Logistical and Transportation Planning Methods

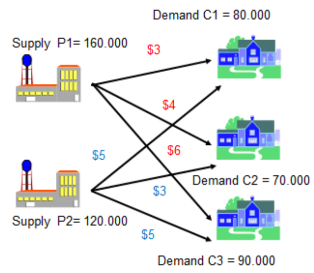
Prof. Hai Jiang

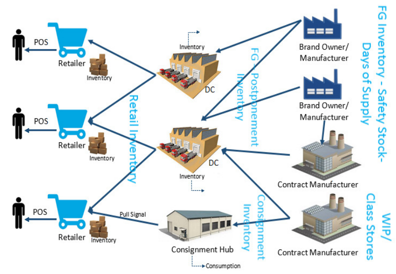
Department of Industrial Engineering

Tsinghua University

This is What We Used to Work With

- Transportation planning
- Multi echelon inventory planning

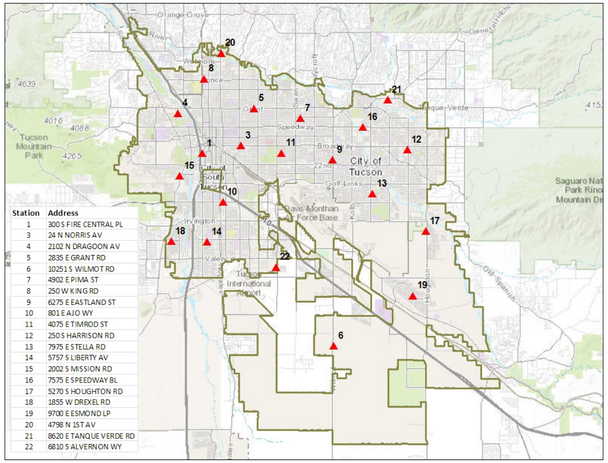




Course Objective

- Focus on deployment problems of certain urban service systems
  - Emergency services (police, fire, medical)
  - Pickup and delivery services (mail delivery)
  - Transportation services (taxicabs, buses)
- Provide students with a set of relevant analytical skills to deal with logistical and transportation planning problems arising in the urban setting

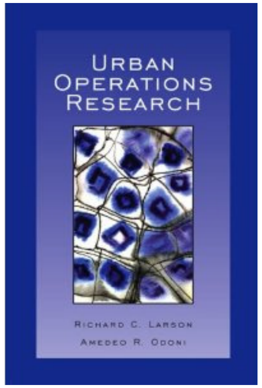
Location Planning of Fire Stations



Characteristics of These Systems

Uncertainty appears in the time/location/duration of service	• Probabilistic analysis
Service requests are distributed spatially	• Geometrical probability
While providing service, congestion is likely to arise	• Queueing theory

Textbook



- R. Larson, A. R. Odoni (2007) "Urban Operations Research" (2nd Ed), Dynamic Ideas, Charlestown, MA [Chapters 1-5]
- Full text:
- http://web.mit.edu/urban_or_book/www/book/

Class Policies

- You need to attend all classes
- Failure to submit your homework by the due date receives a score of ZERO
- All electronic devices are prohibited in class
- Grading
  - Assignments 10%
  - Exam 40%

Review of Probabilistic Modeling

Prof. Hai Jiang

Tsinghua University

Outline

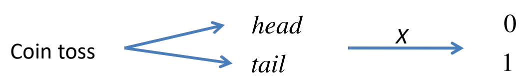
- Review the fundamentals of probabilistic modeling with emphasis on physical situations in an urban setting
- 4-Step Process to calculate event probability
- Random incidence
- Pedestrian crossing problem

Definitions

- Experiment
  - Any nondeterministic process that has a number of distinct possible outcomes
- Experimental trial
  - A particular performance of the experiment yielding exactly one of the outcomes
- Sample space
  - The finest-grained list of outcomes for an experiment
  - {heads, tails}, $\{0 \le x \le 10, 0 \le y \le 10\}$
- Event
  - A collection of elements in the sample space

Random Variables

- Given an experiment with a sample space, a *random variable* is a function that assigns a numerical value to each finest-grained outcome in the sample space



- Random variables are denoted by capital letters, such as, *X*, *Y*, or *Z*
- A particular realization out of an experiment is denoted by the corresponding lowercase letter x, y, z
- Event space: the set of possible values for a RV
- Discrete vs Continuous Random Variables

Continuous Random Variables

- A RV whose event space is distributed over a continuous set of values
- Probability Density Function (PDF)

$$f_X(x)dx \equiv \text{probability that RV } X \text{ assumes a value between } x \text{ and } x+dx \text{ in an experiment trial}$$

- Cumulative Density Function (CDF)

$$F_X(x) = \int_{-\infty}^{x} f_X(y)dy$$

Often-Used PDF's

- Uniform PDF

$$f_U(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

- Exponential PDF

$$f_T(t) = \begin{cases} ae^{-at} & t \ge 0, a > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Normal PDF

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \quad -\infty < y < \infty$$

Discrete Random Variables

- A RV whose event space contains a finite or countably infinite number of values
- Probability Mass Function (PMF)

$$p_X(x) \equiv P\{X = x\}$$
$$\sum_x p_X(x) = 1$$
$$0 \le p_X(x) \le 1$$

- Cumulative Distribution Function (CDF)

$$P_X(x) \equiv P\{X \le x\} = \sum_{y \le x} p_X(y)$$

Often-Used PMF's

- Bernoulli PMF
  - $p_X(0)=1-p$, $p_X(1)=p$
  - $E[X]=p$, $Var(X) = E[X^2]-(E[X])^2 = p(1-p)$
- Poisson PMF

$$p_K(k) = \frac{\mu^k e^{-\mu}}{k!} \quad k = 0,1,2,\ldots; \mu > 0$$

  - $E[K]=\mu$, $Var(K)=\mu$
- Poisson Process
  - The number of Poisson arrivals occurring in a time interval of length t is Poisson-distributed with mean λt

$$P\{N(t) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \qquad k = 0,1,2,\ldots$$

Poisson Process

- Often applied to occurrences of events in time
  - Requests for service
  - Breakdowns of equipment
  - Arrivals of vehicles at an intersection

Interarrival Times

L_k = time of occurrence of the kth arrival, $k = 1, 2, \ldots$

kth order interarrival time distribution f_{L_k} :

$$
\begin{aligned}
f_{L_k}(x)dx &\equiv P\{k\text{th arrival occurs in the interval } x \text{ to } x+dx\} \\
&= P\left\{\begin{array}{l}\text{exactly } k-1 \text{ arrivals in the interval } [0, \mathrm{x}] \\ \text{and exactly one arrival in } [x, x+dx]\end{array}\right\} \\
&= P\{\text{exactly } k-1 \text{ arrivals in the interval } [0, \mathrm{x}]\} \\
&\quad \cdot P\{\text{exactly one arrival in } [x, x+dx]\} \\
&= \frac{(\lambda x)^{k-1} e^{-\lambda x}}{(k-1)!} \cdot [\lambda dx + o(dx)] \\
&= \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} dx
\end{aligned}
$$

Interarrival Times

- Kth-order interarrival time ditribution is a kth-order Erland pdf

$$f_{L_k}(x)dx = \frac{\lambda^k t^{k-1} e^{-\lambda x}}{(k-1)!}, \quad x \geq 0, k = 1,2,\ldots$$

- When k=1, this is exponential pdf

$$f_{L_1}(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

- Poisson process is memoryless, future arrivals do not depend on previous arrivals

Memoryless Property

- Suppose that the inter-arrival time is 10 minutes, and I have already waited 15 minutes, what's the PDF for the additional time I will have to wait

$f_T(t) = \frac{1}{10} e^{-\frac{1}{10}t}$

Let $\mathrm{A} = \{\mathrm{T} \geq 15 \text{ minutes}\}$, we then have

$$\mathrm{f_T}(t \mid A) = \frac{\frac{1}{10} e^{-\frac{1}{10}t}}{P\{A\}} = \frac{\frac{1}{10} e^{-\frac{1}{10}t}}{\int_{15}^{\infty} \frac{1}{10} e^{-\frac{1}{10}t} dt} = \frac{\frac{1}{10} e^{-\frac{1}{10}t}}{\left(-e^{-\frac{1}{10}t}\right)\Big|_{15}^{\infty}} = \frac{1}{10} e^{-\frac{1}{10}(t-15)} \qquad t \geq 15$$

Example: Stick Cutting

- Suppose that two points are marked on a stick of length 1 meter
  1. Define the sample space for this experiment
  2. What is the probability that the second point is to the left of the first one
  3. What is the probability that a triangle can be formed with the resulting three pieces

4-Step Process

- Define the Random Variables
- Identify the joint sample space
- Determine the probability law over the sample space
- Carefully work in the sample space to answer any question of interest

Define Random Variables

Joint PDF

- Uniform over the square

Work Within the Sample Space

- What is the probability that the second point is to the left of the first one
-

Work Within the Sample Space

- What conditions must be satisfied so that a triangle can be formed?

清華大學

Cutting Strategy #2

- Cut the stick at a random point
- Take the left stick piece and cut that at a random point
- What's the probability that a triangle can be formed?

清華大學

4-Step Process

- Define RVs
- Joint sample space

清華大學

Cutting Strategy #3

- Cut the stick at a random point
- Take the longer side and cut again
- What's the probability that a triangle can be formed

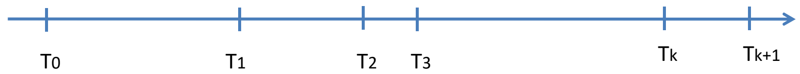
清華大學

4-Step Process Again

- Define RVs
- Joint sample space

Random Incidence

- Suppose we have a stochastic process with identically distributed interarrival times: $Y_k = T_k - T_{k-1}$



- Y_k's can be identically distributed but not necessarily independent
  - Bus headways
  - Police passing by a residence or business

Random Incidence

- If we know $f_Y()$, we are often interested in the following problem
  - A bus passenger looking for a bus, starts observing the process at a ***random time***, what is the time he must wait until the next arrival occurs
  - Similar problems: the waiting time for police car, subway, or elevator
- This is said to be a problem of **random incidence**, because the individual observer is incident to the process at a random time
- What is the pdf of V (the time from the moment of random incidence until the next arrival occurs)?

Derivation of *V*

- Let W be the length of the interarrival gap entered by this random incidence, we have

Example

- Example 1: two gaps with same frequency
  - w_1, $w_2=2w_1$
  - An individual is twice likely to enter the gap of length w_2
- Example 2: frequency of longer gap is half that of the shorter gap
  - $f_Y(w_2)dw=1/2f_Y(w_2)d_w$
  - An individual is equally likely to enter either types of gaps

Once We Enter a Gap

Example 1

- Suppose that buses maintain perfect headway of T_0 minutes, what is $f_V(V)$

$$F_Y(v)=\begin{cases}0, & v<T_0\\ 1, & v\geq T_0\end{cases}$$

$$f_V(v)=\begin{cases}\dfrac{1}{T_0}, & v<T_0\\ 0, & \text{otherwise}\end{cases}$$

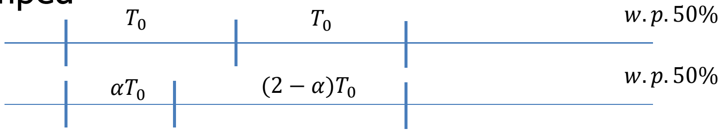
$$E[V]=\int_0^{T_0}\frac{1}{T_0}v\,dv=\frac{T_0}{2}$$

Example 2

- Suppose police cars patrol in a completely random manner with car passings occurring according to a Poisson process with mean rate λ passings per day, what is $f_V(V)$

Example 3

- Suppose that buses on a particular route is on schedule half the time and clumped in pairs the other half the time.
- For 50% of the day (which 50% is unknown) the bus headways are exactly T_0 , for the remaining 50% of the day, buses are clumped



- What is the mean wait time for a bus?

Example 3 (cont'd)

Expected Waiting Time

$$E[V]=\int_0^\infty E[V\mid w]f_W(w)dw$$

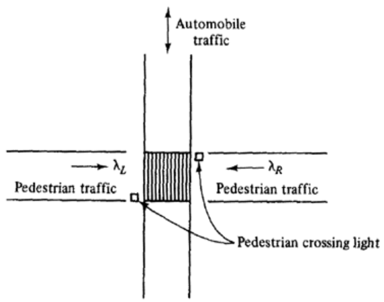
$$=\int_0^\infty \frac{w}{2}\cdot\frac{wf_Y(w)}{E[Y]}dw$$

$$=\frac{E[Y^2]}{2E[Y]}=\frac{\sigma_Y^2+E^2[Y]}{2E[Y]}$$

The mean time from random incidence until the next event depends only on the mean and variance of the inter-event time Y

Example	Calculation Details	E[V]
Buses with perfect headway	$E[Y]=T_0$ $\sigma^2_Y=0$	$T_0/2$
Police patrol	$E[Y]=1/\lambda$ $\sigma^2_Y=1/\lambda^2$	$1/\lambda$
Clumped buses	$E[Y]=T_0$ $E[Y^2]=\frac{(\alpha-1)^2+2}{2}T_0^2$	

Pedestrian Crossing Problem



- Possible Decision Rules:
  - Rule A: Dump every T minutes
  - Rule B: Dump whenever the total number of waiting pedestrians equals N_0
  - Rule C: Dump whenever the first pedestrian to arrive after the previous dump has waited T_0 minutes

Evaluation of Decision Rules

1. The expected number of pedestrians crossing left to right on any dump
2. The probability that zero pedestrians cross left to right on any particular dump
3. The pdf for the time between dumps
4. The expected time that a randomly arriving pedestrian must wait until crossing
5. The expected time that a randomly arriving observer, who is not a pedestrian, will wait until the next dump