

There are N products, index by $i=0, \dots, N-1$

There are M months, index by $j=0, \dots, M-1$

There are K shipping methods: index by $k=0, \dots, K-1$

There are V vendor, index by $V=0, \dots, V-1$

ContainerCap be container capacity (Ocean)

ContainerCost be container cost

M_{big} (larger than all possible value of all decision variables)
 $\rightarrow \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} Demand_{ij} + \sum_{i=0}^{N-1} Init_i$

Demand $_{ij}$: product i at month j demand quantity $i=0, \dots, N-1$ $j=0, \dots, M-1$

Init $_i$: the quantity of product i initial inventory $i=0, \dots, N-1$

BuyCost $_i$: purchase cost of product i $i=0, \dots, N-1$

HoldCost $_i$: holding cost of product i $i=0, \dots, N-1$

Transit $_{ij}$: quantity of in-transit product i that will be delivered at month j . $i=0, \dots, N-1$, $j=0, \dots, M-1$

ShipFixedCost $_k$: fixed cost of shipping method k . $k=0, \dots, K-1$

ShipVarCost $_{ik}$: variable cost of product i of shipping method k . $i=0, \dots, N-1$ $k=0, \dots, K-1$

LeadTime $_{k=(1,2,3)}$: lead time of shipping method k . $k=0, \dots, K-1$

CBM $_i$: volume per cm 3 of product i $i=0, \dots, N-1$

LostSaleCost $_{ij}$: cost of lost sale of product i $i=0, \dots, N-1$

BackOrderCost $_i$: cost of back order of product i $i=0, \dots, N-1$

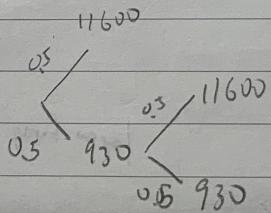
BackOrderProb $_i$: the probability of occurrence of back order of product i $i=0, \dots, N-1$

VendorFixedCost $_v$: fixed cost of buying with vendor v , $v=0, \dots, V-1$

MinOrder $_i$: minimum order quantity of product i if order $i=0, \dots, N-1$

ConflictSet $_\alpha$: pair of conflict products. $\alpha=0, \dots, |ConflictSet|-1$

ProductVendor $_i$: Mapping product i to a index of vendor



Decision Variable

X_{ijk} quantity of order of product i at month j by shipping method k . $i=0, \dots, N-1$ $j=0, \dots, M-1$ $k=0, \dots, K-1$

A_{bijk} be 1 if use shipping method k at month j . $j=0, \dots, M-1$ $k=0, \dots, K-1$

$C_{ContainerCnt_j}$ be container count that needed at month j $j=0, \dots, M-1$

$StockLevel_{ij}$ stock level of product i at month j $i=0, \dots, N-1$ $j=0, \dots, M-1$

$Shortage_{ij}$ shortage of product i at month j $i=0, \dots, N-1$ $j=0, \dots, M-1$

$B_{bin_{ij}}$ 1 if $StockLevel_{ij} \leq 0$ else 0

$C_{bin_{ij}}$ be 1 if order product i at month j

$D_{bin_{ijv}}$ be 1 if have order from vendor v at month j

Expr

$$VolumeInDemand_{Expr_j} = \sum_{i=0}^N X_{ij} \cdot CBM_i \quad j=0, \dots, M-1$$

$$BackOrderCustExpr_{ij} = Shortage_{ij} \cdot BackOrderProb_i$$

$$LotSaleCustExpr_{ij} = Shortage_{ij} (1 - BackOrderProb_i)$$

$$\underline{EndingInventoryExpr_{ij}} = \begin{cases} Init_i - Demand_{i0} & \text{if } j=0 \\ StockLevel_{i0} + X_{i10} + Transit_{i1} - Demand_{i1} - BackOrderCustExpr_{i0} & \text{if } j=1 \\ StockLevel_{i1} + X_{i20} + X_{i10} + Transit_{i2} - Demand_{i2} - BackOrderCustExpr_{i1} & \text{if } j=2 \\ StockLevel_{i2} + X_{i30} + X_{i20} + X_{i10} + X_{i00} - Demand_{i3} - BackOrderCustExpr_{i2} & \text{if } j \geq 3 \end{cases}$$

$$\begin{aligned}
 & \min \quad \text{Total Purchase Cost} + \text{Total ShipFixed Cost} + \text{Total ShipVar Cost} + \text{Total Holding Cost} + \text{Total ContainerCost} + \text{Total BackOrder Cost} + \text{Total lostSale Cost} + \text{Total VendorFixed Cost} \\
 & \text{Total Purchase Cost} = \sum_{k=0}^{K-1} \sum_{j=0}^{N-1} \sum_{i=0}^{M-1} X_{ijk} \cdot \text{BuyCost}_i \\
 & \text{Total ShipFixed Cost} = \sum_{j=0}^{M-1} \sum_{k=0}^{K-1} A_{binjk} \cdot \text{Ship FixedCost}_k \\
 & \text{Total ShipVar Cost} = \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} \sum_{k=0}^{K-1} X_{ijk} \cdot \text{Ship VarCost}_i \\
 & \text{Total Holding Cost} = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \text{StockLevel}_{ij} \cdot \text{HoldingCost}_i \\
 & \text{Total ContainerCost} = \left(\sum_{j=0}^{M-1} \text{ContainerCnt}_j \right) \cdot \text{ContainerCost}
 \end{aligned}$$

$$\text{Total BackOrderCost} = \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} \text{BackOrderCost Expr}_{ij} \cdot \text{BackOrderCost}_i$$

$$\text{Total LostSaleCost} = \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} \text{LostSaleCnt Expr}_{ij} \cdot \text{LostSaleCnt}_i$$

$$\text{Total VendorFixed Cost} = \sum_{j=0}^{M-1} \sum_{v=0}^{V-1} D_{binvj} \cdot \text{VendorFixedCost}_v \quad v = \text{Product Variant}$$

Constrain

let A_{binjk} to be correct

$$\frac{\sum_{i=0}^{N-1} X_{ijk}}{M_{big}} \leq A_{binjk} \quad j=0, \dots, M-1 \quad i=0, \dots, N-1$$

let C_{binij} behave correctly

$$\frac{\sum_{k=0}^{K-1} X_{ijk}}{M_{big}} \leq C_{binij} \quad j=0, \dots, N-1 \quad i=0, \dots, M-1$$

let D_{binvj} behave correctly

$$\frac{\sum_{i=0}^{N-1} \sum_{j=0}^{K-1} X_{ijk}}{M_{big}} \leq D_{binvj} \quad V \in \text{set of products of vendor } V \quad v=0, \dots, V-1 \quad j=0, \dots, M-1$$

Minimum Order Bound

$$C_{binij} \cdot \text{MinOrder}_i \leq \sum_{k=0}^{K-1} X_{ijk} \quad i=0, \dots, N-1 \quad j=0, \dots, M-1$$

let StockLevel_{ij} & Shortage_{ij} behave correctly

$$\text{Stocklevel}_{ij} - \text{Shortage}_{ij} = \text{Init}_{ij} - \text{Demand}_{ij} \quad \forall i, j \quad \# \text{ conflict}$$

$$\text{Stocklevel}_{ij} - \text{Shortage}_{ij} = \text{Stocklevel}_{ij} + X_{i0j0} + \text{Transit}_{j1} - \text{Demand}_{ij} + \text{Shortage}_{ij1} \cdot \text{BackOrderProb}_{ij} \quad \forall i, j$$

$$\text{Stocklevel}_{ij} - \text{Shortage}_{ij} = \text{Stocklevel}_{ij} + X_{i0j0} + X_{i1j1} + \text{Transit}_{j2} - \text{Demand}_{ij2} - \text{Shortage}_{ij2} \cdot \text{BackOrderProb}_{ij} \quad \forall i, j$$

$$\text{Stocklevel}_{ij} - \text{Shortage}_{ij} = \text{Stocklevel}_{ij} + X_{i0j0} + X_{i1j1} + X_{i2j2} - \text{Demand}_{ij} - \text{Shortage}_{ij} \cdot \text{BackOrderProb}_{ij} \quad \forall i, j$$

$$\text{Stocklevel}_{ij} \leq M_{big} (1 - B_{binij}) \quad \forall i, j$$

$$\text{Shortage}_{ij} \leq M_{big} B_{binij} \quad \forall i, j$$

conflict

$$C_{binij} + C_{binij} \leq 1 \quad \forall j, \forall (a, b) \in \text{ConflictSetAlpha}$$

