Supplier selection and order allocation problem using a two-phase fuzzy multi-objective linear programming

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ABSTRACT

The aim of this paper is to solve a supplier selection problem under multi-price level and multi-product using interactive two-phase fuzzy multi-objective linear programming (FMOLP) model. The proposed model attempts to simultaneously minimize total purchasing and ordering costs, a number of defective units, and late delivered units ordered from suppliers. The piecewise linear membership functions are applied to represent the decision maker's fuzzy goals for the supplier selection and order allocation problem, and can be resulted in more flexibility via an interactive decision-making process. To demonstrate effectiveness of the proposed model, results of applying the proposed model are shown by a numerical example. The analytical results show that the proposed approach is effective in uncertain environments and provide a reliable decision tool for integrated multi-objective supplier selection problems.

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1. Introduction

In today's competitiveness world, most organizations attempt to meet demand, increase quality, and decrease cost. In most industries, the cost of raw materials and component parts forms the major part of production cost, e.g. up to 70% [1]. According to Weber et al. [2], the raw material cost may increase to 80% of total cost in hi-tech production environment. Generally, the costs of raw materials and components comprise the main quota of the final cost of a product. Selecting a proper supplier can significantly reduce purchasing costs, decrease production lead time, increase customer satisfaction, and strengthen corporate competitiveness [3].

In the supply chain scope, organizations should select the most appropriate suppliers for considerable products based on production capacity of available suppliers during the planning horizon. In a value chain, suppliers have a potential capability to increase customers' satisfaction. Hence, the supplier selection problem (SSP) is one of critical activity of the purchasing department in an organization and it can intensively affect other processes within organization. In this problem, the number and type of supplier, and the order quantities allocated to these suppliers should simultaneously be determined. Indeed, selection of suppliers and allocation of orders' quantity to each selected supplier are strategic purchasing decisions [4].

Regarding how many suppliers can be considered to supply the required materials, the supplier selection problem can be categorized into two types as follows [5]:

- Selecting the best supplier from the pool of available suppliers that can satisfy all buyer's requirements such as demand, quality, and delivery, etc. (single sourcing).

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- Selecting two or more suppliers to meet demands as none of suppliers can individually meet all buyers' requirements (multiple sourcing). In such situation, we face order allocation problem where the best suppliers should be selected and the optimal order quantities should be assigned to each of them.

Deciding on the order allocation is a strategic purchasing decision that will impact the firm's relationship with suppliers [4]. In multiple sourcing, the buyer has an opportunity to receive lower prices and shipping costs from a multiple-sourcing strategy [6]. Selecting suppliers provide the lowest price in a given industry is a challenge for purchasing managers, specifically when suppliers offer multiple products and volume-based discount pricing schedules [7]. In such case, the supplier offers discounts on the total quantity of sales volume in a given period of time. In general, purchasing multiple items from a supplier [8] and quantity discounts [9] represent a standard business practice.

Since different criteria can be considered during the decision making process for Supplier selection decision, this problem is a more complex in presence of volume discounts and multiple items. These criteria include qualitative and quantitative factors. Therefore, this problem is important for purchasing managers and they should determine the trade-off among the several factors. Improper selection of suppliers may unfavorably affect the company's competiveness strategy. Thus, this problem is naturally a multi-objective decision-making problem with several conflicting factors such as cost, quality, and delivery. Mathematical programming techniques can be applied to determine the optimal solutions of this problem where the criteria are formulated as the objective functions or constraints.

In practice, decision-making in SSP includes a high degree of different types of fuzziness [10]. In real-world SSPs, the input information (e.g. demand, quality, and cost) and the objective function are often uncertain or fuzzy since most of the input information is not precisely known or complete or achievable. The fuzzy set theory is one of the best tools to handle uncertainty and vagueness. Obviously, traditional mathematical programming cannot handle the fuzzy programming problems. The supplier selection model of the present paper under multiple products and multiple price levels represents the role of fuzzy theory where information, objective functions and parameters are imprecise.

The fuzzy sets theory was initially introduced by Zadeh [11]. Zimmermann [12,13], first extended his fuzzy linear programming (FLP) approach to a conventional multi-objective linear programming (MOLP) problem. For each objective functions of this problem, assume that the decision maker (DM) has a fuzzy goal such as 'the objective functions should be essentially less than or equal to some value'. Then, the corresponding linear membership function is defined and the minimum operator proposed by Bellman and Zadeh [14] is applied to combine all the objective functions. By introducing auxiliary variables, this problem can be transformed into an equivalent conventional LP problem and can easily be solved by the simplex method. Subsequent works on fuzzy goal programming (FGP) include Hannan [15], Leberling [16], Luhandjula [17], and Shanker and Vrat [18].

Due to the inherent conflict among the three objectives total purchasing and ordering costs, the number of defective units and late delivered units ordered from suppliers, a fuzzy goal programming approach is proposed in the current research to solve an extended mathematical model of a SSP under multi-price level and multi-product.

The present paper aims to develop an interactive fuzzy multi-objective linear programming (FMOLP) model to solve the multi-objective SSP under multi-price level and multi-product in the fuzzy environment. To do so, an MOLP model of a multi-objective SSP under multi-price level and multi-product is firstly constructed. The model attempts to minimize the total purchasing and ordering costs, the numbers of defective units and late delivered units ordered from suppliers. Then, the model is converted into an FMOLP model by an integration fuzzy sets concept and multiple objective programming approaches.

The remaining of the current paper is structured as follows: Section 2 describes the literature review related to supplier selection and order allocation problem. In Section 3, the MOLP mathematical formulation model of SSP under multi-price level and multi-product is presented. In Section 4, Interactive two-phase FMOLP mathematical models are developed to generate optimal solutions in the fuzzy environment of the problem. Section 5 presents a numerical example and reports the results of computational experiments to demonstrate the efficiency of the proposed interactive two-phase FMOLP model for supplier selection and order allocation problem under multi-price level and multi-product. Finally, conclusion part of the present paper is presented in Section 6.

2. Literature review

Researchers have introduced and examined different criteria for the supplier selection problem since 1960s. Dickson [19] identified 23 criteria based on a survey of 170 purchasing managers involved in various SSPs. The criteria such as price, delivery performance, and quality were the most important criteria in evaluating suppliers. Weber et al. [2] reviewed 74 articles to catch supplier selection criteria. They also concluded that the important criteria are quality, delivery performance and cost. They stressed that supplier selection not only depend on the criterion cost, but also it would depend on other criteria such as quality and delivery performance.

Many papers in the literature have investigated supplier selection and evaluation methods [20]. De Boer et al. [21] and Ho et al. [22] conducted a comprehensive survey of methods used for solving SSP. In this section, direction of literature review will essentially be conducted in the mathematical programming models used for supplier selection and order allocation decisions.

Among 78 journal articles studied by Ho et al. [22], only nine papers (11.54%) formulated the supplier selection problem in form of various mathematical programming models. Hong and Hayya [23] formulated the multiple sourcing as a mathematical model and solved it to obtain the optimal solution of suppliers and the size of the split orders. Talluri and Narasimhan [24] proposed a linear programming model based on data envelopment analysis (DEA) for effective supplier sourcing where multiple strategic and operational factors in the evaluation process are considered. Ng [25] developed a weighted linear programming model for the multi-criteria supplier selection problem.

Different mixed-integer programming approaches have been applied by researchers for the SSP. Talluri [26] presented a binary integer linear programming model in selecting an optimal set of bids that satisfy the buyer's demand requirements to evaluate supplier bids based on the ideal targets set by the buyer. Hong et al. [27] developed a mixed-integer linear programming model for the SSP. The outputs of the Hong et al. [27]' model were the optimal number of suppliers, and the optimal order quantity to maximize revenue while satisfying the procurement condition and maintaining the supplier-relationship for a longer time period. Ghodsypour and O'Brien [28] developed a mixed integer non-linear programming model taking into account the total logistics costs (net price, storage, transportation, and ordering costs) and the buyer limitations (budget, quality, service, etc.) to solve the multiple sourcing problems. Basnet and Leung [29] developed a mixed integer programming model to solve the SSP with multi-period multi-product lot sizing. The objective function included the transaction cost, the purchasing cost, and holding cost for each product in the inventory in each period.

Order quantities with quantity discounts problem has studied from the point of view of supplier selection and order quantity allocation by Bender et al. [30], Chaudhry et al. [31], Ghodsypour and O'Brien [5], Amid et al. [32]. Chaudhry et al. [31] formulated a linear mixed-integer programming model to minimize the aggregate price by considering both cumulative and incremental discount. The selection process was influenced by the price, delivery, and quality objectives of the buyer, as well as by the production or rationing constraints of vendors.

It is well known that the optimal solution of single-objective models can be quite different from the models consisting of multiple objectives. In fact, the decision maker (DM) often wants to minimize the total purchasing and ordering costs, the net number of rejected items from the suppliers and the net number of late delivered items. Each of these objectives is valid from a general point of view. Since these objectives conflict with one another, a solution may perform well for one objective but may give inferior results for others. For this reason, SSP problems have a multi-objective nature [3]. As mentioned, since considering one criterion rarely occurs in practice (single objective function), MOLP is a suitable approach to solve the SSP with taking account into quantity discount.

The multi-objective programming techniques in evaluating suppliers in multiple sourcing problems have not received as much attention as single objective problem in single sourcing problem in the literature [4]. Weber and Current [33] presented a multi-objective approach to systematically analyze the inherent tradeoffs involved in multi-criteria supplier selection problems. Weber et al. [34] presented an approach based on multi-objective programming and data envelopment analysis to determine the number of suppliers in a multi-vendor and single product purchasing environment. Xia and Wu [35] proposed an integrated approach of analytical hierarchy process improved by rough sets theory and multi-objective mixed integer programming simultaneously determine the number of suppliers to employ and the order quantity allocated to these suppliers in the case of multiple sourcing, multiple products, with multiple criteria and with supplier's capacity constraints. These models have not considered price discount or multi-price level.

Narasimhan et al. [36] proposed a multi-objective programming model for dealing with the problem in multi-product and discount environment (multi-price level) environment while considering the competitive bidding mechanism for supplier selection. Wadhwa and Ravindran [37] modeled the SSP as a multi-objective optimization problem, where price, lead-time, and rejects are explicitly considered as the three conflicting criteria to be minimized simultaneously. Because the real cases are actually full of ambiguities or in simplest term fuzzy, some authors have combined the fuzzy set theory to handle such uncertainties and ambiguities.

Amid et al. [38] proposed a FMOLP model (weighted additive) for SSP to tackle with information's vagueness and to help decision makers to find out the appropriate ordering from each supplier. Amid et al. [32] formulated the SSP under price breaks in a supply Chain. Their model included three objective functions minimizing the net cost, minimizing the net rejected items, and minimizing the net late deliveries, while satisfying capacity and demand requirement constraints. Amid et al. [39] developed a simple weighted max–min fuzzy model for a fuzzy multi-objective SSP which taking into account the cost, quality, and service. Kumar et al. [40] formulated the SSP as a fuzzy mixed integer goal programming with three primary goals minimizing the net cost, minimizing the net rejections, and minimizing the net late deliveries subject to realistic constraints regarding the buyer's demand, vendors' capacity, vendors' quota flexibility, purchase value of items, budget allocation to individual vendor, etc. Esfandiari and Seifbarghy [41] proposed a multi-objective model for supplier quota allocation problem while demand was dependent on the offered price by suppliers. They solved their models using genetic algorithm and simulated annealing. Lee et al. [42] tried to construct a lot-sizing model with multi-suppliers and quantity discounts to minimize total cost over the planning horizon as a single-objective problem. The objective was to minimize total costs, where the costs include ordering cost, holding cost, purchase cost, and transportation cost.

As it is evident in the literature, most studies have rarely paid attention to SSP models that simultaneously consider uncertainty in information (incompleteness) and several conflicting criteria under conditions of multiple product and discount environment (multi-price level) environment and multiple sourcing. The main purposes of this paper are outlined: (1) to propose an extended mixed-integer linear programming model including new aspects of suppliers that offer various price levels for selling their products, new aspects of buyers that want to consider several conflicting criteria such as min-

imizing total purchasing and ordering costs, the net number of rejected items, and net number of late delivered items ordered from suppliers subject to real constraints regarding buyer's demand, suppliers' capacity, suppliers' quota flexibility, and purchase value of items, (2) Due to the inherent conflict of the three objectives consisting of the total purchasing and ordering costs, the net number of rejected items from the suppliers, and the net number of late delivered items, we propose an interactive two-phase fuzzy multi-objective linear programming approach to solve an extended mathematical model of a supplier selection and order allocation problem.

3. Model development

The SSP is typically a MOLP problem [43]. Any decision maker often wants to optimize his/her criterion of interest. Also, each objective may have a desired range of aspiration value with different accomplishment price level. Hence, in this section, we tend to model the SSP as a MOLP problem.

The SSP considered in the current paper is as follows: given a set of suppliers offer multi-price levels for multi-product(s), the buyer seeks the pareto optimal solution that minimizes total purchasing and ordering costs, the net number of rejected items, the net number of late delivered items to allocate orders of items while satisfy a set of constraints such as overall demand of all items, capacity of the each supplier for each items, flexibility needed with the suppliers' quota, rating value of supplier, and service level on each item from the supplier.

Assumptions, decision variables, and parameters considered in the model are as follows:

(i) Shortage of each item is not allowed from each supplier.
(ii) One or more items can be supplied from each supplier.
(iii) Quantity discount is offered by each supplier.
(iv) Demand of the items, unit cost, price and other considered parameters are all constant and known.
(v) Capacity of each supplier is finite.

3.1. Notations

The following notations are used to describe the SSP.
Index set
i: Index of items, $i = 1,2,\ldots,m$;
j: Index of suppliers, $j = 1,2,\ldots,n$;
k: Index of a price level offered by the jth suppliers, $k = 1,2,\ldots,m_j$;
l: Index of objectives, for all $l = 1,2,\ldots,L$;
Parameters
m: Number of total items for planning;
n: Number of total suppliers for ordering;
n_i: Maximum number of potential suppliers for the ith item;
m_j: Maximum number of available price level of the jth supplier;
D_i: Total demand of the ith item;
b_{ijk}: Upper limit purchased volume for the ith item of the jth supplier at the kth price level, $0 = b_{ij0} < b_{ij1} < L < b_{ij.m_j}$;
p_{ijk}: Unit price ith item of jth supplier at kth price level;
o_{ij}: Ordering cost for ith item of jth supplier;
q_{ijk}: Percentage of rejected unites in ith item of jth supplier at kth price level;
d_{ijk}: Percentage of late delivered unites of ith item ordered jth supplier at kth price level;
c_{ij}: Capacity of jth supplier for ith item.
f_{ij}: Flexibility of supplier quota allocation of jth supplier for ith item;
F_i: Lower bound of quota flexibility required by ith item;
s_{ij}: Service level of jth supplier for ith item;
S_i: Lower bound of service level required for ith item;
r_{ij}: Rating value of jth supplier for ith item;
R_i: Lower bound of rating value on ith item;
Decision variables
x_{ijk}: Number of ith item ordered from jth supplier at kth price level

$$y_{ijk} = \begin{cases} 1 & \text{if item } i \text{ provided by supplier } j \text{ at level } k, \\ 0 & \text{otherwise}. \end{cases}$$

$$y_j = \begin{cases} 1 & \text{if at least one item is provided by supplier } j, \\ 0 & \text{no any item is provided by supplier } j. \end{cases}$$

3.2. Mathematical model

In the current paper, the objective is to solve a SSP under multi-product and multi-price level with minimizing total purchasing and ordering costs (i.e., Z_1), the net number of rejected items from the suppliers (i.e., Z_2), the net number of late delivered items (i.e., Z_3). These objectives were considered most often in the SSP's literature Weber and Current [33].

$$\text{Min } Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} p_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n} o_{ij}y_j, \tag{1}$$

$$\text{Min } Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} q_{ijk}x_{ijk}, \tag{2}$$

$$\text{Min } Z_3 = \sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} d_{ijk}x_{ijk}. \tag{3}$$

Subject to

$$\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} x_{ijk} = D_i, \quad i = 1, 2, \ldots, m, \tag{4}$$

$$1 \times y_{ijk} \leqslant x_{ijk} \leqslant c_{ij}y_{ijk}, \quad i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n_i; \ k = 1, 2, \ldots, m_j, \tag{5}$$

$$\sum_{k=1}^{m_j} y_{ijk} \leqslant 1, \quad i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n_i, \tag{6}$$

$$\sum_{j=1}^{n_i} f_{ij}\sum_{k=1}^{m_j} x_{ijk} \geqslant F_i D_i, \quad i = 1, 2, \ldots, m, \tag{7}$$

$$\sum_{j=1}^{n_i} s_{ij}\sum_{k=1}^{m_j} x_{ijk} \geqslant S_i D_i, \quad i = 1, 2, \ldots, m, \tag{8}$$

$$\sum_{j=1}^{n_i} r_{ij}\sum_{k=1}^{m_j} x_{ijk} \geqslant R_i D_i, \ i = 1, 2, \ldots, m, \tag{9}$$

$$b_{ij,k-1}y_{ijk} \leqslant x_{ijk} \leqslant b_{ijk}y_{ijk}, \quad i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n_i; \ k = 1, 2, \ldots, m_j, \tag{10}$$

$$y_j \leqslant \sum_{i=1}^{m}\sum_{k=1}^{m_j} y_{ijk} \leqslant my_j, \quad j = 1, 2, \ldots, n_j, \tag{11}$$

$$x_{ijk} \geqslant 0 \text{ and integer}, \quad i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n_i; \ k = 1, 2, \ldots, m_j, \tag{12}$$

$$y_{ijk} \text{ and } y_j \in \{0, 1\}. \tag{13}$$

Constraint (4) shows a constraint on the overall demand of each item. This constraint also guarantees that the overall demand on all the items must be met. Constraint (5) specifies the limitation of supply capacity for each supplier. Constraint (6) shows that only one or none price level can be chosen to order if item *i* is purchased from supplier *j*. Constraints (7)–(9) represent that the quota flexibility, service level and rating values must exceed a given level. Constraint (10) describes that a quantity ordered from the supplier at a given price level within the level interval offered. Constraint (11) means all the products purchased from the same supplier are placed in one order (for calculating ordering cost in objective function (1)).

4. Interactive two-phase fuzzy multi-objective linear programming (FMOLP) model

The original MOLP model can be converted to an interactive two-phase FMOLP model using the piecewise linear membership function given in Hannan [15] in order to represent the fuzzy goals of the DM in the MOLP model given in Bellman and Zadeh [14]. In general, a piecewise linear membership function given in Bellman and Zadeh [14] can be suggested in order to convert the problem to be solved into an ordinary LP problem. The algorithm includes the following steps:

4.1. Algorithm

Step 1: Specifying a degree of a membership function for several values of each objective function Z_l ($l = 1$–3) (see Table 1).

Step 2: Drawing the piecewise linear membership function.

Step 3: Formulating the linear equations for each of the piecewise linear membership functions $f_l(Z_l)$ ($l = 1$–3).

The intervals for possible values of each objective function Z_l was specified by the user as $[T_{l,v_l+1}, T_{l0}]$ implicating a piecewise membership function (see Table 1). In generally, piecewise membership functions can be divided into two main intervals. The first interval, $[0, T_{l,v_l+1}]$, represents "*risk free*" values in the sense that a solution should almost be implementable and realistic. On the other hand, the second interval, $[T_{l,v_l+1}, T_{l0}]$, represents "*full risk*" values that are most certainly unrealistic, impossible, and the solution obtained by these values is not implemental. While moving from "*risk free*" toward "*full risk*" values, it is moved from solutions with a high degree to those with a low degree [44]. In general, T_{l,v_l+1} and T_{l0} indicate the optimistic and pessimistic viewpoints of the DM, respectively.

Step 3.1: Converting the membership functions $f_l(Z_l)$ into the form

$$f_l(Z_l) = \sum_{b=1}^{P_l} \alpha_{lb}|Z_l - T_{lb}| + \beta_l Z_l + \theta_l, \quad l = 1\text{–}3; \tag{14}$$

where

$$\alpha_{lb} = -\frac{\gamma_{l,b+1} - \gamma_l}{2}; \quad \beta_l = \frac{\gamma_{l,V_l+1} + \gamma_{l1}}{2}; \quad \theta_l = \frac{S_{l,V_l+1} + S_{l1}}{2}. \tag{15}$$

Assume that $f_l(Z_l) = \gamma_{lr}Z_l + S_{lr}$ for each segment $T_{l,r-1} \leqslant Z_l \leqslant T_{lr}$, where γ_{lr} denotes the slope and S_{lr} is the y-intercept of the line segment on $[T_{l,\,r-1}, T_{lr}]$ in the piecewise linear membership function. Hence, we have:

$$f_l(Z_l) = -\left(\frac{\gamma_{l2} - \gamma_{l1}}{2}\right)|Z_l - T_{l1}| - \left(\frac{\gamma_{l3} - \gamma_{l2}}{2}\right)|Z_i - T_{i2}| - \cdots - \left(\frac{\gamma_{l,V_l+1} - \gamma_{lV_l}}{2}\right)|Z_l - T_{lv_l}| + \left(\frac{\gamma_{l,V_l+1} + \gamma_{l1}}{2}\right)Z_l$$

$$+ \frac{S_{l,V_l+1} + S_{l1}}{2}; \quad \left(\frac{\gamma_{l,b+1} - \gamma_{lb}}{2}\right) \neq 0, \quad l = 1, 2, 3; \ b = 1, 2, \ldots, V_l, \tag{16}$$

where

$$\gamma_{l1} = \left(\frac{u_{l1} - 0}{T_{l1} - T_{l0}}\right), \quad \gamma_{i2} = \left(\frac{u_{l2} - u_{l1}}{T_{l2} - T_{l1}}\right) \cdots \quad \gamma_{l,V_l+1} = \left(\frac{1 - u_{lV_l}}{T_{l,V_l+1} - T_{lV_l}}\right). \tag{17}$$

V_l is the number of broken points of the lth objective function and S_{l,V_l+1} is the y-intercept for the section of the line segment on $[T_{lv_l}, T_{l,v_l+1}]$

Step 3.2: Introducing the following nonnegative variables.

$$Z_l + d_{lb}^- - d_{lb}^+ = T_{lb}; \quad l = 1\text{–}3; \ b = 1, 2, \ldots, V_l, \tag{18}$$

where d_{lb}^+ and d_{lb}^- denote the deviational variables in positive and negative directions at the lth point and T_{lb} represents the values of the lth objective function at the lth point.

Step 3.3: Substituting equation (18) into (16) yields the following equation.

$$f_l(Z_l) = -\left(\frac{\gamma_{l2} - \gamma_{l1}}{2}\right)(d_{l1}^- - d_{l1}^+) - \left(\frac{\gamma_{l3} - \gamma_{l2}}{2}\right)(d_{l2}^- - d_{l2}^+) - \cdots - \left(\frac{\gamma_{l,V_l+1} - \gamma_{lV_l}}{2}\right)(d_{lV_l}^- - d_{lV_l}^+)$$

$$+ \left(\frac{\gamma_{l,V_l+1} - \gamma_{l1}}{2}\right)Z_l + \frac{S_{l,V_l+1} + S_{l1}}{2}, \quad l = 1\text{–}3. \tag{19}$$

Step 4: Applying the two-phase approach to introduce the auxiliary variable φ and then the problem can be transformed into the equivalent ordinary LP problem. The variable φ can be interpreted as representing an overall degree of satisfaction with the DM's multiple fuzzy goals.

Table 1
Membership function $f_l(Z_l)$.

Z_1	$>T_{10}$	T_{10}	T_{11}	T_{12}	...	T_{1v_1}	T_{1v_1+1}	$<T_{1v_1+1}$
$f_1(Z_1)$	0	0	u_{11}	u_{12}	...	u_{1v_1}	1	1
Z_2	$>T_{20}$	T_{20}	T_{21}	T_{22}	...	T_{2v_2}	T_{2v_2+1}	$<T_{2v_2+1}$
$f_2(Z_2)$	0	0	u_{21}	u_{22}	...	u_{2v_2}	1	1
Z_3	$>T_{30}$	T_{30}	T_{31}	T_{32}	...	T_{3v_3}	T_{3v_3+1}	$<T_{3v_3+1}$
$f_3(Z_3)$	0	0	u_{31}	u_{32}	...	u_{3v_3}	1	1

Note: ($0 \leqslant u_{lb} \leqslant 1, u_{lb} \leqslant u_{lb+1}, l = 1$–$3, b = 1, 2, \ldots, v_l$).

Phase 1: Using the *max–min* operator [14] and φ_0 satisfaction degree, the FMOLP problem can be solved as a single objective problem:

$$\text{Max } \varphi_0 \tag{20}$$

s.t.

$$\varphi_0 \leqslant -\left(\frac{\gamma_{l2} - \gamma_{l1}}{2}\right)(d_{l1}^- - d_{l1}^+) - \left(\frac{\gamma_{l3} - \gamma_{l2}}{2}\right)(d_{l2}^- - d_{l2}^+) - \cdots - \left(\frac{\gamma_{l,V_l+1} - \gamma_{lV_l}}{2}\right)(d_{lV_l}^- - d_{lV_l}^+)$$
$$+ \left(\frac{\gamma_{l,V_l+1} + \gamma_{l1}}{2}\right)Z_l + \frac{S_{l,V_l+1} + S_{l1}}{2}, \quad l = 1-3. \tag{21}$$

$$Z_l + d_{lb}^- - d_{lb}^+ = Y_{lb}, \quad l = 1-3; b = 1, 2, \ldots, V_l. \tag{22}$$

Constraints (4)–(13).

Phase 2: Applying the result of the previous model to overcome disadvantages of phase 1. In this phase, the solution is forced to get improved and modified upon and dominate the solution of *max–min* operator, adding constraints and a new auxiliary objective function to phase 2 to achieve at least the satisfaction degree obtained in phase 1. The proposed phase 2 of the problem is as follows:

$$\text{Max } \frac{1}{3}\left(\sum_{l=1}^{3}\varphi_l\right) \tag{23}$$

s.t.

$$\varphi_0 \leqslant \varphi_l$$
$$\leqslant -\left(\frac{\gamma_{l2} - \gamma_{l1}}{2}\right)(d_{l1}^- - d_{l1}^+) - \left(\frac{\gamma_{l3} - \gamma_{l2}}{2}\right)(d_{l2}^- - d_{l2}^+) - \cdots - \left(\frac{\gamma_{l,V_l+1} - \gamma_{lV_l}}{2}\right)(d_{lV_l}^- - d_{lV_l}^+) + \left(\frac{\gamma_{l,V_l+1} + \gamma_{l1}}{2}\right)Z_l$$
$$+ \frac{S_{l,V_l+1} + S_{l1}}{2}, \quad l$$
$$= 1-3. \tag{24}$$

$$Z_l + d_{lb}^- - d_{lb}^+ = Y_{lb}, \quad l = 1-3; b = 1, 2, \ldots, V_l. \tag{25}$$

Constraints (4)–(13).

Step 5: Executing and modifying the interactive decision process. If the DM is not satisfied with the initial solution, the model must change until finding a satisfactory solution.

Fig. 1 illustrates the block diagram of the interactive two-phase FMOLP model development.

5. Numerical example and performance analysis of interactive two-phase FMOLP

5.1. Basic data for numerical example

In this section, a numerical example and its solutions are presented to show the superiority of the proposed model. Suppose a buyer plans to purchase five items (products) from four suppliers. Each supplier can provide three to four types items. Table 2 shows the parameters related to the SSP discussed in the current paper such as suppliers' capacity and demand for each item. Three price levels are considered for each supplier in the numerical example. Tables 2 and 3 summarize the data used for numerical example.

5.2. Formulate the interactive two-phase FMOLP model

First, initial solutions for each objective function are determined using the conventional mixed integer linear model. Results are obtained by $Z_1 = 1846$ and $Z_2 = 163$, $Z_3 = 39.3$. Then, we formulate the FMOLP model using the initial solutions and the MOLP model presented in Section 4. Table 4 gives the piecewise linear membership functions of the proposed model for the SSP considered in the numerical example. Figs. 2–4 illustrate the corresponding shapes of the piecewise linear membership functions for the numerical example.

Phase 1: Complete FMOLP model using the *max–min* operator. The crisp formulation of the FMOLP for the SSP in the numerical example can be as follows:

$$\text{Max } \varphi_0 \tag{26}$$

s.t.

$$\varphi_0 \leqslant -0.0000654(d_{11}^- - d_{11}^+) - 0.0000327(d_{12}^- - d_{12}^+) - 0.0002287 \times \left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}p_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}o_{ij}y_j\right) + 6.5846, \tag{27}$$
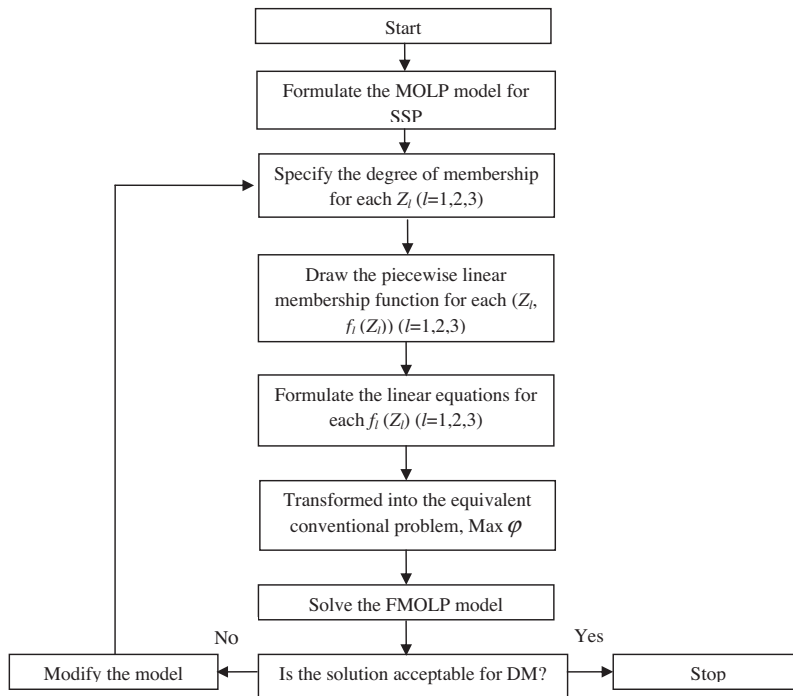
Fig. 1. The block diagram of the interactive two-phase FMOLP model development.

Table 2
Collected data for numerical example.

Items	D_i	Suppliers	$o_{ij}(\$)$	q_{ijk} (%)	d_{ijk} (%)	c_{ij}	F_i (%)	S_i (%)	R_i (%)	f_{ij} (%)	s_{ij} (%)	r_{ij} (%)
1	700	1	800	4	1	1300	2	90	84	3	94	92
		2	750	3	2	1100				2	90	95
		3	600	4	1	1000				2	94	90
		4	650	3	2	900				5	94	96
2	600	1	800	4	2	1400	2	88	89	3	91	95
		3	600	2	3	1400				3	95	96
		4	650	1	2	1400				4	96	96
3	450	2	750	2	2	1300	2	85	91	5	95	91
		3	600	5	1	1200				4	95	92
4	400	1	800	2	3	1000	3	92	84	4	92	93
		2	750	1	3	1000				4	92	93
		3	600	1	4	1100				1	95	92
		4	650	0	2	800				1	94	93
5	380	1	800	2	1	1200	2	90	88	4	91	90
		2	750	2	2	1300				4	96	91

Table 3
Quantity level and price offered by each supplier.

Items	Suppliers	b_{ij0}	$p_{ij1}(\$)$	b_{ij1}	$p_{ij2}(\$)$	b_{ij2}	$p_{ij3}(\$)$
1	1	0	18	100	17.5	200	17
	2	0	17	120	16.5	220	16
	3	0	15	150	14.5	300	14
	4	0	16	90	15.5	180	15
2	1	0	6.5	80	6	170	5.5
	3	0	4	60	3.5	190	3
	4	0	5	90	4.5	210	4
3	2	0	10	75	9.5	180	9
	3	0	11	60	10.5	130	10
4	1	0	8	100	7.5	180	7
	2	0	12	150	11.5	300	11
	3	0	10	90	9.5	160	9
	4	0	13	150	12.5	240	12
5	1	0	6	120	5.5	220	5
	2	0	5	100	4.5	200	4

Table 4
Membership functions for numerical example.

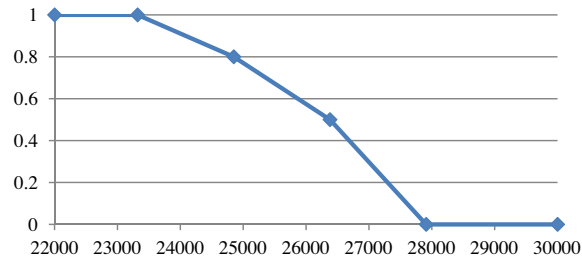| Z_1	>27910	27910	26380	24850	23320	<23320
$f_1(Z_1)$	0	0	0.5	0.8	1	1
Z_2	>89	89	75	61	47	<47
$f_2(Z_2)$	0	0	0.4	0.7	1	1
Z_3	>51	51	47	43	39	<39
$f_3(Z_3)$	0	0	0.4	0.7	1	1

Fig. 2. Shape of membership function $(Z_1, f_1(Z_1))$.
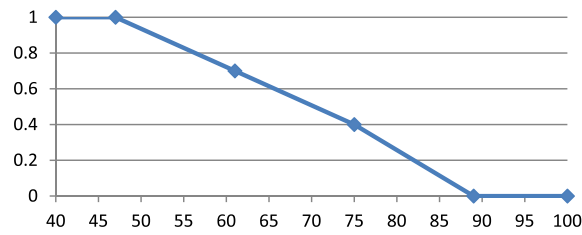

Fig. 3. Shape of membership function $(Z_2, f_2(Z_2))$.
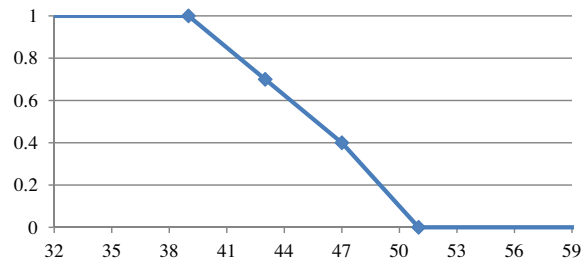

Fig. 4. Shape of membership function $(Z_3, f_3(Z_3))$.

$$\varphi_0 \leqslant -0.00357(d_{21}^- - d_{21}^+) - 0.025 \times \left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} q_{ijk}x_{ijk} \right) + 2.275; \tag{28}$$

$$\varphi_0 \leqslant -0.0125(d_{31}^- - d_{31}^+) - 0.0875 \times \left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} d_{ijk}x_{ijk} \right) + 4.5125; \tag{29}$$

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} p_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n} o_{ij}y_j \right) + d_{11}^- - d_{11}^+ = 26380, \tag{30}$$

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} p_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n} o_{ij}y_j \right) + d_{12}^- - d_{12}^+ = 24850, \tag{31}$$

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} q_{ijk}x_{ijk} \right) + d_{21}^- - d_{21}^+ = 75, \tag{32}$$

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} d_{ijk}x_{ijk} \right) + d_{31}^- - d_{31}^+ = 47; \tag{33}$$

Constraints (4)–(13).

In Constraints (18)–(21), $d_{11}^-, d_{11}^+, d_{12}^-, d_{12}^+, d_{21}^-, d_{21}^+, d_{31}^-$ and d_{31}^+ denote the deviational variables at the first, second, and third points. In addition, $26380(T_{11})$, $24850(T_{12})$, $75(T_{21})$ and $47(T_{31})$ represent the values of the first, second, and third objective function at the first and second point.

The GAMS software system is used to run the FMOLP model for the objectives as $Z_1 = 25165.5$, $Z_2 = 57.81$, $Z_3 = 41.56$, and the overall degree of satisfaction (φ_0) with the DM's multiple fuzzy goals as 0.761.

After getting the optimal solution from phase 1, according to the optimal objective function value φ_0 can be used in phase 2. The equation below represents the proposed FMOLP for the SSP in the numerical example that is transferred from phase 1.

$$\text{Max} \quad \varphi_{Total} = \frac{1}{3}(\varphi_1 + \varphi_2 + \varphi_3), \tag{34}$$

s.t.

$$0.761 \leqslant \varphi_1$$
$$\leqslant -0.0000654(d_{11}^- - d_{11}^+) - 0.0000327(d_{12}^- - d_{12}^+) - 0.0002287 \times \left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} p_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n} o_{ij}y_j \right)$$
$$+ 6.5846, \tag{35}$$

$$0.761 \leqslant \varphi_2 \leqslant -0.00357(d_{21}^- - d_{21}^+) - 0.025 \times \left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} q_{ijk}x_{ijk} \right) + 2.275; \tag{36}$$

$$0.761 \leqslant \varphi_3 \leqslant -0.0125(d_{31}^- - d_{31}^+) - 0.0875 \times \left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} d_{ijk}x_{ijk} \right) + 4.5125; \tag{37}$$

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} p_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n} o_{ij}y_j \right) + d_{11}^- - d_{11}^+ = 26380, \tag{38}$$

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} p_{ijk}x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n} o_{ij}y_j \right) + d_{12}^- - d_{12}^+ = 24850, \tag{39}$$

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} q_{ijk}x_{ijk} \right) + d_{21}^- - d_{21}^+ = 75, \tag{40}$$

$$\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j} d_{ijk}x_{ijk} \right) + d_{31}^- - d_{31}^+ = 47; \tag{41}$$

Constraints (4)–(13).

5.3. Output solutions

The GAMS software system is used to run the proposed interactive two-phase FMOLP model on an Intel® 2.4 GHZ Processor with 4 GB RAM. The results for the numerical example are as follows: $Z_1 = 24606$, $Z_2 = 56.81$, $Z_3 = 42.18$. In addition, the overall degree of satisfaction with the DM's multiple fuzzy goals is 0.794. Table 5 presents computational solutions of the numerical example for each decision variable within 0.785 s of CPU times.

To manipulate different alternatives for the purpose of sensitivity analysis of decision parameters, the interactive two-phase FMOLP model of the preceding numerical example is subjected to the following five scenarios:

- *Scenario 1*: Remove Z_3 (the net number of late delivered items) and consider only Z_1 (total purchasing and ordering costs) and Z_2 (the net number of rejected items) simultaneously.
- *Scenario 2*: Remove Z_2 (the net number of rejected items) and consider only Z_1 (total purchasing and ordering costs) and Z_3 (the net number of late delivered items) simultaneously.
- *Scenario 3*: Set $(Z_2, f_2(Z_2))$ and $(Z_3, f_3(Z_3))$ to their original values in the numerical example and vary $(Z_1, f_1(Z_1))$. Table 6 presents the data and results of implementing Scenario 3.
- *Scenario 4*: Set $(Z_1, f_1(Z_1))$ and $(Z_3, f_3(Z_3))$ to their original values in the numerical example and vary $(Z_2, f_2(Z_2))$. Table 7 presents the data and results of implementing Scenario 4.
- *Scenario 5*: Set $(Z_1, f_1(Z_1))$ and $(Z_2, f_2(Z_2))$ to their original values in the numerical example and vary $(Z_3, f_3(Z_3))$. Table 8 presents the data and results of implementing Scenario 5.

Table 9 presents the results of implementing scenarios 1 and 2. Tables 10–12 present the results of simultaneously implementing scenario 3–5.

Significant managerial implications regarding the practical applications of the proposed model are as follows:

(1) Comparing scenarios 1 and 2 for the numerical example (run # 3 in scenarios 3–5) demonstrates the trade-offs and conflicts among dependent objective functions. Consequently, the proposed model can satisfy the requirement for the practical application because it aims to minimize total purchasing and ordering costs, the net number of rejected items, and the net number of late delivered items (see Tables 9–12).

(2) The results of scenarios 3–5 show that the overall level of satisfaction and output solutions for each decision variable strongly is affected by the specific degree of membership for each objective function. Indeed, we can propose two significant implications. First, the most important task of the DM is to determine the rational degree of membership for each objective function; second, the DM may arbitrarily modify the range of values of the degree of membership to yield satisfactory solutions. Fig. 5–8 depict the changes for φ_{Total} and objectives values of scenarios 3–5, respectively.

(3) The proposed interactive two-phase FMOLP method is based on Hannan's [15] fuzzy programming method, which, assuming that the DM finds the minimum operator a suitable operator for making a decision, aggregates fuzzy sets using logical 'and' operations. It follows that the maximization of two or more membership functions is best completed by maximizing the minimum membership degree. The original fuzzy multi-objective SSP formulated in our work is converted into an equivalent ordinary LP form by the minimum operator to aggregate all fuzzy sets. Hence, the proposed interactive two-phase FMOLP method is preferable where the DM seeks to make optimal values of membership function approximately equal or the DM decides that the minimum operator is suitable for the problem.

Table 5
FMOLP model solutions.

Objective function values	$Z_1 = 24606$, $Z_2 = 56.81$, $Z_3 = 42.18$
x_{ijk}	$x_{111} = 0$, $x_{112} = 0$, $x_{113} = 0$, $x_{121} = 0$, $x_{122} = 0$, $x_{123} = 0$, $x_{131} = 0$, $x_{132} = 0$, $x_{133} = 700$, $x_{141} = 0$, $x_{142} = 0$, $x_{143} = 0$;
	$x_{211} = 0$, $x_{212} = 0$, $x_{213} = 0$, $x_{231} = 0$, $x_{232} = 0$, $x_{233} = 0$, $x_{241} = 0$, $x_{242} = 0$, $x_{243} = 600$; $x_{321} = 0$, $x_{322} = 0$, $x_{323} = 421$, $x_{331} = 29$, $x_{332} = 0$, $x_{333} = 0$;
	$x_{411} = 0$, $x_{412} = 0$, $x_{413} = 0$, $x_{421} = 0$, $x_{422} = 0$, $x_{423} = 0$, $x_{431} = 267$, $x_{432} = 0$, $x_{433} = 700$, $x_{441} = 133$, $x_{442} = 0$, $x_{443} = 0$;
	$x_{511} = 0$, $x_{512} = 0$, $x_{513} = 380$, $x_{521} = 0$, $x_{522} = 0$, $x_{523} = 0$;
Degree of satisfaction	$\varphi_{Total} = 0.794$, $\varphi_1 = 0.832$, $\varphi_2 = 0.79$, $\varphi_3 = 0.761$

Table 6
Data of scenario 3.

Z_2		>89	89	75	61	47	<47	Original levels
$f_2(Z_2)$		0	0	0.4	0.7	1	1	
Z_3		>51	51	47	43	39	<39	Original levels
$f_3(Z_3)$		0	0	0.4	0.7	1	1	
Run 1	Z_1	>24850	24850	23320	21790	20260	<20260	
	$f_1(Z_1)$	0	0	0.5	0.8	1	1	
Run 2	Z_1	>26380	26380	24850	23320	21790	<21790	
	$f_1(Z_1)$	0	0	0.5	0.8	1	1	
Run 3	Z_1	>27910	27910	26380	24850	23320	<23320	Original levels
	$f_1(Z_1)$	0	0	0.5	0.8	1	1	
Run4	Z_1	>29440	29440	27910	26380	24850	<24850	
	$f_1(Z_1)$	0	0	0.5	0.8	1	1	
Run 5	Z_1	>30970	30970	29440	27910	26380	<26380	
	$f_1(Z_1)$	0	0	0.5	0.8	1	1	

Table 7
Data of scenario 4.

Z_1		>27910	27910	26380	24850	23320	<23320	Original levels
$f_1(Z_1)$		0	0	0.5	0.8	1	1	
Z_3		>51	51	47	43	39	<39	Original levels
$f_3(Z_3)$		0	0	0.4	0.7	1	1	
Run 1	Z_2	>61	61	47	33	19	<19	
	$f_2(Z_2)$	0	0	0.4	0.7	1	1	
Run 2	Z_2	>75	75	61	47	33	<33	
	$f_2(Z_2)$	0	0	0.4	0.7	1	1	
Run 3	Z_2	>89	89	75	61	47	<47	Original levels
	$f_2(Z_2)$	0	0	0.4	0.7	1	1	
Run4	Z_2	>103	103	89	75	61	<61	
	$f_2(Z_2)$	0	0	0.4	0.7	1	1	
Run 5	Z_2	>117	117	103	89	75	<75	
	$f_2(Z_2)$	0	0	0.4	0.7	1	1	

Table 8
Data of scenario 5.

Z_1		>27910	27910	26380	24850	23320	<23320	Original levels
$f_1(Z_1)$		0	0	0.5	0.8	1	1	
Z_2		>89	89	75	61	47	<47	Original levels
$f_2(Z_2)$		0	0	0.4	0.7	1	1	
Run 1	Z_3	>43	43	39	35	31	<31	
	$f_3(Z_3)$	0	0	0.4	0.7	1	1	
Run 2	Z_3	>47	47	43	39	35	<35	
	$f_3(Z_3)$	0	0	0.4	0.7	1	1	
Run 3	Z_3	>51	51	47	43	39	<39	Original levels
	$f_3(Z_3)$	0	0	0.4	0.7	1	1	
Run4	Z_3	>55	55	51	47	43	<43	
	$f_3(Z_3)$	0	0	0.4	0.7	1	1	
Run 5	Z_3	>59	59	55	51	47	<47	
	$f_3(Z_3)$	0	0	0.4	0.7	1	1	

Table 9
Results of implementation the Scenario 1 and Scenario 2.

	Z_1	Z_2	Z_3	φ_0	φ_1	φ_2	φ_3	φ_{Total}
Scenario 1	23860	50.3	-	0.929	0.929	0.929	–	0.929
Scenario 2	23450	-	39.3	0.977	0.983	–	0.977	0.98

Table 10
Results of implementation the Scenario 3.

Scenario 3	Z_1	Z_2	Z_3	φ_0	φ_1	φ_2	φ_3	φ_{Total}
Run 1	23700	58.6	44.8	0.546	0.55	0.751	0.565	0.622
Run 2	23918	60.24	43.12	0.691	0.722	0.716	0.691	0.71
Run 3	24606	56.81	42.18	0.761	0.832	0.79	0.761	0.794
Run 4	26048	55.94	42.47	0.737	0.843	0.808	0.74	0.797
Run 5	25920.5	56.69	41.33	0.792	1	0.792	0.825	0.873

(4) In real-world SSPs, the DM typically face with multiple imprecise in-conflict objectives required to be optimized simultaneously by the DM in the framework of fuzzy satisfaction levels. Thus, applying the fuzzy set theory for SSPs offers more effectiveness and flexibility for the interactive two-phase FMOLP method. As a result, the proposed interactive two-phase FMOLP method satisfies practical application requirements for solving SSPs by its simultaneous minimization of the three objectives: total purchasing and ordering costs the net number of rejected items, and the net number of late delivered items.

Table 11
Results of implementation the Scenario 4.

Scenario 4	Z_1	Z_2	Z_3	φ_0	φ_1	φ_2	φ_3	φ_{Total}
Run 1	26129.5	49.47	46.27	0.347	0.633	0.347	0.455	0.478
Run 2	25914.5	52.17	43.57	0.589	0.661	0.589	0.657	0.636
Run 3	24606	56.81	42.18	0.761	0.832	0.79	0.761	0.794
Run 4	24270.5	66.83	40.64	0.875	0.876	0.875	0.877	0.876
Run 5	23450	72.1	39.3	0.977	0.983	1	0.977	0.987

Table 12
Results of implementation the Scenario 5.

Scenario 5	Z_1	Z_2	Z_3	φ_0	φ_1	φ_2	φ_3	φ_{Total}
Run 1	24248	69.44	37.97	0.477	0.879	0.519	0.477	0.625
Run 2	24757	62.21	40.38	0.596	0.812	0.674	0.596	0.694
Run 3	24606	56.81	42.18	0.761	0.832	0.79	0.761	0.794
Run 4	24680	54.3	44.35	0.822	0.822	0.844	0.899	0.855
Run 5	24460	53.64	47	0.795	0.851	0.858	1	0.903
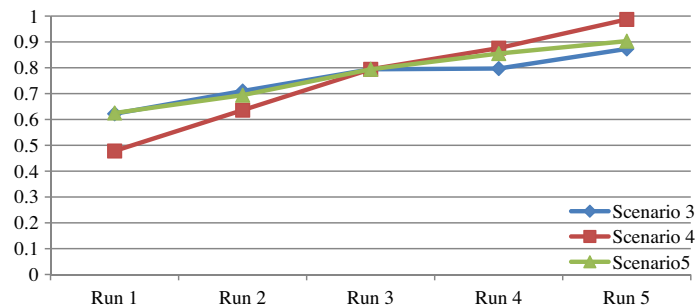
Fig. 5. φ_{Total} values of scenarios 3, 4 and 5.
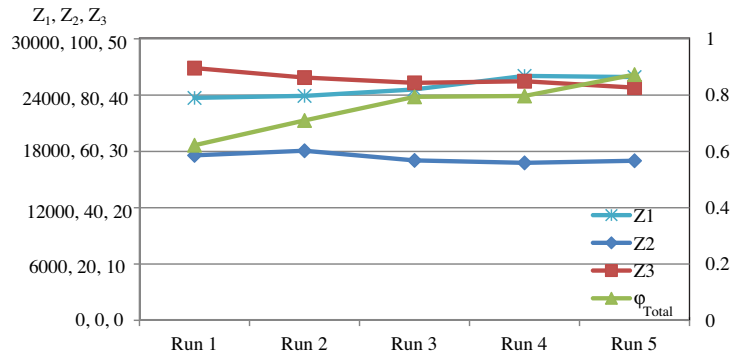
Fig. 6. Objective and φ_{Total} values of Scenario 3.

5.4. Performance analysis

To evaluate the credibility and efficiency of the proposed approach, the results of the proposed model are compared with results of the existing models in the literature. Table 13 compares results of single-objective LP model and Wang and Liang's [45] approach with the results of the proposed FMOLP method for the given example. In single-objective LP model, minimizing the total purchasing and ordering costs (Z_1), minimizing the net number of rejected items (Z_2), and minimizing the net number of late delivered items (Z_3) lead to optimal values 23320, 47.6, and 39.3, respectively. Alternatively, Wang and Liang's [45] approach achieves the following results: $Z_1 = 25146.5$, $Z_2 = 57.81$, $Z_3 = 41.56$ and overall degree of the DM's satisfaction = 0.761. The proposed FMOLP method results in $Z_1 = 24606$, $Z_2 = 56.81$, $Z_3 = 42.18$, and the overall degree of the
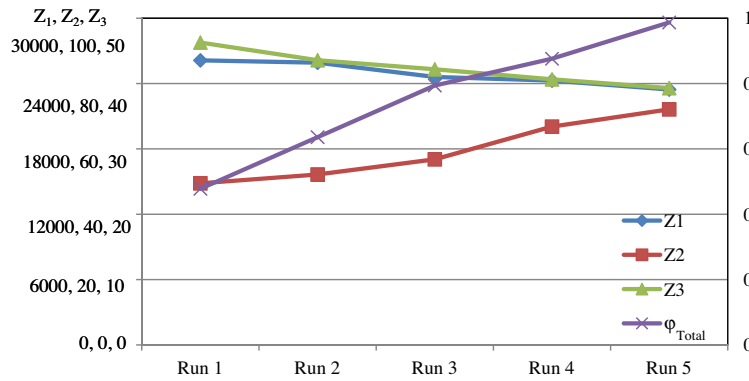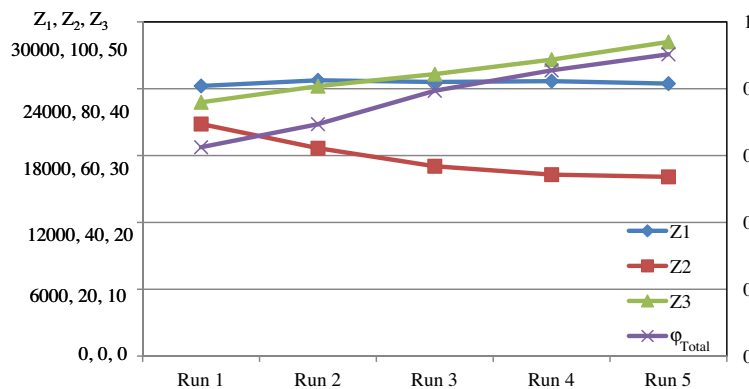
Fig. 7. Objective and φ_{Total} values of Scenario 4.

Fig. 8. Objective and φ_{Total} values of Scenario 5.

Table 13
Comparison results for the numerical example.

	LP-1	LP-2	LP-2	Wang and Liang's [45] method	Proposed method (FMOLP)
Objective function	Min Z_1	Min Z_2	Min Z_3	Max φ	Max φ
φ	100	100	100	0.761	0.794
Z_1	23320	26150	27900	25146.5	24606
Z_2	86.3	47.6	68.1	57.81	56.81
Z_3	41.2	50.8	39.3	41.56	42.18

DM's satisfaction = 0.794. Table 13 indicates the results of the proposed FMOLP method under an acceptable degree of the DM's satisfaction in a fuzzy environment.

To determine the degree of the interactive two-phase FMOLP approach's closeness to the ideal solution, we have used the following family of distance functions [46]:

$$D_p(w, L) = \left[\sum_{l=1}^{L} w_l^p (1 - d_l)^p \right]^{\frac{1}{p}},$$ (42)

where d_k represents the degree of closeness of the preferred compromise solution vector to the optimal solution vector with respect to the kth objective function. $w = (w_1, w_2, \ldots, w_l)$ is the vector of the relative importance of the lth objective function. The power p represents a distance parameter $1 \leqslant p \leqslant \infty$. Assuming $\sum_{l=1}^{L} w_l = 1$, we can write $D_p(w, L)$ with $p = 1, 2$ and ∞ as follows:

$$D_1(w, L) = 1 - \sum_{l=1}^{L} w_l d_l \quad \text{(The Manhattan distance)},$$ (43)

Table 14
Comparisons of solutions.

	Wang and Liang's [45] approach	Proposed interactive two-phaseFMOLP approach
D_1	0.10121	0.09422
D_2	0.06619	0.06117
D_∞	0.05887	0.05404

$$D_2(w,L) = \left[\sum_{l=1}^{L} w_l^2 (1-d_l)^2 \right]^{\frac{1}{2}} \quad \text{(The Euclidean distance)}, \tag{44}$$

$$D_\infty(w,L) = \max_l \{ w_l(1-d_l) \} \quad \text{(The Tchebycheff distance)}, \tag{45}$$

where in minimization problems, d_l takes the form: d_l = (the optimal solution of Z_l)/(the preferred compromise solution Z_l).

However, we can infer that the approach which can derive a preferred compromise solution is better than the others if: Min $D_p(w,L)$ is achieved by its solution with respect to some p as discussed in Abd El-Wahed and Lee [46]. Comparison of the degree of closeness of the Wang and Liang's [45] approach and the proposed interactive two-phase FMOLP model results to the ideal solution is summarized in Table 14. From Table 13, it is clear that the suggested interactive two-phase FMOLP approach gave a preferred compromise solution which is better than the solution by the approaches in Wang and Liang's [45] for all the distance functions D_1, D_2, and D_∞. This comparison shows the proposed interactive two-phase FMOLP is superior to Wang and Liang's approach.

6. Conclusion

In this paper an extended mixed-integer linear programming model for supplier selection and order allocation problems is introduced along with considering the dependence of price level to the order quantities in multi-product environment. We developed an interactive two-phase fuzzy multi-objective linear programming method for solving supplier selection and allocation problem with multiple fuzzy objectives and piecewise linear membership functions. The proposed methodology attempts to simultaneously minimize the total purchasing and ordering costs, the number of defective units, and the number of late delivered units ordered from suppliers. A numerical example is used to show the feasibility and effectiveness of applying the proposed interactive two-phase fuzzy-multi-objective linear programming method to the supplier selection and order allocation problem under fuzzy and uncertain environment. The result of sensitivity analysis for varying objective functions indicated that the trade-offs and conflicts among dependent objective functions. Furthermore, the results and performance analysis showed that the proposed approach is capable to handle uncertain environments and provide a systematic decision tool for logistic managers and practitioners to adopt appropriate strategies and policies in the purchasing activities. The proposed model could be applied as a suitable decision support system for practitioners to investigate different options in the supplier selection and order allocation decision process in real cases. Therefore, the presented model and approach was still open for incorporating uncertain demands in supplier selection, applying other linear and/or non-linear membership functions, adopting the various weight calculation methods such as AHP, fuzzy AHP and other multi-criteria decision making methods for suitable weighting in the second phase of problem, and like that suggested for future research.

Acknowledgement

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