

Lesson Plan
SI Session #10
August 29, 2017

SI Leader: Eason Chang

Course: Math 18
Academic Quarter: Summer Session 2 2017
Instructor: Professor Drimbe

Topics Covered:
Col Space and Nul Space



Opener Activity:

5:05pm - 5:10pm

Talk about: topics that were hard on the midterm, and topics that you comfortable or not comfortable with now.

Activity 1

5:10pm - 5:30pm

Matrix A with f rows and g columns: $f \times g$

- $\text{Col}(A)$ is a subspace of \mathbb{R}^f
- $\text{Nul}(A)$ is a subspace of \mathbb{R}^g
 - $\text{Nul}(A)$ is the solution to $Ax=0$, which are the x 's, x is $g \times 1$ so you need to match the x of rows of x to the # of columns of A

Practice Problem 1a:

EXAMPLE 5 Let

$$A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$$

- If the column space of A is a subspace of \mathbb{R}^k , what is k ?
- If the null space of A is a subspace of \mathbb{R}^k , what is k ?

Practice Problem 1a Solutions:

SOLUTION

- The columns of A each have three entries, so $\text{Col } A$ is a subspace of \mathbb{R}^k , where $k = 3$.
- A vector \mathbf{x} such that $A\mathbf{x}$ is defined must have four entries, so $\text{Nul } A$ is a subspace of \mathbb{R}^k , where $k = 4$. ■

Practice problem 1b:

Find the vector \mathbf{x} determined by coordinate $[\mathbf{x}]_{\beta}$ and the given basis β

$$\mathcal{B} = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\}, [\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -4 \\ 8 \\ -7 \end{bmatrix}$$

Practice Problem solution 1b:

We calculate that

$$\mathbf{x} = (-4) \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + 8 \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix} + (-7) \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}.$$

Activity 2

5:30pm - 5:45pm

Practice Problem 2a:

(36 pts.) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & -2 & -2 \\ 2 & 1 & 1 & 4 & 5 \\ 1 & 0 & 0 & 3 & 3 \end{bmatrix}$. I found that $\begin{bmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is the reduced echelon form of A . Give bases and dimensions for the following three spaces.

Col A Nul A Row A .

Solution to Practice Problem 2a:

Since columns 1, 2 and 5 are pivot columns, we can compute the dimensions:

$$\dim(\text{Col } A) = \dim(\text{Row } A) = 3 \quad \text{and} \quad \dim(\text{Nul } A) = 2.$$

The first, second and fifth columns of A are a basis for $\text{Col } A$.

The nonzero rows of the reduced echelon form are a basis for $\text{Row } A$.

A basis for $\text{Nul } A$ is $\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

Practice Problem 2b:

Find a basis for the space spanned by the given vectors v_1, \dots, v_5

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}$$

Solution to Practice Problem 2b:

This problem is equivalent to finding a basis for $\text{Col } A$, where $A = [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{v}_4 \ \mathbf{v}_5]$. Since the reduced echelon form of A is

$$\begin{bmatrix} 1 & -2 & 6 & 5 & 0 \\ 0 & 1 & -1 & -3 & 3 \\ 0 & -1 & 2 & 3 & -1 \\ 1 & 1 & -1 & -4 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & -3 & 5 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

we see that the first, second, and third columns of A are its pivot columns. Thus a basis for the space spanned by the given vectors is

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix} \right\}.$$

Closure- Survey/ Feedback

5:45pm- 5:50pm

- Wrap-up:

- Please share with the group one thing you gained understanding of through the session today.

- Make a note to yourself/ write down anything you need to review/ do more practice problems on.

- Survey/ Feedback:

1. How fun was the session? (1-10)
2. How useful was the session? (1-10)
3. Would you come back? (yes or no)
4. Optional: Comments (pace of the activity), questions, concerns, suggestions, feedback on the back or wherever

Please recommend SI to your friends/ peers if you found the session useful! Thanks for coming and have a great day :)

PLANNING THE SI SESSION

SI Leader:

Session Date & Day of Week:

Course:

Course Instructor:

Warm-up/ Opening: (2-4 min.)	Content to cover:	Collaborative Learning Technique	Strategy to be used:

Please provide a **DETAILED BREAKDOWN** of warm-up activity **OR** attach corresponding document(s)

Cool-down/ Closing: (2-4 min.)	Content to cover:	Collaborative Learning Technique	Strategy to be used:

Please provide a **DETAILED BREAKDOWN** of cool-down activity **OR** attach corresponding document(s)

Workout: (44-46 min.)	Content to cover:	Collaborative Learning Technique(s)	Strategy(ies) to be used:

Please provide a **DETAILED BREAKDOWN** of workout activity **OR** attach corresponding document(s)