Math 20B Lecture 1 7/3/2017

Professor
SI Leader: Eason Chang

Midterm 1 July 17 (Monday) During Lectures
Midterm 2 July 27 (Thursday)
Final Aug 4

Review (5.2 – 5.5).

$$F(x) = ?? \xleftarrow{\text{integrate}} f(x) = x^3 \underset{\text{integrate}}{\overset{\text{diff}}{\rightleftarrows}} f'(x) = 3x^2 \underset{\text{integrate}}{\overset{\text{diff}}{\rightleftarrows}} f''(x) = 6x$$

$F(x) = \frac{1}{4}x^4 + C$ (any constant)

$F'(x) = f(x) \Longleftrightarrow \int f(x)\,dx = F(x) + C$

Basic Table

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$$

$$\int_{(a>0)} a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin kx\,dx = \frac{-\cos kx}{k} + C$$

$$\int \cos kx\,dx = \frac{\sin kx}{k} + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$

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$$
\begin{tikzcd}
F(x)=?? & f(x)=x^3 \arrow[l, bend left, "\text{integrate}"] \arrow[r, bend left, "\text{diff}"] & f'(x)=3x^2 \arrow[l, bend left, "\text{integrate}"] \arrow[r, bend left, "\text{diff}"] & f''(x)=6x \arrow[l, bend left, "\text{integrate}"]
\end{tikzcd}
$$

$F(x) = \frac{1}{4}x^4 + C$ (any constant)

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Ex. Evaluate the indefinite integral

$$\int \frac{x-4}{\sqrt[3]{x}}\,dx = \int \frac{x}{x^{\frac{1}{3}}} - \frac{4}{\sqrt[3]{x}}\,dx$$

$$= \int x^{\frac{2}{3}} - 4x^{-\frac{1}{3}}\,dx$$

$$= \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} - \frac{4x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + C$$

$$= \frac{3}{5}x^{\frac{5}{3}} - 4\cdot\frac{3}{2}x^{\frac{2}{3}} + C$$

$$= \frac{3}{5}x^{5/3} - 6x^{\frac{2}{3}} + C$$

Solve the initial value problem

$$\frac{dy}{dt} = t + 2e^{3t} \qquad y = y(t)$$

$$y(0) = 4$$

$$y = \int t + 2e^{3t}\,dt$$

$$= \frac{t^2}{2} + 2\frac{e^{3t}}{3} + C$$

$$y = \frac{t^2}{2} + \frac{2}{3}e^{3t} + \frac{10}{3}$$

$$4 = 0 + \frac{2}{3} + C$$

$$C = \frac{10}{3}$$

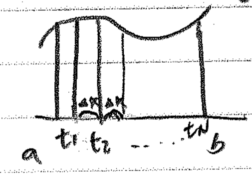
Fundamental Theorem of Calculus (I)

Let $f(t)$ be continuous on $[a, b]$. Then

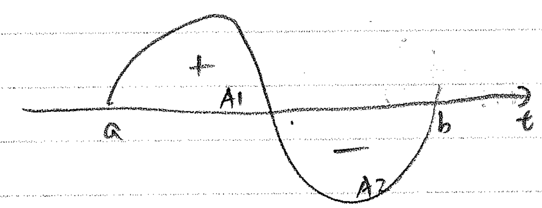
$$\int_a^b f(t)\,dt = F(t)\Big]_a^b = F(b) - F(a)$$

Recall $\int_a^b f(t)\,dt = \lim_{N\to\infty} f(t_1)\Delta x + f(t_2)\Delta x \dots f(t_N)\Delta x$

= signed area



Signed Area



$\int_a^b f(t)\,dt = A_1 - A_2.$

5.6 Net Changes as the Integral of a Rate of Change
Fundamental Theorem of Calculus (I)

$$\int_a^b \underbrace{F'(x)}_{\text{the rate of changes of } F} dx = \underbrace{F(b) - F(a)}_{\text{Net change}}$$

Ex. (I) Water flows into an empty bucket at rate of 1.5 liters / second. How much water is in the bucket after 4 seconds?

Water amount after t seconds $= S(t)$

1.5 lit/sec $= S'(t)$

After 4 seconds, $1.5 \times 4 = 6$ liters.

Using Net Change Theorem

$$\int_0^4 S'(t)\,dt = S(4) - S(0)$$

$$\int_0^4 1.5\,dt = S(4) - 0 = 1.5t\Big]_0^4 = 1.5 \cdot 4 - 0 = 6$$

(II) After 4 sec, water flows into the bucket at rate of $3+2t$ lit/s
How much water under is the bucket after 9 sec

$$\int_4^7 3+2t\,dt = S(7) - S(4)$$

$$= S(7) - 6$$

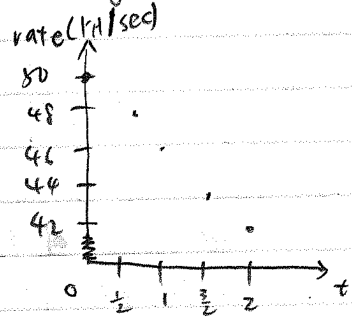
$$= 42$$

$$S(7) = 48$$

(III) The rate (lit/sec) of water flows is recorded as the following

t (sec)	0	0.5	1	1.5	2
lit/sec	50	48	46	44	42

Estimate total amount of water in the bucket in 2 sec using average of the left - and - right endpoint.



average $\frac{50+48}{2} = 49$

$$\text{Total} = 49 \cdot \frac{1}{2} + 47 \cdot \frac{1}{2} + 45 \cdot \frac{1}{2} + 43 \cdot \frac{1}{2} = \frac{1}{2}(49+47+45+43)$$

$$= \frac{1}{2}(184)$$

$$= 92 \text{ liters}$$

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velocity $v(t) = p'(t)$

$\int_a^b v(t)\,dt = P(b) - P(a)$ displacement

$\int_a^b |v(t)|\,dt$ total distance travelled.