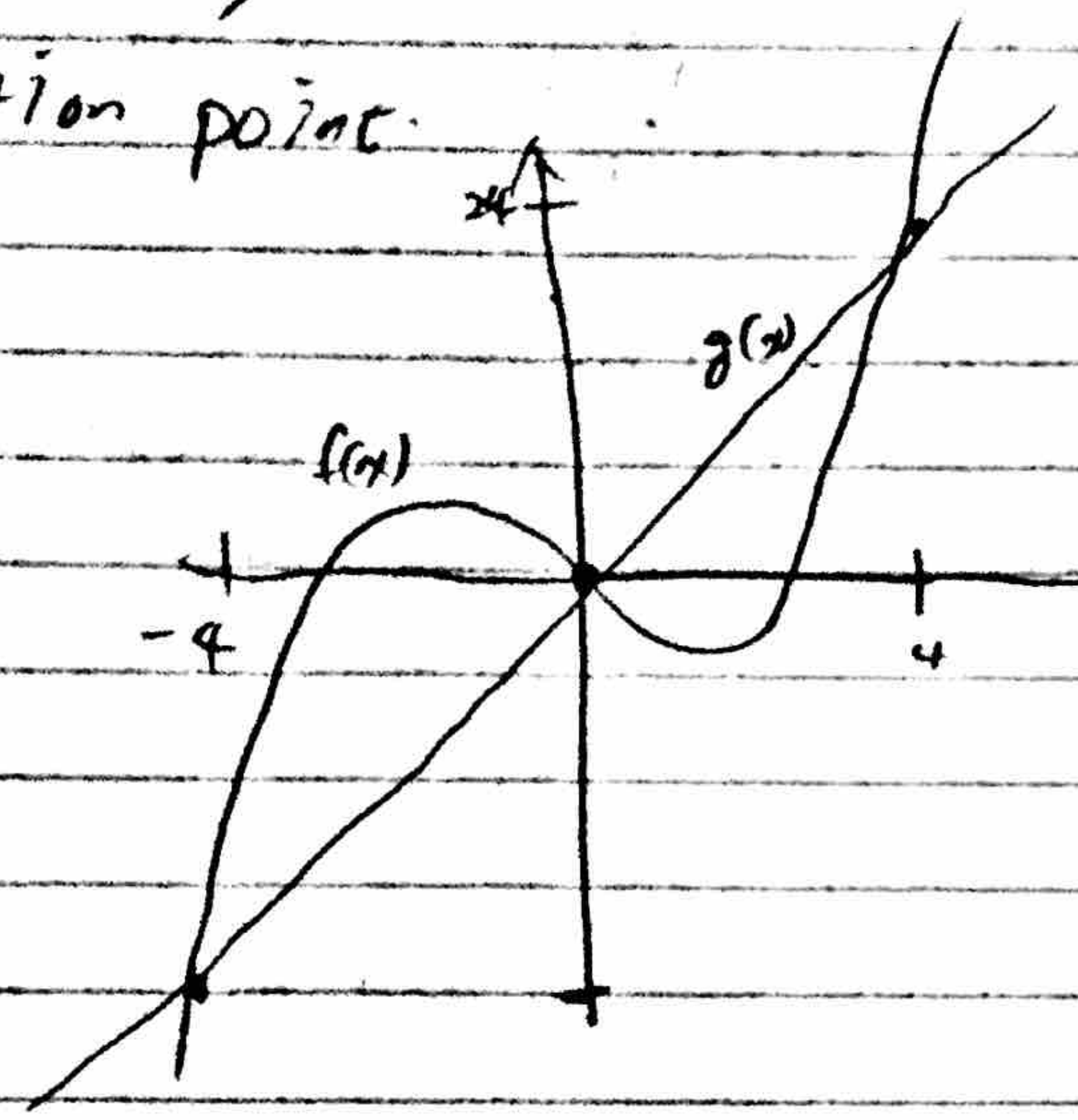
$x^3 - 10x = 6x$

$x^3 - 16x = 0$

$x(x^2 - 16) = 0$

$x(x-4)(x+4)$

$x = 0, \pm 4$

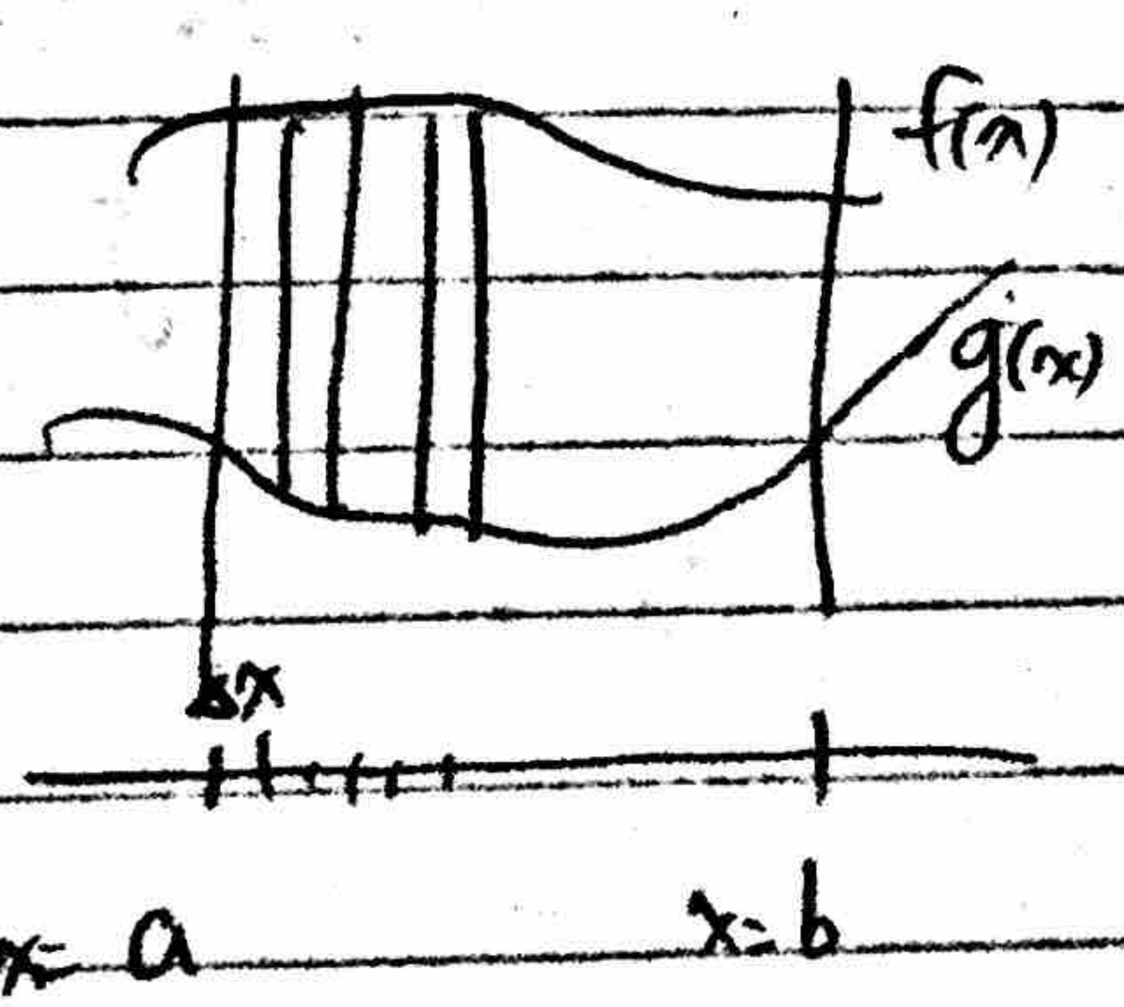


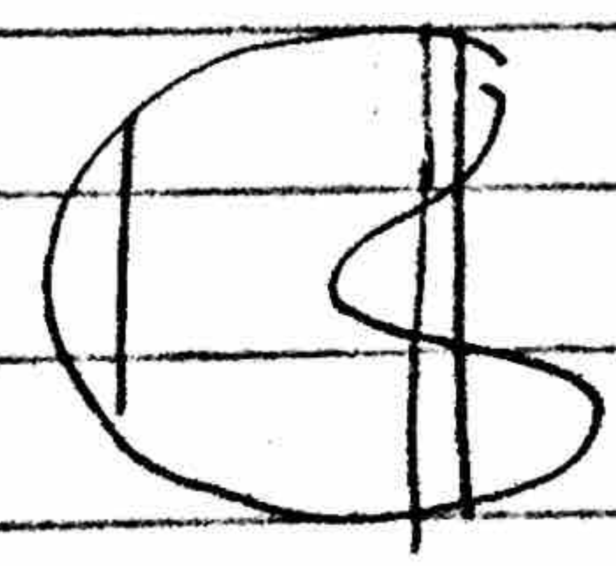
$$\int_{-4}^{0} (x^3-10x) - (6x)\,dx + \int_{0}^{4} (6x) - (x^3-10x)\,dx$$

$$= \int_{-4}^{0} x^3 - 16x\,dx + \int_{0}^{4} 16x - x^3\,dx$$

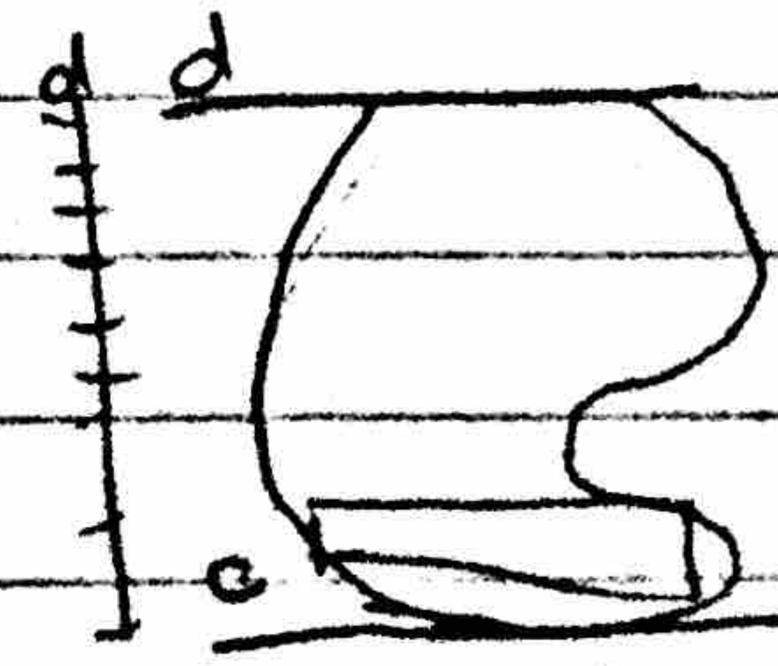
$$= \frac{1}{4}x^4 - \frac{16x^2}{2}\Big]_{-4}^{0} + \Big[\frac{16x^2}{2} - \frac{x^4}{4}\Big]_{0}^{4}$$

$$= 0 - \left(\frac{1}{4}4^4 - 8\cdot 16\right) + \left(8\cdot 4^2 - 4^3\right) = 128$$





can't do it

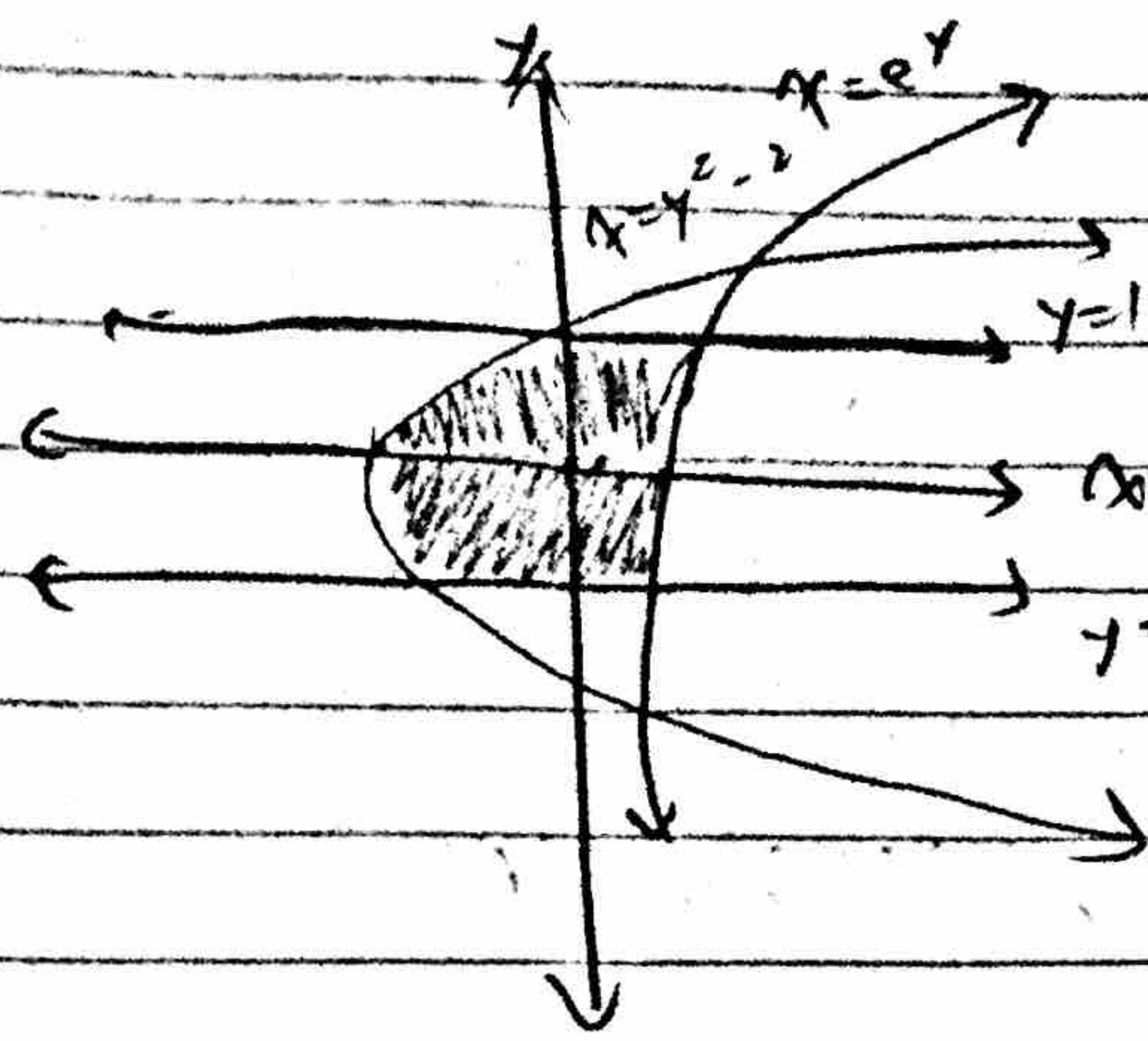


$$\text{Area} = \int_{y=c}^{d} x_{right} - x_{left}$$

$$= \int_{c}^{d} f(y) - g(y)\,dy$$

If horizontal line intersects f & g

Ex. Find the area of the region enclosed by $x=y^2-2$, $x=e^y$, $y=\pm 1$



$$\text{Area} = \int_{y=}^{y=} x_{right} - x_{left}\, dy$$

$$= \int_{-1}^{1} e^y - (y^2-2)\, dy$$

$$= \int_{-1}^{1} e^y - y^2 + 2\, dy$$

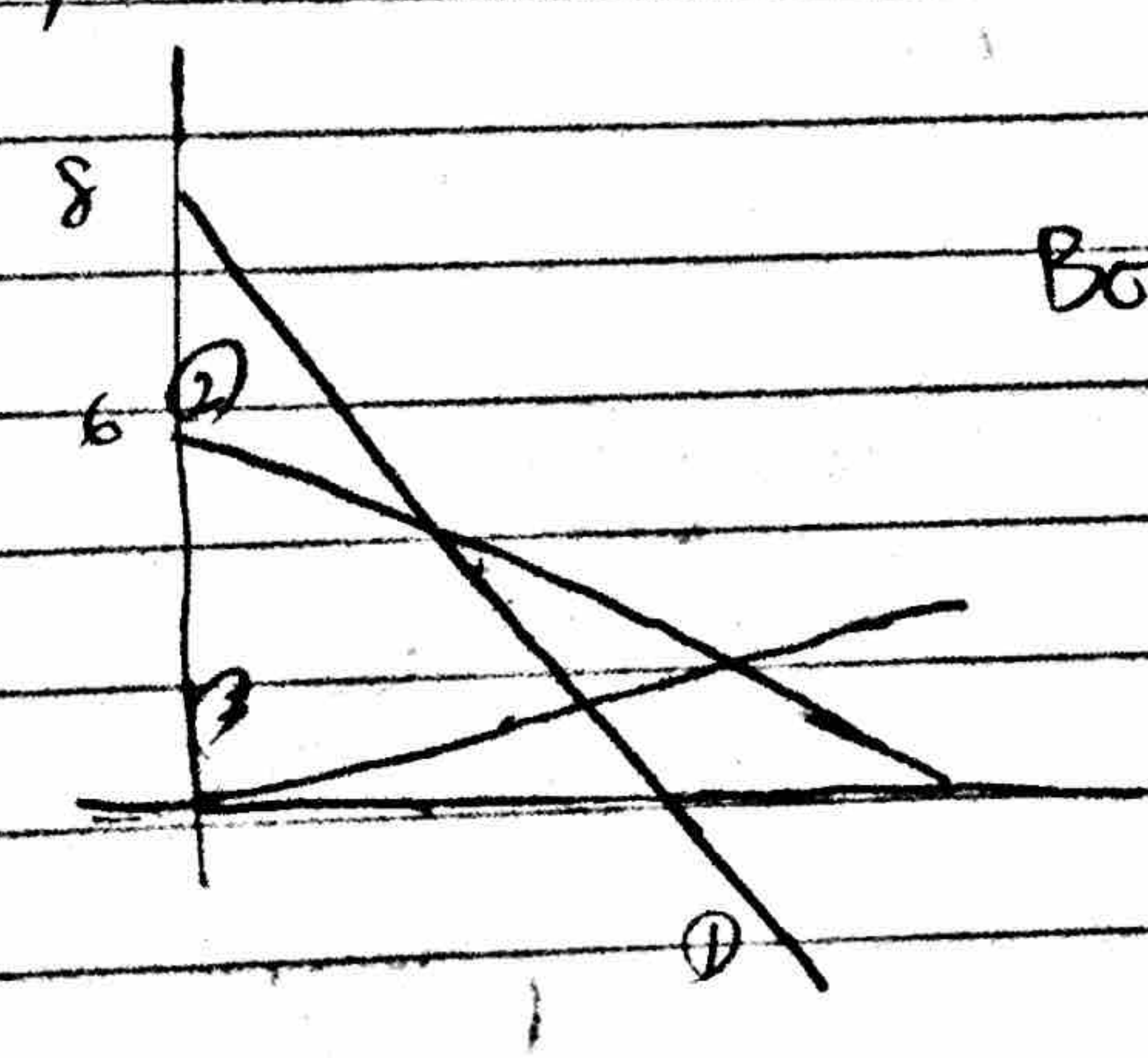
$$= e^y - \frac{y^3}{3} + 2y \Big]_{-1}^{1}$$

$$= e - \frac{1}{3} + 2 - (e^{-1} + \frac{1}{3} - 2)$$

$$= e - \frac{1}{3} + 2 - \frac{1}{e} - \frac{1}{3} + 2$$

$$= e - \frac{1}{e} + \frac{10}{3}$$

Ex ① $y = 8 - 3x$ ② $y = 6 - x$ ③ $y = \frac{1}{2}x$



Both vertical or horizontal works

$8 - 3x = \frac{1}{2}x \rightarrow 8 = \frac{7}{2}x \Rightarrow x = \frac{16}{7}$

$8 - 3x = 6 - x \rightarrow 2 = 2x \Rightarrow x = 1$

$6 - x = \frac{1}{2}x \quad 6 = \frac{3}{2}x \rightarrow x = 4$

$$\int_{1}^{16/7} (6-x) - (8-3x)\, dx + \int_{16/7}^{4} (6-x) - (\tfrac{1}{2}x)$$