均匀分布：$f=\frac{1}{b-a}(a\le x\le b), E=\frac{a+b}{2}, Var\ =\frac{(a-b)^2}{12}$

指数分布：$f=ae^{-ax}(x\ge 0), E=\frac{1}{a}, Var\ =\frac{1}{a^2}$

Poisson分布：$p(x=k)=\frac{\lambda^k e^{-\lambda}}{k!}, E=\lambda, Var\ =\lambda$(泊松过程将$\lambda$换为$\lambda t$)

K阶Erlang分布：$f_{L_k}(x)dx=\frac{\lambda^k x^{k-1}e^{-\lambda x}}{(k-1)!}(k=1,2,\ldots), E=\frac{k}{\lambda}, Var\ =\frac{k}{\lambda^2}$,

泊松过程第k次到达的时间服从k阶埃尔朗分布

瑞雷(Rayleigh)分布: $f_R(r)=\frac{r}{\sigma^2}e^{-\frac{r^2}{2\sigma^2}}(r>0), E=\sigma\sqrt{\frac{\pi}{2}}, Var=(2-\frac{\pi}{2})\sigma^2$

$$E\left[\frac{D}{S}\right]=E[D]E\left[\frac{1}{S}\right]\ge\frac{E[D]}{E[S]}$$

4-step process：（1）定义RV，（2）识别联合样本空间，（3）确定样本空间上的概率分布规则，即计算联合概率分布，（4）求解相应的问题

Random incidence：Y是两次事件的时间间隔，V是顾客需要等待的时间，W是顾客进入系统时事件时间间隔的长度，$f_W(w)=\frac{wf_Y(w)}{E(Y)}$，$f_{V|W}(v|w)=\frac{1}{w}(0\le v\le w), f(v,w)=\frac{f_Y(w)}{E(Y)}$，$f_V(v)=\frac{1-F_Y(v)}{E(Y)}, E(V)=\frac{\sigma_Y^2+E^2(Y)}{2E(Y)}$ (应用：行人过马路问题)

导出分布: (1)先求出原变量的联合概率密度$f_{x_1,x_2,\ldots}(\cdot)$，(2)求出导出变量的累计概率分布$F_{Y_1,Y_2,\ldots}(\cdot)$，(3)$f_{Y_1,Y_2,\ldots}(\cdot)=\frac{\partial^m}{\partial Y_1\ldots\partial Y_m}F_{Y_1,Y_2,\ldots}(\cdot)$.

二项分布变量可以表示成：$f_X(x)=(1-p)\mu_0(x)+p\mu_0(x-1)$，其中$\mu_0(x)$定义为$\int_{-a}^{b}\mu_0(x)=1, \forall a,b>0$.

方形区域$[0-X_0, 0-Y_0]$内两个随机均匀分布变量的**直角距离**，$E[D]=\frac{1}{3}(X_0+Y_0), Var[D]=\frac{1}{18}(X_0^2+Y_0^2)$，直角距离A，直线距离B,$A=(sin\theta+cos\theta)B=\sqrt{2}\cos\left(\theta-\frac{\pi}{4}\right)B$,$F_R(r)=1-\frac{4}{\pi}\cos^{-1}\left(\frac{r}{\sqrt{2}}\right)$,$r\in[1,\sqrt{2}]$,$F_R(r)=\frac{4}{\pi}\frac{1}{\sqrt{2-r^2}}$,$E[R]=\frac{4}{\pi}\approx 1.273$

干扰（Perturbation）：(1)在(b,0)处增加一个长为a的路障，$E[D']=E[D]+E[D_e]=\frac{1}{3}(X_0+Y_0)+\frac{2a}{3}\cdot 2(\frac{b}{X_0}\frac{a}{Y_0})[\frac{(X_0-b)}{X_0}\frac{a}{Y_0}]$

(2)对于PDF的扰动，$f_{X'}(x)=f_X(x)-g(x)+h(x)$，$\int_{-\infty}^{+\infty}g(x)=\int_{-\infty}^{+\infty}f(x)=P_\Delta, \frac{g(x)}{P_\Delta}\to X_g, \frac{h(x)}{P_\Delta}\to X_h$,

$E[X']=E[X]-P_\Delta(E[X_g]-E[X_h]), E[X'^2]=E[X^2]-P_\Delta(E[X_g^2]-E[X_h^2])$

(3)如果是样本空间的扰动，$S'=S-S_\Delta^1+S_\Delta^2$，则$P_\Delta=\frac{S_\Delta}{S}$，$E_{S'}[g(X,Y)]=E_S[g(X,Y)]-P_\Delta(E_{S_\Delta^1}[g(X,Y)]-E_{S_\Delta^2}[g(X,Y)])$.

Server分布服从**空间泊松过程**(Spatial Poisson Process)，规定距离D内至少存在响应（即响应距离），D服从**瑞雷分布**

排队论相关公式:

λ到达率；μ服务率；$E[S]=\frac{1}{\mu}$服务时间的期望；$\bar{W}$稳态下用户的平均等待时长；$\bar{W_q}$稳态下用户的平均排队时间；$\bar{L}$稳态下系统内的平均用户数量；$\overline{L_q}$稳态下队伍中的平均用户数量；$\bar{W}=E[S]+\bar{W_q}$，$\bar{L}=\lambda\bar{W}$，$\overline{L_q}=\lambda\bar{W_q}$，$\rho=\frac{\lambda}{\mu}$

M/M/1/∞：$P_0=1-\rho$，$P_n=\rho^n(1-\rho)$，$\bar{L}=\frac{\lambda}{\mu-\lambda}$，系统一次busy period的时长为：$E(IP)=\frac{1}{\lambda}, E(BP)=\frac{1}{\mu-\lambda}$

M/M/ ∞ /∞：$\bar{L}=\frac{\lambda}{\mu}$，M/M/1/K：$\overline{L_q}=\frac{\rho}{1-\rho}-\frac{\rho(1+K\rho^K)}{1-\rho^{K+1}}$

- N: number of calls in the queueing system just after t_{i-1}, when service to the (i-1)th patient is completed
- R: number of new calls that arrive during the period when patient i receives service
- N': number of callers in the system just after t_i, when service to the ith patient is completed

$$N'=\begin{cases}N+R-1 & \text{if } N>0\\ R & \text{if } N=0\end{cases}$$

- If we define δ as

$$\delta=\begin{cases}0 & \text{if } N>0\\ 1 & \text{if } N=0\end{cases}$$

- We have $N'=N+R-1+\delta \quad N\ge 0$

Suppose that the service time to patient i lasts s, we have

$$E[R\mid S=s]=\lambda s$$

$$E[R]=\int_0^\infty E[R\mid S=s]f_S(s)ds=\int_0^\infty \lambda s f_S(s)ds=\lambda E[S]=\frac{\lambda}{\mu}=\rho$$

Since $N'=N+R-1+\delta$, we have $E[N']=E[N]+E[R]-1+E[\delta] \quad N\ge 0$

In steady state, we have $E[N']=E[N]$, hence $E[\delta]=1-E[R]=1-\frac{\lambda}{\mu}=1-\rho$

$$E[R^2\mid S=s]=Var(R\mid S=s)+(E[R\mid S=s])^2=\lambda s+(\lambda s)^2$$

$$E[R^2]=\int_0^\infty E[R^2\mid S=s]f_S(s)ds=\int_0^\infty[\lambda s+(\lambda s)^2]f_S(s)ds$$

$$=\lambda E[S]+\lambda^2E[S^2]=\frac{\lambda}{\mu}+\lambda^2\left(\sigma_S^2+\frac{1}{\mu^2}\right)=\rho+\lambda^2\sigma_S^2+\rho^2$$

Important results

$P_0=E[\delta]=1-\rho$

$\bar{L}=\rho+\frac{\rho^2+\lambda^2\sigma_S^2}{2(1-\rho)}$

$\bar{W}=\frac{\bar{L}}{\lambda}, \quad \bar{W}_q=\bar{W}-\frac{1}{\mu}, \quad \bar{L}_q=\lambda\bar{W}_q$

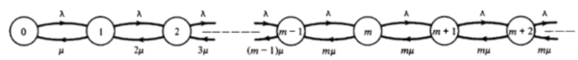
$W_q=\frac{\lambda\left[\frac{1}{\mu^2}+\sigma_S^2\right]}{2(1-\rho)}$

$\frac{E[BP]}{E[BP]+E[IP]}=1-P_0=\rho$

$\frac{E[BP]}{E[BP]+\frac{1}{\lambda}}=1-P_0=\rho$

$E[BP]=\frac{1}{\mu-\lambda}$

Same as M/M/1/∞



$$\lambda_n=\lambda \qquad n=0,1,2,\ldots$$
$$\mu_n=n\mu \qquad n=1,2,3,\ldots,m-1$$
$$\mu_n=m\mu \qquad n=m,m+1,m+2,\ldots$$

$$P_n=\begin{cases}\frac{(\lambda/\mu)^n}{n!}P_0 & \text{for } n=0,1,\ldots,m-1\\ \frac{(\lambda/\mu)^n}{m^{n-m}\cdot m!}P_0 & \text{for } n=m,m+1,m+2,\ldots\end{cases}$$

$$P_0=\left[\sum_{n=0}^{m-1}\frac{(\lambda/\mu)^n}{n!}+\frac{(\lambda/\mu)^m}{m!}\frac{1}{1-(\lambda/m\mu)}\right]^{-1}$$

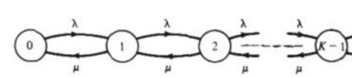
M/M/∞ /∞

$$P_n=\frac{(\lambda/\mu)^n}{n!}P_0 \qquad \text{for } n=1,2,3,\ldots$$
$$1=\sum_{n=0}^{\infty}P_n=P_0\sum_{n=0}^{\infty}\frac{(\lambda/\mu)^n}{n!}=P_0e^{\lambda/\mu}$$
$$P_n=\frac{(\lambda/\mu)^ne^{-\lambda/\mu}}{n!} \qquad \text{for } n=0,1,2,\ldots$$

Steady state probability is Poisson with parameter λ/μ

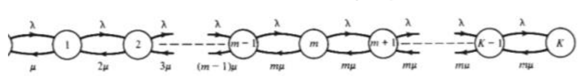
$\bar{L}=E[N]=\frac{\lambda}{\mu} \qquad \bar{W}=\frac{1}{\mu}$

$\bar{W}_q=\bar{L}_q=0$



$$\lambda_n=\begin{cases}\lambda & \text{for } n=0,1,2,\ldots,K-1\\ 0 & \text{otherwise}\end{cases}$$
$$\mu_n=\begin{cases}\mu & \text{for } n=1,2,\ldots,K\\ 0 & \text{otherwise}\end{cases}$$
$$P_n=\begin{cases}\left(\frac{\lambda}{\mu}\right)^nP_0=\rho^nP_0 & \text{for } n=0,1,2,\ldots,K\\ 0 & \text{otherwise}\end{cases}$$
$$1=\sum_{n=0}^{K}P_n=P_0(1+\rho+\rho^2+\cdots+\rho^K)=P_0\frac{1-\rho^{K+1}}{1-\rho}$$
$$P_n=\begin{cases}\frac{\rho^n(1-\rho)}{1-\rho^{K+1}} & \text{for } n=0,1,2,\ldots,K\\ 0 & \text{otherwise}\end{cases}$$

$\bar{L}_q$

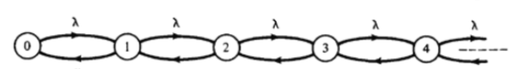


ous calculations lead to:

$$P_0=\left[\sum_{n=0}^{m}\frac{(\lambda/\mu)^n}{n!}+\frac{(\lambda/\mu)^m}{m!}\sum_{n=m+1}^{K}\left(\frac{\lambda}{m\mu}\right)^{n-m}\right]^{-1} \qquad \left(\text{for } \frac{\lambda}{m\mu}<1\right)$$

$$P_n=\begin{cases}\frac{(\lambda/\mu)^n}{n!}P_0 & \text{for } n=0,1,2,\ldots,m\\ \frac{(\lambda/\mu)^n}{m!\,m^{n-m}}P_0 & \text{for } n=m+1,m+2,\ldots,K\end{cases}$$

$$\bar{L}_q=\frac{P_0(\lambda/\mu)^m(\lambda/m\mu)}{m!(1-\lambda/m\mu)^2}\left[1-\left(\frac{\lambda}{m\mu}\right)^{K-m}-(K-m)\left(\frac{\lambda}{m\mu}\right)^{K-m}\left(1-\frac{\lambda}{m\mu}\right)\right]$$



$$P_0=\frac{1}{1+\sum_{n=1}^{\infty}K_n}=\frac{1}{1+\sum_{n=1}^{\infty}\rho^n}=1-\rho$$

$$P_n=\rho^n(1-\rho)$$

$$\bar{L}=\sum_{n=0}^{\infty}nP_n=\sum_{n=0}^{\infty}n\rho^n(1-\rho)=\rho(1-\rho)\sum_{n=0}^{\infty}n\rho^{n-1}=\rho(1-\rho)\sum_{n=0}^{\infty}\frac{d}{d\rho}(\rho^n)$$
$$=\rho(1-\rho)\frac{d}{d\rho}\left(\sum_{n=0}^{\infty}\rho^n\right)=\rho(1-\rho)\frac{d}{d\rho}\left(\frac{1}{1-\rho}\right)=\rho(1-\rho)\frac{1}{(1-\rho)^2}=\frac{\rho}{1-\rho}=\frac{\lambda}{\mu-\lambda}$$

空间分布式队列 (spatially distributed queue)，是一个M/G/1/ ∞排队系统，稳态下满足$E[N^2]=E\left[N'^2\right]$，$E[N]=E[N']$,

$$E[N]=\bar{L}=\rho+\frac{\rho^2+\lambda^2\sigma_S^2}{2(1-\rho)}$$

$$(N')^2=N^2+(R-1)^2+\delta^2+2N(R-1)+2N\delta+2(R-1)\delta$$
$$=N^2+(R-1)^2+\delta+2N(R-1)+0+2(R-1)\delta$$
$$=N^2+(R-1)^2+2N(R-1)+(2R-1)\delta$$

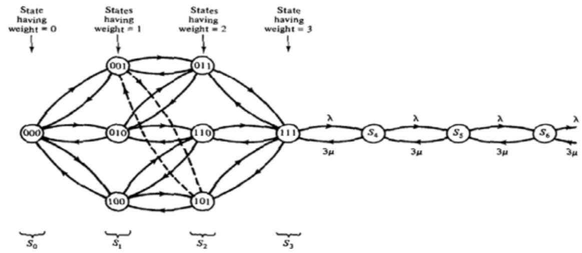
$$E[(N')^2]=E[N^2]+E[(R-1)^2]+2E[N]E[R-1]+E[2R-1]E[\delta]$$
$$0=E[(R-1)^2]+2E[N]E[R-1]+E[2R-1]E[\delta]$$
③ ① ① ②

Hence, we have

$$0=E[(R-1)^2]+2E[N]E[R-1]+E[2R-1]E[\delta]$$
$$0=0+(\lambda^2\sigma_S^2+\rho^2+\rho-2\rho+1)+2E[N](\rho-1)+(2\rho-1)(1-\rho)$$
$$E[N]=\rho+\frac{\rho^2+\lambda^2\sigma_S^2}{2(1-\rho)}$$

f_{nj}:服务器n服务区域j内的顾客的概率；$T_n(j)$服务器n到服务区域j的旅行时间；T^*整个系统的平均旅行时间；$T^* = f_{11}T_1(A) + f_{22}T_2(B-A) + f_{12}T_1(B-A) + f_{21}T_2(A)$

服务器的最优边界到两个服务器的时间差满足：$s_2 - s_1 = \frac{2\eta}{2\eta+1}[T_2(B) - T_1(B)]$



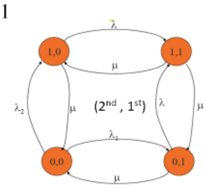
$$\begin{cases}\text{For state } 00: & P_{00}(\lambda_1+\lambda_2) = (P_{01}+P_{10})\mu \\ \text{For state } 01: & P_{01}(\lambda+\mu) = P_{00}\lambda_1 + P_{11}\mu \\ \text{For state } 11: & P_{11}2\mu = P_{01}\lambda + P_{10}\lambda \\ & P_{00}+P_{01}+P_{10}+P_{11} = 1\end{cases}$$

$$P_{00} = \frac{1}{1+\lambda/\mu+\lambda^2/(2\mu^2)} = \frac{1}{1+2\eta+2\eta^2}$$

$$P_{11} = \frac{\lambda^2}{2\mu^2}P_{00} = 2\eta^2 P_{00}$$

$$P_{01} = \frac{\lambda_1+\lambda^2/(2\mu)}{\lambda+\mu}P_{00} = \frac{\eta_1+2\eta^2}{1+2\eta}P_{00}$$

$$P_{10} = \frac{\lambda_2+\lambda^2/(2\mu)}{\lambda+\mu}P_{00} = \frac{\eta_2+2\eta^2}{1+2\eta}P_{00}$$



$$\text{where } \eta = \frac{\lambda}{2\mu}, \eta_1 = \frac{\lambda_1}{\mu}, \eta_2 = \frac{\lambda_2}{\mu}$$

Scenarios	# of requests	f_{nj}
unit 1 serves area 2	$TP_{10}\lambda_2$	$P_{10}\lambda_2/(1-P_{11})(\lambda_1+\lambda_2)$
unit 1 serves area 1	$T(P_{00}+P_{10})\lambda_1$	$(P_{00}+P_{10})\lambda_1/(1-P_{11})(\lambda_1+\lambda_2)$
unit 2 serves area 1	$TP_{01}\lambda_1$	$P_{01}\lambda_1/(1-P_{11})(\lambda_1+\lambda_2)$
unit 2 serves area 2	$T(P_{00}+P_{01})\lambda_2$	$(P_{00}+P_{01})\lambda_2/(1-P_{11})(\lambda_1+\lambda_2)$
Total	$T(1-P_{11})(\lambda_1+\lambda_2)$	

$$T_1(B) = \frac{1}{2}\times\frac{1/4}{2} + \frac{1}{2}\times\frac{1+3/4}{2} = \frac{1}{2}$$

Note that B refers t

$$T_2(B) = \frac{1}{2}\left(\frac{3/4}{7/4}\times\frac{3/4}{2} + \frac{1}{7/4}\times\frac{1}{2}\right) + \frac{1}{2}\times\frac{1+5/4}{2} = \frac{11}{14}$$

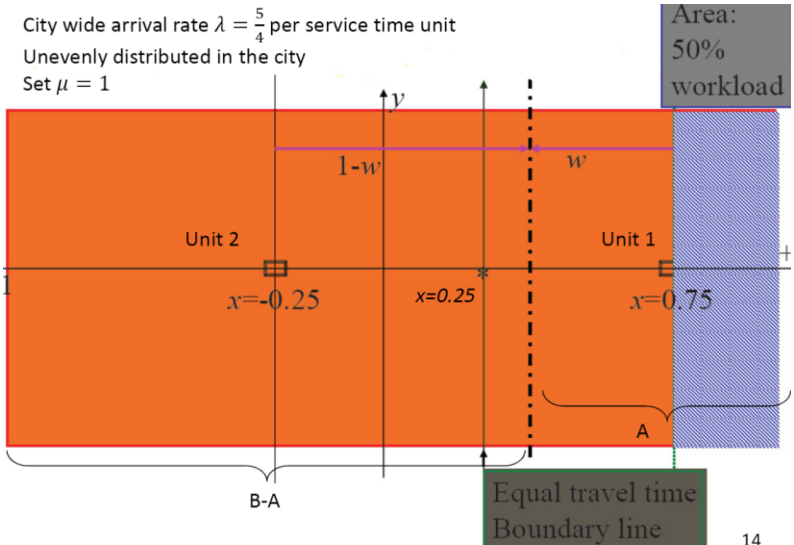
$$T_1(A) = \frac{1/2}{1/2+1/2\frac{w}{7/4}}\times\frac{1}{8} + \frac{1/2\frac{w}{7/4}}{1/2+1/2\frac{w}{7/4}}\times\frac{w}{2} = \frac{\frac{1}{8}+\frac{2}{7}w^2}{1+\frac{4w}{7}}$$

$$T_2(B-A) = \frac{3/4}{3/4+1-w}\times\frac{3/4}{2} + \frac{1-w}{3/4+1-w}\times\frac{1-w}{2} = \frac{\frac{9}{32}+\frac{1}{2}(1-w)^2}{7/4-w}$$

City wide arrival rate $\lambda = \frac{5}{4}$ per service time unit

Unevenly distributed in the city

Set $\mu = 1$



Original set: $\{X_1, X_2, \ldots, X_N\}$ with joint CDF $F_{X_1, X_2, \ldots X_N}()$

Second set: $\{Y_1, Y_2, \ldots, Y_M\}$ and $Y_i = g_i(X_1, X_2, \ldots, X_N)$

Calculate CDF

$F_{Y_1, Y_2, \ldots Y_M}(y_1, y_2, \ldots, y_M)$

$=P\{Y_1 \le y_1, Y_2 \le y_2, \ldots, Y_M \le y_M\}$

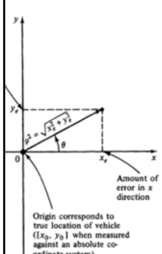
$=P\{Y_i = g_i(X_1, X_2, \ldots, X_N) \le y_i, i=1, 2, \ldots, M\}$

For continuous RVs

$$f_{Y_1,Y_2,\ldots,Y_M(y_1,y_2,\ldots,y_M)} = \frac{\partial^M}{\partial y_1\partial y_2\cdots\partial y_M}F_{Y_1,Y_2,\ldots,Y_M(y_1,y_2,\ldots,y_M)}$$

For discrete RVs

Subtract appropriate CDF values



1. RVs: X_e, Y_e

2. Joint sample space: $-\infty \le X_e, Y_e \le +\infty$

3. PDF: $f_{X_e,Y_e}(x,y) = \frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}} \quad -\infty \le X_e, Y_e \le +\infty$

4. $F_R(r) = P\{R \le r\} = P\{\sqrt{X_e^2+Y_e^2} \le r\}$

$$= \iint_{\text{circle of radius } r} \frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dxdy = \int_0^{2\pi}\int_0^r \frac{1}{2\pi\sigma^2}e^{-\frac{\rho^2}{2\sigma^2}}\rho d\rho d\theta$$

$$= 2\pi\int_0^r \frac{1}{2\pi\sigma^2}e^{-\frac{\rho^2}{2\sigma^2}}\rho d\rho = -\int_0^r e^{-\frac{\rho^2}{2\sigma^2}}d\left(-\frac{\rho^2}{2\sigma^2}\right)$$

$$= \left[\left(-\exp\left(\frac{\rho^2}{2\sigma^2}\right)\right)\right]_0^r = 1 - e^{-\frac{r^2}{2\sigma^2}}$$

5. $f_R(r) = \frac{r}{\sigma^2}e^{-\frac{r^2}{2\sigma^2}} \quad r \ge 0$

$$E[D_e] = P\{A_1\}E[D_e \mid A_1] + P\{A_2\}E[D_e \mid A_2]$$

A_1: scenarios where $D_e > 0$

A_2: scenarios where $D_e = 0$

Event A_1 requires that the response unit and the customer be on opposite side of the barrier, that is,

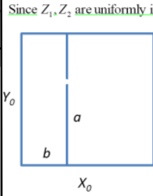
$$Y_1 < a, Y_2 < a, \min(X_1, X_2) \le b, \max(X_1, X_2) > b$$

$$P\{A_1\} = 2\left(\frac{b}{X_0}\frac{a}{Y_0}\right)\left(\frac{X_0-b}{X_0}\frac{a}{Y_0}\right)$$

Given A_1, the extra distance travelled is

$$\{D_e \mid A_1\} = \{2\min(\underbrace{a-Y_1}_{Z_1}, \underbrace{a-Y_2}_{Z_2}) \mid A_1\}$$

Since Z_1, Z_2 are uniformly independently distributed on $[0,a]$, we have $E[D_e \mid A_1] = 2\times\frac{a}{3} = \frac{2a}{3}$



The extra distance travelled:

$$E[D_e] = P\{A_{1a}\}E[D_e \mid A_{1a}] + P\{A_{1b}\}E[D_e \mid A_{1b}] + P\{A_2\}E[D_e \mid A_2]$$

A_{1a}: scenario where $D_e > 0$ and $Y_1, Y_2 > a$

A_{1b}: scenario where $D_e > 0$ and $Y_1, Y_2 \le a$

A_2: scenario where $D_e = 0$

$$E[D_e] = 2\left(\frac{b}{X_0}\frac{Y_0-a}{Y_0}\right)\left(\frac{X_0-b}{X_0}\frac{Y_0-a}{Y_0}\right)\cdot\frac{2}{3}(Y_0-a) + 2\left(\frac{b}{X_0}\frac{a}{Y_0}\right)\left(\frac{X_0-b}{X_0}\frac{a}{Y_0}\right)\frac{2a}{3}$$

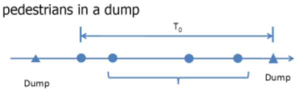
$$= \frac{4b(X_0-b)}{3X_0^2Y_0^2}\left[a^3 + (Y_0-a)^3\right]$$

4. Expected time a randomly arriving pedestrian must wait until crossing (1)

- Rule A is straight forward: $\overline{W}_A = \frac{T}{2}$
- For Rule B: the probability that a random arriving pedestrian is the k^{th} to arrive since the last dump is $1/N_0$
- Given he is the k^{th} to arrive
  - His conditional waiting time is an (N_0-k) order Erlang RV with parameter $\lambda_L+\lambda_R$
  - His expected wait time is $(N_0-k)/(\lambda_L+\lambda_R)$
- Hence

$$\overline{W}_B = \sum_{k=1}^{N_0}\left(\frac{N_0-k}{\lambda_L+\lambda_R}\right)\frac{1}{N_0} = \frac{N_0-1}{2(\lambda_L+\lambda_R)}$$

For Rule 3: Condition on the total number of pedestrians in a dump



Conditional expected wait of a randomly arriving pedestrian, given that exactly k+1 pedestrians are dumped

$$(W_C \mid k+1) = \frac{1}{k+1}T_0 + \frac{k}{k+1}\frac{T_0}{2} = \frac{2+k}{2(k+1)}T_0$$

Define random event S_{k+1}: a random chosen pedestrian has been dumped in a group of k+1 pedestrians (including himself)

$P(S_{k+1}) \propto (k+1)\times P(N(T_0) = k \mid \text{there is already one pedestrian})$

$$P(S_{k+1}) \propto (k+1)\times\frac{[(\lambda_L+\lambda_R)T_0]^k e^{-(\lambda_L+\lambda_R)T_0}}{k!}$$

$$P(S_{k+1}) = (k+1)\times\frac{[(\lambda_L+\lambda_R)T_0]^k e^{-(\lambda_L+\lambda_R)T_0}}{k!}\times\frac{1}{(\lambda_L+\lambda_R)T_0+1}$$

$$\overline{W}_C = \sum_{k=0}^{\infty}P(S_{k+1})\frac{2+k}{2(k+1)}T_0 = \frac{T_0}{2}\left[1+\frac{1}{1+(\lambda_L+\lambda_R)T_0}\right]$$

3. PDF for the time between dumps

$$f_{t_A}(x) = \begin{cases}1, & \text{if } x = T \\ 0, & \text{otherwise}\end{cases}$$

$f_{t_B}(x) = N_0$th-order interarrival time pdf for a Poisson process with pooled arrival rate $\lambda_L+\lambda_R$

$$= \frac{(\lambda_L+\lambda_R)^{N_0}x^{N_0-1}e^{-(\lambda_L+\lambda_R)x}}{(N_0-1)!} \quad x \ge 0$$

X_c = time till first pedestrian arrives $+ T_0$

$$f(x) = (\lambda_L+\lambda_R)e^{-(\lambda_L+\lambda_R)x}$$

$$f_{t_C}(x) = (\lambda_L+\lambda_R)e^{-(\lambda_L+\lambda_R)(x-T_0)} \quad x \ge T_0$$

5. Expected wait time for a randomly arriving observer

- For Rule A

$E[X_A] = T_0$ X is the inter-dump time

$\sigma_{X_A}^2 = 0$

$\overline{V}_A = \frac{T_0^2}{2T_0} = \frac{T_0}{2}$

- For Rule B

$E[X_B] = \frac{N_0}{\lambda_L+\lambda_R}$

$\sigma_{X_B}^2 = Var(Y_1+Y_2+\cdots+Y_{N_0}) = N_0 Var(Y_1) = \frac{N_0}{(\lambda_L+\lambda_R)^2}$

$\overline{V}_B = \frac{N_0+1}{2(\lambda_L+\lambda_R)}$

For Rule C

$E[X_C] = E[Y_1] + T_0 = \frac{1}{\lambda_L+\lambda_R} + T_0$

$\sigma_{X_C}^2 = Var(Y_1+T_0) = Var(Y_1) = \frac{1}{(\lambda_L+\lambda_R)^2}$

$$\overline{V}_C = \frac{T_0}{2} + \frac{1}{2(\lambda_L+\lambda_R)} + \frac{1}{2(\lambda_L+\lambda_R)[1+T_0(\lambda_L+\lambda_R)]}$$

Limiting cases

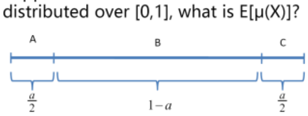
$\lim_{T_0\to 0}\overline{V}_C = \frac{1}{\lambda_L+\lambda_R}$ Expected time of the next pedestrian's arrival

$\lim_{T_0\to\infty}\overline{V}_C = \frac{1}{2}\left[T_0 + \frac{1}{\lambda_L+\lambda_R}\right]$

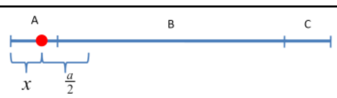
Possible Decision Rules:

- Rule A: Dump every T minutes
- Rule B: Dump whenever the total number of waiting pedestrians equals N_0
- Rule C: Dump whenever the first pedestrian to arrive after the previous dump has waited T_0 minutes

Suppose that the ambulances are distributed over [0,1], what is $E[\mu(X)]$?



$$E[\mu(X)] = E[\mu_A(X) + \mu_B(X) + \mu_C(X)]$$
$$= 2E[\mu_A(X)] + E[\mu_B(X)]$$
$$E[\mu_B(X)] = (1-a)\left[1-(1-a)^N\right]$$

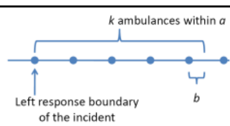


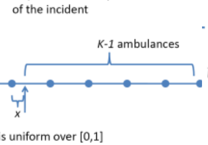
Probability for x to be covered by a particular ambulance is $\frac{a}{2}+x$

Probability for x to be covered by any ambulance is $1-(1-\frac{a}{2}-x)^N$

$$E[\mu_A(X)] = E\left[\int_0^{a/2}S(x)dx\right] = \int_0^{a/2}E[S(x)]dx = \int_0^{a/2}\left(1-(1-\tfrac{a}{2}-x)^N\right)dx$$

$$E[\mu(X)] = (1-a)\left[1-(1-a)^N\right] + 2\int_0^{a/2}\left(1-(1-\tfrac{a}{2}-x)^N\right)dx$$





Probability	# of Ambulances
1-b	k-1
b	k

$$E[N] = bk + (1-b)(k-1)$$
$$= k - 1 + b = a$$

$$Var(N) = E[N^2] - (E[N])^2$$
$$= (k-1)^2(1-b) + k^2b - (k-1+b)^2$$
$$= b(1-b)$$