Lesson Plan
SI Session #12
September 1, 2017

SI Leader: Eason Chang

Course: Math 18
Academic Quarter: Summer Session 2 2017
Instructor: Professor Drimbe

Topics Covered: Determinants, Onto, One to One Linear Transformation, Coordinates

TEACHING+LEARNING
COMMONS

Opener Activity:

5:05pm - 5:10pm

Talk about: Review what onto and one-to-one linear transformation is.

Theorem. *Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. The following are equivalent:*

1. *T is onto.*
2. *The equation $T(\mathbf{x}) = \mathbf{b}$ has solutions for every $\mathbf{b} \in \mathbb{R}^m$.*
3. *If A is the standard matrix of T, then the columns of A span $\mathbb{R}^m$.* That is: every $\mathbf{b} \in \mathbb{R}^m$ is a linear combination of the columns of A.

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3. *If A is the standard matrix of T, then the columns of A span $\mathbb{R}^m$.* That is: every $\mathbf{b} \in \mathbb{R}^m$ is a linear combination of the columns of A.
4. $\mathrm{Im}(A) = \mathbb{R}^m$.
5. $\mathrm{rank}(A) = m$.
6. $\mathrm{nullity}(A) = n - m$.

Activity 1

5:10pm - 5:30pm

Practice Problem 1a:

T: R4 → R5

Is T is one-to-one?

Solution. We look at the reduced row echelon form of homogeneous system $A\,|\,0$:

$$\left(\begin{array}{cccc|c} -1 & 2 & 1 & 2 & 0 \\ 2 & -1 & 2 & 6 & 0 \\ -3 & 4 & 4 & 3 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 1 & 0 & 2 & 5 & 0 \end{array}\right) \to \text{rref} \to \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

We see that $AX = 0$ has infinitely many solutions; thus, T is not one-to-one. (Matrix A does not reduce to the identity with additional rows of 0.)

Practice Problem 1a Solutions:

Theorem 2.2. Let T: $R^n \to R^m$ be a linear transformation with matrix representation A. If $m < n$, then T cannot be one-to-one.

Theorem 2.6. Let T: $R^n \to R^m$ be a linear transformation with matrix representation A. If $m > n$, then T cannot be onto.

T: $R^5 \to R^4$, 4x5 matrix, cannot be one-to-one
T: $R^4 \to R^5$, 5x5 matrix cannot be onto

Practice problem 1b:
T: R3 → R4

Solution. We let A be the matrix representation and look at the reduced row echelon form of homogeneous system $A|0$:

$$\left(\begin{array}{ccc|c} 2 & -2 & 3 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & 4 & 0 & 0 \\ -4 & -6 & 3 & 0 \end{array}\right) \to \text{rref} \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The only solution to $AX = 0$ is $X = 0$; thus, T is one-to-one. (A reduces to the identity with an additional row of 0's.)

Practice Problem 1c:

Example 8. Let T_A: $R^4 \to R^3$ be defined by the matrix representation

$$A = \begin{pmatrix} 1 & 0 & -2 & 4 \\ 1 & -2 & 1 & -3 \\ -2 & 1 & 0 & 2 \end{pmatrix}$$

where $T_A(x) = AX$. Is T_A one-to-one and/or onto?

Solutions Practice Problem 1c:

Solution. Because R^3 is a "smaller" set than R^4, T_A cannot be one-to-one (Theorem 2). To determine if T_A is onto, we consider the reduced row echelon form of A:

$$\begin{pmatrix} 1 & 0 & 0 & -1.2 \\ 0 & 1 & 0 & -0.4 \\ 0 & 0 & 1 & -2.6 \end{pmatrix}$$

Because no rows became all 0, there will never be an inconsistency. So every system $AX = B$ will have a solution; therefore, T_A is onto.

Activity 2

5:30pm - 5:45pm

Practice Problem 2a:

$$|B| = det(B) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = a\begin{vmatrix} e & f \\ h & k \end{vmatrix} - b\begin{vmatrix} d & f \\ g & k \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

The determinant of the matrix

$$\begin{bmatrix} 3 & -5 & 3 \\ 2 & 1 & -1 \\ 1 & 0 & 4 \end{bmatrix}$$

Solution to Practice Problem 2a:

$$3\begin{vmatrix} 1 & -1 \\ 0 & 4 \end{vmatrix} - (-5)\begin{vmatrix} 2 & -1 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 54$$

Practice Problem 2b:

$$\det\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Solution to Practice Problem 2b:

$$= 1 \cdot \det\begin{pmatrix} 5 & 6 \\ 8 & 9 \end{pmatrix} - 2 \cdot \det\begin{pmatrix} 4 & 6 \\ 7 & 9 \end{pmatrix} + 3 \cdot \det\begin{pmatrix} 4 & 5 \\ 7 & 8 \end{pmatrix}$$

$= 1 * (-3) - 2(-6) + 3(-3)$
$= 0$

Closure- Survey/ Feedback

5:45pm- 5:50pm

- Wrap-up:
    - Please share with the group one thing you gained understanding of through the session today.
    - Make a note to yourself/ write down anything you need to review/ do more practice problems on.

- Survey/ Feedback:
    1. How fun was the session? (1-10)
    2. How useful was the session? (1-10)
    3. Would you come back? (yes or no)
    4. Optional: Comments (pace of the activity), questions, concerns, suggestions, feedback on the back or wherever

Please recommend SI to your friends/ peers if you found the session useful! Thanks for coming and have a great day :)

PLANNING THE SI SESSION

SI Leader:
Session Date & Day of Week:
Course:
Course Instructor:

Warm-up/ Opening: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of warm-up activity **OR** attach corresponding document(s)

Cool-down/ Closing: **(2-4 min.)**	**Content to cover:**	**Collaborative Learning Technique**	**Strategy to be used:**

Please provide a **DETAILED BREAKDOWN** of cool-down activity **OR** attach corresponding document(s)

Workout: **(44-46 min.)**	**Content to cover:**	**Collaborative Learning Technique(s)**	**Strategy(ies) to be used:**

Please provide a **DETAILED BREAKDOWN** of workout activity **OR** attach corresponding document(s)