CS553 HW1 – Riley Easton

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1 HW1 – First PolyBench/C GEMM optimization

1.0.1 Background

This report covers changes made to the GEMM kernel in PolyBench/C 4.2.1-beta, a benchmarking suite built by Louis-Noel Pouchet and Tomofumi Yuki. PolyBench/C is copyright of The Ohio State University and can be found on Sourceforge here.

This experiment is being performed on lab machine `pollock`, an Intel i7-12700K 12-core 20-thread system. The version of `gcc` available on this machine is 8.5.0 by default. `clang` 16.0.6 is also available, and generally produces better performing results, but is not used here due to familiarity and to make this report compatible with others' work.

[1]:
```python
import os
```

[2]:
```
! lscpu
```

```
Architecture:        x86_64
CPU op-mode(s):      32-bit, 64-bit
Byte Order:          Little Endian
CPU(s):              20
On-line CPU(s) list: 0-19
Thread(s) per core:  1
Core(s) per socket:  12
Socket(s):           1
NUMA node(s):        1
Vendor ID:           GenuineIntel
CPU family:          6
Model:               151
Model name:          12th Gen Intel(R) Core(TM) i7-12700K
Stepping:            2
CPU MHz:             3600.000
CPU max MHz:         5000.0000
CPU min MHz:         800.0000
BogoMIPS:            7219.20
Virtualization:      VT-x
L1d cache:           48K
L1i cache:           32K
L2 cache:            1280K
```

```
L3 cache:              25600K
NUMA node0 CPU(s):     0-19
Flags:                 fpu vme de pse tsc msr pae mce cx8 apic sep mtrr pge mca
cmov pat pse36 clflush dts acpi mmx fxsr sse sse2 ss ht tm pbe syscall nx
pdpe1gb rdtscp lm constant_tsc art arch_perfmon pebs bts rep_good nopl xtopology
nonstop_tsc cpuid aperfmperf tsc_known_freq pni pclmulqdq dtes64 monitor ds_cpl
vmx smx est tm2 ssse3 sdbg fma cx16 xtpr pdcm sse4_1 sse4_2 x2apic movbe popcnt
tsc_deadline_timer aes xsave avx f16c rdrand lahf_lm abm 3dnowprefetch
cpuid_fault epb ssbd ibrs ibpb stibp ibrs_enhanced tpr_shadow vnmi flexpriority
ept vpid ept_ad fsgsbase tsc_adjust bmi1 avx2 smep bmi2 erms invpcid rdseed adx
smap clflushopt clwb intel_pt sha_ni xsaveopt xsavec xgetbv1 xsaves
split_lock_detect avx_vnni dtherm ida arat pln pts hwp hwp_notify hwp_act_window
hwp_epp hwp_pkg_req hfi umip pku ospke waitpkg gfni vaes vpclmulqdq tme rdpid
movdiri movdir64b fsrm md_clear serialize pconfig arch_lbr flush_l1d
arch_capabilities
```

[3]:
```
! gcc -v
```

```
Using built-in specs.
COLLECT_GCC=gcc
COLLECT_LTO_WRAPPER=/usr/libexec/gcc/x86_64-redhat-linux/8/lto-wrapper
OFFLOAD_TARGET_NAMES=nvptx-none
OFFLOAD_TARGET_DEFAULT=1
Target: x86_64-redhat-linux
Configured with: ../configure --enable-bootstrap --enable-
languages=c,c++,fortran,lto --prefix=/usr --mandir=/usr/share/man
--infodir=/usr/share/info --with-bugurl=http://bugs.almalinux.org/ --enable-
shared --enable-threads=posix --enable-checking=release --enable-multilib
--with-system-zlib --enable-__cxa_atexit --disable-libunwind-exceptions
--enable-gnu-unique-object --enable-linker-build-id --with-gcc-major-version-
only --with-linker-hash-style=gnu --enable-plugin --enable-initfini-array
--with-isl --disable-libmpx --enable-offload-targets=nvptx-none --without-cuda-
driver --enable-gnu-indirect-function --enable-cet --with-tune=generic --with-
arch_32=x86-64 --build=x86_64-redhat-linux
Thread model: posix
gcc version 8.5.0 20210514 (Red Hat 8.5.0-20) (GCC)
```

1.0.2 Testing Methodology

For this experiment, the following autotuning script was built to test GEMM iterations across various targets. Portions of this script and data returned from these runs will be used throughout this report. PB/C's GEMM is designed to print out a benchmark time of its call to the kernel, i.e. the portion of the script where the matrix multiplication is processed. This output is scraped by this autotuner and saved as a float.

[2]:
```
import subprocess
import numpy
import time
```

```python
def autotune_batcher(binary='my_binary', epoch=20, verbose=True):
    if verbose: print('Running ' + binary + ' for ' + str(epoch) + ' epoch...')
    times = []
    for _ in range(epoch):
        times.append(float(subprocess.run(['./' + binary], stdout=subprocess.PIPE).stdout.decode('utf-8').rstrip()))
    return times

def autotune_builder(optargs=['-O3'], filename='gemm.c', sz=None): # refactor? too GEMM specific for reuse
    command = ['gcc']
    command += optargs
    command += ['-I', 'utilities', '-I', 'linear-algebra/blas/gemm', 'utilities/polybench.c',
                'linear-algebra/blas/gemm/' + filename, '-DPOLYBENCH_TIME']
    if sz is not None:
        command += ['-DNI=' + sz, '-DNJ=' + sz, '-DNK=' + sz, '-o', sz]
    else:
        command += ['-o', 'my_binary']
    subprocess.run(command)
    return command[-1]

def autotune_sizes(szs=['512', '513', '1000', '1024', '2000', '2048'],
                   filename='gemm.c',
                   epoch=20,
                   optargs=['-O3'],
                   verbose=True):
    results = {}
    for sz in szs:
        if verbose: print('Initializing ' + filename + ' using ' + str(optargs) + ' on size ' + sz)
        results[sz] = autotune_batcher(autotune_builder(optargs=optargs, filename=filename, sz=sz), epoch=epoch, verbose=verbose)
    return results

def autotune_full(szs=['512', '513', '1000', '1024', '2000', '2048'],
                  filename='gemm.c',
                  epoch=20,
                  optargss=[['-O0'], ['-O1'], ['-O2'], ['-O3'], ['-O3', '-fno-tree-vectorize'], ['-Ofast']],
                  verbose=False):
    print('Begin full autotune...')
    t = time.time()
    results = {}
    for optargs in optargss:
```

```
        results[optargs[-1]] = autotune_sizes(szs=szs, filename=filename,␣
↪epoch=epoch, optargs=optargs, verbose=verbose)
    t = time.time() - t
    print('Full autotune took ' + str(t) + ' sec.')
    return results

def calc_gflops_gemm_3(fSeconds, iI, iJ, iK):
    iBallpark_ops = (iI*iJ*iK)*3 + iI*iJ # float mul in first loop (n^2), 2x␣
↪float mul + float add in second loop (n^3)
    fFlops = iBallpark_ops / fSeconds
    return (fFlops/1000000000)

def calc_gflops_gemm(fSeconds, iN):
    return calc_gflops_gemm_3(fSeconds, iN, iN, iN)

def report_builder_single(results):
    gflops = {}
    for k in results.keys():
        iK = int(k)
        avg = numpy.average(results[k])
        print('For size ' + k + ', the average time was ' + str(round(avg,5)) +␣
↪' seconds (' + str(round(min(results[k]), 5)) + ' minimum/' +␣
↪str(round(max(results[k]), 5)) + ' maximum)')
        gflops[k] = calc_gflops_gemm(avg, iK)
    return gflops


def report_builder_full(results):
    gflops = {}
    for k in results.keys():
        print('The following times were collected using the ' + k + '␣
↪optimization flag:')
        curr_gflops = report_builder_single(results[k])
        for e in curr_gflops.keys():
            if e not in gflops or gflops[e][1] < curr_gflops[e]:
                gflops[e] = (k, curr_gflops[e])
        print()
    for k in gflops.keys():
        print('The best average result for size ' + k + ' was with optimization␣
↪flag ' + gflops[k][0] + ', demonstrating approximately ' +␣
↪str(round(gflops[k][1], 3)) + ' GFLOPS.')
```

1.0.3 Optimizations

Two optimization attempts on the core were built and tested. For reference, the initial GEMM kernel is the following:

```
#pragma scop
```

```
  for (i = 0; i < _PB_NI; i++) {
    for (j = 0; j < _PB_NJ; j++)
    C[i][j] *= beta;
    for (k = 0; k < _PB_NK; k++) {
       for (j = 0; j < _PB_NJ; j++)
      C[i][j] += alpha * A[i][k] * B[k][j];
    }
  }
#pragma endscop
```

[5]:

```
# BASE CODE
times_base = autotune_batcher(binary=autotune_builder(filename='gemm.c'))
avg_base = numpy.average(times_base)
print(str(round(avg_base,5)) + ' sec on average.')
print(str(round(calc_gflops_gemm_3(avg_base, 1000, 1100, 1200),3)) + ' GFLOPS␣
 ↪on average.') # large_dataset default see gemm.h
```

```
Running my_binary for 20 epoch…
0.24478 sec on average.
16.182 GFLOPS on average.
```

[6]:

```
! gcc -O3 -ftree-vectorize -fopt-info-vec-optimized -I utilities -I␣
 ↪linear-algebra/blas/gemm utilities/polybench.c linear-algebra/blas/gemm/gemm.
 ↪c -DPOLYBENCH_TIME -o my_binary
```

```
utilities/polybench.c:121:3: note: loop vectorized
utilities/polybench.c:121:3: note: loop vectorized
utilities/polybench.c:121:3: note: loop vectorized
linear-algebra/blas/gemm/gemm.c:93:8: note: loop vectorized
linear-algebra/blas/gemm/gemm.c:93:8: note: loop versioned for vectorization
because of possible aliasing
linear-algebra/blas/gemm/gemm.c:90:5: note: loop vectorized
linear-algebra/blas/gemm/gemm.c:44:5: note: loop vectorized
linear-algebra/blas/gemm/gemm.c:41:5: note: loop vectorized
linear-algebra/blas/gemm/gemm.c:38:5: note: loop vectorized
```

Overall, my goal with my optimizations was to simplify the loops such that the compiler can more readily analyze the goal of the program. The base code already is in a state where some loop vectorization can take place, namely the two j loops, but the script is fairly complex. In the initial revision of this code, two tasks involving i & j are present.

The first, the C*beta step, can be moved out of the nested loop. This portion is irrelevant to the triple-nested loop doing the alpha*A*B segment of GEMM & none of the second task's goals occur before the first. In the original code, each i row is calculated for C*beta before alpha*A*B; the second iteration simply calculates the entire array before moving on to the second step. While there are now sequential i loops within this code, the benefits demonstrated far outweigh any potential minor costs. For reference, the first revision looped code of the GEMM kernel is the following:

```
#pragma scop
  for (i = 0; i < _PB_NI; i++) {
```

```
    for (j = 0; j < _PB_NJ; j++)
        C[i][j] *= beta;
  }
  for (i = 0; i < _PB_NI; i++) {
    for (k = 0; k < _PB_NK; k++) {
       for (j = 0; j < _PB_NJ; j++)
      C[i][j] += alpha * A[i][k] * B[k][j];
    }
  }
#pragma endscop
```

[7]:
```
# REVISION ONE
times_1 = autotune_batcher(binary=autotune_builder(filename='gemm1.c'))
avg_1 = numpy.average(times_1)
print(str(round(avg_1,5)) + ' sec on average.')
print(str(round(calc_gflops_gemm_3(avg_1, 1000, 1100, 1200),3)) + ' GFLOPS on␣
 ↪average.') # large_dataset default see gemm.h
```

```
Running my_binary for 20 epoch…
0.18407 sec on average.
21.52 GFLOPS on average.
```

[8]:
```
! gcc -O3 -ftree-vectorize -fopt-info-vec-optimized -I utilities -I␣
 ↪linear-algebra/blas/gemm utilities/polybench.c linear-algebra/blas/gemm/
 ↪gemm1.c -DPOLYBENCH_TIME -o my_binary
```

```
utilities/polybench.c:121:3: note: loop vectorized
utilities/polybench.c:121:3: note: loop vectorized
utilities/polybench.c:121:3: note: loop vectorized
linear-algebra/blas/gemm/gemm1.c:95:8: note: loop vectorized
linear-algebra/blas/gemm/gemm1.c:95:8: note: loop versioned for vectorization
because of possible aliasing
linear-algebra/blas/gemm/gemm1.c:90:5: note: loop vectorized
linear-algebra/blas/gemm/gemm1.c:44:5: note: loop vectorized
linear-algebra/blas/gemm/gemm1.c:41:5: note: loop vectorized
linear-algebra/blas/gemm/gemm1.c:38:5: note: loop vectorized
```

My second iteration of this code was partial unrolling of our alpha*A*B calculation. Unfortunately, my reasoning for *why* this works isn't as sound as I would like it to be, but I made this decision based on the k loop's unvectorized status. Complex access patterns are often ignored by `gcc`. While the second j loop is vectorized, fetches defined from k leave performance on the table. A[i][k] will get an entire cache line rather than a single value. I suspected that because B[k][j] is non-sequential, `gcc` defaulted to fetching on A again for each iteration of k. I switched to strides of length 4 for the k loop and added an additional loop to handle the remainder. For reference, the second revision looped code of the GEMM kernel is the following:

```
#pragma scop
  for (i = 0; i < _PB_NI; ++i) {
    for (j = 0; j < _PB_NJ; ++j)
      C[i][j] *= beta;
```

```
  }
  int _PB_NK_SF = _PB_NK - _PB_NK % 4;
  for (i = 0; i < _PB_NI; ++i) {
    for (k = 0; k < _PB_NK_SF; k += 4) {
      for (j = 0; j < _PB_NJ; ++j) {
        C[i][j] += alpha * A[i][k] * B[k][j];
        C[i][j] += alpha * A[i][k+1] * B[k+1][j];
        C[i][j] += alpha * A[i][k+2] * B[k+2][j];
        C[i][j] += alpha * A[i][k+3] * B[k+3][j];
      }
    }
  }
  for (i = 0; i < _PB_NI; ++i) {
    for (k = _PB_NK_SF; k < _PB_NK; ++k) {
      for (j = 0; j < _PB_NJ; ++j) {
        C[i][j] += alpha * A[i][k] * B[k][j];
      }
    }
  }
#pragma endscop
```

[9]:

```
# REVISION TWO
times_2 = autotune_batcher(binary=autotune_builder(filename='gemm2.c'))
avg_2 = numpy.average(times_2)
print(str(round(avg_2,5)) + ' sec on average.')
print(str(round(calc_gflops_gemm_3(avg_2, 1000, 1100, 1200),3)) + ' GFLOPS on␣
↪average.') # large_dataset default see gemm.h
```

```
Running my_binary for 20 epoch…
0.13903 sec on average.
28.49 GFLOPS on average.
```

[10]:

```
! gcc -O3 -ftree-vectorize -fopt-info-vec-optimized -I utilities -I␣
↪linear-algebra/blas/gemm utilities/polybench.c linear-algebra/blas/gemm/
↪gemm2.c -DPOLYBENCH_TIME -o my_binary
```

```
utilities/polybench.c:121:3: note: loop vectorized
utilities/polybench.c:121:3: note: loop vectorized
utilities/polybench.c:121:3: note: loop vectorized
linear-algebra/blas/gemm/gemm2.c:96:7: note: loop vectorized
linear-algebra/blas/gemm/gemm2.c:96:7: note: loop versioned for vectorization
because of possible aliasing
linear-algebra/blas/gemm/gemm2.c:90:5: note: loop vectorized
linear-algebra/blas/gemm/gemm2.c:44:5: note: loop vectorized
linear-algebra/blas/gemm/gemm2.c:41:5: note: loop vectorized
linear-algebra/blas/gemm/gemm2.c:38:5: note: loop vectorized
```

The following two full autotunes, one for the original version and the second revision of the GEMM core, will be the source of data analyzed in the next sections. The dictionaries will be serialized &

their pickles can be loaded to prevent rerunning the script; these will be provided as supplement to this report.

[11]:
```
import pickle
autotunedfull = autotune_full()
autotunedfull2 = autotune_full(filename='gemm2.c')

file = open('gemm.pickle', 'wb')
pickle.dump(autotunedfull, file)
file.close()

file = open('gemm2.pickle', 'wb')
pickle.dump(autotunedfull2, file)
file.close()
```

```
Begin full autotune…
Full autotune took 1288.6435747146606 sec.
Begin full autotune…
Full autotune took 1057.1508729457855 sec.
```

[1]:
```
# To reload the data, run this cell.
import pickle

file = open('gemm.pickle', 'rb')
autotunedfull = pickle.load(file)
file.close()

file = open('gemm2.pickle', 'rb')
autotunedfull2 = pickle.load(file)
file.close()
```

1.0.4 Autotuning for array sizes

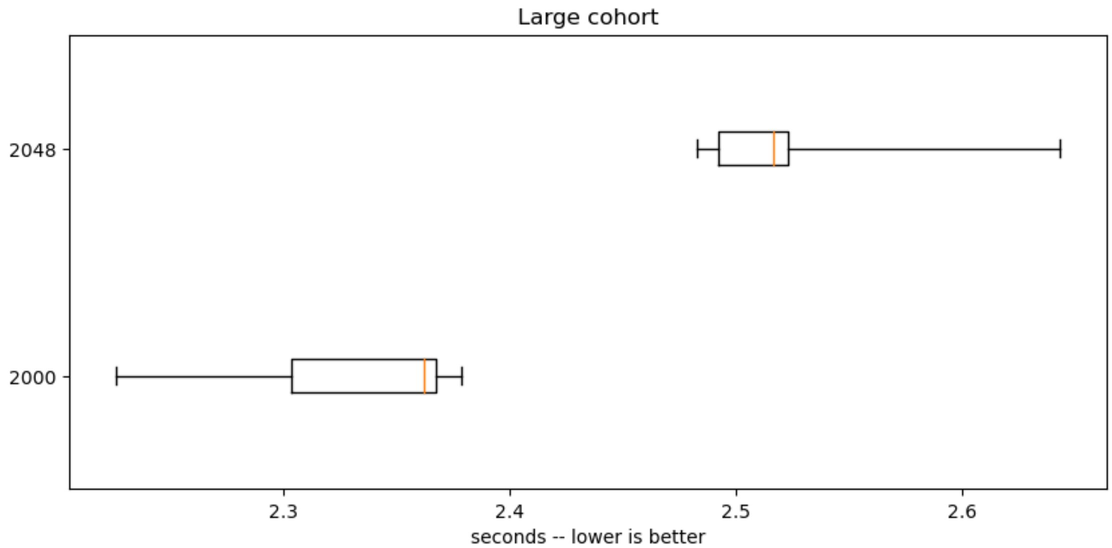
The following analysis will focus the `-O3`-compiled run of the original GEMM program; this data is pulled from the full autotune.

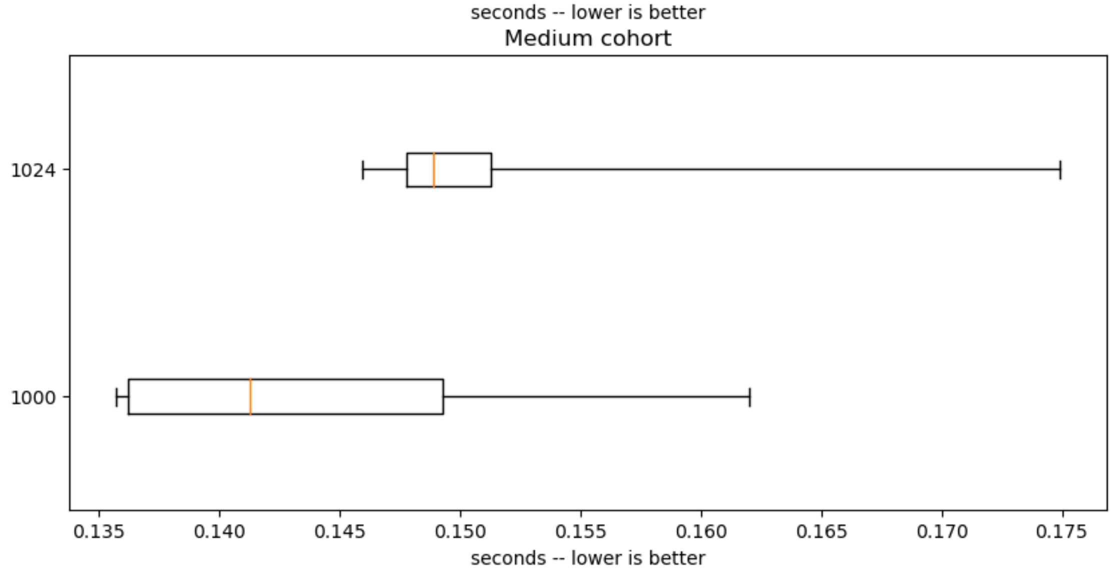
[3]:
```
autotuned4 = autotunedfull['-O3']
report_builder_single(autotuned4)
print('Distribution of times and GFLOPS are shown below:')
```

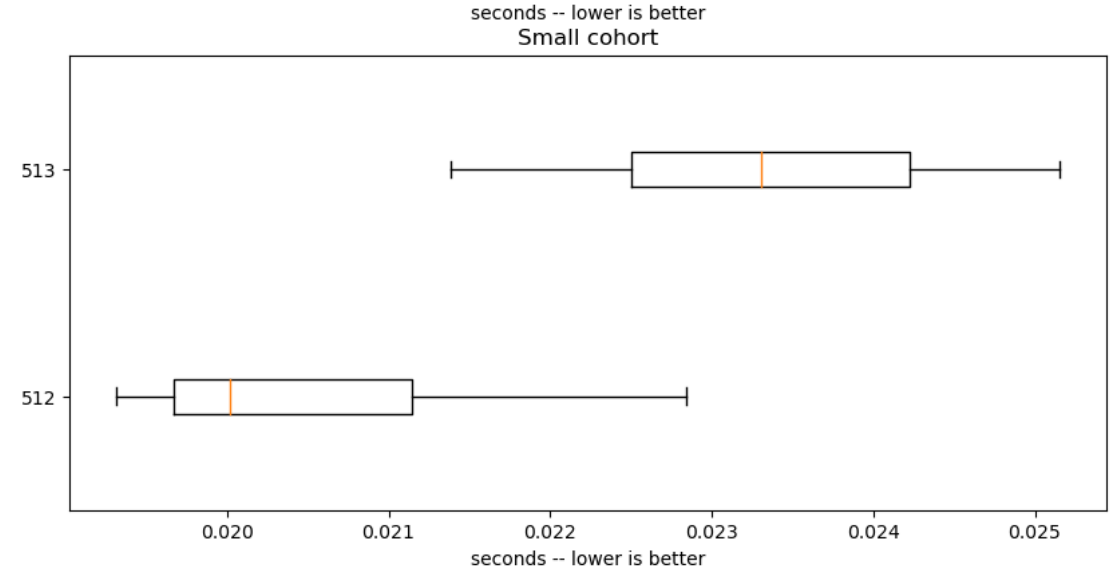
```
For size 512, the average time was 0.02049 seconds (0.01932 minimum/0.02284
maximum)
For size 513, the average time was 0.02331 seconds (0.02138 minimum/0.02515
maximum)
For size 1000, the average time was 0.14471 seconds (0.13574 minimum/0.16201
maximum)
For size 1024, the average time was 0.15276 seconds (0.14597 minimum/0.17492
maximum)
For size 2000, the average time was 2.34091 seconds (2.22656 minimum/2.37895
```

maximum)
For size 2048, the average time was 2.52323 seconds (2.48286 minimum/2.64326 maximum)
Distribution of times and GFLOPS are shown below:

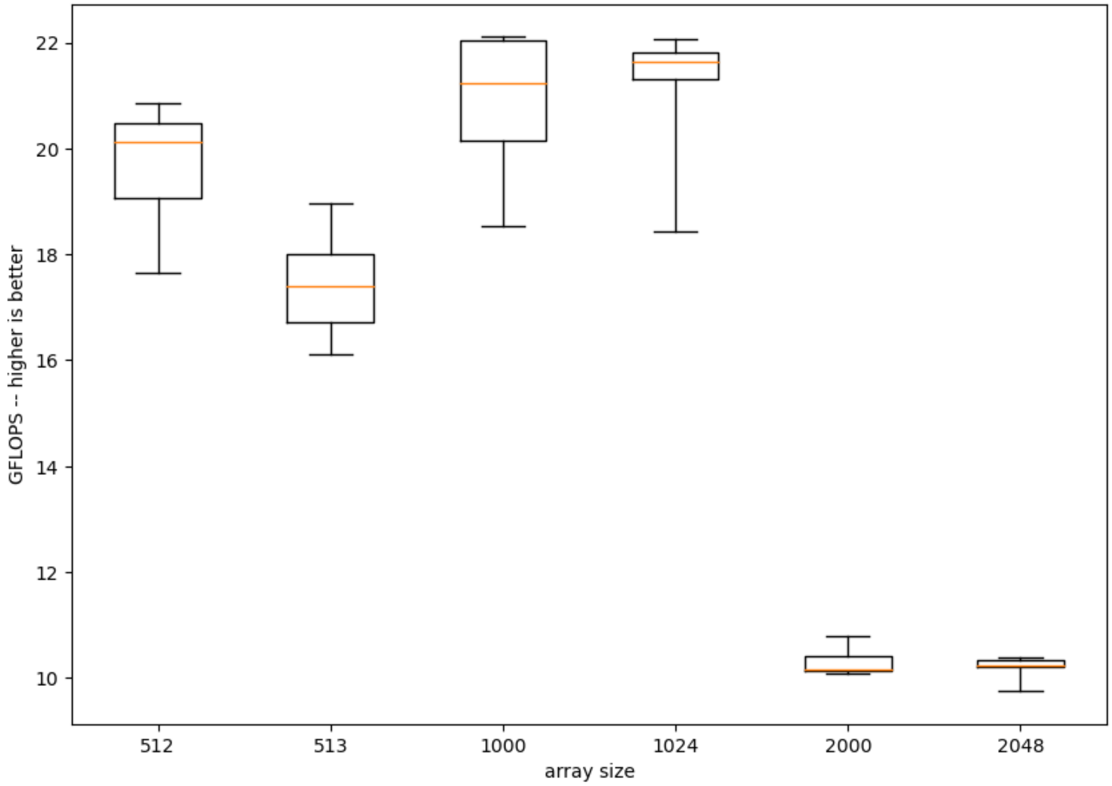
[4]:
```
import matplotlib.pyplot as plt
import pandas as pd
figure, axis = plt.subplots(3,1, figsize=(10, 15))

axis[0].boxplot(list(autotuned4.values())[4:], vert = 0, whis=50000)
axis[0].set_yticklabels(['2000', '2048'])
axis[0].set_title('Large cohort')
axis[1].boxplot(list(autotuned4.values())[2:4], vert = 0, whis=50000)
axis[1].set_yticklabels(['1000', '1024'])
axis[1].set_title('Medium cohort')
axis[2].boxplot(list(autotuned4.values())[:2], vert = 0, whis=50000)
axis[2].set_yticklabels(['512', '513'])
axis[2].set_title('Small cohort')
for a in axis:
    a.set_xlabel('seconds -- lower is better')

plt.show()
```


Large cohort
2048
2000
2.3
2.4
2.5
2.6
seconds -- lower is better


Medium cohort
1024
1000
0.135
0.140
0.145
0.150
0.155
0.160
0.165
0.170
0.175
seconds -- lower is better


Small cohort
513
512
0.020
0.021
0.022
0.023
0.024
0.025
seconds -- lower is better

[5]:
```
flops_seen = []
for size, results in zip([512, 513, 1000, 1024, 2000, 2048], autotuned4.
 ↪values()):
    flops_seen.append([calc_gflops_gemm(seconds, size) for seconds in results])
fig = plt.figure(figsize =(10, 7))
plt.boxplot(flops_seen, whis=50000)
plt.xticks([1, 2, 3, 4, 5, 6], ['512', '513', '1000', '1024', '2000', '2048'])
plt.ylabel('GFLOPS -- higher is better')
plt.xlabel('array size')
plt.show()
```



[6]:
```
for size, results in zip(['512', '513', '1000', '1024', '2000', '2048'],␣
 ↪flops_seen):
    print('The maximum GFLOPS seen for size ' + size + ' was ' +␣
 ↪str(round(max(results),3)) + ' GFLOPS.')
```

```
The maximum GFLOPS seen for size 512 was 20.857 GFLOPS.
The maximum GFLOPS seen for size 513 was 18.955 GFLOPS.
The maximum GFLOPS seen for size 1000 was 22.109 GFLOPS.
The maximum GFLOPS seen for size 1024 was 22.075 GFLOPS.
```

```
The maximum GFLOPS seen for size 2000 was 10.781 GFLOPS.
The maximum GFLOPS seen for size 2048 was 10.381 GFLOPS.
```

The time variation across runs for the same value of N can be caused by a number of factors. This program is running on a machine I have limited control over. While I did schedule these tests in the middle of the night, these are machines that can handle multiple users at once. The autotuner may have had lower priority at times for the scheduler; this is particularly relevant for 1024. A small number of 1024 runs were significant outliers and ran at approximately 4 GFLOPS worse than other attempts. Additionally, the processor may have downclocked on some runs due to load. Boost is not disabled on these machines. While it's likely the CPU would be on the higher end of speeds available to it, the frequency could be anywhere from 800 MHz up to 5 GHz. This inconsistency is out of my control and likely leads to most of the variation demonstrated here, beyond the significant outliers.

For me, this variation is less interesting than the relation between various sizes. Immediately, there's a significant drop in performance from 1024 to 2000 size loads. It's likely communication speed significantly slowed these runs, as these batches are larger than the L2 cache size on the 12700K. The dip from 512 to 513 is also interesting, my guess here is that the serial versioned branch of the second j loop is taken at higher indices due to the odd size of j.

As established, on `-O3`, the program is SIMD vectorized for both j loops.

1.0.5 Compiler Options Autotune

Below is a report of the full autotune for the orginal version of GEMM. I chose to report the average GFLOPS here as that seems more relevant in an uncontrolled system rather than the absolute highest demonstrated. `-O3` and `-Ofast` use SIMD vectorization for the loops.

[7]:
```
report_builder_full(autotunedfull)
```

```
The following times were collected using the -O0 optimization flag:
For size 512, the average time was 0.21123 seconds (0.20873 minimum/0.22053
maximum)
For size 513, the average time was 0.26168 seconds (0.26027 minimum/0.26472
maximum)
For size 1000, the average time was 1.49219 seconds (1.48777 minimum/1.50063
maximum)
For size 1024, the average time was 1.65012 seconds (1.64529 minimum/1.65751
maximum)
For size 2000, the average time was 11.95958 seconds (11.8494 minimum/12.27431
maximum)
For size 2048, the average time was 13.23654 seconds (13.18058 minimum/13.28119
maximum)

The following times were collected using the -O1 optimization flag:
For size 512, the average time was 0.05304 seconds (0.04818 minimum/0.05614
maximum)
For size 513, the average time was 0.03921 seconds (0.03659 minimum/0.04861
maximum)
For size 1000, the average time was 0.3373 seconds (0.3162 minimum/0.37099
```

maximum)
For size 1024, the average time was 0.41156 seconds (0.38404 minimum/0.42994 maximum)
For size 2000, the average time was 3.61468 seconds (3.54925 minimum/3.74945 maximum)
For size 2048, the average time was 3.92872 seconds (3.83492 minimum/4.1624 maximum)

The following times were collected using the -O2 optimization flag:
For size 512, the average time was 0.04406 seconds (0.0387 minimum/0.05802 maximum)
For size 513, the average time was 0.04066 seconds (0.03739 minimum/0.04994 maximum)
For size 1000, the average time was 0.2924 seconds (0.26564 minimum/0.3496 maximum)
For size 1024, the average time was 0.33115 seconds (0.2996 minimum/0.39132 maximum)
For size 2000, the average time was 3.41093 seconds (3.27745 minimum/3.66295 maximum)
For size 2048, the average time was 3.90555 seconds (3.81334 minimum/4.11697 maximum)

The following times were collected using the -O3 optimization flag:
For size 512, the average time was 0.02049 seconds (0.01932 minimum/0.02284 maximum)
For size 513, the average time was 0.02331 seconds (0.02138 minimum/0.02515 maximum)
For size 1000, the average time was 0.14471 seconds (0.13574 minimum/0.16201 maximum)
For size 1024, the average time was 0.15276 seconds (0.14597 minimum/0.17492 maximum)
For size 2000, the average time was 2.34091 seconds (2.22656 minimum/2.37895 maximum)
For size 2048, the average time was 2.52323 seconds (2.48286 minimum/2.64326 maximum)

The following times were collected using the -fno-tree-vectorize optimization flag:
For size 512, the average time was 0.04395 seconds (0.03855 minimum/0.05049 maximum)
For size 513, the average time was 0.04197 seconds (0.03738 minimum/0.04956 maximum)
For size 1000, the average time was 0.30656 seconds (0.26591 minimum/0.35119 maximum)
For size 1024, the average time was 0.33654 seconds (0.29983 minimum/0.39386 maximum)
For size 2000, the average time was 3.38036 seconds (3.32088 minimum/3.44359 maximum)

```
For size 2048, the average time was 3.8666 seconds (3.75868 minimum/4.11535
maximum)

The following times were collected using the -Ofast optimization flag:
For size 512, the average time was 0.01998 seconds (0.01921 minimum/0.023
maximum)
For size 513, the average time was 0.02384 seconds (0.0212 minimum/0.02687
maximum)
For size 1000, the average time was 0.14322 seconds (0.1351 minimum/0.16191
maximum)
For size 1024, the average time was 0.14968 seconds (0.14594 minimum/0.17447
maximum)
For size 2000, the average time was 2.36604 seconds (2.34482 minimum/2.43449
maximum)
For size 2048, the average time was 2.59376 seconds (2.56788 minimum/2.66256
maximum)

The best average result for size 512 was with optimization flag -Ofast,
demonstrating approximately 20.17 GFLOPS.
The best average result for size 513 was with optimization flag -O3,
demonstrating approximately 17.389 GFLOPS.
The best average result for size 1000 was with optimization flag -Ofast,
demonstrating approximately 20.953 GFLOPS.
The best average result for size 1024 was with optimization flag -Ofast,
demonstrating approximately 21.528 GFLOPS.
The best average result for size 2000 was with optimization flag -O3,
demonstrating approximately 10.254 GFLOPS.
The best average result for size 2048 was with optimization flag -O3,
demonstrating approximately 10.215 GFLOPS.
```

1.0.6 Alternative Version Autotune

Below is a report of the full autotune for the second iteration of GEMM. The same optimization levels for `gcc` are used here; the same SIMD status applies to each option.

[8]:
```
report_builder_full(autotunedfull2)
```

```
The following times were collected using the -O0 optimization flag:
For size 512, the average time was 0.20156 seconds (0.2006 minimum/0.20436
maximum)
For size 513, the average time was 0.25413 seconds (0.25294 minimum/0.25588
maximum)
For size 1000, the average time was 1.40226 seconds (1.39964 minimum/1.41139
maximum)
For size 1024, the average time was 1.60152 seconds (1.59969 minimum/1.60607
maximum)
For size 2000, the average time was 11.44771 seconds (11.41683 minimum/11.59127
maximum)
```

For size 2048, the average time was 13.13609 seconds (13.09668 minimum/13.20725 maximum)

The following times were collected using the -O1 optimization flag:
For size 512, the average time was 0.03681 seconds (0.03424 minimum/0.04455 maximum)
For size 513, the average time was 0.03423 seconds (0.03217 minimum/0.04249 maximum)
For size 1000, the average time was 0.24115 seconds (0.22519 minimum/0.29466 maximum)
For size 1024, the average time was 0.29704 seconds (0.27425 minimum/0.35346 maximum)
For size 2000, the average time was 2.10306 seconds (2.06527 minimum/2.49904 maximum)
For size 2048, the average time was 2.78125 seconds (2.68916 minimum/3.23977 maximum)

The following times were collected using the -O2 optimization flag:
For size 512, the average time was 0.03637 seconds (0.03436 minimum/0.04451 maximum)
For size 513, the average time was 0.03198 seconds (0.03146 minimum/0.03297 maximum)
For size 1000, the average time was 0.22978 seconds (0.22481 minimum/0.29564 maximum)
For size 1024, the average time was 0.29363 seconds (0.27467 minimum/0.35408 maximum)
For size 2000, the average time was 2.17876 seconds (2.10488 minimum/2.57642 maximum)
For size 2048, the average time was 2.80628 seconds (2.73822 minimum/3.26627 maximum)

The following times were collected using the -O3 optimization flag:
For size 512, the average time was 0.01733 seconds (0.01679 minimum/0.02024 maximum)
For size 513, the average time was 0.01823 seconds (0.01727 minimum/0.02012 maximum)
For size 1000, the average time was 0.11171 seconds (0.10553 minimum/0.12379 maximum)
For size 1024, the average time was 0.13688 seconds (0.13386 minimum/0.15532 maximum)
For size 2000, the average time was 1.51458 seconds (1.48182 minimum/1.57782 maximum)
For size 2048, the average time was 1.86734 seconds (1.79049 minimum/1.91664 maximum)

The following times were collected using the -fno-tree-vectorize optimization flag:
For size 512, the average time was 0.03673 seconds (0.03434 minimum/0.04418

```
maximum)
For size 513, the average time was 0.0339 seconds (0.03138 minimum/0.04054
maximum)
For size 1000, the average time was 0.23586 seconds (0.22512 minimum/0.2955
maximum)
For size 1024, the average time was 0.29184 seconds (0.2751 minimum/0.35419
maximum)
For size 2000, the average time was 2.16417 seconds (2.10999 minimum/2.58992
maximum)
For size 2048, the average time was 2.82972 seconds (2.72666 minimum/3.28225
maximum)

The following times were collected using the -Ofast optimization flag:
For size 512, the average time was 0.01762 seconds (0.01665 minimum/0.02089
maximum)
For size 513, the average time was 0.0177 seconds (0.01734 minimum/0.0184
maximum)
For size 1000, the average time was 0.10901 seconds (0.10531 minimum/0.11796
maximum)
For size 1024, the average time was 0.141 seconds (0.13344 minimum/0.15806
maximum)
For size 2000, the average time was 1.5674 seconds (1.49886 minimum/1.62513
maximum)
For size 2048, the average time was 1.86129 seconds (1.82894 minimum/1.9012
maximum)

The best average result for size 512 was with optimization flag -O3,
demonstrating approximately 23.248 GFLOPS.
The best average result for size 513 was with optimization flag -Ofast,
demonstrating approximately 22.891 GFLOPS.
The best average result for size 1000 was with optimization flag -Ofast,
demonstrating approximately 27.529 GFLOPS.
The best average result for size 1024 was with optimization flag -O3,
demonstrating approximately 23.542 GFLOPS.
The best average result for size 2000 was with optimization flag -O3,
demonstrating approximately 15.849 GFLOPS.
The best average result for size 2048 was with optimization flag -Ofast,
demonstrating approximately 13.847 GFLOPS.
```