

There are  $N$  product. index by  $i=0, \dots, N-1$

There are  $M$  month. index by  $j=0, \dots, M-1$

There are  $K$  shipping method: index by  $k=0, \dots, K-1$

there are  $V$  vendor. index by  $V=0, \dots, V-1$

ContainerCap be container capacity (Ocean)

ContainerCost be container cost

$$\underline{M_{big}} \quad (\text{larger than all possible value of all decision variable})$$
$$\sum_{j=0}^{M-1} \sum_{i=0}^{N-1} \text{Demand}_{ij} + \sum_{i=0}^{N-1} \text{Init}_i$$

Demand<sub>ij</sub>: product  $i$  at month  $j$  demand quantity  $i=0, \dots, N-1$   $j=0, \dots, M-1$

Init<sub>i</sub>: the quantity of product  $i$  initial inventory  $i=0, \dots, N-1$

BuyCost<sub>i</sub>: purchase cost of product  $i$   $i=0, \dots, N-1$

HoldCost<sub>i</sub>: holding cost of product  $i$   $i=0, \dots, N-1$

Transit<sub>ij</sub>: quantity of in-transit product  $i$  that will be delivered at month  $j$ .  $i=0, \dots, N-1$ ,  $j=0, \dots, M-1$

ShipFixedCost<sub>k</sub>: fixed cost of shipping method  $k$ .  $k=0, \dots, K-1$

ShipVarCost<sub>ik</sub>: variable cost of product  $i$  of shipping method  $k$ .  $i=0, \dots, N-1$   $k=0, \dots, K-1$

LeadTime<sub>k=(1,2,3)</sub>: lead time of shipping method  $k$ .  $k=0, \dots, K-1$

CBM<sub>i</sub>: volume per cm<sup>3</sup> of product  $i$   $i=0, \dots, N-1$

LostSaleCost<sub>i</sub>: cost of lost sale of product  $i$   $i=0, \dots, N-1$

BackOrderCost<sub>i</sub>: cost of back order of product  $i$   $i=0, \dots, N-1$

BackOrderProb<sub>i</sub>: the probability of occurrence of back order of product  $i$   $i=0, \dots, N-1$

VendorFixedCost<sub>v</sub>: fixed cost of buying with vendor  $v$ ,  $v=0, \dots, V-1$

MinOrder<sub>i</sub>: minimum order quantity of product  $i$  if order  $i=0, \dots, N-1$

ConflictSet<sub>d</sub>: pair of conflict products.  $d=0, \dots, |\text{ConflictSet}| - 1$

ProductVendor: Mapping product  $i$  to a index of vendor

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13

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49

$x_{ijk}$  quantity of order of product  $i$  at month  $j$  by shipping method  $k$ .  $i=0, \dots, N-1$   $j=0, \dots, M-1$   $k=0, \dots, K-1$

$a_{binjk}$  be 1 if use shipping method  $k$  at month  $j$ .  $j=0, \dots, M-1$   $k=0, \dots, K-1$

$c_{ContainerCntj}$  be container count that needed at month  $j$   $j=0, \dots, M-1$

$StockLevel_{ij}$  stock level of product  $i$  at month  $j$   $i=0, \dots, N-1$   $j=0, \dots, M-1$

$UnfulDemand_{ij}$  unfulfill demand of product  $i$  at month  $j$   $i=0, \dots, N-1$   $j=0, \dots, M-1$

$b_{binij}$  helper variable for let

$$StockLevel_{ij} = \max \{ 0, \text{Ending Inventory Expr.}_j \}$$

$$UnfulDemand_{ij} = \max \{ 0, -\text{Ending Inventory Expr.}_j \}$$

$c_{binij}$  be 1 if order product  $i$  at month  $j$

$d_{binv_j}$  be 1 if have order from vendor  $v$  at month  $j$

Expr

$$\text{VolumeInOrder Expr.}_j = \sum_{i=0}^N x_{ij} \cdot CBM_i \quad j=0, \dots, M-1$$

$$\text{BackOrderCntr Expr.}_j = UnfulDemand_{rj} \cdot \text{BackOrderProb}_j$$

$$\text{LostSaleCntr Expr.}_j = UnfulDemand_{rj} (1 - \text{BackOrderProb}_j)$$

$$\text{EndingInventory Expr.}_j = \begin{cases} \text{Init}_i - \text{Demand}_{r0} & \text{if } j=0 \\ \end{cases}$$

$$\begin{cases} StockLevel_{r0} + x_{i0} + \text{Transit}_{r1} - \text{Demand}_{r1} - \text{BackOrderCntr Expr.}_0 & \text{if } j=1 \end{cases}$$

$$\begin{cases} StockLevel_{r1} + x_{i1} + x_{i0} + \text{Transit}_{r2} - \text{Demand}_{r2} - \text{BackOrderCntr Expr.}_1 & \text{if } j=2 \end{cases}$$

$$\begin{cases} StockLevel_{r(j-1)} + x_{i(j-1)} + x_{i(j-2)} + x_{i(j-3)} - \text{Demand}_{rj} - \text{BackOrderCntr Expr.}_{(j-1)} & \text{if } j \geq 3 \end{cases}$$

## Analyze Data

$$\min \text{Total Purchase Cost} + \text{Total Purchase Cost} = \sum_{k=0}^{K-1} \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} x_{ijk} \cdot \text{BuyCost}_i$$

Total ShipFixed Cost +

Total ShipVar Cost +

Total Holding Cost +

Total ContainerCost +

Total BackOrder Cost +

Total LostSale Cost +

Total VendorFixed Cost

$$\text{Total ShipFixed Cost} = \sum_{j=0}^{M-1} \sum_{k=0}^{K-1} A_{bin,jk} \cdot \text{ShipFixedCost}_k$$

$$\text{Total ShipVar Cost} = \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} \sum_{k=0}^{K-1} x_{ijk} \cdot \text{ShipVarCost}_i$$

$$\text{Total Holding Cost} = \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} \text{StockLevel}_{ij} \cdot \text{HoldingCost}_i$$

$$\text{Total Container Cost} = \left( \sum_{j=0}^{M-1} \text{ContainerCnt}_{ij} \right) \cdot \text{ContainerCost}$$

$$\text{Total BackOrderCost} = \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} \text{BackOrderCntExpr}_{ij} \cdot \text{BackOrderCost}_i$$

$$\text{Total LostSale Cost} = \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} \text{LostSaleCntExpr}_{ij} \cdot \text{LostSaleCnt}_i$$

$$\text{Total VendorFixed Cost} = \sum_{j=0}^{M-1} \sum_{v=0}^{V-1} D_{bin,vj} \cdot \text{VendorFixedCost}_v \quad v = \text{Product\_V}$$

### Constraint

# let  $A_{bin,jk}$  to be correct

$$\frac{\sum_{k=0}^{K-1} X_{ijk}}{M_{big}} \leq A_{bin,jk} \quad j=0, \dots, M-1 \\ k=0, \dots, K-1$$

# let  $C_{bin,ij}$  behave correctly

$$\frac{\sum_{k=0}^{K-1} X_{ijk}}{M_{big}} \leq C_{bin,ij} \quad i=0, \dots, N-1 \\ j=0, \dots, M-1$$

# let  $D_{bin,vj}$  behave correctly

$$\frac{\sum_{j=0}^{M-1} \sum_{k=0}^{K-1} X_{ijk}}{M_{big}} \leq D_{bin,vj} \quad w \text{ be set of products of vendor } V \\ v=0, \dots, V-1 \quad j=0, \dots, M-1$$

# Minimum Order Bound

$$C_{bin,ij} \cdot \text{MinOrder}_i \leq \sum_{k=0}^{K-1} X_{ijk} \quad i=0, \dots, N-1 \\ j=0, \dots, M-1$$

# let StockLevel<sub>ij</sub> & UnfulDemand<sub>ij</sub> behave correctly

$$0 \leq \text{StockLevel}_{ij}, \text{EndingInventory} \leq \text{StockLevel}_{ij}$$

$$0 \leq \text{UnfulDemand}_{ij}, -\text{EndingInventory} \leq \text{UnfulDemand}_{ij}$$

# conflict

$$C_{bin,a} + C_{bin,b} \leq 1 \quad j=0, \dots, M-1, \forall (a, b) \in \text{conflictSet}_2$$

$$\text{StockLevel}_{ij} \leq 0 + M_{big} \cdot B_{bin,ij}, \text{StockLevel}_{ij} \leq \text{EndingInventory} + M_{big} (1 - B_{bin,ij})$$

$$\text{UnfulDemand}_{ij} \leq 0 + M_{big} (1 - B_{bin,ij}), \text{UnfulDemand}_{ij} \leq -\text{EndingInventory} + M_{big} \cdot B_{bin,ij}$$

$$i=0, \dots, N-1 \\ j=0, \dots, M-1$$

