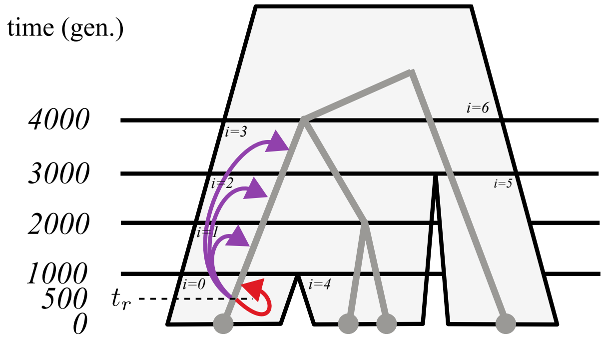
# Example: Calculating the probability of no-change




The probability of a no-change event given recombination on a specific branch at a specific time:

$$\mathbb{P}(\text{no-change}|\mathcal{S},\mathcal{G},b,t_r) = \frac{1}{k_i} + f(i,i)\exp\left\{\frac{k_i}{2n_i}t_r\right\} + \sum_{j\in\mathcal{J}_b(i)} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\}$$

In this example, recombination occurs on branch $b$ within interval 0 ($i$=0) at time $t_r$=500. Let's also assume, for this example that $N_e$=1000. Let's start by solving the constant ($\exp\left\{\frac{k_i}{2n_i}t_r\right\}$) which we can then assign to the variable X to make the equation much simpler.

$$\mathrm{X} = \exp\left\{\frac{k_i}{2n_i}t_r\right\} = \exp\left\{\frac{1}{2(1000)}500\right\} = 1.284$$

$$\mathbb{P}(\text{no-change}|\mathcal{S},\mathcal{G},b,t_r) = \frac{1}{k_i} + f(i,i)\,\mathrm{X} + \sum_{j\in\mathcal{J}_b(i)} f(i,j)\,\mathrm{X}$$

Next, we can substitute 0 for $i$, and define the set $J$ of intervals on $b$ above i as {1, 2, 3}. Also, we can solve $1/k_i$ which here is just 1. This gives the following:

$$\mathbb{P}(\text{no-change}|\mathcal{S},\mathcal{G},b,t_r) = 1 + f(0,0)\mathrm{X} + \sum_{j\in\{1,2,3\}} f(0,j)\,\mathrm{X}$$

All that is left to do is to expand the piecewise constant function *f(i,j)* for each interval and solve:

Equation: $f(i,i) = -\frac{1}{k_i}\exp\left\{-\frac{k_i}{2n_i}\mu_i\right\}$

With data: $f(0,0) = -\frac{1}{1}\exp\left\{-\frac{1}{2(1000)}1000\right\} = -0.6065$

Equation: $f(i,j) = \frac{1}{k_j}\left(1-\exp\left\{-\frac{k_j}{2n_j}d_j\right\}\right)\exp\left\{-\frac{k_i}{2n_i}\mu_i - \sum_{q\in\mathcal{Q}_b(i,j)}\frac{k_q}{2n_q}d_q\right\}$

With data: $f(0,1) = \frac{1}{3}\left(1-\exp\left\{-\frac{3}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000\right\} = 0.1571$

$$f(0,2) = \frac{1}{2}\left(1-\exp\left\{-\frac{2}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000-\left(\frac{3}{2(1000)}1000\right)\right\} = 0.0428$$

$$f(0,3) = \frac{1}{3}\left(1-\exp\left\{-\frac{3}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000-\left(\frac{3}{2(1000)}1000+\frac{2}{2(1000)}1000\right)\right\} = 0.0129$$

Finally, we sum the components to get the final result (colored to correspond with the figure above):

$$\begin{aligned}\mathbb{P}(\text{no-change}|\mathcal{S},\mathcal{G},b,t_r) &= 1 + f(0,0)\,\mathrm{X} + \sum_{j\in\{1,2,3\}} f(0,j)\,\mathrm{X} \\ &= 1 + (-0.6065\times 1.284) + (0.1571\times 1.284) + (0.0428\times 1.284) + (0.0129\times 1.284) \\ &= 0.4944\end{aligned}$$