Professor Um
SI: Eason

Math 20B Lecture 6 7/12/2017

Exam 1

Monday July 17

5.2 - 5.7 (5.6)

6.1 - 6.3

7.1

11.3 11.4

7.2 7.4, 7.5

Ex $\int \sin^3 x \, dx$ $u = \cos x$, $du = -\sin x \, dx$

$= -\int \sin^2 x \underline{\sin x \, dx}$

$= -\int (1-\cos^2 x) \, du$

$= -\int 1 - u^2 \, du$

$= \int u^2 - 1 \, du$

$= \frac{u^3}{3} - u + C$

$= \frac{\cos^3 x}{3} - \cos x + C$

Trig ID

1. $\sin^2 x + \cos^2 x = 1$
2. $1 + \tan^2 x = \sec^2 x$
3. $\sin^2 x = \frac{1-\cos 2x}{2}$
4. $\cos^2 x = \frac{1+\cos 2x}{2}$

Ex $\int \sin^2 x \cos^2 x \, dx$

$= \int \sin^2 x (1 - \sin^2 x) \, dx$

$= \int \sin^2 x - \sin^4 x \, dx$

$= \int \frac{1-\cos(2x)}{2} - \left(\frac{1-\cos(2x)}{2}\right)^2 dx$

$= \int \frac{1}{2} - \frac{1}{2}\cos(2x) - \frac{1 - 2\cos(2x) + \cos^2(2x)}{4} \, dx$

$= \int \left(\frac{1}{2} - \frac{1}{4}\right) + \underbrace{\left(-\frac{1}{2} + \frac{2}{4}\right)}_{0} \cos(2x) - \frac{1}{4}\cos^2(2x) \, dx$

$= \int \frac{1}{4} - \frac{1}{4}\cos^2(2x) \, dx$

$(\tan' x) = \sec^2 x$

$(\sec' x) = \sec x \tan x$

$= \int \frac{1}{4} - \frac{1}{4} \frac{1+\cos(4x)}{2} dx$

$= \int \frac{1}{4} - \frac{1}{8} - \frac{1}{8} \cos(4x) dx$

$= \frac{1}{8} x - \frac{1}{32} \sin 4x + C$

Ex. $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$

$u = \cos x$

$du = -\sin x \, dx$

$= \int \frac{1}{u} (-du)$

$= -\ln|u| + C$

$= \cancel{\ln|\cos x| + C}$

$= \ln|\cos x|^{-1} + C$

$= \boxed{\ln|\sec x| + C}$ since $\frac{1}{\cos x} = \sec x$

Ex. $\int \sec x \, dx$

$= \int \frac{\cancel{\sec x(\sec x + \tan x)}}{(\sec x + \tan x)} dx$

$u = \sec x + \tan x$

$du = (\sec x \tan x + \sec^2 x) dx$

$= \int \frac{\sec x \cdot \tan x + \sec^2 x}{\sec x + \tan x} dx$

$= \int \frac{1}{u} du$

$= \ln|u| + C$

$= \ln|\sec x + \tan x| + C$

Ex. $\int \frac{e^x + 2x}{e^x + x^2 + 1} dx$ ($e^x + 2x \, dx = du$, $e^x + x^2 + 1 = u$) $= \ln|u| + C$

$= \ln|e^x + x^2 + 1| + C$

Ex (i) $\int \tan^2 x \cdot \sec^4 x \, dx$ $\quad$ $u = \tan x$

$du = \sec^2 x \, dx$

$\int \tan^2 x \cdot \sec^2 x \; \underline{\sec^2 x \, dx}$

$= \int \tan^2 x (1 + \tan^2 x) \; du$

$= \int \tan^2 x + \tan^4 x \; du$

$= \int u^2 + u^4 \, du$

$= \frac{u^3}{3} + \frac{u^5}{5} + C \Rightarrow \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$

(ii) $\int \tan^3 x \cdot \sec^3 x \, dx$

$u = \sec x$

$= \int \tan^2 x \cdot \sec^2 x \cdot \sec x \tan x \; dx$ $\quad$ $du = \sec x \cdot \tan x \, dx$

$= \int (\sec^2 x - 1) \sec^2 x \, dx$

$= \int (u^2 - 1) u^2 \, du$

Ex $\int \tan^2 x \cdot \sec x \, dx$

$= \int (\sec^2 x - 1)(\sec x) \, dx$

$= \int \sec^3 x - \sec x \, dx$

$= \int \sec^3 x \, dx - \int \sec x \, dx$

$\int \tan^{\text{odd}} x \sec^{\text{even}} x$, $u = \tan x$

$\int \tan^{\text{odd}} x \sec^{\text{any}} x$, $u = \sec x$

$u = \sec x \quad dv = \sec^2 x$

$du = \sec x \tan x \, dx \quad v = \tan x$

$\int \sec^3 x \, dx$

$= \int \sec x \cdot \sec^2 x \, dx$

$\sec x \tan x - \int \tan^2 x \sec x \, dx$

7.4 Integrals involving Hyperbolic functions

Def: $\sinh x \overset{\text{def}}{=} \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$(\sinh x)' = \cosh x$

$(\cosh x)' = \sinh x$

$(\tanh x)' = \operatorname{sech}^2 x$

$(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$

$\cosh^2 x - \sinh^2 x = 1$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$\sinh^2 x = \frac{-1 + \cosh(2x)}{2}$

$\cosh^2 x = \frac{1 + \cosh(2x)}{2}$

Ex. $\int \sinh^3 x \cosh^2 x \, dx$ $\quad u = \cosh x$

$du = \sinh x$

$= \int \underbrace{\sinh^2 x}_{\substack{\cosh^2 x - 1 \\ u^2 - 1}} \underbrace{\cosh^2 x}_{u^2} \underbrace{\sinh x \, dx}_{du}$

$= \int (u^2 - 1) u^2 \, du$

$= u^4 - u^2 \, du$

$= \frac{u^5}{5} - \frac{u^3}{3} \, du$

$= \frac{1}{5} (\cosh x)^5 - \frac{1}{3} (\cosh x)^3 + C$

7.5 The method of partial fractions.

$\int$ Rational Function $= \left(\frac{\text{poly}}{\text{poly}}\right) dx$

by ~~simplify the denominator~~

Ex: $\int \frac{2}{(x+1)(2x+1)} dx$

$$= \frac{A}{x+1} + \frac{B}{2x+1}$$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int \frac{1}{x+a} dx = \int \frac{1}{u} du = \ln|x+a| + C$

$2 = A(2x+1) + B(x+1)$ same function for any x

$x=-1$, $2 = -A \rightarrow A = -2$

$x = -\frac{1}{2}$ $2 = \frac{1}{2} B \rightarrow B = 4$

$$\int \frac{2}{(x+1)(2x+1)} dx = -\int \frac{2}{x+1} dx + \int \frac{4}{2x+1} dx$$

$$= -2\int \frac{1}{x+1} dx + 2\int \frac{1}{x+\frac{1}{2}} dx$$

$$= -2\ln|x+1| + 2\ln|x+\tfrac{1}{2}| + C$$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int \frac{1}{x+a} dx = \int \frac{1}{u} du$ $\quad u = x+a$, $du = dx$

$= \ln|x+a| + C$