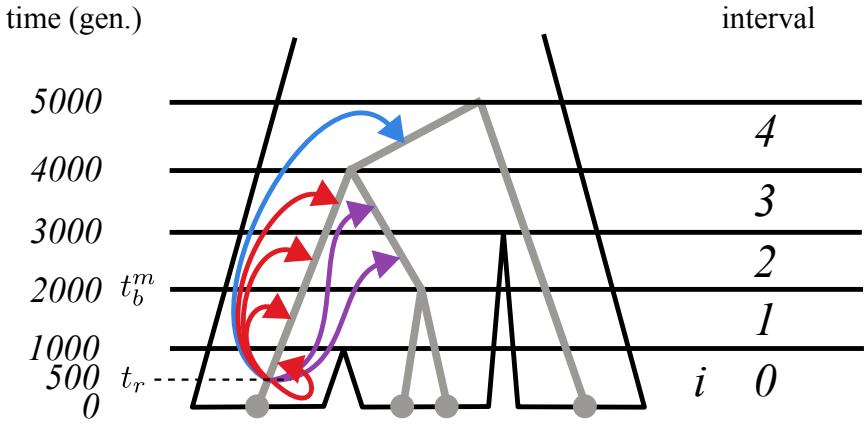
Example: Calculating the probability of topology-unchanged



The probability of topology-unchanged given the branch and timing of recombination:

$$\mathbb{P}(\text{topology-unchanged}|\mathcal{S},\mathcal{G},b,t_r) =$$

$$\begin{cases} \frac{1}{k_i} + \sum_{j\in\mathcal{I}_{bc}} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\} + \sum_{j\in\mathcal{M}_b} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\}, & \text{if } t_r < t_b^m \\ 2\left(\frac{1}{k_i} + \sum_{j\in\mathcal{I}_b} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\}\right) + \sum_{j\in\mathcal{I}_c} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\}, & \text{if } t_r \geq t_b^m \end{cases}$$

In this example, recombination occurs on branch *b* in interval 0 (*i*=0) at time t_r=500, and we will assume all N_e=1000. Because $t_r < t_b^m$, we apply the first case above:

$$\mathbb{P}(\text{topology-unchanged}|\mathcal{S},\mathcal{G},b,t_r) = \frac{1}{k_i} + \sum_{j\in\mathcal{I}_{bc}} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\} + \sum_{j\in\mathcal{M}_b} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\}$$

$$= \frac{1}{1} + \sum_{j\in\{0,1,2,3,4\}} f(i,j)\exp\left\{\frac{1}{2(1000)}500\right\} + \sum_{j\in\{2,3\}} f(i,j)\exp\left\{\frac{1}{2(1000)}500\right\}$$

Expand piece-wise constant functions *f(i,j)* for each interval over branches *b* and *c*:

$$f(0,0) = -\frac{1}{1}\exp\left\{-\frac{1}{2(1000)}1000\right\}$$

$$f(0,1) = \frac{1}{3}\left(1-\exp\left\{-\frac{3}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000\right\}$$

$$f(0,2) = \frac{1}{2}\left(1-\exp\left\{-\frac{2}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000-\left(\frac{3}{2(1000)}1000\right)\right\}$$

$$f(0,3) = \frac{1}{3}\left(1-\exp\left\{-\frac{3}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000-\left(\frac{3}{2(1000)}1000+\frac{2}{2(1000)}1000\right)\right\}$$

$$f(0,4) = \frac{1}{2}\left(1-\exp\left\{-\frac{2}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000-\left(\frac{3}{2(1000)}1000+\frac{2}{2(1000)}1000+\frac{3}{2(1000)}1000\right)\right\}$$

Yields a final result (colored to correspond with the figure above):

$$= 1+(-0.6065\times1.284)+(0.1571\times1.284)+(0.0428\times1.284)+(0.0129\times1.284)+(0.0035\times1.284)+(0.0428\times1.284)+(0.0129\times1.284)$$

$$= 0.5703$$