λ $X \geq_{st} Y \Rightarrow X^+ \geq_{st} Y^+$ and $X^- \leq_{st} Y^-$

Proof: $P\{X^- > a\} = P\{0 > a \mid X \geq 0\} P\{X \geq 0\} + P\{-X > a \mid X < 0\} P\{X < 0\}$

~~① $a \geq 0$~~ $P\{X^+ > a\} = P\{X > a \mid X \geq 0\} P\{X \geq 0\} + P\{0 > a \mid X < 0\} P\{X < 0\}$

① $a \geq 0$ $P\{X^+ > a\} = P\{X > a\} \geq P\{Y > a\} = P\{Y^+ > a\}$

$P\{X^- > a\} = P\{X < -a\} = F(-a)$

Since $\bar{F}(a) \geq \bar{G}(a) \Rightarrow F(a) \leq G(a)$ 对 $F \cdot G$ 单调.

$\Rightarrow P\{X^- > a\} = F(-a) \leq G(-a) = P\{Y^- > a\}$

② $a < 0$, $P\{X^+ > a\} = 1 = P\{Y^+ > a\}$

$P\{X^- > a\} = 1 = P\{Y^- > a\}$

综上: $P\{X^+ > a\} \geq P\{Y^+ > a\}$ $P\{X^- > a\} \leq P\{Y^- > a\}$.

$\Rightarrow$ $X^+ \geq_{st} Y^+$ $X^- \leq_{st} Y^-$

2, Counterexample when independency assumption is dropped?

Example 2

$X_1 \sim B(1, \frac{1}{2})$ $\qquad X_2 = 1 - X_1$

$Y_1 \sim B(1, \frac{1}{2})$ $\qquad Y_2 \sim B(1, \frac{1}{2})$

$$f(x_1, x_2) = \begin{cases} 0 & x_1 + x_2 \le 1 \\ 1 & \text{else} \end{cases}$$

$X_i \geq_{st} Y_i$

$$E[f(X_1, X_2)] = 0 \le E[f(Y_1, Y_2)] = \frac{1}{4}$$

构成反例.