Operation Research HW01

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1.

(a)

$\sum_{k=1}^{t}\sum_{j=1}^{k} x_{ijk} \geq \sum_{k=1}^{t} D_{ik} - I_i \quad \forall t,\ \forall i$

(b)

$I_i + \sum_{j=1}^{t}(\sum_{k=j}^{T} x_{ijk} - D_{ij})$

(c)

min $\sum_{i=1}^{N} C_i \sum_{k=1}^{T}\sum_{j=1}^{T} x_{ijk} + \sum_{t=1}^{T}\sum_{i=1}^{N} H_i(I_i + \sum_{j=1}^{t}(\sum_{k=j}^{T} x_{ijk} - D_{ij}))$

s.t.

$\sum_{i=1}^{N}\sum_{k=j}^{T} x_{ijk} \leq K \quad \forall j$

$\sum_{k=1}^{t}\sum_{j=1}^{k} x_{ijk} \geq \sum_{k=1}^{t} D_{ik} - I_i \quad \forall t,\ \forall i$

$x_{ijt} \geq 0 \quad \forall i, \forall j, \forall t$

(d)

$\sum_{i=1}^{N}\sum_{j=1}^{T-E}\sum_{k=j+E}^{T} x_{ijk} \leq 0$

$\sum_{j=1}^{t} x_{ijt} \geq D_{it} \quad t = E, .., T$ (Can't use initial inventory after day E)

2.

(a)

let x_{ij} be the amount of barrels(in million) refine at refinery i and sell at distribution point j per year.

let y_i be the amount of the expanding capacity of refinery i

let w_i be 1 if $y_i \geq 0$ else 0

let $y_{max} = \sum_{j=1}^{M} D_j$

max $\sum_{j=1}^{M} \sum_{i=1}^{N} x_{ij} P_{ij} - \sum_{i=1}^{N} (C_i Y_i + w_i F_i)$

s.t.

$\sum_{j=1}^{M} x_{ij} \leq K_i + Y_i \quad \forall i$

$\sum_{i=1}^{N} x_{ij} \leq D_i \quad \forall j$

$w_i \geq \dfrac{y_i}{y_{max}} \quad \forall i$

$w_i \in \{0, 1\} \quad \forall i$

(b)

let x_{ijk} be the amount of barrels(in million) refine at refinery i and sell at distribution point j per year in scenario k.

let y_i be the amount of the expanding capacity of refinery i

let $y_{max} = \sum_{j=1}^{M} D_j$

max $\sum_{k=1}^{S} Q_k (\sum_{j=1}^{M} \sum_{i=1}^{N} x_{ijk} P_{ij}) - \sum_{i=1}^{N} (C_i Y_i)$

s.t.

$\sum_{j=1}^{M} x_{ijk} \leq K_i + Y_i \quad \forall i, \forall k$

$\sum_{i=1}^{N} x_{ijk} \leq D_i \quad \forall j, \forall k$

3.

(a)

let x_{jt} be 1 if hour t is used to process product j else 0

let w_j be the shortage amount of product j

$w_j = max\{D_j - \sum_{t=1}^{T} Q_{jt} x_{jt},\ 0\} \quad \forall j$

min $\sum_{j=1}^{N} w_j$

s.t.

$\sum_{j=1}^{N} x_{jt} \leq 1 \quad \forall t$

$w_j \geq D_j - \sum_{t=1}^{T} Q_{jt} x_{jt} \quad \forall j$

$w_j \geq 0 \quad \forall j$

$x_{jt} \in \{0,\ 1\} \quad \forall j,\ \forall t$

(b)

let x_{jt} be 1 if hour t is used to process product j else 0

let w_j be 1 if fulfill demand of product j, else 0

let M be $\sum_{j=1}^{N} \sum_{t=1}^{T} Q_{jt}$

max $\sum_{j=1}^{N} w_j R_j$

s.t.

$\sum_{j=1}^{N} x_{jt} \leq 1 \quad \forall t$

$(1+M) w_j \leq \sum_{t=1}^{T} Q_{jt} x_{jt} - D_j + 1 + M \quad \forall j$

$w_j \in \{0,\ 1\} \quad \forall j$

$x_{jt} \in \{0,\ 1\} \quad \forall j,\ \forall t$

(c)

let x_{jt} be 1 if hour t is used to process product j else 0

let x_{0t} be 1 if hour t is maintaining the machine else 0

let w_j be 1 if fulfill demand of product j, else 0

let M be $\sum_{j=1}^{N} \sum_{t=1}^{T} Q_{jt}$

let z_{kl} be 1 if x_{0k} and x_{0l} is both 1 else 0, $\forall l = \{1, ..., T-1\}, \quad \forall k = \{l+1, ..., T\}$

max $\sum_{j=0}^{T} w_j$

s.t.

$\sum_{j=0}^{N} x_{jt} \leq 1 \quad \forall t$

$(1+M)w_j \leq \sum_{t=1}^{T} Q_{jt}x_{jt} - D_j + 1 + M \quad \forall j$

$\sum_{t=1}^{T} x_{0t} \geq H$

$k \times x_{0k} - l \times x_{0l} - (G+1)z_{kl} \geq 0 \quad \forall k,\ \forall l$

$z_{kl} \geq x_{0k} + x_{0j} - 1 \quad \forall k,\ \forall l$

$w_j \in \{0,\ 1\} \quad \forall j$

$x_{jt} \in \{0,\ 1\} \quad \forall j,\ \forall t$

$z_{kl} \in \{0,\ 1\} \quad \forall k,\ \forall l$

(d)

let x_{jt} be 1 if hour t is used to process product j else 0

let w_j be 1 if fulfill demand of product j, else 0

let M be $\sum_{j=1}^{N} \sum_{t=1}^{T} Q_{jt}$

max $\sum_{j=0}^{T} w_j$

s.t.

$\sum_{j=1}^{N} x_{jt} \leq 1 \quad \forall t \sum_{j=0}^{N} x_{jt} \leq 1 \quad \forall t$

$(1+M)w_j \leq \sum_{t=1}^{T} Q_{jt}x_{jt} - D_j + 1 + M \quad \forall j$

$\sum_{k=1}^{L_j} x_{jk} \geq 1 \quad \forall j$ s.t. $L_j \neq T$

$x_{jt} \in \{0,\ 1\} \quad \forall j,\ \forall t$

$w_j \in \{0,\ 1\} \quad \forall j$

4.

max $2w + 3x + k$

s.t.

$k \leq 5y$

$$k \leq 7z$$

$$k \leq 9$$

$$2w + 5z \leq 10$$

$$3x + 8y \leq 10$$

$$w \leq 50$$

$$w \leq 5xz$$

$$w \geq 0,\ x \geq 0$$

$$y \in \{0, 1\},\ z \in \{0, 1\}$$