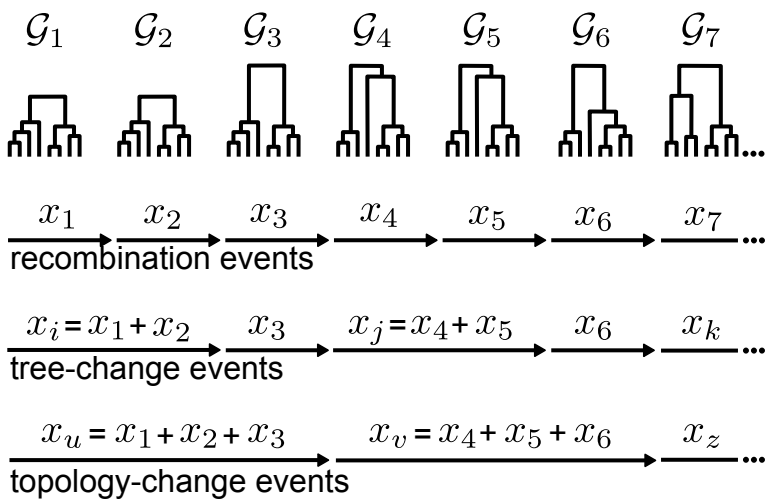
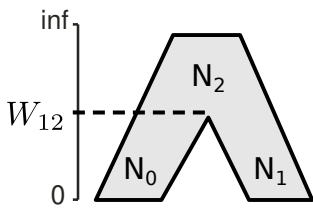


## (A) Proposed ARG



## (B) Parameterized MSC model ( $\mathcal{S}$ )



## (C) Extract genealogies between each event type

$$\begin{aligned}
 G_r &= [\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, \dots] && \text{(recombination events)} \\
 G_g &= [\mathcal{G}_1, \mathcal{G}_3, \mathcal{G}_4, \dots] && \text{(tree-change events)} \\
 G_t &= [\mathcal{G}_1, \mathcal{G}_4, \mathcal{G}_7, \dots] && \text{(topology-change events)}
 \end{aligned}$$

## (D) Extract waiting distances for each event type

$$\begin{aligned}
 X_r &= [x_1, x_2, x_3, \dots] && \text{(recombination events)} \\
 X_g &= [x_i, x_3, x_j, \dots] && \text{(tree-change events)} \\
 X_t &= [x_u, x_v, x_z, \dots] && \text{(topology-change events)}
 \end{aligned}$$

## (E) Calculate MS-SMC rate parameters ( $\lambda$ ) for each event type given $\mathcal{S}$ , $\mathcal{G}$ , and recomb rate

$$\begin{aligned}
 \Lambda_r &= [\lambda_{r1}, \lambda_{r2}, \lambda_{r3}, \dots] && \text{(recombination events)} \\
 \Lambda_g &= [\lambda_{g1}, \lambda_{g3}, \lambda_{g4}, \dots] && \text{(tree-change events)} \\
 \Lambda_t &= [\lambda_{t1}, \lambda_{t4}, \lambda_{t7}, \dots] && \text{(topology-change events)}
 \end{aligned}$$

## (F) Calculate likelihood of waiting distances

$$\begin{aligned}
 \mathcal{L}(X_r | \Lambda_r) &&& \text{(recombination events)} \\
 \mathcal{L}(X_g | \Lambda_g) &&& \text{(tree-change events)} \\
 \mathcal{L}(X_t | \Lambda_t) &&& \text{(topology-change events)} \\
 \mathcal{L}(X_g | \Lambda_g) * \mathcal{L}(X_t | \Lambda_t) &&& \text{(tree \& topo-change events)}
 \end{aligned}$$