Math 20B Lecture 3 7/6/2017

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$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

let $u = 1 - e^{2x}$

$\frac{du}{-2e^x} = \frac{-2e^{2x}}{-2e^x}\,dx$ getting a harder problem

$= e^x\,dx$

$= \int \frac{1}{\sqrt{u}} \frac{du}{-2e^x}$

$= \int \frac{1}{\sqrt{1-u^2}}\,du$ let $u = e^x$

$du = e^x\,dx$

$= \sin^{-1}(u) + C$

$= \sin^{-1}(e^x) + C$

Exercise: see if we can use u-sub

① $\int (x^2+1)e^{-x}\,dx$ (Y/N)

② $\int (-x)e^{x^2+1}\,dx$ (Y/N) $u = x^2+1$, $du = 2x\,dx$

③ $\int \frac{\ln y}{\sqrt{y}}\,dy$ (N)

④ $\int \frac{1}{y\sqrt{\ln y}}\,dy$ (Y) $u = \ln y$, $du = \frac{1}{y}\,dy$

⑤ $\int \frac{3x^2+2}{(x^3+2x+5)^3}\,dx$ (Y)

⑥ $\int \frac{3x^2+1}{(x^3+2x+5)}\,dx$ (N)

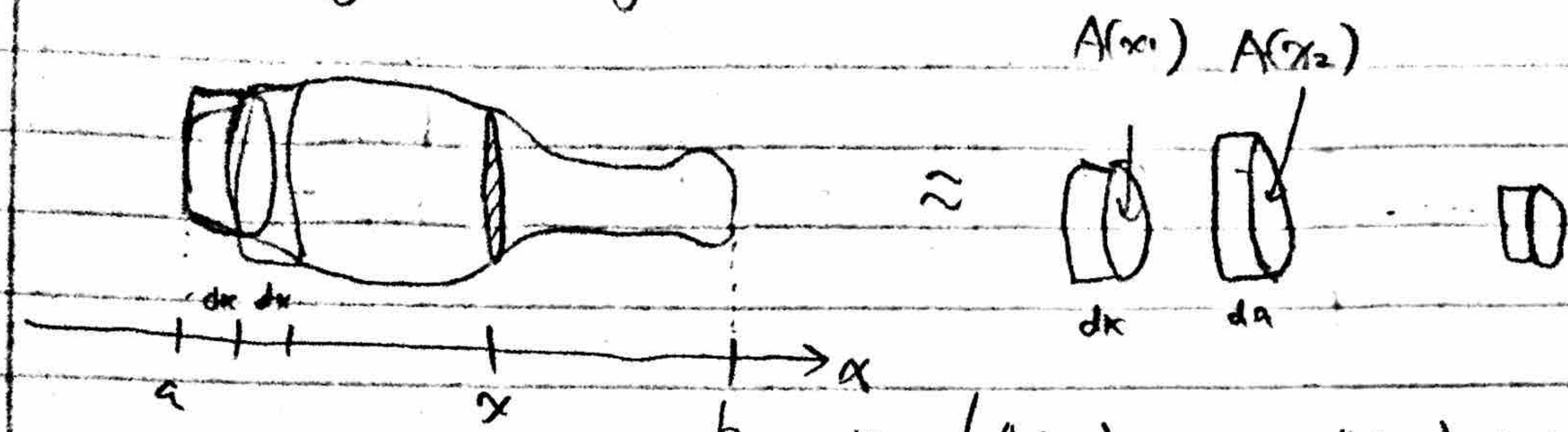
⑦ $\int \frac{\tan^{-1}x}{1+x^2}\,dx$ $u = \tan^{-1}x$, $du = \frac{1}{1+x^2}\,dx$

$= \int u\,du$

$= \frac{u^2}{2} + C$

$= \frac{(\tan^{-1}x)^2}{2} + C$

6.2 Setting Up Integrals for volume



$$Vol = \lim_{N\to\infty} \left(A(x_1)\cdot\Delta x + A(x_2)\cdot\Delta x \ldots A(x_N)\cdot\Delta x\right)$$

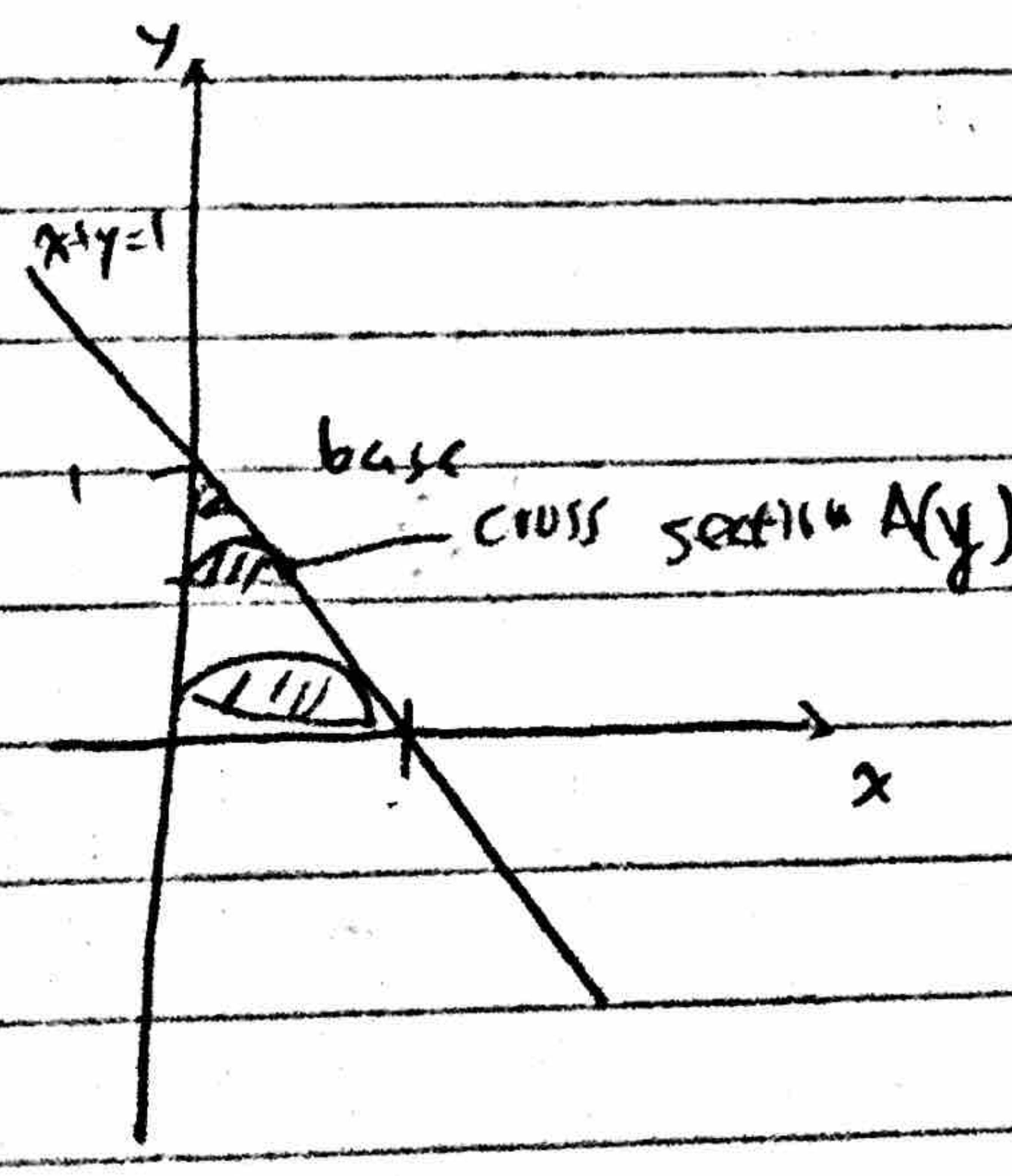
Recall $\displaystyle\int_a^b f(x)\,dx \overset{\text{def.}}{=} \lim_{N\to\infty} \left(f(x_1)\Delta x + f(x_2)\Delta x \ldots + f(x_N)\Delta x\right)$

$$Vol = \int_a^b A(x)\,dx$$ when $A(x)$ the area of the vertical cross section at x

or

$$\int_{y=a}^{y=b} A(y)\,dy$$ when $A(y)$ the " horizontal " y

Ex. Find the volume of the solid whose base is the triangle enclosed by $x+y=1$, the x-axis and the y-axis.
The cross section perpendicular to the y-axis are semicircles.



$$Vol = \int_0^1 A(y)\,dy$$

$$= \int_0^1 \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 dy$$

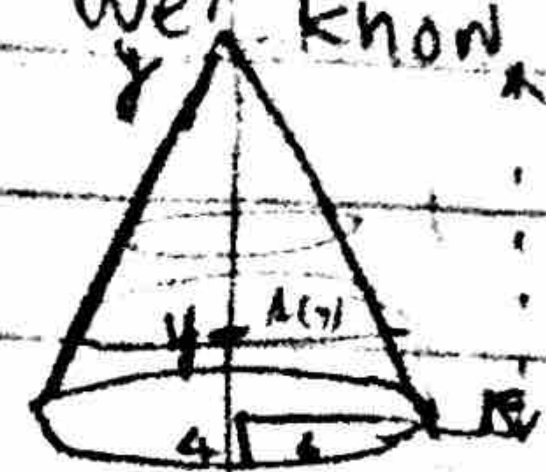
$$= \int \frac{1}{2}\pi\left(\frac{1-y}{2}\right)^2 dy$$

$$= \frac{\pi}{8}\int_0^1 1-2y+y^2\,dy$$

$$= \frac{\pi}{8}\left[y - y^2 + \frac{y^3}{3}\right]_0^1$$

$$= \frac{\pi}{8}\left(1-1+\frac{1}{3}\right) = \frac{\pi}{24}$$

Ex. We know the area of ellipse



$= \pi a b$

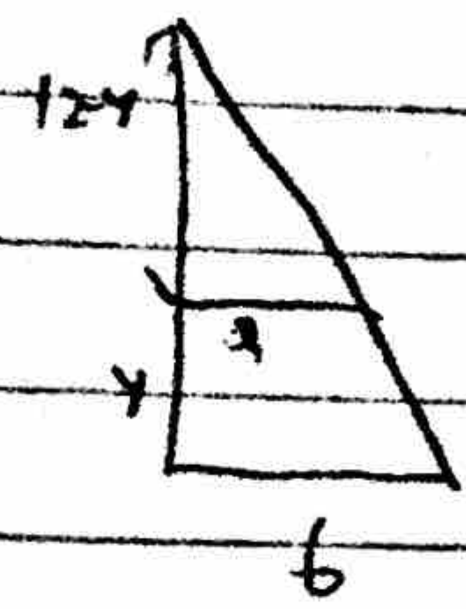
formula: $\frac{1}{3}(\pi \cdot 6 \cdot 4)(12) = 96\pi$

$\text{Vol} = \int_0^{12} A(y)\, dy$

$A(y) = \pi l m$

$= \pi \frac{1}{6}(12-y)^2$

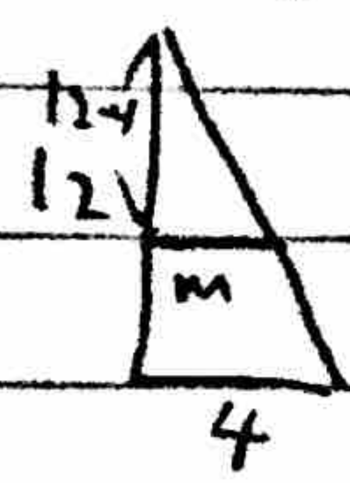
$= \frac{1}{6}\pi(12-y)^2$



$\frac{l}{12-y} = \frac{6}{12}$

$12l = 72 - 6y$

$l = 6 - \frac{1}{2}y$



$m = \frac{4}{12}(12-y)$

$m = \frac{1}{3}(12-y)$

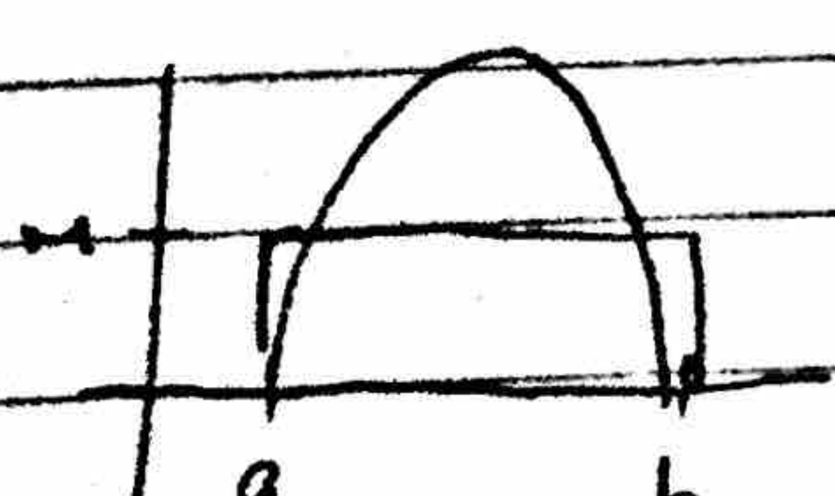
$\text{Vol} = \int_0^{12} \frac{\pi}{6}(12-y)^2\, dy$

$= \frac{\pi}{6}\int_0^{12} 144 - 24y + y^2\, dy$

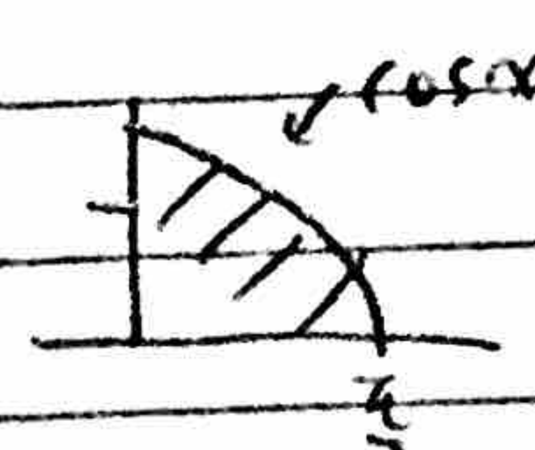
$= \frac{\pi}{6}\left(144y - \frac{24y^2}{2} + \frac{y^3}{3}\right)\Big|_0^{12}$

$= 96\pi$

Average value of f on $[a, b] = \frac{1}{b-a}\int_a^b f(x)\,dx = M \Rightarrow \int_a^b f(x)\,dx = (b-a)M$



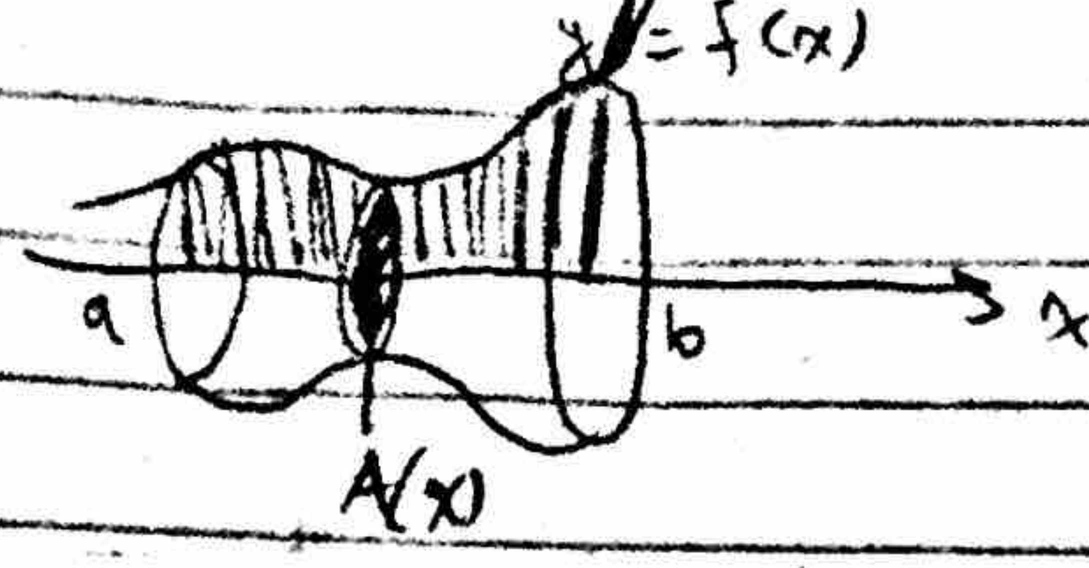
Ex. Av val of $f(x) = \cos x$



$= \frac{1}{\frac{\pi}{2} - 0}\int_0^{\frac{\pi}{2}} \cos x\, dx$

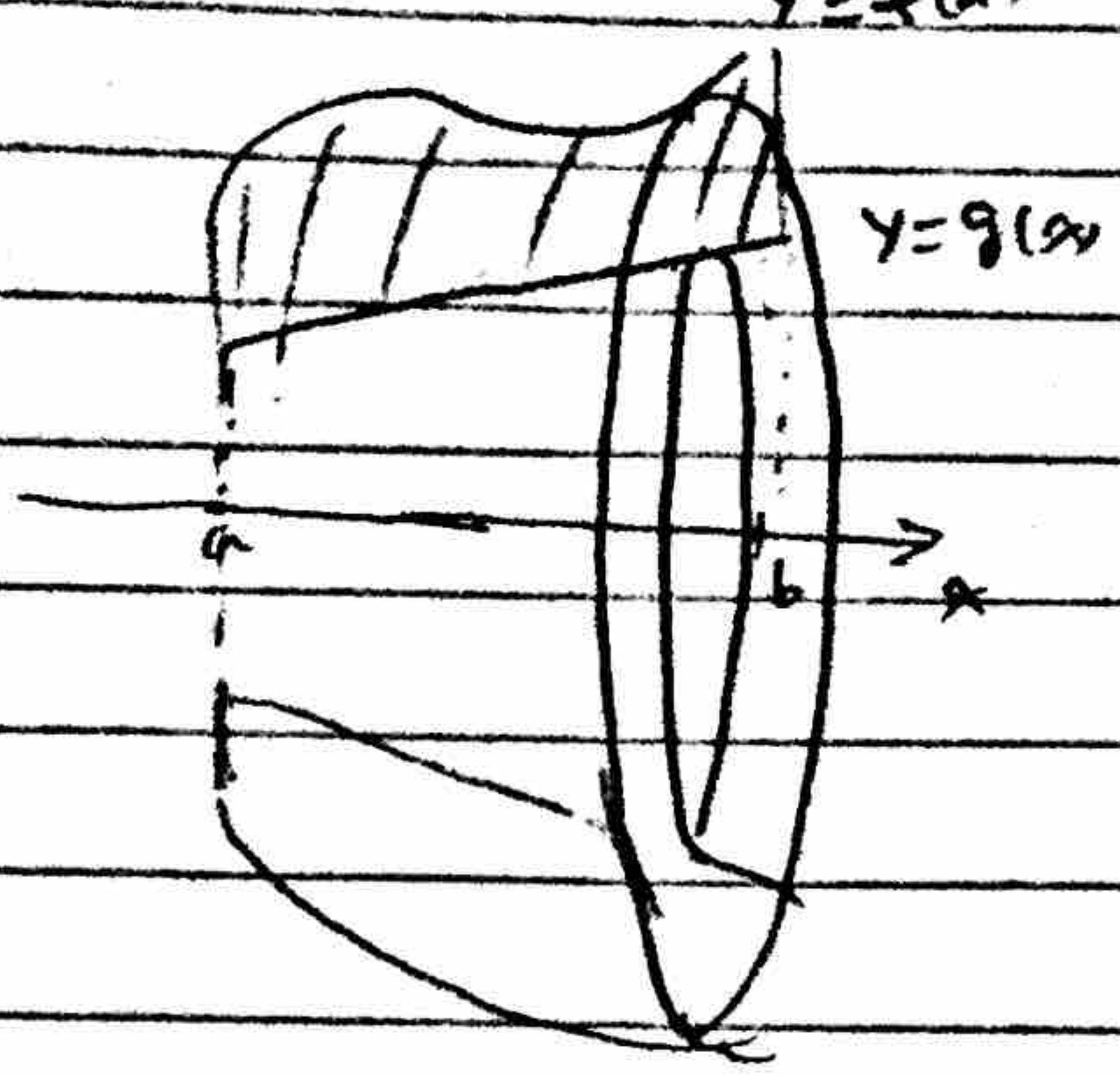
$= \frac{2}{\pi}\left[\sin x\right]_0^{\frac{\pi}{2}} = \frac{2}{\pi}(1) = \frac{2}{\pi}$

6.3 Volume of Revolution



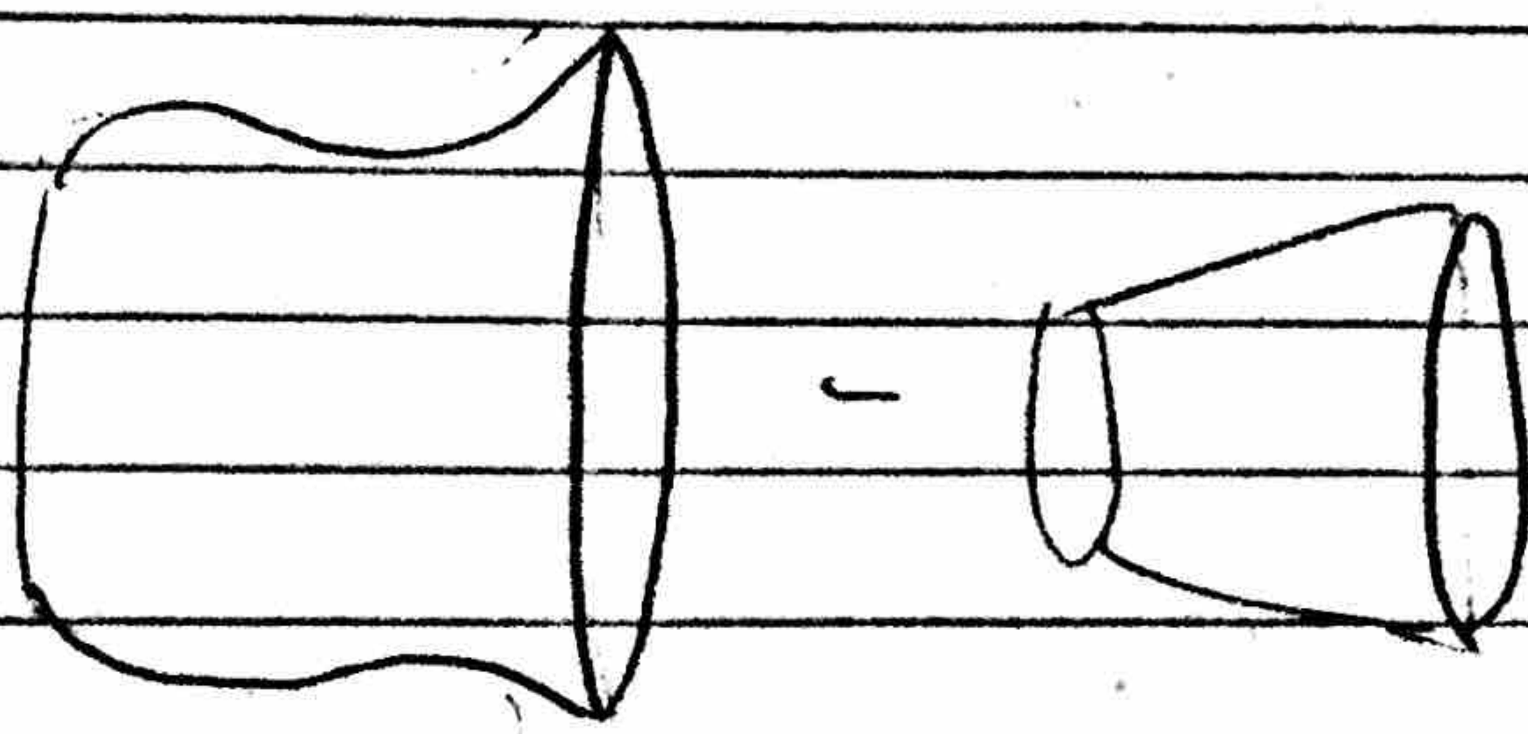
$$\text{Vol} = \int_{x=a}^{b} A(x)\, dx$$

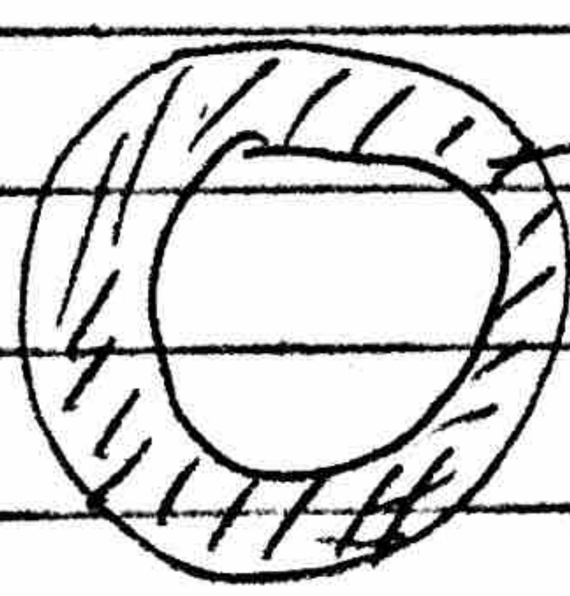
$$= \int_a^b \pi(f(x))^2\, dx$$



$$\text{Vol} = \int_a^b \pi(f(x))^2\, dx - \int_a^b \pi(g(x))^2\, dx$$

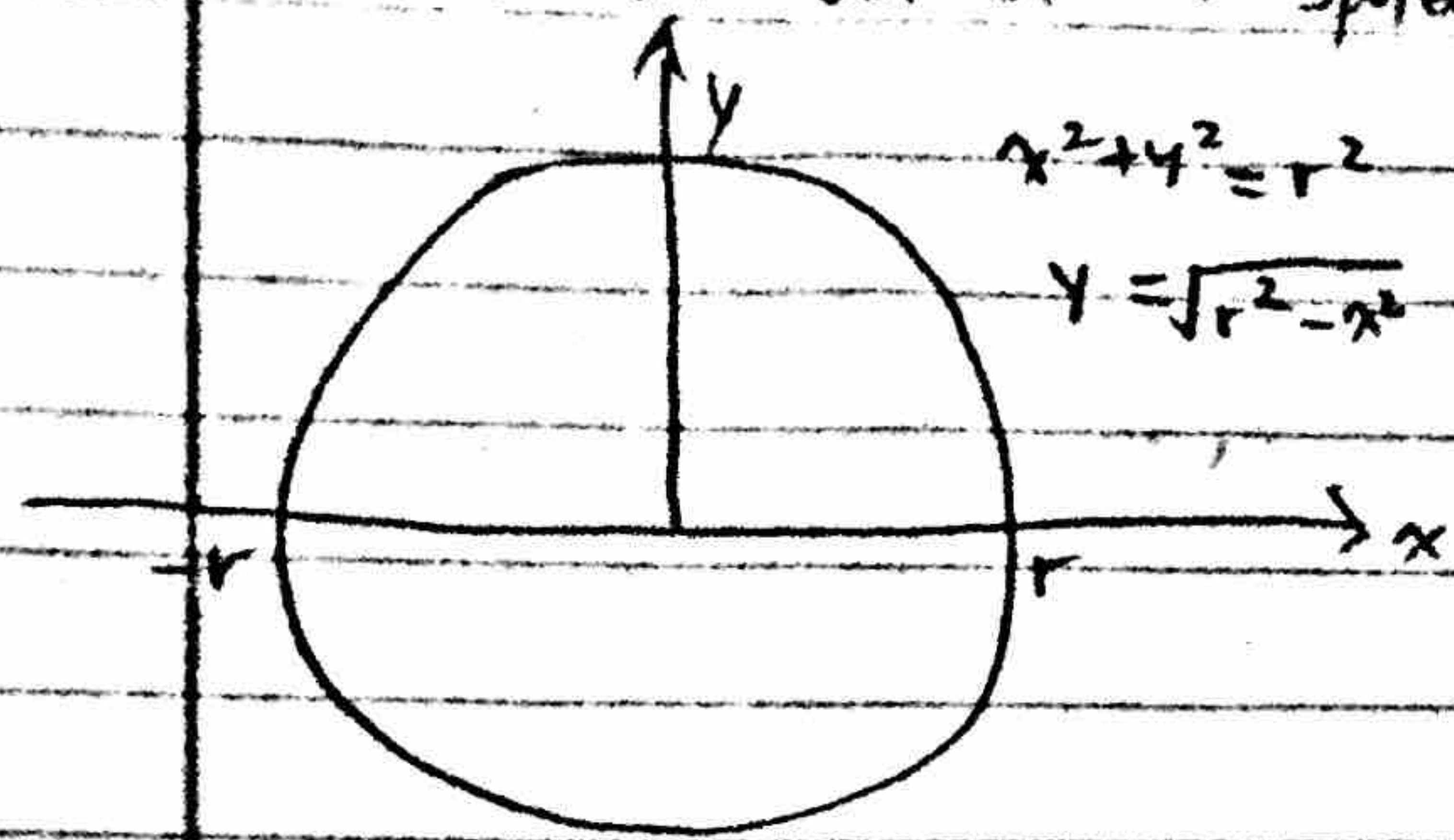
$$= \int_a^b \pi\left(f(x)^2 - g(x)^2\right) dx$$





$$A(x) = \pi(R_{out})^2 - \pi(R_{in})^2$$

Show the vol of a sphere of radius r is $\frac{4}{3}\pi r^3$



$x^2+y^2=r^2$

$y=\sqrt{r^2-x^2}$

$$vol = \int_{-r}^{r} \pi (f(x))^2 \, dx$$

$$= 2\int_0^r \pi \left(\sqrt{r^2-x^2}\right)^2 dx$$

$$= 2\pi\int_0^r r^2-x^2 \, dx$$

$$= 2\pi\left[r^2x - \frac{x^3}{3}\right]_0^r$$

$$= 2\pi\left[r^3 - \frac{r^3}{3}\right]$$

$$= 2\pi \, \frac{3r^3-r^3}{3}$$

$$= 2\pi \, \frac{2r^3}{3}$$

$$= \frac{4\pi r^3}{3}$$