



The probability the topology is unchanged given recombination on a given branch at a given timee:

$$\mathbb{P}(\text{topology-unchanged}|\mathcal{S}, \mathcal{G}, b, t_r) = \begin{cases} \frac{1}{a_i} + \sum_{\substack{j \in \mathcal{I}_{bc} \\ \text{red}}} f(i, j) \exp \left\{ \frac{a_i}{2n_i} t_r \right\} + \sum_{\substack{j \in \mathcal{M}_b \\ \text{purple}}} f(i, j) \exp \left\{ \frac{a_i}{2n_i} t_r \right\}, & \text{if } t_r < t_b^m \\ 2 \left( \frac{1}{a_i} + \sum_{\substack{j \in \mathcal{I}_b \\ \text{red}}} f(i, j) \exp \left\{ \frac{a_i}{2n_i} t_r \right\} \right) + \sum_{\substack{j \in \mathcal{I}_c \\ \text{blue}}} f(i, j) \exp \left\{ \frac{a_i}{2n_i} t_r \right\}, & \text{if } t_r \geq t_b^m \end{cases}$$

In this example, recombination occurs on branch  $b$  in interval 0 ( $i=0$ ) at time  $t_r=500$ , and we will assume all  $N_e=1000$ . Because  $t_r < t_b^m$ , we apply the first case above:

$$\begin{aligned} \mathbb{P}(\text{topology-unchanged}|\mathcal{S}, \mathcal{G}, b, t_r) &= \frac{1}{a_i} + \sum_{j \in \mathcal{I}_{bc}} f(i, j) \exp \left\{ \frac{a_i}{2n_i} t_r \right\} + \sum_{j \in \mathcal{M}_b} f(i, j) \exp \left\{ \frac{a_i}{2n_i} t_r \right\} \\ &= \frac{1}{1} + \sum_{j \in \{0,1,2,3,4\}} f(i, j) \exp \left\{ \frac{1}{2(1000)} 500 \right\} + \sum_{j \in \{2,3\}} f(i, j) \exp \left\{ \frac{1}{2(1000)} 500 \right\} \end{aligned}$$

Expand piece-wise constant functions  $f(i, j)$  for each interval over branches  $b$  and  $c$ :

$$\begin{aligned} f(i, j) &= \frac{1}{a_j} \left( 1 - \exp \left\{ -\frac{a_j}{2n_j} d_j \right\} \right) \exp \left\{ -\frac{a_i}{2n_i} \mu_i - \sum_{q \in \mathcal{Q}_b} \frac{a_q}{2n_q} d_q \right\} \\ f(0, 0) &= -\frac{1}{1} \exp \left\{ -\frac{1}{2(1000)} 1000 \right\} \\ f(0, 1) &= \frac{1}{3} \left( 1 - \exp \left\{ -\frac{3}{2(1000)} 1000 \right\} \right) \exp \left\{ -\frac{1}{2(1000)} 1000 \right\} \\ f(0, 2) &= \frac{1}{2} \left( 1 - \exp \left\{ -\frac{2}{2(1000)} 1000 \right\} \right) \exp \left\{ -\frac{1}{2(1000)} 1000 - \left( \frac{3}{2(1000)} 1000 \right) \right\} \\ f(0, 3) &= \frac{1}{3} \left( 1 - \exp \left\{ -\frac{3}{2(1000)} 1000 \right\} \right) \exp \left\{ -\frac{1}{2(1000)} 1000 - \left( \frac{3}{2(1000)} 1000 + \frac{2}{2(1000)} 1000 \right) \right\} \\ f(0, 4) &= \frac{1}{2} \left( 1 - \exp \left\{ -\frac{2}{2(1000)} 1000 \right\} \right) \exp \left\{ -\frac{1}{2(1000)} 1000 - \left( \frac{3}{2(1000)} 1000 + \frac{2}{2(1000)} 1000 + \frac{3}{2(1000)} 1000 \right) \right\} \end{aligned}$$

Yields a final result (colored to correspond with the figure above):

$$\begin{aligned} &= 1 + (-0.6065 \times 1.284) + (0.1571 \times 1.284) + (0.0428 \times 1.284) + (0.0129 \times 1.284) + (0.0035 \times 1.284) + (0.0428 \times 1.284) + (0.0129 \times 1.284) \\ &= 0.5703 \end{aligned}$$