Distribution System Modeling (80160152-0) - Assignment 2

Due date: March 24, 2021
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Instruction: Late submissions receive a score of ZERO.

Problem 1

Consider a square area of side 4, with $(0, 0)$ the lower left corner. A barrier extends from $(0, 1)$ to $(4, 1)$, with a break at $(1, 1)$ through which traffic can pass. Assume that an emergency is equally likely to arise at all points in the square, while the position of the response vehicle is uniformly distributed over the square and independent of the location of the emergency.

(i) Compared to having no barrier, what is the maximum additional travel distance (Manhattan metric) that the barrier could impose on the response vehicle?

(ii) What is the probability that the additional travel distance because of the barrier is greater than 5?

(iii) What is the mean additional travel distance for the response vehicle caused by the barrier?

Problem 2

In the sky-crossing example, what is the probability that an eastbound plane will be in conflict with exactly three northbound planes?

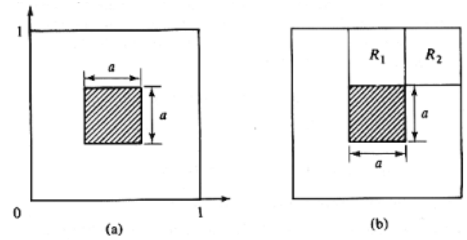
Given such a triple conflict, what is the probability that all three northbound planes are in conflict with one another?

Problem 3: Zero-demand zone

Consider a unit-square response area, as shown in Figure (a). We assume that a response unit and incident (i.e., requests for service) are distributed uniformly, independently over that part of the unit square not contained within the central square having area a^2. Travel occurs according to the right-angle metric, and travel is allowed through the zero-demand zone. We want to use conditioning arguments to derive the expected ravel distance $W(a)$ to a random incident.

Let (X_1, Y_1) and (X_2, Y_2) denote the locations of the response unit and incident, respectively. Let S (S') denote the set of points within (outside) the cental square.

Let $A = \{(X_1, Y_1) \in S\}$ and $B = \{(X_2, Y_2) \in S\}$. $(A' = \{(X_1, Y_1) \in S'\}$ and $B = \{(X_2, Y_2) \in S'\}$.)



Now focus on a unit square on which incidents and the response unit are uniformly, independently distributed over the entire square, yield an expected travel distance $E[D]$.

(a) Show that

$$\begin{aligned} E[D] =& \frac{2}{3} = E[D|A \cap B]P(A \cap B) + 2E[D|A \cap B']P(A \cap B') + E[D|A' \cap B']P(A' \cap B') \\ =& \frac{2}{3}a(a^2)^2 + 2E[D|A \cap B']a^2(1 - a^2) + E[D|A' \cap B'](1 - a^2)^2 \end{aligned}$$

(b) We wish to derive $E[D|A' \cap B'] = W(a)$. The relationship above allows us to compute this quantity by finding the easier-to-compute quantity $E[D|A \cap B']$. (Note the similarity of approach to Crofton's method.)

(i) To find $E[D|A \cap B']$, argue that one need only consider the incident to be located in R_1 or R_2, as shown in Figure (b).

(ii) Show that

$$P[(X_2, Y_2) \in R_1 | (X_2, Y_2) \in R_1 \cup R_2] = \frac{2a}{a + 1}$$

(iii) Show that

$$\begin{aligned} E[D|A \cap R_1] &= \frac{1}{4} + \frac{7}{12}a \\ E[D|A \cap R_2] &= \frac{1}{2} + \frac{1}{2}a \end{aligned}$$

(c) Finally, find $W(a)$. As a check, $W(0) = \frac{2}{3}$, $W(1) = \frac{11}{12}$. (Why?)

Problem 4

Suppose that emergency vehicles are distributed under a spatial Poisson process with parameter λ, and let $x(k)$ be the straight-line distance between some point O and the k-th nearest vehicle.

(a) Obtain an expression for the mean of $x(2)$.

(b) Find the probability that $x(5) > 3x(2)$.