R Workshop

Multilevel Modelling with R
Interpretation of more complex models: adjusted predictions

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Association between IV and DV depending on (moderated by) another predictor

Interaction terms

Model without interaction term

- Objective: Association between age of family carers and their subjective burden.
- What is the underlying assumption of this model?

```
data(efc, package = "ggeffects")
efc$c161sex <- datawizard::to_factor(efc$c161sex)
model <- lm(neg_c_7 ~ barthtot + c160age + c161sex, data = efc)
model_parameters(model)
#> Parameter        | Coefficient |       SE |         95% CI | t(868) |      p
#> ---------------------------------------------------------------------------
#> (Intercept)      |       14.39 |     0.67 | [13.08, 15.70] |  21.56 | < .001
#> barthtot         |       -0.05 | 4.17e-03 | [-0.06, -0.04] | -12.49 | < .001
#> c160age          |    8.75e-03 | 9.24e-03 | [-0.01,  0.03] |   0.95 | 0.344
#> c161sex [Female] |        0.45 |     0.28 | [-0.10,  1.00] |   1.61 | 0.108
```

Model without interaction term

- Objective: Association between age of family carers and their subjective burden.
- What is the underlying assumption of this model?
  - For instance, the association between age and subjective burden is the same (constant) for female and male family carers.

```
model <- lm(neg_c_7 ~ barthtot + c160age + c161sex, data = efc)
model_parameters(model)
#> Parameter          | Coefficient |       SE |         95% CI | t(868) |      p
#> ------------------------------------------------------------------------------
#> (Intercept)        |       14.39 |     0.67 | [13.08, 15.70] |  21.56 | < .001
#> barthtot           |       -0.05 | 4.17e-03 | [-0.06, -0.04] | -12.49 | < .001
#> c160age            |    8.75e-03 | 9.24e-03 | [-0.01,  0.03] |   0.95 | 0.344
#> c161sex [Female]   |        0.45 |     0.28 | [-0.10,  1.00] |   1.61 | 0.108
```

Interactions

- Also called “moderation”.
- The association between x and y is “moderated” by z.
- That means, the association between x and y varies, depending on the value of z.

Regression formula with interaction terms

- Interaction terms in regression formulas are denoted by `*` instead of `+`.

- The following model assumes that the association between age and subjective burden depends on the family carers' gender, so the association varies between female and male family carers.

```
# interaction term: numeric * categorical variable
model <- lm(neg_c_7 ~ barthtot + c160age * c161sex, data = efc)
```

Regression formula with interaction terms

```
# interaction term: numeric * categorical variable
model <- lm(neg_c_7 ~ barthtot + c160age * c161sex, data = efc)
model_parameters(model)
#> Parameter                   | Coefficient |       SE |         95% CI | t(867) |      p
#> ----------------------------------------------------------------------------------------
#> (Intercept)                 |       15.78 |     0.94 | [13.93, 17.63] |  16.74 | < .001
#> barthtot                    |       -0.05 | 4.16e-03 | [-0.06, -0.04] | -12.55 | < .001
#> c160age                     |       -0.02 |     0.02 | [-0.05,  0.01] |  -1.11 | 0.268
#> c161sex [Female]            |       -1.63 |     1.04 | [-3.68,  0.41] |  -1.57 | 0.117
#> c160age × c161sex [Female]  |        0.04 |     0.02 | [ 0.00,  0.08] |   2.08 | 0.038
```

Regression formula with interaction terms

```
#> Parameter                  | Coefficient |
#> ----------------------------------------
#> (Intercept)                |       15.78 |
#> barthtot                   |       -0.05 |
#> c160age                    |       -0.02 |
#> c161sex [Female]           |       -1.63 |
#> c160age × c161sex [Female] |        0.04 |
```

- Interaction terms are denoted by a `*` (or by `:` when calling `summary()`).
- In case of interaction, the effect of a predictor is no longer represented only by its main effect.
- When interpreting interaction terms, the coefficients both of the main and the interaction effect have to be taken into account.

Regression formula with interaction terms

```
#> Parameter                         | Coefficient |
#> ------------------------------------------------
#> (Intercept)                       |       15.78 |
#> barthtot                          |       -0.05 |
#> c160age                           |       -0.02 |
#> c161sex [Female]                  |       -1.63 |
#> c160age × c161sex [Female]        |        0.04 |
```

- When interpreting interaction terms, the coefficients both of the main and the interaction effect have to be taken into account.
- There is no “total” effect of age…

Regression formula with interaction terms

```
#> Parameter                          | Coefficient |
#> ---------------------------------------------------
#> (Intercept)                        |       15.78 |
#> barthtot                           |       -0.05 |
#> c160age                            |       -0.02 |
#> c161sex [Female]                   |       -1.63 |
#> c160age × c161sex [Female]         |        0.04 |
```

- When interpreting interaction terms, the coefficients both of the main and the interaction effect have to be taken into account.

- There is no “total” effect of age…
- …but the effect of age varies by sex.

What is the predicted outcome for respondents at age 50 by gender?

- … when we hold other variables constant? - Remember the dragon pictures!

```
#> Parameter                     | Coefficient
#> ----------------------------------------
#> (Intercept)                   |       15.78
#> barthtot                      |       -0.05
#> c160age                       |       -0.02
#> c161sex [Female]              |       -1.63
#> c160age × c161sex [Female] |        0.04
```

- The predictors of *interest* are also called focal predictors (or *focal terms*).
- The *remaining* predictors that are held constant at certain values. For continuous variables, they are usually held constant at their mean. For categorical variables, at their “reference” level.

Predicted outcome for respondents at age 50 by gender

- But what does this exactly mean? How do we predict our outcome and compare different "categories" of our interaction term?
- A bit algebra, let's look at the regression formula:

$$y = \beta_0 + \beta_1 * \text{barthtot} + \beta_2 * \text{c160age} + \beta_3 * \text{c161sex} + \beta_4 * \text{c160age} * \text{c161sex}$$

```
#> Parameter                       | Coefficient
#> -----------------------------------------------
#> (Intercept)                     |       15.78
#> barthtot                        |       -0.05
#> c160age                         |       -0.02
#> c161sex [Female]                |       -1.63
#> c160age × c161sex [Female]      |        0.04
```

Predicted outcome for respondents at age 50 by gender

- Our "non-focal" term (which we are currently not interested in) is *barthtot*. Furthermore, we know the value of β_0, which is the intercept.
- The mean of *barthtot* is 64.66. We plug in this number into the formula.

```
#> Parameter                   | Coefficient
#> ---------------------------------------
#> (Intercept)                 |       15.78
#> barthtot                    |       -0.05
#> c160age                     |       -0.02
#> c161sex [Female]            |       -1.63
#> c160age × c161sex [Female]  |        0.04
```

$$y = \beta_0 + \beta_1 * \text{barthtot} + \beta_2 * \text{c160age} + \beta_3 * \text{c161sex} + \beta_4 * \text{c160age} * \text{c161sex}$$

$$y = 15.78 + -0.05 * 64.66 + \beta_2 * \text{c160age} + \beta_3 * \text{c161sex} + \beta_4 * \text{c160age} * \text{c161sex}$$

$$y = 12.547 + \beta_2 * \text{c160age} + \beta_3 * \text{c161sex} + \beta_4 * \text{c160age} * \text{c161sex}$$

Predicted outcome for respondents at age 50 by gender

- Our “focal terms” are the interaction terms, *age* and *sex*. We want to predict our outcome for respondents at the age of 50.

```
#> Parameter                    | Coefficient
#> ----------------------------------------
#> (Intercept)                  |       15.78
#> barthtot                     |       -0.05
#> c160age                      |       -0.02
#> c161sex [Female]             |       -1.63
#> c160age × c161sex [Female]   |        0.04
```

$$y = 12.547 + -0.02 * 50 + \beta_3 * \text{c161sex} + 0.04 * 50 * \text{c161sex}$$

$$y = 12.547 + -1 + \beta_3 * \text{c161sex} + 2 * \text{c161sex}$$

$$y = 11.547 + \beta_3 * \text{c161sex} + 2 * \text{c161sex}$$

Predicted outcome for respondents at age 50 by gender

- But which value do we use for the categorical variable `c161sex`, our gender-variable?

- By default, R uses treatment- (or dummy) coding, where each level of the categorical variable is contrasted to a specified reference level.

- In our example, the coefficient of `c161sex [Female]` (-1.63) means that the average difference in our outcome, `neg_c_7`, is -1.63 points *lower* for *female* compared to male persons.

```
#> Parameter                     | Coefficient
#> -----------------------------------------
#> (Intercept)                   |       15.78
#> barthtot                      |       -0.05
#> c160age                       |       -0.02
#> c161sex [Female]              |       -1.63
#> c160age × c161sex [Female]    |        0.04
```

Predicted outcome for respondents at age 50 by gender

- But which value do we use for the categorical variable `c161sex`, our gender-variable?
- By default, R uses treatment- (or dummy) coding, where each level of the categorical variable is contrasted to a specified reference level.
- In our example, the coefficient of `c161sex [Female]` (-1.63) means that the average difference in our outcome, `neg_c_7`, is -1.63 points *lower* for *female* compared to male persons - when `barthtot` and `c160age` are set to zero!

```
#> Parameter                      | Coefficient
#> ------------------------------------------
#> (Intercept)                    |       15.78
#> barthtot                       |       -0.05
#> c160age                        |       -0.02
#> c161sex [Female]               |       -1.63
#> c160age × c161sex [Female]     |        0.04
```

Predicted outcome for respondents at age 50 by gender

- For our formula, this means that if we want to know the predicted outcome for *male* persons (the *reference category*), we simply use a 0, i.e., we drop that term from the formula. Else, we use a 1, i.e., include the coefficient value.

```
#> Parameter                       | Coefficient
#> ---------------------------------------------
#> (Intercept)                     |       15.78
#> barthtot                        |       -0.05
#> c160age                         |       -0.02
#> c161sex [Female]                |       -1.63
#> c160age × c161sex [Female]      |        0.04
```

- For male respondents:

$$y = 11.547 + \ -1.63 * 0 + 2 * 0$$

$$y = 11.547$$

The average negative impact of care, `neg_c_7`, for 50-year-old male respondents (with an Barthel-Index equal to the sample mean) is 11.547 points

Predicted outcome for respondents at age 50 by gender

- For our formula, this means that if we want to know the predicted outcome for *male* persons (the *reference category*), we simply use a 0, i.e., we drop that term from the formula. Else, we use a 1, i.e., include the coefficient value.

```
#> Parameter                     | Coefficient
#> ------------------------------------------
#> (Intercept)                   |       15.78
#> barthtot                      |       -0.05
#> c160age                       |       -0.02
#> c161sex [Female]              |       -1.63
#> c160age × c161sex [Female]    |        0.04
```

- For female respondents:

$$y = 11.547 + \ -1.63 * 1 + 2*1$$

$$y = 11.547 - 1.63 + 2 = 11.917$$

The average negative impact of care, `neg_c_7`, for 50-year-old female respondents (with an Barthel-Index equal to the sample mean) is 11.917 points

But isn't there an easier way to do this?

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https://github.com/easystats/modelbased

The modelbased package:
Estimated Marginal Means and Adjusted Predictions

Estimated Marginal Means and Adjusted Predictions

- After fitting a model, it is useful generate model-based estimates. Usually, there are three quantities of interest:
  - marginal means: to better understand the relationship of predictors with the outcome. These are the average expected values, or *adjusted predictions,* of the response variable for different combinations of predictor values.
  - marginal effects: to evaluate the average strength of an effect (slope, average coefficient).
  - marginal contrasts: to look for (statistically significant) differences between groups, which helps us, for instance, analyzing “social inequalities”.

Estimated Marginal Means and Adjusted Predictions

- The key difference:
  - *marginal means (or: adjusted predictions)* return averages of the outcome variable, which allows you to say for instance “the average health score for a person at the age of sixty is 80 points”.
  - *marginal effects* return averages of coefficients, which allows you to say, for instance, “the average effect of age (or ageing) on the health score is an decrease 5 points”.
  - *marginal contrasts* return the average difference between two groups, typically indicated by different factor levels, which allows you to say “the average difference in health scores for lower and higher status groups at the same age is 15 points”.

Estimated Marginal Means and Adjusted Predictions

- Briefly:
  - marginal means show the association of the *dependent variable* and one or more *independent variables*;
  - marginal effects show you the *average strength* of a coefficient;
  - marginal contrasts show you average differences in the outcome between groups

- In contrast to raw mean values or differences, the above mentioned estimates are adjusted for the non-focal terms (co-variates in your model) and thereby potentially address confounding issues or imbalanced data.

Estimated Marginal Means and Adjusted Predictions

- We will use the **modelbased** package to calculate adjusted predictions for our model.

- A basic workflow, after fitting the model, is as follows:
  - Understand your results with marginal means and marginal effects: `estimate_means()` and `estimate_slopes()`
  - Check "importance" (read: do we find relevant inequalities?) of results with contrasts or pairwise comparisons: `estimate_contrasts()`
  - Communicate your results with plotting: `plot()`

Estimated Marginal Means and Adjusted Predictions

- We will use the **modelbased** package to calculate adjusted predictions for our model.

- We just need to define our predictors of interest, so called *focal terms*.
  - The functions in the **modelbased** package use the `by` argument to specify focal predictors and their *representative values,* at which adjusted predictions are evaluated.
  - There are plenty examples and description of all options in the documentation of `?insight::get_datagrid` and `?estimate_means`.

https://easystats.github.io/modelbased/reference/estimate_means.html
https://easystats.github.io/insight/reference/get_datagrid.html
https://easystats.github.io/modelbased/articles/visualisation_matrix.html

Estimated Marginal Means and Adjusted Predictions

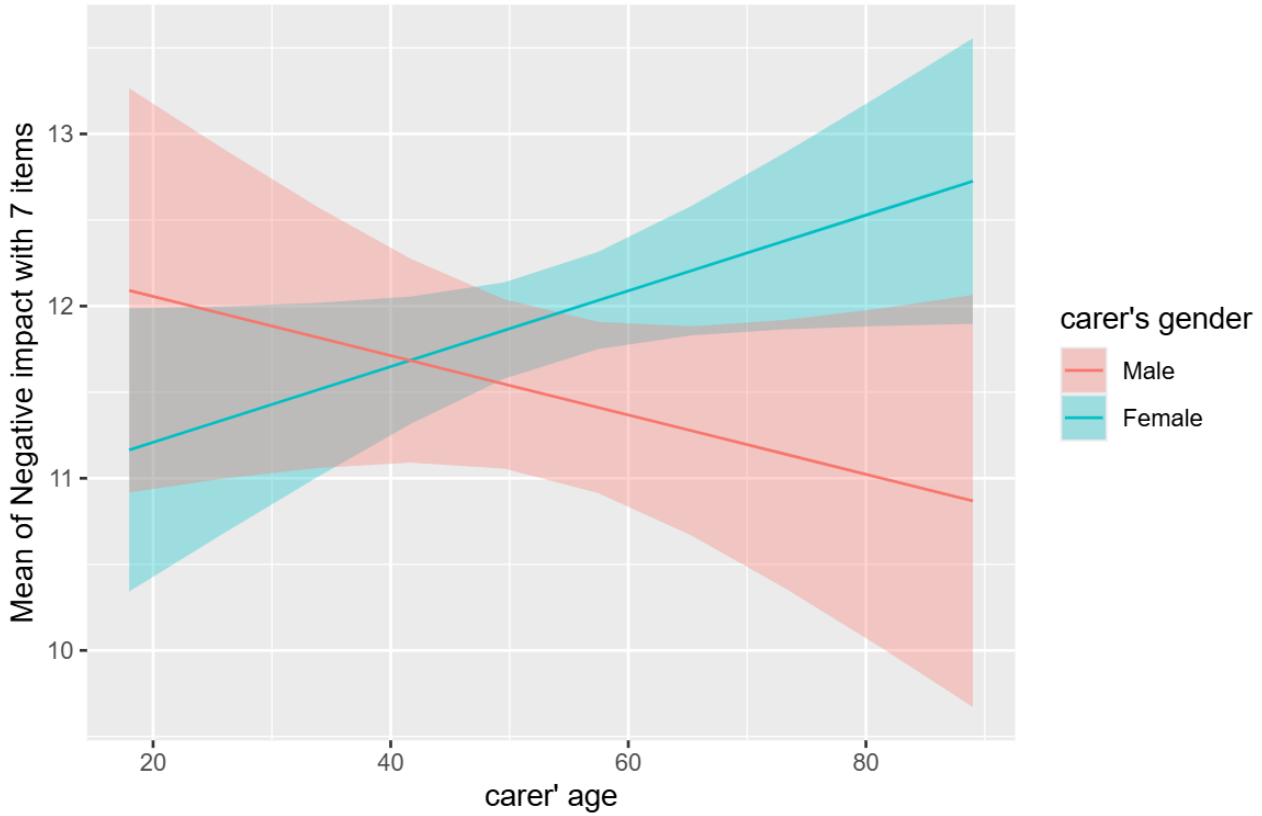
```
library(modelbased)
# predicted outcome at age of 50, by gender
estimate_means(model, by = c("c160age=50", "c161sex"))
#> Estimated Marginal Means
#>
#> c160age | c161sex |  Mean |   SE |          95% CI | t(867)
#> ------------------------------------------------------------
#> 50      | Male    | 11.54 | 0.25 | [11.05, 12.03] |  46.38
#> 50      | Female  | 11.87 | 0.14 | [11.59, 12.15] |  84.10
#>
#> Variable predicted: neg_c_7
#> Predictors modulated: c160age=50, c161sex
#> Predictors averaged: barthtot
```

Estimated Marginal Means and Adjusted Predictions

```
# at different age values
estimate_means(model, by = c("c160age=[pretty]", "c161sex"))
#> Estimated Marginal Means
#>
#> c160age | c161sex |  Mean |   SE |          95% CI | t(867)
#> ------------------------------------------------------------
#> 0       | Male    | 12.40 | 0.86 | [10.71, 14.09] |  14.41
#> 20      | Male    | 12.06 | 0.57 | [10.94, 13.17] |  21.18
#> 40      | Male    | 11.71 | 0.32 | [11.09, 12.34] |  36.87
#> 60      | Male    | 11.37 | 0.27 | [10.84, 11.89] |  42.60
#> 80      | Male    | 11.02 | 0.48 | [10.07, 11.97] |  22.75
#> 100     | Male    | 10.68 | 0.77 | [ 9.17, 12.19] |  13.88
#> 0       | Female  | 10.77 | 0.61 | [ 9.57, 11.97] |  17.56
#> 20      | Female  | 11.21 | 0.40 | [10.43, 11.99] |  28.19
#> 40      | Female  | 11.65 | 0.20 | [11.25, 12.05] |  57.66
#> 60      | Female  | 12.09 | 0.16 | [11.78, 12.39] |  77.77
#> 80      | Female  | 12.53 | 0.33 | [11.88, 13.17] |  38.10
#> 100     | Female  | 12.97 | 0.54 | [11.91, 14.03] |  23.97
#>
#> Variable predicted: neg_c_7
```

Estimated Marginal Means and Adjusted Predictions

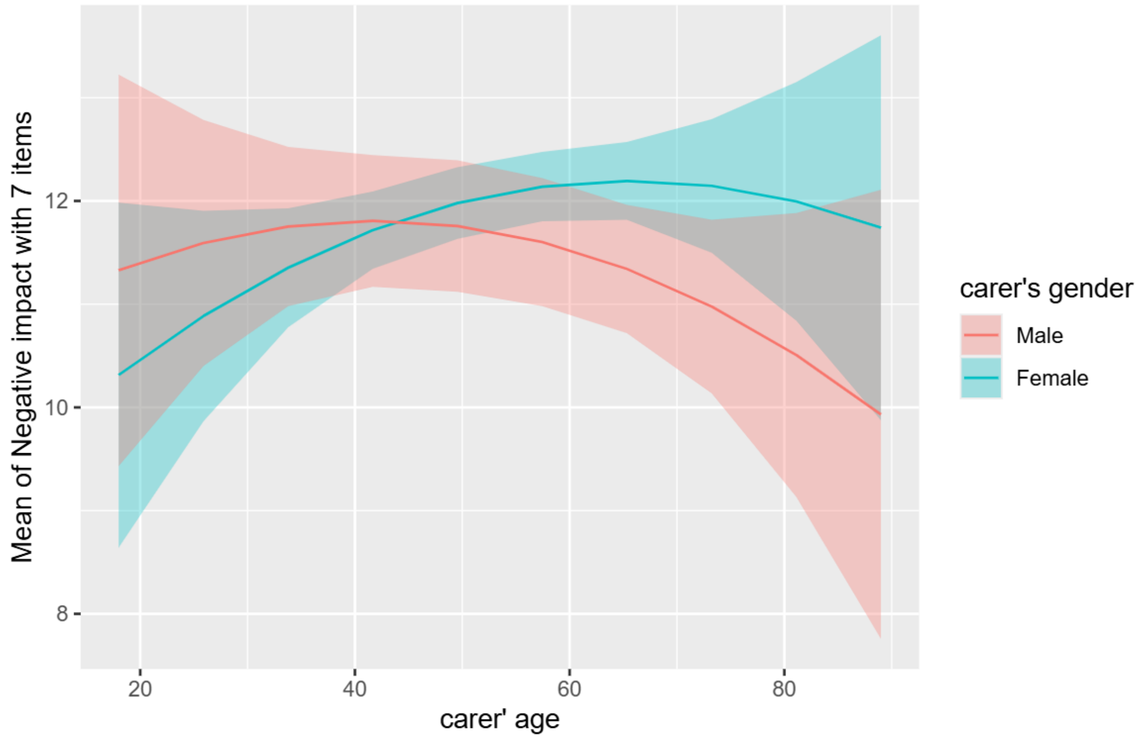
```
# plot
estimate_means(model, by = c("c160age", "c161sex")) |> plot()
```



Estimated Marginal Means and Adjusted Predictions

- Adjusted predictions are in particular helpful to understand the relationship between predictors and outcome in more complex models.

```
# quadratic relationship between
# age and negative impact
model <- lm(
  neg_c_7 ~ barthtot + c160age *
    c161sex + I(c160age^2) * c161sex,
  data = efc
)
estimate_means(
  model,
  by = c("c160age", "c161sex")
) |> plot()
```



Estimated Marginal Means and Adjusted Predictions

- Let's get back to the example with an ordinal factor.

```
library(parameters)
data(diamonds, package = "ggplot2")
diamonds$cut_f <- factor(diamonds$cut, ordered = FALSE)
m1 <- lm(price ~ cut, data = diamonds)   # ordinal factor
m2 <- lm(price ~ cut_f, data = diamonds) # regular (nominal) factor
# ordinal factors harder to interpret
compare_parameters(m1, m2)
#> Parameter         |                          m1 |                           m2
#> ------------------------------------------------------------------------------
#> (Intercept)       | 4062.24 (4012.45, 4112.02) | 4358.76 ( 4165.13, 4552.38)
#> cut (linear)      | -362.73 (-496.08, -229.37) |
#> cut (quadratic)   | -225.58 (-344.45, -106.71) |
#> cut (cubic)       | -699.50 (-802.94, -596.05) |
#> cut (4th degree)  | -280.36 (-363.77, -196.94) |
#> cut f (Good)      |                            | -429.89 ( -653.04, -206.75)
#> cut f (Very Good) |                            | -377.00 ( -583.12, -170.88)
#> cut f (Premium)   |                            |  225.50 (   20.88,  430.12)
#> cut f (Ideal)     |                            | -901.22 (-1101.94, -700.49)
#> ------------------------------------------------------------------------------
#> Observations      |                       53940 |                        53940
```

Estimated Marginal Means and Adjusted Predictions

- Ordinal or nominal doesn't matter when we calculate adjusted predictions – interpretation is much easier!

```
# ordinal factor
estimate_means(m1, "cut")
#> Estimated Marginal Means
#>
#> cut       |    Mean |               95% CI
#> ------------------------------------------
#> Fair      | 4358.76 | [4165.13, 4552.38]
#> Good      | 3928.86 | [3817.94, 4039.78]
#> Very Good | 3981.76 | [3911.08, 4052.44]
#> Premium   | 4584.26 | [4518.10, 4650.41]
#> Ideal     | 3457.54 | [3404.62, 3510.46]
```

```
# nominal factor
estimate_means(m2, "cut_f")
#> Estimated Marginal Means
#>
#> cut_f     |    Mean |               95% CI
#> ------------------------------------------
#> Fair      | 4358.76 | [4165.13, 4552.38]
#> Good      | 3928.86 | [3817.94, 4039.78]
#> Very Good | 3981.76 | [3911.08, 4052.44]
#> Premium   | 4584.26 | [4518.10, 4650.41]
#> Ideal     | 3457.54 | [3404.62, 3510.46]
```

- We see that, given that our model is correctly specified, and our data is good, the average price of a diamond with Fair cut is about 4359 USD.

What are the differences?

Adjusted predictions, estimated marginal means, counterfactual predictions

Adjusted predictions - adjusted for what?

- `estimate_means()` predicts the average expected response at representative values or levels of your *focal terms*, i.e., you specify the predictors you are mainly interested in, using the `by` argument.

- The `estimate` argument indicates the *type of target population* predictions refer to, and how to *marginalize* over the *non-focal predictors*, i.e., those variables that are not specified in `by`. Each option answers slightly different questions.

Adjusted predictions

`estimate="specific"`

- Numeric values are set to their mean, factors to their reference level.
- Estimated means represent a rather "theoretical" view on the data, which does not necessarily exactly reflect the characteristics of the sample.
- It refers to a "specific individual" from the sample (i.e., a specific combination of predictor values).
- It helps answer the question: *What is the expected average value of the response at meaningful values or levels of my focal terms for a 'specific' observation in my data?*, where 'specific' refers to certain characteristics of the remaining predictors.

Estimated marginal means

`estimate="typical"`

- Predictions are made for observations that are represented by a data grid, which is built from all combinations of the predictor levels of the focal terms.
- Then, takes the mean value for non-focal numeric predictors and marginalizes over the factor levels of non-focal terms, which computes a kind of “weighted average”.
- These predictions are useful for comparing defined “groups” and are still a good representation of the sample.
- It answers the question, *“What would be the average outcome for a ‘typical’ observation?”* where ‘typical’ refers to subjects represented by (i.e., that share the characteristics from) the data grid.

Average predictions

`estimate="average"`

- Predictions are made for each observation in the sample.
- Then, the average of all predictions is calculated within all groups (or levels) of the focal terms defined in `by`.
- These predictions are the closest representation of the sample, because `estimate = "average"` averages across the full sample, where groups (in `by`) are not represented by a *balanced data grid*, but rather by the empirical distributions of the characteristics of the sample.
- It answers the question, *"What is the predicted value for an average observation (from a certain group in by) in my data?"*.

Average counterfactual prediction

`estimate="population"`

- Non-focal predictors are marginalized over the observations in the sample, where the sample is replicated multiple times to produce "counterfactuals", and then takes the average of these predicted values (aggregated/grouped by the focal terms).
- It can be considered as extrapolation to a hypothetical target population. Counterfactual predictions are useful, insofar as the results can also be transferred to other contexts (Dickerman and Hernán, 2020).
- It answers the question, *What is the predicted response value for the 'average' observation in the broader target population?* It does not only refer to the actual data in your observed sample, but also "what would be if" we had more data, or if we had data from a different sample.

Dickerman, Barbra A., and Miguel A. Hernán. 2020. "Counterfactual Prediction Is Not Only for Causal Inference." European Journal of Epidemiology 35 (7): 615–17.

Examples

- Let's show an example for the different `marginalize` options.

```
# setup
data(efc, package = "modelbased")
efc <- datawizard::to_factor(efc, c("c161sex", "c172code", "e16sex", "e42dep"))
levels(efc$c172code) <- c("low", "mid", "high")

# model:
# c172code -> education (low, mid high)
# neg_c_7 -> negative-burden-score
m <- lm(barthtot ~ c161sex + c172code + neg_c_7, data = efc)
```

Adjusted predictions

- What is the difference between *or the expected outcome of* a (specific) male and female person, when they are low educated (reference) with an average burden?

```
estimate_means(m, "c161sex", estimate = "specific")
#> Model-based Predictions
#>
#> c161sex |  Mean |   SE |         95% CI | t(810)
#> ---------------------------------------------------
#> Male    | 61.15 | 2.66 | [55.93, 66.38] |  22.98
#> Female  | 60.61 | 2.11 | [56.46, 64.75] |  28.71
```

Estimated marginal means

- What is the difference between *or the expected outcome of* a (typical) male and female person (with an “average” educational level and average burden score)?

```
estimate_means(m, "c161sex", estimate = "typical")
#> Estimated Marginal Means
#>
#> c161sex |  Mean |   SE |          95% CI | t(810)
#> ------------------------------------------------
#> Male    | 64.61 | 1.98 | [60.72, 68.50] |  32.60
#> Female  | 64.07 | 1.22 | [61.67, 66.47] |  52.44
```

Average predictions

- What is the difference between *or the expected outcome of* an average male and average female person in the sample?
  - (i.e. the average difference between the mean outcome of all males and the mean outcome of all females)

```
estimate_means(m, "c161sex", estimate = "average")
#> Estimated Marginal Means
#>
#> c161sex |  Mean |   SE |         95% CI | t(810)
#> ------------------------------------------------
#> Male    | 67.05 | 1.93 | [63.26, 70.84] |  34.72
#> Female  | 64.03 | 1.08 | [61.91, 66.16] |  59.12
```

Average counterfactual predictions

- We know that female persons share other characteristics than male persons (e.g., less higher educated, more willing/required to do care work, ...). What if we also had male persons that share the same characteristics as the female persons, and if we also had more female persons that have the same characteristics as our male sample?
- If we have this data: What would be the average difference between *or the expected outcome of* male and female persons (when the data is “unbiased”)?

```
estimate_means(m, "c161sex", estimate = "population")
#> Estimated Marginal Means
#>
#> c161sex |  Mean |   SE |         95% CI | t(810)
#> ------------------------------------------------
#> Male    | 65.17 | 1.94 | [61.36, 68.98] |  33.56
#> Female  | 64.62 | 1.08 | [62.49, 66.75] |  59.56
```

Summary

- We can roughly distinguish these types into *modelbased means* (data grid based) and *empirical means* (sample/population based).
- modelbased means (`specific` and `typical`):
  - are useful to look at differences between groups, e.g. “male” versus “female” characteristics
  - for visualization
- empirical means (`average` and `population`):
  - Average predictions are useful is useful if you want a realistic picture of your sample, assuming that it is representative for a special population
  - Counterfactual predictions are useful for “what-if” scenarios, especially if you want to make unbiased comparisons (g-computation)

And what about…

Marginal Effects

Marginal effects – the average effects of a coefficient

- In certain (simple?) regression models, a regression coefficient can directly be used to interpret the strength of an association between a predictor and the outcome.
- But sometimes – e.g. when interaction terms are involved, or when we have other models than linear regression – quantifying the "average effect" (marginal effect) of a predictor is less straightforward.
- These are situations where we might need *marginal effects*.

Marginal effects for interaction terms

- Let's look at an example with an interaction term.
- We see that the "effect" of Petal Length is not identical across Species.

```
library(parameters)
m <- lm(Sepal.Length ~ Species * Petal.Length, data = iris)
model_parameters(m)
#> Parameter                              | Coefficient |   SE |         95% CI | t(144) |      p
#> ------------------------------------------------------------------------------------------------
#> (Intercept)                            |        4.21 | 0.41 | [ 3.41,  5.01] |  10.34 | < .001
#> Species [versicolor]                   |       -1.81 | 0.60 | [-2.98, -0.63] |  -3.02 | 0.003
#> Species [virginica]                    |       -3.15 | 0.63 | [-4.40, -1.91] |  -4.97 | < .001
#> Petal Length                           |        0.54 | 0.28 | [ 0.00,  1.08] |   1.96 | 0.050
#> Species [versicolor] × Petal Length    |        0.29 | 0.30 | [-0.29,  0.86] |   0.97 | 0.332
#> Species [virginica] × Petal Length     |        0.45 | 0.29 | [-0.12,  1.02] |   1.56 | 0.118
#>
#> Uncertainty intervals (profile-likelihood) and p-values (two-tailed)
#>   computed using a Wald t-distribution approximation.
```

Marginal effects for interaction terms

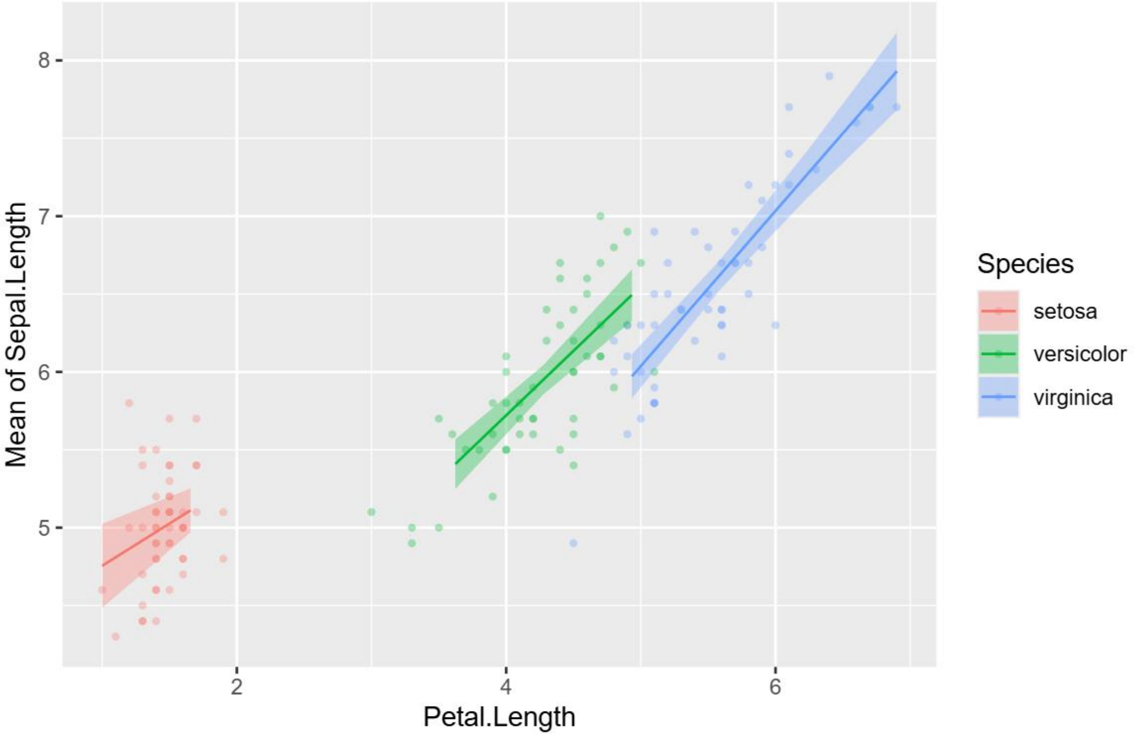
- We see that the “effect” of Petal Length is not identical across Species (expected outcome for different Petal Length values varies by Species).

```
estimate_means(m, c("Petal.Length", "Species"), preserve_range = TRUE)
#> Estimated Marginal Means
#>
#> Petal.Length | Species    | Mean |   SE |       95% CI | t(144)
#> ---------------------------------------------------------------
#> 1.00         | setosa     | 4.76 | 0.14 | [4.49, 5.03] |  34.86
#> 1.66         | setosa     | 5.11 | 0.07 | [4.97, 5.25] |  71.33
#> 3.62         | versicolor | 5.41 | 0.08 | [5.25, 5.57] |  66.97
#> 4.28         | versicolor | 5.95 | 0.05 | [5.86, 6.04] | 124.97
#> 4.93         | versicolor | 6.49 | 0.08 | [6.33, 6.66] |  77.57
#> 4.93         | virginica  | 5.97 | 0.07 | [5.83, 6.11] |  83.08
#> 5.59         | virginica  | 6.62 | 0.05 | [6.53, 6.72] | 138.91
#> 6.24         | virginica  | 7.28 | 0.08 | [7.13, 7.43] |  94.74
#> 6.90         | virginica  | 7.93 | 0.13 | [7.68, 8.18] |  62.60
```

Marginal effects for interaction terms

- We can also visualize this model, by calculating estimated marginal means for all values of Petal Length by Species.

```
library(modelbased)
m <- lm(
  Sepal.Length ~ Species * Petal.Length,
  data = iris
)
means <- estimate_means(
  m,
  c("Petal.Length", "Species"),
  preserve_range = TRUE
)
plot(means, show_data = TRUE)
```

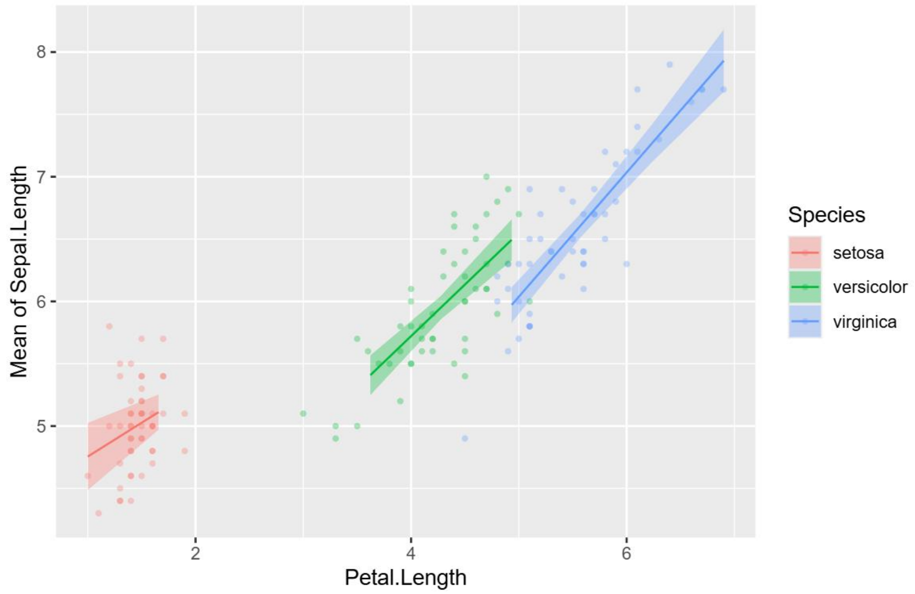


But what is the “overall” or “average” effect of Petal Length in this model?

Marginal effects for interaction terms

- We can see that the average effect of Petal Length, *marginalized over Species*, is 0.79!

```
estimate_slopes(m, trend = "Petal.Length")
#> Estimated Marginal Effects
#>
#> Slope |   SE |       95% CI |    t |      p
#> -------------------------------------------
#> 0.79  | 0.10 | [0.59, 0.99] | 7.69 | < .001
#>
#> Marginal effects estimated for Petal.Length
#> Type of slope was dY/dX
```



Marginal effects for logistic regression

- Now an example with logistic regression.

```
# we simulate some data for this model...
set.seed(5)
x1 <- rnorm(200)
z <- 0.5 + rnorm(200, 1, 0.2) * x1
pr <- 1 / (1 + exp(-z))
y <- rbinom(200, 1, pr)
d <- data.frame(y = y, x1 = x1)
m <- glm(y ~ x1, data = d, family = binomial())
# 'fivenum' selects Tukey Five-Number summaries as representative values
estimate_means(m, "x1 = [fivenum]")
#> Estimated Marginal Means
#>
#> x1    | Probability |       95% CI
#> ----------------------------------
#> -2.62 |        0.06 | [0.02, 0.16]
#> -0.60 |        0.42 | [0.33, 0.51]
#> -0.06 |        0.58 | [0.50, 0.65]
#> 0.78  |        0.79 | [0.69, 0.85]
#> 2.60  |        0.97 | [0.91, 0.99]
```

Marginal effects for logistic regression

```
# we simulate some data for this model...
set.seed(5)
n <- 200
x1 <- rnorm(n)
z <- 0.5 + rnorm(n, 1, 0.2) * x1
pr <- 1 / (1 + exp(-z))
y <- rbinom(n, 1, pr)
d <- data.frame(y = y, x1 = x1)
m <- glm(y ~ x1, data = d, family = binomial())
estimate_means(m, "x1 = [fivenum]")
#> Estimated Marginal Means
#>
#> x1    | Probability |       95% CI
#> ----------------------------------
#> -2.62 |        0.06 | [0.02, 0.16]
#> -0.60 |        0.42 | [0.33, 0.51]
#> -0.06 |        0.58 | [0.50, 0.65]
#> 0.78  |        0.79 | [0.69, 0.85]
#> 2.60  |        0.97 | [0.91, 0.99]
```

- As we can see, the predicted probability for P(y=1) is 6% when x1 is -2.62.

Marginal effects for logistic regression

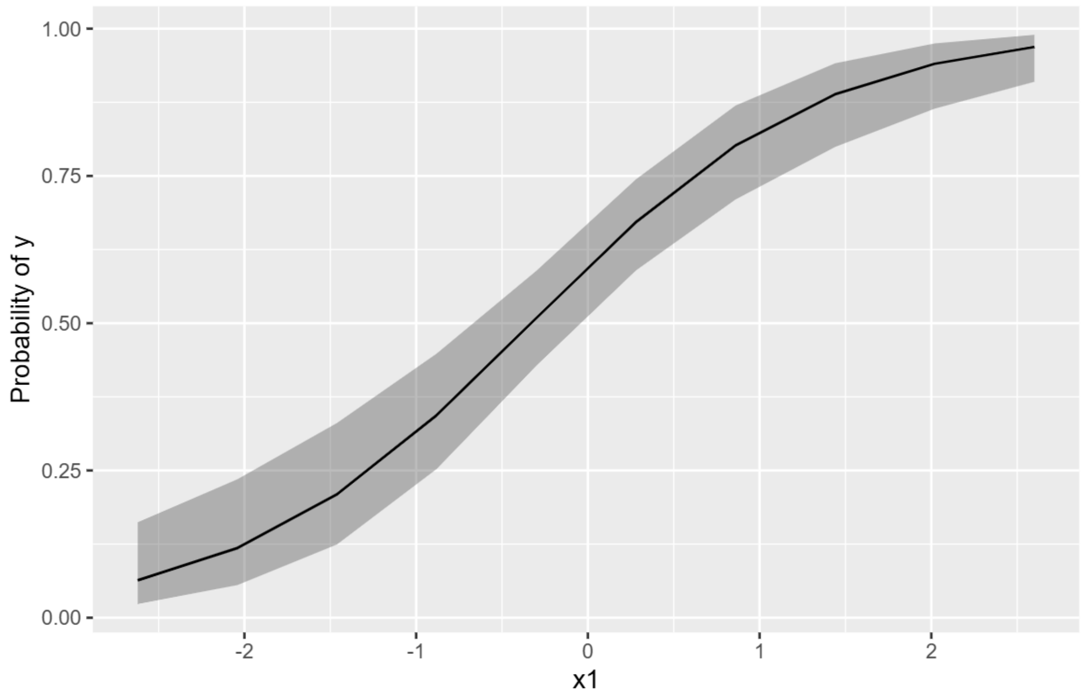
```
# we simulate some data for this model...
set.seed(5)
n <- 200
x1 <- rnorm(n)
z <- 0.5 + rnorm(n, 1, 0.2) * x1
pr <- 1 / (1 + exp(-z))
y <- rbinom(n, 1, pr)
d <- data.frame(y = y, x1 = x1)
m <- glm(y ~ x1, data = d, family = binomial())
estimate_means(m, "x1 = [fivenum]")
#> Estimated Marginal Means
#>
#> x1    | Probability |       95% CI
#> ----------------------------------
#> -2.62 |        0.06 | [0.02, 0.16]
#> -0.60 |        0.42 | [0.33, 0.51]
#> -0.06 |        0.58 | [0.50, 0.65]
#> 0.78  |        0.79 | [0.69, 0.85]
#> 2.60  |        0.97 | [0.91, 0.99]
```

- As we can see, the predicted probability for P(y=1) is 6% when x1 is -2.62.
- When x1 = 0.78, the predicted probability of P(y=1) is about 79%.

Marginal effects for logistic regression

- But what is the *average* effect of x1 for y = 1 (i.e. P(y=1)?
- We can clearly see that the association (slope) between x1 and y is not constant across values of x1.

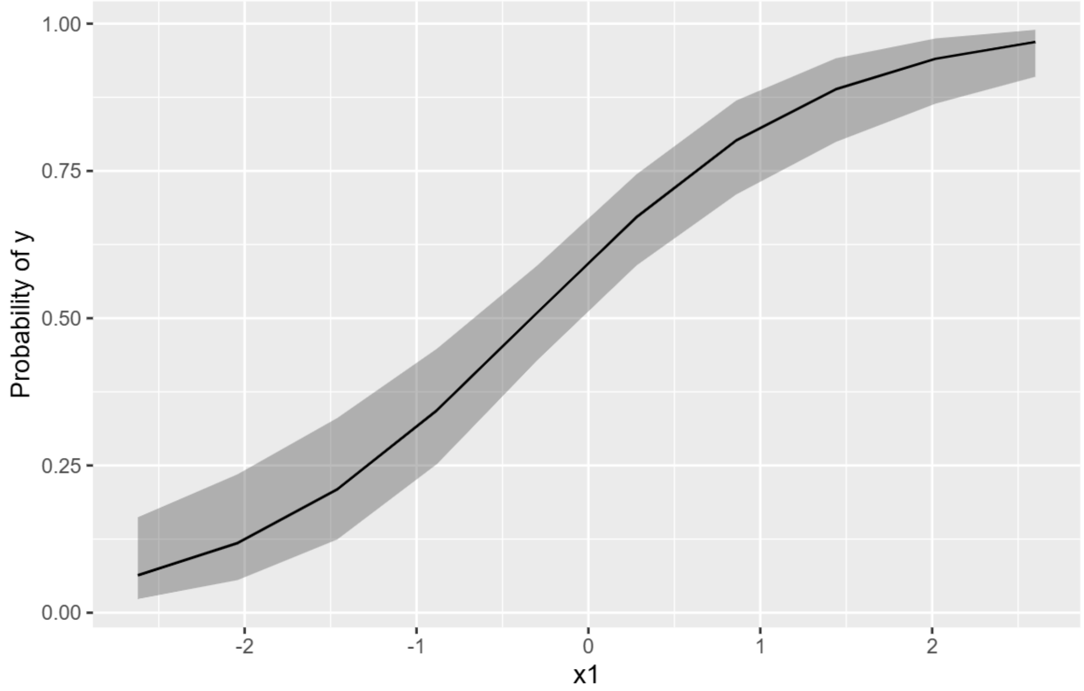
```
estimate_means(m, "x1 = [fivenum]")
#> Estimated Marginal Means
#>
#> x1    | Probability |       95% CI
#> ----------------------------------
#> -2.62 |        0.06 | [0.02, 0.16]
#> -0.60 |        0.42 | [0.33, 0.51]
#> -0.06 |        0.58 | [0.50, 0.65]
#> 0.78  |        0.79 | [0.69, 0.85]
#> 2.60  |        0.97 | [0.91, 0.99]

estimate_means(m, "x1") |> plot()
```



Marginal effects for logistic regression

- Again, we use `estimate_slopes()` to calculate the marginal effect for our predictor of interest, and we can see that, on average, P(y=1) is about 23% across all values of x1.

```
estimate_slopes(m, "x1")
#> Estimated Marginal Effects
#>
#> Slope |   SE |        95% CI |    z |      p
#> ---------------------------------------------
#> 0.23  | 0.03 | [0.18, 0.27] | 8.93 | < .001
#>
#> Marginal effects estimated for x1
#> Type of slope was dY/dX
```



Marginal effects of interaction terms – former example

- Remember when we said:
  - There is no "total" effect of age…
  - …but the effect of age varies by sex.

```
#> Parameter                           | Coefficient |
#> -------------------------------------------------
#> (Intercept)                         |       15.78 |
#> barthtot                            |       -0.05 |
#> c160age                             |       -0.02 |
#> c161sex [Female]                    |       -1.63 |
#> c160age × c161sex [Female]          |        0.04 |
```

- With marginal effects, we can indeed calculate a "total" effect (or better: the *average* effect) of age,

```
estimate_slopes(model, "c160age")
#> Estimated Marginal Effects
#>
#> Slope |       SE |        95% CI |     p
#> ---------------------------------------
#> 0.01  | 9.42e-03 | [-0.01, 0.03] | 0.178
```

Questions?