

Lecture 04

Syntax Analyzer (Parser)

Part 1: Context Free Grammars

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Overview



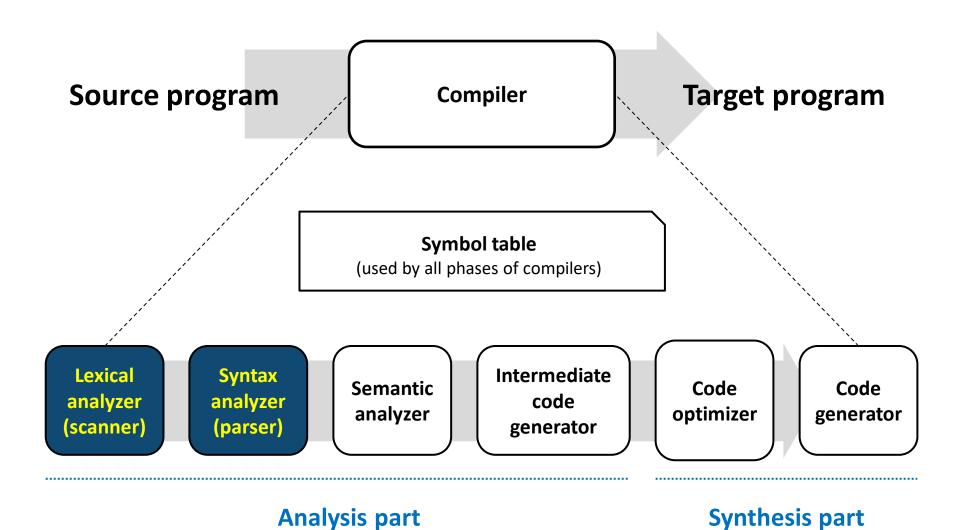
In this lecture, you will learn

1. What a syntax analyzer is & how it works

2. How to specify the syntax of programming languages

Overview









What does a syntax analyzer do?

In / this / course / , / you / will / learn / how / to / design / and / implement / compilers



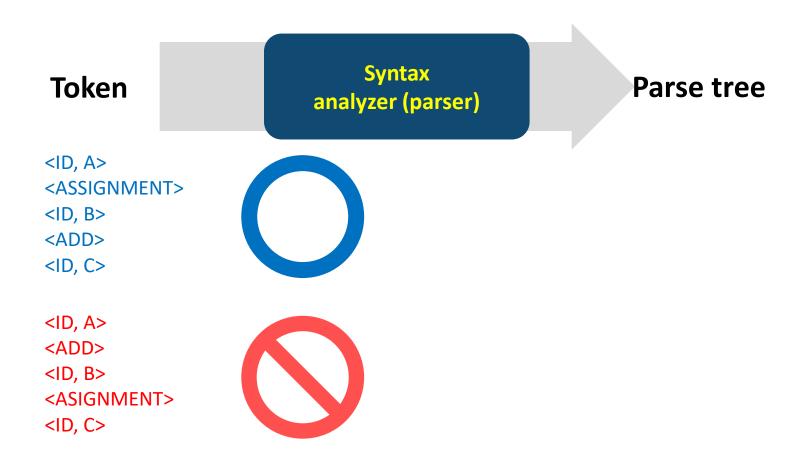
Does this sentence have a valid structure?

e.g., verb after noun





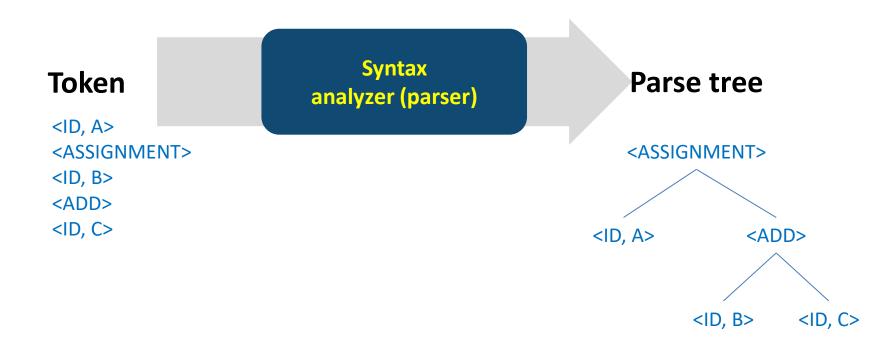
1. Decides whether a given set of tokens is valid or not





Syntax analyzer

2. Creates a tree-like intermediate representation (e.g., parse tree) that depicts the grammatical structure of the token stream







1. Decides whether a given set of tokens is valid or not



- Q. How to specify the rule for deciding valid token set?
- Q. How to distinguish between valid and invalid token sets?

Context free grammar!!



Why don't we use regular expressions?

It is not sufficient to depict the syntax of programming languages

An expression is a regular expression

If and only if it can be described by using the basic regular expressions only

Regular expression	Expressed regular language
ϵ	$L(\epsilon) = \{\epsilon\}$
а	$L(a)=\{a\}$, where a is a symbol in alphabet Σ
$r_1 r_2$	$L(r_1) \cup L(r_2)$, where r_1 and r_2 are regular expressions
r_1r_2	$L(r_1r_2) = \{ab a \in L(r_1) \text{ and } b \in L(r_2)\}$
r *	$L(r^*) = \bigcup_{i \ge 0} L(r^i)$



Why don't we use regular expressions?

It is not sufficient to depict the syntax of programming languages

An expression is a regular expression

If and only if it can be described by using the basic regular expressions only

 $\binom{n}{n}^n$ ($0 \le n \le \infty$) is not a regular expression over alphabet $\Sigma = \{(,)\}$

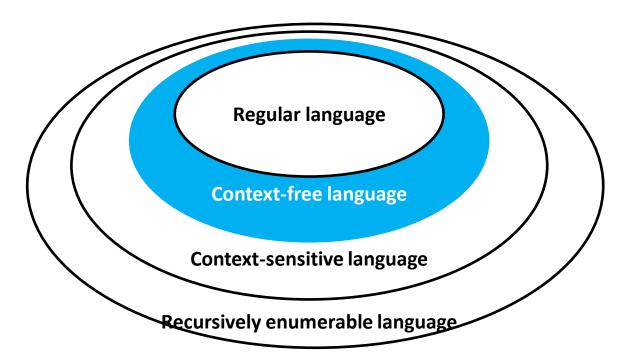


Why don't we use regular expressions?

It is not sufficient to depict the syntax of programming languages

Thus, instead of using regular expressions, we use context free grammars!!

The coverage of formal languages





Context free grammars (CFG)

A notation for describing context free languages

A CFG consists of

- Terminals: the basic symbols (usually, token name = terminal)
 Terminals can not be replaced
- Non-terminals: syntactic variables
 Non-terminals can be replaced by other non-terminals or terminals
- A start symbol: one non-terminal (usually, the non-terminal of the first rule)
- **Productions** (\rightarrow) : a rule for replacement





Context free grammars (CFG)

A notation for describing context free languages

A CFG consists of terminals, non-terminals, a start symbol, and productions

A CFG follows the rule

• $A \rightarrow \alpha$, $B \rightarrow \beta$

A and B are non-terminals and A is a start symbol (the non-terminal of the first production) α and β are any sequence of non-terminals, terminals, and ϵ

Example

Terminal = {+, id}, non-terminals = {S, E}, a start symbol = S, two productions

$$S \rightarrow E + E$$
, $E \rightarrow id$



NOTE: In this class,

We will use the following notational conventions

- Non-terminals are written in upper-case
- Terminals are written in lower-case
- A sequence of non-terminals, terminals, and ϵ is written in α , β , ω
 - e.g., $\alpha = aABBBcddef$

Based on the CFG rules and notational conventions, an expression $\binom{n}{r}$ $(0 \le n \le \infty)$ can be described as follows:

 $BALANCED \rightarrow (BALANCED)|\epsilon$



Context free grammars (CFG)

It is good at expressing the recursive structure of a program

In our programming languages, recursive structures are very frequently observed.

```
STMT \rightarrow if \; (EXPR) \; STMT \; else \; STMT \; if \; (xxx) \; \{ \ | \; if \; (EXPR) \; STMT \; if \; (yyy) \; \{ \ | \; if \; (...) \; \{ \; ... \; \} \}
```





Let's express simple arithmetic operations

• Terminals: id, +,*,(,) e.g., id + id, id * id,(id), id + id * id, (id + id) * id

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$





Let's express simple function calls

Terminals: id, (,)
 e.g., func(arg1, arg2), print(arg), func2() ...

$$S \rightarrow id(E) \mid id()$$

 $E \rightarrow id, E \mid id$





Let's express a simple while statement

• Terminals: while, id, comparison, number, (,)e.g., while(A > B), while(A), while(0), while(3 > 2), while(A <= 1), while()

 $S \rightarrow while(E)$ $E \rightarrow TcomparisonT|T|\epsilon$ $T \rightarrow id \mid number$





1. Decides whether a given set of tokens is valid or not

Token

Syntax
analyzer (parser)

Parse tree

Q. How to specify the rule for deciding valid token set?

Make a context free grammar G based on the rule of a programming language

Q. How to distinguish between valid and invalid token sets?





A derivation (⇒) is a sequence of replacement

⇒*: Do derivations zero or more times

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

$$\mathbf{E} \Rightarrow \mathbf{E} + E \Rightarrow E * E + E$$
$$\mathbf{E} \Rightarrow^* \mathbf{E} * \mathbf{E} + \mathbf{E}$$

$$E \Rightarrow (E) \Rightarrow (E + E) \Rightarrow (id + E) \Rightarrow (id + id)$$
$$E \Rightarrow^* (id + id)$$





A rule for derivations

- Leftmost (\Rightarrow_{lm}) : replace the lest-most non-terminal first
- Rightmost (\Rightarrow_{rm}) : replace the right-most non-terminal first

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

$$\mathbf{E} \Rightarrow_{lm} \mathbf{E} + E \Rightarrow_{lm} \mathbf{E} * E + E \Rightarrow_{lm} id * \mathbf{E} + E \Rightarrow_{lm} id * id + \mathbf{E} \Rightarrow_{lm} id * id + id$$

$$\mathbf{E} \Rightarrow_{lm}^{*} id * id + id$$

$$E \Rightarrow_{rm} E + E \Rightarrow_{rm} E + id \Rightarrow_{rm} E * E + id \Rightarrow_{rm} E * id + id \Rightarrow_{rm} id * id + id$$

$$E \Rightarrow_{rm}^{*} id * id + id$$





Definition: A sentinel form of a CFG G

- α is a sentinel form of G, if $A \Rightarrow^* \alpha$, where A is the start symbol of G
 - If $A \Rightarrow_{lm}^* \alpha$ or $A \Rightarrow_{rm}^* \alpha$, α is a (left or right) sentinel form of G

Definition: A sentence of a CFG G

• lpha is a sentence form of G, if lpha is a sentinel form of a CFG G which consists of terminals only

Definition: A language of a CFG G

- L(G) is a language of a CFG G (context-free language)
- $L(G) = \{\alpha | \alpha \text{ is a sentence of } G\}$

If an input string (e.g., a token set) is in L(G), we can say that it is valid in G





1. Decides whether a given set of tokens is valid or not

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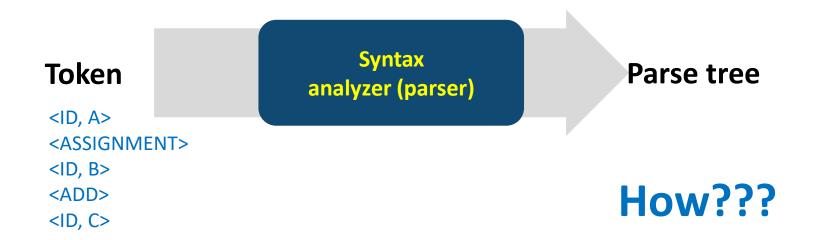
Check whether the given token set can be derived from the context free grammar G





Syntax analyzer

2. Creates a tree-like intermediate representation (e.g., syntax tree) that depicts the grammatical structure of the token stream





$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

For
$$id * id + id$$

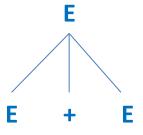
E

• *E*



$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

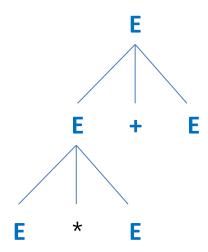
- E
- $\Rightarrow_{lm} E + E$





$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

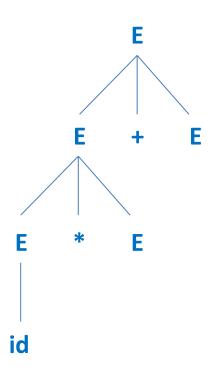
- E
- $\Rightarrow_{lm} E + E$
- $\Rightarrow_{lm} E * E + E$





$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

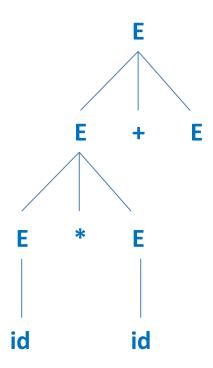
- E
- $\Rightarrow_{lm} E + E$
- $\Rightarrow_{lm} E * E + E$
- $\Rightarrow_{lm} id * E + E$





$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

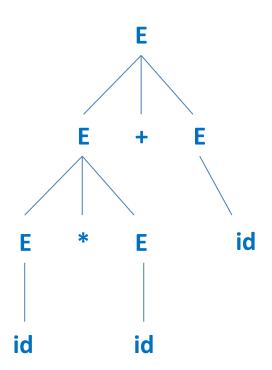
- E
- $\Rightarrow_{lm} E + E$
- $\Rightarrow_{lm} E * E + E$
- $\Rightarrow_{lm} id * E + E$
- $\Rightarrow_{lm} id * id + E$





$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

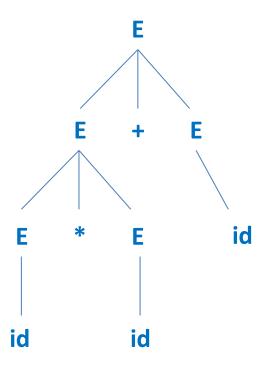
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A parse tree

- Leaf nodes = terminals
- Non-leaf nodes = non-terminals
- An inorder traversal of a parse tree = original strings



X The same tree is created regardless of the derivation method





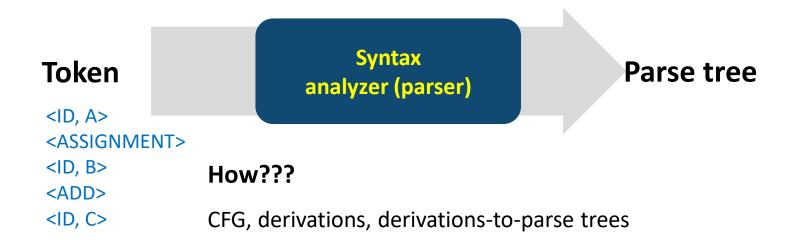
$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

- E
- $\Rightarrow_{rm} E + E$
- $\Rightarrow_{rm} E + id$
- $\Rightarrow_{rm} E * E + id$
- $\Rightarrow_{rm} E * id + id$
- $\Rightarrow_{rm} id * id + id$



Syntax analyzer

2. Creates a tree-like intermediate representation (e.g., syntax tree) that depicts the grammatical structure of the token stream







Syntax analyzer

- 1. Decides whether a given set of tokens is valid or not
- 2. Creates a tree-like intermediate representation (e.g., syntax tree) that depicts the grammatical structure of the token stream



Then, how to do these processes 1) efficiently and 2) automatically?



For efficient parsing: creating a good CFG

1. A good CFG is non-ambiguous

2. A good CFG has no left recursion

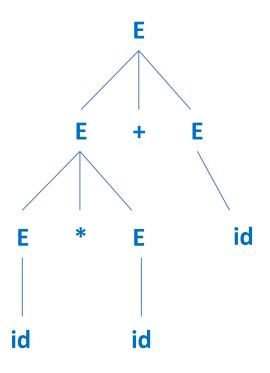
3. For each non-terminal, a good CFG has only one choice of production starting from a specific input symbol

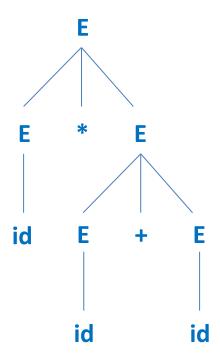




One input string can have multiple different parse trees

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$



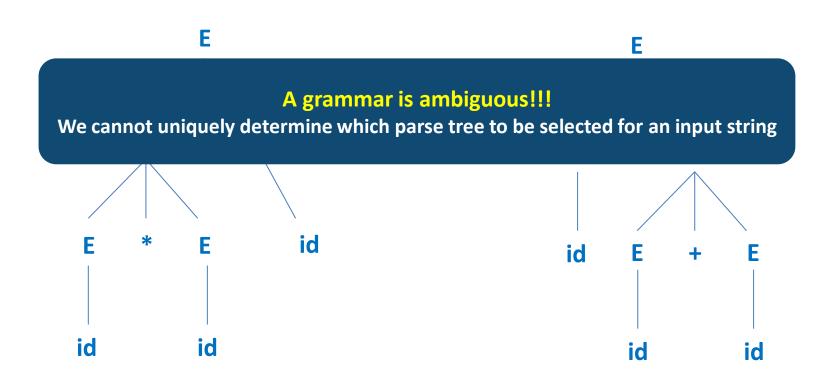






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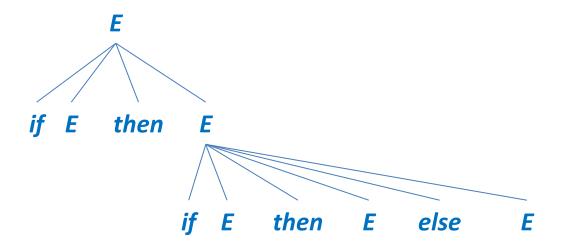


Just rewrite the ambiguous grammars based on disambiguating rules

The most common ambiguity problem: dangling-else

 $E \rightarrow if E then E \mid if E then E else E \mid other$

For if other then if other then other else other,



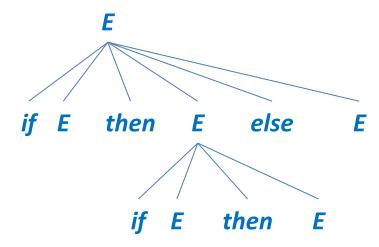


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Just rewrite the ambiguous grammars based on disambiguating rules

• The most common ambiguity problem: dangling-else

 $E \rightarrow if E then E \mid if E then E else E \mid other$

For if other then if other then other else other,

Disambiguating rule for the dangling-else problem

"Match each else with the closest unmatched then"

 $E \rightarrow MATCHED \mid UNMATCHED$

 $MATCHED \rightarrow if MATCHED then MATCHED else MATCHED | other$

 $UNMATCHED \rightarrow if E then E \mid if E then MATCHED else UNMATCHED$

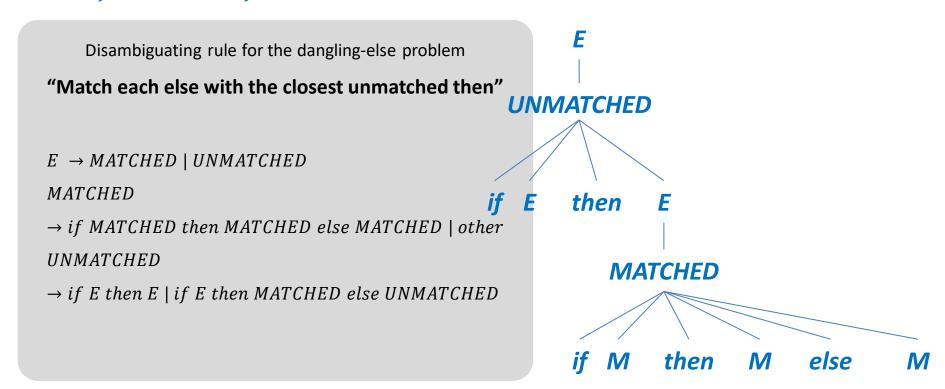


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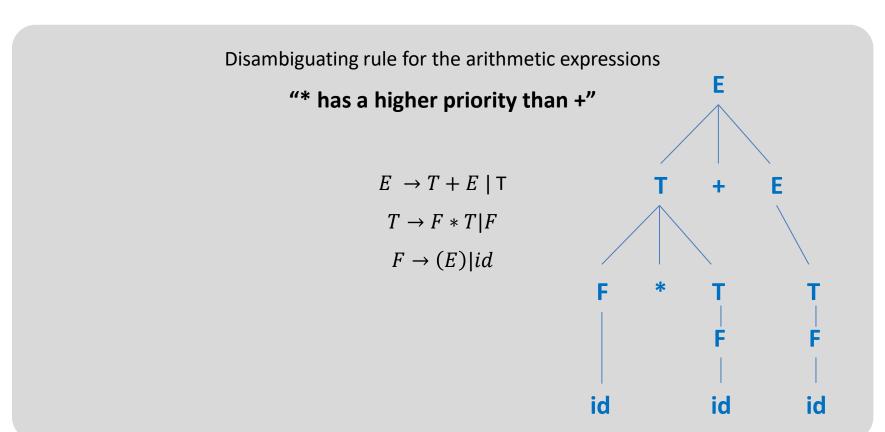




Just rewrite the ambiguous grammars based on disambiguating rules

Another example: arithmetic expressions

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$







Some parsing techniques (called top-down parsing) cannot handle

left recursive grammars because they use a leftmost derivation policy

- A grammar is **left recursive** if it has a **nonterminal** A with a derivation $A \Rightarrow^+ A\alpha$, where α is any sequence of non-terminals and terminals
 - $S \rightarrow Sa|b$ is a left recursive grammar
 - e.g., try a leftmost derivation for an input string a, $S \Rightarrow_{lm} Sa \Rightarrow_{lm} Saa \Rightarrow_{lm} Saaa \Rightarrow_{lm} ... \textbf{Infinite loop!!}$
 - Q. Is $S \to Aa|b, A \to Sb$ a left recursive grammar?





Answer: rewrite using right-recursion!!

• e.g., $S \rightarrow Sa|b$ can be rewritten as:

$$S \rightarrow bA$$
, $A \rightarrow aA|\epsilon$

- e.g., $S \to S\alpha_1 |S\alpha_2| \dots |S\alpha_m|\beta_1|\beta_2| \dots |\beta_n|$ can be rewritten as: $(\alpha_i \ and \ \beta_i \ are \ any \ sequence \ of \ terminals \ and \ non-terminals)$
 - Step 1: Make a new nonterminal A and add a production rule $lpha_i A$ for all $lpha_i$ and ϵ
 - $A \rightarrow \alpha_1 A |\alpha_2 A| ... |\alpha_m A| \epsilon$
 - Step 2: For a nonterminal S, add a production rule $\beta_i A$ for all β_i and discard other rules
 - $S \rightarrow \beta_1 A |\beta_2 A| \dots |\beta_n A$, $A \rightarrow \alpha_1 A |\alpha_2 A| \dots |\alpha_m A| \epsilon$



Left factoring

For a non-terminal, if there are two or more productions which start with the same input symbol....

• e.g., $E \to T + E|T, T \to F * T|F, F \to (E)|id$ Then, which production should be selected?

We need left factoring to discard this confusion



The procedure of left factoring

$$E \rightarrow T + E|T$$
, $T \rightarrow F * T|F$, $F \rightarrow (E)|id$

- Step 1: For each non-terminal A, find the longest common prefix of productions α
 - e.g., for E, $\alpha = T$
- Step 2: Discard all productions which have the form of $A \to \alpha \beta$, and add $A \to \alpha A'$
 - e.g., $E \rightarrow TE'$
- Step 3: For the new non-terminal A', add $A' \rightarrow \beta$ for all discarded productions in step 2
 - e.g., $E' \rightarrow +E | \epsilon$
- Step 4: Repeat step 1 ~ 3 until there is no more common prefix for all non-terminals
 - $E \to TE'$, $E' \to +E|\epsilon$, $T \to FT'$, $T' \to *T|\epsilon$, $F \to (E)|id$



For efficient parsing: creating a good CFG

1. A good CFG is non-ambiguous

- We can achieve this by defining disambiguating rules
- But, it's not easy...

2. A good CFG has no left recursion

We can easily achieve this by rewriting with right recursion

3. For each non-terminal, a good CFG has only one choice of production starting from a specific input symbol

We can easily achieve this through left factoring



For efficient parsing: creating a good CFG

Examples

Let's rewrite a CFG G: $DECL o DECL \ type \ id$; $|DECL \ type \ id = id$; $|\epsilon|$

(G is non-ambiguous)

Step 1: rewrite G by using right recursion

Step 2: rewrite G by using left factoring



Syntax analyzer

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Then, how to do these processes 1) efficiently and 2) automatically?



Abstract syntax trees look like parse trees, but without some parsing details

Example

For an input stream (A + B) * C

$$(A + B) * C$$

Lexical analyzer

$$(id + id) * id$$



Abstract syntax trees look like parse trees, but without some parsing details

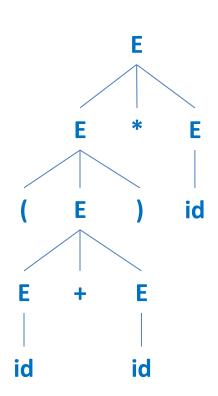
Example

For a token stream (id + id) * id with a CFG $G: E \rightarrow E + E|E * E|(E)|id$

An example sequence of derivations

$$E \Rightarrow_{lm} E * E \Rightarrow_{lm} (E) * E$$
$$\Rightarrow_{lm} (E + E) * E \Rightarrow_{lm}^{*} (id + id) * id$$

- A parse tree for (id + id) * id describes
 - The sequence of derivations
 - The nesting structure
 - But, too much information...
- Q) Which nodes can be reduced?

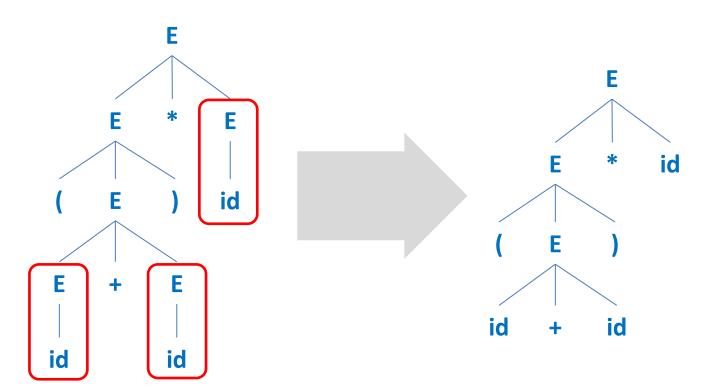




Abstract syntax trees look like parse trees, but without some parsing details

Q) Which nodes can be reduced?

1. Single-successor nodes which have exactly one child node Our main focus is their single child, not the parent nodes.



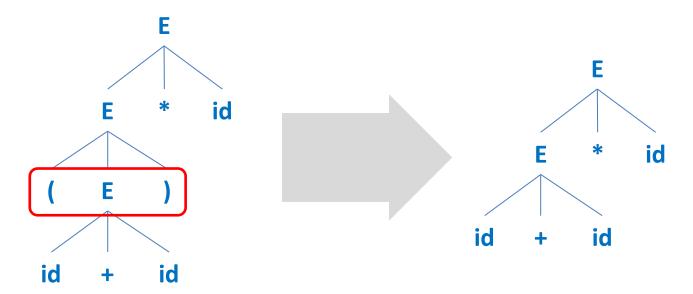


Abstract syntax trees look like parse trees, but without some parsing details

Q) Which nodes can be reduced?

- **1. Single-successor nodes** which have exactly one child node Our main focus is their single child, not the parent nodes.
- 2. Symbols for describing syntactic details (e.g., parenthesis, comma)

A parse tree already describes such syntactic information



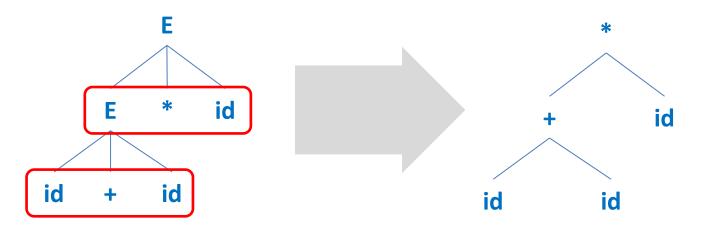


Abstract syntax trees look like parse trees, but without some parsing details

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 A parse tree already describes such syntactic information
- 3. Non-terminals with an operator and arguments as their child nodes





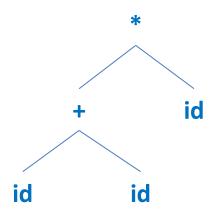
Abstract syntax trees look like parse trees, but without some parsing details

AST for (id * id) + id describes

The nesting structure (core syntactic information)

Compared to a parse tree

- More compact
- Easier to use and understand







AST construction

Make semantic actions for each production of a CFG G

Semantic action? An action related with grammar productions It is also used for type checking, code generation, ...

Example

For a CFG
$$G: E \rightarrow E + E|E * E|(E)|id$$

Production	Semantic action
$E \to E_1 + E_2$	$E.node = new Node('+', E_1.node, E_2.node)$
$E \to E_1 * E_2$	$E.node = new Node('*', E_1.node, E_2.node)$
$E \to (E_1)$	$E.node = E_1.node$
$E \rightarrow id$	$E.node = new \ Leaf(id, id. value)$

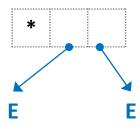




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$E \to (E_1)$	$E.node = E_1.node$
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$$E \Rightarrow_{lm} E * E$$



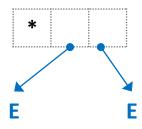




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$$E \Rightarrow_{lm} E * E \Rightarrow_{lm} (E) * E$$



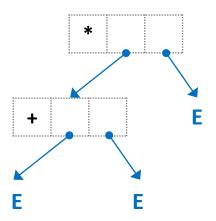




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$$E \Rightarrow_{lm} E * E \Rightarrow_{lm} (E) * E$$
$$\Rightarrow_{lm} (E + E) * E$$







AST construction

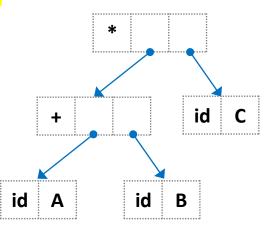
Example

For a CFG
$$G: E \rightarrow E + E|E * E|(E)|id$$

Production	Semantic action
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$E \to (E_1)$	$E.node = E_1.node$
$E \rightarrow id$	E.node = new Leaf(id, id. value)

$$E \Rightarrow_{lm} E * E \Rightarrow_{lm} (E) * E$$

$$\Rightarrow_{lm} (E + E) * E \Rightarrow_{lm}^{*} (id + id) * id$$







For a CFG
$$G: S \rightarrow while(C)\{B\}$$
, $C \rightarrow id \ comp \ id$, $B \rightarrow type \ id; | id();$

Production	Semantic action
$S \rightarrow while(C)\{B\}$	S.node = new Node('while', C.node, B.node)
$C \rightarrow id_1 \ comp \ id_2$	$C.node = new\ Node('cond', new\ Node('comp', comp.value, new\ Leaf(id_1, id_1.value), new\ Leaf(id_2, id_2.value)))$
$B \rightarrow type id;$	B.node = new Node('block', new Node('declaration', new Leaf(type, type.value), new Leaf(id, id.value)))
$B \rightarrow id();$	B.node = new Node('block', new Node('call', new Leaf(id, id. value))

Let's construct AST for while $(leftVar < rightVar)\{int \ a;\}$

- After lexical analysis: while(id comp id){type id;}
- A sequence of derivations for the input string

$$S \Rightarrow_{lm} while(C)\{B\} \Rightarrow_{lm} while(id\ comp\ id)\{B\} \Rightarrow_{lm} while(id\ comp\ id)\{type\ id;\}$$



Summary: AST

Abstract syntax trees look like parse trees, but without some parsing details

We can eliminate the following nodes in parse trees

- 1. Single-successor nodes
- 2. Symbols for describing syntactic details
- 3. Non-terminals with an operator and arguments as their child nodes

AST can be constructed by using semantic actions

The semantic actions can be also used for type checking, code generation, ...





