

# Welcome!

*Energy and Human Ambition on a Finite Planet*

# Hello!

- Logistics
  - M, W, F 1:30-3:00 pm Eastern Time
  - 7 sessions from Aug 9-23
- EdStem
- Introductions

# Syllabus

## Expectations:

- Be respectful to the fellow participants and the instructor.
- Actively engage in lectures
- EdStem will be used to post and discuss optional problem sets. Students are highly encouraged to participate!

## Course Content:

- The course will rapidly but not comprehensively cover the content of the textbook of the same name by Tom Murphy (2020), which is available online for free.

Class #, Date	Class Topic(s)
1, 8/9	<ul style="list-style-type: none"><li>•Exponential growth</li><li>•Energy</li><li>•Economic growth</li><li>•Population</li></ul>
2, 8/11	<ul style="list-style-type: none"><li>•Population continued</li><li>•Space colonization</li></ul>
3, 8/13	<ul style="list-style-type: none"><li>•Energy</li><li>•US energy sources and breakdown</li><li>•Energy trends</li><li>•Fossil fuels overview</li></ul>
4, 8/16	<ul style="list-style-type: none"><li>•Fossil fuels current and future state (continued)</li><li>•Climate change</li></ul>
5, 8/18	<ul style="list-style-type: none"><li>•Renewable energy<ul style="list-style-type: none"><li>• Hydroenergy</li><li>• Wind</li><li>• Solar</li><li>• Biological</li></ul></li></ul>
6, 8/20	<ul style="list-style-type: none"><li>•Nuclear energy</li><li>•Future planning</li><li>•Economic regimes</li></ul>
7, 8/23	<ul style="list-style-type: none"><li>•Adaptation strategies</li><li>•Reflection</li></ul>



# 1: Exponential Growth

Emily Wang

Adapted from *Energy and Human Ambition on a Finite Planet* by Tom Murphy

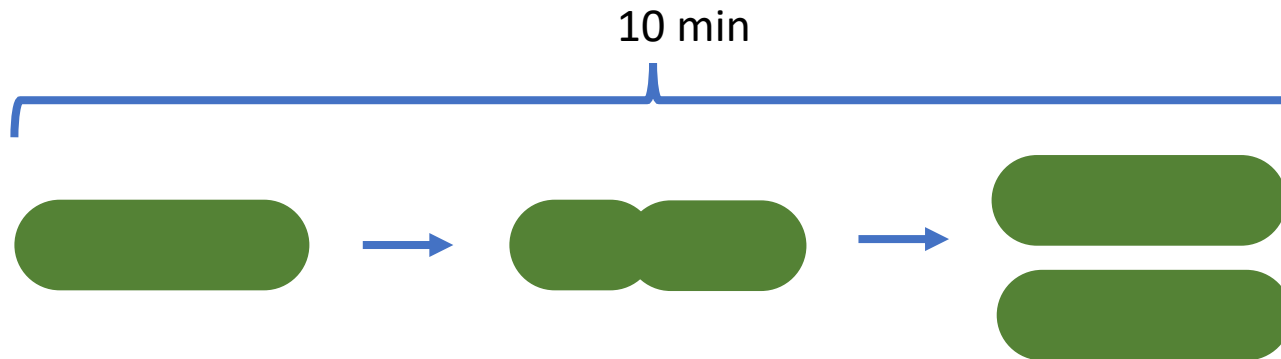


# What is exponential growth?

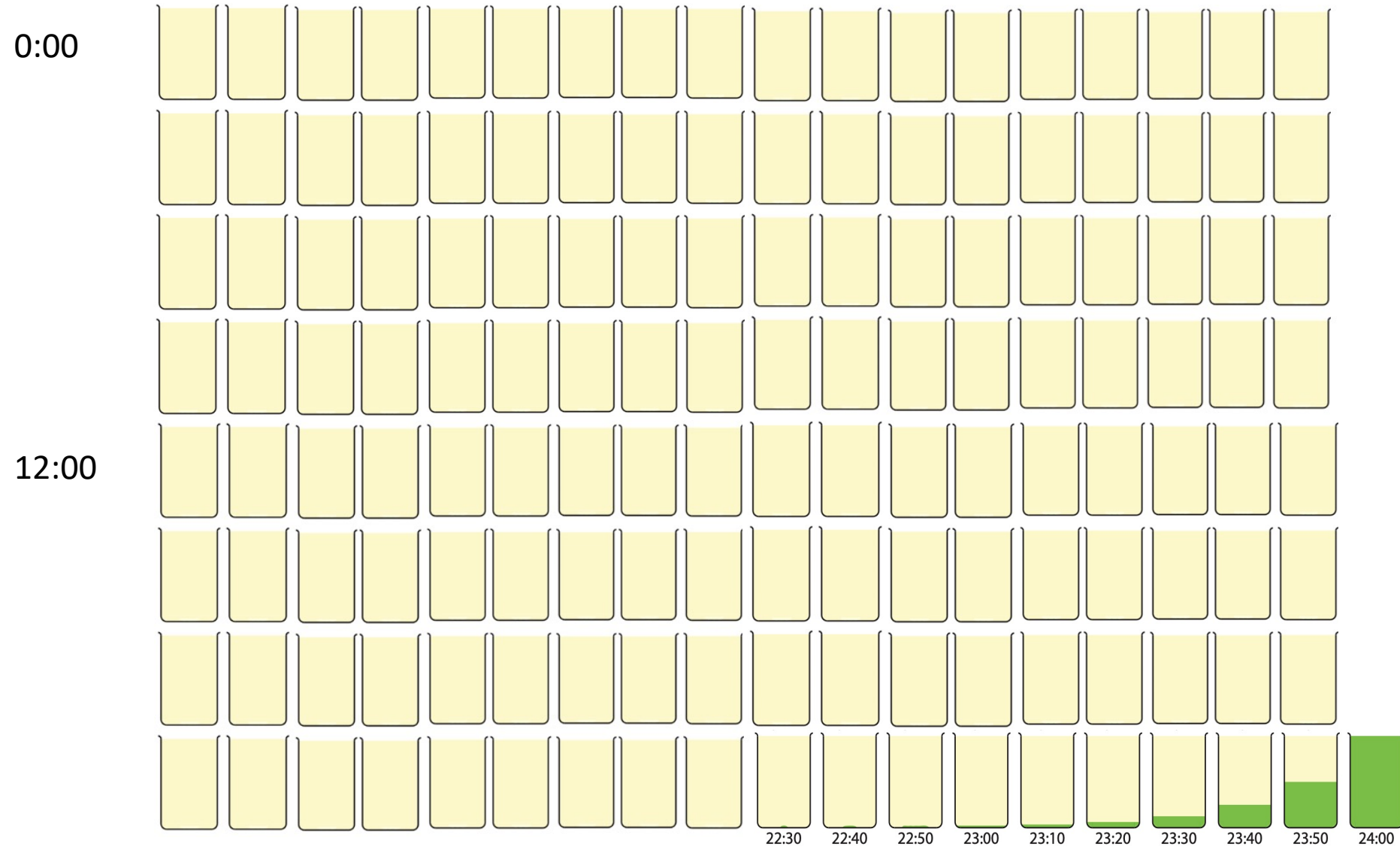
- Describe in your own words
- What are some examples from your life?

# Bacteria in a Jar

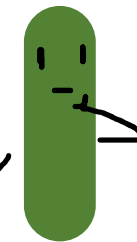
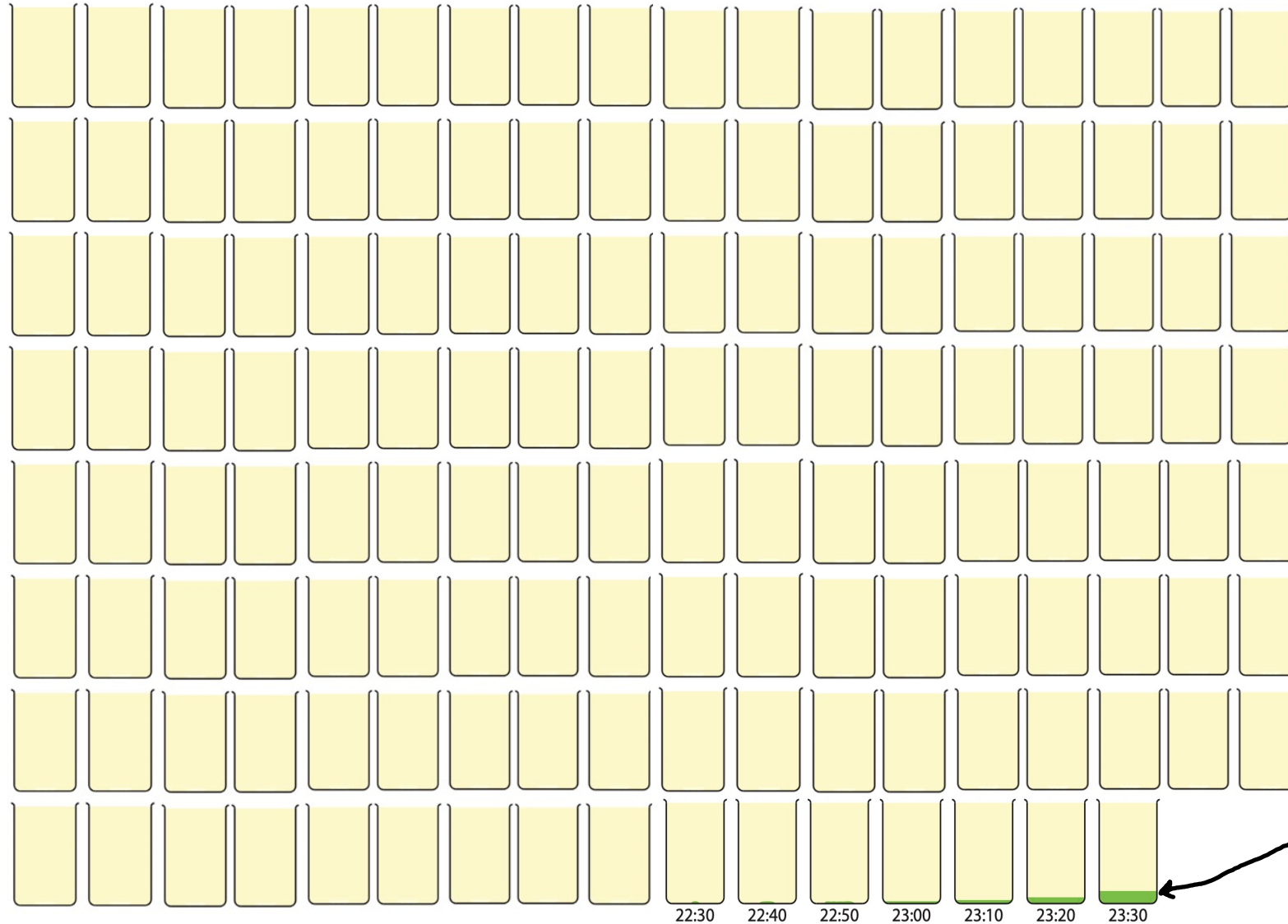
- The jar contains just the right amount of nutrients that if each bacterium splits every 10 minutes, the jar will become full of bacteria in 24 hours.
- The experiment starts at midnight, with 1 bacterium in the jar
- At what time will the jar be half full?



# Bacteria in a Jar



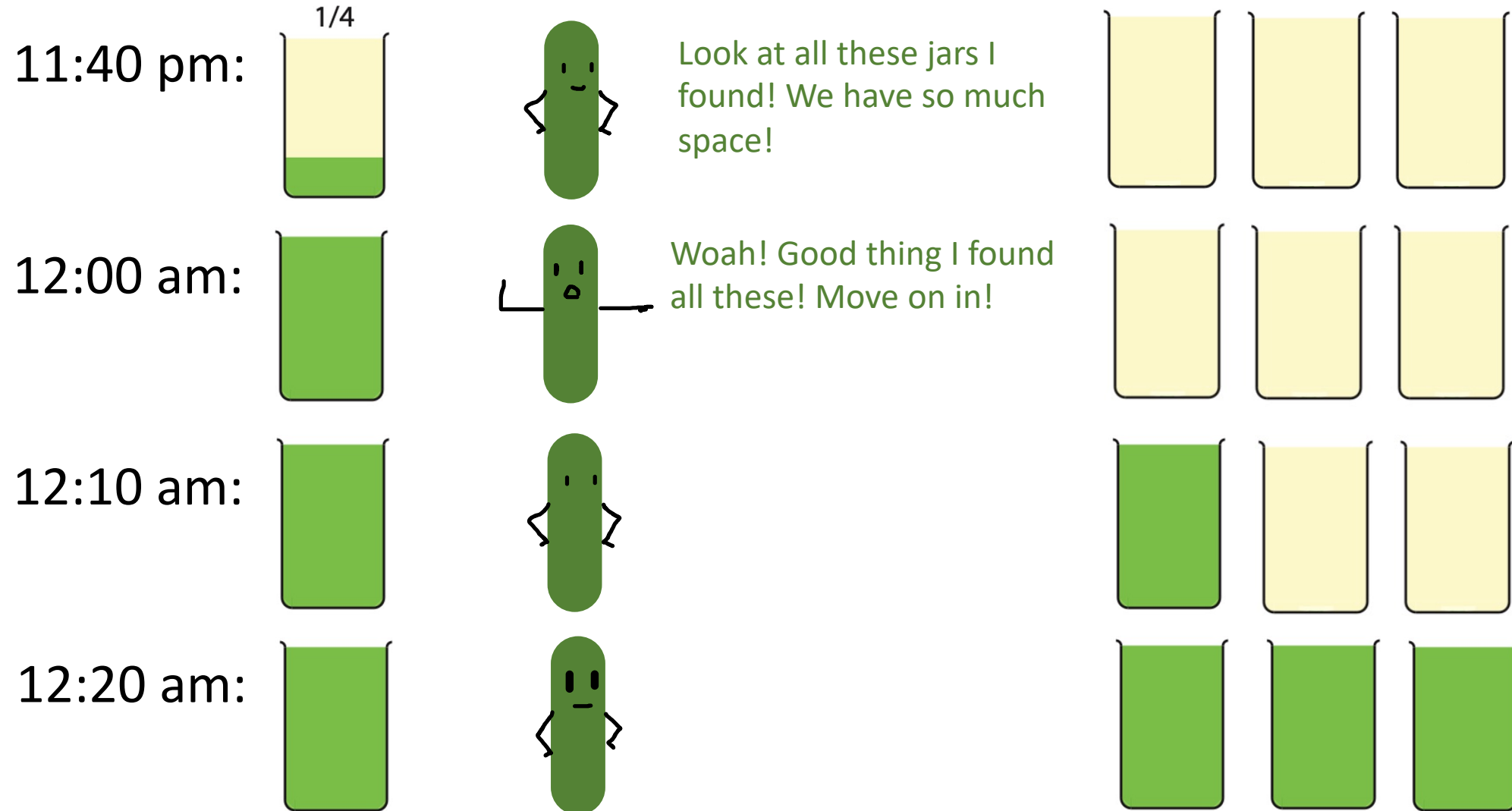
# Benny the Bacterium



For generations this jar has been nearly empty, but maybe it will fill up in the far future. I'll go look for more!



# Benny the Bacterium



# We Are Bacteria and Earth is the Jar

- How full is the jar?
- How much longer until it is full?

b = base (growth rate)  
x = exponent (repetitions)  
y = multiplicative factor

# Exponential Math

- Classic example: bank account with interest
- 2% interest means you earn 2% of your balance every year
- i = initial balance

$$\text{balance}(\text{year } 0) = i$$

$$\text{balance}(\text{year } 1) = i + 0.02i = (1 + 0.02)i = 1.02i$$

$$\text{balance}(\text{year } 2) = 1.02 * \text{balance}(\text{year } 1) = 1.02 * 1.02i$$

...

$$\text{balance}(\text{year } 4) = 1.02 * 1.02 * 1.02 * 1.02i$$

$$\text{balance}(\text{year } x) = 1.02^x * i$$

The annual growth rate is the **base** and the number of repetitions is the **exponent**

Interesting part of the equation

Let's define it as multiplicative factor y where  **$y = b^x$**

b = base (growth rate)  
x = exponent (repetitions)  
y = multiplicative factor

# Opposite of Exponential:

- The exponential function takes in the exponent  $x$  and spits out  $b^x$ 
  - $f(x) = b^x$
- **Inverse functions** undo each other
  - If  $f(x)$  and  $g(x)$  are inverses, then  $f(g(x)) = x$  and  $g(f(x)) = x$
- Inverse of exp is **logarithm**  $\log(x)$ 
  - Since an exponential needs to be defined with a base, so does log
  - Like we made  $b^n$ , we decided to write logs as  $\log_b(x)$
  - $\log_x(b^x) = x$  and  $b^{\log_b(x)} = x$
  - The irrational number  $e$  (2.7182...) is so special that  $\log_e(x)$  is written  **$\ln(x)$**

b = base (growth rate)  
x = exponent (repetitions)  
y = multiplicative factor

# Inverting our growth function

- I want a function that where I specify  $y$ ,  $b$  and it **spits out  $x$**  such that  $y = b^x$ 
  - Such as: **how many years  $x$**  will it take for my bank account balance to get  **$y$  times bigger** at an annual interest rate of  $b$ ?
- Let's use the property  $b^{\log_b(x)} = x$
- Let's sub  $e$  for  $b$  and  $b$  for  $x$  to get  $e^{\ln(b)} = b$
- Take our exponential function  $y = b^x$
- Plug that to get  $y = (e^{\ln(b)})^x = e^{x \ln(b)}$
- Undo the exponent with  $\ln$  like this:  $\ln(y) = x \ln(b)$
- Rearrange to get  $x = \ln(y) / \ln(b)$





b = base (growth rate)  
x = exponent (repetitions)  
y = multiplicative factor

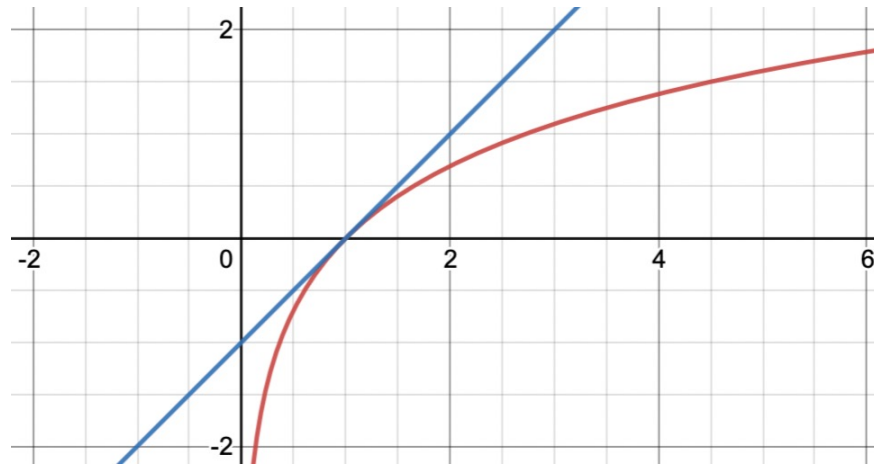
# Rule of 70

- $x = \ln(y)/\ln(b)$  is useful
- The “rule of 70” says that the doubling time of something growing at p percent is approximately  $70/p$

$$\frac{\ln(2)}{\ln(1+p/100)} \approx \frac{0.693}{p/100} \approx \frac{70}{p}$$

- Only works well for small b, because it relies on  $\ln(1 + p/100) \approx p/100$
- Useful for making quick calculations to model exp. growth mentally – supplementing intuition with math!

1		$y = \ln(x)$
2		$y = x - 1$



# Try it!

- What is the doubling rate of 3.5% growth?

# Try it!

- What is the doubling rate of 3.5% growth?

Rule of 70 estimation:

$$\text{Doubling rate} = 70/3.5 = 20$$

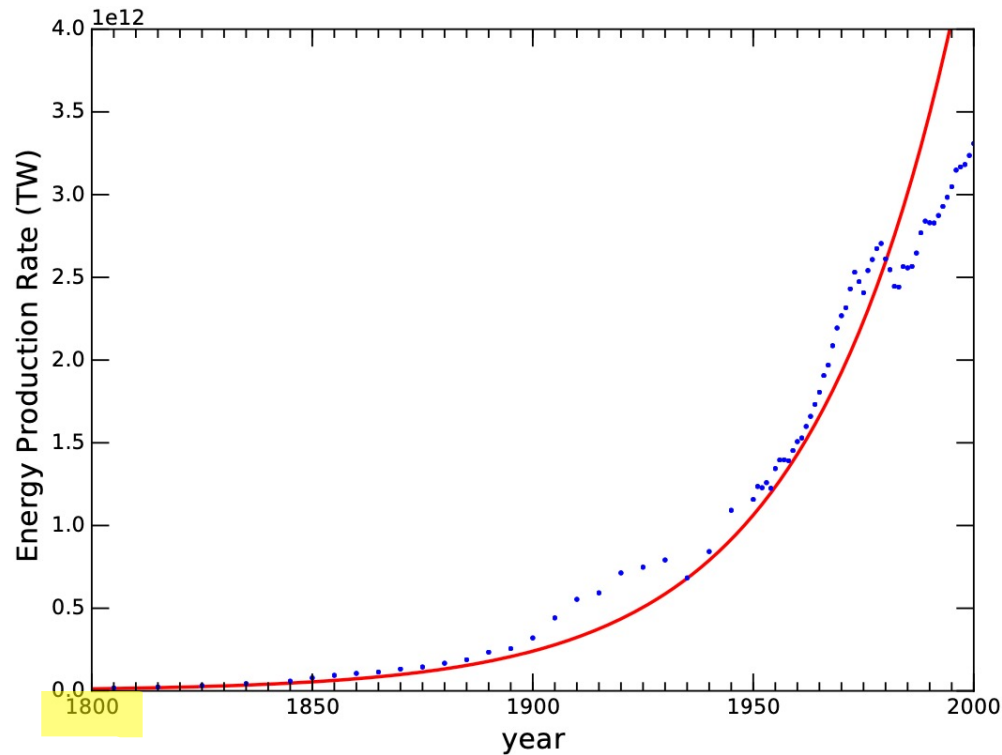
**= 20 years**

Analytical solution:

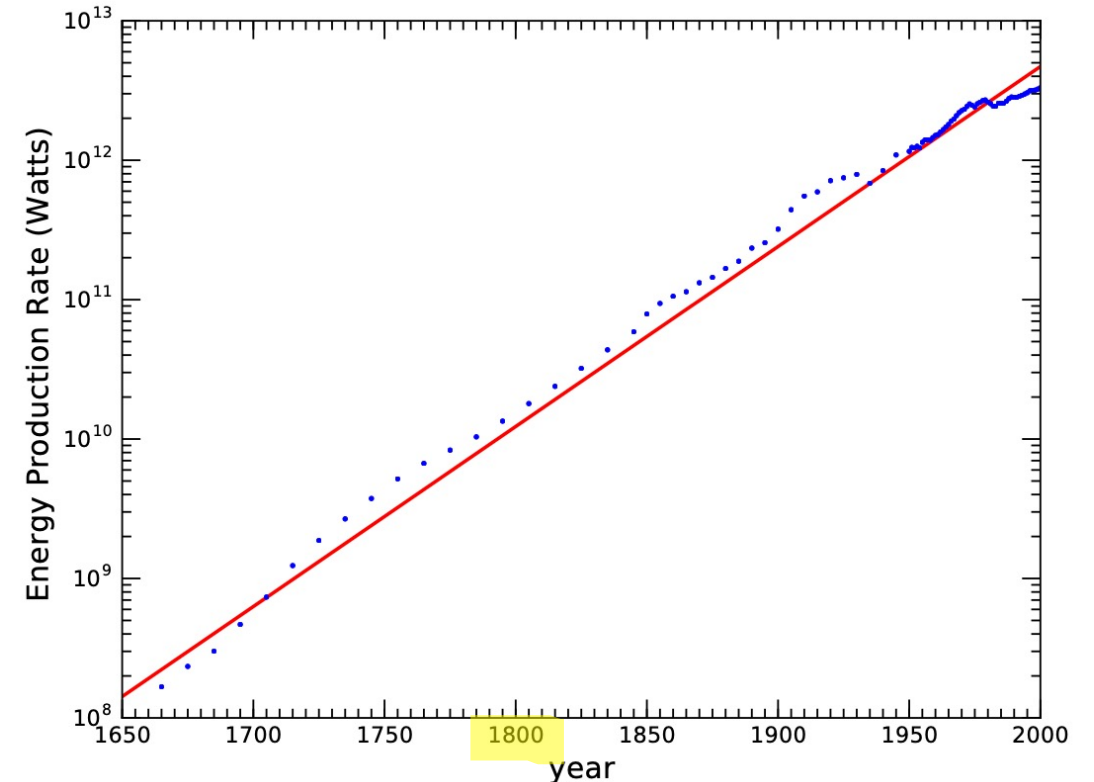
$$x = \ln(y)/\ln(b)$$
$$\text{Doubling rate} = \ln(2)/\ln(1.035)$$
$$= 0.693/0.0344$$

**= 20.15 years**

# Exponential Energy Use Extrapolation



**Figure 1.2:** U.S. energy over 200 years, showing a dramatic rise due almost entirely to fossil fuels. The red curve is an exponential fit tuned to cover the broader period shown in Figure 1.3.



**Figure 1.3:** Energy trajectory in the U.S. over a long period. The red line is an exponential at a **2.9% growth rate**, which appears linear on a logarithmic plot.

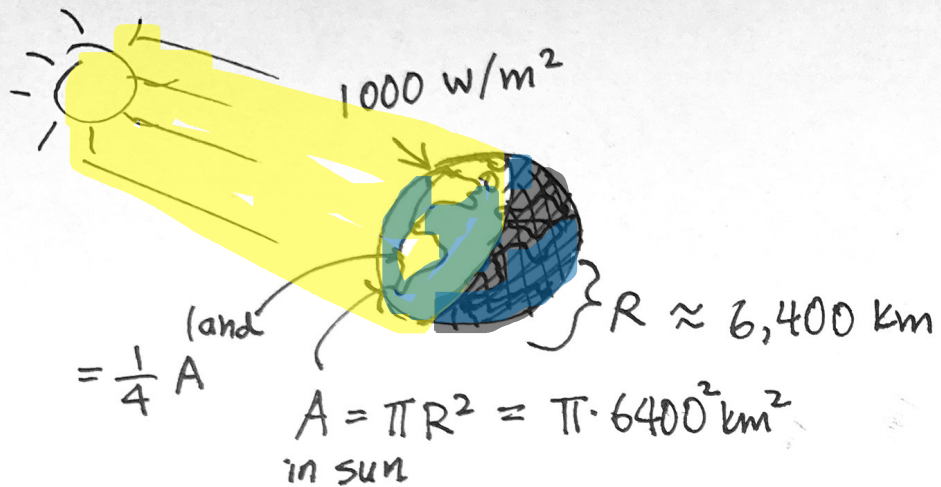
# Limits of Energy

We currently globally use ~18 TW/year (a terawatt is  $10^{12}$  watts)

Let's use a modest 2.3% projected rate which also happens to correspond to a 10x increase every century

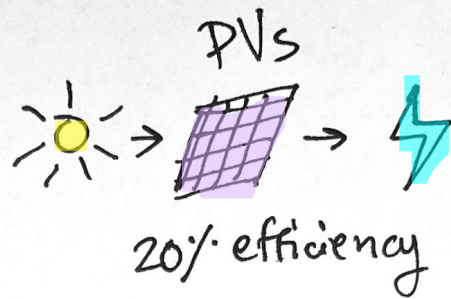
How far will solar energy get us?



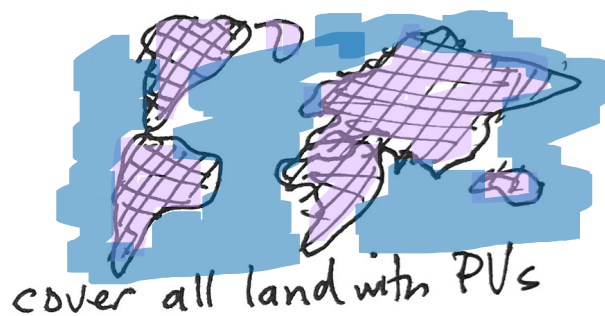


How long will that cover?

$$\underbrace{\frac{6 \cdot 10^{15} \text{ W}}{18 \cdot 10^{12} \text{ W}}}_{\text{multiplicative factor}} = 1.023^x$$



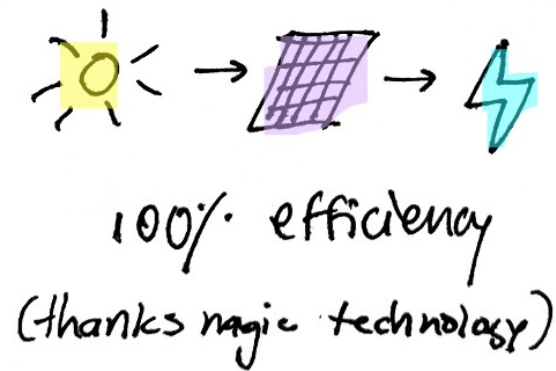
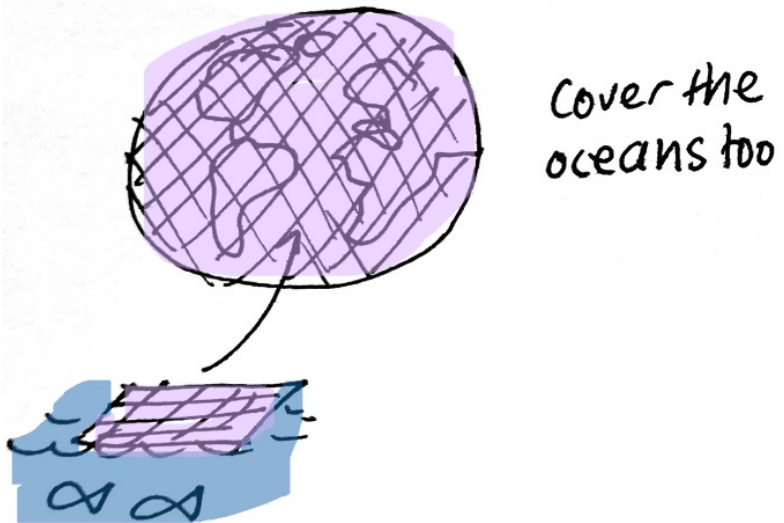
$\rightarrow 6 \cdot 10^{15} \text{ W collected!}$



$$x = \frac{\ln\left(\frac{60}{18}\right)}{\ln(1.023)} = \boxed{255 \text{ years}}$$

or estimating  $\frac{6 \cdot 10^{15}}{18 \cdot 10^{12}} \approx 10^2 - 10^3$   $\xrightarrow{\text{if } 10\times \text{ every } 100 \text{ years}}$  200-300 years

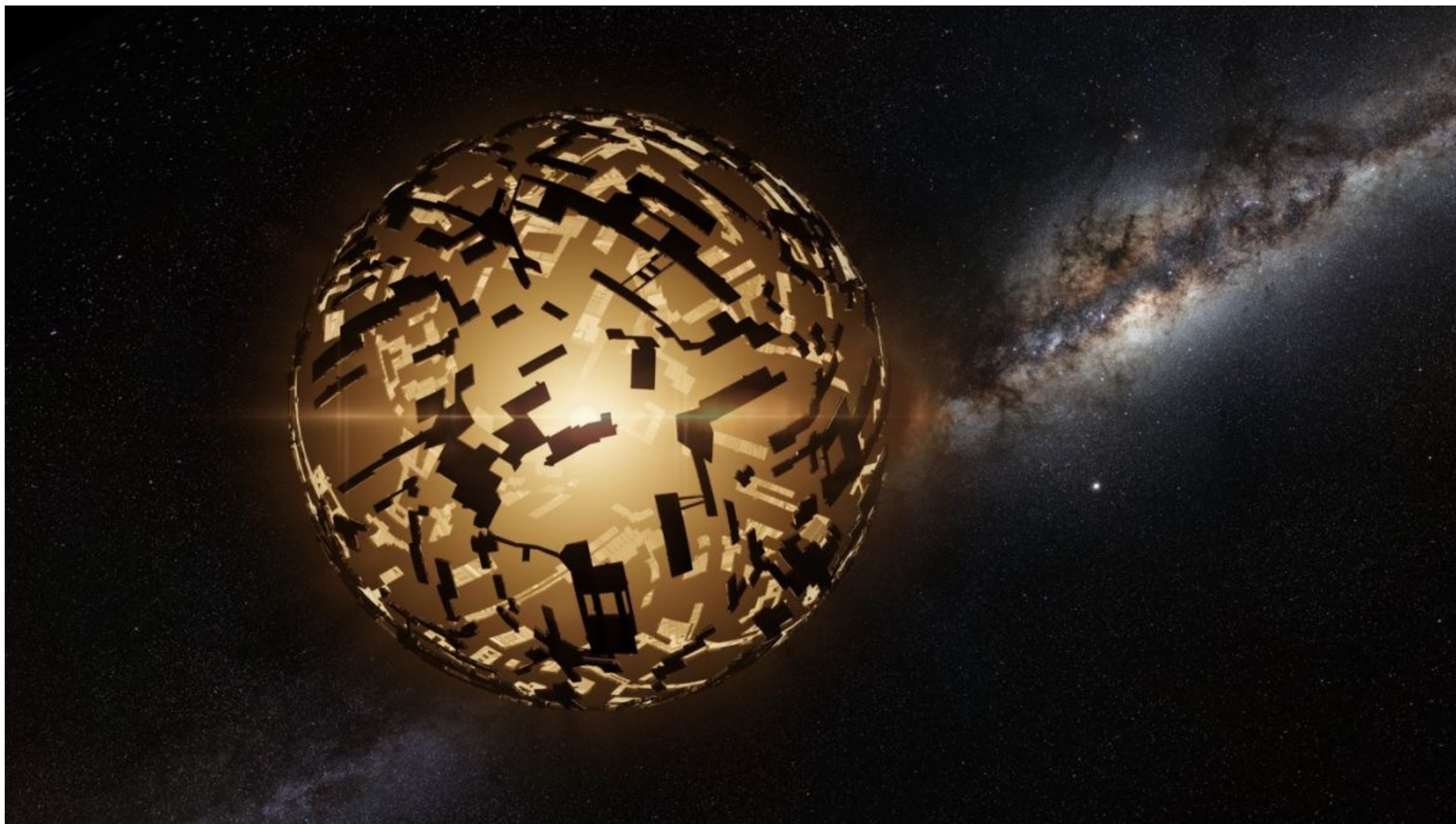
# Why stop there?



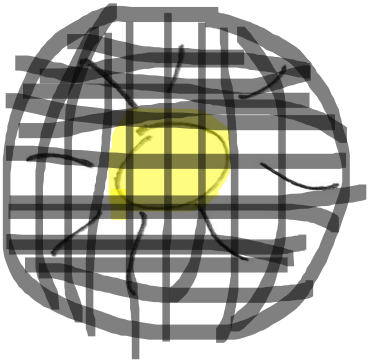
$$\frac{6 \cdot 10^{15} \cdot \underbrace{4 \cdot 5}_{20 \times}}{\ln\left(\frac{6 \cdot 4 \cdot 5 \cdot 10^{15}}{18 \cdot 10^{12}}\right)} = \boxed{387 \text{ years}}$$

$\ln(1.023)$

Why stop there???



Why stop there???



Dyson sphere

Sun output =  $4 \times 10^{26}$  W

$$\frac{4 \times 10^{26}}{18 \times 10^{12}}$$

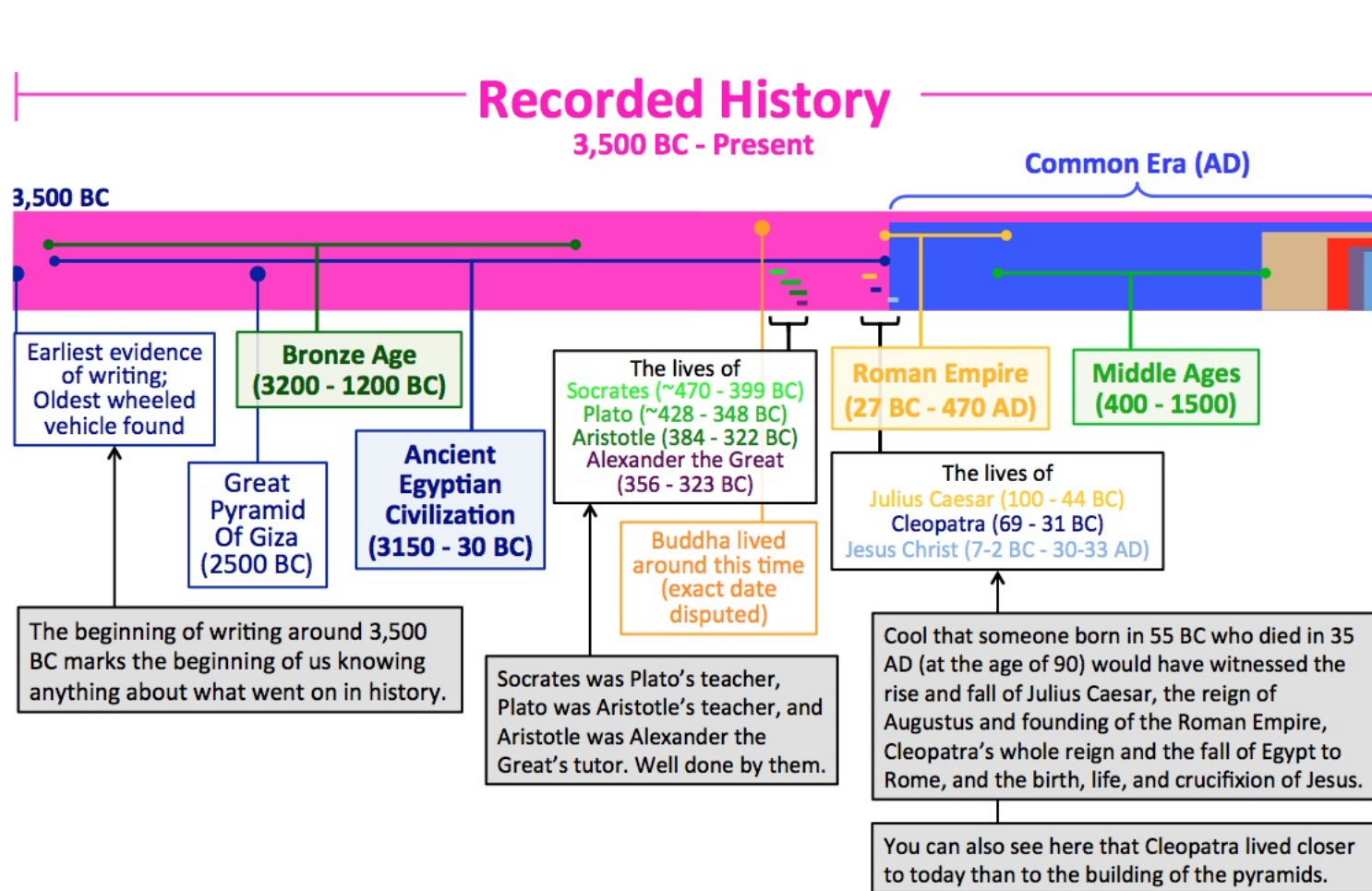
$\approx 10^{14}$

2.3% growth does  
10x in 100 years →

1,400 years



# Time in perspective



How much longer we could grow energy usage at 2.3% per year if we made a Dyson sphere



# Why stop there???

**Table 1.3:** Energy limit timescales.

Utilizing	years until
Solar, land, 20%	250
Solar, earth, 100%	390
Entire Sun	1,400
Entire Galaxy	2,500
Light in Universe	3,600
Mass in Universe	5,000

# Thermodynamic Constraints

- Essentially all of our energy expenditures end up as heat.
  - Directly: heaters, ovens, toasters, laundry dryer
  - Indirectly: Moving objects stir the air and braking objects heat surfaces
  - Metabolic: Eat food to stay warm and alive
  - Small exceptions: Beaming energy out into space or storing potential energy
- How does Earth maintain equilibrium?
  - Earth radiates heat into space!
  - Stefan-Boltzmann law:
    - “Black body radiation”
    - $T [=]$  Kelvins

$$P = A_{\text{surf}} \sigma (T_{\text{hot}}^4 - T_{\text{cold}}^4)$$

Temperature of space –  
negligible compared to  $T_{\text{hot}}$

$$\sigma = \frac{2\pi^5 k_B^4}{15h^3 c^2} = 5.670374419 \times 10^{-8} \frac{W}{m^2 K^4}$$

$\sigma$  = Stefan-Boltzmann constant

# Thermodynamic Effect of 2.3% Growth

- In 400 years we'd be boiling
- This has nothing to do with the energy source – just the use

Years	Power Density (W/m <sup>2</sup> )	T (K)	ΔT (C)
100	1.4	288.1	0.1
200	14	288.9	~1
300	140	296.9	~9
400	1,400	344	56
417	2,070	373	100
1,000	$1.4 \times 10^9$	8,600	8,300

Table 1.3: Energy limit timescales.

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Conclusion: Continued  
exponential growth of energy  
consumption = not feasible

5 minute break



# Economic Growth Limits

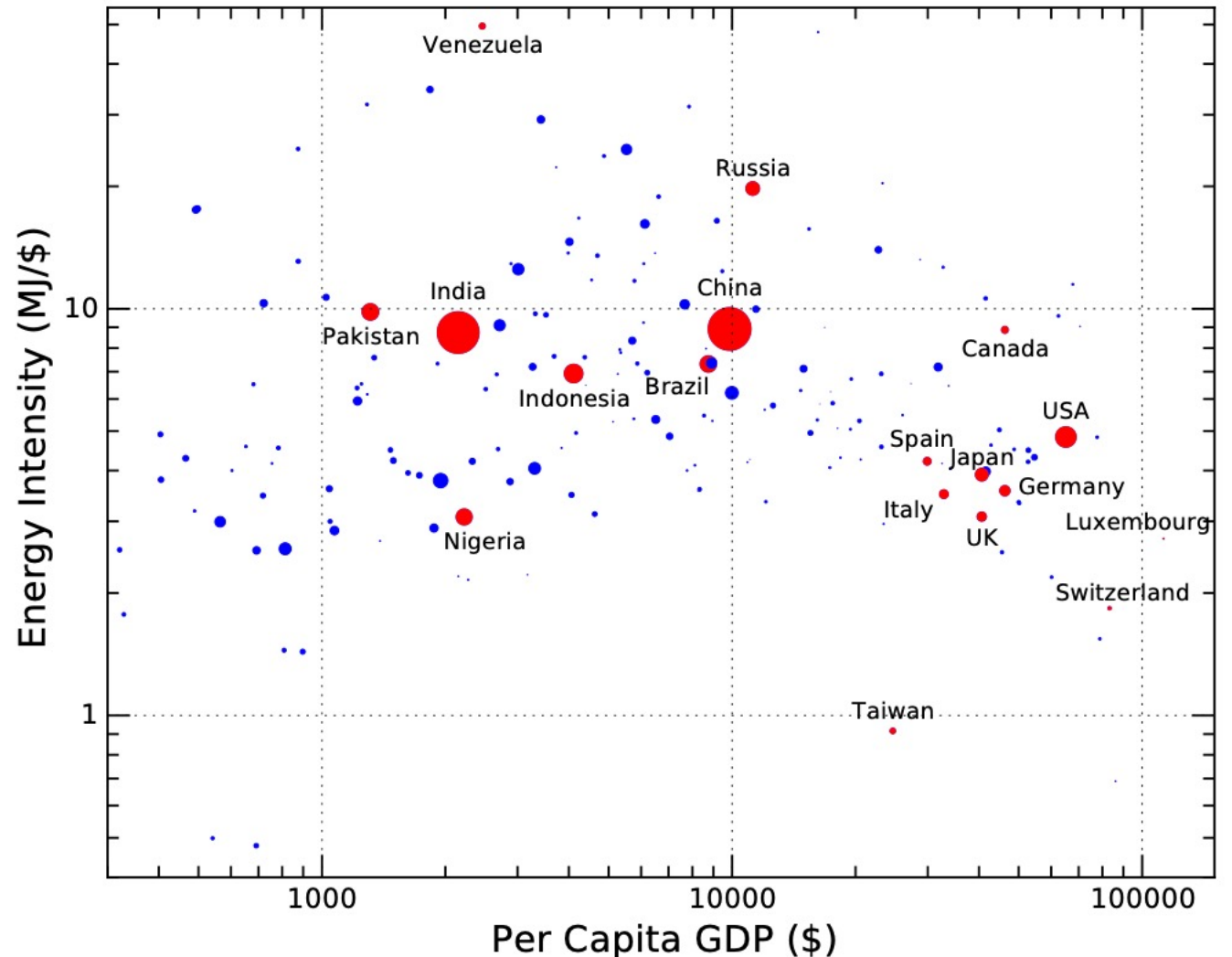
- We just figured out that we cannot have exponential growth of energy use forever – what about economic growth?
- What is economic growth?
  - Producing “more”
    - Goods
      - Limited by physical resources (like iron ore, coal, minerals)
      - Limited by energy (as we just discussed!)
    - Services
      - Are they limited?
- Why would you want infinite exponential economic growth?
  - So we can have infinitely more goods and services
  - What goods and services would you want infinitely more of?
- On Earth we clearly cannot have infinitely more material goods

# Decoupling – The Silver Bullet?

- High energy and resource: Steel, food, buildings
- Low energy and resource: Fine art, performing art, personal coaching
- Since our material resources are clearly limited, maybe we can still have infinite growth if we **substitute** a growing portion of our GDP with **services** rather than goods
- Focusing on energy usage, we can use energy intensity to measure “decoupledness” – the lower the better!
- Is decoupling observed in the real world?

# Energy Intensity

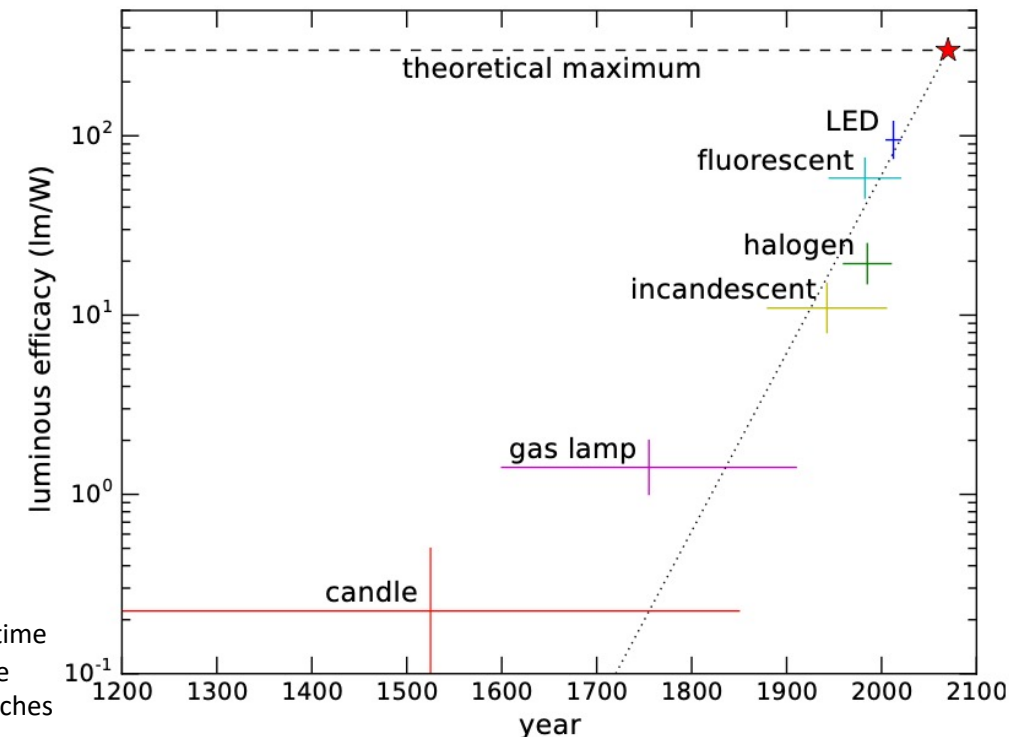
- Energy Intensity = Energy/GDP
- As countries develop and become more prosperous, does intensity *decrease*, as we would want it to do as a signal of decoupling?
- On the large scale, any effect is modest. Going from India to the U.S. affords only a factor-of-two improvement in intensity, while spanning most of the horizontal extent in personal prosperity (a factor of 30 in per capita GDP). That's pretty weak.
- Getting richer isn't going to solve our energy consumption problem



**Figure 2.2:** Energy intensity of countries, on a log–log plot. The vertical axis shows how energetically “hungry” each country is in relation to its economic output, while the horizontal axis sorts countries by economic output per person. A few instructive cases (red dots) are labeled. The dot areas are scaled to population. Prosperous countries tend to have lower intensity than developing countries, but part of this may relate to moving manufacturing from the former to the latter [6–8].

# Energy Efficiency

- How much more efficient can we be?
- How much more efficient would we need to be to make exponential growth possible?
- Energy efficiency has physical limits too
- Example: lighting efficiency
- What other technologies have progressed a lot in efficiency but may be near physical limits?



**Figure 2.3:** Historical progress of lighting efficiency on a logarithmic plot, using bars to indicate the approximate range of time and performance. The dashed line at top represents the maximum theoretical luminous efficacy for white light (no waste heat). **The dotted line rises by our customary factor of ten per century (2.3% annual rate).** Note that the guiding line reaches the theoretical maximum mid-century (red star), indicating that this centuries-long ride cannot continue much longer

# Why Energy Efficiency Won't Solve Demand

1. Most current efficiencies are already within a factor-of-two of theoretical limits. A motor or generator operating at 90% efficiency has **little room to improve**. If efficiencies were typically far smaller than 1%, it would be reasonable to seek improvements as a “resource” for some time to come, but that is not the lay of the land.
2. Energy efficiency **improvements tend to be slow**: ~1% per year, or sometimes 2%. Doubling times are therefore measured in decades, which combined with the previous point suggests an end to this train ride within the century.
3. Efficiency improvements can **backfire**, in a process called the **Jevons paradox** or the **rebound effect**. Increased demand for the more efficient technology results in *greater* demand for the underlying resource. For example, improvements in refrigerator efficiency resulted in larger refrigerators and more of them, for a net increase in energy devoted to refrigeration. Consider that per-capita global energy and material resource use has climbed inexorably amidst a backdrop of substantial efficiency improvements over the last century.

# Exponential Economic Growth Scenario

- What happens when total GDP grows exponentially while the amount of physical goods are limited by finite resources?
  - What fraction of total expenditure do physical goods eventually take up?
  - Does that make sense?

## Box 2.5: Economic Growth Limits

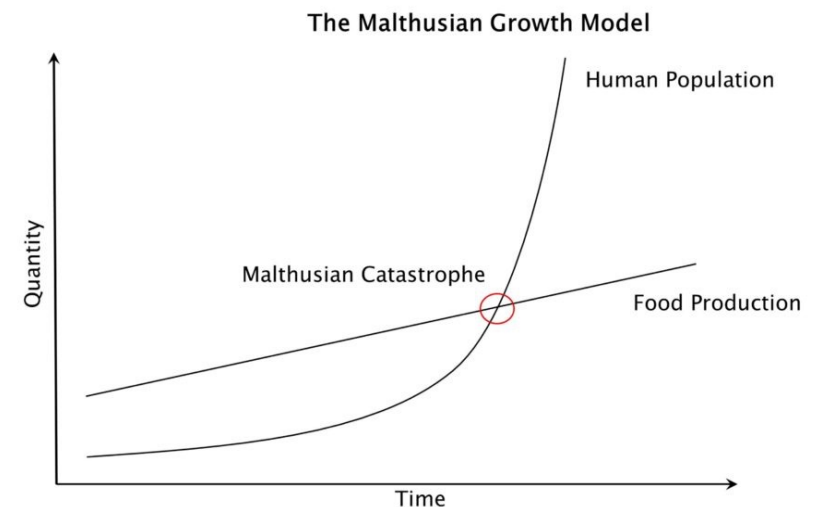
1. Physical resources (ex. energy) ultimately stabilize to a fixed annual amount.
2. Non-physical sectors of the economy must assume responsibility for continued economic growth, if growth is to continue.
3. The economy comes to be dominated by non-physical sectors.
4. Physical sectors are relegated to an ever-smaller fraction of the economy, ultimately vanishing if exponential growth is to hold.
5. In this scenario, physical goods become arbitrarily cheap
6. This situation is impossible and does not respect common-sense supply/demand notions: a finite, limited but absolutely vital resource will never become arbitrarily cheap in a market system.
7. At some point, physical resources will “saturate” to a minimum fraction of the economy, at which point overall growth in non-physical sectors must also cease.

# No-Growth World: Uncommon Common Sense

- If exponential growth is not possible in the long term, why do we act like it is?
  - Interest rates
  - Investments, loans, and banking
  - Social safety net systems and retirement plans

# False Victory over Early Economists

- Classical economists and early thinkers like Smith, Ricardo, Malthus, Stuart Mill did not anticipate indefinite exponential growth
- Growth was seen as a temporary phase bounded by limited land
- But **fossil fuels** came along and with a bang of tremendous energy, fooled us into thinking these conclusions were wrong
- If we were truly clever, we would *start thinking about a world that does not depend on growth, and how to live compatibly within planetary limits.*





# Exponential Growth Takeaway:

Exponential growth of energy consumption and “the economy” is a **temporary trend** that cannot extend into the far future due the laws of physics.

# Reminders

- Check out Ed for
  - Discussion questions and problems
  - These slides

See you on Wednesday!

# Fun resources if you're interested!

Some great posts explaining exponentials, logarithms,  $e$ , and exponential growth

<https://betterexplained.com/articles/an-intuitive-guide-to-exponential-functions-e/>

<https://betterexplained.com/articles/think-with-exponents/>

<https://betterexplained.com/articles/understanding-discrete-vs-continuous-growth/>