Lemma 6.15

Proof:

$\forall \lambda \in (0,1)$,

$$V[\lambda p_1 + (1-\lambda) p_2] = \min_{\iota \in \Delta} V_\iota(\lambda p_1 + (1-\lambda) p_2)$$

Since polieses in Δ are independent of initial probability, it follows from the interpretation of the state as a probability that for $\iota \in \Delta$

$$V_\iota[\lambda p_1 + (1-\lambda) p_2] = \lambda V_\iota(p_1) + (1-\lambda) V_\iota(p_2)$$

However,

$$V[\lambda p_1 + (1-\lambda) p_2] = \min_{\iota \in \Delta} \{\lambda V_\iota(p_1) + (1-\lambda) V_\iota(p_2)\}$$

$$\geq \min_{\iota \in \Delta} \lambda V_\iota(p_1) + \min_{\iota \in \Delta} (1-\lambda) V_\iota(p_2)$$

$$= \lambda V(p_1) + (1-\lambda) V(p_2).$$

Then the lemma is proven.