Image Processing - CS 474
Programming Assignment 4
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- Anthony Silva

Theory

Introduction

This assignment explores interesting applications of the Fourier Transform in image processing, such as image enhancement and restoration. This report investigates four areas: frequency domain filtering for noise removal, validation of the convolution theorem, homomorphic filtering for dynamic range compression, and image restoration techniques for degraded images.

Frequency Domain Filtering

Frequency domain filtering works on the principle that certain image features correspond to specific frequency components. Periodic noise manifests as distinct impulses in the frequency spectrum, making it possible to selectively remove these components without affecting the rest of the image content.

Band reject filters attenuate frequencies within a specified band, D_0, while passing all others. The Gaussian band reject filter is defined as:

$$H(u, v) = 1 - e^{-\left[\frac{D^2 - D_0^{\,2}}{D \cdot W}\right]^2}$$

Where D is D(u, v), representing the distance from the center of the frequency domain. D_0 is the center of frequency of the specified band, and W is the band width. Its Gaussian nature helps it smoothly transition between pass and reject regions avoiding the ringing artifacts associated with ideal filters.

Band pass filters are the complement of band reject filters, passing only frequencies within the specified band:

$$H(u, v) = e^{-\left[\frac{D^2 - D_0^{\,2}}{D \cdot W}\right]^2}$$

Notch Filters provide more precise control by rejecting frequencies at specific locations rather than entire bands. The Gaussian notch reject filter is defined as:

$$H(u, v) = 1 - e^{\frac{-(D_1 \cdot D_2)}{2 \cdot D_0^{\,2}}}$$

Where D_1 and D_2 are the distances from the notch centers at (u_0, v_0) and ($-u_0$, $-v_0$) respectively. The symmetric placement ensures that both positive and negative frequency components of periodic noise are eliminated, as required by the conjugate symmetry of real-valued images.

Convolution Theorem

The Convolution Theorem establishes a fundamental relationship between spatial domain convolution and frequency domain multiplication:

$$f(x, y) * h(x, y) \leftrightarrow F(u, v) \cdot H(u, v)$$

This states that convolution in the spatial domain is equivalent to element-wise multiplication in the frequency domain, and vice versa. This relationship is very important to utilize in common cases where frequency domain multiplication can be more efficient than direct spatial convolution.

Special attention must be given when implementing this fact as certain theoretical properties must be maintained to ensure that the Convolution Theorem is maintained. Firstly, a true convolution requires the kernel to be rotated 180 degrees before the operation, which means when coming from a discrete mask, you have to rotate it. Also, for odd-symmetric filters like the Sobel mask, you must take extra steps to ensure that the symmetry is preserved. This entails adding a leading row and column of zeros to create an even dimensioned array that can have its center aligned to the expanded mask. It also requires zeroing out the real component after the Fourier Transform since odd-symmetric functions should have only imaginary outputs in their Fourier Transforms.

Homomorphic Filtering

Homomorphic filtering addresses images with awkward illumination by operation on the multiplicative components of image formation. An image’s brightness values can be modeled as:

$$f(x, y) = i(x, y) \cdot r(x, y)$$

where i(x, y) is the illumination component with slowly varying values (low frequency) and r(x, y) is the reflectance component with rapidly varying values (high frequency). Taking the logarithm of these components makes the relationship additive:

$$ln(f(x, y)) = ln(i(x, y)) + ln(r(x, y))$$

The High Frequency Emphasis (HFE) filter can then be applied in the frequency domain:

$$H(u, v) = (\gamma_H - \gamma_L)\left[1 - e^{\left(\frac{-c \cdot D^2(u, v)}{D_0^{\,2}}\right)}\right] + \gamma_L$$

where γ_L controls the attenuation of low frequencies (illumination), γ_H controls the amplification of high frequencies (reflectance), c controls the sharpness of the transition, and D_0 is the cutoff frequency. After filtering, the exponential function is applied to return to the original intensity domain. This compresses the dynamic range of illumination variations while enhancing reflectance details.

Image Restoration

Image restoration attempts to recover an original image from a degraded observation. The degradation model in the frequency domain is:

$$G(u, v) = F(u, v) \cdot H(u, v) + N(u, v)$$

where G is the degraded image, F is the original image, H is the degradation function, and N is additive noise.

Motion blur occurs when there is relative motion between the camera and scene during exposure. For uniform linear motion, the degradation function is:

$$H(u, v) = \left(\frac{T}{\pi(ua + vb)}\right) \cdot sin(\pi(ua + vb)) \cdot e^{(-j\pi(ua + vb))}$$

where T is the exposure time, and a and b describe the motion in the x and y directions respectively. This function contains zeros (where sin equals zero), which creates challenges for restoration.

Inverse Filtering directly inverts the degradation:

$$\widehat{F}(u, v) = G(u, v) / H(u, v)$$

Due to its simpleness, inverse filtering is highly sensitive to noise. When H(u,v) is small, division amplifies any noise present in G(u,v). A practical modification limits the inverse filtering to frequencies within a radius where H(u,v) maintains sufficient magnitude.

Wiener Filtering addresses the noise sensitivity of inverse filtering by incorporating knowledge of the noise characteristics:

$$\widehat{F}(u, v) = \left[H^{*}(u, v) / \left(|H(u, v)|^{2} + K\right)\right] \cdot G(u, v)$$

where H* is the complex conjugate of H, and K represents the noise-to-signal power ratio. When $|H|^2$ is much larger than K, the filter approximates inverse filtering; when $|H|^2$ is much smaller than K, the filter suppresses the output, preventing noise amplification. The parameter K provides a trade-off between noise suppression and restoration fidelity. Basically, larger K values produce smoother results with less noise but also less detail recovery.

Results

Frequency Domain Filtering

Sinusoidal Noise has been added to this image. We can isolate it and remove it through frequency domain transformations like band filter and notch filter. The image and its frequency are shown below. It should be noted that there are four small bright spots located in the vertices of a rectangle visible in the frequency domain that align with the periodic noise added to the image.

Image A. Noisy Boy Image

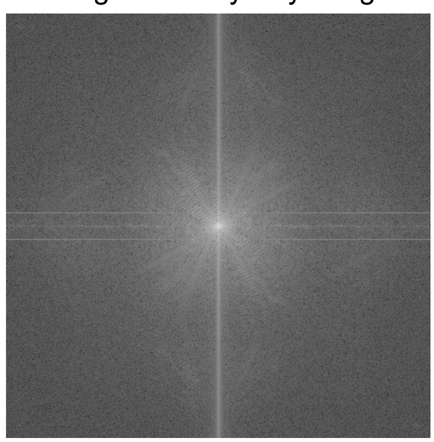


Image B. Noisy Boy Frequency Representation

A band pass, band reject, and a notch reject filter were all added to the image in the frequency domain through element wise multiplication, and their real parts were taken back into the spatial domain to be analyzed.

Filter	Applied	Result

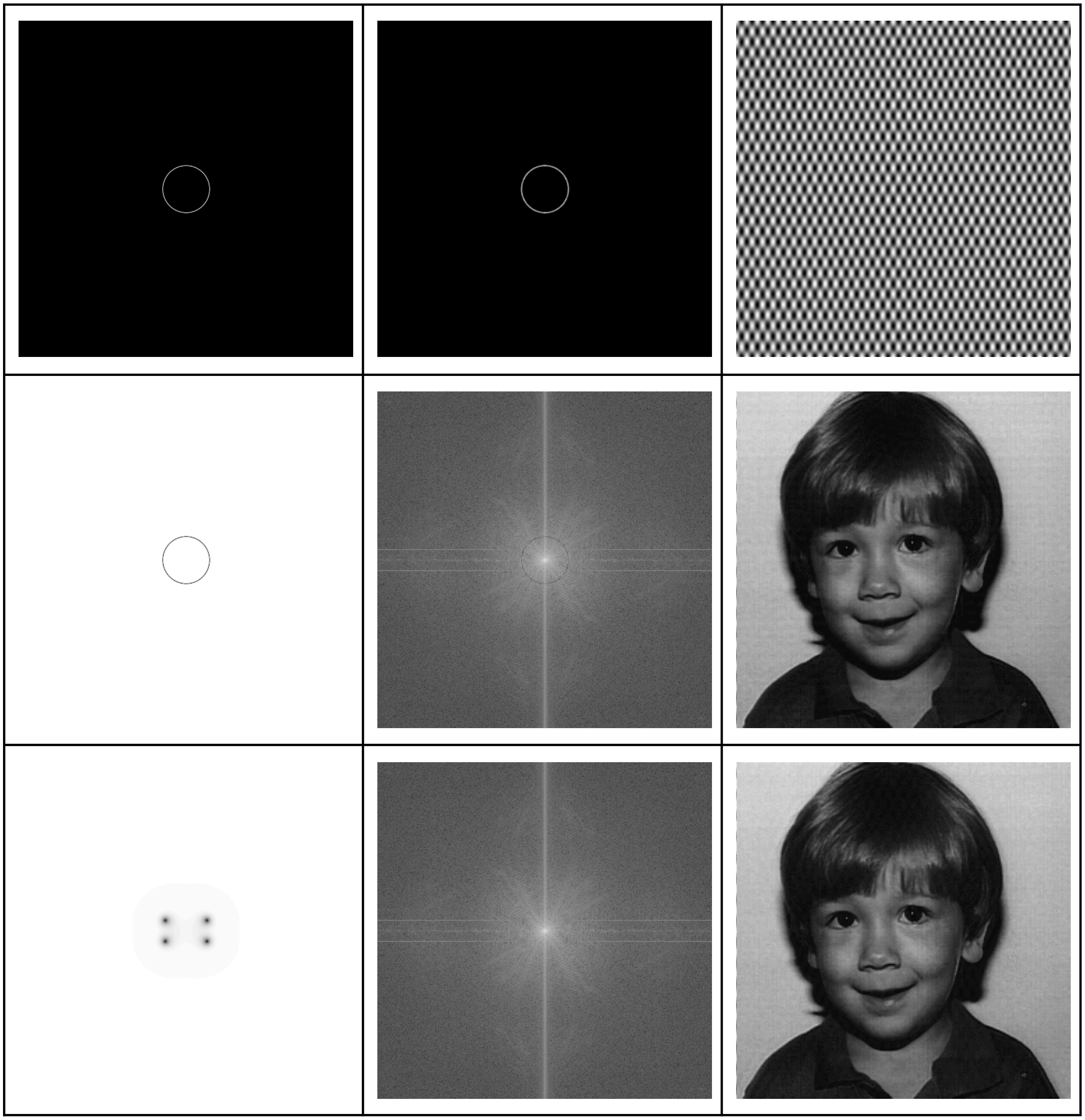


Table A. Frequency Isolation Results

The band pass shows the periodic noise by itself while the band and notch rejects focused on the additional periodic noise show the boy image without noise. There are subtle differences between the notch and band reject due to the small differences in frequencies rejected between the band or the notches, but overall they both seem effective. The effectiveness of these filters come from the fundamental property that spatial periodic noise appears as impulses in the frequency domain. These impulses are then easily removed through reject filters smoothly by Gaussian based filters that minimize ringing artifacts.

Experiment 1, Part C, Free Response Question

Face verification and recognition is a growing important application of computer science, computer vision, and image processing. Because of that, the stakes of proper image processing grow higher due to the growing integration of these algorithms within societal functions. Failing to reduce noise in images or introducing artifacts due to algorithmic errors due to inadequate image processing can be extremely detrimental to someone's social, economic, and business functions. A few examples of high stake face verification and recognition include device access technologies like FaceID and security surveillance incorporating face recognition technologies.

Device Access through facial recognition is crucial to get right, because technologies like FaceID are being increasingly used for authentication in various fields. An improper facial recognition algorithm would either make it difficult for a user to access their device when they need to or allow for a person to impersonate that user. In regards to cellphones, this risks proper social integration due to the growing requirement of a cellphone to connect with people, as well as more serious consequences relating to finances and work as many banking and work apps can be accessed through a device's facial recognition capabilities.

Security surveillance can also be costly for an organization if they get their facial recognition algorithms wrong. A growing use case for image processing and facial recognition relates to identifying people who are allowed and not allowed to be within certain perimeters. Failing to properly allow a person who should be allowed in a certain area based on facial recognition can be problematic for people trying to complete work, while allowing a person who should not be granted access can also be very problematic depending on the context. One case locally in Reno is a great example of this. A UPS driver was arrested at the Peppermill Casino while trying to deliver packages because their AI facial recognition system falsely identified him as someone who was previously found trespassing on their premises previously. Despite having proper identification for his job and motives for being there, he was detained by security. The result of this altercation led to a lawsuit and a lot of headaches that did not need to happen, all because of an improper facial recognition algorithm.

Overall, these examples show how choices and execution of image processing in facial recognition systems carry consequences more than just efficiency, but also affect the lives of the people who use these technologies. Specifically for identification/authentication technologies like facial recognition, the security implications are grand and can affect a person's social, financial, and business spheres of their lives.

Convolution Theorem

The convolution theorem was explored by applying a x partial derivative sobel mask in the spatial domain through a convolution, and as a multiplication in the frequency domain between the frequency representations of both. The convolution theorem says they should be the same. The first row of the following table shows the spatial convolution, and the second row shows the frequency multiplication.

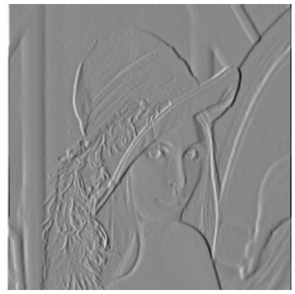
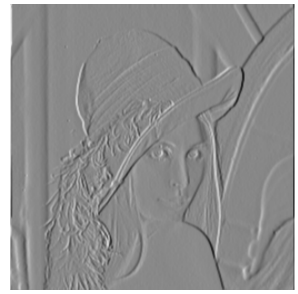
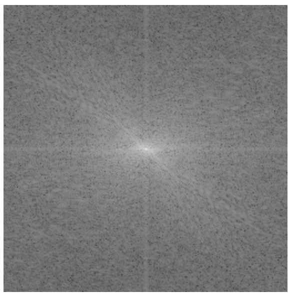
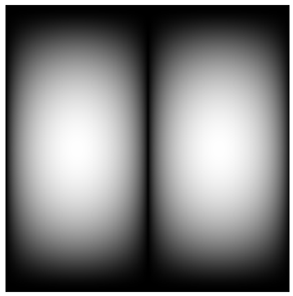
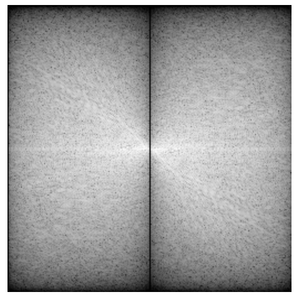
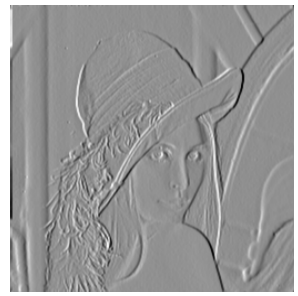
Image	Mask	Applied	Result
			
			

Table B. Convolution Theorem Exploration

It can be seen that the results of both transformations are identical, showing that spatial domain convolution and frequency domain multiplication are mathematically equivalent operations related through the Fourier Transform. This equivalence provides computational advantages where FFT-based multiplication (O(N^2logN)) outperforms direct convolution (O(N^2 * M^2)) for an MxM kernel).

Homomorphic Filtering

Homomorphic filtering was applied by taking an awkwardly exposed image with a bright center and a dark perimeter and using a high frequency emphasis filter to alter lower and higher frequencies differently. The original image and its frequency representation are shown below.

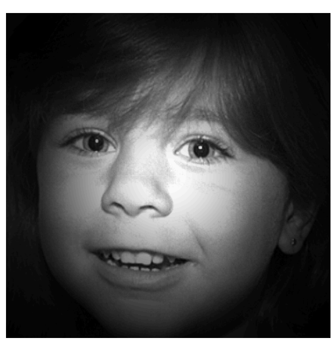


Image C. Girl Image

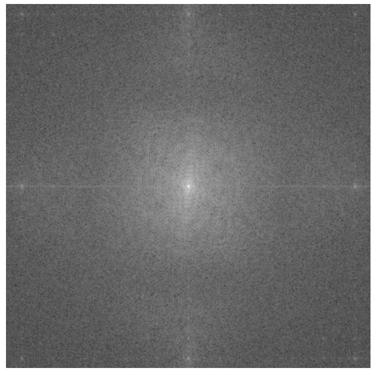


Image D. Girl Frequency Representation

The equation is logarithmic and bounded between two constants, a gamma low and a gamma high. A total of nine filters were created with combinations of gamma low and gamma high shown below.

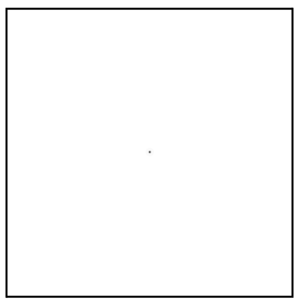
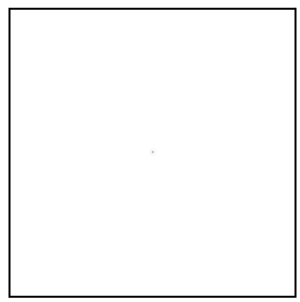
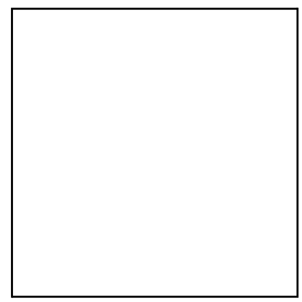
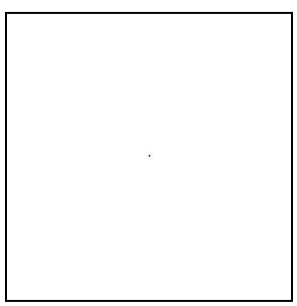
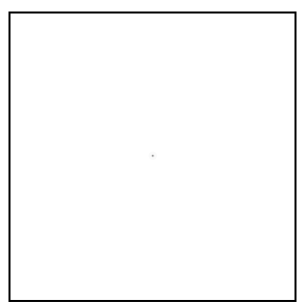
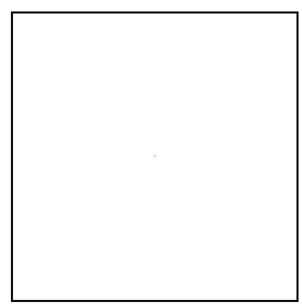
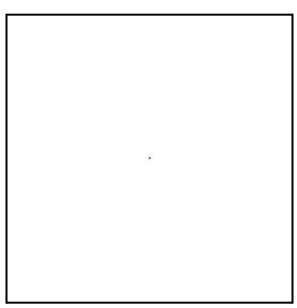
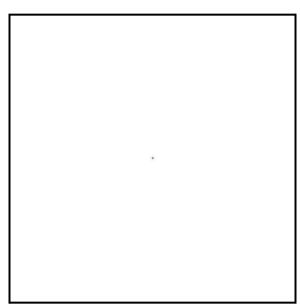
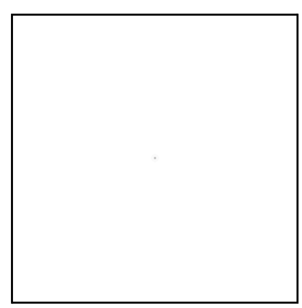
	gammaL = 0.0	gammaL = 0.5	gammaL = 1.0
gammaH = 1.0			
gammaH = 1.5			
gammaH = 2.0			

Table C. High Frequency Emphasis Filters

The multiplicative result of the original girl frequency and each high frequency emphasis mask are shown below.

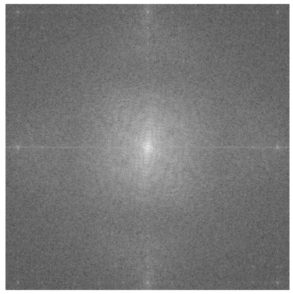
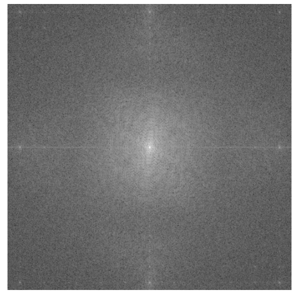
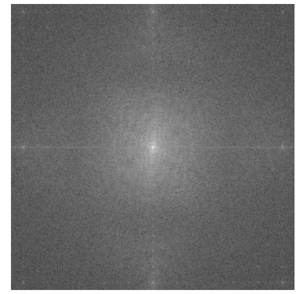
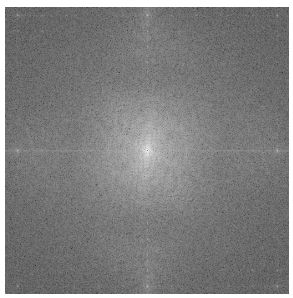
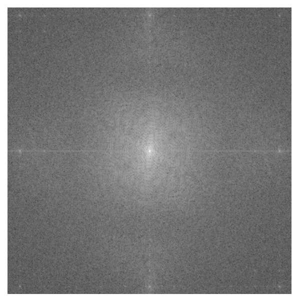
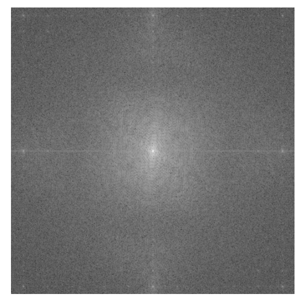
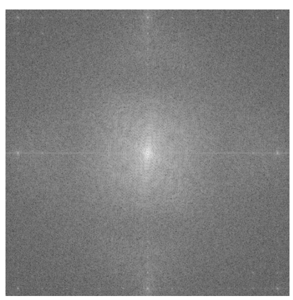
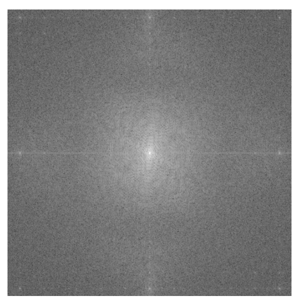
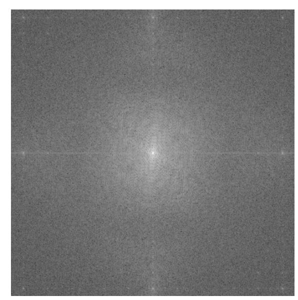
	gammaL = 0.0	gammaL = 0.5	gammaL = 1.0
gammaH = 1.0			
gammaH = 1.5			
gammaH = 2.0			

Table D. Applied HFE Filters to Girl Image

The resulting images from a reverse fourier transform of that transformation are shown in Table E.

	gammaL = 0.0	gammaL = 0.5	gammaL = 1.0

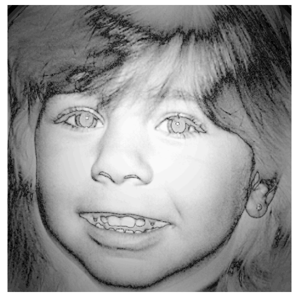
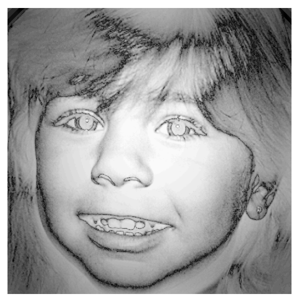
gammaH = 1.0			
gammaH = 1.5			
gammaH = 2.0			

Table E. Spatial Domain Representation of Transformed Images

The results demonstrate that having a large enough delta between gamma low and gamma high is important to provide sufficient contrast between illumination suppression and reflectance enhancement, and having a large enough gamma low is important as well to allow for the preservation of baseline image information without the complete elimination of low frequency content. The results show that a gamma low of 1.0 and gamma high between 1.0 and 1.5 produces the most visually balanced results. This aligns with theory because gamma low attenuates the slowly varying illumination component that causes the uneven brightness, while gamma high > 1 amplifies the reflectance component containing edge and texture details. When gamma low is too low, low frequency information is eliminated, resulting in an edge map appearance. When there is not enough difference between the two parameters, there is not enough contrast.

Image Restoration

Image restoration was explored through the motion blur transformation and gaussian noise. The transformation was sampled in the frequency domain and the application of that transformation to the given image is shown in Table F.

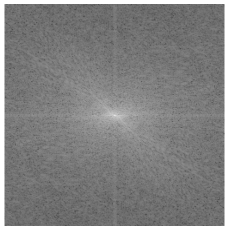
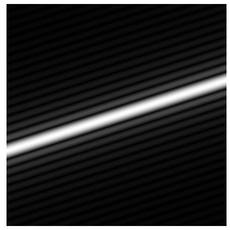
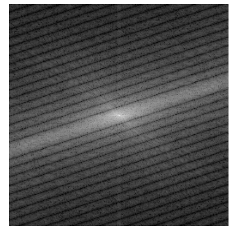
Original Image	Frequency Representation	Motion Blur Transformation	Blurred Frequency Representation	Blurred Image
				

Table F. Application of Motion Blur to Image

Then, varying degrees of gaussian noise were added to the blurred image in the frequency domain through the box muller transformation. The results are shown in Table G.

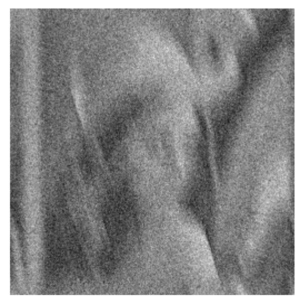
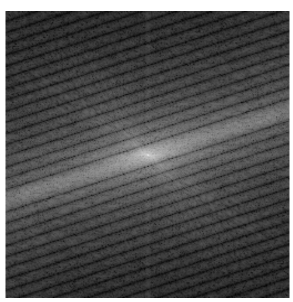
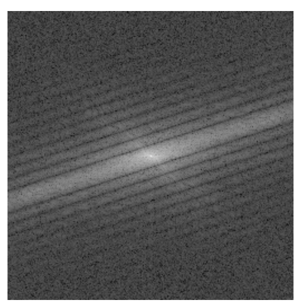
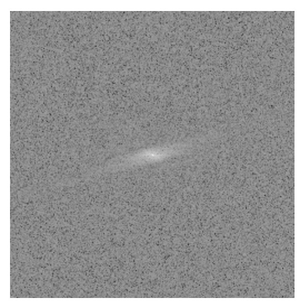
	Noise = (0,1)	Noise = (0,10)	Noise = (0,100)
Lenna (Spatial)			
Lenna (Frequency Magnitude)			

Table G. Addition of Noise to Motion Blurred Image.

Then, Inverse Filtering's ability to undo the degradation was explored. The results using various frequency radius cutoffs on the different noised images are shown in Table F.

	Noise = (0,1)	Noise = (0,10)	Noise = (0,100)

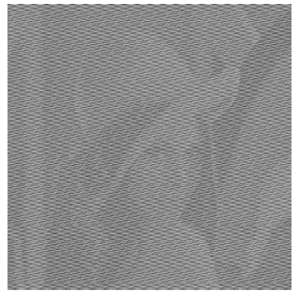
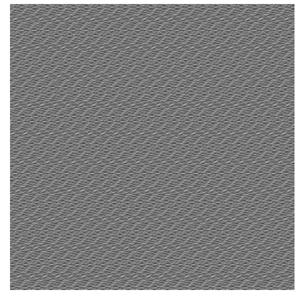
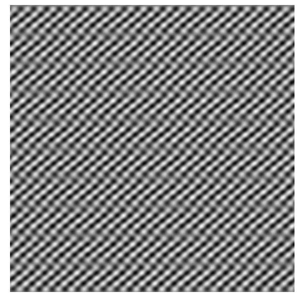
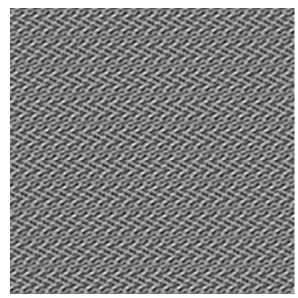
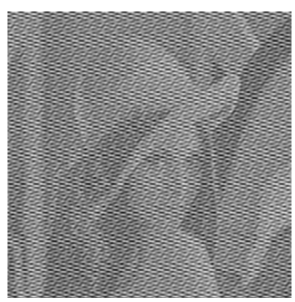
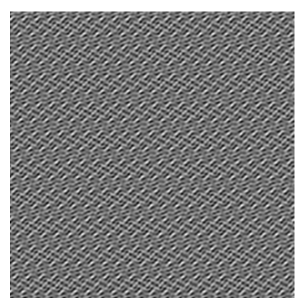
‘Infinite’ Radius			
Radius = 40			
Radius = 70			
Radius = 85			

Table H. Inverse Filtering Results

Inverse filtering seems to work well when noise is negligible, with large radiuses looking very close to the original image. But when noise gets added to the picture, the restored images begin to lose a lot of quality. In these cases, smaller radiuses seem to display image features better while large radiuses let in more noise. In the extreme noise case, the image is not recollectable. This makes sense because inverse filtering divides by H(u, v), and at high frequencies where H approaches zero (at the zeros of the sinc function), any noise present grows towards infinity. The radius limitation acts as a low pass filter to accept some blur in exchange for reduced noise. With minimal noise (sigma=1), the inverse filter successfully recovered the image because the noise was too small to amplify. As the noise increased, the amplified noise got too overbearing and reduced the restoration process' success.

Then, Wiener Filtering was explored which should be better due to its consideration of noise in its restoration process. The results with varying K values are shown below in Table G.

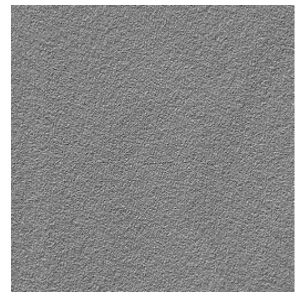
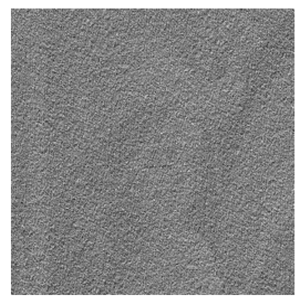
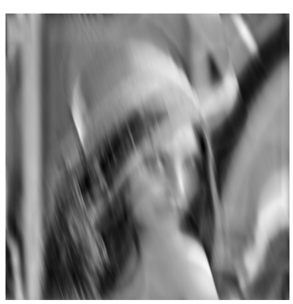
	Noise = (0,1)	Noise = (0,10)	Noise = (0,100)
K = 0.0001			
K = 0.001			
K = 0.01			
K = 0.1			

Table I. Wiener Filtering Results

It can be seen that smaller K values work best until noise becomes an issue, where larger Ks are able to retain more of the image by ignoring more of the noise. This makes sense because the K parameter in the Wiener filter acts as a regularization term that prevents division by small values of $|H|^2$. When K is small relative to the $|H|^2$ term, the filter approximates inverse filtering and recovers fine details. When noise is significant, a larger K prevents the noise amplification that ruins inverse filtering by ignoring frequencies where the signal-to-noise ratio is poor. The optimal K then depends on the actual signal-to-noise ratio. Underestimating K leads to more noise amplification, while overestimating K results in too much smoothing.

Conclusion

This assignment explored advanced frequency domain techniques for image enhancement and restoration.

The frequency filtering experiments confirmed that periodic noise can be effectively isolated and removed using band reject and notch filters. The ability to target specific frequency components while preserving others represents a significant advantage over spatial domain filtering, which cannot achieve selective frequency manipulation.

The comparison between band reject and notch filters were not strongly illustrated in the results of this experiment, but typically notch filters provide more targeted removal but require accurate identification of noise frequencies.

The convolution theorem validation demonstrated the mathematical equivalence between spatial convolution and frequency multiplication. Beyond theoretical interest, this equivalence has practical implications for computational efficiency and provides insight into how spatial operations affect frequency content. The successful matching of spatial and frequency domain results also validated the implementation details required for odd-symmetric filters.

Homomorphic filtering proved effective for images with non-uniform illumination, successfully separating and independently processing the illumination and reflectance components. The parameter exploration revealed that moderate values of γL (1.0) and γH (1.0-1.5) produce the most visually pleasing results, balancing illumination normalization against detail preservation.

The image restoration experiments revealed the fundamental challenges of recovering images from degraded observations. Inverse filtering, while theoretically optimal for noise-free cases, proved extremely sensitive to even small amounts of noise. The Wiener filter's incorporation of noise statistics (signal-to-noise) provided substantially better results.

These results show that real-world image restoration requires a strong understanding of the degradation process and careful consideration of the noise characteristics for balancing restoration quality.

Together, these experiments demonstrate that frequency domain analysis provides powerful tools for image processing, but effective application requires understanding both the theoretical foundations and practical limitations of each technique. Implementing these methods helped solidify concepts that would otherwise remain abstract..