ME3525 Syllabus

Monday, January 15, 2024 9:24 PM

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Syllabus for
HEAT TRANSFER ME 3525

Introduction and Course Description

SYLLABUS FOR HEAT TRANSFER (ME 3525)	
Semester: Spring 2024	School: Missouri S&T
Course Information: Heat Transfer (ME 3525)	Instructor Name: Zhi Liang
Units: 3 lecture hours	Office Number: Toomey 229
Time: TTh 11:00-12:15 PM	E-Mail: zlch5@mst.edu
Location: Computer Science 216	Telephone: 573-341-4982
Website: canvas.umsystem.edu	Office Hours: MW 3:30 PM – 5:30 PM.

Zoom link:
https://umsystem.zoom.us/j/4265942278?pwd=V1U1QU54b1ZJOUxPQ1hXNEZTbEpwUT09

COVID Update on Classroom Instruction

- For the Spring 2024 semester, in-person courses and assessments are scheduled without distancing between students.
- Staying home when you are sick and seeking testing when you have symptoms of COVID-19 are measures to help reduce the spread of the virus.
- Students who are ill should contact Student Health Services (mstshs@mst.edu), 573-341-4284.
- **If a student is isolating**, the student will receive an absence note from Student Health. **The student will be responsible of forwarding the absence note to their instructors**.
- **To protect against serious illness from COVID-19, everyone aged 5 years and older is recommended to get one dose of an updated COVID-19 vaccination.**
- The instructor will send an email to students to explain how the course will continue in the event of instructor emergency.

COURSE DESCRIPTION

Course description: Fundamental principles of heat transmission by radiation, conduction, and convection; application of these principles to the solution of engineering problems.

Prerequisites

A grade of "C" or better each of Comp Sci 1570 or 1970 or 1971 or 1972, Math 3304, ME 2519.

Required Textbook and Materials

Heat and Mass Transfer: Fundamentals & Applications, Yunus A. Çengel and Afshin J. Ghajar, McGraw-Hill, $5^{th}/6^{th}$ Edition.

AutoAccess

This course is part of our AutoAccess program designed to reduce the cost of course materials for students. You will be able to access the digital content for this course through Canvas on the first day of class automatically. Your student account will be charged for the cost of the digital course material. We have helped save students over $54 million by providing digital content over the last 9 years. The lowest cost content has been sourced. If you choose to opt out of the content, please do so by **January 30, 2024** to receive a refund**.** You will be sent an AutoAccess Welcome Email that will provide charge amounts, the opt-out process and any additional information needed for your AutoAccess course(s) beginning January 3, 2024

Your AutoAccess course may have a Print Upgrade available as an additional purchase. This is a low-cost version of the printed text made available by the publisher at a reduced cost. It is the publisher's requirement that in order to purchase the additional print, you must be opted in for the AutoAccess digital required material. If you have questions about Print Upgrades and opting out, please contact us at autoaccess.thesandtstore.com. If you have questions please contact call 573-341-4705, email autoaccess@mst.edu or visit autoaccess.thesandtstore.com.

Examinations and Major Assignments

Examinations:

There will be two midterm tests (40% of course) and one comprehensive final exam (25% of course) scheduled. The dates for the tests are shown in the **tentative** course schedule on P8 and P9.

Make-up Policy:

Attendance at exams and lectures is mandatory. A makeup exam will be given only under certain extreme circumstances because it is very hard to give a makeup exam that has the same difficulty level as the regular exam.

Major Assignments:

Homework assignments (5% of course) will be assigned roughly once a week and will be due, typically, one week after assignment. You may do your homework with others if you like, but your motivation must be to understand the material and not simply to copy what someone else has struggled with (exam grades are a strong reflection of how well you understood homework problems).

Homework Requirements: Homework assignments will be posted to Canvas. Homework will be graded for neatness. You must clearly identify what is given in the problem, what you are required to find, and show your units and assumptions clearly through the entire problem. Work that is not legible or does not include units will not receive credit. Homework will be requested at the beginning of class. Late homework will only be accepted when a student has a reasonable excuse.

Participation (20% of course): To earn participation points, a student must answer questions in class. By giving a correct answer to a question asked by the instructor, a student can earn 5 points each time. To receive a full grade of participation, a student must earn 100 or more points by the end of the semester. At the beginning of the semester, the instructor gives every student 20 participation points. If a student cannot give a correct answer to a **review question** which is provided on Canvas before the lecture, the instructor will take 5 points off. In this case, to get the 5 points back, the student must provide correct written answers to all review questions of that chapter. If a student cannot give a correct answer to a **regular question**, the instructor will not take points off. By the end of the semester, if a student gets more than 100 participation points, he/she will get (participation points - 100)/5 bonus points to his/her final exam.

If a student is absent for <u>three or more times</u>, he/she will fail this course. An absence can be excused if a student can provide a doctor's note or a proof of other emergencies.

Project: One project (10% of course) is planned for the semester. Final computer-generated reports with complete lists of references are to be prepared by the student.

Study Expectations. Consider using the following statement:

It is usually expected that students will spend approximately 2 hours of study time outside of class for every one hour in class. Since this is a 3-unit class, you should expect to study an average of 6 hours outside of class each week. Some students may need more outside study time and some less.

Grading

<u>Semester Grade:</u> Traditional letter grading will be assigned at the end of the semester according to the following weighting:

Homework	5%
Participation	20%
Project	10%
2 Exams	40%
Final Exam	25%
Student's Total Score	100%

All grades will be assigned as follows:

Students' Total Scores	Letter Grade
90.0-100%	A
80.0-90.0%	B
70.0-80.0%	C
60.0-70.0%	D
Below 60.0%	F

The instructor reserves the right to curve exam scores as appropriate.

Contingency Information

- If a student becomes ill and is unable to attend class, the student should contact Student Health Services (mstshs@mst.edu), 573-341-4284. In this case, the student will receive an absence note from Student Health and not Student Support and Community Standards (Care Management). The student will be responsible of forwarding the absence note to their instructors.
- In the event of instructor emergency, the in-person lecture will be switched to virtual lecture. The instructor will send a zoom link for the virtual lecture.

Accessibility and Accommodations

It is the university's goal that learning experiences be as accessible as possible. If you anticipate or experience physical or academic barriers based on a disability, please contact Student Accessibility and Testing at (573) 341-6655, email dss@mst.edu, or visit https://saat.mst.edu/ for information.

Student Support and Community Standards is your "Google Maps" for support. During your time at S&T, you or a friend may need help navigating their student experience, facing a barrier, or experiencing a challenge. You are not alone!

Student Support has a dedicated team and numerous resources such as UCARE and the student emergency fund to help you navigate the S&T experience and support your success. This includes support to address barriers related to academic, personal, emotional, medical, financial, or any other needs. All students can learn and grow from challenges or setbacks, they are stepping stones to success and we are here to help.

Writing and Communication Center

The Writing and Communication Center's mission is to assist all students in their efforts to become better writers, communicators, and critical thinkers. The Center's peer consultants and coaches provide free individualized one-on-one and small-group conversations to offer meaningful feedback and guidance to students across all disciplines. More information can be found on our website, through email: writing@mst.edu or stop by Curtis Laws Wilson Library 314–315.

Student Success Center

The Student Success Center (SSC) supports student development through peer Academic Mentoring focusing primarily on STEM courses, peer-to-peer soft skill coaching which can also act as an accountability buddy, and campus programming – all while providing free coffee and hot beverages! All undergraduate students are encouraged to utilize the SSC's free services to get timely support and to enhance their S&T Miner Experience. Contact us at success@mst.edu OR 573-341-7590. To see the course offerings and times for SSC Academic Mentoring, visit https://studentsuccess.mst.edu/academicmentoring/.

Knack Tutoring (https://mst.joinknack.com/)

Enrolled S&T undergraduate students can receive complimentary **FREE** tutoring assistance from peers who have successfully completed the course, available round the clock. You have the option to connect via the Knack platform online or in person on campus. If you've excelled in a course, consider becoming a Knack Tutor to support your fellow Miners!

Student Honor Code and Academic Integrity

- All students are expected to follow the Honor Code.
- Student Academic Regulations describes the student standard of conduct relative to the University of Missouri System's Collected Rules and Regulations section 200.010, and offers descriptions of academic dishonesty including cheating, plagiarism, and sabotage, any of which will be reported to the Vice Provost for Undergraduate Education.
- Other resources for students regarding academic integrity can be found online.

I have a zero-tolerance policy for any form of academic dishonesty. **Student(s) will be penalized with a failing grade (i.e., zero points)** for the assignment, quiz, project or examination in which the infringement occurred and all violations will be reported to the department chair and the Vice Provost of Undergraduate Education.

Student Support and Community Standards has a dedicated team and numerous resources such as UCARE and the student emergency fund to help students navigate the S&T experience and support their success. This includes support to address barriers related to academic, personal, emotional, medical, financial, or any other needs. All students can learn and grow from challenges or setbacks, they are stepping stones to success and we are here to help.

Course Goals and Primary Learning Outcomes

Course Goals and Principles Covered:

1. Modes of Heat Transfer
2. Conservation of Energy
3. Conduction, Convection and Radiation Heat Transfer
4. Extended Surfaces
5. Finite Difference Method
6. Heat Exchangers
7. Multi-Mode Heat Transfer

Course Learning Objectives (CLOs):

Upon completion of this course, students should:

1) Understand the concepts in conduction, convection, and radiation heat transfer. (ME Program Student Outcomes 1)
2) Understand and apply analytical and **numerical** techniques used in steady-state and transient conduction heat transfer analysis. (ME Program Student Outcome 1)
3) Understand and apply in real life settings concepts used in fin design problems. (ME Program Student Outcome 1, 2, 5, 7)
4) Understand and apply in real life settings concepts used in heat exchanger design. (a ME Program Student Outcome 1, 2)
5) Understand the concepts used in multi-surface radiant exchange. (ME Program Student Outcome 1,2)

Overall, this course satisfies ME Programmatic Student Outcomes 1 and 2.

Student Outcomes (SOs):

1. an ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics.
2. an ability to apply engineering design to produce solutions that meet specified needs with consideration of public health, safety, and welfare, as well as global, cultural, social, environmental, and economic factors.
3. an ability to communicate effectively with a range of audiences.
4. an ability to recognize ethical and professional responsibilities in engineering situations and make informed judgments, which must consider the impact of engineering solutions in global, economic, environmental, and societal contexts.
5. an ability to function effectively on a team whose members together provide leadership, create a collaborative and inclusive environment, establish goals, plan tasks, and meet objectives
6. an ability to develop and conduct appropriate experimentation, analyze and interpret data, and use engineering judgment to draw conclusions.
7. an ability to acquire and apply new knowledge as needed, using appropriate learning strategies.

Mapping of CLOs and SOs (H=high, M=medium, L=low connection of CLO to SO)	CLO 1 Understand the concepts in conduction, convection, and radiation heat transfer	CLO 2 Understand and apply analytical and numerical techniques	CLO 3 Understand and apply in real life settings concepts used in fin design property diagrams	CLO 4 Understand and apply in real life settings concepts used in heat exchanger	CLO 5 Understand the concepts used in multi-surface radiant exchange
SO 1 identify formulate, and solve complex eng. problems by applying principles of eng., sci., and math.	**H**	**H**	**H**	**H**	**H**
SO 2 apply eng. des. to prod. sol. that meet specif. needs with consid. of public health, safety, and welfare, as well as global, cultural, social, envir., and econ. factors			**H**	**H**	**H**
SO 3 communicate effectively with a range of audiences					
SO 4 recognize ethical and prof. resp. in eng. sit. and make inform. judgments, which must consider the impact of eng. sol. in global, econ., environ., and societal contexts					
SO 5 funct. effect. on a team whose mem. together prov. leadership, create a collab. and inclusive environ., establish goals, plan tasks, and meet obj.			**M**		
SO 6 dev. and conduct appr. exp., analyze and interpret data, and use eng. judg. to draw conclusions					
SO 7 acquire and apply new know. as needed, using appropriate learning strategies			**M**		

Tentative Course Schedule

The schedule and procedures for this course are subject to change.

	Date	Reading Assignment	Topic
1	Tu., Jan. 16	1.1 – 1.4	Introduction/Thermodynamics and Heat Transfer
2	Th., Jan. 18	1.5 – 1.8	Heat Transfer Mechanisms
3	Tu., Jan. 23	1.9	Heat Transfer Mechanisms (Continued)
4	Th., Jan. 25	2.1 – 2.4	One-Dimensional Heat Conduction Equation
5	Tu., Jan. 30	2.5 – 2.7	Steady One-Dimensional Heat Conduction
6	Th., Feb. 1	3.1 – 3.3	Steady Heat Conduction, Thermal Resistance Networks
7	Tu., Feb. 6	3.4, 3.5	Heat Conduction in Cylinders and Spheres
8	Th., Feb. 8	4.1	Transient Heat Conduction in Lumped systems
9	Tu., Feb. 13	4.2	Transient Heat Conduction in 1-D systems
10	Th., Feb. 15	4.3 – 4.4	Transient Heat Conduction in Semi-infinite and 2-D/3-D systems
11	Tu., Feb. 20	5.1	Numerical Method in Heat Conduction (**Career Fair, watch video**)
12	Th., Feb. 22	5.2	Numerical Heat Transfer, Finite Difference Formulation (**Project 1**)
13	Tu., Feb. 27		**TEST 1**
14	Th., Feb. 29	3.6	Heat Transfer from Finned Surfaces
15	Tu., Mar. 5	6.1 – 6.11	Fundamentals of Convection
16	Th., Mar. 7	7.1 – 7.2	Heat Transfer in External Flow, Parallel Flow over Flat Plates
17	Tu., Mar. 12	7.3	Flow across Cylinders and Spheres
18	Th., Mar. 14		**Spring Recess (No Lecture)**
19	Tu., Mar. 19	8.1 – 8.3	Internal Forced Convection, Introduction
20	Th., Mar. 21	8.4	General Thermal Analysis of Flow in Tubes
21	Tu., Mar. 26		**Spring Break (No Lecture)**
22	Th., Mar. 28		**Spring Break (No Lecture)**
23	Tu., Apr. 2	8.5, 8.6	Laminar and Turbulent Flows in Tubes
24	Th., Apr. 4	11.1, 11.2	Introduction to Heat Exchangers
25	Tu., Apr. 9	11.3	Analysis of Heat Exchangers

26	Th., Apr. 11	11.4 – 11.6	Design of Heat Exchangers
27	Tu., Apr. 16	12.1 – 12.3	Thermal Radiation, Blackbody Radiation
28	Th., Apr. 18		**TEST 2**
29	Tu., Apr. 23	12.5	Radiative properties
30	Th., Apr. 25	12.6	Atmosphere and Solar Radiation
31	Tu., Apr. 30	13.1-13.2	View factor
32	Th., May 2	13.3-13.4	Radiation Heat Transfer

Finals week	**Days**	**Dates**	**Time**
Final Semester Examinations	Monday-Friday	May 6-10	
Final Exam in this course	Thursday	May 9	10:00am-12:00pm

Intro to Heat Transfer

Tuesday, January 16, 2024 11:28 AM

Q amount of heat transfer

$\dot{Q} = \frac{dQ}{dt}$ rate of heat transfer

$\dot{Q}_{avg} = \frac{Q}{\Delta t}$

1st Law of Thermodynamics: Energy conservation

$E_{in} - E_{out} = \Delta E_{sys}$

Heat transfer + work + mass transfer

$\Delta U + \Delta KE + \Delta PE$

const pressure, const volume

c_p for ΔU, c_v for Δh specific heat

$\dot{q} = \frac{\dot{Q}}{A}$ heat flux

EXAMPLE 1–1 Heating of a Copper Ball

A 10-cm-diameter copper ball is to be heated from 100°C to an average temperature of 150°C in 30 min (Fig. 1–14). Taking the average density and specific heat of copper in this temperature range to be $\rho = 8950\ \text{kg/m}^3$ and $c_p = 0.395\ \text{kJ/kg·°C}$, respectively, determine (a) the total amount of heat transfer to the copper ball, (b) the average rate of heat transfer to the ball, and (c) the average heat flux.

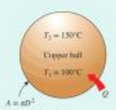


FIGURE 1–14 Schematic for Example 1–1.

SOLUTION The copper ball is to be heated from 100°C to 150°C. The total heat transfer, the average rate of heat transfer, and the average heat flux are to be determined.

Assumptions Constant properties can be used for copper at the average temperature.

Properties The average density and specific heat of copper are given to be $\rho = 8950\ \text{kg/m}^3$ and $c_p = 0.395\ \text{kJ/kg·°C}$.

Analysis (a) The amount of heat transferred to the copper ball is simply the change in its internal energy and is determined from

$$\text{Energy transfer to the system} = \text{Energy increase of the system}$$
$$Q = \Delta U = mc_{avg}(T_2 - T_1)$$

$Q + \cancel{W} + \cancel{E_{mass}} = \cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U$

where

$$m = \rho V = \frac{\pi}{6}\rho D^3 = \frac{\pi}{6}(8950\ \text{kg/m}^3)(0.1\ \text{m})^3 = 4.686\ \text{kg}$$

Substituting,

$$Q = (4.686\ \text{kg})(0.395\ \text{kJ/kg·°C})(150 - 100)\text{°C} = 92.6\ \text{kJ}$$

Therefore, 92.6 kJ of heat needs to be transferred to the copper ball to heat it from 100°C to 150°C.

(b) The rate of heat transfer normally changes during a process with time. However, we can determine the *average* rate of heat transfer by dividing the total amount of heat transfer by the time interval. Therefore,

$$\dot{Q}_{avg} = \frac{Q}{\Delta t} = \frac{92.6\ \text{kJ}}{1800\ \text{s}} = 0.0514\ \text{kJ/s} = 51.4\ \text{W}$$

(c) Heat flux is defined as the heat transfer per unit time per unit area, or the rate of heat transfer per unit area. Therefore, the average heat flux in this case is

$$\dot{q}_{avg} = \frac{\dot{Q}_{avg}}{A} = \frac{\dot{Q}_{avg}}{\pi D^2} = \frac{51.4\ \text{W}}{\pi(0.1\ \text{m})^2} = 1636\ \text{W/m}^2$$

Discussion Note that heat flux may vary with location on a surface. The value calculated here is the *average* heat flux over the entire surface of the ball.

Stationary system, no work:

$Q = mc_v\Delta T$

$\dot{Q}_{in} - \cancel{\dot{W}_{out}} = \dot{m}(\cancel{\Delta ke} + \cancel{\Delta pe} + \Delta h)$

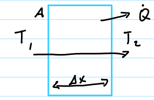
EXAMPLE 1–3 Heat Loss from Heating Ducts in a Basement

A 5-m-long section of an air heating system of a house passes through an unheated space in the basement (Fig. 1–21). The cross section of the rectangular duct of the heating system is 20 cm × 25 cm. Hot air enters the duct at 100 kPa and 60°C at an average velocity of 5 m/s. The temperature of the air in the duct drops to 54°C as a result of heat loss to the cool space in the basement. Determine the rate of heat loss from the air in the duct to the basement under steady conditions. Also, determine the cost of this heat loss per hour if the house is heated by a natural gas furnace that has an efficiency of 80 percent and the cost of the natural gas in that area is \$1.60/therm(1 therm = 100,000

Heat transfer modes:

Conduction

particle interaction (vibrations + free flow of electrons in solids, collisions + diffusion in fluids) thermal conductivity (material property)



$\boxed{\dot{Q}_{cond} = -kA\frac{dT}{dx}}$ Fourier's law of heat conduction

EXAMPLE 1–5 The Cost of Heat Loss Through a Roof

The roof of an electrically heated home is 6 m long, 8 m wide, and 0.25 m thick, and it is made of a flat layer of concrete whose thermal conductivity is $k = 0.8\ \text{W/m·K}$ (Fig. 1–27). The temperatures of the inner and the outer surfaces of the roof one night are measured to be 15°C and 4°C, respectively, for a period of 10 h. Determine (a) the rate of heat loss through the roof that night and (b) the cost of that heat loss to the homeowner if the cost of electricity is \$0.08/kWh.

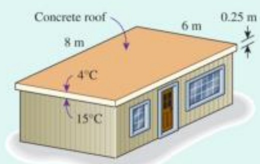


FIGURE 1–27 Schematic for Example 1–5.

SOLUTION The inner and outer surfaces of the flat concrete roof of an electrically heated home are maintained at specified temperatures during a night. The heat loss through the roof and its cost that night are to be determined.

Assumptions 1 Steady operating conditions exist during the entire night since the surface temperatures of the roof remain constant at the specified values. 2 Constant properties can be used for the roof.

Properties The thermal conductivity of the roof is given to be $k = 0.8\ \text{W/m·K}$.

Analysis (a) Noting that heat transfer through the roof is by conduction and the area of the roof is $A = 6\ \text{m} \times 8\ \text{m} = 48\ \text{m}^2$, the steady rate of heat transfer through the roof is

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8\ \text{W/m·K})(48\ \text{m}^2)\frac{(15-4)\text{°C}}{0.25\ \text{m}} = 1690\ \text{W} = 1.69\ \text{kW}$$

(b) The amount of heat lost through the roof during a 10-h period and its cost are

$$Q = \dot{Q}\Delta t = (1.69\ \text{kW})(10\ \text{h}) = 16.9\ \text{kWh}$$
$$\text{Cost} = (\text{Amount of energy})(\text{Unit cost of energy}) = (16.9\ \text{kWh})(\$0.08/\text{kWh}) = \$1.35$$

Discussion The cost to the homeowner of the heat loss through the roof that night was \$1.35. The total heating bill of the house will be much larger since the heat losses through the walls are not considered in these calculations.

TABLE 1–1

The thermal conductivities of some materials at room temperature

Material	k, W/m·K
Diamond	2300 ← poor electrical conductor
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.607
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

good electrical conductor (Silver – Iron)

TABLE 1–2

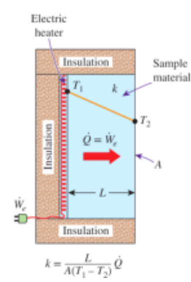
The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

Pure Metal or Alloy	k, W/m·K, at 300 K
Copper	401
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze (90% Cu, 10% Al)	52

TABLE 1–3

Thermal conductivities of materials vary with temperature

	k, W/m·K	
T, K	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218



Experiment to determine k

Convection

fluid in motion contacts solid

Newton's law of cooling

$\boxed{\dot{Q}_{conv} = hA_s(T_{surface} - T_\infty)}$ ← temp of fluid far from surface

convection heat transfer coefficient

TABLE 1–5

Typical values of convection heat transfer coefficient

Type of Convection	h, W/m²·K*
Free convection of gases	2–25
Free convection of liquids	10–1000
Forced convection of gases	25–250
Forced convection of liquids	50–20,000
Boiling and condensation	2500–100,000

EXAMPLE 1–8 Measuring Convection Heat Transfer Coefficient

A 2-m-long, 0.3-cm-diameter electrical wire extends across a room at 15°C, as shown in Fig. 1–37. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 60 V and 1.5 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

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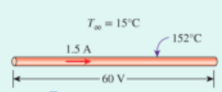

FIGURE 1–37 Schematic for Example 1–8.

the cool space in the basement. Determine the rate of heat loss from the air in the duct to the basement under steady conditions. Also, determine the cost of this heat loss per hour if the house is heated by a natural gas furnace that has an efficiency of 80 percent and the cost of the natural gas in that area is $\$1.60/\text{therm}$ ($1\ \text{therm} = 100{,}000\ \text{Btu} = 105{,}500\ \text{kJ}$).

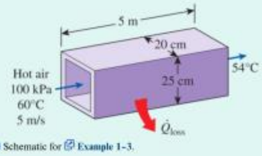

FIGURE 1–21 Schematic for Example 1–3.

SOLUTION The temperature of the air in the heating duct of a house drops as a result of heat loss to the cool space in the basement. The rate of heat loss from the hot air and its cost are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air can be treated as an ideal gas with constant properties at room temperature.

Properties The constant-pressure specific heat of air at the average temperature of $(54 + 60)/2 = 57°\text{C}$ is $1.007\ \text{kJ/kg·K}$ (Table A–15).

Analysis We take the basement section of the heating system as our system, which is a steady-flow system. The rate of heat loss from the air in the duct can be determined from

$$\dot{Q} = \dot{m} c_p \Delta T$$

where $\dot{m}$ is the mass flow rate and ΔT is the temperature drop. The density of air at the inlet conditions is

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$$\rho = \frac{P}{RT} = \frac{100\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg·K})(60+273)\text{K}} = 1.046\ \text{kg/m}^3$$

The cross-sectional area of the duct is

$$A_c = (0.20\ \text{m})(0.25\ \text{m}) = 0.05\ \text{m}^2$$

Then the mass flow rate of air through the duct and the rate of heat loss become

$$\dot{m} = \rho V A_c = (1.046\ \text{kg/m}^3)(5\ \text{m/s})(0.05\ \text{m}^2) = 0.2615\ \text{kg/s}$$

and

$$\begin{aligned}\dot{Q}_{loss} &= \dot{m} c_p (T_{in} - T_{out}) \\ &= (0.2615\ \text{kg/s})(1.007\ \text{kJ/kg·°C})(60-54)°\text{C} \\ &= 1.58\ \text{kJ/s}\end{aligned}$$

or $5688\ \text{kJ/h}$. The cost of this heat loss to the homeowner is

$$\begin{aligned}\text{Cost of heat loss} &= \frac{(\text{Rate of heat loss})(\text{Unit cost of energy input})}{\text{Furnace efficiency}} \\ &= \frac{(5688\ \text{kJ/h})(\$1.60/\text{therm})}{0.80}\left(\frac{1\ \text{therm}}{105{,}500\ \text{kJ}}\right) \\ &= \$0.108/\text{h}\end{aligned}$$

Discussion The heat loss from the heating ducts in the basement is costing the homeowner 10.8 cents per hour. Assuming the heater operates 2000 hours during a heating season, the annual cost of this heat loss adds up to \$216. Most of this money can be saved by insulating the heating ducts in the unheated areas.

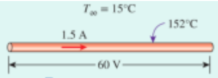


FIGURE 1–37 Schematic for Example 1–8.

SOLUTION The convection heat transfer coefficient for heat transfer from an electrically heated wire to air is to be determined by measuring temperatures when steady operating conditions are reached and the electric power consumed.

Assumptions **1** Steady operating conditions exist since the temperature readings do not change with time. **2** Radiation heat transfer is negligible.

Analysis When steady operating conditions are reached, the rate of heat loss from the wire equals the rate of heat generation in the wire as a result of resistance heating. That is,

$$\dot{Q} = \dot{E}_{generated} = VI = (60\text{V})(1.5\text{A}) = 90\ \text{W}$$

The surface area of the wire is

$$A_s = \pi D L = \pi(0.003\ \text{m})(2\ \text{m}) = 0.01885\ \text{m}^2$$

Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q}_{conv} = h A_s (T_s - T_\infty)$$

Disregarding any heat transfer by radiation and thus assuming all the heat loss from the wire occurs by convection, the convection heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}_{conv}}{A_s(T_s - T_\infty)} = \frac{90\ \text{W}}{(0.01885\ \text{m}^2)(152-15)°\text{C}} = 34.9\ \text{W/m}^2\cdot\text{K}$$

Discussion Note that the simple setup described above can be used to determine the average heat transfer coefficients from a variety of surfaces in air. Also, heat transfer by radiation can be eliminated by keeping the surrounding surfaces at the temperature of the wire.

Radiation

energy emitted by electromagnetic waves / photons

Stefan - Bolteman law

$$\boxed{\dot{Q}_{emit,max} = \sigma A_s T_s^4}$$

$\sigma = 5.670 \times 10^{-8}\ W/m^2\cdot K^4$ or $.0174 \times 10^{-8}\ Btu/h\ ft^2\ R^4$

black body: idealized surface that emits radiation at a max rate
($\varepsilon = 1$, $\alpha = 1$)

$\dot{Q}_{emit} = \varepsilon \sigma A_s T_s^4$

emissivity

$0 \le \varepsilon \le 1$

$\dot{Q}_{absorbed} = \alpha \dot{Q}_{incident}$

absorptivity $0 \le \alpha \le 1$

if surrounded by air or other gas:

$\dot{Q}_{rad,net} = \varepsilon \sigma A_s (T_s^4 - T_{surroundings}^4)$

TABLE 1–6

Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

Chap 1 Review

Thursday, January 18, 2024 12:58 PM

Chap.+1

Chapter 1 Review questions:

1. In the 1st law of thermodynamics $E_{in} - E_{out} = \Delta E$, what are the meanings of E_{in}, E_{out}, and ΔE, respectively?

2. What are the three forms of energy transfer across a system boundary? What are the three forms of energy usually considered in a system?

3. By definition, what is the difference between a closed system and an open system?

4. Give an example of a stationary closed system with no work. What is the equation when the 1st law of thermodynamics is applied to such a system?

5. Give an example of a steady flow open system with no work. What is the equation when the 1st law of thermodynamics is applied to such a system?

6. In heat transfer, what are the meanings of Q, $\dot{Q}$, and $\dot{q}$, respectively? How are these three quantities defined? What are the units of Q, $\dot{Q}$, and $\dot{q}$, respectively?

7. Which law is used to determine the heat conduction rate? What is the meaning and unit of k in this law?

8. Is the heat transfer between a solid and the adjacent fluid always by convection?

9. Which law is used to determine the heat convection rate? What is the meaning and unit of h in this law?

10. What is the definition of a blackbody surface? ε = ? at a blackbody surface.

11. Which law is used to determine the thermal radiation rate? Which temperature must be used in this law? What are the meanings and units of ε and σ in this law?

12. When is the equation $\dot{Q}_{rad.,net} = \varepsilon\sigma A_s\left(T_s^4 - T_{surr.}^4\right)$ valid?

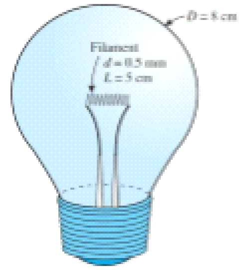
HW1

Thursday, January 18, 2024 12:58 PM

HW1

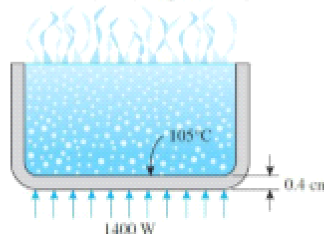
1. Read Chapter 1.

2. Consider a 150-W incandescent lamp. The filament of the lamp is 5-cm long and has a diameter of 0.5 mm. The diameter of the glass bulb of the lamp is 8 cm. Determine the heat flux, in W/m^2, (*a*) on the surface of the filament and (*b*) on the surface of the glass bulb, and (*c*) calculate how much it will cost per year to keep that lamp on for eight hours a day every day if the unit cost of electricity is $0.08/kWh.

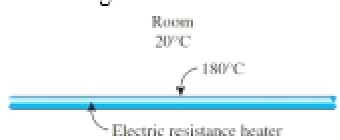


3. Water is heated in an insulated, constant diameter tube by a 5-kW electric resistance heater. If the water enters the heater steadily at 15°C and leaves at 60°C, determine the mass flow rate of water.

4. An aluminum pan whose thermal conductivity is 237 W/m·K has a flat bottom with diameter 15 cm and thickness 0.4 cm. Heat is transferred steadily to boiling water in the pan through its bottom at a rate of 1400 W. If the inner surface of the bottom of the pan is at 105°C, determine the temperature of the outer surface of the bottom of the pan.



5. A 2.1-m-long, 0.2-cm-diameter electrical wire extends across a room that is maintained at 20°C. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 180°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 110 V and 3 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.



6. Consider a person whose exposed surface area is 1.7 m^2, emissivity is 0.5, and surface temperature is 32°C. Determine the rate of heat loss from that person by radiation in a large room having walls at a temperature of (*a*) 27°C and (*b*) 7°C.

2. $\dot{Q} = 150w$

a. $\dot{q} = \frac{\dot{Q}}{A}$

$A_{cyl} = \pi d L + \frac{1}{2}\pi d^2$

$= \pi(.0005m)(.05m) + \frac{1}{2}\pi(.0005m)^2$

$= .000079\ m^2$

$\dot{q} = \frac{150w}{.000079m^2} = 1,900,000\ \frac{w}{m^2}$

b. $A_{sph} = \pi D^2 = (.08m)^2\pi = .0201\ m^2$

$\dot{q} = \frac{150w}{.0201} = 7460.39\ w/m^2$

c. $.15\ kW \ \frac{.08\$}{1 kW\cdot hr}\ \frac{8hr}{1d}\ \frac{365d}{1y} = 35.04\ \frac{\$}{y}$

3. $\dot{Q} = \dot{m} c_p \Delta T$

$\dot{m} = \frac{\dot{Q}}{C_p(T_2 - T_1)}$

$T_{avg} = \frac{60+15}{2} = 37.5$

C_p liquid water @37.5°C = 4178.5 J/kg·k

Table A-9

$\dot{m} = \frac{5000w}{(4178.5 J/kg\cdot k)(60-15)} = 53.1887\ \frac{kg}{s}$

4. $\dot{Q} = kA\frac{(T_1 - T_2)}{L}$

$A = \frac{\pi(.15m)^2}{4} = .017671\ m^2$

$L = .004\ m$

$k = 237\ W/m\cdot k$

$\dot{Q} = -1400w$

$T_1 = 105°C = 378K$

$T_2 = T_1 - \frac{\dot{Q}L}{kA}$

$= 378k - \frac{-1400w(.004m)}{237 W/m\cdot k(.017671m^2)}$

$T_2 = 376.337\ K$

5.

$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$

$A_s = \pi D L = \pi(.002m)(2.1m) = .013195\ m^2$

$h = \frac{\dot{Q}_{conv}}{A_s(T_s - T_\infty)}$

$\dot{Q} = (110V)(3A) = 330\ w$

$h = \frac{330w}{.013195m^2(180-20)k} = 156.313\ W/m^2\cdot k$

6. $\dot{Q}_{rad,net} = \varepsilon \sigma A_s (T_s^4 - T_{surroundings}^4)$

$\sigma = 5.670 \times 10^{-8}\ W/m^2 \cdot K^4$

$\varepsilon = .5$

$A_s = 1.7 m^2$

$T_s = 32°C = 305\ K$

a. $T_{surr} = 27°C = 300K$

$\dot{Q}_{rad} = .5\ (5.670 \times 10^{-8}\ W/m^2 \cdot K^4)(1.7m^2)((305\ K)^4 - (300\ K)^4)$

$= 26.68\ W$

b. $T_{surr} = 7°C = 280\ K$

$\dot{Q}_{rad} = .5\ (5.670 \times 10^{-8}\ W/m^2 \cdot K^4)(1.7m^2)((305\ K)^4 - (280\ K)^4)$

$= 120.87\ W$

Heat Conduction Equation

Tuesday, January 23, 2024 11:35 AM

$\dot{e}_{gen} = \frac{\dot{E}_{gen}}{V}$ ← heat generation rate
← volume

EXAMPLE 2–1 Heat Generation in a Hair Dryer

The resistance wire of a 1200-W hair dryer is 80 cm long and has a diameter of $D = 0.3cm$ (Fig. 2–11). Determine the rate of heat generation in the wire per unit volume, in W/cm^3, and the heat flux on the outer surface of the wire as a result of this heat generation.

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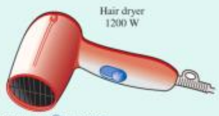


FIGURE 2–11 Schematic for Example 2–1.

SOLUTION The power consumed by the resistance wire of a hair dryer is given. The heat generation and the heat flux are to be determined.

Assumptions Heat is generated uniformly in the resistance wire.

Analysis A 1200-W hair dryer converts electrical energy into heat in the wire at a rate of 1200 W. Therefore, the rate of heat generation in a resistance wire is equal to the power consumption of a resistance heater. Then the rate of heat generation in the wire per unit volume is determined by dividing the total rate of heat generation by the volume of the wire,

$$\dot{e}_{gen} = \frac{\dot{E}_{gen}}{V_{wire}} = \frac{\dot{E}_{gen}}{(\pi D^2/4)L} = \frac{1200\ W}{[\pi(0.3cm)^2/4](80cm)} = 212\ W/cm^3$$

Similarly, heat flux on the outer surface of the wire as a result of this heat generation is determined by dividing the total rate of heat generation by the surface area of the wire,

$$\dot{Q}_s = \frac{\dot{E}_{gen}}{A_{wire}} = \frac{\dot{E}_{gen}}{\pi DL} = \frac{1200\ W}{\pi(0.3cm)(80cm)} = 15.9\ W/cm^2$$

One dimensional heat conduction:

const conductivity: from 1st law of thermo and Fourier's law of heat conduction

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$\alpha = \frac{k}{\rho c}$ = thermal diffusivity = how fast heat propagates $[m^2/s]$

k ← thermal conductivity

c ← specific heat

1. steady state: $\frac{\partial T}{\partial t} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{k} = 0$$

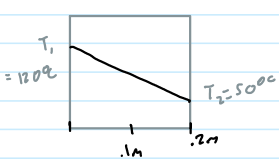
2. no heat generation: $\dot{e}_{gen} = 0$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

3. Both

$$\frac{\partial^2 T}{\partial x^2} = 0$$

Ex.



Steady state
no heat generation

$$\frac{\partial^2 T}{\partial x^2} = 0$$

$T = C_1 x + C_2$

$T(x=0) = T_1 = 120 = C_2$

$T(x=L) = C_1 L + C_2 = T_2$

$C_1 = \frac{50-120}{.2} = -350$

$T(x=L) = C_1 L + C_2 = T_2$

$C_1 = \frac{50-120}{.2} = -350$

$T = -350x + 120$

Ex 2

EXAMPLE 2–12 Heat Conduction in the Base Plate of an Iron

Consider the base plate of a 1200-W household iron that has a thickness of $L = 0.5\,\text{cm}$, base area of $A = 300\,\text{cm}^2$, and thermal conductivity of $k = 15\,\text{W/m·K}$. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside, and the outer surface loses heat to the surroundings at $T_\infty = 20^\circ\text{C}$ by convection, as shown in Fig. 2–45. Taking the convection heat transfer coefficient to be $h = 80\,\text{W/m}^2\text{·K}$ and disregarding heat loss by radiation, obtain an expression for the variation of temperature in the base plate, and evaluate the temperatures at the inner and the outer surfaces.

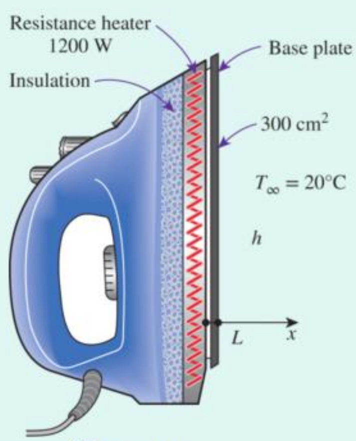


FIGURE 2–45 Schematic for Example 2-12.

SOLUTION The base plate of an iron is considered. The variation of temperature in the plate and the surface temperatures are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since the surface area of the base plate is large relative to its thickness, and the thermal conditions on both sides are uniform. **3** Thermal conductivity is constant. **4** There is no heat generation in the medium. **5** Heat transfer by radiation is negligible. **6** The upper part of the iron is well insulated, so all heat generated in the resistance wires is transferred to the base plate through its inner surface.

Properties The thermal conductivity is given to be $k = 15\,\text{W/m·K}$.

Analysis The inner surface of the base plate is subjected to uniform heat flux at a rate of

$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{1200\,W}{0.03\,\text{m}^2} = 40{,}000\,\text{W/m}^2$$

The outer side of the plate is subjected to the convection condition. Taking the direction normal to the surface of the wall as the x-direction with its origin on the inner surface, the differential equation for this problem can be expressed as (Fig. 2–46)

$$\frac{d^2T}{dx^2} = 0$$

with the boundary conditions

$$-k\frac{dT(0)}{dx} = \dot{q}_0 = 40{,}000\,\text{W/m}^2$$

$$-k\frac{dT(L)}{dx} = h\left[T(L) - T_\infty\right]$$

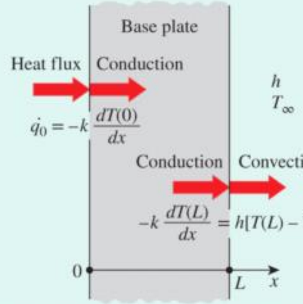


FIGURE 2–46 The boundary conditions on the base plate of the iron discussed in Examp

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The general solution to the differential equation is again obtained by two successive integrations to be

$$\frac{dT}{dx} = C_1$$

and

$$T(x) = C_1x + C_2 \qquad (a)$$

where C_1 and C_2 are arbitrary constants. Applying the first boundary condition,

$$-k\frac{dT(0)}{dx} = \dot{q}_0 \rightarrow -kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

Noting that $dT/dx = C_1$ and $T(L) = C_1L + C_2$, the application of the second boundary condition gives

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty] \rightarrow -kC_1 = h[(C_1L + C_2) - T_\infty]$$

Substituting $C_1 = -\dot{q}_0/k$ and solving for C_2, we obtain

$$C_2 = T_\infty + \frac{\dot{q}_0}{h} + \frac{\dot{q}_0}{k}L$$

Now substituting C_1 and C_2 into the general solution (a) gives

$$T(x) = T_\infty + \dot{q}_0\left(\frac{L-x}{k} + \frac{1}{h}\right) \qquad (b)$$

which is the solution for the variation of the temperature in the plate. The temperatures at the inner and outer surfaces of the plate are determined by substituting $x = 0$ and $x = L$, respectively, into the relation (b):

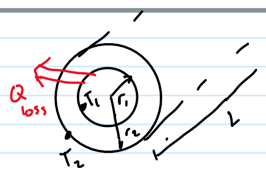
$$\begin{aligned} T(0) &= T_\infty + \dot{q}_0\left(\frac{L}{k} + \frac{1}{h}\right) \\ &= 20^\circ\text{C} + (40{,}000\ \text{W/m}^2)\left(\frac{0.005\ \text{m}}{15\ \text{W/m}\cdot\text{K}} + \frac{1}{80\ \text{W/m}^2\cdot\text{K}}\right) = 533^\circ\text{C} \end{aligned}$$

and

$$T(L) = T_\infty + \dot{q}_0\left(0 + \frac{1}{h}\right) = 20^\circ\text{C} + \frac{40{,}000\ \text{W/m}^2}{80\ \text{W/m}^2\cdot\text{K}} = 520^\circ\text{C}$$

Discussion Note that the temperature of the inner surface of the base plate is 13°C higher than the temperature of the outer surface when steady operating conditions are reached. Also note that this heat transfer analysis enables us to calculate the temperatures of surfaces that we cannot even reach. This example demonstrates how the heat flux and convection boundary conditions are applied to heat transfer problems.

Ex.



$r_1 = .06\ m$
$r_2 = .08\ m$
$L = 20\ m$
$T_1 = 150°C$
$T_2 = 60°C$
$k = 20\ W/m\cdot K$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{\partial T}{\partial r}\right) + \cancel{\frac{\dot{e}_{gen}}{k}}^{\,0} \quad \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}^{\,0}$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$T(r) = c_1 \ln(r) + c_2$$

$$C_1 = \frac{T_2 - T_1}{\ln r_2 - \ln r_1} \qquad C_2 = T_1 - \frac{T_2 - T_1}{\ln r_2 - \ln r_1}\ln r_1$$

$$\dot{Q}_{loss} = -kA\frac{dT}{dr} = -k(2\pi r L)\frac{C_1}{r} = -k\,2\pi L\,\frac{T_1 - T_2}{\ln r_2 - \ln r_1}$$

$$\dot{Q}_{loss} = 786\ kW$$

$T_s = 105°C$

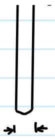
$L = 5\ m$

$T_\infty = 100°C$

Find T @ wire center

$k = 15\ W/m\cdot K$
$\dot{W}_e = 2000\ W$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{gen}}{k} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}^{\,0}$$



$T_w = 100°C$

$D = .004\,m$ $\quad R = .002\,m$

$W_e = 6000\ W$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\cancelto{0}{\frac{\partial T}{\partial t}}$$

$$\dot{e}_{gen} = \frac{\dot{E}_{gen}}{V} = \frac{\dot{W}_e}{\frac{\pi}{4}D^2 L} = 3.18\times10^{8}\ W/m^3$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{e}_{gen}}{k}r$$

$$r\frac{dT}{dr} = -\frac{\dot{e}_{gen}}{2k}r^2 + C_1$$

BC 1: $T(r=R) = T_s$

BC 2: $\dot{Q}_{left}(r=R) = \dot{Q}_{right}(r=R)$

Thermal Symmetry $\rightarrow C_1 = 0$

$$\frac{dT}{dr} = -\frac{\dot{e}_{gen}}{2k}r$$

$$T(r) = -\frac{\dot{e}_{gen}}{4k}r^2 + C_2$$

$$C_2 = T_s + \frac{\dot{e}_{gen}}{4k}R^2$$

$$T(r) = \frac{\dot{e}_{gen}}{4k}r^2 + T_s + \frac{\dot{e}_{gen}}{4k}R^2$$

$$T(r=0) = T_s + \frac{\dot{e}_{gen}}{4k}R^2 = 126°C$$

Chap. 2 Review

Thursday, January 25, 2024 11:14 AM

Chap.+2

Chapter 2 Review questions:

1. What is the meaning of **one-dimensional** heat conduction?

2. What are the meanings and units of $\dot{E}_{gen}$ and $\dot{e}_{gen}$, respectively?

3. What is the 1-D heat conduction equation in a rectangular coordinate system? Which law(s) do we use to derive this equation?

4. What is α in the 1-D heat conduction equation? What is the definition, unit, and physical meaning of α?

5. What does steady state mean in the 1-D heat conduction equation?

6. How many boundary conditions do we need to solve the 1-D heat conduction equation?

7. If the temperature at a system boundary is not given or unknown, can we still use the temperature boundary condition to solve the 1-D heat conduction equation? If not, what boundary condition(s) should we use in this case?

8. What is the 1-D heat conduction equation in a cylindrical coordinate system?

9. The **1-D** heat conduction equation in a cylindrical coordinate system can be used to study heat conduction in a long cylinder. Why can we approximate a long cylinder as a **1-D** system?

10. If diameter is given, how to calculate the perimeter and area of a circle? If diameter is given, how to calculate the surface area and volume of a sphere? If diameter and length are given, how to calculate the area of the side wall and the volume of a cylinder?

HW2

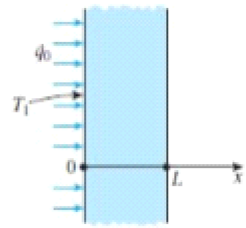
Thursday, February 1, 2024 9:13 AM

HW2

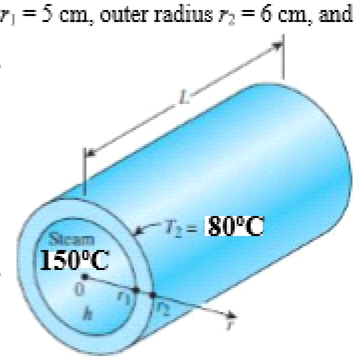
1. Read Chapter 2.

2. Consider a large stainless steel plate with a thickness of L. Heat is generated uniformly in the plate at a rate of $\dot{e}_{gen}$. Assuming the plate is losing heat from both sides, prove that the heat flux on the surface of the plate during steady operation equals to $\dot{e}_{gen}L/2$.

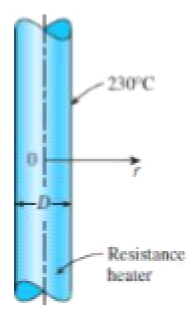
3. Consider a large plane wall of thickness L = 0.3 m, thermal conductivity k = 2.5 W/m·K, and surface area A = 12 m². The left side of the wall at x = 0 is subjected to a net heat flux of $\dot{q}_0$ = 700 W/m² while the temperature at that surface is measured to be T_1 = 80°C. Assuming constant thermal conductivity and no heat generation in the wall, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the temperature of the right surface of the wall at $x = L$.



4. Consider a steam pipe of length L = 9 m, inner radius r_1 = 5 cm, outer radius r_2 = 6 cm, and thermal conductivity k = 12.5 W/m·°C. Steam is flowing through the pipe at an average temperature of 150°C, and the average convection heat transfer coefficient on the inner surface is given to be h = 70 W/m²·°C. If the average temperature on the outer surfaces of the pipe is T_2 = 80°C, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the pipe, (*b*) obtain a relation for the variation of temperature in the pipe by solving the differential equation, and (*c*) evaluate the rate of heat loss from the steam through the pipe.



5. A 2-kW resistance heater wire with thermal conductivity of k = 20 W/m·K, a diameter of D = 4 mm, and a length of L = 0.9 m is used to boil water. If the outer surface temperature of the resistance wire is T_s = 230°C, **solve the heat conduction equation** to determine the temperature at **r = 1 mm**. (You must express the differential equation and the boundary conditions and solve the differential equation to determine the temperature. Otherwise, your HW2 grade will be zero!)



2. 1st law: $\dot{Q} = \dot{E}$

2. 1st law: $\dot{Q}_{loss} = \dot{E}_{gen}$

$$\dot{q} = \frac{\dot{Q}}{A} \Rightarrow \dot{Q} = \dot{q}A$$

$$\dot{e}_{gen} = \frac{\dot{E}}{V} \Rightarrow \dot{E} = V\dot{e}_{gen}$$

$$\dot{q}A = V\dot{e}_{gen}$$

$$\dot{q} = \frac{V}{A}\dot{e}_{gen}$$

$$\dot{q} = \frac{A \cdot L}{A \cdot 2}\dot{e}_{gen}$$

$$\dot{q} = \dot{e}_{gen}\frac{L}{2}$$

3. a. no heat generation steady state

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\dot{e}_{gen}}{k}}^{\,0} = \cancel{\frac{1}{\alpha}\frac{dT}{dt}}^{\,0}$$

$$\frac{\partial^2 T}{\partial x^2} = 0$$

BC$_1$. $T(x=0) = T_1 = 80°C$

BC$_2$: $\dot{q}_{left}(x=0) = \dot{q}_{right}(x=0)$

b. $\dot{q} = -k \left.\frac{dT}{dx}\right|_{x=0}$

$$\frac{dT}{dx} = C_1 = -\frac{\dot{q}}{k}$$

$$T(x=0) = C_1 x + C_2$$

$$80°C = C_2 = 353\,K$$

$$T(x) = -\frac{700}{2.5}x + 353$$

c. $$T(L) = \frac{-700}{2.5}(.3) + 353$$

$$T_2 = 269\,K = -4°C$$

4. a. $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\cancel{\frac{\dot{e}_{gen}}{k}}^{0}=\cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}^{0}$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

$$T(r)=c_1\ln(r)+c_2$$

BC1: $\dot{Q}_{left}(r_1)=\dot{Q}_{right}(r_1)$

$hA(T_\infty-T(r_1))=-kA\frac{dT}{dr}\Big|_{r_1}$

BC2: $T(r=r_2)=T_2$

b. $r=r_1$: $-k\frac{c_1}{r_1}=h\left[T_\infty-(c_1\ln r_1+c_2)\right]$

$r=r_2$: $T(r_2)=c_1\ln r_2+c_2=T_2$

Solving for c_1, c_2:

$$c_1=\frac{T_2-T_\infty}{\ln\frac{r_2}{r_1}+\frac{k}{hr_1}}$$

$$c_2=T_2-c_1\ln r_2=T_2-\frac{T_2-T_\infty}{\ln\frac{r_2}{r_1}+\frac{k}{hr_1}}\ln r_2$$

$$T(r)=\frac{T_2-T_\infty}{\ln\frac{r_2}{r_1}+\frac{k}{hr_1}}\ln\frac{r}{r_2}+T_2$$

$$T(.05)=\frac{80-150}{\ln\left(\frac{.06}{.05}\right)+\frac{12.5}{70(.05)}}\ln\left(\frac{.05}{.06}\right)+80$$

$$\boxed{T=83.4\ ^\circ C}$$

c.

$$\dot{Q}=-kA\frac{dT}{dr}=-k(2\pi rL)\frac{c_1}{r}=-2\pi Lk\frac{T_2-T_\infty}{\ln\frac{r_2}{r_1}+\frac{k}{hr_1}}$$

$$\dot{Q}=-2\pi(4)(12.5)\frac{80-150}{\ln\left(\frac{.06}{.05}\right)+\frac{12.5}{70(.05)}}$$

$$\boxed{\dot{Q}=13181.5\ W}$$

5.

$k = 20 \ W/m \cdot K$

$\dot{W}_e = 2000 \ W$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\cancel{\frac{\partial T}{\partial t}}^{0}$$

$$\dot{e}_{gen} = \frac{\dot{E}_{gen}}{V} = \frac{\dot{W}_e}{\frac{\pi}{4}D^2L} = 3.18\times10^8 \ W/m^3$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{e}_{gen}}{k}r$$

$$r\frac{dT}{dr} = -\frac{\dot{e}_{gen}}{2k}r^2 + C_1$$

BC 1: $T(r=R) = T_s$

BC 2: $\dot{Q}_{left}(r=R) = \dot{Q}_{right}(r=R)$

Thermal symmetry $\rightarrow C_1 = 0$

$$\frac{dT}{dr} = -\frac{\dot{e}_{gen}}{2k}r$$

$$T(r) = -\frac{\dot{e}_{gen}}{4k}r^2 + C_2$$

$$C_2 = T_s + \frac{\dot{e}_{gen}}{4k}R^2$$

$$T(r) = \frac{\dot{e}_{gen}}{4k}r^2 + T_s + \frac{\dot{e}_{gen}}{4k}R^2$$

$$T(r=.001) = \frac{3.18\times10^8}{4(20)}(.001)^2 + 230 + \frac{(3.18\times10^8)(.002)^2}{4(20)}$$

$$\boxed{= 249.875 \ °C}$$

Steady Heat Conduction

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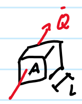
1-D, steady state, $\dot{e}_{gen}=0$

$$\dot{Q} = \frac{\Delta T}{R_{tot}}$$

$$R_{cond} = \frac{L}{kA}$$



$$R_{conv} = \frac{1}{hA}$$

EXAMPLE 3–2 Heat Loss Through a Single-Pane Window

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m·K. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is -10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2·K and $h_2 = 40$ W/m^2·K, which includes the effects of radiation.

SOLUTION Heat loss through a window glass is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Assumptions **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer through the wall is one-dimensional since any significant temperature gradients exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

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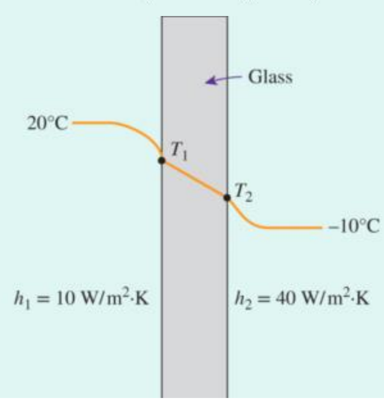
Properties The thermal conductivity is given to be $k = 0.78$ W/m·K.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and it can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in **Fig. 3–12**. Noting that the area of the window is $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.08333°\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m}\cdot\text{K}, 1.2 \text{ m}^2)} = 0.00855°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.02083°\text{C/W}$$



EXAMPLE 3–3 Heat Loss Through Double-Pane Windows

Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass ($k = 0.78$ W/m·K) separated by a 10-mm-wide stagnant airspace ($k = 0.026$ W/m·K). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is -10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2·K and $h_2 = 40$ W/m^2·K, which includes the effects of radiation.

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SOLUTION A double-pane window is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

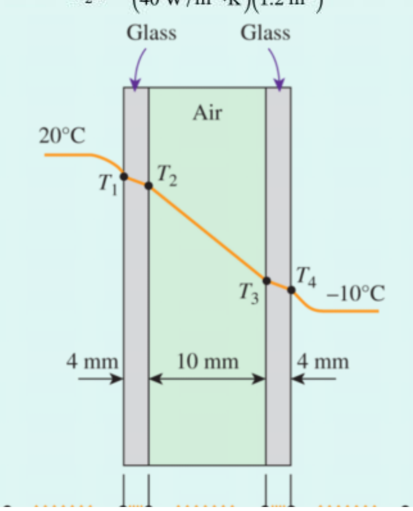
Analysis This example problem is identical to the previous one except that the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that enclose a 10-mm-wide stagnant airspace. Therefore, the thermal resistance network of this problem involves two additional conduction resistances corresponding to the two additional layers, as shown in **Fig. 3–13**. Noting that the area of the window is again $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.08333°\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004 \text{ m}}{(0.78 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.00427°\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01 \text{ m}}{(0.026 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.3205°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.02083°\text{C/W}$$



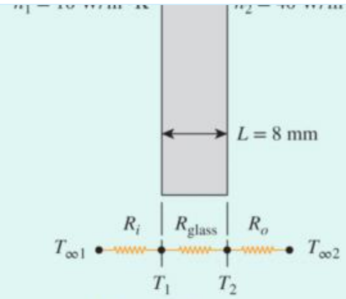


FIGURE 3–12 Schematic for Example 3-2.

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv},1} + R_{\text{glass}} + R_{\text{conv},2} = 0.08333 + 0.00855 + 0.02083 \\ &= 0.1127^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.1127^\circ\text{C/W}} = 266\text{ W}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\begin{aligned} \dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}} \longrightarrow T_1 &= T_{\infty 1} - \dot{Q}R_{\text{conv},1} \\ &= 20^\circ\text{C} - (266\text{ W})(0.08333^\circ\text{C/W}) \\ &= -2.2^\circ\text{C} \end{aligned}$$

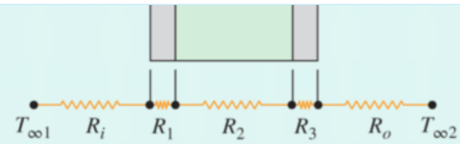


FIGURE 3–13 Schematic for Example 3-3.

Noting that all five resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv},1} + R_{\text{glass},1} + R_{\text{air}} + R_{\text{glass},2} + R_{\text{conv},2} \\ &= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 \\ &= 0.4332^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.4332^\circ\text{C/W}} = 69.2\text{ W}$$

which is about one-fourth of the result obtained in **Example 3-2**. This explains the popularity of the double- and even triple-pane windows in cold climates. The drastic reduction in the heat transfer rate in this case is due to the large thermal resistance of the air layer between the glasses.

The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty 1} - \dot{Q}R_{\text{conv},1} = 20^\circ\text{C} - (69.2\text{ W})(0.08333^\circ\text{C/W}) = 14.2^\circ\text{C}$$

EXAMPLE 3–6 Heat Loss Through a Composite Wall

A 3-m-high and 5-m-wide wall consists of long 16-cm × 22-cm cross section horizontal bricks ($k = 0.72\ \mathrm{W/m \cdot K}$) separated by 3-cm-thick plaster layers ($k = 0.22\ \mathrm{W/m \cdot K}$). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026\ \mathrm{W/m \cdot K}$) on the inner side of the wall, as shown in **Fig. 3-21**. The indoor and the outdoor temperatures are 20°C and −10°C, respectively, and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10\ \mathrm{W/m^2 \cdot K}$ and $h_2 = 25\ \mathrm{W/m^2 \cdot K}$, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

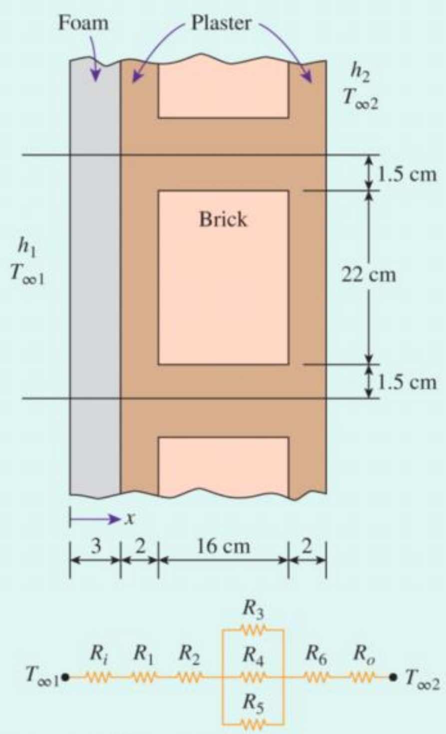


FIGURE 3–21 Schematic for **Example 3-6**.

SOLUTION The composition of a composite wall is given. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the *x*-direction. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivities are given to be $k = 0.72\ \mathrm{W/m \cdot K}$ for bricks, $k = 0.22\ \mathrm{W/m \cdot K}$ for plaster layers, and $k = 0.026\ \mathrm{W/m \cdot K}$ for the rigid foam.

Analysis There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

Assuming any cross section of the wall normal to the *x*-direction to be *isothermal*, the thermal resistance network for the representative section of the wall becomes as shown in **Fig. 3-21**. The individual resistances are evaluated as:

EXAMPLE 3–9 Heat Loss from an Insulated Electric Wire

A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is $k = 0.15\ \mathrm{W/m \cdot K}$. Electrical measurements indicate that a current of 10 A passes through the wire, and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30°\mathrm{C}$ with a heat transfer coefficient of $h = 12\ \mathrm{W/m^2 \cdot K}$, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient incorporates the radiation effects, if any.

Properties The thermal conductivity of plastic is given to be $k = 0.15\ \mathrm{W/m \cdot K}$.

Analysis Heat is generated in the wire, and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}e = \mathbf{V}I = (8\ \mathrm{V})(10\ \mathrm{A}) = 80\ \mathrm{W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in **Fig. 3-32**. The values of these two resistances are

$$\begin{aligned} A_2 &= (2\pi r_2)L = 2\pi(0.0035\ \mathrm{m})(5\ \mathrm{m}) = 0.110\ \mathrm{m^2} \\ R_{\text{conv}} &= \frac{1}{hA_2} = \frac{1}{(12\ \mathrm{W/m^2 \cdot K})(0.110\ \mathrm{m^2})} = 0.76°\mathrm{C/W} \\ R_{\text{plastic}} &= \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\ \mathrm{W/m \cdot K})(5\ \mathrm{m})} = 0.18°\mathrm{C/W} \end{aligned}$$

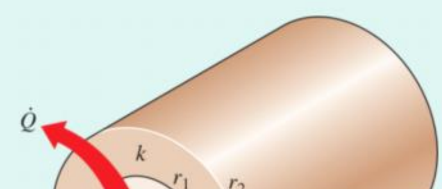
and therefore

$$R_{\text{total}} = R_{\text{plastic}} + R_{\text{conv}} = 0.76 + 0.18 = 0.94°\mathrm{C/W}$$

Then the interface temperature can be determined from

$$\begin{aligned} \dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 &= T_\infty + \dot{Q}R_{\text{total}} \\ &= 30°\mathrm{C} + (80\ \mathrm{W})(0.94°\mathrm{C/W}) = 105°\mathrm{C} \end{aligned}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.



isothermal, the thermal resistance network for the representative section of the wall becomes as shown in Fig. 3-21. The individual resistances are evaluated as:

$$R_i = R_{\text{conv},1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 0.40°\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{\text{kA}} = \frac{0.03\ \text{m}}{(0.026\ \text{W/m}\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 4.62°\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{\text{kA}} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m}\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 0.36°\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{\text{kA}} = \frac{0.16\ \text{m}}{(0.22\ \text{W/m}\cdot\text{K})(0.015 \times 1\ \text{m}^2)} = 48.48°\text{C/W}$$

$$R_4 = R_{\text{brick}} = \frac{L}{\text{kA}} = \frac{0.16\ \text{m}}{(0.72\ \text{W/m}\cdot\text{K})(0.22 \times 1\ \text{m}^2)} = 1.01°\text{C/W}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_2 A} = \frac{1}{(25\ \text{W/m}^2\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 0.16°\text{C/W}$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

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$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03\ \text{W/°C}$$

which gives

$$R_{\text{mid}} = 0.97°\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o \\ &= 0.40 + 4.62 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.87°\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{6.87°\text{C/W}} = 4.37\ \text{W} \quad (\text{per } 0.25\ \text{m}^2 \text{ surface area})$$

or $4.37/0.25 = 17.5$ W per m^2 area. The total area of the wall is $A = 3\ \text{m} \times 5\ \text{m} = 15\ \text{m}^2$. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{total}} = (1.75\ \text{W/m}^2)(15\ \text{m}^2) = 263\ \text{W}$$

Of course, this result is approximate, since we assumed the temperature within the wall to vary in one direction only and ignored any temperature change (and thus heat transfer) in the other two directions.

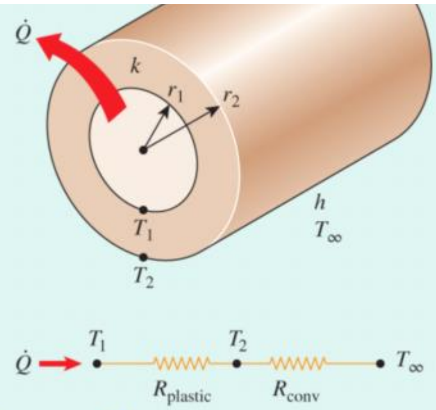


FIGURE 3–32 Schematic for Example 3-9.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 3-50 to be

$$r_{\text{cr}} = \frac{k}{h} = \frac{0.15\ \text{W/m}\cdot\text{K}}{12\ \text{W/m}^2\cdot\text{K}} = 0.0125\ \text{m} = 12.5\ \text{mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

Discussion It can be shown by repeating the calculations for a 4-mm-thick plastic cover that the interface temperature drops to 90.6°C when the thickness of the plastic cover is doubled. It can also be shown in a similar manner that the interface reaches a minimum temperature of 83°C when the outer radius of the plastic cover equals the critical radius.

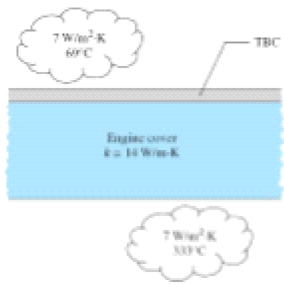
HW3

Tuesday, February 6, 2024 11:25 AM

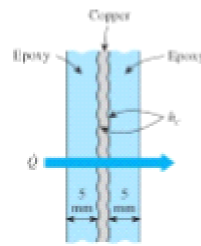
HW3

1. A 1.0 m × 1.5 m double-pane window consists of two 4-mm-thick layers of glass (k = 0.78 W/m·K) that are separated by a 5-mm air gap (k_{air} = 0.025 W/m·K). The heat flow through the air gap is assumed to be by conduction. The inside and outside air temperatures are 20°C and −20°C, respectively, and the inside and outside heat transfer coefficients are 40 and 20 W/m^2·K. Determine (*a*) the daily rate of heat loss through the window in steady operation and (*b*) the temperature difference across the largest thermal resistance.

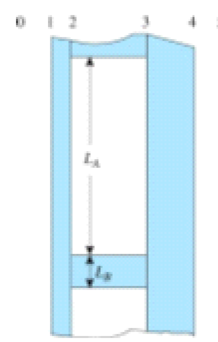
2. The outer surface of an engine is situated in a place where oil leakage can occur. When leaked oil comes in contact with a hot surface that has a temperature above its autoignition temperature, the oil can ignite spontaneously. Consider an engine cover that is made of a stainless steel plate with a thickness of 1 cm and a thermal conductivity of 14 W/m·K. The inner surface of the engine cover is exposed to hot air with a convection heat transfer coefficient of 7 W/m^2·K at 333°C. The outer surface is exposed to an environment where the ambient air is 69°C with a convection heat transfer coefficient of 7 W/m^2·K. To prevent fire hazard in the event of oil leak on the engine cover, a layer of thermal barrier coating (TBC) with a thermal conductivity of 1.1 W/m·K is applied on the engine cover outer surface. Would a TBC layer of 4 mm in thickness be sufficient to keep the engine cover surface below autoignition temperature of 200°C to prevent fire hazard?



3. A 1-mm-thick copper plate (k = 386 W/m·K) is sandwiched between two 5-mm-thick epoxy boards (k = 0.26 W/m·K) that are 15 cm × 20 cm in size. If the thermal contact conductance (h_c) on both sides of the copper plate is estimated to be 6000 W/m^2·K, determine the error involved in the total thermal resistance of the plate if the thermal contact resistances ($R_{contact} = 1/h_cA_c$) are ignored.



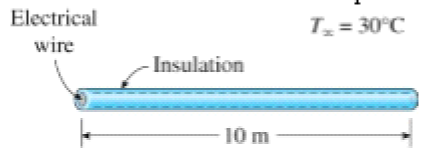
4. A typical section of a building wall is shown in right figure. This section extends in and out of the page and is repeated in the vertical direction. The wall support members are made of steel (k = 50 W/m·K). The support members are 8 cm (t_{23}) × 0.5 cm (L_B). The remainder of the inner wall space is filled with insulation (k = 0.03 W/m·K) and measures 8 cm (t_{23}) × 60 cm (L_A). The inner wall is made of gypsum board (k = 0.5 W/m·K) that is 1 cm thick (t_{12}) and the outer wall is made of brick (k = 1.0 W/m·K) that is 10 cm thick (t_{34}). What is the average heat flux through this wall when T_1 = 20°C and T_4 = 35°C?



5. Watch the lecture video and answer the following concept questions:

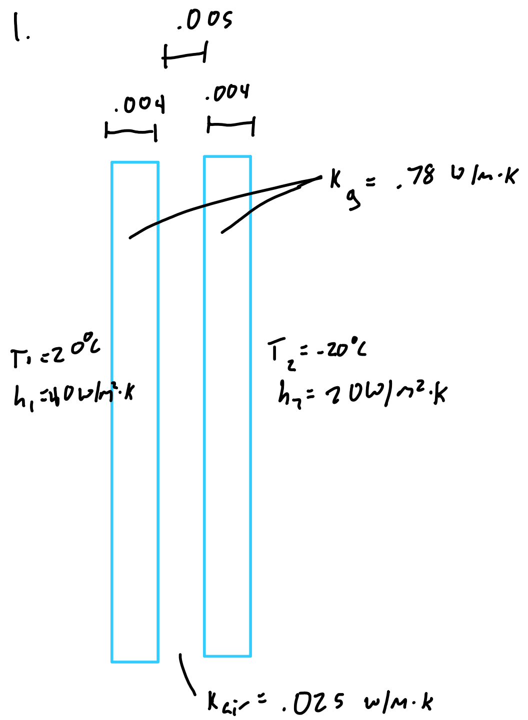
(a) What is the meaning of critical thickness?
(b) Is there a critical thickness in a planar wall? Explain.

6. A 2.2-mm-diameter and 10-m-long electric wire is tightly wrapped with a 1-mm-thick plastic cover whose thermal conductivity is k = 0.15 W/m·K. Electrical measurements indicate that a current of 13 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at T_∞ = 30°C with a heat transfer coefficient of h = 24 W/m²·K, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine if doubling the thickness of the plastic cover will increase or decrease this interface temperature.



1. .005

1.



$A = 1.5 \times 1 = 1.5 m^2$

$R_1 = \frac{1}{h_1 A} = \frac{1}{40(1.5)} = .01667$

$R_2 = R_4 = \frac{L_1}{k_g A} = \frac{.004}{.78(1.5)} = .0034$

$R_3 = \frac{L_2}{K_{air} A} = .1333$

$R_5 = \frac{1}{h_2 A} = 0333$

$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R = .1867$

$\dot{Q} = \frac{T_1 - T_2}{R_{eq}} = \frac{20 - -20}{.2174} = 214.2 W$

$= 214.2 \frac{J}{s} \cdot \frac{60 s}{1 min} \cdot \frac{60 min}{1 hr} \cdot \frac{24 hr}{1 d} = \boxed{1.8506 \times 10^7 J/day}$

$\Delta T = \dot{Q} R_3 = 214.2 (.1333) = \boxed{28.56 °C}$

2. $A = 1 m^2$

$R_1 = \frac{1}{h_a A} = \frac{1}{7} = .1429$

$R_2 = \frac{L_{Fe}}{K_{Fe} A} = \frac{.01}{14} = .000714$

$R_3 = \frac{L_{TOC}}{K_{TOC} A} = \frac{.004}{1.1} = .00364$

$R_4 = \frac{1}{h_a A} = \frac{1}{7} = .1429$

$R_{eq} = R_1 + R_2 + R_3 + R_4 = .29$

$\dot{Q} = \frac{T_{1\infty} - T_{2\infty}}{R_{eq}} = \frac{333 - 69}{.29} = 910.41 W$

$\dot{Q} = \frac{T_2 - T_{2\infty}}{R_4} \Rightarrow T_2 = \dot{Q} R_4 + T_{2\infty} = 910.41 (.1429) + 69 = \boxed{199.02 °C}$

Marginally, yes the temp is less than 200°C.
No room for error however.

3.

$A = .15 \times .2 = .03\ m^2$

$R_1 = \frac{L_e}{K_e A} = R_3 = \frac{.005}{.26(.03)} = .64103$

$R_2 = \frac{L_c}{K_c A} = \frac{.001}{386(.03)} = .000086$

$R_{contact} = \frac{1}{h_c A_c} = \frac{1}{6000(.03)} = .00556$

$R_{eq,ignore} = R_1 + R_2 + R_3 = 1.28214$

$R_{eq,contact} = R_1 + R_{contact} + R_2 + R_{contact} + R_3 = 1.29325$

$\%\ Error = \frac{R_{eq,contact} - R_{eq,ignore}}{R_{eq,ignore}} = \frac{1.29325 - 1.28214}{1.28214} = \boxed{.867\%\ error}$

4.

$A = .08(.605) = .0484\ m^2$

$R_1 = \frac{L_1}{k_g A} = \frac{.01}{.5(.0484)} = .41322$

$\frac{1}{R_2} = \frac{1}{R_A} + \frac{1}{R_B} = \frac{1}{\left(\frac{L_A}{K_A A_A}\right)} + \frac{1}{\left(\frac{L_B}{K_B A_B}\right)}$

$R_3 = \frac{L_3}{K_{brick} A} = \frac{.1}{1(.0484)} = 2.06612$

$R_2 = \left(\left(\frac{1}{\frac{.08}{.07(1 \cdot .6)}}\right) + \left(\frac{1}{\frac{.08}{50(1 \cdot .005)}}\right)\right)^{-1}$

$= .298507$

$R_{eq} = R_1 + R_2 + R_3 = 2.7779$

$\dot{Q} = \frac{T_4 - T_1}{R_{eq}} = \frac{35 - 20}{2.7779} = 5.3999\ W$

$\dot{q} = \frac{\dot{Q}}{A} = \frac{5.3999}{.605(1)} = \boxed{8.9254\ W/m^2}$

5.

a. Adding insulation to a cylindrical pipe increases conduction resistance but decreases convection resistance because the surface area increases. The critical radius is the outside radius for which the thermal resistance is minimum.

b. There is no critical thickness in a planar wall because the heat transfer area A is constant. Adding insulation always increases thermal resistance without increasing convection resistance.

6.

$\dot{Q} = VI = 8V(13A) = 104\ W$

$A_2 = 2\pi r_2 L = 2\pi(.001 + .0022/2)(10) = .13195\ m^2$

$R_{conv} = \dfrac{1}{hA_2} = \dfrac{1}{24(.131955)} = .31578$

$R_{plastic} = \dfrac{\ln(r_2/r_1)}{2\pi k L} = \dfrac{\ln(.0021/.0011)}{2\pi(.15)(10)} = .06861$

$R_{eq} = R_{plastic} + R_{conv} = .384393$

$\dot{Q} = \dfrac{T_1 - T_\infty}{R_{eq}} \Rightarrow T_1 = T_\infty + \dot{Q} R_{eq}$

$T_1 = 70°C + 104(.384393)$

$\boxed{T_1 = 69.977\ °C}$

$r_{cr} = \dfrac{k}{h} = \dfrac{.15}{24} = .00625\ m$

r_{cr} is larger than thickness of plastic cover, therefore increasing thickness of cover will enhance heat transfer until $r_2 = 6.25$ mm, meaning that T_1 decreases when plastic thickness is doubled.

Chap 3 Review

Tuesday, February 6, 2024 11:47 AM

Chap.+3

Chapter 3 Review questions:

1. Under what conditions can we use the concept of thermal resistance to solve heat transfer problems?

2. What is the general equation used to calculate the heat transfer rate from thermal resistance?

3. What are the expressions and units of conduction resistance and convection resistance, respectively?

4. What is the meaning of thermal contact resistance? What causes thermal contact resistance? How can we reduce thermal contact resistance?

5. What is the expression of the thermal resistance of a cylindrical wall?

6. What is the meaning of critical thickness? Is there a critical thickness in a planar wall?

Transient Heat Conduction

Thursday, February 8, 2024 11:44 AM

characteristic length: $L_c = \frac{V}{A_s}$

Biot number: $B_i = \frac{hL_c}{k} = \frac{\text{Convection at surface}}{\text{Conduction within body}}$

Lumped system analysis is valid when $B_i \leq 1$

Lumped system analysis:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$b = \frac{hA_s}{\rho V c_p}$

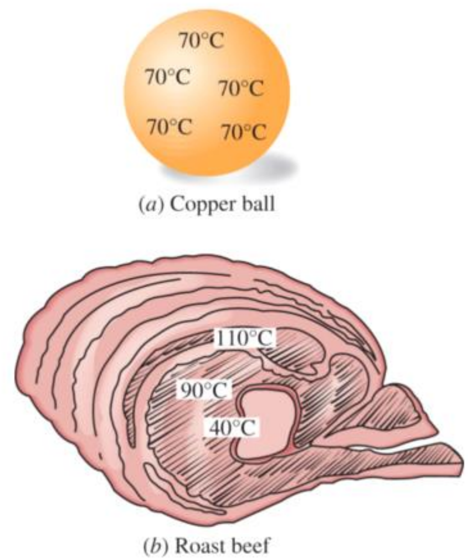


FIGURE 4–1 A small copper ball can be modeled as a lumped system, but a roast beef cannot.

EXAMPLE 4–5 **Boiling Eggs**

Page 268

An ordinary egg can be approximated as a 5-cm-diameter sphere (**Fig. 4–21**). The egg is initially at a uniform temperature of 5°C and is dropped into boiling water at 95°C. Taking the convection heat transfer coefficient to be $h = 1200\ \mathrm{W/m^2{\cdot}K}$, determine how long it will take for the center of the egg to reach 70°C.

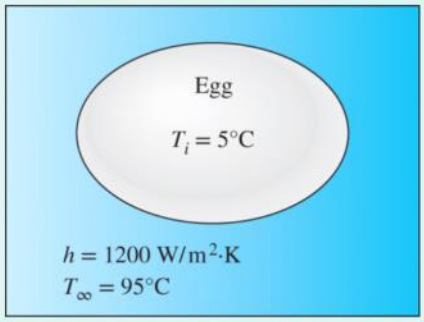


FIGURE 4–21 Schematic for **Example 4–5**.

SOLUTION An egg is cooked in boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical in shape with a radius of $r_o = 2.5$ cm. **2** Heat conduction in the egg is one-dimensional because of thermal symmetry about the midpoint. **3** The thermal properties of the egg and the heat transfer coefficient are constant. **4** The Fourier number is $\tau > 0.2$, so the one-term approximate solutions are applicable.

Properties The water content of eggs is about 74 percent, and thus the thermal conductivity and diffusivity of eggs can be approximated by those of water at the average temperature of $(5 + 70)/2 = 37.5°C$; $k = 0.627\ \mathrm{W/m{\cdot}K}$ and $\alpha = k/\rho c_p = 0.151 \times 10^{-6}\ \mathrm{m^2/s}$ (**Table A–9**).

Analysis Egg white begins to thicken at 63°C and turns solid at 65°C. The yolk begins to thicken at 65°C and sets at 70°C. The whole egg sets at temperatures above 70°C. Therefore, the egg in this case will qualify as hard boiled. The temperature within the egg varies with radial distance as well as time, and the temperature at a specified location at a given time can be determined from the one-term approximate solution. The Biot number for this problem is

$$\text{Bi} = \frac{hr_o}{k} = \frac{(1200\ \text{W/m}^2\cdot\text{K})(0.025\ \text{m})}{0.627\ \text{W/m}\cdot\text{K}} = 47.8$$

which is much greater than 0.1, and thus the lumped system analysis is not applicable. The coefficients λ_1 and A_1 for a sphere corresponding to this Bi are, from **Table 4–2**,

$$\lambda_1 = 3.0754,\ A_1 = 1.9958$$

Substituting these and other values into **Eq. 4–28** and solving for τ gives

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70-95}{5-95} = 1.9958 e^{-(3.0754)^2 \tau} \longrightarrow \tau = 0.209$$

which is greater than 0.2, and thus the one-term approximate solution is applicable with an error of less than 2 percent. Then the cooking time is determined from the definition of the Fourier number to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.209)(0.025\ \text{m})^2}{0.151 \times 10^{-6}\ \text{m}^2/\text{s}} = 865\ \text{s} \approx 14.4\ \text{min}$$

Therefore, it will take about 15 min for the center of the egg to be heated from 5°C to 70°C.

Discussion Note that the Biot number in lumped system analysis was defined differently as Bi $= hL_c/k = h(r_o/3)/k$. However, either definition can be used in determining the applicability of the lumped system analysis unless Bi ≈ 0.1.

this **one-term approximation**, given as

$$\text{Plane wall: } \theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L),\ \tau > 0.2 \qquad \textbf{(4–23)}$$

$$\text{Cylinder: } \theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o),\ \tau > 0.2 \qquad \textbf{(4–24)}$$

$$\text{Sphere: } \theta_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o},\ \tau > 0.2 \qquad \textbf{(4–25)}$$

where the constants A_1 and λ_1 are functions of the Bi number only, and their values are listed in **Table 4–2** against the Bi number for all three geometries. The function J_0 is the zeroth-order Bessel function of the first kind, whose value can be determined from **Table 4–3**. Noting that $\cos(0) = J_0(0) = 1$ and the limit of $(\sin x)/x$ is also 1, these relations

EXAMPLE 4–6 **Minimum Burial Depth of Water Pipes to Avoid Freezing**

Page 275

In areas where the air temperature remains below 0°C for prolonged periods of time, the freezing of water in underground pipes is a major concern. Fortunately, the soil remains relatively warm during those periods, and it takes weeks for the subfreezing temperatures to reach the water mains in the ground. Thus, the soil effectively serves as insulation to protect the water from subfreezing temperatures in winter.

The ground at a particular location is covered with snowpack at $-10°C$ for a continuous period of three months, and the average soil properties at that location are $k = 0.4\ \mathrm{W/m \cdot K}$ and $\alpha = 0.15 \times 10^{-6} \mathrm{m^2/s}$ (**Fig. 4–29**). Assuming an initial uniform temperature of 15°C for the ground, determine the minimum burial depth to prevent the water pipes from freezing.

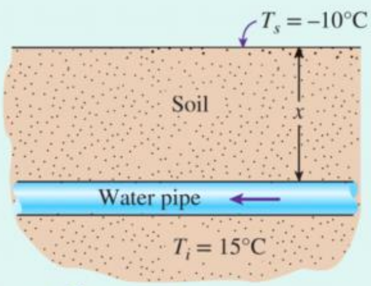


FIGURE 4–29 Schematic for **Example 4–6**.

SOLUTION The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium. **2** The thermal properties of the soil are constant.

Properties The properties of the soil are as given in the problem statement.

Analysis The temperature of the soil surrounding the pipes will be 0°C after three months in the case of minimum burial depth. Therefore, from **Fig. 4–27**, we have

$$\left.\begin{aligned} &\frac{h\sqrt{\alpha t}}{k} = \infty \qquad (\text{since } h \to \infty) \\ &\frac{T(x,t) - T_i}{T_\infty - T_i} = \frac{0 - 15}{-10 - 15} = 0.6 \end{aligned}\right\} \eta = \frac{x}{2\sqrt{\alpha t}} = 0.36$$

We note that

$$t = (90 \text{ days})(24 \text{ h/day})(3600 \text{ s/h}) = 7.78 \times 10^6 \text{ s}$$

and thus

$$x = 2\eta\sqrt{\alpha t} = 2 \times 0.36\sqrt{(0.15 \times 10^{-6} \text{ m}^2/\text{s})(7.78 \times 10^6 \text{ s})} = 0.78 \text{ m}$$

Therefore, the water pipes must be buried to a depth of at least 78 cm to avoid freezing under the specified harsh winter conditions.

Therefore, the water pipes must be buried to a depth of at least 78 cm to avoid freezing under the specified harsh winter conditions.

ALTERNATIVE SOLUTION The solution of this problem could also be determined from Eq. 4-45:

$$\frac{T(x,t)-T_i}{T_s-T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad \longrightarrow \quad \frac{0-15}{-10-15} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) = 0.60$$

The argument that corresponds to this value of the complementary error function is determined from Table 4-4 to be $\eta = 0.37$. Therefore,

$$x = 2\eta\sqrt{\alpha t} = 2 \times 0.37\sqrt{\left(0.15 \times 10^{-6}\ \text{m}^2/\text{s}\right)\left(7.78 \times 10^{6}\ \text{s}\right)} = 0.80\ \text{m}$$

Again, the slight difference is due to the reading error of the chart.

Chap 4 Review

Thursday, February 8, 2024 12:01 PM

Chap.+4

Chapter 4 Review questions:

1. What is the meaning of transient heat conduction in the heat conduction equation? What is the simplest case of transient heat conduction?

2. When is the lumped system approximation valid?

3. What is the definition, unit, and physical meaning of Biot number?

4. Which equation do we use for lumped system analysis?

5. What are the one-term approximate solutions for transient heat conduction in a large planar wall, a long cylinder, and a sphere, respectively? When are these approximate solutions valid?

6. How is the dimensionless time τ defined in the one-term approximate solutions?

7. How do we determine A_1, λ_1, and J_0 in one-term approximate solutions? Is the definition of Bi in one-term approximate solutions the same as that in the lumped system analysis?

8. What is the solution to the 1-D transient heat conduction in a semi-infinite solid with a constant temperature at the surface? What is the meaning of erfc(η) in the equation? Which table do we use to find erfc(η)?

9. When is the product solution to the 2-D/3-D transient heat conduction problems valid?

HW4

Thursday, February 15, 2024 11:57 AM

HW4

1. Carbon steel balls (k = 54 W/m·K, ρ = 7833 kg/m^3, c_p = 0.465 kJ/kg·°C) 8 mm in diameter are annealed by heating them first to 900°C in a furnace and then allowing them to cool slowly to 100°C in ambient air at 35°C. If the average heat transfer coefficient is 75 W/m^2·K, determine how long the cooling process with take.

2. A person is found dead at 5PM in a room whose temperature is 20°C. The surface temperature of the body (k = 0.47 W/m·K and $\alpha = 0.13 \times 10^{-6}$ m^2/s) is measured to be 25°C when found, and the heat transfer coefficient is estimated to be 8 W/m^2·K. Modeling the body as a 30-cm-diameter **long cylinder** and assume the body was initially at a uniform temperature of 37°C, estimate the time of death of that person.

3. An ordinary egg can be approximated as a 5.5-cm-diameter sphere whose properties are roughly k = 0.6 W/m·K and $\alpha = 0.14 \times 10^{-6}$ m^2/s. The egg is initially at a uniform temperature of 8°C and is dropped into boiling water at 97°C. Take the convection heat transfer coefficient to be h = 1400 W/m^2·K, determine how long it will take for the center of the egg to reach 70°C. Also determine the surface temperature of the egg when the center of the egg reaches 70°C.

4. A highway made of asphalt is initially at a uniform temperature of 55°C. Suddenly the highway surface temperature is reduced to 25°C by rain. Determine the temperature at the depth of 3 cm from the highway surface after 60 minutes. Assume the highway surface temperature is maintained at 25°C.

5. Consider a cylindrical block whose height and diameter are both 5 cm. The block is initially at 20°C and are made of granite (k = 2.5 W/m·K and $\alpha = 1.15 \times 10^{-6}$ m^2/s). Now the block is exposed to hot gases at 500°C in a furnace on all of their surfaces with a heat transfer coefficient of 40 W/m^2·K. Determine the center temperature of the block after 10, 20, and 60 min.

1.

$$L_c = \frac{V}{A_s} = \frac{\cancel{\frac{4}{3}}\pi \frac{D^3}{8}}{\cancel{4}\pi \frac{D^2}{4}} = \frac{D}{6} = .00133 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{75(.00133)}{54} = .00185 < .1 \quad \checkmark$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$b = \frac{hA_s}{\rho V c_p} = \frac{75(\pi(.008)^2)}{7833(\pi(.008)^3/6)(.465)}$$

$$b = \frac{h}{\rho c_p L_s} = 15.4433$$

$$\frac{\ln\left(\frac{T(t)-T_\infty}{T_i - T_\infty}\right)}{-b} = t$$

$$t = \frac{\ln\left(\frac{100-35}{900-35}\right)}{-15.4433}$$

$$\boxed{t = .1676 \text{ s}}$$

2.

$$Bi = \frac{hr_o}{k} = \frac{8(.15)}{.47} = 2.55$$

Table 4-2: $\quad y = y_1 + \frac{(x-x_1)(y_2-y_1)}{(x_2-x_1)}$

$\lambda_1 = 1.7036$

$A_1 = 1.3828$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o) \qquad \tau > .2$$

$$\tau = \frac{\alpha t}{r_o^2}$$

$$t = \frac{r_o^2}{-\lambda_1^2 \alpha} \ln\left(\frac{\left(\frac{T(t)-T_\infty}{T_i - T_\infty}\right)}{A_1 J_0(\lambda_1 r/r_o)}\right)$$

$\tau = 1.06 > .2 \quad \checkmark$

$J_0(1.7) = .398$

$$t = \frac{.15^2}{-1.7^2(.13\times10^{-6})} \ln\left(\frac{\left(\frac{25-20}{37-20}\right)}{1.3828(.398)}\right)$$

$t = 37366.5$ s

$t = 10.38$ hours $\qquad$ 5 pm

$t = 37366.5$ s

$t = 10.38$ hours   5 pm

$\boxed{TOD = 6:13 \text{ AM same day}}$

3.

$$Bi = \frac{hr_o}{k} = \frac{1400(.0275)}{.6} = 64.1667$$

$\lambda_1 = 3.1$

$A_1 = 1.9975$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \qquad \tau = \frac{t\alpha}{r_0^2}$$

$$\tau = \frac{1}{-\lambda_1^2} \ln\left(\frac{\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)}{A_1}\right)$$

$$\tau = \frac{1}{-(3.1)^2} \ln\left(\frac{\left(\frac{70-97}{8-97}\right)}{1.9975}\right)$$

$\tau = .196118 \not> .2$

$$t = \frac{\tau r_0^2}{\alpha} = \frac{1.96118(.0225)^2}{.14 \times 10^{-6}}$$

$\boxed{t = 709 \text{ s}}$

$\boxed{t = 11.82 \text{ min}}$

$$\frac{T_s - T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2 \tau}\right) \frac{\sin(\lambda_1 r/r_0)}{\lambda_1 r/r_0} \qquad r = r_0$$

$$T_s = \left[\left(A_1 e^{-\lambda_1^2 \tau}\right) \frac{\sin(\lambda_1 r/r_0)}{\lambda_1 r/r_0}\right](T_i - T_\infty) + T_\infty$$

$$T_s = \left[1.9975 \left(e^{-3.1^2(.196118)}\right) \frac{\sin(3.1)}{3.1}\right](8-97) + 97$$

$\boxed{T_s = 96.6378 \ ^\circ C}$

4.

Table A-8:

$\rho = 2115\ kg/m^3$

$c_p = 920\ J/kg$

$k = .062\ W/m\cdot K$

$$\alpha = \frac{k}{\rho c_p} = 3.186\times10^{-8}$$

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\frac{x}{2\alpha t} = \frac{.03}{2\sqrt{3.186\times10^{-8}(60\ 60)}} = 1.4$$

Table 4-4: $erfc(1.4) = .04772$

$$T(.03m, 3600s) = (T_s - T_i)\,erfc(1.4) + T_i$$

$$\boxed{T = (25-55)(.04772)+55 = 53.6\ ^\circ C}$$

5.

$$B_i = \frac{hr}{k} = \frac{40(.025)}{2.5} = .4$$

Table 4-2

Plane wall:

$\lambda_1 = .5932$

$A_1 = 1.058$

Cylinder:

$\lambda_1 = .8516$

$A_1 = 1.0931$

10 minutes:

$$\tau = \frac{\alpha t}{L^2} = \frac{1.15\cdot10^{-6}(10\cdot60)}{.025^2} = 1.104 \quad > .2$$

$$\frac{T-T_\infty}{T_i-T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{wall}\left(A_1 e^{-\lambda_1^2\tau}\right)_{cyl}$$

$$T = \left(A_1 e^{-\lambda_1^2\tau}\right)_{wall}\left(A_1 e^{-\lambda_1^2\tau}\right)_{cyl}(T_i - T_\infty) + T_\infty$$

$$T = \left(1.058\, e^{-(.5932)^2(1.104)}\right)\left(1.0931\, e^{-(.8516)^2(1.104)}\right)(20-500)+500$$

$$\boxed{T = 331\ ^\circ C}$$

20 minutes:

$\tau = \frac{\alpha t}{L^2} = \frac{1.15 \cdot 10^{-6}(20 \cdot 60)}{.025^2} = 2.208$

$$T = \left(1.058\, e^{-(.5932)^2(2.208)}\right)\left(1.0931\, e^{-(.8516)^2(2.208)}\right)(20-500)+500$$

$\boxed{T = 449^\circ C}$

60 minutes:

$\tau = \frac{\alpha t}{L^2} = \frac{1.15 \cdot 10^{-6}(60 \cdot 60)}{.025^2} = 6.624$

$$T = \left(1.058\, e^{-(.5932)^2(6.624)}\right)\left(1.0931\, e^{-(.8516)^2(6.624)}\right)(20-500)+500$$

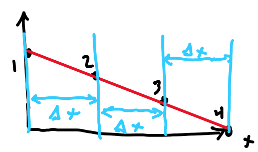
$\boxed{T = 500^\circ C}$

Numerical Solution

Saturday, February 17, 2024 5:42 PM

$$\left.\frac{d^2T}{dx^2}\right|_m = \frac{T(x_m+\Delta x) - 2T(x_m) + T(x_m - \Delta x)}{(\Delta x)^2}$$

4 node case:



analytical:
$T(x) = \frac{-600\,°C}{m} x + 100\,°C$

$\Delta x = L/3 = 3.33\,cm$

$T_1 = 100\,°C$
$T_2 = ?$
$T_3 = ?$
$T_4 = 40\,°C$

$$\left.\frac{d^2T}{dx^2}\right|_2 = \frac{T_3 - 2T_2 + T_1}{(.0333)^2} = 0 \Rightarrow T_3 - 2T_2 = -100$$

$$\left.\frac{d^2T}{dx^2}\right|_3 = \frac{T_4 - 2T_3 + T_2}{(.0333)^2} = 0 \Rightarrow T_2 - 2T_3 = -40$$

System of eqn
$T_2 = 80$
$T_3 = 60$

node #	1	2	3	4	
x_m (m)	0	.0333	.0667	.1	
$T_{m,a}$ (°C)	100	80	60	40	← analytical
$T_{m,n}$ (°C)	100	80	60	40	← numerical

Exam 1 Study guide

Thursday, February 22, 2024 10:54 AM

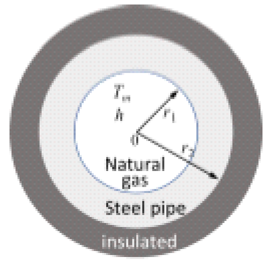
Exam+1+-+
Study+gui...

ME 3525 Exam 1 Name:

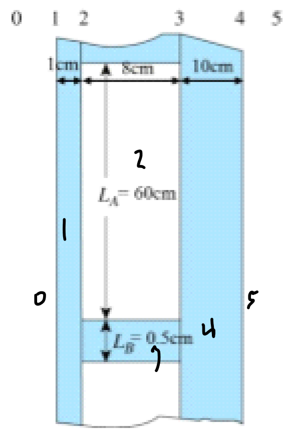
1. A spherical heating element of diameter d = 2 cm generates heat at a rate of 200 W/cm^3. Determine the heat flux at the surface of the heating element in steady operation. (**66.7 W/cm^2**)

2. A heating plate whose surface is maintained at constant temperature of 50°C is exposed to the ambient air at 30°C in a room. The heat transfer coefficient of air on the heating plate is 10 W/m^2·K. Determine the total heat loss rate from the heating plate if the plate surface area is 0.1 m^2. Take the emissivity of the plate to be 0.9 and assume the temperature of the inner surfaces of the room to be the same as the air temperature. (**32.5 W**)

3. A long steel pipe (k = 15 W/m·K) with an inner radius r_1 = 10 cm and outer radius r_2 = 15 cm is used to transport natural gas at a mean fluid temperature of T_m = 25°C with a heat transfer coefficient of h = 100 W/m^2·K. The outer surface of the pipe is insulated as shown in the figure below. (a) Express the differential equation and the boundary conditions for steady one-dimensional heat conduction across the pipe wall, and (b) solve the differential equation to find the temperature at the outer surface of the pipe at steady state. (**25 °C**) (c) Can we determine the temperature at the outer surface of the pipe using the thermal resistance concept? Why?



4. A typical section of a building wall is shown in right figure. This section extends in and out of the page and is repeated in the vertical direction. The wall support members are made of steel (k_B = 50 W/m·K). The support members are 8 cm (t_{23}) × 0.5 cm (L_B). The remainder of the inner wall space is filled with insulation (k_A = 0.03 W/m·K) and measures 8 cm (t_{23}) × 60 cm (L_A). The inner wall is made of gypsum board (k_{12} = 0.5 W/m·K) that is 1 cm thick (t_{12}) and the outer wall is made of brick (k_{34} = 1.0 W/m·K) that is 10 cm thick (t_{34}). The indoor and the outdoor temperatures are T_0 = 22°C and T_5 = -4°C, and convection heat transfer coefficients on the inner and the outer sides are h_0 = 10 W/m^2·K and h_5 = 20 W/m^2·K, respectively. (a) Draw the thermal resistance network. (b) Determine the average heat flux through this wall. (**57.7 W/m^2**)



5. An ostrich egg can be approximated as a 14-cm-diameter solid sphere. The ostrich egg is initially at a uniform temperature of 15°C and is to be cooked in an oven maintained at 220°C with a heat transfer coefficient of 80 W/m^2·K. With this idealization, determine the temperature difference between the surface of the ostrich egg and the center of the ostrich egg after a 2-hr heating period. Take the thermal conductivity of the ostrich egg to be 0.63 W/m·K and the thermal diffusivity of the ostrich egg to be 1.5×10^{-7} m^2/s. (**61 °C**)

TABLE 4–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres ($Bi = hL/k$ for a plane wall of thickness $2L$, and $Bi = hr_o/k$ for a cylinder or sphere of radius r_o)

	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 4–3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

TABLE 4–4

The complementary error function

η	erfc (η)	η	erfc (η)	η	erfc (η)	η	erfc (η)	η	erfc (η)	η	erfc (η)
0.00	1.00000	0.38	0.5910	0.76	0.2825	1.14	0.1069	1.52	0.03159	1.90	0.00721
0.02	0.9774	0.40	0.5716	0.78	0.2700	1.16	0.10090	1.54	0.02941	1.92	0.00662
0.04	0.9549	0.42	0.5525	0.80	0.2579	1.18	0.09516	1.56	0.02737	1.94	0.00608
0.06	0.9324	0.44	0.5338	0.82	0.2462	1.20	0.08969	1.58	0.02545	1.96	0.00557
0.08	0.9099	0.46	0.5153	0.84	0.2349	1.22	0.08447	1.60	0.02365	1.98	0.00511
0.10	0.8875	0.48	0.4973	0.86	0.2239	1.24	0.07950	1.62	0.02196	2.00	0.00468
0.12	0.8652	0.50	0.4795	0.88	0.2133	1.26	0.07476	1.64	0.02038	2.10	0.00298
0.14	0.8431	0.52	0.4621	0.90	0.2031	1.28	0.07027	1.66	0.01890	2.20	0.00186
0.16	0.8210	0.54	0.4451	0.92	0.1932	1.30	0.06599	1.68	0.01751	2.30	0.00114
0.18	0.7991	0.56	0.4284	0.94	0.1837	1.32	0.06194	1.70	0.01612	2.40	0.00069
0.20	0.7773	0.58	0.4121	0.96	0.1746	1.34	0.05809	1.72	0.01500	2.50	0.00041
0.22	0.7557	0.60	0.3961	0.98	0.1658	1.36	0.05444	1.74	0.01387	2.60	0.00024
0.24	0.7343	0.62	0.3806	1.00	0.1573	1.38	0.05098	1.76	0.01281	2.70	0.00013
0.26	0.7131	0.64	0.3654	1.02	0.1492	1.40	0.04772	1.78	0.01183	2.80	0.00008
0.28	0.6921	0.66	0.3506	1.04	0.1413	1.42	0.04462	1.80	0.01091	2.90	0.00004
0.30	0.6714	0.68	0.3362	1.06	0.1339	1.44	0.04170	1.82	0.01006	3.00	0.00002
0.32	0.6509	0.70	0.3222	1.08	0.1267	1.46	0.03895	1.84	0.00926	3.20	0.00001
0.34	0.6306	0.72	0.3086	1.10	0.1198	1.48	0.03635	1.86	0.00853	3.40	0.00000
0.36	0.6107	0.74	0.2953	1.12	0.1132	1.50	0.03390	1.88	0.00784	3.60	0.00000

1.

$\dot{q} = \frac{\dot{Q}}{A} = \frac{\dot{E}_{gen}}{A} = \frac{\dot{e}_{gen} V}{A} = \dot{e}_{gen} \frac{\cancel{\frac{4}{3}} \pi \frac{D^3}{8}}{\cancel{4} \pi \frac{D^2}{4}} = \dot{e}_{gen} \frac{D}{6} = \frac{200 \text{ W/cm}^3 (2\text{cm})}{6} = \boxed{66.7 \text{ W/cm}^2}$

2.

$\dot{Q}_{loss} = \dot{Q}_{conv} + \dot{Q}_{net,rad}$

$= h A_s (T_s - T_\infty) + \varepsilon \sigma A_s (T_s^4 - T_{surr}^4)$

$T_s = 50°C = 323 \text{ K}$

$T_{surr} = 30°C = 303 \text{ K}$

$\dot{Q}_{loss} = (10 \text{ W/m}^2 \cdot K)(.1\text{m}^2)(50-30) + .9(5.670 \times 10^{-8} \text{ W/m}^2 \cdot K^4)(.1\text{m}^2)(323^4 - 303^4) K$

$\boxed{\dot{Q}_{loss} = 32.5 \text{ W}}$

3. a. $\frac{1}{r} \frac{\partial}{\partial t}\left(r \frac{dT}{dr}\right) + \cancel{\frac{\dot{e}_{gen}}{k}}^{\,0} = \cancel{\frac{1}{\alpha} \frac{dT}{dt}}^{\,0}$ steady

BC1: $\dot{Q}_{left}(r_1) = \dot{Q}_{right}(r_1)$

$hA(T_\infty - T(r=r_1)) = -kA \frac{dT}{dr}(r=r_1)$

BC2: $\dot{Q}_{left}(r_2) = \cancel{\dot{Q}_{right}(r_2)}^{\,0}$ insulated

$-kA \frac{dT}{dr}(r=r_2) = 0$

b.

$T(r) = C_1 \ln r + C_2$

$r = r_1$:

$-\frac{kC_1}{r_1} = h(T_\infty - (C_1 \ln r + C_2))$

$r = r_2$

$C_1 = \frac{dT}{dR} = 0$

$C_2 = T_\infty$

$T(r) = T_\infty = \boxed{23°C}$

c. 1. 1D ✓
2. Steady state ✓
3. no heat generation ✓

Therefore, we can use the thermal resistance concept.

4.

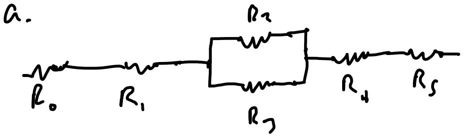
$R_{cond} = \frac{L}{KA}$

$R_{conv} = \frac{1}{hA}$

$R_0 = \frac{1}{10(.605)} = .165$

Assume 1m wide

a.



$R_1 = \frac{.01}{.5(.605)} = .033$

$R_2 = \frac{.08}{.03(6)} = 4.444$

$R_3 \quad \frac{.08}{50(.005)} = .32$

$R_4 = \frac{.1}{1}(.605) = 165$

$R_5 = \frac{1}{20}(.605) = .083$

$R_{23,comb} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{4.444} + \frac{1}{.32}\right)^{-1}$

$= .299$

$R_{eq} = R_0 + R_1 + R_{23,comb} + R_4 + R_5 = \quad .745$

$\dot{Q} = \frac{T_0 - T_5}{R_{eq}} = \frac{22 - -4}{.745} = 34.909$

$\dot{q} = \frac{\dot{Q}}{A} = \frac{34.909}{.605} = \boxed{57.7 \text{ W/m}^2}$

5.

$Bi = \frac{h L_c}{K} = \frac{80(.0233)}{.63} = 3.11 \not< .1$

$L_c = \frac{V}{A_s} = \frac{\cancel{4}/3 \pi D^3/8}{\cancel{4}\pi D^2/4} = \frac{D}{6} = \frac{.14}{6} = .0233$

$Bi = \frac{hr}{K} = \frac{80(.07)}{.63} = 8.88$

$\lambda_1 = 2.8 \qquad Bi \approx 9$

$A_1 = 1.91$

$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \qquad \tau = \frac{t\alpha}{r_0^2}$

$\tau = 2hr \; \frac{60min}{1hr} \; \frac{60s}{1min} \; \frac{(15 \times 10^{-7})}{(.07)^2} = .2204$

$T_0 = (A_1 e^{-\lambda_1^2 \tau})(T_i - T_\infty) + T_\infty = 1.91\, e^{-(2.8)^2(.2204)} (15 - 220) + 220$

$T_0 = 154.083\ °C$

$\frac{T_s - T_\infty}{T_i - T_\infty} = A_1\, e^{-\lambda_1^2 \tau} \; \frac{\sin(\lambda_1 r/r_1)}{\lambda_1 r/r_1}$

$T_s = \left(A_1 e^{-\lambda_1^2 \tau} \frac{\sin \lambda_1}{\lambda_1}\right)(T_i - T_\infty) + T_\infty = 1.91\, e^{-(2.8)^2(.2204)} \frac{\sin 2.8}{2.8} (15 - 220) + 220$

$\lambda_1 \, r/r_1$

$T_s = \left(A_1 e^{-\lambda_1^2 \tau} \frac{\sin \lambda_1}{\lambda_1}\right)(T_i - T_\infty) + T_\infty = 1.91\, e^{-(2.8)^2(.2204)} \frac{\sin 2.8}{2.8}(15-220)+220$

$T_s = 212.114$

$T_s - T_o = 212.114 - 154.087$

$\boxed{\Delta T = 58\,°C}$ ≈ 61, didn't interpolate for θ_i, bit of error

Project+description

Thursday, February 22, 2024 11:31 AM

Project+description

Heat transfer in an electrical wire

An electrical current is passed through a horizontal copper rod 2 mm in diameter and 30 cm long, located in an air stream at 0 °C. The ends of the rod are maintained at 20 °C and the convective heat transfer coefficient is estimated to be 30 W/m²·K. Neglecting heat loss by radiation, determine the temperature distribution along the electric wire under steady conditions.

(1) Draw a schematic diagram of the system. **(10 pts)**

(2) Obtain the governing equation for steady state heat transfer in the electric wire. **(10 pts)**

(3) Obtain the analytical solution for the temperature distribution along the electric wire under steady conditions if the electrical current is i) 0 Amperes, ii) 40 Amperes, and iii) 60 Amperes. **(20 pts)**

(4) Consider a total of three, five, seven and eleven (four cases) equally spaced nodes along the electric wire, obtain the finite difference formulation of each interior node. **(20 pts)**

(5) When the electrical current is 60 Amperes, obtain the numerical solution to the temperatures at the nodes and validate the numerical results with the prediction from the analytical solution. How does the number of nodes affect the numerical results? **(30 pts)**

(6) Obtain the governing equation if heat loss by radiation is considered. Can we use the same numerical method to obtain the temperatures at each node? Why? **(10 pts)**

Must submit a computer-generated report. No photos in the report.

Use equation editor to type equations. For example, the second derivative of temperature is $\frac{d^2T}{dx^2}$. Do not type it as d^2T\dx^2.

Students can finish the project individually or in a team of two students.

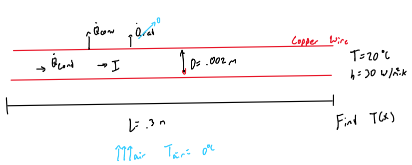
Common problems in the project report:

1. A complete technical report should have a cover page, a table of contents, abstract, introduction (problem description), different sections presenting governing equations and modeling results, conclusions, and references.

2. A report should have pages numbers. You also need to assign an equation number to each equation.

3. When you use material properties such as k and ρ_e of copper in the calculation, you should show the reference for these properties.

4. Analytical results give you T at any x. Therefore, you should use lines for analytical results in figures. Numerical results only give you T at each node. Therefore, you should use scatters (no lines) for numerical results.



Numerical

$$\frac{d^2T}{dx^2} - \frac{hp}{KA_c}(T-T_\infty) + \frac{I^2\rho_e}{KA_c} = 0$$

$$\frac{d^2T}{dx^2} = \frac{T_3 - 2T_2 - T_1}{(\Delta x)^2}$$

$$\frac{T_3 - 2T_2 + T_1}{(\Delta x)^2} - \frac{hp}{KA_c}(T_2 - T_\infty) + \frac{I^2\rho_e}{KA_c^2} = 0$$

Governing Equation (1st law)

$$\dot{Q}_x - \dot{Q}_{x+dx} - \dot{Q}_{conv} + \dot{E}_{gen} = mC_p \cancel{\frac{dT}{dt}}^{\,0 \text{ steady}}$$

$$-KA\frac{dT}{dx}\Big|_x - \left(-KA\frac{dT}{dx}\Big|_{x+dx}\right) - hA_s(T-T_\infty) + I^2R_e = 0$$

copper electrical resistance $R_e = \rho_e \frac{dx}{A_c}$ electrical resistivity

$p\,dx$ perimeter

divide by dx

$$KA_c \frac{\frac{dT}{dx}\Big|_{x+dx} - \frac{dT}{dx}\Big|_x}{dx} - hp(T-T_\infty) + \frac{I_2\rho_e}{A_c} = 0$$

$$KA_c\frac{d^2T}{dx^2} - hp(T-T_\infty) + \frac{I^2\rho_e}{A_c} = 0$$

$p = \pi D$

$A_c = \frac{\pi}{4}D^2$

$$\boxed{\frac{d^2T}{dx^2} - \frac{hp}{KA_c}(T-T_\infty) + \frac{I^2\rho_e}{A_c^2} = 0}$$ governing eq

Two BC's

$T(x=0) = 20°C$

$T(x=L) = 20°C$

Analytical Solution

$$\frac{d^2T}{dx^2} - \frac{hp}{KA_c}\left(T - T_\infty - \frac{I^2\rho_e}{pA_ch}\right) = 0$$

$m^2 = \frac{hp}{KA_c}$

$\theta(x) = T(x) - T_\infty - \frac{I^2\rho_e}{pA_ch}$

$\frac{d^2\theta}{dx^2} - m^2\theta = 0$

$\theta(x) = C_1e^{mx} + C_2e^{-mx}$

BC1: $x=0$ $T=20°C$

$T_1 - T_\infty - \frac{I^2\rho_e}{pA_ch} = C_1e^{mx} + C_2e^{-mx}$

$T_1 - T_\infty - \frac{I^2\rho_e}{pA_ch} = C_1e^0 + C_2e^{-0}$

$C_1 = T_1 - T_\infty - \frac{I^2\rho_e}{pA_ch} - C_2$

$\frac{I^2\rho_e}{pA_ch} = B = \frac{I^2 \times 1.7\times10^{-8}}{6.283\times10^{-3}(3.14159\times10^{-6})(30)} = .028708\,I^2$

$T_1 - T_\infty - B = C_1 + C_2$

$T_1 - T_\infty - B = C_1e^{mL} + C_2e^{-mL}$

$C_1 = \frac{T_1 - T_\infty - B - C_2e^{-mL}}{e^{mL}}$

$T_1 - T_\infty - B = \frac{T_1 - T_\infty - B - C_2e^{-mL}}{e^{mL}} + C_2$

$e^{mL}(T_1 - T_\infty - B) = T_1 - T_\infty - B - C_2e^{-mL} + C_2e^{mL}$

$\frac{e^{mL}(T_1 - T_\infty - B) - (T_1 - T_\infty - B)}{(e^{mL} - e^{-mL})} = C_2$

BC2: $x=L$ $T=20°C$

$T_2 - T_\infty - \frac{I^2\rho_e}{pA_ch} = C_1e^{mL} + C_2e^{-mL}$

$C_2 = \frac{T_2 - T_\infty - \frac{I^2\rho_e}{pA_ch} - C_1e^{mL}}{e^{-mL}}$

$C_2 = T_2 - T_\infty - \frac{I^2\rho_e}{pA_ch} - \left(T_1 - T_\infty - \frac{I^2\rho_e}{pA_ch} - C_2\right)e^{mL}$

$C_2 + C_1e^{mL} = T_2 - T_\infty - \frac{I^2\rho_e}{pA_ch} - e^{mL}\left(T_1 - T_\infty - \frac{I^2\rho_e}{pA_ch}\right)$

$C_2 = \frac{T_2 - T_\infty - \frac{I^2\rho_e}{pA_ch} - e^{mL}\left(T_1 - T_\infty - \frac{I^2\rho_e}{pA_ch}\right)}{1 + e^{mL}}$

$C_2 = \frac{(T_1 - T_\infty - B)(e^{mL} - 1)}{e^{mL} - e^{-mL}} =$

$C_1 = \frac{T_1 - T_\infty - B - \frac{(T_1 - T_\infty - B)(e^{mL} - 1)}{e^{mL} - e^{-mL}}\left(e^{-mL}\right)}{e^{mL}}$

$$C_2 + C_1 e^{mL} = T_2 - T_\infty - \frac{I^2 \rho_e}{pA_c h} - e^{mL}\left(T_1 - T_\infty - \frac{I^2 \rho_e}{pA_c h}\right)$$

$$C_2 = \frac{T_2 - T_\infty - \frac{I^2 \rho_e}{pA_c h} - e^{mL}\left(T_1 - T_\infty - \frac{I^2 \rho_e}{pA_c h}\right)}{1 + e^{mL}}$$

$$C_2 = \frac{T_e\left(1 - e^{mL}\right) - T_\infty\left(1 - e^{mL}\right) - \frac{I^2 \rho_e}{pA_c h}\left(1 - e^{mL}\right)}{1 + e^{mL}}$$

$$C_2 = \frac{\left(1 - e^{mL}\right)\left(T_e - T_\infty - \frac{I^2 \rho_e}{pA_c h}\right)}{1 + e^{mL}}$$

$$e^{mL} - e^{-mL}$$

$$C_1 = \frac{T_1 - T_\infty - \theta - \frac{\left(T_1 - T_\infty - \theta\right)\left(e^{mL} - 1\right)}{e^{mL} - e^{-mL}}\left(e^{-mL}\right)}{e^{mL}}$$

$$C_1 = \frac{\left(T_1 - T_\infty - \theta\right)\left(1 - \frac{\left(e^{mL} - 1\right)\left(e^{-mL}\right)}{e^{mL} - e^{-mL}}\right)}{e^{mL}}$$

$$C_1 = T_1 - T_\infty - \frac{I^2 \rho_e}{pA_c h} - C_2$$

$$C_1 = T_e - T_\infty - \frac{I^2 \rho_e}{pA_c h} - \left[\frac{\left(1 - e^{mL}\right)\left(T_e - T_\infty - \frac{I^2 \rho_e}{pA_c h}\right)}{1 + e^{mL}}\right]$$

$$C_1 = T_e - T_\infty - \frac{I^2 \rho_e}{pA_c h} - \quad \left(T_e - T_\infty - \frac{I^2 \rho_e}{pA_c h}\right) \quad \frac{\left(1 - e^{mL}\right)}{1 + e^{mL}}$$

$$C_1 = T_e\left(1 - \frac{\left(1 - e^{mL}\right)}{1 + e^{mL}}\right) - T_\infty\left(1 - \frac{\left(1 - e^{mL}\right)}{1 + e^{mL}}\right) - \frac{I^2 \rho_e}{pA_c h}\left(1 - \frac{\left(1 - e^{mL}\right)}{1 + e^{mL}}\right)$$

$\rightarrow \theta(x) =$

$\rightarrow T(x) =$

$I_1 =$

$I_2 =$

$I_3 =$

plot w/ matlab for each I

$$C_1 = \left(1 - \frac{\left(1 - e^{mL}\right)}{1 + e^{mL}}\right)\left(T_e - T_\infty - \frac{I^2 \rho_e}{pA_c h}\right)$$

$C_1 = 1 \cdot \quad 1 - e$

$p = \pi D = 6.283 \times 10^{-3}$

$$A_c = \frac{\pi D^2}{4} = \pi \frac{(.002)^2}{4} = 3.14159 \times 10^{-6}$$

$$m = \sqrt{\frac{hp}{KA_c}} = \sqrt{\frac{20\left(6.283 \times 10^{-3}\right)}{401\left(3.14159 \times 10^{-6}\right)}} = 12.2322$$

$T_e = 20$

$T_\infty = 0$

$\rho_e = 17 \times 10^{-8}$

$L = .3$

$$T(x) = T_\infty + \frac{I^2 \rho_e}{pA_c h} + C_1 e^{mL} + C_2 e^{-mL}$$

$$T_x = \frac{I^2 \rho_e}{pA_c h} + \frac{\left(T_1 - T_\infty - \theta\right)\left(1 - \frac{\left(e^{mL} - 1\right)\left(e^{-mL}\right)}{e^{mL} - e^{-mL}}\right)}{e^{mL}} e^{mL} + \frac{\left(T_1 - T_\infty - \theta\right)\left(e^{mL} - 1\right)}{e^{mL} - e^{-mL}} e^{-mL}$$

$$\left(T - \theta\right)\left(1 - \frac{(J - 1)K}{(J - K)}\right)J$$

$e^{mL} = J \qquad e^{-mL} = K$

Exam 1

Tuesday, February 27, 2024 11:57 AM

ME 3525 Exam I Name: Easton Ingram

1. A 6-mm-diameter electrical transmission line carries an electric current of 50 A and has a resistance of 0.002 ohm per meter length. Determine the heat flux at the surface of the wire in steady state. **(8 pts)**

$V = IR = 50(.002) = .1\ V$

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{\dot{E}\forall}{A} = \frac{\dot{e}}{A} = \frac{VI\forall}{A} = \frac{VI\forall}{A} = \frac{(.1V)(50A)}{\pi(.006)^2/4\ m^2}$$

$$\dot{q} = 176.839\ \frac{kW}{m^2}$$

$\dot{E}_{gen} = \frac{\dot{e}_{gen}}{\forall}$

2. Consider a person standing in a room at 18°C. Determine the total rate of heat transfer from this person if the exposed surface area and the skin temperature of the person are 1.7 m^2 and 32°C, respectively, and the convection heat transfer coefficient is 5 W/m^2·K. Take the emissivity of the skin and the clothes to be 0.9, and assume the temperature of the inner surfaces of the room to be the same as the air temperature. **(16 pts)**

$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$

Conduction through Feet is negligible

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = 5\ W/m^2\cdot K(1.7m^2)(32-18)°K$$

$$\dot{Q}_{conv} = 119\ W$$

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = .9(5.67\times10^{-8}\ W/m^2\cdot K^4)(1.7m^2)(305^4 - 291^4)$$

$$\dot{Q}_{rad} = 128.633\ W$$

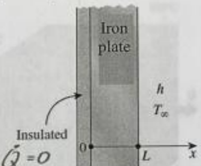
$T_s = 32°C = 305K$

$T_{surr} = 18°C = 291K$

$$\dot{Q}_{tot} = 119 + 128.633$$

$$\dot{Q}_{tot} = 247.633\ W$$

3. A large 7-cm-thick iron plate ($k = 15$ W/m·K) is exposed to an environment at 25°C with a heat transfer coefficient of 100 W/m^2·K. The left surface of the iron plate is insulated. (*a*) Express the differential equation in its simplest form and the boundary conditions for steady one-dimensional heat conduction through the iron plate without heat generation, and (*b*) solve the differential equation to find the temperatures on both surfaces of the iron plate. (c) Can we determine the temperature on the plate surfaces using the thermal resistance concept? Why? (**29 pts**)



$\dot{Q} = 0$

a.

$$\frac{d^2T}{d^2x} + \underbrace{\cancel{\frac{\dot{e}_{gen}}{k}}}_{\text{no heat gen}} = \frac{1}{\alpha}\underbrace{\cancel{\frac{dT}{dt}}}_{\text{steady}}$$

$$\frac{d^2T}{d^2x} = 0$$

BC 1: $\dot{Q}(x=0)_{left} = \dot{Q}(x=0)_{right}$ conduction

"Insulated" $\Rightarrow$ $0 = -kA\frac{dT}{dx}$ $\Rightarrow C_1 = 0$

BC2: $\dot{Q}(x=L)_{left} = \dot{Q}(x=L)_{right}$ convection

conduction

$$-kA\frac{dT}{dx} = hA_s(T_s - T_\infty) \quad \Rightarrow C_2 = T_\infty$$

$T = C_1 x + C_2$

$T = T_\infty$

b. $T_{left} = 25°C$

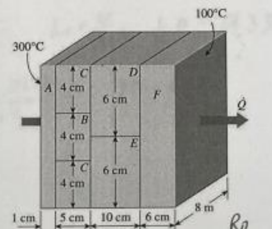
$T_{right} = 25°C$

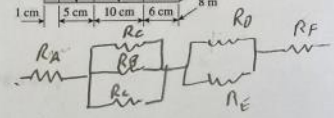
c. Conditions for thermal resistance are

1. 1-D heat conduction ✓
2. Steady state ✓
3. no heat generation ✓

Therefore, thermal resistance concept can be used.

4. Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross section is as given in the figure. The thermal conductivities of various materials used, in W/m·K, are $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$. The left and right surfaces of the wall are maintained at uniform temperatures of 300°C and 100°C, respectively. Assuming heat transfer through the wall to be one-dimensional, determine (*a*) the rate of heat transfer through the wall; (*b*) the temperature at the point where the sections *B*, *D*, and *E* meet. **(27 pts)**





$R_{conv} = \frac{1}{hA}$ $\quad R_{cond} = \frac{L}{KA}$

$R_1 = \frac{.01}{2(.12)} = .0417$

$R_2 = \left(\frac{1}{R_C} + \frac{1}{R_B} + \frac{1}{R_C}\right)^{-1}$

$= \left(\frac{1}{\left(\frac{.05}{20(.04)}\right)} + \frac{1}{\left(\frac{.05}{8(.04)}\right)} + \frac{1}{\frac{.05}{20(.04)}}\right)^{-1}$

$= .0260$

$R_3 = \left(\frac{1}{R_D} + \frac{1}{R_E}\right)^{-1} = \left(\frac{1}{\left(\frac{.1}{15(.06)}\right)} + \frac{1}{\left(\frac{.1}{35(.06)}\right)}\right)^{-1} = .0333$

$R_4 = \frac{L}{KA} = \frac{.06}{2(.12)} = .25$

$R_{eq} = R_1 + R_2 + R_3 + R_4 = .0417 + .026 + .0333 + .25 = .3510$

$\dot{Q} = \frac{\Delta T}{R_{eq}} = \frac{300-100}{.351} = \boxed{569.733\ W}$

$\frac{T_1 - T_{BDE}}{R_1 + R_2} = \dot{Q}$

$\frac{300 - T_{BDE}}{.0417 + .026} = 569.733 \Rightarrow T_{BDE} = 300 - (.0417 + .026)(569.733)$

$\boxed{T_{BDE} = 261.424\ °C}$

5. Citrus fruits are very susceptible to cold weather, and extended exposure to subfreezing temperatures can destroy them. Consider an 8-cm-diameter orange ($\alpha = 1.36\times10^{-7}$ m^2/s, $k = 0.57$ $W/m\cdot K$) that is initially at 15°C. A cold front moves in one night, and the ambient temperature suddenly drops to −6°C, with a heat transfer coefficient of 15 $W/m^2\cdot K$. Assuming the ambient conditions to remain constant for 4 h before the cold front moves out, determine if any part of the orange will freeze that night. **(20 pts)**

Check for lumped system: $B_i = \frac{hL_c}{K}$ $L_c = \frac{V}{A_s} = \frac{1}{6}D = .0133$

Not lumped system $B_i = \frac{15(.0133)}{.57} = .35 \not< .1$

1-term approximation:

$$\frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

The outside will be the first to freeze, so check $r = .04$ m

$$T_s = \left[A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}\right](T_i - T_\infty) + T_\infty$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.36\times10^{-7}\ m^2/s)(4hr)\left(\frac{60min}{1hr}\right)\left(\frac{60s}{1min}\right)}{(.04\ m)^2}$$

From Table 4-2: $\tau = 1.224 > .2$ ✓

$$B_i = \frac{hr_o}{K} = \frac{15(.04)}{.57} = 1.0526 \approx 1.0$$

$\lambda_1 = 1.5708$

$A_1 = 1.2732$ $\frac{r}{r_o} = 1$

$$T_s = \left[1.2732 e^{-(1.5708)^2(1.224)} \frac{\sin(1.5708)}{1.5708}\right](15-(-6)) + (-6)$$

$T_s = -5.169$

Since Temp at surface is less than 0, and an orange is mostly water, the part of the orange will freeze.