Operations Research, Spring 2021 (109-2)
In-class Decision-making Challenge

Instructor: Ling-Chieh Kung
Department of Information Management
National Taiwan University

IEDO is a company which imports products from an overseas manufacturer to sell to domestic consumers. As a reseller, the company's business relies heavily on purchasing and the control of inventory. On one hand, if a consumer cannot get a product when she/he wants, lost sales occurs and hurts the company's profit. On the other hand, if there are too many unsold products, holding costs emerge.

You are newly hired as an operations researcher by the company. Right after you started to work, you realized why the company hired you. In the past, they made all the purchasing decisions, including purchasing quantities, shipping methods, and the timing of ordering, by hands, pencil, paper, spreadsheet software, and experience. That was not bad, but indeed there is room for improvement.

Today, you sat down at your desk and started to organize all the information you collected in the past two weeks. The ordering cycle time is a month. More precisely, at the beginning of each month, the company needs to determine the order quantities and shipping methods of all products and place an order to the manufacturer. While the ordering decision must be made for every month, you have decided to start by considering the next ordering timing, March 1, as an example.

1 Basic ordering process

1.1 Demand and inventory holding costs

The company orders ten products from the manufacturer. Monthly forecast demands as well as the current on-hand inventory levels of all products in the next six months are given in Table 1. For example, for product 2 it is estimated that the monthly demand for April will be 101 units. Note that for some products the initial inventory levels are not enough to cover future demands. Purchasing should thus be considered.

Product	Initial inventory	Monthly forecast demands					
		March	April	May	June	July	August
1	800	138	55	172	194	94	185
2	600	190	101	68	185	13	136
3	425	79	179	21	49	199	200
4	350	142	103	78	131	146	155
5	400	35	62	83	90	197	49
6	524	91	95	107	127	116	183
7	453	105	164	19	116	119	175
8	218	37	155	10	77	168	32
9	673	108	185	188	176	81	172
10	200	46	178	162	200	154	199

Table 1: Monthly forecast demands and initial inventory levels

Inventory should be prepared to face future demands and avoid costs related to inventory holding. Holding one unit of a product by one month incurs a unit holding cost for that product (due to warehousing and interest loss, among others). The company take 2% of the unit purchasing cost as the per unit per month holding cost of a product. The purchasing costs per unit and holding costs per unit per month of all products are listed in Table 2. For example, it costs the company \$2,000 to purchase one unit of product 2, and that means holding one unit of product 2 for one month costs the company \$2,000 $\times$ 0.02 = \$40.

Product	Price (\$)	Costs (\$)	
		Purchasing	Holding
1	10700	5000	100
2	5000	2000	40
3	26900	9000	180
4	19600	9000	180
5	6900	2000	40
6	31500	9000	180
7	14100	7000	140
8	16400	5000	100
9	33000	9000	180
10	18600	7000	140

Table 2: Inventory holding costs

As an example, suppose that your initial inventory for product 1 is 800 units, and you order 38 units of product 1 that will be available for sales in May. It then follows that the ending inventory for product 1 will be 662, 607, 473, 279, 185, 0 for each of the following six months. The total inventory holding cost is thus

$$\$100 \times (662 + 607 + 473 + 279 + 185 + 0) = \$220{,}600.$$

1.2 Shipping methods

There are three shipping methods to choose from: express delivery, air freight, and ocean freight. The three methods have different lead times and costs. For each method, the lead time include the processing time needed by the manufacturer and shipper, custom clearance, and the processing time needed ourselves, etc. In any case, it is the time it needs between the order is placed and the products are ready to be sold to end consumers. To keep things simple, the company says you may assume that an order is always placed on the first day of a month, and products are always ready to be sold on the last day of a month. The lead times of express delivery, air freight, and ocean freight are one, two, and three months, respectively. For example, if you place an express delivery order on March 1, the ordered products will get ready on March 31.

There are three types of shipping costs. A variable shipping cost is charged per unit of product. A fixed shipping cost is charged per order. These two costs exist for express delivery and air freight. For ocean freight, there is no variable cost. The fixed cost of initiating an express delivery order, an air freight, or an ocean freight is \$100, \$80, or \$50, respectively. Table 3 contains variable costs for all products.

As an example, suppose that for each product you order 200 units, where 50 units are shipped by air and 150 units are by sea. The total cost for air freight is

$$\$80 + (\$18 + \$45 + \cdots + \$20) \times 50 = \$17{,}230,$$

which is the sum of the fixed cost per order and variable costs. The total cost for ocean freight is \$50. Finally, as nothing is ordered through express delivery, no fixed or variable cost should be paid for this method. The total shipping cost is thus \$17,230 + \$50 = \$17,280.

Product	Variable cost ($)	
	Express delivery	Air freight
1	44	18
2	89	45
3	86	38
4	91	46
5	50	21
6	51	25
7	83	46
8	96	49
9	80	35
10	49	20

Table 3: Shipping costs

1.3 In-transit products

The inventory in-transit of all products in the next three months are given in Table 4. For example, for product 6, it is estimated that 18 units will be delivered and be available for sales in April, 23 units will be delivered and be available for sales in May. Note that because the maximum lead time is three months (if using the ocean freight), we only consider the inventory in-transit for the following two months.

Product	Inventory in-transit	
	April	May
1	0	0
2	48	0
3	0	20
4	153	0
5	0	0
6	18	23
7	28	45
8	0	0
9	109	34
10	0	0

Table 4: Monthly inventory in-transit

1.4 Problem 1

Given all the data and rules above, you now need to come up with an ordering plan for the next coming six months. In particular, at the beginning of each month, you need to determine for each product how many to order through each of the three shipping methods. Your goal is to minimize total cost while ensuring that all demands are satisfied.

2 Adding container costs

2.1 Information about the container costs

In the previous section, we say that there is only fixed cost for ocean freight. Now, we add a container cost charged per container regarding ocean freight.[1] A container, whose usable capacity is considered as 30 cubic meter (CBM), costs $ 2,750.[2] Other costs remain the same. Table 5 contains volume information for all products.

Product	Volume (CBM)
1	0.073
2	0.005
3	0.043
4	0.063
5	0.045
6	0.086
7	0.079
8	0.082
9	0.068
10	0.098

Table 5: Product volumes

As an example, suppose that for each product you order 200 units, where 50 units are

[1] In practice, a container may be of one of two standard containers: 20 feet or 40 feet. To make the problem easier, in this case assignment we assume all containers are of 20 feet.

[2] The true usable capacity of a 20-foot container is 32.6 CBM. To simplify calculation, your company assumes that all ordered products may be put inside a container as long as their total volume does not exceed 30 CBM.

shipped by air and 150 units are by sea. The total cost for air freight is

$$\$80 + (\$18 + \$45 + \cdots + \$20) \times 50 = \$17{,}230,$$

which is the sum of the fixed cost per order and variable costs. The total cost for ocean freight is

$$\$50 + \$2,750 \left\lceil \frac{(0.073 + 0.005 + \cdots + 0.098) \times 150}{30} \right\rceil = \$50 + \$2,750 \times \lceil 3.21 \rceil = \$11{,}050.$$

Note that those the fourth container is not fully utilized, the company still needs to pay its full cost. Finally, as nothing is ordered through express delivery, no fixed or variable cost should be paid for this method. The total shipping cost is thus \$17,230 + \$11,050 = \$28,280.

2.2 Problem 2

Again, given all the data and rules above, you now need to come up with another ordering plan for the next coming six months. In particular, at the beginning of each month, you need to determine for each product how many to order through each of the three shipping methods. Your goal is to minimize total cost while ensuring that all demands are satisfied.

3 Allowing shortage and assuming all shortages are lost sales

3.1 Information about lost sales

Continue from the previous section with container costs, we now allow shortage to happen. Unit shortage cost incurs in a similar way as the unit inventory cost. Interestingly, two things may happen when a consumer cannot get the product she/he wants: She/he may make a request and wait until the product is available or leave empty-handed. We say there is a backorder in the former case or a lost sales in the latter case. For now, we assume that all shortage become lost sales. When there is a lost sales, the shortage cost is the difference between the sales price and purchasing cost of the product. The lost sales cost per unit per month of all products are listed in Table 6.

As an example, we know from the previous section that your initial inventory for product 3 is 425 units, and you order no product 3. Table 7 records the detailed steps

Product	Costs of lost sales ($)
1	5700
2	3000
3	17900
4	10600
5	4900
6	22500
7	7100
8	11400
9	24000
10	11600

Table 6: Cost of lost sales

of calculating all inventory-related costs. In the first month, you have $425 - 79 = 346$ units left, which is your stocking level at the end of March. For April, May, and June, the stocking levels are all positive. For July, 82 consumers cannot get product 3, and all 82 consumers leave. For August, 200 more consumers cannot get product 3 and thus will leave. The total holding cost is $\$180 \times (346 + 167 + 166 + 117) = \$143{,}280$, and total lost sales cost is $\$17{,}900 \times (82 + 200) = \$5{,}047{,}800$. The total inventory-related costs is \$5,191,080.

	March	April	May	June	July	August
Demand	79	179	21	49	199	200
Inventory in-transit	0	0	20	0	0	0
Ending inventory	346	167	166	117	−82	−200
Stocking level	346	167	166	117	0	0
Lost sales	0	0	0	0	82	200

Table 7: Lost sales example

3.2 Problem 3

Again, given all the data and rules above, you now need to come up with another ordering plan for the next coming six months. In particular, at the beginning of each month, you need to determine for each product how many to order through each of the three shipping methods. As shortage is allowed now, your goal is to minimize total cost by making trade-off between all the related costs.

4 Shortage as lost sales and backorder

4.1 Information about lost sales and backorder

As mentioned, a consumer may make a request and wait until the product is available (which is the case of backorder) or leave empty-handed (which is the case of lost sales). Now, we consider both cases. When there is a backorder, a customer gets 5% of the sales price as a discount for each month she/he waits. The shortage cost of having one backorder is thus 5% of the sales price. When there is a lost sales, the shortage cost is the difference between the sales price and purchasing cost of the product. According to historical records and experience, the backorder percentage of each product is estimated. The backorder costs per unit per month and backorder percentages of all products are listed in Table 8. For example, as the company may sell product 2 in \$5,000, giving out 5% of discount due to let one customer wait for one month results in the shortage cost $\$5{,}000 \times 0.05 = \$250$ per unit per month. Having one unit of lost sales, however, is a profit loss of $\$5{,}000 - \$2{,}000 = \$3{,}000$ as lost sales cost. Luckily, 70% of all consumers will choose to wait when product 2 is out of stock.

Product	Costs of backorder (\$)	Backorder percentage
1	535	0
2	250	0.7
3	1345	0.1
4	980	1
5	345	1
6	1575	0.3
7	705	0.6
8	820	0.2
9	1650	0.1
10	930	0.5

Table 8: Backorder informtaion

As an example, suppose that you own 400 units of product 5 at the beginning of March, and you order nothing for product 5 throughout the six months. It then follows that the ending inventory for product 5 will be 365, 303, 220, 130, -67, and -116 for each of the following six months, where a "negative inventory level" means shortage. Note that for product 5 all consumers will wait upon shortage, and there will be 67 consumers who wait for two months (during this six-month horizon). The total inventory-related

cost is thus[3]

$$\$40 \times (365 + 303 + 220 + 130) + \$345 \times (67 + 116) = \$40,720 + \$63,135 = \$103,855.$$

As an example, we know from the previous section that your initial inventory for product 3 is 425 units, and we assume that you still order no product 3. Table 9 records the detailed steps of calculating all inventory-related costs. In the first month, you have $425-79 = 346$ units left, which is your stocking level at the end of March. For April, May, and June, the stocking levels are all positive. For July, 82 consumers cannot get product 3, and 10% of them (8.2) prefer to wait while 90% of them (73.8) prefer to leave. For August, as 200 more consumers cannot get product 3, in total there are $8.2+200 = 208.2$ consumers who wish to (but cannot) get product 3 in August. Note that this number should be $8.2+200$, not $82+200$, because 73.8 consumers leave at the end of July. Now, at the end of August, $208.2\times0.1 = 20.82$ consumers are still waiting, and $208.2\times0.9 = 187.38$ consumers leave. The total holding cost is $\$180 \times (346 + 167 + 116 + 117) = \$134{,}280$, total backorder cost is $\$1{,}345 \times (8.2 + 20.82) = \$39{,}031.9$, and total lost sales cost is $\$17{,}900 \times (73.8+187.38) = \$4{,}675{,}122$. The total inventory-related costs is \$4,848,433.9.[4]

	March	April	May	June	July	August
Demand	79	179	21	49	199	200
Inventory in-transit	0	0	20	0	0	0
Ending inventory	346	167	166	117	−82	−208.2
Stocking level	346	167	166	117	0	0
Backorder	0	0	0	0	8.2	20.82
Lost sales	0	0	0	0	73.8	187.38

Table 9: Backorder example

4.2 Problem 4

Again, given all the data and rules above, you now need to come up with another ordering plan for the next coming six months. In particular, at the beginning of each month, you need to determine for each product how many to order through each of the three shipping

[3]I guess you also agree that ordering some product 5 is a good idea.

[4]You may wonder how a "number of consumers" may be fractional. This is of course to simply the problem and calculation. After all, even if one insists to round these fractional values, the resulting costs will not be too far from those calculated with no rounding.

methods. As shortage is allowed and there are two conditions when a shortage incurs, now, your goal is to minimize total cost by making trade-off between all the related costs.