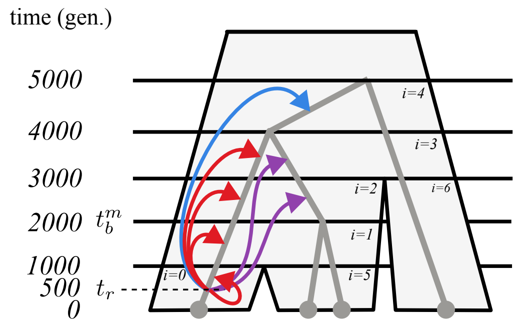
Example: Calculating the probability of topology-unchanged



The probability of topology-unchanged given the branch and timing of recombination:

$$\mathbb{P}(\text{topology-unchanged}|\mathcal{S},\mathcal{G},b,t_r) = \begin{cases} \frac{1}{k_i} + \sum_{j\in\mathcal{I}_{bc}} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\} + \sum_{j\in\mathcal{M}_b} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\}, & \text{if } t_r < t_b^m \\ 2\left(\frac{1}{k_i} + \sum_{j\in\mathcal{I}_b} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\}\right) + \sum_{j\in\mathcal{I}_c} f(i,j)\exp\left\{\frac{k_i}{2n_i}t_r\right\}, & \text{if } t_r \geq t_b^m \end{cases}$$

Because $t_r < t_b^m$, we will apply the first case. In this example, recombination occurs on branch *b* in interval 0 (*i*=0) at time t_r=500, and we will assume all N_e=1000. Let's start by solving the constant ($\exp\left\{\frac{k_i}{2n_i}t_r\right\}$) which we can then assign to the variable X to make the equation much simpler:

$$\text{X} = \exp\left\{\frac{k_i}{2n_i}t_r\right\} = \exp\left\{\frac{1}{2(1000)}500\right\} = 1.284$$

$$\mathbb{P}(\text{topology-unchanged}|\mathcal{S},\mathcal{G},b,t_r) = \frac{1}{k_i} + \sum_{j\in\mathcal{I}_{bc}} f(i,j)\,\mathbf{X} + \sum_{j\in\mathcal{M}_b} f(i,j)\,\mathbf{X}$$

Next, we substitute 0 for *i*, and define the set of intervals (I_{bc}) over branch *b* and its parent as {0, 1, 2, 3, 4}, and define the set of intervals shared by *b* and its sister (M_b) as {2, 3}. Also, we can solve $1/k_i$ which here is just 1.

$$\mathbb{P}(\text{topology-unchanged}|\mathcal{S},\mathcal{G},b,t_r) = 1 + \sum_{j\in\{0,1,2,3,4\}} f(i,j)\,\mathbf{X} + \sum_{j\in\{2,3\}} f(i,j)\,\mathbf{X}$$

All that is left to do is to expand the piecewise constant function *f(i,j)* for each interval and solve:

Equation: $f(i,i) = -\frac{1}{k_i}\exp\left\{-\frac{k_i}{2n_i}\mu_i\right\}$

With data: $f(0,0) = -\frac{1}{1}\exp\left\{-\frac{1}{2(1000)}1000\right\} = -0.6065$

Equation: $f(i,j) = \frac{1}{k_j}\left(1-\exp\left\{-\frac{k_j}{2n_j}d_j\right\}\right)\exp\left\{-\frac{k_i}{2n_i}\mu_i - \sum_{q\in\mathcal{Q}_b(i,j)}\frac{k_q}{2n_q}d_q\right\}$

With data: $f(0,1) = \frac{1}{3}\left(1-\exp\left\{-\frac{3}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000\right\} = 0.1571$

$f(0,2) = \frac{1}{2}\left(1-\exp\left\{-\frac{2}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000 - \left(\frac{3}{2(1000)}1000\right)\right\} = 0.0428$

$f(0,3) = \frac{1}{3}\left(1-\exp\left\{-\frac{3}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000 - \left(\frac{3}{2(1000)}1000 + \frac{2}{2(1000)}1000\right)\right\} = 0.0129$

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$f(0,4) = \frac{1}{2}\left(1-\exp\left\{-\frac{2}{2(1000)}1000\right\}\right)\exp\left\{-\frac{1}{2(1000)}1000 - \left(\frac{3}{2(1000)}1000 + \frac{2}{2(1000)}1000 + \frac{3}{2(1000)}1000\right)\right\} = 0.0035$

Finally, we sum the components to get the final result (colored to correspond with the figure above):

$= 1 + (-0.6065\times 1.284) + (0.1571\times 1.284) + (0.0428\times 1.284) + (0.0129\times 1.284) + (0.0035\times 1.284) + (0.0428\times 1.284) + (0.0129\times 1.284)$

$= 0.5703$