Syllabus

Wednesday, January 17, 2024 7:58 AM

Syllabus

ME/AE 5449: Robotic Manipulators and Mechanisms
Spring 2024

Instructor: Prof. Doug Bristow
Office: 292C Toomey Hall
Mailbox: 194Toomey Hall
Email: dbristow@mst.edu

Lecture Time: MW(F) 8-8:50am
Location: 295 Toomey

Office hours: Tues 4-5 pm
Location: 292C Toomey

Course Website: *Canvas*. All assignments, assignment solutions, exam solutions, and course handouts will be posted on the website.

Text: Spong, Hutchinson, Vidyasagar, Robot Modeling and Control, *2nd ed.*, 2020.
ISBN 9781119523994

Prerequisite: CS 1970 (or equivalent), ME 3313

Grading Policy:

Homework	20%	
Lab Reports	20%	
Project	20%	
Midterm Exam	20%	W, March 13, in class
Final Exam	20%	M, May 6, 12:30-2:30pm

Homework Assignments

There will be 6 homework assignments during the course. The homework will be posted on Canvas two weeks before it is due and collected **in class** on the due date. **Late homework should be turned in to Dr. Bristow's mailbox in 194 Toomey and will be accepted with a 20% point deduction until the Friday following the due date, at 4:00 pm.** No late homework will be accepted after that point. Students are strongly encouraged to work together on the homework, **but use of solutions (text solutions or another student's solutions) is considered cheating and will result in an automatic F for the course.**

1/31	HW 1 due	2/28	HW 3 due	4/17	HW 5 due
2/14	HW 2 due	4/3	HW 4 due	5/1	HW 6 due

Examinations

All examinations should be considered comprehensive, but the most recent course material will be emphasized. Requests for a conflict or make-up examination will be individually evaluated. Only requests that, in the instructor's opinion, are fully justified will be granted.

Absences from Class

There is no attendance policy so you do not need to email the instructor if you are going to be absent from class. You should get a copy of the notes from another student in the class. If you have any questions about the notes, you should speak with other students or the instructor.

Laboratory

Lab Time: Individually arranged
Lab Location: 205 Toomey Hall

TA: Philip Olubodun
Office: 205 Toomey Hall
Email: paocmc@mst.edu

The lab component of the course consists of five lab assignments and one student-defined project. The student will learn to program and use the Fanuc LR Mate 200i 6-axis industrial robot.

Lab Groups
Students will perform lab assignments in groups of 2 or 3. You will need to form your group and sign up for a weekly two-hour timeslot on the canvas discussion thread. **You should sign up by the end of the day on Thursday, January 18.**

Lab Assignments
There will be five lab assignments during the semester. Lab assignments will be posted on Canvas the Wednesday before lab work begins. You should print out a copy of the assignment and bring it with you to the lab. You will have two weeks to work on each lab; the lab report is due in lab in the following week. Only one report should be turned in per group.

Lab #	Week 1 (wk. of)	Week 2 (wk. of)	Report due (wk. of)
Lab 1	1/22	1/29	2/5
Lab 2	2/5	2/12	2/19
Lab 3	2/19	2/26	3/4
Lab 4	3/4	3/11	3/18
Lab 5	3/18	4/1	4/8

The TA will be available for the start of your lab time each week. However, the laboratories are intended to be completed independently. Each lab will involve some reading of the robot manual. You should also refer to the manual anytime questions arise.

Lab Procedures
The laboratory room is a shared space, please be considerate by refraining from loud or disruptive behavior. The room also contains graduate student's desks and research equipment. **Under no circumstances should you touch or disturb any equipment or desks other than those directly involved in the lab.** Please do not leave the room unattended with the door propped open.

Lab Safety
The robot is capable of exerting large forces and moving at very high speeds that can cause serious injury. The first lab will cover safety procedures and a list of lab safety procedures will be posted in the lab. For your safety, it is imperative that you always follow lab safety procedures. **Failure to adhere to lab safety rules will result in an automatic failing grade for the course.**

Course Project
The course project will be student-defined, subject to the instructor's approval. Students are encouraged to perform their project with their lab group, although other arrangements will be considered if justified. The project is generally open to any facet of robotics that the student is interested in exploring. **Project proposals will be due March 20 in class**; more details will be provided later in the semester.

Linear Algebra

Saturday, February 10, 2024 1:05 PM

Matrix Multiplication

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}_{3\times2} = \begin{bmatrix} 58 & \\ & \end{bmatrix}_{2\times2}$$

Row first and column of second must match!

Dot product rows and columns

$1\cdot7 + 2\cdot9 + 3\cdot11 = 58$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ & \end{bmatrix}$$

$1\cdot8 + 2\cdot10 + 3\cdot12 = 64$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & \end{bmatrix}$$

$4\cdot7 + 5\cdot9 + 6\cdot11 = 139$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

$4\ 8 + 5\cdot10 + 6\cdot12 = 154$

$AB \neq BA$ Not commutative

$A(B+C) = AB + AC$ Distributive

$(AB)^T = A^T B^T$ Transpose

$(AB)C = A(BC)$ Associative

Transpose of a Matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times2} \qquad A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}_{2\times3}$$

Inverse of a Matrix

$AA^{-1} = A^{-1}A = I$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

determinant

$$\boxed{\begin{aligned} XA &= B \\ XAA^{-1} &= BA^{-1} \\ XI &= BA^{-1} \\ X &= BA^{-1} \end{aligned}}$$

$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times 2} \qquad \begin{bmatrix} 2 & 4 & 6 \end{bmatrix}_{2\times 3}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Identity Matrix

$$I_1 = \begin{bmatrix} 1 \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AI = IA = A$$

Intro

Friday, January 19, 2024 8:06 AM

Robot: programmed actuated mechanism with a degree of autonomy to perform locomotion, manipulation or positioning

Where am I?

- Kinematics

How do I get there?

- Spatial → path planning
- temporal → trajectory planning

Where am I going?

- Inverse kinematics

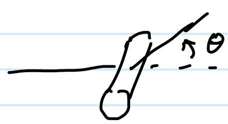
How do I make the robot get there?

- Controls

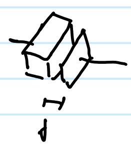
Robot geometries:

Joint types.

Revolute



Prismatic



Rotations

Friday, January 19, 2024 8:49 AM

coordinate frames: consist of an origin point and n orthogonal unit vectors arranged in a right hand configuration

name: $O_0\, x_0\, y_0$

points: defined w.r.t. coordinate frames

$$p^0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{matrix} \leftarrow x^0 \\ \leftarrow y^0 \end{matrix}$$

vector: length and direction (no location). Described by the end position, with the base at the origin of the c.f.

θ has discontinuities at $0°$
Does not scale well to 3D

$$x_1^0 = \begin{bmatrix} x_1 \cdot x_0 \\ x_1 \cdot y_0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$y_1^0 = \begin{bmatrix} y_1 \cdot x_0 \\ y_1 \cdot y_0 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

Rotation Matrix: $R_1^0 = [x_1^0\ y_1^0] = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 \end{bmatrix}$

Conveniently, $V^A = R_B^A V^B$

$$R_B^A = (R_A^B)^{-1} = (R_A^B)^T$$

if A, B share an origin, $p^A = R_B^A p^B$

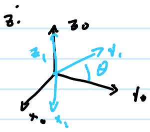
3D:

$$R_1^0 = [x_1^0\ \ y_1^0\ \ z_1^0] = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$$

Basic Rotations

New notation:
$c_\theta = \cos\theta$
$s_\theta = \sin\theta$
$R_{z,\theta}$ = Rotation about z by θ

1. about z:



$$R_1^0 = \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

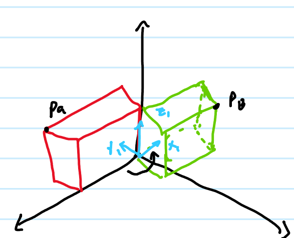
Conveniently, $R_{z,0} = I$

$$R_{z,\theta} R_{z,\phi} = R_{z,\theta+\phi}$$

$$(R_{z,\theta})^{-1} = R_{z,-\theta}$$

2. $R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\theta & -s_\theta \\ 0 & s_\theta & c_\theta \end{bmatrix}$

3. $R_{y,\theta} = \begin{bmatrix} c_\theta & 0 & -s_\theta \\ 0 & 1 & 0 \\ s_\theta & 0 & c_\theta \end{bmatrix}$

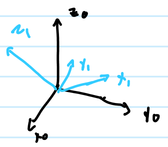


1. Attach CF to block
2. $p_b^0 = R_1^0 p_b^1$
3. $p_b^1 = p_A^0$
$\Rightarrow p_b^0 = R p_A^0$

There are 3 uses for a rotation matrix:

$$\begin{bmatrix} 0 & 1 & 0 \\ s_\theta & 0 & c_\theta \end{bmatrix}$$

Rotational Transformations:



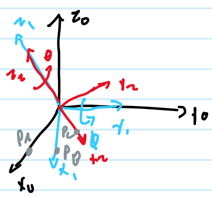
There are 3 uses for a rotation matrix:

1. Represents orientation of one frame w.r.t another $\quad R_A^B = [x_A^B \; y_A^B \; z_A^B]$
2. Relates coordinates of points and vectors between frames $\quad p^B = R_A^B p^A$
3. Rotational operator taking point/vector and rotates to create a new point/vector $\quad p_B^0 = R p_A^0$

Compound Rotations:

Two types:

1. about current frame



$R = R_{y,\phi} = R_1^0$

$p_B^1 = p_A^0$

$p_B^0 = R p_A^0$

$R = R_{z_1,\theta} = R_2^1$

$p_C^2 = p_B^1 = p_A^0$

$p_C^1 = R_2^1 p_C^2$

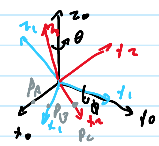
$p_C^0 = R_1^0 p_C^1$

$p_C^0 = R_1^0 R_2^1 p_A^0$

$= R_{y,\phi} R_{z,\theta} p_A^0$

$R_2^0 = R_1^0 R_2^1$

2. fixed frame



$p_C^2 = p_B^1 = p_A^0$

$p_C^0 = R_2^0 p_C^2$

$p_C^0 = R_1^0 p_C^1 = R_1^0 p_A$

$p_C^0 = R_{z,\theta} p_B^0$

$p_C^0 = R_{z,\theta} R_{y,\phi} p_A^0$

$R_2^0 = R R_1^0$

$R_2^0 = R_1^0 R_2^1$

Parameterization

3D rotation has 3 DOF, which are not unique

Euler Angles

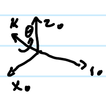
$R_{zyz} = R_{z,\phi} R_{y,\theta} R_{z,\psi}$

$$= \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta - c_\phi s_\psi & -s_\phi s_\theta s_\psi - c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Roll, Pitch, Yaw

$R = R_{x,\phi} \; R_{y,\theta} \; R_{x,\psi}$

Yaw $\quad$ Pitch $\quad$ Roll



$k = [k_x \quad k_y \quad k_z]^T$

$$R_{k,\theta} = \begin{bmatrix} k_x^2(1-c_\theta)+c_\theta & k_x k_y(1-c_\theta)-k_z s_\theta & k_x k_z(1-c_\theta)+k_y s_\theta \\ k_x k_y(1-c_\theta)+k_z s_\theta & k_y^2(1-c_\theta)+c_\theta & k_y k_z(1-c_\theta)-k_x s_\theta \\ k_x k_y(1-c_\theta)+k_y s_\theta & k_y k_z(1-c_\theta)+k_x s_\theta & k_z(1-c_\theta)+c_\theta \end{bmatrix}$$

Lab+Assignment+1

Tuesday, January 23, 2024 10:26 AM

Lab+Assignment+1

Laboratory Assignment #1 Intro to Robot Programing with Pick and Place Application

Safety Procedures While Operating Robot

1. **Under NO circumstances is anyone to be within the work cell while the robot is activated!**
2. There must be at least two people physically in the room whenever the robot is powered on.
3. All programs must first be verified by stepping through them at 10% speed before being executed.
4. The speed of the robot should never exceed 20%.
5. The TA must inspect any hardware or wiring changes before powering on the robot.
6. Do not adjust the robot axis limits.
7. No food or drink is allowed near the robot.

Reading Assignment (Robot Manual) and Learning Objectives

- Safety – Safety Section
- Robot Jogging and Jogging Frames – Chapter 2
- Basic Programing - Chapter 5 (pages 5-18 through 5-29)
- Robot Motion Commands - Chapter 6 (pages 6-12 through 6-15)
- Robot Digital I/O – Chapter 6 (pages 6-110 through 6-111)
- Position Register Offsets – Chapter 6 (page 6-42)

Part I (first week): Robot Jogging, Basic Programing, and Motion Commands

1. Read and understand the above safety rules, and the safety section in the robot manual.
2. Start the robot and turn on the TEACH Pendant (Procedure 2-1 on page 2-3).
3. Develop familiarity with the JOINT and WORLD coordinate systems by manually jogging the robot around within the work cell (Chapter 2).
4. Place two upright blocks at the first pick and place locations on the on white pallet (Figure 1).
5. Use procedure 5-1 on page 5-12 (starting at step 7) to open a new robot program with the following information:
    a. Program Name: "G#LAB1" where "#" is your group number.
    b. Sub Type: None
    c. Comments: point to point
    d. Write Protection: OFF
    e. Ignore pause: OFF
6. Continue using procedure 5-1 to create the following program:
    a. Manually jog the robot so that the arrow on the end effector is lined up about 1-2 cm above the first block (Figure 2).
    b. Record the point using the J P[] 100% FINE motion type.
    c. Repeat for point above the second block (Figure 2).
    d. Add a line to call Go_Home at the beginning and end of the program.
7. After creating your program, switch to step mode by selecting the STEP button (a green indicator light will show up on the left-hand side of the teach pendant). Verify that the robot's override speed is at 10%.
8. While pressing/holding the DEADMAN switch and shift button, step through the program one command at a time with the FWD button.
    a. Record the locations of the points in both the Joint and World coordinate systems. (Note: the position relative to various frames is found by pressing the POSN key on the pendant).
9. Change the motion type of both points to Linear and step through program once again.
10. Use tripod to place phone directly in front of pallet and record the program motion from step 6 and step 9.

Pseudo Code for Part I

1. Go_Home (note this is a program that your program will reference externally with the Call: instruction)
2. Point 1 (above first block)
3. Point 2 (above second block)
4. Go_Home

Table of Recorded Points

Joint	Axis 1	Axis 2	Axis 3	Axis 4	Axis 5	Axis 6
P1						
P2						

World	X	Y	Z	W	P	R
P1						
P2						

Part II (second week): Robot Digital I/O and Position Register Offsets

1. Delete points from previous program.
2. Reposition two blocks at the first and second pickup locations on the white pallet (Figure 3).
1. Manually jog to each pickup and drop off point and record their positions in your program. (Four total new positions P[1] – P[4]). The gripper only needs to contact the top half of the block, so it should never be closer than 1 cm above the table.
3. Add RO[1]=on from digital I/O instructions after pickup points.
4. Add RO[1]=off from digital I/O instructions after drop off points.
5. Incorporate WAIT 0.5 (sec) commands into your program to give the end effector time to open or close.
6. Setup a Position Register (PR[#]) in the data menu such that Z=200 mm and all other axis are 0 mm.
   a. Note: A position register is basically a variable that stores positional data in all six axes.
7. Use the WinTPE "transfer and edit procedure" to open your program in WinTPE.
8. Add a point before and after each of your points (P[1] – P[4])). Add an OFFSET,PR[#] motion instruction to the duplicated points. Pick the position register number you set up in step 1.
   b. Note: the point number (P[#]) of your offset point selects which point you are offsetting from.
9. Transfer your program back to the robot. (You may need to delete your original program on the teach pendant).
10. STEP through your program at 10% speed to make sure it operates properly. Then you can run the program at 20%.
11. Use tripod to record entire pick and place procedure.

Incomplete Code for Second Week

```
CALL GO_HOME
RO[1]=(on/off)
J P[1] 100% FINE Offset,PR[#]
L P[1] 2000mm/s FINE
RO[1]=(on/off)
WAIT 0.5 (sec)
L P[1] 2000mm/s FINE Offset,PR[#]
…
*insert missing code here*
…
J P[4] 100% FINE Offset,PR[#]
L P[4] 2000mm/s FINE
RO[1]=(on/off)
WAIT 0.5 (sec)
L P[4] 2000mm/s FINE Offset,PR[#]
CALL GO_HOME
```

Figures

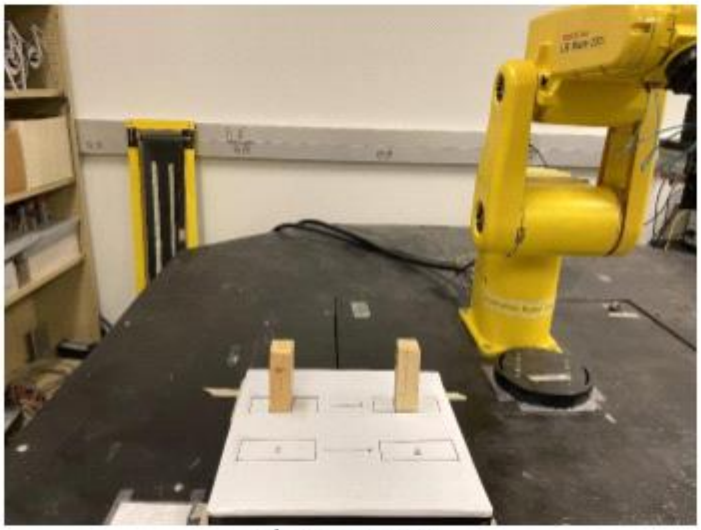


Figure 1: Part I Setup

Figure 2: Part I P[1] (left) and P[2] (right).

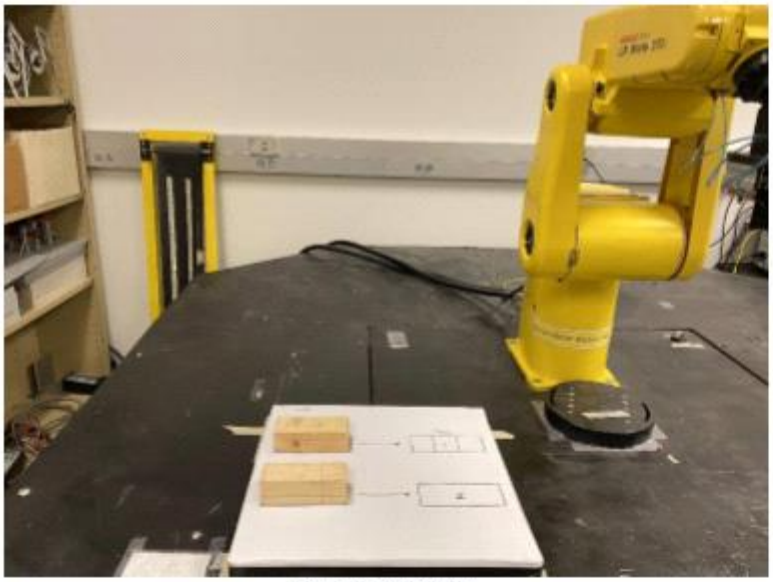


Figure 3: Part II Setup

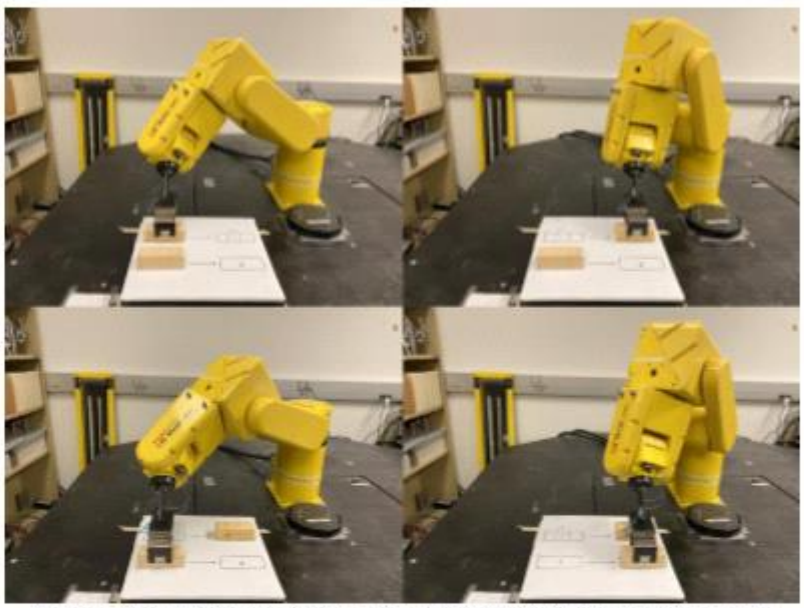


Figure 4: Part II P[1] (top left), P[2] (top right), P[3] (bottom left), and P[4] (bottom right).

<u>Laboratory Write-up and Submission</u>
Your report should include the following:

- The safely rules that must be followed while operating the robot.
- The coordinates for each of the points in the joint and world coordinate systems from Part I.
- A printout of your program from Part II (use WinTPE).
- Answers to the following questions (from lab experiments and reading):
    1. Describe the coordinate systems (Joint, World, Tool) that can be used for jogging.
    2. Explain how using different coordinate systems can make jogging to points easier.
    3. What are the available motion types and how do they differ (compare your videos from Part I)?
    4. What does RO[] stand for?
    5. How do the position register offsets modify the programmed position?
- Grading sheet with all sections listed on it.

Your canvas submission should include the following:

- Report
- Video of joint and linear motion types from Part I
- Video of pick and place operation from Part II

HW01

Monday, January 29, 2024 12:05 PM

HW01

AE/ME 5449
Homework 1: Introduction and Coordinate Transforms

Assigned: January 17, 2024 Due: January 31, 2024

Review the following concepts discussed in lecture:

1) Chapter 1: Introduction
2) Position and Rotation Transformations (2.1-2.4)

Work the following problems from the text. Unless otherwise stated, work all problem parts. Problems labeled (A.x) are additional problems of my own design.

Introductory Material
1-2
1-13
1-15
1-16
1-20

Transformations
2-6
2-10
2-11
2-15

(A.1) A point p is located at $p^0=[1\ 1\ 1]^T$ with respect to a world frame $o_0x_0y_0z_0$. A second frame $o_1x_1y_1z_1$ is defined by the sequence of rotations given in problem (2-10) with $\varphi = 15°$, $\theta=30°$, and $\psi=0°$. What is p^1?

1-2

Forward kinematics: Determine position and orientation of end effector in terms of joint variables

Inverse kinematics: Solve for joint variables in terms of x and y coordinates of a location

Trajectory planning: Determine a path in task space to move robot while avoiding collisions. Ignores timing

Workspace: Total volume accessible by end effector

accuracy: How close the manipulator can come to a given point within work space

Repeatability: How close the manipulator can return to a previously taught point

Resolution: Smallest increment of motion that a controller can sense

Joint variable: relative displacement between links
θ for revolute joints, d for prismatic

Spherical wrist: wrist whose three joint axis intersect at a common point

End effector: holds tool / performs task

1-13 Resolution $= d / 2^n \leftarrow$ number of bits

$d = \pi \cdot .5\ m = 1.5708\, m$

$$Res = \frac{1.5708}{2^8} = .006136\ m/bit$$

1-15 Repeatability is affected by mainly controller resolution because the encoder values are known. Accuracy errors are generally greater because it is affected by computational errors, backlash, rigidity, and other effects.

1-16 End point sensing can be used to close the control loop and sense errors between desired position and current position and correct. A popular method is vision-based control. This method increases robot cost and complexity, however.

1-20 Reducing the mass of distal links would decrease momentum of the link. This would result in less dynamic errors, less stress on gears, and reducing gravitational forces that motors have to fight against. Ways to do this include making motors lighter, but this reduces maximum possible torque or increases cost. You could reduce material used to construct links, but this decreases rigidity as well. Another method would be to use pulleys to move the mass of the motors closer to the base, but this introduces backlash into the system.

2-6 Verify Eqn. 2.4 - 2.6

$$R_{z,0} = R_1^0 = \begin{bmatrix} \cos 0 & -\sin 0 & 0 \\ \sin 0 & \cos 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$R_{z,\theta} R_{z,\phi} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta c\phi - s\theta s\phi & -c\theta s\phi - s\theta c\phi & 0 \\ s\theta c\phi + c\theta s\phi & -s\theta s\phi + c\theta c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Trig sum identities:

$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$$= \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= R_{z,\theta+\phi}$$

$$(R_{z,\theta})^{-1} = (R_{z,\theta})^T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Trig even/odd identities:

$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos\theta$

$$= \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= R_{z,-\theta}$$

2-10

$$R_{x,\phi} = R_1^0$$

$$R_{z,\theta} = R_2^1$$

$$R_2^0 = R_1^0 R_2^1$$

$$R_{y,\psi} = R_{32}$$

$$R_3^0 = R_{y,\psi} R_2^0$$

$$R_3^0 = R_{y,\psi}(R_1^0 R_2^1)$$

2-11 $R_{x,\phi} = R_1^0$

$$R_2^0 = R_{z,\theta} R_1^0$$

$$R_3^2 = R_{x,\psi}$$

$$R_3^0 = R_2^0 R_3^2$$

$$R_3^0 = (R_{z,\theta} \quad R_1^0) R_3^2$$

2-15

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, \quad R_3^1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R_3^1 = R_2^1 R_3^2$$

$$R_3^2 = (R_2^1)^{-1} R_3^1 = (R_2^1)^T R_3^1$$

$$(R_1^2)^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$R_3^2 = \begin{bmatrix} 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \end{bmatrix}$$

A-1

$$P_0 = [1 \quad 1 \quad 0]^T$$

$$\phi = 15^\circ$$
$$\theta = 30^\circ$$
$$\psi = 0$$

$$R_1^0 = R_{x,\phi} \quad R_{z,\theta}$$

$$p^1 = R_1^0 p^0$$

$$R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \times \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta & s\theta & 0 \\ -c\phi s\theta & c\phi c\theta & -s\phi \\ -s\phi s\theta & s\phi c\theta & c\phi \end{bmatrix}$$

$$p^1 = \begin{bmatrix} c\theta & s\theta & 0 \\ -c\phi s\theta & c\phi c\theta & -s\phi \\ -s\phi s\theta & s\phi c\theta & c\phi \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

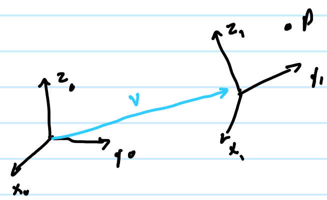
$$p' = \begin{bmatrix} c\theta + s\theta \\ -c\phi s\theta + c\phi c\theta - s\phi \\ -s\phi s\theta + s\phi c\theta + c\phi \end{bmatrix}$$

$$p' = \begin{bmatrix} 1.36603 \\ .094734 \\ 1.06066 \end{bmatrix}$$

Rigid Motions

Wednesday, January 31, 2024 8:29 AM

A Pure translation and a pure rotation



Given p^1

find p^0

$$v^0 = O_1^0$$

$$p^0 R_1^0 p^1 + O_1^0$$

$$R_1^0 = [\, x_1^0 \quad y_1^0 \quad z_1^0 \,]$$

Homogeneous Transformations

$$H = \begin{bmatrix} R^{3\times3} & d^{3\times1} \\ \underset{1\times3}{0} & \underset{1\times1}{1} \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} R_1^0 & O_1^0 \\ 0 & 1 \end{bmatrix} = \begin{matrix} x_1^0 & y_1^0 & z_1^0 & O_1^0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$H_2^1 = \begin{bmatrix} R_2^1 & O_2^1 \\ 0 & 1 \end{bmatrix}$$

$$H_1^0 H_2^1 = \begin{bmatrix} R_1^0 R_2^1 & R_1^0 O_2^1 + O_1^0 \\ 0 & 1 \end{bmatrix} = H_2^0$$

$$H_A^0 = (H_0^A)^{-1} \neq H^T$$

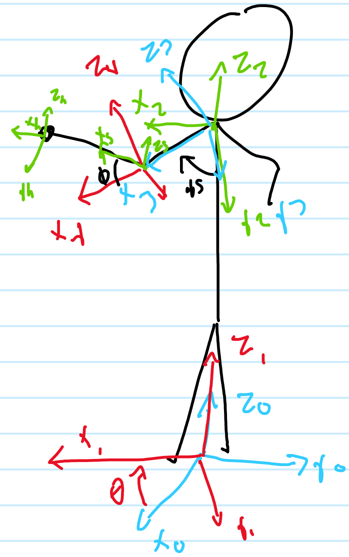
$$H^{-1} = \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix}$$

Basic Homogeneous Transformations

$$\text{Trans}_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha & 0 \\ 0 & s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ex.



$$H_1^0 = Rot_{z,\theta}$$

$$H_2^1 = Trans_{z,5}$$

$$H_3^2 = Rot_{y,45}$$

$$H_4^3 = Trans_{x,1}$$

$$H_5^4 = Rot_{y,-\phi}$$

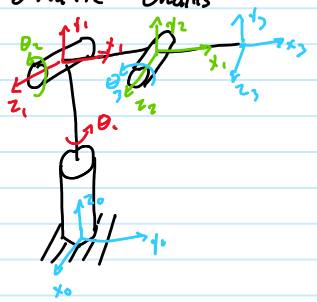
$$H_6^5 = Trans_{x,1}$$

$$H_6^0 = H_1^0 \, H_2^1 \, H_3^2 \, H_4^3 \, H_5^4 \, H_6^5$$

Forward Kinematics

Wednesday, February 7, 2024 7:06 AM

Kinematic Chains



A robot with n joints has n+1 links

Joint variable $= q_i$

$$H_i^{i-1} = A_i = A_i(q_i)$$

$$T_j^i = \begin{cases} A_{i+1} A_{i+2} \cdots A_j & \text{if } i<j \\ I & \text{if } i=j \\ (T_i^j)^{-1} & \text{if } i>j \end{cases}$$

$$H = T_n^0 = A_1(q_1)\, A_2(q_2) \dots A_n(q_n)$$

Denavit - Hartenberg Convention

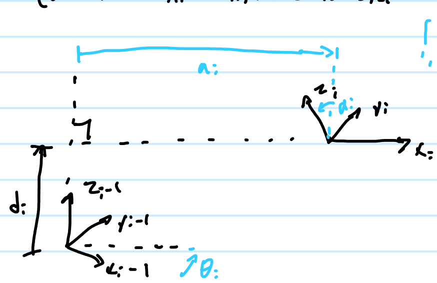
Represent joint transformations $i \to i+1$ as 4 variables (6 deg. of freedom)

D-H convention:

z_{i-1} in direction of q_i

(DH1) $x_i \perp z_{i-1}$

(DH2) x_i intersects z_{i-1}



$\theta_i =$ angle from x_{i-1} to x_i measured around z_{i-1}

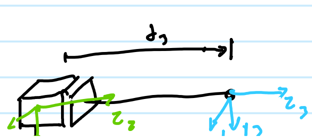
$d_i =$ distance from O_{i-1} to O_i along z_{i-1}

$a_i =$ distance from O_{i-1} to O_i along x_i

$\alpha_i =$ angle from z_{i-1} to z_i around x_i

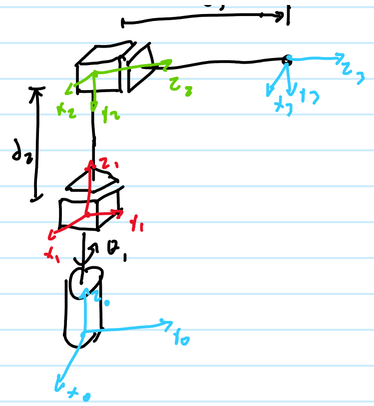
$$A_i = Rot_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, Rot_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



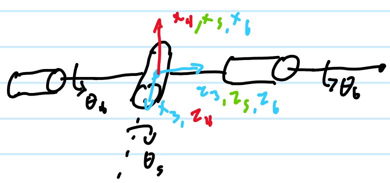
$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$

Ex



Link	θ_i	d_i	a_i	α_i
1	θ_1	d_1	0	0
2	0	d_2	0	$-\frac{\pi}{2}$
3	0	d_3	0	0

Ex



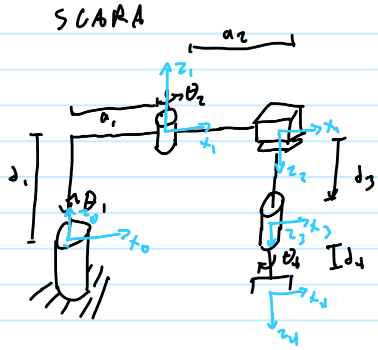
Link	θ_i	d_i	a_i	α_i
4	$\theta_4 - \frac{\pi}{2}$	0	0	$-\pi/2$
5	$-\theta_5$	0	0	$\pi/2$
6	θ_6	0	0	0

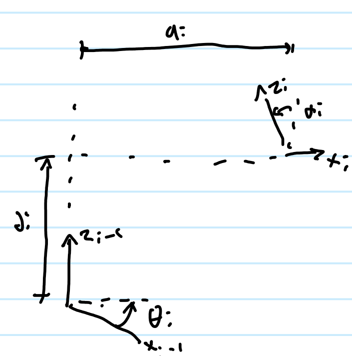
$$A_4 = Rot_{z,\theta - \frac{\pi}{2}} \; Rot_{x,-\pi/2}$$

$$A_5 = Rot_{z,-\theta_5} \; Rot_{x,\pi/2}$$

$$A_6 = Rot_{z,\theta_6}$$

Ex. SCARA





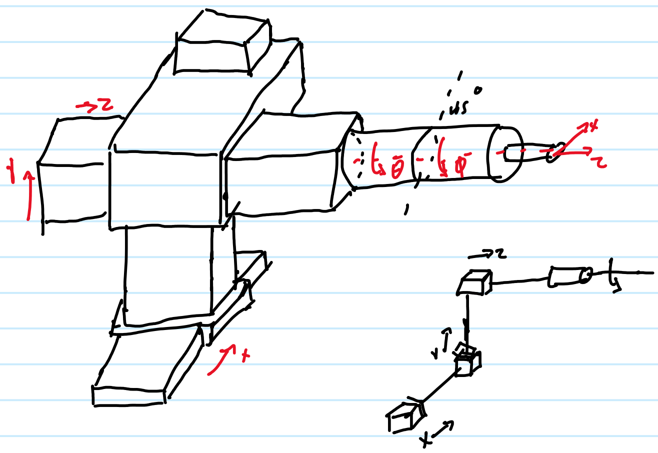
$$A_i = Rot_{z,\theta_i} \; Trans_{z,d_i} \; Trans_{x,a_i} \; Rot_{x,\alpha_i}$$

Link	θ_i	d_i	a_i	α_i
1	θ_1	d_1	a_1	0
2	θ_2	0	a_2	π
3	0	d_3	0	0
4	θ_4	d_4	0	0

$A_3 = T_3^2 = T_{z,d_3}$

$A_4 = T_4^3 = Rot_{z,\theta_4} T_{z,d_4}$

Ex.



Lab+Assignment+2

Wednesday, February 7, 2024 4:02 PM

Lab+Assignment+2

Laboratory Assignment #2 Trajectory Planning

Safety Procedures While Operating Robot

1. **Under NO circumstances is anyone to be within the work cell while the robot is activated!**
2. There must be at least two people physically in the room whenever the robot is powered on.
3. All programs must first be verified by stepping through them at 10% speed before being executed.
4. The speed of the robot should never exceed 20%.
5. The TA must inspect any hardware or wiring changes before powering on the robot.
6. Do not adjust the robot axis limits.
7. No food or drink is allowed near the robot.

Reading Assignment (Robot Manual) and Learning Objectives

- Termination Type (6-37 through 6-38 and 6-55)
- Speed (6-27 through 6-36)
- Trajectory Planning (6-53)

Part I (first week): Continuous Termination Type

1. Print out four Lab 2 Experiment Worksheets.
2. Place and tape down a "Lab 2 Experiment Worksheet" on the white pallet in the specified location (**Figure 1**). Label this worksheet "Fine and Continuous Termination Types".
3. Create a new program named G#Lab2.
4. Add a line to call the GO_HOME program at the beginning and end of the program.
5. Jog the robot to the three labeled points (P[1], P[2], and P[3] in **Figure 2**) with the pen pushed slightly into the paper, and the front face of the end effector in the same direction (e.g. Facing forward).
    a. Use the LINEAR motion type for all three points.
    b. Initially, use the FINE termination type.
    c. Set P[1] to 2000 mm/s.
    d. Set P[2] and P[3] to 200 mm/s
    e. Note: All points should be at approximately the same height.
6. Duplicate P[1] and P[3] and add position register offsets to the duplicates (before P[1] and after P[3]). Use the position registers you created in Lab 1. Verify that they have not been changed.
7. Step through your program at 10% speed to insure it runs safely. It should start at home, move to an offset P[1], draw an "L" on the worksheet, move to an offset P[3], and return to home.
8. Once the program motion has been verified, turn off step mode.
9. Run your program at 20% speed.
10. Change the termination type of P[2] and P[3] (without offset) to CNT0.
11. With the same piece of paper, run the program several more times for continuous termination types: CNT0, CNT20, CNT40, CNT60, CNT80, and CNT100. Label each curve with the termination type used. Note any trends you see.
12. Use the calipers to measure the deviation between the corner and the curve where it intersects the black dashed line (**Figure 3**) and record in the provided table.
13. Once measured, compute the difference between the curve intersections (CNT20-CNT0, CNT40-CNT20, …, etc.) and record in the provided table.

Part II (first week): Corner Distance Termination Type

1. Place and tape down a new "Lab 2 Experiment Worksheet" over the first worksheet. Label this worksheet "Corner Distance Termination Type"
2. Change the termination type of P[2] and P[3] (without offset) to CD0
3. With the new worksheet, run the program several times for corner distance termination types: CD0, CD5, CD10, CD15, and CD20. Label each curve with the termination type used. Note any trends you see.

14\. Use the calipers to measure the deviation between the corner and the curve where it intersects the black dashed line (**Figure 3**) and record in the provided table.

4\. Once measured, compute the difference between the curve intersections (CD5-CD0, CD10-CD5, …, etc.).

Part III (second week): Program Speed

1. Place and tape down a new "Lab 2 Experiment Worksheet" on the white pallet in the specified location (**Figure 1**). Label this worksheet "Program Speed"
2. Re-teach points P[1], P[2], and P[3] (**Figure 2**) if necessary (use the touch-up feature on the teach pendant).
3. Change the termination type of P[2] and P[3] (without offset) to CNT100 and set the program speed to 100mm/s.
4. With the new worksheet, run the program several times for program speeds: 100mm/s, 200mm/s, 300mm/s, 400mm/s, 500mm/s, 600mm/s, 1000mm/s, and 2000mm/s. Label each curve with the program speed used. Note any trends you see.
5. Use the calipers to measure the deviation between the corner and the curve where it intersects the black dashed line (**Figure 3**) and record in the provided table.

Part IV (second week): Override Speed

1. Place and tape down a new "Lab 2 Experiment Worksheet" over the second worksheet. Label this worksheet "Override Speed"
2. Change the termination type of P[2] and P[3] (without offset) to CNT100 and set the program speed to 200mm/s.
3. With the new worksheet, run the program several more times for override speeds (± arrows on teach pendant): 5%, 10%, 15%, and 20% without changing the program speed. Label each curve with the program speed used. Note any trends you see.
4. Use the calipers to measure the deviation between the corner and the curve where it intersects the black dashed line (**Figure 3**) and record in the provided table.

Pseudo Code (all parts)

1. Go_Home
2. Point 1 (joint move with position register offset)
3. Point 1 (linear move, always with FINE termination type)
4. Point 2 (linear move)
5. Point 3 (linear move)
6. Point 3 (linear move with position register offset and always with FINE termination type)
7. Go_Home

Table of Measured Distances and Differences

Termination Type	**CNT0**	**CNT20**	**CNT40**	**CNT60**	**CNT80**	**CNT100**
Deviation (mm)						
Difference (mm)	N/A					

Termination Type	**CD0**	**CD5**	**CD10**	**CD15**	**CD20**
Deviation (mm)					
Difference (mm)	N/A				

Program Speed (mm/s)	**100**	**200**	**300**	**400**	**500**	**600**	**1000**	**2000**
Deviation (mm)								

Override Speed (%)	**5**	**10**	**15**	**20**
Deviation (mm)				

Figures

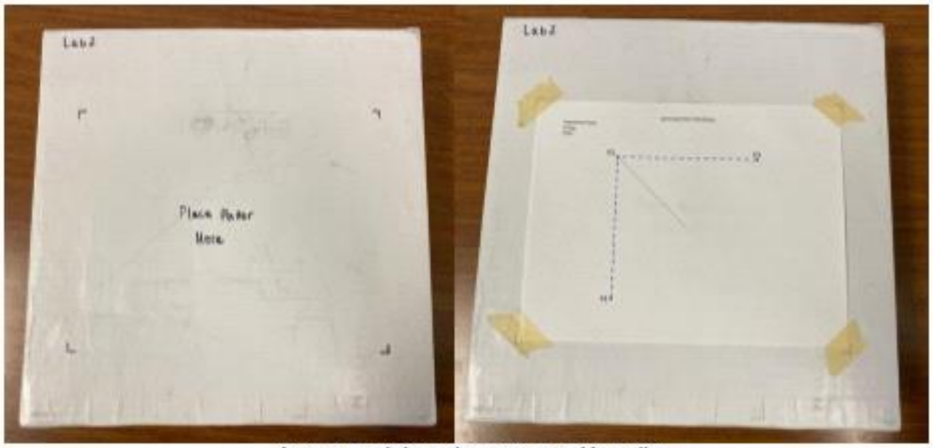


Figure 1: Worksheet Placement on White Pallet

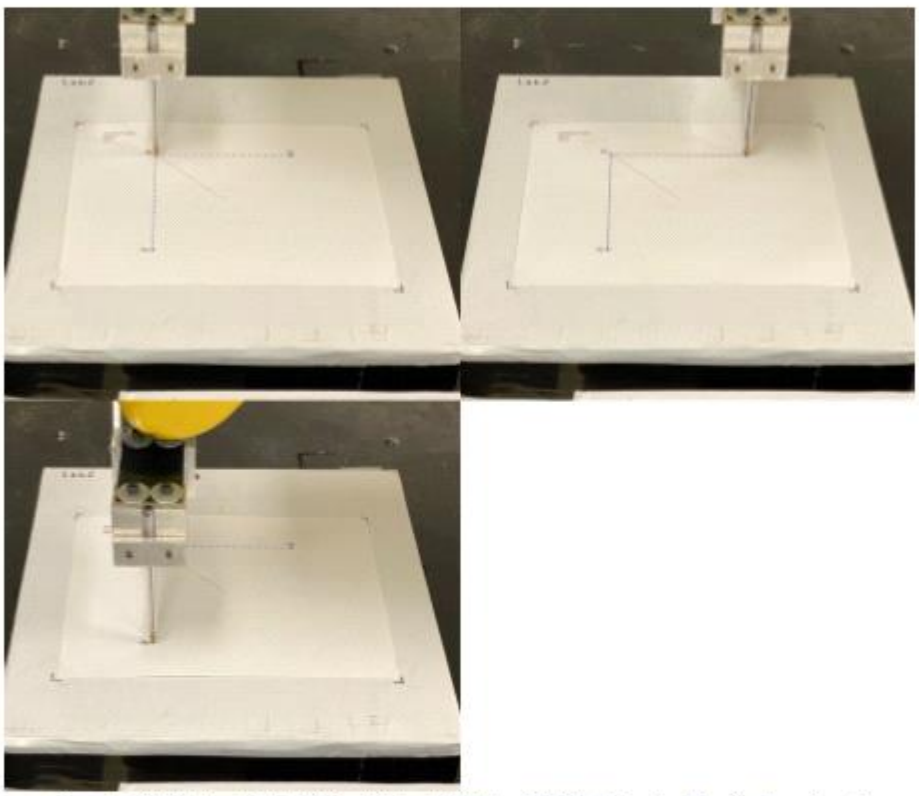


Figure 2: P[1] (bottom left), P[2] (top left), and P[3] (top right) for Trajectory Planning Experiments

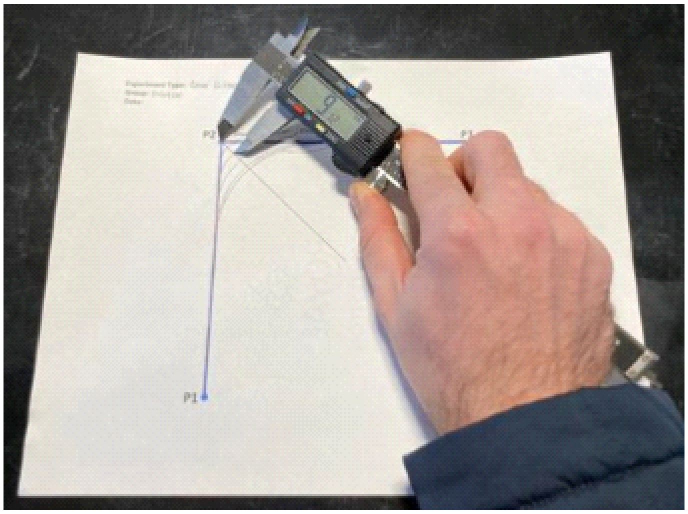


Figure 3: How to Measure Deviation of Trajectory Planning Experiments

<u>Laboratory Write-up and Submission</u>
Your report should include the following:

- A printout of your program from Part IV (use WinTPE).
- The deviation and difference tables from each of the four experiments.
- Your "Lab 2 Experiment Worksheet" for each of the four experiments.
- A Deviation vs. Speed plot of the data you collected in Part III.
- Answers to the following questions (from lab experiments and reading):
  1. Explain in detail what each item in the following line of code means (use the manual).

     ```
     2:  J PR[1] R[30]% CNT50
     ```

  2. Is the continuous termination type the same as the corner rounding termination type? Explain (use manual).
  3. How does the corner rounding termination type number (CD#) correlate to the deviation and difference data collected in the table? What does this say about the robot's accuracy and repeatability?
  4. In Parts III and IV, you changed the speed that the robot moved through the taught trajectory; how do these methods (program vs override speed) affect the robot's planning of the taught trajectory?
  5. What kind of trend(s) do you see in your Deviation vs Speed plot? Do they match what you would expect? If not, explain what you think the cause is (hint on 6-53)?
- Grading sheet with all sections listed on it.

Your canvas submission should include the following:

- Report (no videos this week)

HW02

Saturday, February 10, 2024 12:57 PM

HW02

AE/ME 5449

Homework 2: Rigid Motions and D-H Convention

Assigned: January 31, 2024 Due: February 14, 2024

Review the following concepts discussed in lecture:

1) Rotation Parameterizations and Rigid Motions (2.5-2.6)
2) Denavit-Hartenberg Convention (Chapter 3)

Work the following problems from the text. Unless otherwise stated, work all problem parts. Problems labeled (A-x) are additional problems of my own design.

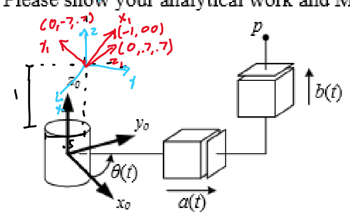
<u>Homogeneous Transformations</u>

2-38

2-40

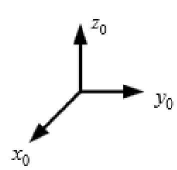
$H_1^0 = R_{ot\,z,\theta(t)} T_{rans\,x,a(t)} T_{rans\,z,b(t)}$

(A-1) Consider the robot shown in the schematic below. The joints follow the trajectories, $\theta(t) = 90t$ (in degrees), $a(t) = t(20-t)$, and $b(t) = t$. Use MATLAB (or any other software) to plot the trajectory of p^0 for $0 \le t \le 20$ seconds. Please show your analytical work and MATLAB code.



(A.2) Given the transformation below, sketch frame 1.

$$T_1^0 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -0.7 & 0.7 & 1 \\ 0 & 0.7 & 0.7 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

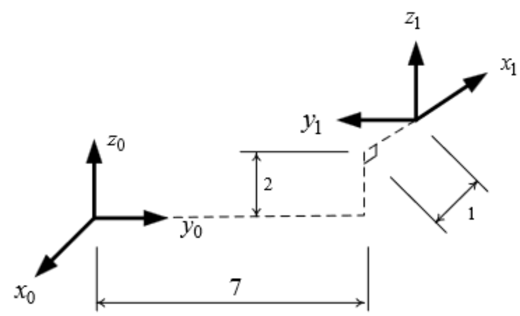


(A.3) The transformation of Frame A with respect to Frame B is given below. What is the transformation of Frame B with respect to Frame A?

$$T_A^B = \begin{bmatrix} 0 & 0 & 1 & 2 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Additional Problems on next page)

(A.4) Two frames are shown below. Find T_1^0



D-H Convention
3-1
3-5
3-7

2-38

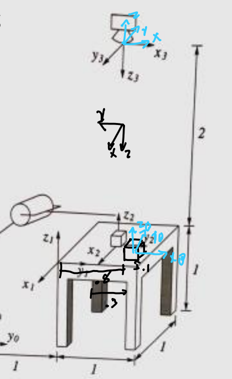


Figure 2.14: Diagram for Problem 2-38.

$$H_1^0 = Trans_{y,1}\, Trans_{z,1}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = Trans_{y,1.5}\, Trans_{x,-.5}\, Trans_{z,1}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = Trans_{y,1.5}\, Trans_{x,-.5}\, Trans_{z,.3}\, Rot_{x,\pi}\, Rot_{z,-\pi/2}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_\pi & -S_\pi & 0 \\ 0 & S_\pi & C_\pi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{-\pi/2} & -S_{-\pi/2} & 0 & 0 \\ S_{-\pi/2} & C_{-\pi/2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & -.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = Rot_{x,\pi}\, Rot_{z,-\pi/2}$$

$$= \begin{bmatrix} C_{-\pi/2} & -C_{\pi/2} & 0 & 0 \\ C_\pi S_{-\pi/2} & C_\pi C_{-\pi/2} & -S_\pi & 0 \\ S_\pi S_{-\pi/2} & S_\pi C_{-\pi/2} & C_\pi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.40

Block to camera

$$H_0^3 = Rot_{x,\pi}\, Trans_{z,-2}\, Trans_{z,.1}\, Trans_{x,.5}\, Trans_{y,.3}$$

$$\ldots \quad Trans_{z,-1.9} \ldots$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_\pi & -S_\pi & 0 \\ 0 & S_\pi & C_\pi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & .3 \\ 0 & 1 & 0 & -.5 \\ 0 & 0 & 1 & -1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & .3 \\ 0 & -1 & 0 & .5 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Block to base

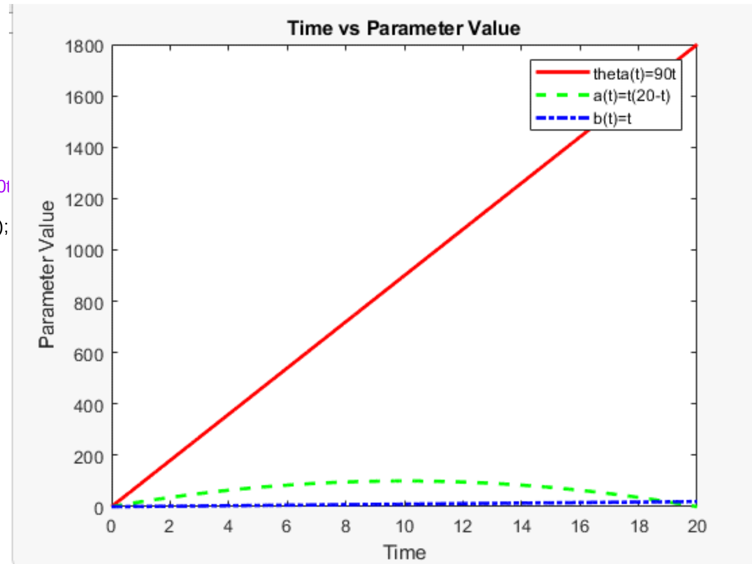
$$H_B^0 = Trans_{y,1}\, Trans_{z,1}\, Trans_{y,.8}\, Trans_{z,.1}\, Rot_{z,\pi/2}$$

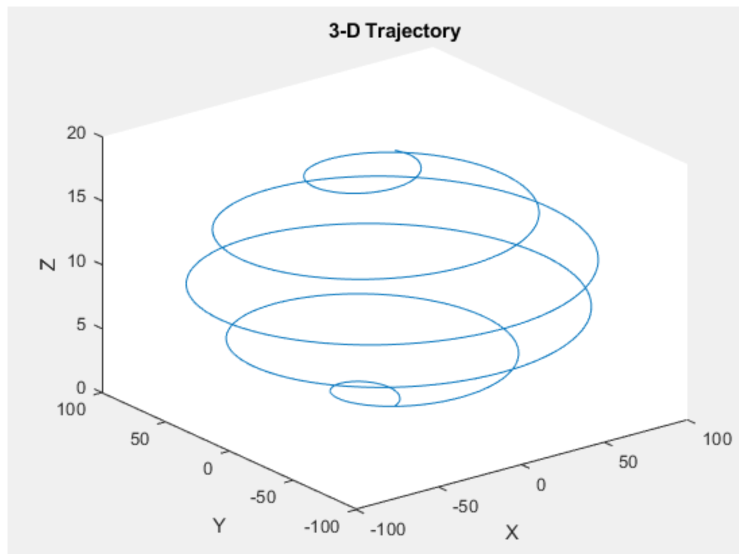
$$= Trans_{y,1.8}\, Trans_{z,1.1}\, Rot_{z,\pi/2}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1.8 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{\pi/2} & -S_{\pi/2} & 0 & 0 \\ S_{\pi/2} & C_{\pi/2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1.8 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
clc
clear
t = 0:0.01:20;  % time vector
theta = 90*t;
a = t.*(20-t);
b=t;
figure;
plot(t,theta,'r-', 'LineWidth', 2, 'DisplayName', 'theta(t)=90t
hold on;
plot(t, a, 'g--', 'LineWidth', 2, 'DisplayName', 'a(t)=t(20-t)');
plot(t, b, 'b-.', 'LineWidth', 2, 'DisplayName', 'b(t)=t');
title('Time vs Parameter Value')
xlabel('Time')
ylabel('Parameter Value')
legend('show')
hold off

%% 3-D Trajectory
% p is fixed to frame h
ph = [0 0 0 1]';
% p0 is the [x y z]' location of p w.r.t. frame 0
p0 = zeros(4,length(t));    % initialize as zeros
for i=1:length(t)
    p0(:,i) = RotZ(theta(i))*TransX(a(i))*TransZ(b(i))*ph;
end
p0x = p0(1,:);  % time series for p0's x position
p0y = p0(2,:);  % time series for p0's y position
p0z = p0(3,:);  % time series for p0's z position
figure;
plot3(p0x,p0y,p0z)
title('3-D Trajectory')
xlabel('X')
ylabel('Y')
zlabel('Z')


%% Trying to make the pretty animations
h1=figure(1);
h2=figure(2);
pause
for i=2:length(t)
    figure(h1)
    title("Top Down Trajectory")
    plot3(p0x(1:i),p0y(1:i),p0z(1:i),'b')



    xlabel('X')
    ylabel('Y')
    view(2)
    hold off
    figure(h2)
    plot3(p0x(1:i),p0y(1:i),p0z(1:i),'b')
    view(3)
    hold on
    pause(0.0001);
end
```





A.7

$$T_B^A = \left(T_A^B\right)^{-1} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

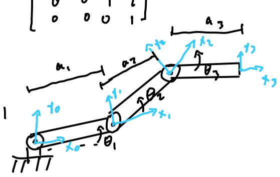
A.4

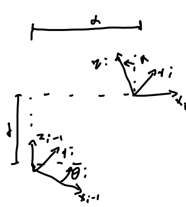
$$T_1^0 = \mathrm{Trans}_{y,7}\ \mathrm{Trans}_{z,2}\ \mathrm{Trans}_{x,-1}\ \mathrm{Rot}_{z,\pi}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ & 1 & 0 & 7 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\pi & -s_\pi & 0 & 0 \\ s_\pi & c_\pi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.1



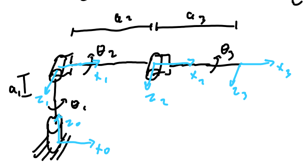


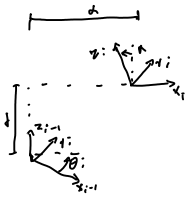
$$A_i = \mathrm{Rot}_{z,\theta_i}\ \mathrm{Trans}_{z,d_i}\ \mathrm{Rot}_{x,\alpha_i}\ \mathrm{Trans}_{x,a_i}$$

Link	θ_i	d_i	α_i	a_i
1	θ_1	0	0	a_1
2	θ_2	0	0	a_2
3	θ_3	0	0	a_3

$$T_3^0 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3-5





$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Rot_{x,\alpha} Trans_{x,a_i}$

Link	θ_i	d_i	α_i	a_i
1	θ_1	a_1	$\pi/2$	0
2	θ_2	0	0	a_2
3	θ_3	0	0	a_3

$T_3^0 = A_1A_2A_3$

$= (Rot_{z,\theta_1} Trans_{z,a_1} Rot_{x,\pi/2})(Rot_{z,\theta_2} Trans_{x,a_2})(Rot_{z,\theta_3} Trans_{x,a_3})$

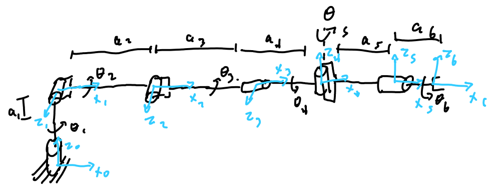
$$= \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3c_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

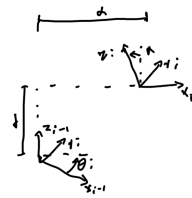
```
clear
clc
syms th1 th2 th3 a1 a2 a3

A1=[cos(th1) -sin(th1) 0 0 ;sin(th1) cos(th1) 0 0;0 0 1 0 ;0 0 0 1]*[1 0 0 0 ;0 1 0 0;0 0 1 a1 ;0 0 0 1]*[1 0 0 0 ;0 cosd(90) -sind(90) 0;0 sind(90) cosd(90) 0 ;0 0 0 1]
A2= [cos(th2) -sin(th2) 0 0 ;sin(th2) cos(th2) 0 0;0 0 1 0 ;0 0 0 1]*[1 0 0 a2 ;0 1 0 0;0 0 1 0 ;0 0 0 1]
A3= [cos(th3) -sin(th3) 0 0 ;sin(th3) cos(th3) 0 0;0 0 1 0 ;0 0 0 1]*[1 0 0 a3 ;0 1 0 0;0 0 1 0 ;0 0 0 1]
T=A1*A2*A3
```

Command Window

```
A1 =

[cos(th1), 0,  sin(th1),  0]
[sin(th1), 0, -cos(th1),  0]
[       0, 1,         0, a1]
[       0, 0,         0,  1]


A2 =

[cos(th2), -sin(th2), 0, a2*cos(th2)]
[sin(th2),  cos(th2), 0, a2*sin(th2)]
[       0,         0, 1,           0]
[       0,         0, 0,           1]


A3 =

[cos(th3), -sin(th3), 0, a3*cos(th3)]
[sin(th3),  cos(th3), 0, a3*sin(th3)]
[       0,         0, 1,           0]
[       0,         0, 0,           1]


T =

[cos(th1)*cos(th2)*cos(th3) - cos(th1)*sin(th2)*sin(th3), - cos(th1)*cos(th2)*sin(th3) - cos(th1)*cos(th3)*sin(th2),  sin(th1), a2*cos(th1)*cos(th2) + a3*cos(th1)*cos(th2)*cos(th3) - a3*cos(th1)*sin(th2)*sin(th3)]
[cos(th2)*cos(th3)*sin(th1) - sin(th1)*sin(th2)*sin(th3), - cos(th2)*sin(th1)*sin(th3) - cos(th3)*sin(th1)*sin(th2), -cos(th1), a2*cos(th2)*sin(th1) + a3*cos(th2)*cos(th3)*sin(th1) - a3*sin(th1)*sin(th2)*sin(th3)]
[                     cos(th2)*sin(th3) + cos(th3)*sin(th2),                       cos(th2)*cos(th3) - sin(th2)*sin(th3),         0,                     a1 + a2*sin(th2) + a3*cos(th2)*sin(th3) + a3*cos(th3)*sin(th2)]
[                                                        0,                                                          0,         0,                                                                                  1]
```

3-7





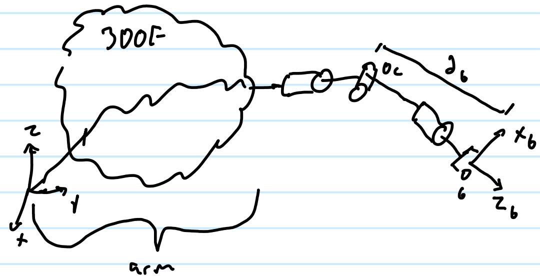
$A_i = Rot_{z,\theta_i} \, Trans_{z,d_i} \, Rot_{x,\alpha_i} \, Trans_{x,a_i}$

Link	θ_i	d_i	α_i	a_i
1	θ_1	a_1	$\pi/2$	0
2	θ_2	0	0	a_2
3	θ_3	0	0	a_3
4	$-\pi/2$	0	θ_4	a_4
5	0	0	θ_5	a_5
6	0	0	θ_6	a_6

Inverse Kinematics

Wednesday, February 14, 2024 8:38 AM

6 DOF robot w/ spherical wrist



Given H, or

O - desired end effector position

R - desired end effector orientation

Note that $o^6 = o_c^0 + R\begin{bmatrix}0\\0\\d_6\end{bmatrix} \Rightarrow o_c = o^6 - R\begin{bmatrix}0\\0\\d_6\end{bmatrix}$

$$\begin{bmatrix}x_c\\y_c\\z_c\end{bmatrix} = \begin{bmatrix}o_x - r_{13}d_6\\o_y - r_{23}d_6\\o_z - r_{33}d_6\end{bmatrix}$$

1. Use $[x_c \; y_c \; z_c]^T$ to find $q_1 \, q_2 \, q_3$ of the arm

Solve $T_3^0(q_1, q_2, q_3) = A_1(q_1)A_2(q_2)A_3(q_3)$

$= \begin{bmatrix} R_3^0(q_1, q_2, q_3) & d_3^0(q_1, q_2, q_3) \end{bmatrix}$

$R_6^0 = R_3^0 R_6^3 = R$

$R_6^3 = (R_3^0)^T R$

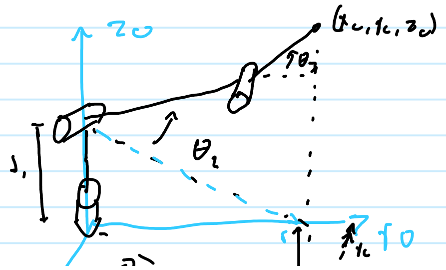
2. Euler ZYZ

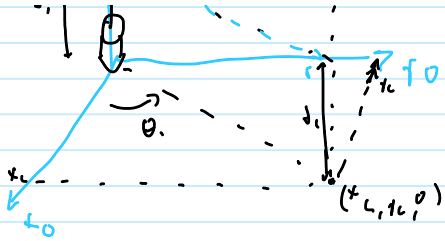
Solve inverse

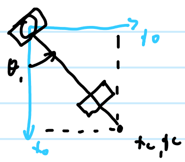
$\rightarrow \theta_4$

θ_5

θ_6

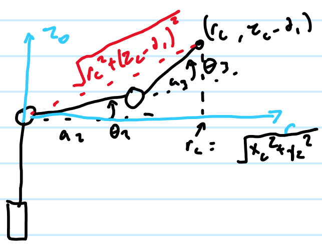




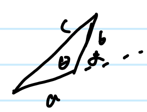


$$\text{atan2}(y, x) = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right), & \text{if } x > 0 \\ 180 - \tan^{-1}\left(\frac{y}{x}\right), & \text{if } x < 0 \end{cases}$$

$$\boxed{\begin{array}{ll} \theta_1 = \text{atan2}(x_c, y_c) & \text{forward facing} \\ \quad \text{or} & \\ \quad 180 + \text{atan2}(x_c, y_c) & \text{backward facing} \\ \quad \text{or} & \\ \quad x_c = y_c = 0 \quad \text{then} \quad \theta_1 = \text{anything} & \end{array}}$$



Law of Cosines



$c^2 = a^2 + b^2 - 2ab\cos\theta$

$\cos\alpha = \cos(180 - \theta)$

$= \cos 180 \cos(-\theta) - \sin(\cancel{180})\sin(-\theta)$

$= -\cos\theta$

$c^2 - a^2 - b^2 = 2ab\cos\alpha$

$$\cos\alpha = \frac{c^2 - a^2 - b^2}{2ab}$$

$$\cos\theta_3 = \frac{r_c^2 + (z_c - d_1)^2 - a_2^2 - a_3^2}{2a_2 a_3}$$

$$= \frac{x_c^2 + y_c^2 + (z_c - d_1)^2 - a_2^2 - a_3^2}{2a_2 a_3} = D$$

$$\boxed{\theta_3 = \text{atan2}\left(D, \pm\sqrt{1 - D^2}\right)}$$

$$\boxed{\theta_2 = \text{atan2}\left(\sqrt{x_c^2 + y_c^2}, z_c - d_1\right) - \text{atan2}\left(a_2 + a_3\cos\theta_3, a_3\sin\theta_3\right)}$$

4 solutions:

fwd facing elbow down
fwd facing elbow up
backward facing elbow down

fwd facing elbow down
fwd facing elbow up
backward facing elbow down
backward facing elbow up

$R_3^0 R_6^3 = R$

now known

$R_3^6 = (R_3^0)^T R$

$$R_3^6 = \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 \\ -s_5 c_6 & s_5 s_6 & c_5 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Case I $\quad r_{13}$ & r_{23} are not both zero

$\Downarrow$

$s_5 \neq 0$

r_{31} & r_{32} are not both zero

$\Downarrow$

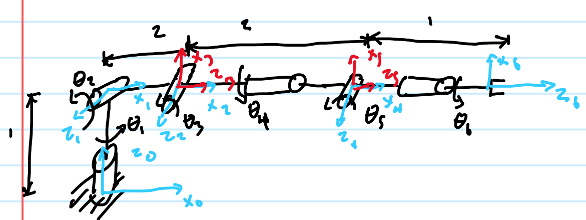
$r_{33} \neq \pm 1$

$c_5 = r_{33}, \quad s_5 = \pm\sqrt{1 - r_{33}^2}$

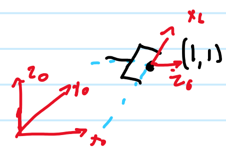
$+ \qquad -$

positive wrist	negative wrist
$\theta_5 = \text{atan2}(r_{33}, \sqrt{1-r_{33}^2})$	$\theta_5 = \text{atan2}(r_{33}, -\sqrt{1-r_{33}^2})$
$\theta_6 = \text{atan2}(-r_{31}, r_{32})$	$\theta_6 = \text{atan2}(r_{31}, -r_{32})$
$\theta_4 = \text{atan2}(r_{13}, r_{23})$	$\theta_4 = \text{atan2}(-r_{13}, -r_{23})$

Complete example:



Find all θs to grab an item @ $(x_0, y_0) = (1,1)$ on the table approaching from x_0 direction



$d = O_6^0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad R_6^0 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$O_6 = O_c + R\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = O_c + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$O_c = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$D = \dfrac{x_c^2 + y_c^2 + (z_c - d_1)^2 - a_2^2 - a_3^2}{2 a_2 a_3} = -\dfrac{3}{4} \qquad |D| < 1$ to be within reach ✓

$$D = \frac{x_c^2 + y_c^2 + (z_c - d_1)^2 - a_2^2 - a_3^2}{2a_2a_3} = -\frac{3}{4}$$

$|D| < 1$ to be within reach ✓

✓ choice

$x_c = 0$ $\quad d_1 = 1$

$y_c = 1$ $\quad a_2 = 2$

$z_c = 0$ $\quad a_3 = 2$

Pick forward facing elbow up

$\theta_1 = \text{atan2}(x_c, y_c) = \text{atan2}(0, 1) = \boxed{90°}$

$\theta_3 = \text{atan2}(D, -\sqrt{1-D^2}) = \text{atan2}\left(-\frac{3}{4}, -\sqrt{1-\left(\frac{3}{4}\right)^2}\right) = \boxed{-139°}$

↖ elbow up

$\theta_2 = \text{atan2}\left(\sqrt{x_c^2 + y_c^2}, z_c - d_1\right) - \text{atan2}(a_2 + a_3\cos\theta_3, a_3\sin\theta_3) = \boxed{24°}$

$R_6^0 = R_3^0 R_6^3$

$R_3^0 = A_1(90) A_2(24) A_3(-139)$

$$R_3^0 = \begin{bmatrix} 0 & 1 & 0 \\ .9 & 0 & -.4 \\ -.4 & 0 & -.9 \end{bmatrix}$$

$R_6^3 = (R_3^0)^T R_6^0$

$$R_6^3 = \begin{bmatrix} .9 & -.4 & 0 \\ 0 & 0 & 1 \\ -.4 & -.9 & 0 \end{bmatrix}$$

Pick positive wrist

$\theta_5 = \text{atan2}(r_{33}, \sqrt{1-r_{33}^2}) = \text{atan2}(0, 1) = \boxed{90°}$

$\theta_4 = \text{atan2}(r_{13}, r_{23}) = \text{atan2}(0, 1) = \boxed{90°}$

$\theta_6 = \text{atan2}(-r_{31}, r_{32}) = \text{atan2}(+.4, -.9) = \boxed{-66°}$

Velocity Kinematics

Friday, February 23, 2024 8:40 AM

Skew Matricies

- A matrix is a skew symmetric iff

$S + S^T = 0$

$\Rightarrow s_{ij} = -s_{ji}$

and $j=i$

$s_{ii} = 0$

$$\begin{bmatrix} 0 & -s_3 & s_2 \\ s_3 & 0 & -s_1 \\ -s_2 & s_1 & 0 \end{bmatrix}$$

Given $a = [a_x \ a_y \ a_z]^T$

$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Properties:

$\alpha, \beta \in \mathbb{R}$; $a, b \in \mathbb{R}^{3\times1}$

1. $S(\alpha a + \beta b)$
   $\alpha S(a) + \beta S(b)$ Linearity

2. $S(a)\, b = a \times b$

3. $R\, S(a) R^T = S(Ra)$

4. for any $x \in \mathbb{R}^{3\times1}$

   $x^T S x = 0$

Derivative of the Rotation Matrix

$$\frac{d}{dt} R(\theta) = S R(\theta)$$

$$\frac{d}{d\theta} R_{x,\theta} = S(i) R_{x,\theta} \qquad \dot{R}_A^B = S(w_{B,A}^B)\, R_A^B$$

$$\frac{d}{d\theta} R_{u,\theta} = S(u) R_{u,\theta}$$

Angular Velocity

Ex. $R_{z,\theta(t)}$, $p^1 = [1 \ 0 \ 0]^T$

$$\dot{R}_{z,\theta} = S(\dot{\theta} k) R_{z,\theta}$$

$$\dot{p}^0(t) = S(\dot{\theta} k)\, R_{z,\theta}\, p^1$$

$$= \begin{bmatrix} 0 & -\dot{\theta} & 0 \\ \dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\dot{\theta} s_\theta \\ \dot{\theta} c_\theta \\ 0 \end{bmatrix}$$

$$p^0(t) = R_{z,\theta}\, p^1 = \begin{bmatrix} c_\theta \\ s_\theta \\ 0 \end{bmatrix}$$

Linear velocity of a point translating and Rotating

$$\dot{p} = w \times r + v$$

$$H_1^0(t) = \begin{bmatrix} R_1^0(t) & O_1^0(t) \\ 0 & 1 \end{bmatrix}$$

$p^1 = \text{const}$

$$p^0 = R_{10}\, p^1 + O_1^0$$

$$\dot{p}^0 = \dot{R}_1^0 p^1 + \cancel{R_1^0 \dot{p}^1}^{\,0} + \dot{o}_1^0$$

$$= S(\omega(t)) p^1 + v_1^0(t) \leftarrow \dot{o}_1^0(t)$$

$$= \omega(t) \times p^1 + v$$

Compound Angular Velocities

Frame 1 rotates about frame 0

by $R_1^0(t)$ and frame 2 rotating
about frame 1 by $R_2^1(t)$

$\omega_{i,j}^k$ = angular velocity vector

of R_j^i w.r.t. frame k

$$R_2^0(t) = R_1^0(t) R_2^1(t)$$

$$\underbrace{\dot{R}_2^0(t)}_{3} = \underbrace{\dot{R}_1(t) R_2^1(t)}_{1} + \underbrace{R_1^0(t) \dot{R}_2^1(t)}_{2}$$

① $\dot{R}_1^0(t) R_2^1(t) = S(\omega_{0,1}^0) R_2^1(t) = S(\omega_{0,1}^0) R_2^0(t)$

② $R_1^0 \dot{R}_2^1 = R_1^0(t)\, S(\omega_{1,2}^1) R_2^1(t) = R_1^0\, S(\omega_{1,2}^1) \underbrace{R_1^{0T} R_1^0}_{I} R_2^1$

$$= R_1^0\, S(\omega_{1,2}^1) R_1^{0T} \cdot R_2^0$$

$$= S(R_1^0 \omega_{1,2}^1)\, R_2^0$$

$$S(\omega_{1,2}^0)\, R_2^0$$

③ $\dot{R}_2^0 = S(\omega_{0,2}^0) R_2^0$

$$S(\omega_{0,2}^0) R_2^0 = S(\omega_{0,1}^0) R_2^0 + S(\omega_{1,2}^0) R_2^0$$

or

$$S(\omega_{0,2}^0) = S(\omega_{0,1}^0) + S(\omega_{1,2}^0)$$

or

$$\omega_{0,2}^0 = \omega_{0,1}^0 + \omega_{1,2}^0$$

$$\boxed{\text{In general } \dot{R}_n^0 = S(\omega_{0,n}^0) R_n^0 \text{ where } \omega_{0,n}^0 = \omega_{0,1}^0 + R_1^0 \omega_{1,2}^1 + \dots + R_{n-1}^0 \omega_{n-1,n}^{n-1}}$$

Ex.

What is $\omega_{0,2}^0(t)$ for $R_{z,\theta(t)}$ followed by $R_{y,\phi(t)}$ in current frame?

$$R_2^0 = R_{z,\theta} R_{y,\phi}$$

$$\dot{R}_2^0 = S(\omega_{0,2}^0) R_2^0$$

$$\omega_{0,2}^0 = \omega_{0,1}^0 + R_1^0 \omega_{1,2}^1$$

$$= \dot{\theta} k + R_{z,\theta} \dot{\phi} j$$

$$= \dot{\theta} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \dot{\phi} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\dot{\phi} s\theta \\ -\dot{\phi} c\theta \\ \dot{\theta} \end{bmatrix}$$

Derivative of the Jacobian

n-link manipulator

$$T_n^0 (q, q_1, q_2, q_3 \cdots q_n)(t) = \begin{bmatrix} R_n^0 (q, q_1, \ldots q_n)(t) & o_n (q_1, q_2, \ldots, q_n)(t) \\ 0 & 1 \end{bmatrix}$$

Tool at end effector:

$$p_T^n = \text{const.}$$

$$p_T^0 = R_n^0 p_T^n + o_n^0$$

$$\dot{p}_T^0 = \underline{\dot{R}_n^0}\, p_T^n + \underline{\dot{o}_n^0}$$

we need these two

$$\dot{R}_n^0 = S(\omega_{0,n}^0) R_n^0$$

or this and V_n^0

$$\underset{3\times 1}{V_n^0} = \underset{3\times n}{J_v (q_1, q_2, \ldots, q_n)(t)}\, \underset{n\times 1}{\dot{q}(t)}$$

$$\omega_{0,n}^0 = J_\omega (q_1, q_2 \ldots, q_n)(t)\, \dot{q}(t)$$

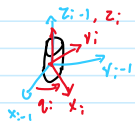
or

$$\underset{6\times 1}{\begin{bmatrix} V_n^0 \\ \omega_{0,n}^0 \end{bmatrix}} = \underbrace{\underset{6\times n}{\begin{bmatrix} J_v (q_1, q_2, \ldots, q_n)(t) \\ J_\omega (q_1, q_2 \ldots, q_n)(t) \end{bmatrix}}}_{\text{Jacobian}} \dot{q}(t)$$

Angular velocity Jacobian, J_ω.

Revolute

$$\omega_{i-1,i}^{i-1} = \dot{q}_i z_{i-1}^{i-1} = \dot{q}_i k$$



Prismatic

$$\omega_{i-1,i}^{i-1} = 0$$

Then,

$$\omega_{0,n}^0 = \rho_1 \dot{q}_1 k + R_1^0 \rho_2 \dot{q}_2 k + \ldots + R_{n-1}^0 \rho_n \dot{q}_n k$$

Math...

$$J_\omega = [\rho_1 z_0^0 \quad \rho_2 z_1^0 \quad \ldots \quad \rho_n z_{n-1}^0]$$

Velocity Jacobian, J_v

Prismatic

$$v_n^0 = \underbrace{z_{i-1}^0}_{J_{v_i}} \dot{q}_i$$

Revolute

$$v_n^0 = \dot{q}_i z_{i-1}^0 \times (o_n^0 - o_{i-1}^0)$$

Revolute

$$v_n^0 = \dot{q}_i\, z_{i-1}^0 \times (o_n^0 - o_{i-1}^0)$$

$$v_n^0 = \underbrace{\left(z_{i-1}^0 \times (o_n^0 - o_{i-1}^0)\right)}_{J_{v_i}} \dot{q}_i$$

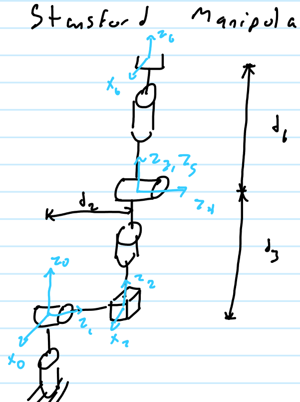
Combine:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_{v_1} & J_{v_2} & \cdots & J_{v_n} \\ J_{\omega_1} & J_{\omega_2} & \cdots & J_{\omega_n} \end{bmatrix}$$

$$J_{v_i} = \begin{cases} z_{i-1}^0 \times (o_n^0 - o_{i-1}^0), & \text{for revolute joint } i \\ z_{i-1}^0 & \text{prismatic} \end{cases}$$

$$J_{\omega_i} = \begin{cases} z_{i-1}^0, & \text{for revolute joint } i \\ 0, & \text{prismatic} \end{cases}$$

Ex: Stanford Manipulator



$$J_{6\times n} = \begin{bmatrix} z_0^0 \times (o_6^0 - o_0^0) & z_1^0 \times (o_6^0 - o_1^0) & z_2^0 & z_3^0 \times (o_6^0 - o_3^0) & z_4^0 \times (o_6^0 - o_4^0) & z_5^0 \times (o_6^0 - o_5^0) \\ z_0^0 & z_1^0 & 0 & z_3^0 & z_4^0 & z_5 \end{bmatrix}$$

$$o_0^0 = o_1^0 = [\,0 \;\; 0 \;\; 0\,]^T$$

$$o_3^0 = o_4^0 = o_5^0 = o_c^0$$

$$T_i^0 = A_1 A_2 \ldots A_i = \begin{bmatrix} R_i^0 & o_i^0 \\ 0 & 1 \end{bmatrix}$$

$$o_c = T_3^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

or

$$T_3^0 = \begin{bmatrix} x_3^0 & y_3^0 & z_3^0 & \overset{o_c^0}{\overset{\shortparallel}{o_3^0}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$o_c^0 = \begin{bmatrix} c_1 s_1 d_3 - s_1 d_2 \\ s_1 s_2 d_3 + c_1 d_2 \\ c_2 d_3 \end{bmatrix}$$

$$o_6^0 = T_6^0 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \ldots$$

$$J_1 = \begin{bmatrix} z_0^0 \times (o_6^0 - o_0^0) \\ z_0^0 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(o_6^0 - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right) \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_0^0 \times (O_6^0 - O_0^0) \\ z_0^0 \end{bmatrix} = \begin{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(O_6^0 - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{pmatrix}$$

Singularities

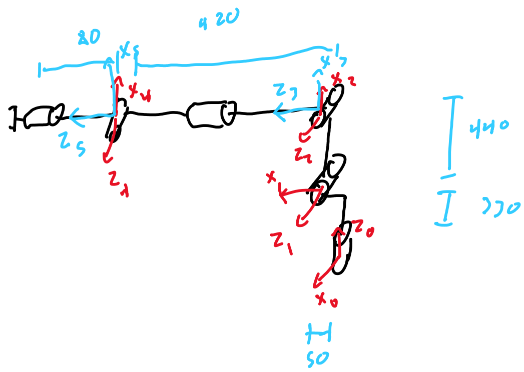
$$\begin{bmatrix} v_n^0 \\ w_{0,n}^0 \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

Direction that the robot cannot move instantaneously

Velocity example

Friday, November 1, 2024 6:38 PM

Fanuc LR mate 200iD 7L



HW03

Friday, February 23, 2024 12:57 PM

HW03

AE/ME 5449
Homework 3: Inverse Kinematics

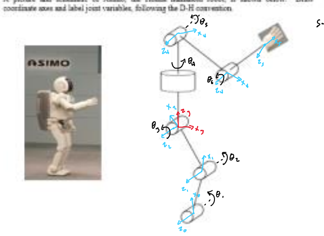
Assigned: February 14, 2024 Due: February 28, 2024

Review the following concepts discussed in lecture:
1) Inverse Kinematics (Chapter 5)

Work the following problems from the text. Unless otherwise stated, work all problem parts. Problems labeled (A-x) are additional problems of my own design.

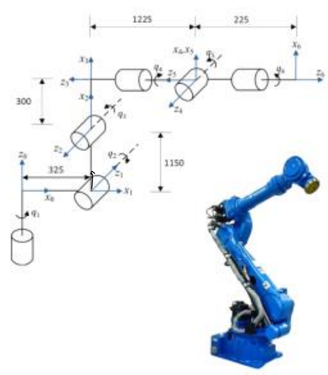
Inverse Kinematics
5-1
5-3
5-5
5-8

D-H Convention (review problems)
(A.1) A picture and schematic of Asimo, the Honda humanoid robot, is shown below. Draw coordinate axes and label joint variables, following the D-H convention.



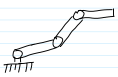
(Additional problems on next page)

(A.2) Consider the schematic of a Yaskawa MH180 robot shown below. Coordinate axes, following the D-H convention, are already drawn on the robot for you (note the positive direction of axes indicated in the diagram). Create a table with the D-H convention link variables.



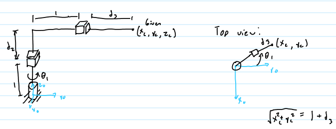
5-1

5



Given only a desired end effector position, there are infinite solutions to the inverse kinematics. If the end effector orientation is also specified, there are two solutions, elbow up and elbow down. If the distance from the base to the desired position is equal to the sum of joint lengths, there would only be one solution. If the desired position is out of reach, there is no solution.

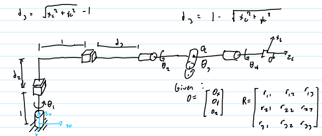
5-3



$\sqrt{x_c^2+y_c^2} = 1 + d_3$

Forward facing	Backward facing
$\theta_1 = \text{Atan2}(x_c, y_c)$	$\theta_1 = \text{Atan2}(x_c, y_c) + 180$
$d_2 = z_c - 1$	$d_2 = z_c - 1$
$d_3 = \sqrt{x_c^2+y_c^2} - 1$	$d_3 = 1 - \sqrt{x_c^2+y_c^2}$

5-5



Given: $O = \begin{bmatrix} O_x \\ O_y \\ O_z \end{bmatrix}$ $R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

$x_c = O_x - d_6 r_{13}$

$y_c = O_2 - d_6 r_{23}$

$z_c = O_z - d_6 r_{33}$

Forward facing	Backward facing
$\theta_1 = \text{Atan2}(x_c, y_c)$	$\theta_1 = \text{Atan2}(x_c, y_c) + 180$
$d_2 = z_c - 1$	$d_2 = z_c - 1$
$d_3 = \sqrt{x_c^2+y_c^2} - 1$	$d_3 = 1 - \sqrt{x_c^2+y_c^2}$

$R_6^0 = R_3^0 R_6^3$

$R_6^3 = (R_3^0)^T R_6^0$

$R_3^0 = A_{\theta_1} = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

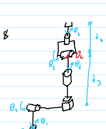
$R_6^3 = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

$R_6^3 = \begin{bmatrix} c_1 r_{11} + s_1 r_{21} & c_1 r_{12} + s_1 r_{22} & c_1 r_{13} + s_1 r_{23} \\ -s_1 r_{11} + c_1 r_{21} & -s_1 r_{12} + c_1 r_{22} & -s_1 r_{13} + c_1 r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

Euler angle solution

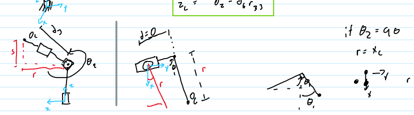
$\theta_5 = \text{atan2}(r_{33}, \sqrt{1-r_{33}^2})$	$\theta_5 = \text{atan2}(r_{33}, -\sqrt{1-r_{33}^2})$
$\theta_6 = \text{atan2}(-r_{31}, r_{32})$	$\theta_6 = \text{atan2}(r_{31}, -r_{32})$
$\theta_4 = \text{atan2}(r_{13}, r_{23})$	$\theta_4 = \text{atan2}(-r_{13}, -r_{23})$
positive wrist	negative wrist

5-8



Given: $O = \begin{bmatrix} O_x \\ O_y \\ O_z \end{bmatrix}$ $R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

$$\boxed{\begin{aligned} x_c &= O_x - d_6 r_{13} \\ y_c &= O_y - d_6 r_{23} \\ z_c &= O_z - d_6 r_{33} \end{aligned}}$$



if $\theta_2 = 90$

$r = x_c$

$s = \sqrt{d_3^2 - r^2}$ $r = \sqrt{x_c^2 + z_c^2}$

$\theta_2 = \text{Atan2}(\ r\ ,\ s\ )$

$z_c = s = \sqrt{d_3^2 - r^2}$

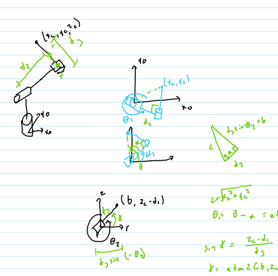
$d_3 = \sqrt{z_c^2 + r^2}$

$\theta_1 = \text{Atan2}(x_c, y_c)$

$R_3^0 = [R_{ot\,z}, R_{ot\,y}]^T R$

$R_3^0 = \left\{ \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{bmatrix} \right\}^T \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

$R_3^0 = \Big[(c_1c_2 - s_1s_2)r_{11} + (s_1c_2 + c_1s_2)r_{21} \quad (c_1c_2 - s_1s_2)r_{12} + (s_1c_2 + c_1s_2)r_{22} \quad (c_1c_2 - s_1s_2)r_{13} + (s_1c_2 + c_1s_2)r_{23}$
$-s_1 r_{11} + c_1 r_{21} \quad -s_1 r_{12} + c_1 r_{22} \quad -s_1 r_{13} + c_1 r_{23}$



$b = \sqrt{c^2 - d^2}$

$b = \sqrt{x^2 + y^2 - d^2} = d_3 \sin\theta_2$

$c = \sqrt{x_c^2 + y_c^2}$

$\theta_1 = \beta - \alpha = \text{atan2}(x_c, y_c) - \text{acos}\left(\frac{d_2}{c}\right)$

$\sin\gamma = \frac{z_c - d_1}{d_3}$

$\gamma = \text{atan2}(b, z_c - d_1)$

$d_3 = \sqrt{b^2 + (z_c - d_1)^2}$

$d_3 = \sqrt{x_c^2 + y_c^2 - d_2^2 + (z_c - d_1)^2}$

$\theta_2 = 270 + \gamma$

$= \text{atan2}\left(\sqrt{x_c^2 + y_c^2 - d_2^2}, z_c - d_1\right) - 90$

c. (21 pts) Frame 2 rotates around Frame 1 with an angle θ and point p rotates around Frame 3 with an angle ϕ. Find p^2, the location of point p with respect to Frame 2.

$= \operatorname{atan2}\left(\sqrt{x_c^2+y_c^2-d_2^2},\ z_c-d_1\right)-90$

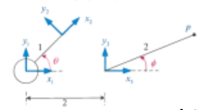
$$R_3^0 = \left\{ \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{bmatrix} \right\}^T \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$R_3^0 = \begin{bmatrix} (c_1c_2-s_1s_2)r_{11}+(s_1c_2+c_1s_2)r_{21} & (c_1c_2-s_1s_2)r_{12}+(s_1c_2+c_1s_2)r_{22} & (c_1c_2-s_1s_2)r_{13}+(s_1c_2+c_1s_2)r_{23} \\ -s_1r_{11}+c_1r_{21} & -s_1r_{12}+c_1r_{22} & -s_1r_{13}+c_1r_{23} \\ (c_1s_2+c_2s_1)r_{11}+(s_1s_2-c_1c_2)r_{21} & (c_1s_2+c_2s_1)r_{12}+(s_1s_2-c_1c_2)r_{22} & (c_1s_2+c_2s_1)r_{13}+(s_1s_2-c_1c_2)r_{23} \end{bmatrix}$$

Euler angle solution

$\theta_5 = \operatorname{atan2}(r_{33}, \sqrt{1-r_{33}^2})$	$\theta_5 = \operatorname{atan2}(r_{33}, -\sqrt{1-r_{33}^2})$
$\theta_6 = \operatorname{atan2}(-r_{31}, r_{32})$	$\theta_6 = \operatorname{atan2}(r_{31}, -r_{32})$
$\theta_4 = \operatorname{atan2}(r_{13}, r_{23})$	$\theta_4 = \operatorname{atan2}(-r_{12}, -r_{23})$
positive wrist	negative wrist

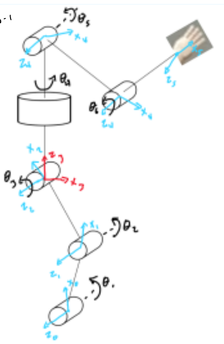
c. (21 pts) Frame 2 rotates around Frame 1 with an angle θ and point p rotates around Frame 3 with an angle ϕ. Find p^2, the location of point p with respect to Frame 2.



$$p^2 = R_{z,\phi}\begin{bmatrix}2\\0\\0\end{bmatrix} = \begin{bmatrix}2c\phi\\2s\phi\\0\end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} R_{z,\theta} & \begin{bmatrix}1\\0\\0\end{bmatrix} \\ 0 & 1 \end{bmatrix}$$

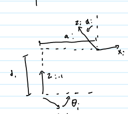
A-1



A-2

$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Rot_{x,\alpha_i} Trans_{x,a_i}$

Link	θ_i	d_i	α_i	a_i
1	q_1	0	-90	325
2	q_2-90	0	180	1150
3	q_3	0	-90	300
4	q_4	-1125	90	0
5	q_5	0	-90	0
6	q_6	-225	180	0



Random Lectures

Friday, March 1, 2024 8:01 AM

Mechanical Design

Acceleration = $\frac{1}{\text{Inertia}}$ Torque

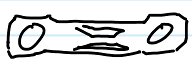
Stiffness and precision

$$\delta = \frac{FL^3}{3EI}$$

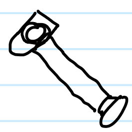
stiff materials - E
stiff mechanical design - I

Structural Design

1. Trusses



2. Tube



Harmonic Drives
- Converts speed to torque
  300:1
- light weight

Transmissions

Calibration

Error sources

Static errors
- Machining tolerances
- Assembly tolerances
- Encoder accuracy
- zero location (mastering)
- Bearing/gear wear
- Flexural errors from robot mass

Quasi-static
- Flexural errors from load of tool
- Thermal errors from surrounding environment

Accuracy >> Repeatability

Zeroing the encoders | Mastering

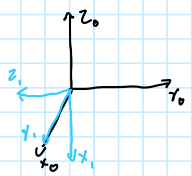
Calibration Model

Exam Prep

Tuesday, March 12, 2024 4:21 PM

1.

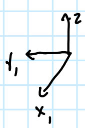
$$R_1^0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$ Draw



b. Valid coordinate frame?

$$R_A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$ No, x_c not unit vector

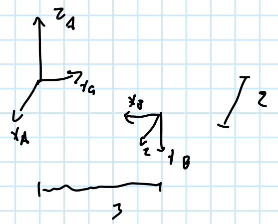
$$R_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



, Not Right-handed

$$R_C = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ zc Not unit vector

c.



Find T_A^B

Trans x, Trans z,-2 Rot x, -90 Rot

Not possible

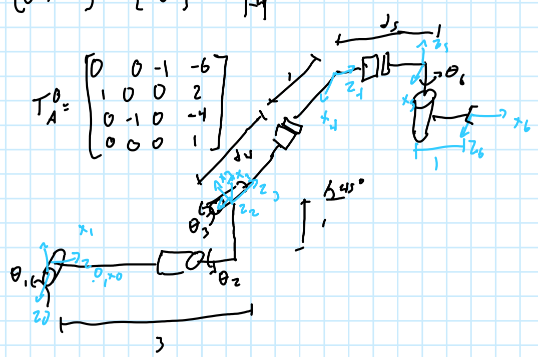
d. $T_B^A = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Find T_B^A

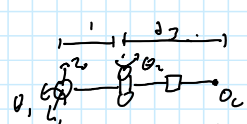
$$R_b^a = (R_a^b)^{-1} = (R_a^b)^T, \quad H_b^a = (H_a^b)^{-1} = \begin{matrix} (R_a^b)^T & -(R_a^b)^T o_a^b \\ 0 & 1 \end{matrix}$$

$$-\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \\ -4 \end{bmatrix}$$

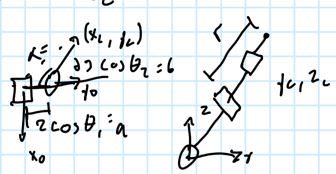
$$T_A^B = \begin{bmatrix} 0 & 0 & -1 & -6 \\ 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2





θ_1, θ_2, d_3

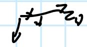


$r = d_3 \cos\theta_2$

$180 - \theta_2$

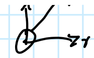
Link	θ_i	d_i	a_i	α_i
1	$\theta_1 + 90$	0	0	90

Link	θ_i	d_i	a_i	α_i
1	θ_1+90	0	0	90
2	θ_2	1	3	-90
3	θ_3+45	0	0	90
4	90	d_4+1	0	45
5	0	d_5	0	90
6	$90+\theta_6$	1	1	90



$2\cos\theta_1 \cdot a$

x_0



$180-\theta_2$

$r = d_3 \cos\theta_2$

$\sqrt{y_c^2+z_c^2} = r+2$

$\cos 180-\theta_2 = \frac{a^2+b^2-c^2}{2ab}$

$\theta_1 = \operatorname{atan2}(y_c, z_c)$

HW04

Saturday, March 30, 2024 6:44 PM

HW04

AE/ME 5449
Homework 4: Velocity Kinematics and the Jacobian

Assigned: March 13, 2024 Due: April 3, 2024

Review the following concepts discussed in lecture:

1) Chapter 4

Work the following problems from the text. Unless otherwise stated, work all problem parts. Problems labeled (A-x) are additional problems of my own design.

4-1
4-5
4-6
4-7 (there is a typo – it should be $R = R_{z,\psi}R_{y,\theta}R_{z,\phi}$)
4-10
4-11
4-12 (Note the picture is drawn incorrectly. Frame 0 should be located at Frame 1, and x_0 should be in the same direction as x_1).

4-1

Verify $S(\alpha a + \beta b) = \alpha S(a) + \beta S(b)$

$a = (a_x, a_y, a_z)$

$b = (b_x, b_y, b_z)$

$\alpha a + \beta b = (\alpha a_x + \beta b_x, \alpha a_y + \beta b_y, \alpha a_z + \beta b_z)$

$$S(\alpha a + \beta b) = \begin{bmatrix} 0 & -(\alpha a_z + \beta b_z) & \alpha a_y + \beta b_y \\ \alpha a_z + \beta b_z & 0 & -(\alpha a_x + \beta b_x) \\ -(\alpha a_y + \beta b_y) & \alpha a_x + \beta b_x & 0 \end{bmatrix}$$

$\alpha S(a) + \beta S(b)$

$$\alpha S(a) = \begin{bmatrix} 0 & -\alpha a_z & \alpha a_y \\ \alpha a_z & 0 & -\alpha a_x \\ -\alpha a_y & \alpha a_x & 0 \end{bmatrix}$$

$$\beta S(b) = \begin{bmatrix} 0 & -\beta b_z & \beta b_y \\ \beta b_z & 0 & -\beta b_x \\ -\beta b_y & \beta b_x & 0 \end{bmatrix}$$

$$\alpha S(a) + \beta S(b) = \begin{bmatrix} 0 & -(\alpha a_z + \beta b_z) & \alpha a_y + \beta b_y \\ \alpha a_z + \beta b_z & 0 & -(\alpha a_x + \beta b_x) \\ -(\alpha a_y + \beta b_y) & \alpha a_x + \beta b_x & 0 \end{bmatrix}$$

$$= S(\alpha a + \beta b)$$

4-5 $a = (1, -1, 2)$, $R = R_{x,90}$, show $R S(a) R^T = S(Ra)$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{90} & -s_{90} \\ 0 & s_{90} & c_{90} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$S(a) = \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$R S(a) R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{matrix} 0 & 1 & -2 \\ -1 & 0 & -1 \\ 2 & 1 & 0 \end{matrix}$$

$$Ra = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$S(Ra) = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$R S(a) R^T = S(Ra)$$

4-6 Given $R = R_{x,\theta} R_{y,\phi}$, compute $\frac{\partial R}{\partial \phi}$. Evaluate $\frac{\partial R}{\partial \phi}$ at $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\theta & -s_\theta \\ 0 & s_\theta & c_\theta \end{bmatrix}$$

$$R_{y,\phi} = \begin{bmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{bmatrix}$$

$$R = R_{x,\theta} R_{y,\phi} = \begin{bmatrix} c_\phi & 0 & s_\phi \\ s_\phi s_\theta & c_\theta & -c_\phi s_\theta \\ -c_\theta s_\phi & s_\theta & c_\phi c_\theta \end{bmatrix}$$

$$\frac{\partial R}{\partial \phi} = \begin{bmatrix} -s_\phi & 0 & c_\phi \\ c_\phi s_\theta & 0 & s_\phi s_\theta \\ -c_\theta c_\phi & 0 & -s_\phi c_\theta \end{bmatrix}$$

$$\frac{\partial R}{\partial \phi}(\theta, \phi) = \frac{\partial R}{\partial \phi}\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

4-7

$$R = R_{z,\psi} R_{y,\theta} R_{z,\phi}$$

show $\frac{d}{dt} R = S(\omega) R$ where

$$\omega = (c_\psi s_\theta \dot{\phi} - s_\psi \dot{\theta}) i + (s_\psi s_\theta \dot{\phi} + c_\psi \dot{\theta}) j + (\dot{\psi} + c_\theta \dot{\phi}) k$$

$$R = \begin{bmatrix} c_\phi c_\psi c_\theta - s_\phi s_\psi & -c_\phi s_\psi - c_\psi c_\theta s_\phi & c_\psi s_\theta \\ c_\psi s_\theta + c_\phi c_\theta s_\psi & c_\phi c_\psi - c_\theta s_\phi s_\psi & s_\phi s_\theta \\ -c_\phi s_\theta & s_\phi s_\theta & c_\theta \end{bmatrix}$$

$$\frac{d}{dt} R = \begin{bmatrix} (-c_\phi s_\psi \dot{\psi} - c_\psi \dot{\phi} s_\phi) c_\theta + (-c_\phi c_\psi s_\theta \dot{\theta}) - (c_\phi \dot{\phi} s_\psi + c_\psi \dot{\psi} s_\phi) & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

4-10 Frames $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by

$$H = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$v_1(t) = (3,1,0)$ relative to frame 1

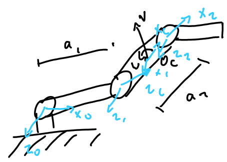
find v_1 w.r.t. frame 0

$$v_1^0 = H \begin{bmatrix} v_1(t)^T \\ 0 \end{bmatrix}$$

$$v_1^0 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$v_1^0 = (-1, 3, 0)$$

4-11



	θ_i	d_i	a_i	α_i
1	θ_1	0	a_1	0
2	θ_2	0	$\frac{a_2}{2}$	0

$$T_c^0 = Rot_{z,\theta_1} Trans_{x,a_1} Rot_{z,\theta_2} Trans_{x,\frac{a_2}{2}}$$

$$T_c^0 = \begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - c_2s_1 & 0 & a_2s_1s_2/2 \\ c_1s_2 + c_2s_1 & c_1c_2 - s_1s_2 & 0 & a_2c_2s_1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{\begin{matrix} a_2s_1s_2/2 \\ a_2c_2s_1/2 \\ 0 \end{matrix}} = O_c$$

$$J(q) = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \times (o_3 - o_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = z_1 = z_2$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad o_1 = \begin{bmatrix} a_1c_1 \\ a_2s_1 \\ 0 \end{bmatrix} \qquad o_2 = \begin{bmatrix} a_1c_1 + a_2c_{12} \\ a_1s_1 + a_2s_{12} \\ 0 \end{bmatrix} \qquad o_3 = T_3^0 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$T_3^0 = A_1A_2A_3$$

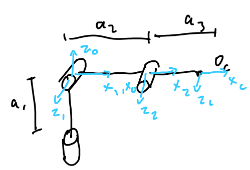
$o_3 =$

```
syms th1 th2 th3 a1 a2 a3;

R1=[cos(th1) -sin(th1) 0 0; sin(th1) cos(th1) 0 0; 0 0 1 0; 0 0 0 1];
T1=[1 0 0 a1;0 1 0 0;0 0 1 0; 0 0 0 1];

R2=[cos(th2) -sin(th2) 0 0; sin(th2) cos(th2) 0 0; 0 0 1 0; 0 0 0 1];
T2=[1 0 0 a2;0 1 0 0;0 0 1 0; 0 0 0 1];

R3=[cos(th3) -sin(th3) 0 0; sin(th3) cos(th3) 0 0; 0 0 1 0; 0 0 0 1];
T3=[1 0 0 a3;0 1 0 0;0 0 1 0; 0 0 0 1];

A1=R1*T1;
A2=R2*T2;
A3=R3*T3;

T30=A1*A2*A3;
o3=T30*[0;0;0;1];
o33=o3(1:3)

z0 = [0;0;1];
z1=z0;
z2=z0;

o0= [0; 0; 0];
o2 = [a1*cos(th1)+a2*cos(th1+th2);a1*sin(th1)+a2*sin(th1+th2); 0];
o1 = [a1*cos(th1);a2*sin(th1);0];

J1=cross(z0,(o33-o0))
J2=cross(z1,(o33-01))
J3=cross(z2,o33-o2)
```

Command Window

```
o33 =

a1*cos(th1) + a3*cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a3*sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a2*cos(th1)*cos(th2) - a2*sin(th1)*sin(th2)
a1*sin(th1) + a3*cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a3*sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) + a2*cos(th1)*sin(th2) + a2*cos(th2)*sin(th1)
                                                                                                                                                                    0


J1 =

- a1*sin(th1) - a3*cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) - a3*sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a2*cos(th1)*sin(th2) - a2*cos(th2)*sin(th1)
  a1*cos(th1) + a3*cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a3*sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a2*cos(th1)*cos(th2) - a2*sin(th1)*sin(th2)
                                                                                                                                                                      0


J2 =

1 - a3*cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) - a3*sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a2*cos(th1)*sin(th2) - a2*cos(th2)*sin(th1) - a1*sin(th1)
a1*cos(th1) + a3*cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a3*sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a2*cos(th1)*cos(th2) - a2*sin(th1)*sin(th2) - 1
                                                                                                                                                                        0


J3 =

a2*sin(th1 + th2) - a3*cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) - a3*sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a2*cos(th1)*sin(th2) - a2*cos(th2)*sin(th1)
a3*cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a2*cos(th1 + th2) - a3*sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a2*cos(th1)*cos(th2) - a2*sin(th1)*sin(th2)
                                                                                                                                                                         0
```

4-12



$A_1 = Rot_{z,\theta_1} Rot_{x,a0}$

$A_2 = Rot_{z,\theta_2} Trans_{x,a_2}$

$A_3 = Rot_{z,\theta_3} Trans_{x,a_3}$

$T_3^0 = A_1 A_2 A_3$

$$J(q) = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \times (o_3 - o_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$o_c = T_3^0 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

```
syms th1 th2 th3 a1 a2 a3;

R1=[cos(th1) -sin(th1) 0 0; sin(th1) cos(th1) 0 0; 0 0 1 0; 0 0 0 1];
T1=[1 0 0 a1;0 1 0 0;0 0 1 0; 0 0 0 1];

R2=[cos(th2) -sin(th2) 0 0; sin(th2) cos(th2) 0 0; 0 0 1 0; 0 0 0 1];
T2=[1 0 0 a2;0 1 0 0;0 0 1 0; 0 0 0 1];

R3=[cos(th3) -sin(th3) 0 0; sin(th3) cos(th3) 0 0; 0 0 1 0; 0 0 0 1];
T3=[1 0 0 a3;0 1 0 0;0 0 1 0; 0 0 0 1];

A1=R1*T1;
A2=R2*T2;
A3=R3*T3;

T30=A1*A2*A3;
o3=T30*[0;0;0;1];
o33=o3(1:3)

z0 = [0;0;1];
z1=z0;
z2=z0;

o0= [0; 0; 0];
o2 = [a1*cos(th1)+a2*cos(th1+th2);a1*sin(th1)+a2*sin(th1+th2); 0];
o1 = [a1*cos(th1);a2*sin(th1);0];

J1=cross(z0,(o33-o0))
J2=cross(z1,(o33-01))
J3=cross(z2,o33-o2)
```

Command Window

```
o33 =

a1*cos(th1) + a3*cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a3*sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a2*cos(th1)*cos(th2) - a2*sin(th1)*sin(th2)
a1*sin(th1) + a3*cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a3*sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) + a2*cos(th1)*sin(th2) + a2*cos(th2)*sin(th1)
                                                                                                                                                                    0


J1 =

- a1*sin(th1) - a3*cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) - a3*sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a2*cos(th1)*sin(th2) - a2*cos(th2)*sin(th1)
  a1*cos(th1) + a3*cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a3*sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a2*cos(th1)*cos(th2) - a2*sin(th1)*sin(th2)
                                                                                                                                                                      0


J2 =

1 - a3*cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) - a3*sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a2*cos(th1)*sin(th2) - a2*cos(th2)*sin(th1) - a1*sin(th1)
a1*cos(th1) + a3*cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a3*sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a2*cos(th1)*cos(th2) - a2*sin(th1)*sin(th2) - 1
                                                                                                                                                                        0


J3 =

a2*sin(th1 + th2) - a3*cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) - a3*sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a2*cos(th1)*sin(th2) - a2*cos(th2)*sin(th1)
a3*cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - a2*cos(th1 + th2) - a3*sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + a2*cos(th1)*cos(th2) - a2*sin(th1)*sin(th2)
                                                                                                                                                                          0
fx
```

Psuedoinverse

Wednesday, April 10, 2024 8:06 AM

Psudo inverse

$x = A^{+} y \quad \Rightarrow$ smallest solution for x

If $n > m$ (tall A)

no solutions

$$A^{+} = (A^T A)^{-1} A^T$$

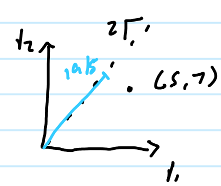
$\tilde{x} = A^{+} y$

$\tilde{x}$ is the closest x to y

$$\begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} x$$

$$\tilde{x} = \left(\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\tilde{x} = \frac{19}{5}$$

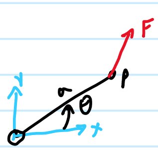


Static Torque/Force Relationships

$\tau = \begin{bmatrix} \tau_1 & \tau_2 & \dots & \tau_n \end{bmatrix}^T$ joint torques/forces

$F = [\underbrace{f_x \quad f_y \quad f_z}_{\text{forces}} \quad \underbrace{n_x \quad n_y \quad n_z}_{\text{moments}}]^T$ at end effector

Ex.



$$\tau = p \times F = \begin{bmatrix} a c_\theta \\ a s_\theta \end{bmatrix} \times \begin{bmatrix} f_x \\ f_y \end{bmatrix} = -a s_\theta f_x + a c_\theta f_y$$

$$F = \begin{bmatrix} -\frac{1}{a} s_\theta \tau \\ \frac{1}{a} c_\theta \tau \end{bmatrix}$$

$$dq^T \cdot \tau = dx^T F$$

$$dx = J(q)\, dq$$

$$dx = J(q)\, dq$$

$$dq^T \tau = (J(q)\, dq)^T F$$

$$dq^T \tau = dq^T J(q)^T F$$

$$\tau = J(q)^T F$$

Ex. $J(q) = z_0^0 \times (p - O_0^0)$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a c\theta \\ a s\theta \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a s\theta \\ a c\theta \\ 0 \end{bmatrix}$$

$$\tau = J^T F = \begin{bmatrix} -a s_\theta & a c\theta \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

$$\tau = -a s\theta f_x + a c\theta f_y \qquad \checkmark$$

$$F = (J(q)^T)^+ \tau$$

$$= J(J^T J)^{-1} \tau$$

$$F = \begin{bmatrix} -\frac{1}{a} s_\theta \\ \frac{1}{a} c\theta \end{bmatrix} \tau$$

Manipulability

$$u = |\det(J(q))|$$

Path and Trajectory Planning

Friday, April 12, 2024 8:35 AM

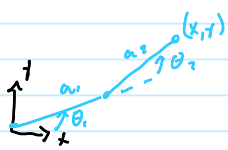
path: spatial progression of q: from start, q_s, to Finish, q_f

trajectory: progression of q along the path in time, $q(t)$

Line path:

$$p(a) = p_s + a(p_f - p_s) \qquad a \in [0,1]$$

Ex:



$$x = a_1 c_1 + a_2 c_{12}$$
$$y = a_1 s_1 + a_2 s_{12}$$

$$D = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2 a_1 a_2}$$

$$\theta_2 = \operatorname{atan2}(D, \pm\sqrt{1-D^2})$$

$$\theta_1 = \operatorname{atan2}(x,y) - \operatorname{atan2}(a_1 + a_2 c_2, a_2 s_2)$$

$$p_s = \begin{bmatrix} x_s \\ y_s \end{bmatrix} \qquad p_f = \begin{bmatrix} x_f \\ y_f \end{bmatrix}$$

$$\boxed{\begin{aligned} x &= x_s + a(x_f - x_s) \\ y &= y_s + a(y_f - y_s) \end{aligned}}$$

line in world space (x,y)
need to pick configuration

$$\boxed{\begin{aligned} q_1(a) &= q_{1s} + a(q_{1f} - q_{1s}) \\ q_2(a) &= q_{2s} + a(q_{2f} - q_{1s}) \end{aligned}}$$

line in joint space (q_1, q_2)

If

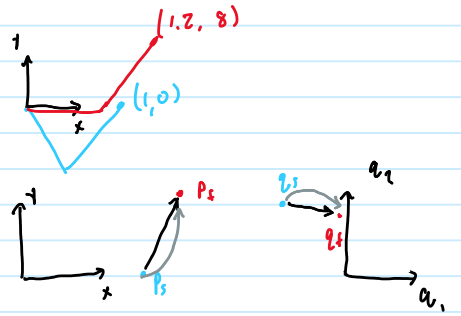
$a_1 = a_2 = 1$

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x_f \\ y_f \end{bmatrix} = \begin{bmatrix} 1.2 \\ .8 \end{bmatrix}$$

$$\Updownarrow$$

$$\begin{bmatrix} q_{1s} \\ q_{2s} \end{bmatrix} = \begin{bmatrix} -60^\circ \\ 120^\circ \end{bmatrix} \qquad \begin{bmatrix} q_{1f} \\ q_{2f} \end{bmatrix} = \begin{bmatrix} -10^\circ \\ 87.8^\circ \end{bmatrix}$$

$$\begin{bmatrix} q_{1s} \\ q_{2s} \end{bmatrix} = \begin{bmatrix} -60^\circ \\ 120^\circ \end{bmatrix} \qquad \begin{bmatrix} q_{1f} \\ q_{2f} \end{bmatrix} = \begin{bmatrix} -10^\circ \\ 87.8^\circ \end{bmatrix}$$



	World space	Joint space
Singularities	need to be tracked	Not a problem
motion efficiency	less	most
orientation changes	constant	changing
config decisions	Yes (bad)	none (good)
locating obstacles	easy	hard
avoiding obstacles	hard	easy

World space is generally preferred for low number of joints ($n = 2, 3$)

Joint space otherwise ($n > 3$)

Trajectory Planning: Joint Space

Path planning generates a desired path, $q(a)$, $a \in [0, 1]$

Goal: get path as function of time, $q(t)$

Constraints:

$q(t_0) = q_0$ start point

$\dot{q}(t_0) = v_0$ start velocity

$q(t_f) = q_0$ end point

$\dot{q}(t_f) = v_f$ end velocity

Use a cubic polynomial

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\underbrace{\begin{bmatrix} q_0 \\ v_0 \\ q_f \\ v_f \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}}_{x}$$

$$x = M^{-1} y$$

Ex. $v_0 = v_f = 0$, $t_0 = 0$ $t_f = 1$

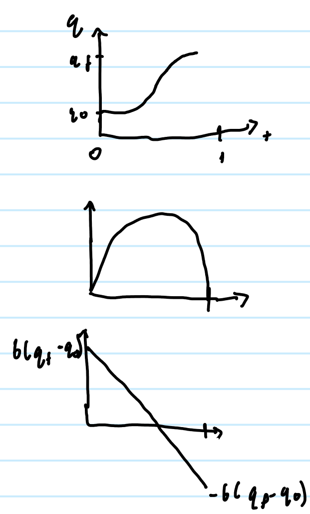
q_0, q_f

$a_0 = q_0$
$a_1 = 0$
$a_2 = 3(q_f - q_0)$
$a_3 = -2(q_f - q_0)$

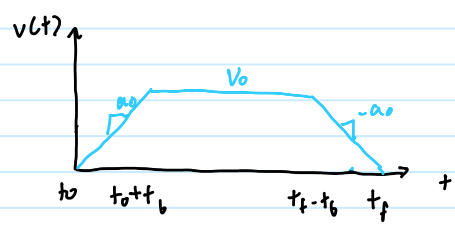
$$q(t) = q_0 + 3(q_f - q_0)t^2 - 2(q_f - q_0)t^3$$

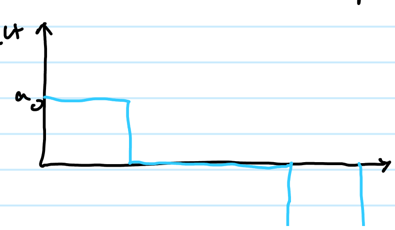
$$\dot{q}(t) = 6(q_f - q_0)t - 6(q_f - q_0)t^2$$

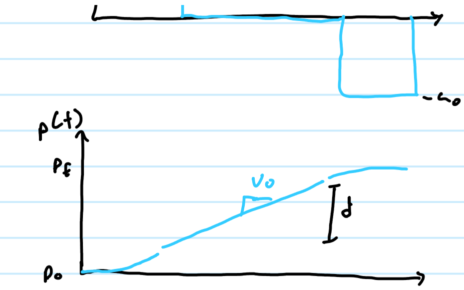
$$\ddot{q}(t) \quad 6(q_f - q_0) - 12(q_f - q_0)t$$



Trapezoidal velocity:
constant velocity







Given q_0, q_f, choose a_0, v_0

let $t_0 = 0$

$0 < t < t_b$:

$\ddot{q}(t) = a_0$

$q(t) = a_0 t$

$q(t) = \frac{1}{2} a_0 t^2 + q_0$

$t = t_b$, $\dot{q}(t_b) = v_0$

$\dot{q}(t_b) = a_0 t_b = v_0$

$t_b = \frac{v_0}{a_0}$

$q(t_b) = \frac{1}{2} a_0 t_b^2 + q_0 = \frac{1}{2} \frac{v_0^2}{a_0} + q_0$

$t_f - t_b < t < t_f$:

distance traveled $= \frac{1}{2} \frac{v_0^2}{a_0}$

const velocity phase,

distance traveled $d = p_f - p_0 - \frac{1}{2}\frac{v_0^2}{a_0} - \frac{1}{2}\frac{v_0^2}{a_0} = d$

$d = v_0 t_d$

$t_d = \frac{d}{v_0} = \frac{p_f - p_0 - \frac{v_0^2}{a_0}}{v_0}$

$t_f = t_b + t_d + t_b = \frac{v_0}{a_0} + \frac{p_f - p_0 - \frac{v_0^2}{a_0}}{v_0} + \frac{v_0}{a_0}$

$t_f = \frac{v_0}{a_0} + \frac{p_f - p_0}{v_0}$

putting it together:

$0 < t < \frac{V_0}{a_0}$

$$\dot{q}(t) = a_0 t$$

$$q(t) = q_0 + \frac{1}{2} a_0 t^2$$

$\frac{V_0}{a_0} < t < \frac{q_f - q_0}{V_0}$

$$\dot{q}(t) = V_0$$

$$q(t) = q_0 + \frac{1}{2} \frac{V_0^2}{a_0} + V_0 \left(t - \frac{V_0}{a_0} \right)$$

$\frac{q_f - q_0}{V_0} < t < \frac{V_0}{a_0} + \frac{q_f - q_0}{V_0}$:

$$\dot{q}(t) = -a_0 \left(t - \frac{q_f - q_0}{V_0} \right)$$

$$q(t) = q_f - \frac{1}{2} \frac{V_0^2}{a_0} - \frac{1}{2} a_0 \left(t - \frac{q_f - q_0}{V_0} \right)^2$$

HW05

Sunday, April 14, 2024 4:37 PM

HW05

AE/ME 5449
Homework 5: Jacobian Applications

Assigned: April 3, 2024 Due: April 17, 2024

Review the following concepts discussed in lecture:

1) Chapter 4

Work the following problems from the text. Unless otherwise stated, work all problem parts. Problems labeled (A-x) are additional problems of my own design.

<u>Jacobian of the SCARA Robot</u>

The following problems deal with the SCARA robot (pg. 95 of the text). The Jacobian for this arm is given in (4.73) of the text. You will need to use MATLAB, or another numerical software package, to answer the following questions. If you are not familiar with MATLAB, please begin early to allow time for office hour visits.

(A.1) The upper 3 rows of the Jacobian, J_v, give the linear velocity of the end effecter. Note that the fourth column is all zeros, indicating that the fourth joint does not contribute to linear velocity of the end effecter. Let's ignore that joint altogether by removing the last column from your J_v matrix (leaving a 3x3 matrix). Find the determinant of the 3x3 J_v. What configurations result in singularities?

(A.2) Now use your 3x3 J_v to determine what configurations give the maximum manipulability? What configurations give the minimum manipulability? Draw a sketch illustrating the configuration at the maximum and minimum.

(A.3) You would like to use this robot for manipulating items on the table. What region of the table would work best for this task? Be specific and explain your answer.

(A.4) A force $F=[F_x\ F_y\ F_z]^T$ is applied to the end effecter. Find the torques/forces that are induced in each of the first three joints.

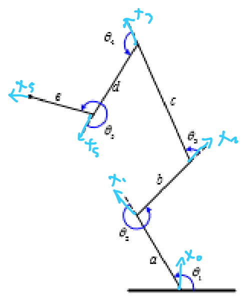
(A.5) The force contains only the x component, $F=[F_x\ 0\ 0]^T$. What configuration will result in the largest induced torque on the first axis?

(A.6) The robot is in the configuration $q=[0°\ 90°\ 0]^T$. What joint torques will generate the end effecter force, $F=[1\ 0\ 0]^T$? What about $F=[1\ 1\ 0]^T$? What about $F=[0\ 0\ 1]^T$?

(continued on next page)

Application to Human Kinematics

Consider the kinematic model of a human profile shown right. A table of standard limb (i.e., link) lengths for an average adult female shown below. Also shown below is a table of two different configurations for picking up an object at approximately the same location. Illustrations and MATLAB plots demonstrating these two configurations are also shown below.



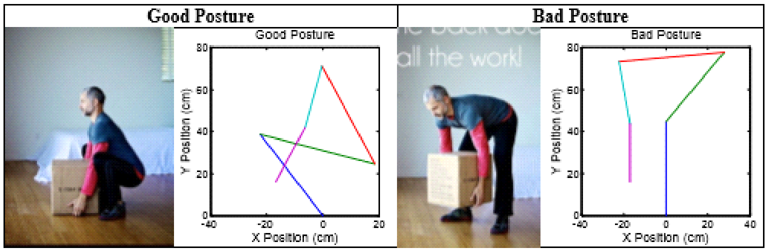
Standard limb lengths for an average adult female.

	a	b	c	d	e
Length (cm)	44.6	43.2	50.3	30.0	27.8

Configurations for picking up an object at approximately (-16,16)

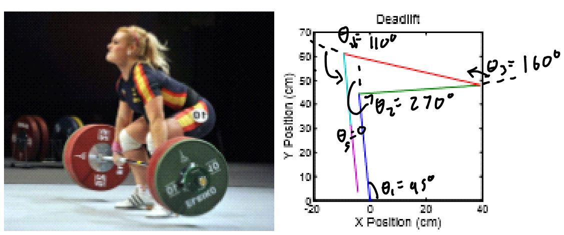
	θ_1	θ_2	θ_3	θ_4	θ_5
Good Posture	120	-139	131	146	-10
Bad Posture	90	-40	135	95	-10

Good Posture | **Bad Posture**



(A.7) Determine the effect of posture on lower back pain by calculating the torque at the hip joint, θ_3, for both postures for a 10 kg load in the negative Y direction. Include the correct units in your answer.

(A.8) Find a configuration that results in a deadlift pose (shown below). Find the torques on each joint for a load of 200 kg in the negative Y direction. Comment on distribution of torques and the suitability of this configuration for lifting very large objects.



A1:

$$\det(J) = a_1 a_2 \cos(\theta_1+\theta_2)\sin\theta_1 - a_1 a_2 \sin(\theta_1+\theta_2)\cos\theta_1$$

if $\theta_2 = \pi, 0$

$$(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)\sin\theta_1 - (\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)\cos\theta_1 = 0$$

$$\sin\theta_1 \cancel{\cos\theta_1\cos\theta_2} - \sin^2\theta_1\sin\theta_2 - \sin\theta_1 \cancel{\cos\theta_1\cos\theta_2} - \cos^2\theta_1\sin\theta_2 = 0$$

$$-\sin^2\theta_1\sin\theta_2 - \cos^2\theta_1\sin\theta_2 = 0$$

$$\sin\theta_2(\sin^2\theta_1 + \cos^2\theta_1) = 0$$

$$\sin\theta_2 = 0$$

Singularity if $\theta_2 = \pi, 0$

A2: $|\sin\theta_2| = [0,1]$

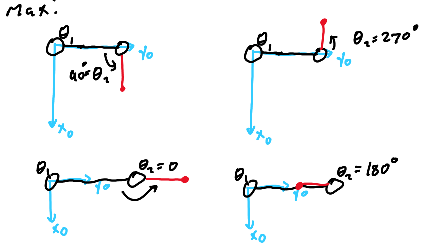
Singularity if $\theta_2 = \pi, 0$

A2: $|\sin\theta_2| = [0,1]$

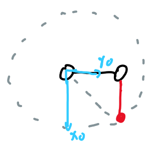
1 if $\theta_2 =$ 90 or 270

0 if $\theta_2 =$ 0, 180

Max:



A3:



anywhere where $\theta_2 =$ 90 or 270 achieves maximum manipulability.

Can be described by a circle where the end effector makes a radius $r = \sqrt{a_1^2 + a_2^2}$

A4:

$$T = J^T(q) F$$

$$F = [F_x \;\; F_y \;\; F_z \;\; 0 \;\; 0 \;\; 0]^T$$

$$J^T(q) = \begin{bmatrix} -a_2 s_{12} - a_1 s_1 & -a_2 s_{12} & 0 & 0 & 0 & 1 \\ a_2 c_{12} + a_1 c_1 & a_2 c_{12} & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} -F_x(a_2 s_{12} - a_1 s_1) - F_y a_2 s_{12} \\ F_x(a_2 c_{12} - a_1 c_1) + F_y a_2 \cos_{12} \end{bmatrix}$$

A5:

$$F = [F_x \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T$$

$$T_1 = J^T(q) F = -F_x(a_2 s_{12} + a_1 s_1)$$

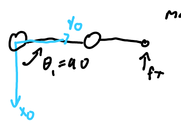
if $s_{12} = s_1 = 1$ or -1

$\theta_1 = 90, -90$

$\theta_2 = 0, 180$

then $T_1 = -F_x(a_2 + a_1)$

max



A6:

$$T = J^T(q) F$$

```
clear all
close all
clc

syms a1 th1 a2 th2 Fx Fy Fz;

J=[-a1*sin(th1)-a2*sin(th1+th2) -a2*sin(th1+th2) 0; a1*cos(th1)+a2*cos(th1+th2) a2*cos(th1+th2) 0; 0 0 -1]

det_J=det(J)

%manip_J= norm(det_J)

J_T=[[- a2*sin(th1 + th2) - a1*sin(th1), -a2*sin(th1 + th2), 0, 0, 0, 1]
[  a2*cos(th1 + th2) + a1*cos(th1),  a2*cos(th1 + th2), 0, 0, 0, 1]]

torques1=J_T*[1; 0; 0; 0; 0; 0]
torques2=J_T*[1; 1; 0; 0; 0; 0]
torques3=J_T*[0; 0; 1; 0; 0; 0]

F1=subs(torques1,[th1 th2], [0 pi/2])
F2=subs(torques2,[th1 th2], [0 pi/2])
F3=subs(torques3,[th1 th2], [0 pi/2])
```

Command Window

```
F1 =

-a2
 a1


F2 =

-2*a2
   a1


F3 =

0
0
```

A7:

$$J_{6\times n} = \begin{bmatrix} z_0^0 \times (O_5^0 - O_0^0) & z_1^0 \times (O_5^0 - O_1^0) & z_2^0 \times (O_5^0 - O_2^0) & z_3^0 \times (O_5^0 - O_3^0) & z_4^0 \times (O_5^0 - O_4^0) \\ z_0^0 & z_1^0 & z_2^0 & z_3^0 & z_4^0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = z_4 = z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} a c_1 \\ a s_1 \\ 0 \end{bmatrix}$$

O_2

```
clear all
close all
clc

syms th1 th2 th3 th4 th5 a b c d e

z0=[0;0;1];
z1=z0;
z2=z0;
z3=z0;
z4=z0;
%{
th1=120;
th2=-139;
th3=131;
th4=146;
th5=-10;
%}
```

```
th1=90;
th2=-40;
th3=135;
th4=95;
th5=-10;

a=44.6;
b=43.2;
c=50.3;
d=30;
e=27.8;

o0=[0;0;0];

o1t=RotZ(th1)*TransX(a);
o1=[a*cos(th1); a*sin(th1);0];

o2t= o1t*RotZ(th2)*TransX(b);
o2=[a*cos(th1)+b*cos(th1+th2); a*sin(th1)+b*sin(th1+th2);0];

o3t=o2t*RotZ(th3)*TransX(c);
o3=o3t*[0;0;0;1];
o3=o3(1:3)

o4t=o3t*RotZ(th4)*TransX(d);
o4=o4t*[0;0;0;1];
o4=o4(1:3)

o5t=o4t*RotZ(th5)*TransX(e);
o5=o5t*[0;0;0;1];
o5=o5(1:3)

J=[cross(z0,o5-o0) cross(z1,o5-o1) cross(z2, o5-o2) cross(z3, o5-o3) cross(z4, o5-o4) [0; 0; 0]; z0 z1 z2 z3 z4 [0; 0; 0]]

torque=J(1:3,:)*[0; -10*9.8; 0; 0; 0; 0]
```

Good posture	Bad posture
torque =	torque =
1.0e+03 *	1.0e+03 *
T_1 = -0.9622	-2.3429
T_2 = 5.2195 ?	-0.2796 ?
T_3 = 0	0

All units in N • cm

A9:

$\theta_1 = 95^\circ$
$\theta_2 = 270^\circ$
$\theta_3 = 160^\circ$
$\theta_4 = 110^\circ$
$\theta_5 = 0$

Same code, but with these approximate angles and $F_y = -200 \cdot 9.81$.

```
torque =

   1.0e+04 *

  -5.2605
   7.2453
        0
```
?

HW06

Tuesday, April 30, 2024 3:27 PM

HW06

AE/ME 5449
Homework 6: Path and Trajectory Planning

Assigned: April 17, 2024 Due: May 1, 2024

Review the following concepts discussed in lecture:

1) Chapter 7

Work the following problems from the text. Unless otherwise stated, work all problem parts. Problems labeled (A-x) are additional problems of my own design.

7-2
7-1

(A.1) Consider the configuration space path planning problem for the two-link planar mechanism in Figure 1.19 with link lengths equal to 1. The mechanism should start at $\theta_1 = \theta_2 = 0°$ and finish at $\theta_1 = \theta_2 = 90°$. Consider an obstacle bounded in the configuration space by a polygon with coordinates $V_1=(20°,60°)$, $V_2=(80°,10°)$, $V_3=(45°,45°)$. a) Using Visibility Graphs, write the sequence for all noncircular paths (without repeating any point). b) Select one path and write an equation for $\theta_1(a)$ and $\theta_2(a)$. Plot them on the same plot. c) Write the equation for the paths $o_1(a)$ and $o_2(a)$ of the first and second joint. Plot them on the same plot.

7-21
7-22 (set the cruise velocity to 1 rad/s and do not decelerate at the end of the trajectory).

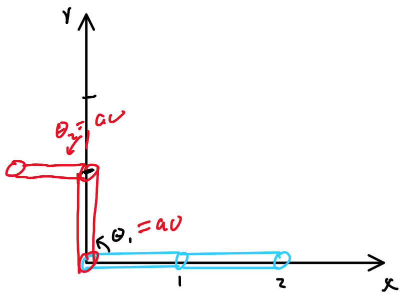
(A.2) A joint on a robot is to go from an initial angle of 20° to an intermediate angle of 40° in 5 seconds and continue to its destination of 125° in another 5 seconds. Use an intermediate velocity of 5 °/s. Calculate the coefficients for a third-order polynomial in joint-space for each segment. Plot the joint angles, velocities, and accelerations.

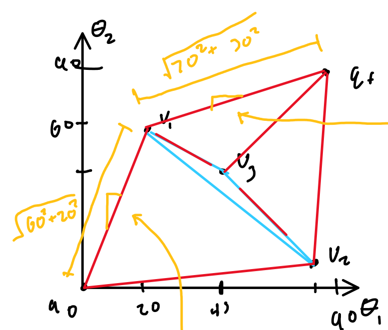
7-2: The configuration space of a 3-link planar arm can be represented by $q = (\theta_1, \theta_2, \theta_3)$

or $Q = S^1 \times S^1 \times S^1 = T^3$.

7-1: Configuration space for a mobile robot is $Q = SE(2)$ or $q = (d_1, d_2, \theta_1)$.

A-1:





(90, 90)

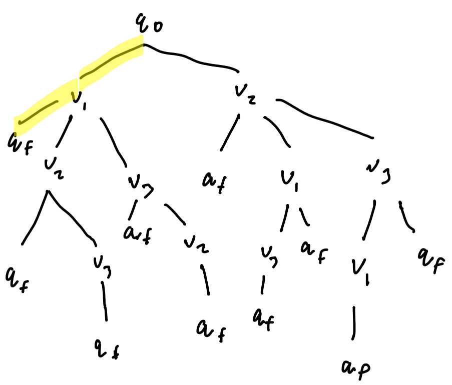
$$\frac{\theta_2(q_f) - \theta_2(v_1)}{\theta_1(q_f) - \theta_1(v_1)} = \frac{90-60}{90-20} = \frac{30}{70} = \frac{3}{7}$$

$$y - 90 = \frac{3}{7}(x - 90)$$

$$\frac{3}{7}x + \frac{360}{7}$$

$$\frac{\theta_2(v_1) - \theta_2(q_0)}{\theta_1(v_1) - \theta_1(q_0)} = \frac{60}{20} = 3$$

$a \in [0, 1]$



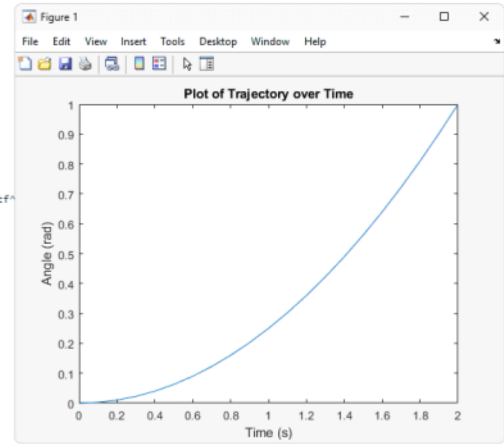
$q_0 \rightarrow v_1 \rightarrow q_f$

$$T_2^0 = \begin{bmatrix} c_{12} & -s_{12} & 0 & c_1 + c_{12} \\ s_{12} & c_{12} & 0 & s_1 + s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$O_1(\theta_1, \theta_2) = c_1 + c_{12}$$
$$O_2(\theta_1, \theta_2) = s_1 + s_{12}$$

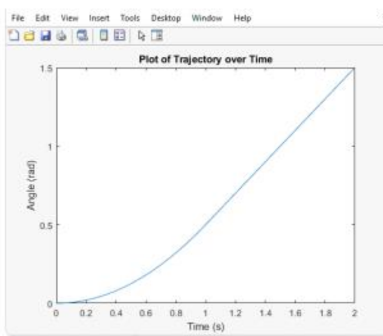
7-21

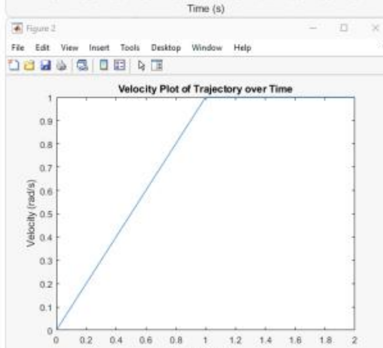
```
clc;
clear;
close all;

syms q0 v0 qf vf t0 tf

tf=2;
vf=1;
v0=0 % at rest
q0=0; % Need assumptions
t0=0;
qf=1;


coefficients=inv([1 t0 t0^2 t0^3;0 1 2*t0 3*t0^2;1 tf tf^2 tf^3;0 1 2*tf 3*tf^

a0 = coefficients(1);
a1 = coefficients(2);
a2 = coefficients(3);
a3 = coefficients(4);

t = 0:0.1:2;

q=a0+a1*t+a2*t.^2+a3*t.^3;

% Plot the function
plot(t, q);

% Label the axes
xlabel('Time (s)');
ylabel('Angle (rad)');
title('Plot of Trajectory over Time');
```

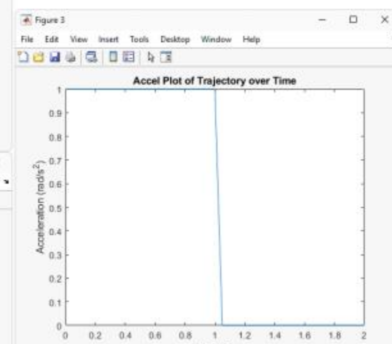


7-22

```
clc;
clear;
close all;

syms t
tf = 2;
vf = 1;
v0 = 0; % at rest
q0 = 0; % Need assumptions
t0 = 0;
qf = 1;
V = 1;

tb = (q0 - qf + V * tf) / V;
alpha = V / tb;

t1 = t0:0.01:tb;

% Define q1 as a function of t
q1 = q0 + alpha * t^2 / 2;
q1_dot=diff(q1)
q1_dot_dot=diff(q1_dot)

t2 = tb + 0.05:0.05:tf;

% Define q2 as a function of t
q2 = (qf + q0 - V * tf) / 2 + V * t;
q2_dot=diff(q2);
q2_dot_dot=diff(q2_dot)

% Substitute values of t into q1 and q2
q1_vals = subs(q1, t, t1);
q2_vals = subs(q2, t, t2);
q1_dot_vals = subs(q1_dot, t, t1);
q2_dot_vals = subs(q2_dot, t, t2);
q1_dot_dot_vals = subs(q1_dot_dot, t, t1);
q2_dot_dot_vals = subs(q2_dot_dot, t, t2);
% Concatenate time and position vectors
t3 = [t1, t2];
q = [q1_vals, q2_vals];

q_dot = [q1_dot_vals, q2_dot_vals];
q_dot_dot = [q1_dot_dot_vals, q2_dot_dot_vals];

% Plot the function

figure(1)
plot(t3, q);


% Label the axes
xlabel('Time (s)');
ylabel('Angle (rad)');
title('Plot of Trajectory over Time');

figure(2)
```







A-2

```
clc;
clear;
close all;

syms t1 t2

tf_1 = 5;
vf_1 = 5;
v0_1 = 0; % at rest
q0_1 = 20;
t0_1 = 0;
qf_1 = 40;

coefficients_1 = inv([1 t0_1 t0_1^2 t0_1^3; 0 1 2*t0_1 3*t0_1^2; 1 tf_1 tf_1^2 tf_1^3; 0 1 2*tf_1 3*tf_1^2]) * [q0_1;
a0_1 = coefficients_1(1);
a1_1 = coefficients_1(2);
a2_1 = coefficients_1(3);
a3_1 = coefficients_1(4);

q_1 = a0_1 + a1_1*t1 + a2_1*t1.^2 + a3_1*t1.^3;
q_dot_1 = diff(q_1);
q_dot_dot_1 = diff(q_dot_1);

t_1 = 0:0.1:tf_1;
q1 = subs(q_1, t1, t_1);
q_dot_1 = subs(q_dot_1, t1, t_1);
q_dot_dot_1 = subs(q_dot_dot_1, t1, t_1);

tf_2 = 10;
vf_2 = 0;
v0_2 = 5;
q0_2 = 40;
t0_2 = 5;
qf_2 = 125;

coefficients_2 = inv([1 t0_2 t0_2^2 t0_2^3; 0 1 2*t0_2 3*t0_2^2; 1 tf_2 tf_2^2 tf_2^3; 0 1 2*tf_2 3*tf_2^2]) * [q0_2;
a0_2 = coefficients_2(1);
a1_2 = coefficients_2(2);
a2_2 = coefficients_2(3);
a3_2 = coefficients_2(4);

q_2 = a0_2 + a1_2*t2 + a2_2*t2.^2 + a3_2*t2.^3;
q_dot_2 = diff(q_2);
q_dot_dot_2 = diff(q_dot_2);

t_2 = 5:0.2:tf_2;
q2 = subs(q_2, t2, t_2);
q_dot_2 = subs(q_dot_2, t2, t_2);
q_dot_dot_2 = subs(q_dot_dot_2, t2, t_2);

q = [q1, q2];
q_dot = [q_dot_1, q_dot_2];
q_dot_dot = [q_dot_dot_1, q_dot_dot_2];
t = [t_1, t_2];

% Plot the function
figure;
plot(t, q);
xlabel('Time (s)');
ylabel('Angle (deg)');
title('Plot of Trajectory over Time');

% Plot velocity
figure;
plot(t, q_dot);
xlabel('Time (s)');
ylabel('Velocity (deg/s)');
title('Plot of Velocity over Time');

% Plot acceleration
figure;
plot(t, q_dot_dot);
xlabel('Time (s)');
ylabel('Acceleration (deg/s^2)');
title('Plot of Acceleration over Time');
```