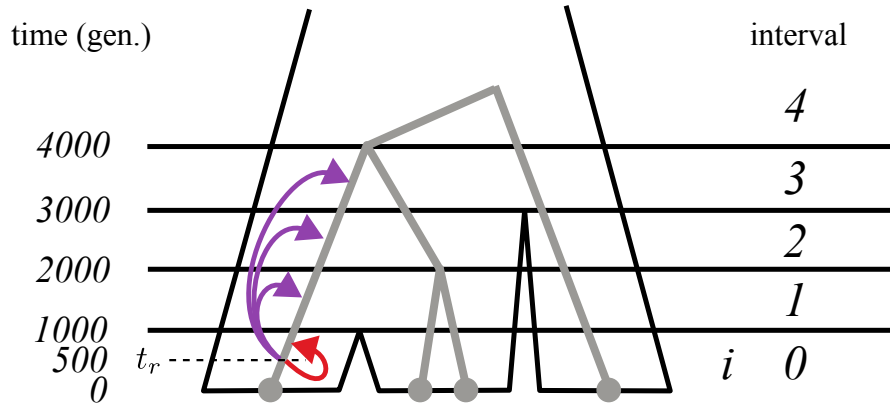


## Example: Calculating the probability of no-change



The probability of no-change given recombination on a given branch at a given time:

$$\mathbb{P}(\text{no-change}|\mathcal{S}, \mathcal{G}, b, t_r) = \frac{1}{k_i} + f(i, i) \exp \left\{ \frac{k_i}{2n_i} t_r \right\} + \sum_{j \in \mathcal{J}_b(i)} f(i, j) \exp \left\{ \frac{k_i}{2n_i} t_r \right\}$$

In this example, recombination occurs on branch  $b$  in interval 0 ( $i=0$ ) at time  $t_r=500$ , and we will assume all  $N_e=1000$ . We can plug these values into the equation:

$$\mathbb{P}(\text{no-change}|\mathcal{S}, \mathcal{G}, b, t_r) = \frac{1}{1} + f(0, 0) \exp \left\{ \frac{1}{2(1000)} 500 \right\} + \sum_{j \in \{1, 2, 3\}} f(0, j) \exp \left\{ \frac{1}{2(1000)} 500 \right\}$$

Then expand the piecewise constant functions  $f(i, j)$  for each interval on  $b$ :

$$f(i, i) = -\frac{1}{k_i} \exp \left\{ -\frac{k_i}{2n_i} \mu_i \right\}$$

$$f(0, 0) = -\frac{1}{1} \exp \left\{ -\frac{1}{2(1000)} 500 \right\}$$

$$f(i, j) = \frac{1}{k_j} \left( 1 - \exp \left\{ -\frac{k_j}{2n_j} d_j \right\} \right) \exp \left\{ -\frac{k_i}{2n_i} \mu_i - \sum_{q \in \mathcal{Q}_b(i, j)} \frac{k_q}{2n_q} d_q \right\}$$

$$f(0, 1) = \frac{1}{3} \left( 1 - \exp \left\{ -\frac{3}{2(1000)} 1000 \right\} \right) \exp \left\{ -\frac{1}{2(1000)} 1000 \right\}$$

$$f(0, 2) = \frac{1}{2} \left( 1 - \exp \left\{ -\frac{2}{2(1000)} 1000 \right\} \right) \exp \left\{ -\frac{1}{2(1000)} 1000 - \left( \frac{3}{2(1000)} 1000 \right) \right\}$$

$$f(0, 3) = \frac{1}{3} \left( 1 - \exp \left\{ -\frac{3}{2(1000)} 1000 \right\} \right) \exp \left\{ -\frac{1}{2(1000)} 1000 - \left( \frac{3}{2(1000)} 1000 + \frac{2}{2(1000)} 1000 \right) \right\}$$

And sum to get final result: (colored to correspond with the figure above):

$$\begin{aligned} \mathbb{P}(\text{no-change}|\mathcal{S}, \mathcal{G}, b, t_r) &= 1 + f(0, 0) \times 1.284 + \sum_{j \in \{1, 2, 3\}} f(0, j) \times 1.284 \\ &= 1 + (-0.6065 \times 1.284) + (0.1571 \times 1.284) + (0.0428 \times 1.284) + (0.0129 \times 1.284) \\ &= 0.4944 \end{aligned}$$