

Lesson Plan
SI Session #6
August 18, 2017

SI Leader: Eason Chang

Course: Math 18
Academic Quarter: Summer Session 2 2017
Instructor: Professor Drimbe

Topics Covered:
Transformation, Matrix Multiplication



Opener Activity:

5:05pm - 5:10pm

- Spend 1 min to review notes, and see who can recall the definitions for transformation and linear transformation, etc.
- Transformation: A **transformation** (or **function** or **mapping**) T from \mathbb{R}^n to \mathbb{R}^m is a rule that assigns to each vector \mathbf{x} in \mathbb{R}^n a vector $T(\mathbf{x})$ in \mathbb{R}^m . The set \mathbb{R}^n is called the **domain** of T , and \mathbb{R}^m is called the **codomain** of T .
- Domain, codomain

A transformation (or mapping) T is **linear** if:

- (i) $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all \mathbf{u}, \mathbf{v} in the domain of T ;
- (ii) $T(c\mathbf{u}) = cT(\mathbf{u})$ for all scalars c and all \mathbf{u} in the domain of T .

- (Optional for this session) shear transformation, contraction and dilation

Activity 1

5:10pm - 5:30pm

Practice Problem 1a:

- How to know when a matrix is onto? If a matrix in its reduced echelon form has a pivot in every row.
- How to know when a matrix is one to one? If T is a linear transformation, $T(\mathbf{x})$ has a unique solution.

(Source: University of Alberta,

<http://www.stat.ualberta.ca/~skalayci/Math%20102/Lecturenotes25-28March2011.pdf>)

Example: Is the matrix transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$, where $T(x, y) = (x, y, x + y)$ is onto?

Practice Problem 1a Solutions:

Solution: T is onto if for any vector $(a, b, c) \in \mathbb{R}^3$ we can find a corresponding $(x, y) \in \mathbb{R}^2$ such that $T(x, y) = (a, b, c)$. From here we get linear system

$$\begin{aligned}x &= a \\y &= b \\x + y &= c\end{aligned}$$

T is onto if this system is consistent for all (a, b, c) . $\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 1 & 1 & c \end{bmatrix} \xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & c - b - a \end{bmatrix}$

So this system is consistent if $c = a + b$. Hence for $(1, 2, 5)$ there is no (x, y) that is mapped to $(1, 2, 5)$ under T . So T is not onto.

Practice problem 1b and Practice Problem solution 1b:

Let the linear operator $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by the following equations $w_1 = 2x - 5y$ and $w_2 = 4x + 3y$. Is this transformation one-to-one?

The standard matrix for this operation is $A = \begin{bmatrix} 2 & -5 \\ 4 & 3 \end{bmatrix}$, and $\det(A) = 26 \neq 0$, so this transformation is one-to-one.

Since there is a pivot in every row when the matrix is row reduced, then the columns of the matrix will span \mathbb{R}^3 .

Activity 2

5:30pm - 5:45pm

Practice Problem 2a and Solution to Practice Problem 2a:

$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

$$T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 + x_2 - x_3 \\ x_1 - x_2 + x_3 \\ 2x_1 + 2x_2 + x_3 \end{bmatrix} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

(2, 2, 6), the system has a unique solution, therefore the transformation is one-to-one

Closure- Survey/ Feedback

5:45pm- 5:50pm

- Wrap-up:

- Please share with the group one thing you gained understanding of through the session today.

- Make a note to yourself/ write down anything you need to review/ do more practice problems on.

- Survey/ Feedback:

1. How fun was the session? (1-10)
2. How useful was the session? (1-10)
3. Would you come back? (yes or no)
4. Optional: Comments (pace of the activity), questions, concerns, suggestions, feedback on the back or wherever

Please recommend SI to your friends/ peers if you found the session useful! Thanks for coming and have a great day :)

PLANNING THE SI SESSION

SI Leader:

Session Date & Day of Week:

Course:

Course Instructor:

Warm-up/ Opening: (2-4 min.)	Content to cover:	Collaborative Learning Technique	Strategy to be used:

Please provide a **DETAILED BREAKDOWN** of warm-up activity **OR** attach corresponding document(s)

Cool-down/ Closing: (2-4 min.)	Content to cover:	Collaborative Learning Technique	Strategy to be used:

Please provide a **DETAILED BREAKDOWN** of cool-down activity **OR** attach corresponding document(s)

Workout: (44-46 min.)	Content to cover:	Collaborative Learning Technique(s)	Strategy(ies) to be used:

Please provide a **DETAILED BREAKDOWN** of workout activity **OR** attach corresponding document(s)