Lesson Plan
SI Session #15
September 7, 2017

SI Leader: Eason Chang

Course: Math 18
Academic Quarter: Summer Session 2 2017
Instructor: Professor Drimbe

Topics Covered: Final Review

Opener Activity:

5:05pm - 5:10pm

4. Let $\mathcal{B}$ and $\mathcal{C}$ be two bases for the vector space $\mathbb{R}^2$.

   (a) If $\mathcal{B} = \{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}-2\\1\end{bmatrix}\}$ and $\mathcal{C} = \{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}-2\\2\end{bmatrix}\}$, find the change of coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$.

Activity 1

5:10pm - 5:30pm

Practice problem 1a:

(b) Prove that the inverse of the change of coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ is the change of coordinates matrix from $\mathcal{C}$ to $\mathcal{B}$.

Solutions for Practice Problem 1a:

4.a) B to C

$$P_{C \leftarrow B}$$

$$= \left[\begin{array}{cc|cc} 1 & -2 & 1 & -2 \\ 1 & 2 & 2 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & -2 & 1 & -2 \\ 0 & 4 & 1 & 3 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & -2 & 1 & -2 \\ 0 & 2 & 1/2 & 3/2 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 3/2 & -1/2 \\ 0 & 2 & 1/2 & 3/2 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 3/2 & -1/2 \\ 0 & 1 & 1/4 & 3/4 \end{array}\right]$$

$$P_{C \leftarrow B} = \begin{bmatrix} 3/2 & -1/2 \\ 1/4 & 3/4 \end{bmatrix}$$

b) $P_{C\leftarrow B}^{-1} = \dfrac{1}{\frac{3}{2}\left(\frac{3}{4}\right)-\left(-\frac{1}{2}\right)\left(\frac{1}{4}\right)} \begin{bmatrix} 3/4 & 1/2 \\ -1/4 & 3/2 \end{bmatrix}$

$= \dfrac{1}{\frac{9}{8}+\frac{1}{8}} \begin{bmatrix} 3/4 & 1/2 \\ -1/4 & 3/2 \end{bmatrix}$

$= \dfrac{4}{5} \begin{bmatrix} 3/4 & 1/2 \\ -1/4 & 3/2 \end{bmatrix}$

$= \begin{bmatrix} 3/5 & 2/5 \\ -1/5 & 6/5 \end{bmatrix}$

$P_{B\leftarrow C} = \left[\begin{array}{cc|cc} 1 & -2 & 1 & -2 \\ 2 & 1 & 1 & 2 \end{array}\right]$

$\sim \left[\begin{array}{cc|cc} 2 & -4 & 2 & -4 \\ 2 & 1 & 1 & 2 \end{array}\right]$

$= \left[\begin{array}{cc|cc} 1 & -2 & 1 & -2 \\ 0 & 5 & -1 & 6 \end{array}\right]$

$= \left[\begin{array}{cc|cc} 1 & -2 & 1 & -2 \\ 0 & 2 & -2/5 & 12/5 \end{array}\right]$

$= \left[\begin{array}{cc|cc} 1 & 0 & 3/5 & 2/5 \\ 0 & 1 & -1/5 & 6/5 \end{array}\right]$

$P_{B\leftarrow C} = \begin{bmatrix} 3/5 & 2/5 \\ -1/5 & 6/5 \end{bmatrix} = P_{C\leftarrow B}^{-1}$ Q.E.D

Practice Problem 1b:

6. Let $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$. (*You should try this problem after Tuesday, after we cover eigenvalues.*)

   (a) Find the eigenvalues of A and the associate eigenvectors.

   (b) Prove that A is diagonalizable and find P invertible D diagonal such that $A = PDP^{-1}$.

Practice Problem 1b Solutions:

6 b) A : 2×2 matrix

2 distinct eigenvalues → match

By theorem, A is thus diagonalizable

For P: set eigenvectors as the column vectors of matrix P

⇒ use P to compute P^{-1}

For D: compute: $P^{-1}AP$ to obtain matrix

→ check work by checking to see if $A = PDP^{-1}$

Activity 2

5:30pm - 5:45pm

Practice Problem 2a:

Diagonalize the matrix $\begin{bmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{bmatrix}$

Solution to Practice Problem 2a:

To find the eigenvalues, compute

$$\det \begin{bmatrix} 3-\lambda & 0 & 0 \\ -3 & 4-\lambda & 9 \\ 0 & 0 & 3-\lambda \end{bmatrix} = (3-\lambda)(4-\lambda)(3-\lambda).$$

So the eigenvalues are $\lambda = 3$ and $\lambda = 4$.

We can find two linearly independent eigenvectors $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$ corresponding to the eigenvalue 3, and one eigenvector $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ with eigenvalue 4. The diagonalized form of the matrix is

$$\begin{bmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -3 \\ -3 & 1 & 9 \end{bmatrix}.$$

Note that if you chose different eigenvectors, your matrices will be different. The middle matrix should have entries 3, 3, 4 in some order, and you should multiply out the product to make sure you have the right answer.

Practice Problem 2b:

(c) Let A be a 2×2 matrix such that $A^2 = I_2$. Is it true that $A = I_2$? Justify your answer.

Solution to Practice Problem 2b:

False; if $A^2 = I_2$, then $A^4 = (A^2)^2 = (I_2)^2 = I_2$.

Practice Problem 2c:

(b) Let A be a $n \times n$ matrix with real entries such that $A^T A = 0$. Prove that $A = 0$.

Solution to 2c:

An idea: if we put $A = (a_{ij})_{1 \le i,j \le n}$, then $A^t = (b_{ij})$, with $b_{ij} = a_{ji}$, so by definition:

$$AA^t = \left(\sum_{k=1}^{n} a_{ik} b_{kj} \right) = \left(\sum_{k=1}^{n} a_{ik} a_{jk} \right)$$

If you now look at the main diagonal's general entry of the above, you get

$$\sum_{k=1}^{n} a_{ik} a_{ik} = \sum_{k=1}^{n} a_{ik}^2$$

So if $AA^t = 0$ then the above diagonal's entries are zero, but a sum of squared *real* numbers is zero iff each number is zero, so...

The same result is true with complex matrices if instead we require $AA^* = 0$, $A^* := \overline{A^t}$

Goal: Review the topics covered in the lecture, to better prepare the students. (Students were given less help so they can apply the knowledge)

Closure- Survey/ Feedback

5:45pm- 5:50pm

- Wrap-up:
    - Please share with the group one thing you gained understanding of through the session today.
    - Make a note to yourself/ write down anything you need to review/ do more practice problems on.

- Survey/ Feedback:
    1. How fun was the session? (1-10)
    2. How useful was the session? (1-10)
    3. Would you come back? (yes or no)
    4. Optional: Comments (pace of the activity), questions, concerns, suggestions, feedback on the back or wherever

Please recommend SI to your friends/ peers if you found the session useful! Thanks for coming and have a great day :)

PLANNING THE SI SESSION

SI Leader:
Session Date & Day of Week:
Course:
Course Instructor:

Warm-up/ Opening: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of warm-up activity **OR** attach corresponding document(s)

Cool-down/ Closing: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of cool-down activity **OR** attach corresponding document(s)

Workout: **(44-46 min.)**	**Content to cover:**	**Collaborative Learning Technique(s)**	**Strategy(ies) to be used:**

Please provide a **DETAILED BREAKDOWN** of workout activity **OR** attach corresponding document(s)