Lesson Plan
SI Session #7
August 22, 2017

SI Leader: Eason Chang

Course: Math 18
Academic Quarter: Summer Session 2 2017
Instructor: Professor Drimbe

Topics Covered:
Transformation and Midterm Review

TEACHING+LEARNING
commons

<u>Opener Activity:</u>

<u>5:05pm - 5:10pm</u>

- Midterm Review, brainstorm on what topics and type of problems may show up on the exam and talk to each other.

<u>Activity 1</u>

<u>5:10pm - 5:30pm</u>

Practice Problem 1a:

- How to know when a matrix is onto? If a matrix in its reduced echelon form has a pivot in every row.
- How to know when a matrix is one to one? If T is a linear transformation, T(X) has a unique solution.

(Source: University of Alberta,
http://www.stat.ualberta.ca/~skalayci/Math%20102/Lecturenotes25-28March2011.pdf)

Example: Is the matrix transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$, where $T(x, y) = (x, y, x + y)$ is onto?

Practice Problem 1a Solutions:

Solution: T is onto if for any vector $(a, b, c) \in \mathbb{R}^3$ we can find a corresponding $(x, y) \in \mathbb{R}^2$ such that $T(x, y) = (a, b, c)$. From here we get linear system

$$\begin{aligned} x &= a \\ y &= b \\ x + y &= c \end{aligned}$$

T is onto if this sytem is consistent for all (a, b, c). $\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 1 & 1 & c \end{bmatrix} \xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & c - b - a \end{bmatrix}$

So this system is consistent if $c = a + b$. Hence for $(1, 2, 5)$ there is no (x, y) that is mapped to $(1, 2, 5)$ under T. So T is not onto.

Practice problem 1b:

Practice Problem solution 1b:

Let the linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the following equations $w_1 = 2x - 5y$ and $w_2 = 4x + 3y$. Is this transformation one-to-one?

The standard matrix for this operation is $A = \begin{bmatrix} 2 & -5 \\ 4 & 3 \end{bmatrix}$, and $\det(A) = 26 \neq 0$, so this transformation is one-to-one.

Since there is a pivot in every row when the matrix is row reduced, then the columns of the matrix will span R^3.

Activity 2

5:30pm - 5:45pm

Midterm Review

Practice Problem 2a:

(45 pts.) The reduced row echelon form for the matrix A is $M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

For each of the following answer either "Yes," "No," or "Not enough information."
Note that all the questions are about A, *not* about M.
To receive credit *you must justify your answers.*

(a) The columns of A are linearly independent.

(b) The equation $A\vec{x} = \vec{0}$ has more than one solution.

(c) The equation $A\vec{x} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ has more than one solution.

Solution to Practice Problem 2a:

(a) No. Linear independence means $A\vec{x} = \vec{0}$ has only the trivial solution. From M we can see that x_4 is free, so there are nontrivial solutions.

(b) Yes. See (a).

(c) Not enough information. By (b) we know that when $\vec{b} = \vec{0}$ there is more than one solution. If we augment A with $\vec{b}$ and carry out reduction, it may happen that the lower right corner will be nonzero. In this case, the equations will be inconsistent and so there will be no solutions. Problem 1 provides an example of how this can happen.

Practice Problem 2b:

Problem 2. Find the value of z for which $\begin{pmatrix} 3 \\ -5 \\ z \end{pmatrix}$ is a linear combination of the two vectors $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ -8 \\ 2 \end{pmatrix}$.

Solution to Practice Problem 2b:

$A = \begin{pmatrix} 1 & -5 \\ 3 & -8 \\ -1 & 2 \end{pmatrix}$, $b = \begin{pmatrix} 3 \\ -5 \\ z \end{pmatrix}$. Asking when $Ax = b$ <u>consistent</u>.

$$(A|b) = \left(\begin{array}{cc|c} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & z \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 7 & -14 \\ 0 & -3 & z+3 \end{array}\right) \sim$$

$$\left(\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & 1 & \frac{-1}{3}(z+3) \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & \underbrace{\frac{-1}{3}(z+3)+2}_{= \frac{-1}{3}(z-3)} \end{array}\right)$$

$$\sim \left(\begin{array}{cc|c} 1 & 0 & -7 \\ 0 & 1 & -2 \\ 0 & 0 & \frac{-1}{3}(z-3) \end{array}\right).$$

- For this to be <u>consistent</u>, it's necessary & sufficient that $\frac{-1}{3}(z-3) = 0$. I.e., that

$$\boxed{z = 3}.$$

Closure- Survey/ Feedback

5:45pm- 5:50pm

- Wrap-up:
    - Please share with the group one thing you gained understanding of through the session today.
    - Make a note to yourself/ write down anything you need to review/ do more practice problems on.

- Survey/ Feedback:
    1. How fun was the session? (1-10)
    2. How useful was the session? (1-10)
    3. Would you come back? (yes or no)
    4. Optional: Comments (pace of the activity), questions, concerns, suggestions, feedback on the back or wherever

Please recommend SI to your friends/ peers if you found the session useful! Thanks for coming and have a great day :)

PLANNING THE SI SESSION

SI Leader:
Session Date & Day of Week:
Course:
Course Instructor:

Warm-up/ Opening: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of warm-up activity **OR** attach corresponding document(s)

Cool-down/ Closing: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of cool-down activity **OR** attach corresponding document(s)

Workout: **(44-46 min.)**	**Content to cover:**	**Collaborative Learning Technique(s)**	**Strategy(ies) to be used:**

Please provide a **DETAILED BREAKDOWN** of workout activity **OR** attach corresponding document(s)