Lesson Plan
SI Session #5
August 16, 2017

SI Leader: Eason Chang

Course: Math 18
Academic Quarter: Summer Session 2 2017
Instructor: Professor Drimbe

Topics Covered:
Transformations; Linear Transformation; Proof

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Opener Activity:

5:05pm - 5:10pm

- Spend 1 min to review notes, and see who can recall the definitions for transformation and linear transformation, etc.
- Transformation: A **transformation** (or **function** or **mapping**) T from R^n to R^m is a rule that assigns to each vector **x** in R^n a vector T (**x**) in R^m. The set R^n is called the **domain** of T, and R^m is called the **codomain** of T.
- Domain, codomain

A transformation (or mapping) T is **linear** if:

(i) $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v}$ in the domain of T;

(ii) $T(c\mathbf{u}) = cT(\mathbf{u})$ for all scalars c and all $\mathbf{u}$ in the domain of T.

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- (Optional for this session) shear transformation, contraction and dilation

Activity 1

5:10pm - 5:30pm

Practice Problem 1a:
(Source: University of Texas, https://www.ma.utexas.edu/users/olenab/Fall-2011-341/341lintranssols.pdf)

Determine whether the following functions are linear transformations. If they are, prove it; if not, provide a counterexample to one of the properties:

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$, with

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ y \end{bmatrix}$$

Practice Problem 1a Solutions:

Solution:

This $\boxed{\text{IS}}$ a linear transformation. Let's check the properties:

(1) $T(\vec{x}+\vec{y}) = T(\vec{x}) + T(\vec{y})$: Let $\vec{x}$ and $\vec{y}$ be vectors in $\mathbb{R}^2$. Then, we can write them as

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

By definition, we have that

$$T(\vec{x}+\vec{y}) = T\begin{bmatrix} x_1+y_1 \\ x_2+y_2 \end{bmatrix} = \begin{bmatrix} x_1+y_1+x_2+y_2 \\ x_2+y_2 \end{bmatrix}$$

and

$$\begin{aligned} T(\vec{x})+T(\vec{y}) &= T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + T\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \\ &= \begin{bmatrix} x_1+x_2 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1+y_2 \\ y_2 \end{bmatrix} \\ &= \begin{bmatrix} x_1+x_2+y_1+y_2 \\ x_2+y_2 \end{bmatrix} \end{aligned}$$

Thus, we see that $T(\vec{x}+\vec{y}) = T(\vec{x})+T(\vec{y})$, so this property holds.

(2) $T(c\vec{x}) = cT(\vec{x})$: Let $\vec{x}$ be as above, and let c be a scalar. Then,

$$T(c\vec{x}) = T\begin{bmatrix} cx_1 \\ cx_2 \end{bmatrix} = \begin{bmatrix} cx_1+cx_2 \\ cx_2 \end{bmatrix}$$

while

$$cT(\vec{x}) = c\begin{bmatrix} x_1+x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} cx_1+cx_2 \\ cx_2 \end{bmatrix}$$

Therefore, $T(c\vec{x}) = cT(\vec{x})$, so this property holds as well.

Practice problem 1b:

$T : \mathbb{R}^2 \to \mathbb{R}^2$, with

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$$

Practice Problem solution 1b:

Solution:

This is $\boxed{\text{NOT}}$ a linear transformation. It can be checked that neither property (1) nor property (2) from above hold. Let's show that property (2) doesn't hold. Let

$$\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and let $c = 2$. Then,

$$T(\vec{x}) = T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and therefore, we have that

$$2T(\vec{x}) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

However, we have

$$T(2\vec{x}) = T \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

Thus, we see that $2T(\vec{x}) \neq T(2\vec{x})$, and hence T is not a linear transformation.

Activity 2

5:30pm - 5:45pm

Practice Problem 2a:

For the following linear transformations $T : \mathbb{R}^n \to \mathbb{R}^n$, find a matrix A such that $T(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathbb{R}^n$.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^3$,

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x - y \\ 3y \\ 4x + 5y \end{bmatrix}$$

Solution to Practice Problem 2a:

Solution:

To figure out the matrix for a linear transformation from $\mathbb{R}^n$, we find the matrix A whose first column is $T(\vec{e}_1)$, whose second column is $T(\vec{e}_2)$ – in general, whose ith column is $T(\vec{e}_i)$. Here, by definition we have that

$$T(\vec{e}_1) = T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, T(\vec{e}_2) = T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$$

Thus,

$$\boxed{A = \begin{bmatrix} 1 & -1 \\ 0 & 3 \\ 4 & 5 \end{bmatrix}}$$

Practice Problem 2b:

- How to know when a matrix is onto? If a matrix in its reduced echelon form has a pivot in every row.
- How to know when a matrix is one to one? If T is a linear transformation, T(X) has a unique solution.

(Source: University of Alberta, http://www.stat.ualberta.ca/~skalayci/Math%20102/Lecturenotes25-28March2011.pdf)

Example: Is the matrix transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$, where $T(x, y) = (x, y, x + y)$ is onto?

Solution to Practice Problem 2b:

Solution: T is onto if for any vector $(a, b, c) \in \mathbb{R}^3$ we can find a corresponding $(x, y) \in \mathbb{R}^2$ such that $T(x, y) = (a, b, c)$. From here we get linear system

$$\begin{aligned} x &= a \\ y &= b \\ x + y &= c \end{aligned}$$

T is onto if this sytem is consistent for all (a, b, c). $\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 1 & 1 & c \end{bmatrix} \xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & c - b - a \end{bmatrix}$

So this system is consistent if $c = a + b$. Hence for $(1, 2, 5)$ there is no (x, y) that is mapped to $(1, 2, 5)$ under T. So T is not onto.

Goal: Review the topics covered in the lecture, to better prepare the students. (Students were given less help so they can apply the knowledge)

Closure- Survey/ Feedback

5:45pm- 5:50pm

- Wrap-up:
    - Please share with the group one thing you gained understanding of through the session today.
    - Make a note to yourself/ write down anything you need to review/ do more practice problems on.

- Survey/ Feedback:
    1. How fun was the session? (1-10)
    2. How useful was the session? (1-10)
    3. Would you come back? (yes or no)
    4. Optional: Comments (pace of the activity), questions, concerns, suggestions, feedback on the back or wherever

Please recommend SI to your friends/ peers if you found the session useful! Thanks for coming and have a great day :)

PLANNING THE SI SESSION

SI Leader:
Session Date & Day of Week:
Course:
Course Instructor:

Warm-up/ Opening: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of warm-up activity **OR** attach corresponding document(s)

Cool-down/ Closing: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of cool-down activity **OR** attach corresponding document(s)

Workout: **(44-46 min.)**	**Content to cover:**	**Collaborative Learning Technique(s)**	**Strategy(ies) to be used:**

Please provide a **DETAILED BREAKDOWN** of workout activity **OR** attach corresponding document(s)