Improvement in the Roche-Wainer-Thissen Stature Prediction Model: A Comparative Study

H.J. KHAMIS AND SHUMEI GUO
Department of Mathematics and Statistics and Department of Community Health, School of Medicine, Wright State University, Dayton, Ohio 45435

ABSTRACT The Roche-Wainer-Thissen (RWT) prediction model, developed in 1975, predicts the adult stature of a child based on age, current stature, current weight, current skeletal age, and the average stature of the parents. Generally, the method has worked well; however, there are certain steps in the procedure that can be improved. Seven variations of the current version of the RWT prediction model are investigated and compared in terms of the accuracy and reliability of prediction, culminating in a recommendation for the prediction of adult stature in Caucasian Americans. The recommended method, called multivariate cubic spline smoothing [$MCS^2(1)$], uses cubic splines in the smoothing part of the RWT prediction model, resulting in a simpler (i.e., fewer steps) method with smaller maximum deviations between predicted and actual adult statures than the current multivariate semi-metric smoothing (MS^2) method.

Accurate prediction of adult stature is an important concern among pediatricians and others. Near the age of puberty, variation from the mean in the statures of many short or tall children becomes more marked, leading to possible adverse psychological effects and concern on the part of children and their parents. There are correlations between shortness in children and hyperactivity, poor concentration, and low attainment in reading; however, the underachievement observed in short children is largely due to the association between social disadvantage and short stature (Voss et al., 1991). The medical and psychological management of children with unusual growth can be enhanced with the reliable prediction of adult stature. For instance, if a reliable prediction can be made, clinicians could reassure many children that their adult statures will be within normal ranges without therapeutic intervention. Reliable prediction of adult stature is an important part of therapy that includes, for example, human growth hormone or anabolic steroids. It is also important for studying the effects of surgery for congenital heart disease, the surgical management of anisomelic children, and improvement in the level of nutrition or reduction in the incidence of disease through intervention programs (see Roche et al., 1975). The Roche-Wainer-Thissen (RWT) prediction model (Roche et al., 1975) has provided a basis for therapeutic selection, regulation of therapeutic agent dosage, and estimates of therapeutic effects on the potential for growth in stature of individuals.

Roche (1980) provides a discussion of comparisons between prediction methods. The RWT method is more accurate than the Bayley-Pinneau (BP, 1952) method when applied to children in either the Fels, Denver, or Harvard growth studies for ages 1–14 years. Also, the RWT procedure is continuous in age while BP is not. Zachman et al. (1978) report that in normal children the standard deviation of the errors tends to be smaller for the RWT method (less than 2 cm at all ages) than for the BP or Tanner-Whitehouse (TW, Tanner et al., 1975) methods except at 16 years in males and 13–14 years in females when they are smaller for the TW method. Lenko and others (Kantero and Lenko, 1976; Lenko, 1979) found that in males, mean errors are largest for BP and smallest for RWT except after 14 years when

Received July 24, 1992; accepted June 24, 1993.

Address reprint requests to H.J. Khamis, Division of Human Biology, Wright State University, 1005 Xenia Ave., Yellow Springs, OH 45387-1695.

the order is reversed. The TW method markedly underpredicted for males from 13–16 years of age.

The RWT and TW methods are satisfactory in endocrinopathies in which appropriate treatment restores normal growth, but they are grossly inaccurate in conditions such as Turner syndrome and precocious puberty. All three methods, RWT, TW, and BP, are reasonably accurate in children with familial tall stature, but Roche and Wettenhall (1969) report that the RWT method is more accurate than the BP method when applied to 28 short males.

More recently, Bramswig et al. (1990) compared stature predictions calculated by the BP, RWT, target height (Tanner, 1962), and TW Mark I and Mark II (Tanner et al., 1975, 1983) methods with final adult stature in 37 males and 32 females who, as children, had short stature and constitutional delay of puberty. Males and females were first seen at average chronological ages of 14.8 years and 12.9 years, respectively. For males, the RWT method gave very accurate results, underestimating adult stature by (mean ± standard deviation) -0.53 ± 4.37 cm. The prediction errors for the other methods were -7.3 ± 4.65 cm (TW Mark II), 3.1 ± 5.49 cm (BP), and 1.7 ± 5.69 cm (target height). For females, the estimation of adult stature was not clearly superior by any method. Another measure of accuracy is the absolute error of estimate, which is independent of each method's tendency to overestimate or underestimate adult stature. The mean and standard deviation of the absolute error of estimate for males was 3.4 ± 2.8 cm with the RWT method, significantly less than for the other prediction methods. For females, the mean absolute errors ranged from 2.9 ± 1.8 cm (TW Mark I) to 3.2 ± 2.9 cm (RWT), where the differences among the prediction methods were not statistically significant (Bramswig et al., 1990).

Generally, the RWT procedure is seen as a superior method for prediction of adult stature (i.e., stature at age 18 years), especially over the age range for which it was developed. Briefly, the procedure calls for 1) multiple linear regressions at each target age, regressing adult stature on four predictor variables (see below), 2) orthonormalization of the resulting least squares estimates of the coefficients, 3) smoothing of the estimates over target age (TA) in the orthonormalized space, and 4) transforming the smoothed estimates back to the original metric.

The RWT method was designed to help overcome certain difficulties in the interpolation of regression coefficients with respect to TA and in prediction. These difficulties can be summarized as follows.

1. Multicolinearity is present in the multiple regression model; in fact, stature and weight have correlation coefficients between 0.66 and 0.80 from ages 3–12 years for both males and females.
2. The sample sizes across TA range from 67–182 for males and from 40–160 for females, yielding wide variation in the accuracy and reliability of estimated regression coefficients.
3. Simple least squares estimates do not necessarily provide resistant predictor estimators (i.e., estimators that lead to accurate and reliable predictions in populations that are different from, but comparable to, the population that was sampled).
4. Regression coefficients for the same variables at adjacent ages should be similar, indicating that some smoothing procedure may be needed.

A special case of the RWT method, called multivariate semi-metric smoothing (MS^2), was developed by Wainer and Thissen (1975). The MS^2 technique can be summarized as a six-step procedure as follows.

A. Multiple linear regressions

Perform multiple linear regressions of adult stature on present stature (STAT), present weight (WT), average stature of parents (midparent stature, MPS), and median skeletal age of the hand-wrist (SA) for each TA.

B. Orthonormalization

Use Cholesky decomposition to orthonormalize the four vectors of coefficients for STAT, WT, MPS, and SA. This is equivalent to performing the Gram-Schmidt procedure on the corresponding matrix of coefficients, where there are 30 target ages (3.0–17.5 [0.5]). Specifically, let F be the 30×4 matrix consisting of the four vectors of coefficients,

$$F = [x_1, x_2, x_3, x_4].$$

Then $A = F'F$ is a symmetric, positive-definite matrix with Cholesky decomposition $A = SS'$, where S is the lower-triangular unique Cholesky factor, and $F^* = F(S^{-1})'$ is orthonormal with respect to columns.

C. Smoothing

For each vector of transformed coefficients in the orthonormalized space, smooth across TA using Tukey's (1972) "53H" method.

D. Final smoothing

Fit the 53H smoothed values in the orthonormalized space with a fifth-degree polynomial.

E. Transformation back to the original metric

Use the unique Cholesky factor from part B to transform the coefficients back to the original variables. Namely, if F_s^* represents the orthonormalized smoothed estimates and F_s represents the smoothed estimates in the original metric, then $F_s = F_s^*S'$.

F. Smoothing of the intercept

Obtain the estimated intercept by using the least squares formula for the intercept with the smoothed coefficients replacing the least squares estimates. Conduct a final smoothing of the intercept across TA using a sixth-degree polynomial.

The influence of each step in the MS^2 technique was investigated by Wainer and Thissen (1975). Omitting the orthonormalization step leads to very serious deterioration in prediction among very young females. Omitting the orthonormalization and the 53H smoothing steps leads to a procedure that performs poorly for older males. Omitting all steps of the MS^2 procedure, so that prediction is based exclusively on the least squares estimates of the regression coefficients, leads to a prediction procedure for which interpolation is difficult and in which the median absolute error is about doubled for very early ages and during adolescence relative to MS^2. Generally, it appears that each step of the MS^2 procedure provides an important contribution toward accurate prediction. Improvements can be made, however, within the individual steps of the procedure. The purpose of this study is to compare several variations in the RWT prediction procedure, where MS^2 is treated as one such variation.

VARIATIONS IN THE RWT PREDICTION MODEL

Each of the six steps of the procedure is reviewed separately to identify where improvements might be made.

A. Multiple linear regressions

The four regressor variables were carefully selected from 80 possible predictors, using a variety of statistical methods, as the most appropriate predictors for the criterion, adult stature (Roche et al., 1975). Consequently, no changes are made in this step.

B. Orthonormalization

Because the four predictor variables are intercorrelated, it is necessary to orthonormalize the vectors of coefficients so that the subsequent smoothing can be done relatively independently. No changes are made in this crucial part of the procedure.

C. Smoothing

Because the regression coefficients for the same variables at adjacent ages should be similar, some smoothing procedure is indicated. Furthermore, the procedure should be nonmetric (i.e., should rely on the ordinal nature of the data) to deal with the wide variation across age in the accuracy of estimation of the regression coefficients and because no valid functional model is readily available. A variety of smoothing procedures can be considered. Wainer and Thissen (1975) used Tukey's 53H procedure for smoothing (MS^2), a relatively obscure procedure that is semi-metric in nature. Other procedures that are principally ordinal or nonparametric in nature may lead to improvements in prediction. In recent years, many new techniques for smoothing have been developed, including spline models (Wold, 1974; Smith, 1979), kernel regression methods (Rosenblatt, 1971; Müller, 1980), and exploratory data analysis methods (Tukey, 1977). Care must be used in not overfitting or underfitting the data; overfitting can result in modeling artifactual changes in the behavior of the coefficients that are not biologically meaningful, and hence would lead to a loss of resistance in the predictor estimators, and underfitting would not identify important biological changes in the behavior of the coefficients, and hence would lead to a loss in the accuracy of prediction.

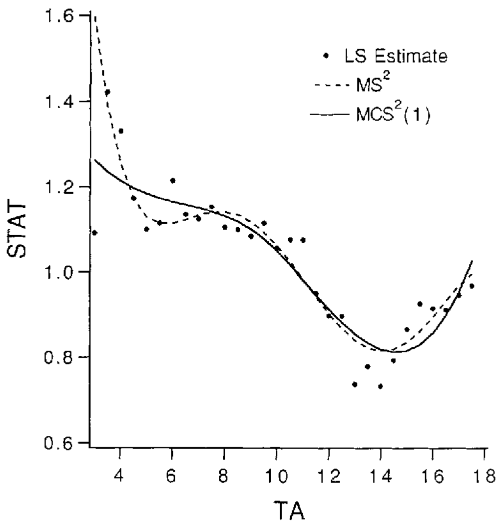

Fig. 1. Plot of STAT coefficients by TA for males.

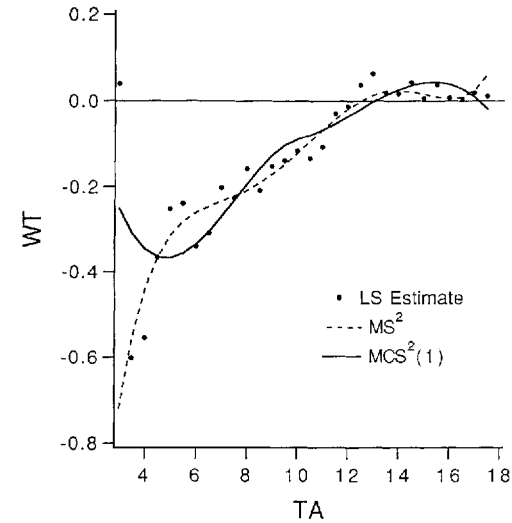

Fig. 2. Plot of WT coefficients by TA for males.

D. Final smoothing

Fitting the smoothed values to a fifth-degree polynomial, as is done in the MS^2 technique, forces the behavior of the coefficient to follow a prescribed functional form that does not have biological justification. Furthermore, the polynomial fitting of the 53H smoothed values may lead to a very poor fit near the endpoints. This is illustrated in Figures 1–8, where the least squares estimates (dots) and MS^2 predicted values (dashed curve) are plotted across TA for each predictor variable for males (Figs. 1–4) and females (Figs. 5–8). The most serious lack of fit for males in the MS^2 smoothing procedure occurs for STAT and WT at TA = 3.0 years, and for MPS at TAs between 3.0 and 5.0 years; for females, note the lack of fit in SA at TAs between 3.0 and 7.0 years and at TA = 17.5 years. In the spirit of step C, a nonparametric technique should be used in the final smoothing process.

E. Transformation back to the original metric

This step was not changed.

F. Smoothing of the intercept

As in part D, the shape of the curve for the intercept should be determined by the data rather than a prescribed polynomial.

Eight variations of the RWT procedure were investigated (including MS^2); in each of the seven new variations, changes were made in parts C, D, and F of the MS^2 technique. These variations were compared with one another and with the least squares estimate of adult stature, which was obtained without smoothing (part A of the procedure only). The seven new variations are:

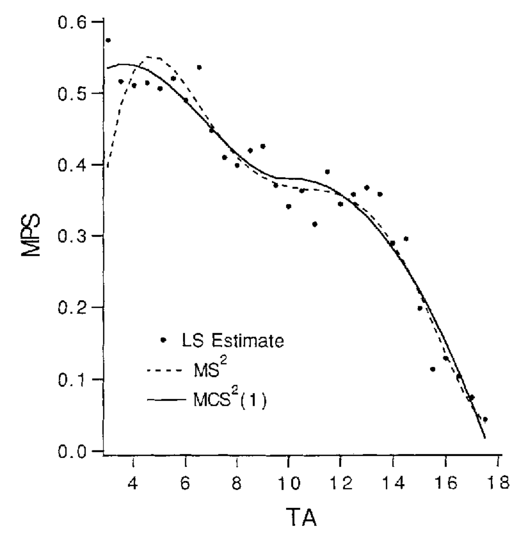

Fig. 3. Plot of MPS coefficients by TA for males.

Multivariate cubic spline smoothing with k knots [$MCS^2(k)$], k = 1, 2, 3, and 4

A cubic spline model with continuous first derivatives and k knots placed uniformly in the range of TA values is used to smooth the

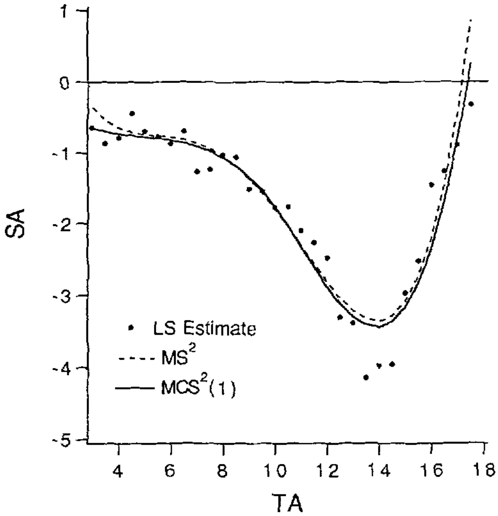

Fig. 4. Plot of SA coefficients by TA for males.

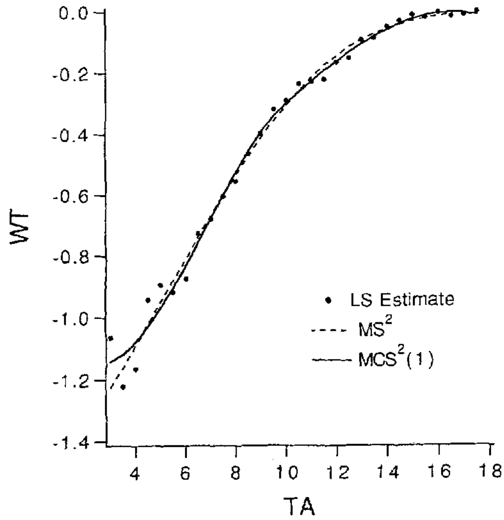

Fig. 6. Plot of WT coefficients by TA for females.

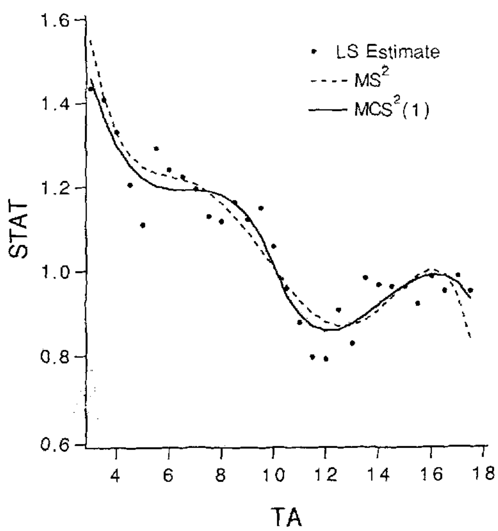

Fig. 5. Plot of STAT coefficients by TA for females.

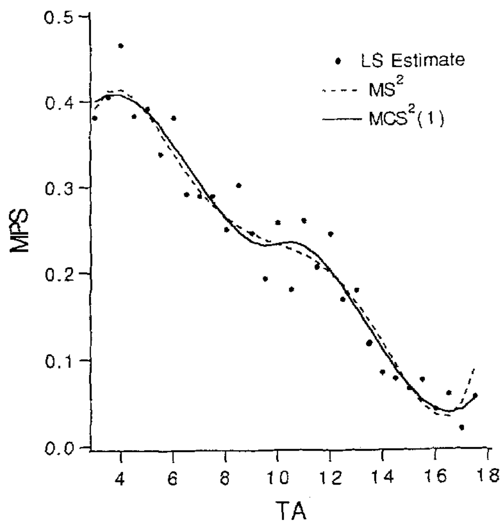

Fig. 7. Plot of MPS coefficients by TA for females.

data in part C; in part F, a four-knot cubic spline is used. Although splines represent smooth curves, they do not suffer the handicap in which behavior in a small region determines behavior everywhere, as occurs with polynomials. Third-degree piecewise polynomials are used because they are of low degree, fairly smooth assuming continuous first derivatives, and yet can incorporate several different trends in the data simply by increasing the number of knots. Furthermore, cubic spline smoothing simplifies the prediction procedure by combining parts C and D. A mathematical description of $MCS^2(k)$ is given in Appendix A.

Multivariate 3R smoothing (M3RS)

This procedure replaces Tukey's 53H smoothing method in part C with Tukey's 3R method (Tukey, 1977, pp. 210–223). Additionally, the smoothing procedures used in

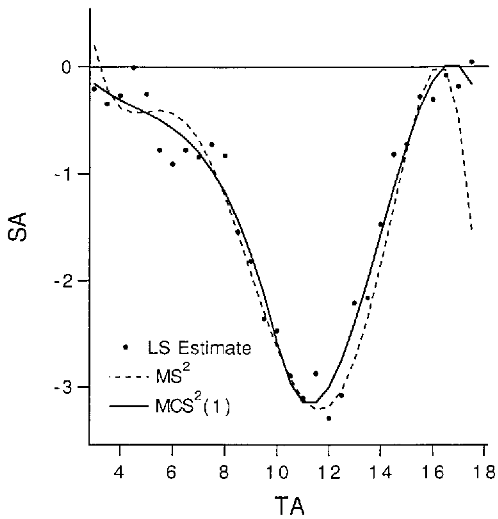

Fig. 8. Plot of SA coefficients by TA for females.

parts D and F are three-knot and four-knot cubic splines, respectively. The 3R method is one of the simplest of Tukey's smoothing procedures, but it has two advantages over the 53H method: it uses only ordinal information (running medians of three), and it has an end-value smoothing component.

Multivariate kernel regression smoothing with bandwidth b [MKRS(b)], b = 3.5, 4.5

Kernel regression, a nonparametric smoothing procedure (see Rosenblatt, 1971; Müller, 1980; Gasser et al., 1984a, b), is used in part C. Part D is omitted and a four-knot cubic spline is used in part F. For the kernel regression, a second-degree polynomial kernel function, as discussed in Guo (1989), is used. In addition, a smoothing parameter called the bandwidth, b, must be selected so that the need for a good curve fit is balanced with the need for a biologically meaningful fit. Bandwidths of 3.5 and 4.5 years are used in the present application. A mathematical description of MKRS(b) is given in Appendix A.

Essentially, these modifications offer alternative smoothing techniques to Tukey's 53H procedure and the high-degree polynomials used in parts C, D, and F of Wainer and Thissen's (1975) MS^2 procedure. The proposed smoothing modifications are more effective, more flexible and, in some cases, easier to implement, thereby leading to a shorter and easier procedure for predicting adult stature accurately and reliably.

TABLE 1. Summary statistics over TA for MAD between observed and predicted adult stature in centimeters

Technique	Mean	Minimum	Maximum
Males			
$MCS^2(4)$	1.91	0.57	2.54
$MCS^2(3)$	1.93	0.47	2.56
$MCS^2(2)$	1.97	0.44	2.55
$MCS^2(1)$	1.97	0.58	2.51
M3RS	1.93	0.47	2.53
MS^2	1.98	0.89	2.67
MKRS(4.5)	1.97	0.51	2.58
MKRS(3.5)	1.94	0.55	2.49
Females			
$MCS^2(4)$	1.64	0.49	2.49
$MCS^2(3)$	1.74	0.39	2.55
$MCS^2(2)$	1.71	0.46	2.57
$MCS^2(1)$	1.69	0.47	2.57
M3RS	1.76	0.33	2.77
MS^2	1.70	0.39	2.51
MKRS(4.5)	1.73	0.43	2.50
MKRS(3.5)	1.69	9.40	2.58

The data used in comparing the eight variations of the RWT method are from the Fels Longitudinal Study, which includes a total of 223 males and 210 females for whom adult stature (cm) at age 18 is recorded. Serial data including STAT, WT, MPS, SA (using FELS hand-wrist skeletal age; Roche et al., 1988), and SEX are recorded for TA values 3–18 (0.5). Excluded from the sample were the few Black participants (N = 15), one randomly chosen member of each monozygous twin pair (N = 4), and participants with serious pathological conditions (N = 14). A demographic description of the study sample is given in Roche et al. (1988).

RESULTS

To assess the effectiveness of the techniques, in terms of accuracy and reliability, four criteria were investigated.

Median absolute deviation

The median of the absolute values of the differences between the observed adult stature and the predicted adult stature, or median absolute deviation (MAD), at each TA is computed to compare the performance of the techniques described. Summary statistics for the 30 TAs (TA = 3.0 − 17.5 [0.5]) are given in Table 1 for all techniques.

For males, the observed mean MAD is slightly smaller for $MCS^2(4)$, $MCS^2(3)$, M3RS, and MKRS(3.5) than for the other techniques; additionally, the observed maximum MAD is larger for MS^2 than for any of the other techniques. For females, the observed mean MAD is smallest for $MCS^2(4)$, and the observed maximum MAD is larger for M3RS than for any of the other techniques. Using TA as a blocking variable, an analysis of variance indicates that there are no significant differences among the three techniques, MS^2, $MCS^2(3)$, and M3RS among males in terms of average MAD ($P = 0.197$). Similarly, there are no significant differences among the techniques MS^2, $MCS^2(1)$, and M3RS for females ($P = 0.241$). Note that among the cubic spline models, $MCS^2(3)$ performs well for males and $MCS^2(1)$ performs well for females in terms of the mean MAD.

The reason that the median of the absolute values of the differences between observed and predicted stature at each TA was taken instead of the arithmetic mean of the differences is that the arithmetic mean does not provide any discrimination among the techniques. In fact, the arithmetic mean of the differences between observed and predicted stature is 0.15 cm for males (underprediction) and −0.42 cm for females (overprediction) for all techniques. Additionally, the accuracy of predictions in individuals as opposed to the group is of interest.

Proximity to the least squares estimates

Another criterion used to assess the effectiveness of the prediction model is a measure of the closeness of the predicted value to the least squares estimate. For each TA, the median absolute difference between the observed and the predicted adult statures is subtracted from the median absolute difference between observed adult statures and the least squares estimate of adult stature:

$$\text{med}(|y - \text{least squares } y|) - \text{med}(|y - \text{predicted } y|).$$

The averages of these differences over the 30 TAs are given in Table 2 for males and females. The observed values for $MCS^2(4)$, $MCS^2(3)$, M3RS, and MKRS(3.5) are almost half those for MS^2 among males. Among females, the observed value for $MCS^2(4)$ is smallest in absolute value and that of M3RS is largest; $MCS^2(2)$, $MCS^2(1)$, MS^2, and MKRS(3.5) are similar to each other.

Root mean square error

For each TA, the square root of the mean square error (RMSE) for the predicted values is computed. Summary statistics over TA are given for all techniques in Table 3. Significant differences among MS^2, $MCS^2(3)$, and M3RS are not apparent in terms of the average RMSE for males ($P = 0.427$). Similarly, no differences are apparent among the techniques MS^2, $MCS^2(1)$, and M3RS for females ($P = 0.160$). It is noted that the maximum RMSE over the 30 TAs is larger for MS^2 than for any of the other techniques among both males (3.86 cm) and females (3.60 cm).

TABLE 2. Average difference over TA between the MAD for the least squares estimate of stature and the MAD for the predicted stature in centimeters: Average of [med (| y − least squares y |) − med(| y − predicted y |)] over TA

Technique	Males	Females
$MCS^2(4)$	−0.05	−0.08
$MCS^2(3)$	−0.07	−0.18
$MCS^2(2)$	−0.11	−0.15
$MCS^2(1)$	−0.11	−0.13
M3RS	−0.07	−0.20
MS^2	−0.12	−0.14
MKRS(4.5)	−0.11	−0.17
MKRS(3.5)	−0.08	−0.14

TABLE 3. Summary statistics over target age for the RMSE of prediction in centimeters

Technique	Mean	Minimum	Maximum
Males			
$MCS^2(4)$	2.84	1.09	3.65
$MCS^2(3)$	2.90	0.87	3.66
$MCS^2(2)$	2.94	0.80	3.75
$MCS^2(1)$	2.89	0.92	3.64
M3RS	2.90	0.86	3.65
MS^2	2.93	1.39	3.86
MKRS(4.5)	2.90	0.86	3.66
MKRS(3.5)	2.92	0.88	3.69
Females			
$MCS^2(4)$	2.44	0.79	3.52
$MCS^2(3)$	2.54	0.70	3.52
$MCS^2(2)$	2.51	0.65	3.52
$MCS^2(1)$	2.49	0.72	3.52
M3RS	2.56	0.69	3.53
MS^2	2.50	0.67	3.60
MKRS(4.5)	2.51	0.67	3.53
MKRS(3.5)	2.50	0.62	3.52

Analysis of prediction failure

At each TA, the frequency of observations for which the absolute value of the deviation between observed adult stature and predicted adult stature exceeds 4 cm is obtained. For a given TA, this frequency can be interpreted as the number of subjects for which the prediction failed. The average fre-

TABLE 4. Average frequency of prediction failure over target age: |y − predicted y| ≥ 4.0 cm

Technique	Males	Females
$MCS^2(4)$	22.6	16.1
$MCS^2(3)$	23.9	17.5
$MCS^2(2)$	25.1	17.0
$MCS^2(1)$	23.0	16.9
M3RS	24.0	17.3
MS^2	23.3	16.1
MKRS(4.5)	24.0	17.6
MKRS(3.5)	24.3	17.3

quency across TA for each technique is given in Table 4. Treating TA as a blocking variable and assuming independence of observations over TA, an analysis of variance reveals a significant difference among the techniques in terms of average frequency of prediction failure for males ($P = 0.0004$). The techniques with the lowest average frequencies are $MCS^2(4)$, $MCS^2(1)$, and MS^2. For females, there is no significant difference among the techniques in terms of average frequency of prediction failure ($P = 0.1271$); however, the same three techniques have the lowest observed averages.

DISCUSSION

As expected, accuracy (MAD) and reliability (RMSE) of prediction tend to improve steadily as TA increases (tables not provided). Also, the quality of prediction is higher in females than in males in terms of MAD, RMSE, and prediction failure rates.

With respect to selection of a prediction technique from among those investigated, there is no substantial evidence that any one technique is superior, except perhaps in terms of prediction failure and the maximum MAD and maximum RMSE. One of the advantages in using the $MCS^2(k)$ technique is its convenience of application. In terms of estimation of parameters, the $MCS^2(k)$ technique can be carried out with any software that performs regression analysis. The parameter k in the cubic spline smoothing procedure determines the amount of detail that is "smoothed out." As k increases, more detail is retained, possibly resulting in an overfitting of the data. Consequently, a balance between the value of k and the degree of accuracy in prediction must be reached. Based on the results of this study, the technique recommended for accurate and reliable prediction of adult stature is $MCS^2(1)$. The reasons for selecting this procedure are as follows.

1. The $MCS^2(1)$ technique is convenient to apply, requiring only regression software for the smoothing part of the procedure.
2. The low value of k indicates that the technique will provide resistant estimates.
3. The prediction procedure is reduced from a six-step procedure (MS^2) to a five-step procedure.
4. The fit of the $MCS^2(1)$ smoothed curves to the plot of least squares coefficients is more aesthetically acceptable than that for MS^2 (see Figs. 1–8). In particular, the fit is better near the ends of the range of TA values.
5. In terms of accuracy and reliability, $MCS^2(1)$ is comparable to MS^2 with regard to all criteria studied except for the maximum MAD for males and the maximum RMSE for both males and females, where it appears to be superior.
6. In terms of prediction failure, $MCS^2(1)$ has one of the lowest average failure rates, especially for males, and is comparable to MS^2.

Generally, $MCS^2(1)$ seems to be comparable in performance to $MCS^2(2)$ and $MCS^2(3)$, but would most likely provide more resistant estimates because of the lower value of k. $MCS^2(4)$ does quite well in terms of the performance criteria but is almost certainly an overfitting of the data because of the high value of k; this is confirmed by plots of the $MCS^2(4)$ curves (not shown). M3RS has an inflated maximum MAD for females. MKRS(b) for b = 3.5 and 4.5 tend to fall between the best and worst techniques for the various criteria, although their average failure rates tend to be relatively high.

Generally, the use of $MCS^2(1)$ in place of MS^2 is justified on the basis of its convenience of application, its superior fit to the least squares coefficients, and the fact that, with respect to all the performance criteria studied, it is never worse than MS^2 and tends to be better than MS^2 in terms of its maximum MAD for males and maximum RMSE for both males and females. In addition, the $MCS^2(k)$ techniques are more useful than MS^2 when applied to special populations, such as children with Turner syndrome and children with short stature

TABLE 5. MCS^2(1) smoothed coefficients for males for each TA

TA	Intercept	STAT	WT	MPS	SA
3.0	−27.2340	1.26246	−0.25019	0.53461	−0.65638
3.5	−28.2574	1.23505	−0.30869	0.53982	−0.70428
4.0	−28.9167	1.21348	−0.34595	0.53855	−0.73831
4.5	−29.2444	1.19675	−0.36458	0.53180	−0.76317
5.0	−29.2727	1.18387	−0.36721	0.52055	−0.78353
5.5	−29.0343	1.17385	−0.35645	0.50581	−0.80409
6.0	−28.5616	1.16573	−0.33493	0.48857	−0.82953
6.5	−27.8955	1.15849	−0.30526	0.46982	−0.86454
7.0	−27.0179	1.15117	−0.27008	0.45056	−0.91381
7.5	−25.8717	1.14277	−0.23200	0.43178	−0.98203
8.0	−24.4000	1.13231	−0.19364	0.41449	−1.07388
8.5	−22.5461	1.11880	−0.15762	0.39967	−1.19404
9.0	−20.2529	1.10126	−0.12657	0.38832	−1.34721
9.5	−17.0286	1.07869	−0.10311	0.38143	−1.53808
10.0	−12.5118	1.05012	−0.08985	0.38001	−1.77132
10.5	−6.8414	1.01698	−0.08062	0.37987	−2.04178
11.0	−0.1564	0.98228	−0.06811	0.37618	−2.32959
11.5	7.4041	0.94744	−0.05323	0.36898	−2.61742
12.0	15.7014	0.91384	−0.03687	0.35828	−2.88793
12.5	24.0267	0.88289	0.01995	0.34412	−3.12378
13.0	31.5226	0.85598	−0.00337	0.32651	−3.30763
13.5	37.8261	0.83452	0.01195	0.30549	−3.42213
14.0	42.5748	0.81989	0.02512	0.28108	−3.44996
14.5	45.4058	0.81349	0.03521	0.25331	−3.37376
15.0	45.9566	0.81674	0.04133	0.22220	−3.17620
15.5	43.7440	0.83101	0.04257	0.18777	−2.83994
16.0	37.8800	0.85772	0.03802	0.15006	−2.34764
16.5	27.3943	0.89825	0.02677	0.10908	−1.68196
17.0	11.3167	0.95402	0.00791	0.06487	−0.82556
17.5	−11.3232	1.02640	−0.01946	0.01745	0.23891

TABLE 6. MCS^2(1) smoothed coefficients for females for each TA

TA	Intercept	STAT	WT	MPS	SA
3.0	−23.9478	1.45753	−1.14127	0.39955	−0.16198
3.5	−20.3292	1.36486	−1.11756	0.40731	−0.24780
4.0	−17.4687	1.29673	−1.07989	0.40680	−0.31738
4.5	−15.2455	1.24936	−1.03034	0.39932	−0.37813
5.0	−13.5388	1.21894	−0.97101	0.38620	−0.43747
5.5	−12.2278	1.20167	−0.90399	0.36875	−0.50281
6.0	−11.1916	1.19378	−0.83135	0.34829	−0.58157
6.5	−10.5424	1.19145	−0.75518	0.32611	−0.68116
7.0	−10.0670	1.19089	−0.67757	0.30355	−0.80899
7.5	−9.1559	1.18832	−0.60062	0.28191	−0.97248
8.0	−7.1997	1.17992	−0.52639	0.26250	−1.17904
8.5	−3.5889	1.16192	−0.45698	0.24665	−1.43608
9.0	2.2858	1.13050	−0.39448	0.23566	−1.75101
9.5	10.7978	1.08188	−0.34097	0.23084	−2.13125
10.0	20.8509	1.01227	−0.29853	0.23352	−2.58422
10.5	30.8503	0.94329	−0.26275	0.23610	−2.96492
11.0	39.2013	0.89702	−0.22799	0.23100	−3.14271
11.5	44.3092	0.87033	−0.19458	0.21950	−3.14657
12.0	44.5791	0.86007	−0.16283	0.20291	−3.00544
12.5	41.0599	0.86311	−0.13304	0.18251	−2.74831
13.0	36.0835	0.87630	−0.10554	0.15961	−2.40411
13.5	29.9801	0.89652	−0.08063	0.13548	−2.00183
14.0	23.0799	0.92063	−0.05862	0.11144	−1.57041
14.5	15.7131	0.94548	−0.03984	0.08878	−1.13882
15.0	8.2098	0.96794	−0.02458	0.06879	−0.73603
15.5	1.6941	0.98488	−0.01317	0.05276	−0.39099
16.0	−2.4822	0.99315	−0.00591	0.04199	−0.13266
16.5	−3.6480	0.98962	−0.00312	0.03778	0.00999
17.0	−1.1320	0.97115	−0.00511	0.04141	0.00801
17.5	5.7371	0.93460	−0.01219	0.05419	−0.16758

and constitutional delay of growth and puberty, because of the flexibility allowed by the choice in the number and location of the knot(s).

It should be pointed out that all of the procedures used in this comparison predict stature at age 18 years rather than final adult stature. To adjust this for the total median increments in stature after this age, 0.8 cm (males) and 0.6 cm (females) should be added to the predicted value (Roche and Davila, 1972; Roche, 1989).

The smoothed coefficients resulting from the $MCS^2(1)$ procedure are provided in Tables 5 and 6. In order to use these tables, the measurement units are centimeters for stature, years for TA and skeletal age, and kilograms for weight. Consider a 10-year-old boy with STAT 135 cm, wt 35 kilograms, MPS 168 cm, and SA 9.0 years. Then, from the line corresponding to TA = 10.0 in Table 5, the predicted adult stature is computed to be

$$-12.5118 + (1.05012)(135) - (0.08985)(35) + (0.38001)(168) - (1.77132)(9.0) = 174.01 \text{ cm.}$$

Generally, the results of this analysis answer an important question: can a well-known, frequently used method of stature prediction be improved? On the average, the $MCS^2(1)$ method is comparable to MS^2 for all of the performance criteria considered; however, $MCS^2(1)$ provides a modest improvement over MS^2 in terms of the maximum MAD in males and the maximum RMSE in both sexes.

ACKNOWLEDGMENTS

The authors wish to thank Mr. Michael Kepler of Academic Computing Resources, Wright State University, for programming assistance and Dr. Alex Roche, Division of Human Biology, Wright State University, for helpful suggestions.

LITERATURE CITED

Bayley N, Pinneau SR (1952) Tables for predicting adult height from skeletal age: Revised for use with Greulich-Pyle hand standards. J. Pediatr. *40:*423–441.

Bramswig JH, Fasse M, Holthoff ML, vonLengerke HJ, vonPetrykowski W, Schellong G (1990) Adult height in boys and girls with untreated short stature and constitutional delay of growth and puberty: Accuracy of five different methods of height prediction. J. Pediatr. *117:*886–891.

Gasser T, Kohler W, Müller HG, Kneip A (1984a) Velocity and acceleration of height growth using kernel estimation. Ann. Hum. Biol. *11:*397–411.

Gasser T, Müller HG, Kohler W, Molinari L, Prader A (1984b) Nonparametric regression analysis of growth curves. Ann. Stat. *12:*210–229.

Guo S (1989) A computer program for smoothing using kernel estimation. Joint Statistical Meetings; American Statistical Association, Biometric Society, and Institute of Mathematical Statistics 52nd Annual Meeting, Proceedings of the Statistical Computing Section, pp. 306–308.

Kantero RL, Lenko HL (1976) Prediction of adult height. In Compte-Rendu de la XIII Reunion, des Equipes Chargees des Etudes sur la Croissance et le Developpement de l'Enfant Normal, Rennes.

Lenko HL (1979) Prediction of adult height with various methods in Finnish children. Acta Paediatr. Scand. *68:*85–92.

Müller HG (1980) Nonparametric regression analysis of longitudinal data. J Berger, S Fienberg, J Gani, K Krickeberg, and B Singer (eds.): Lecture Notes in Statistics 46. New York: Springer-Verlag, pp 131–150.

Roche AF (1980) Prediction. In FE Johnston, AF Roche, and C Susanne (eds.): Human Physical Growth and Maturation, Methodologies and Factors. New York: Plenum, pp. 177–191.

Roche AF (1989) The final stage of growth in stature. Growth Genet Horm *5:*4–6.

Roche AF, Chumlea WC, Thissen D (1988) Assessing the Skeletal Maturity of the Hand-Wrist: FELS Method. Springfield, IL: CC Thomas.

Roche AF, Davila GH (1972) Late adolescent growth in stature. Pediatrics *50:*874–880.

Roche AF, Wainer H, Thissen D (1975) Predicting Adult Stature for Individuals. Basel: Karger.

Roche AF, Wettenhall HNB (1969) The prediction of adult stature in tall girls. Austr. Paediatr. J. *5:*13–22.

Rosenblatt M (1971) Curve estimates. Ann. Math. Stat. *42:*1815–1841.

Smith PL (1979) Splines as a useful and convenient statistical tool. Am. Stat. *33:*57–62.

Tanner JM (1962) Growth at Adolescence, 2nd edition. Oxford: Blackwell Scientific Publications.

Tanner JM, Landt KW, Cameron N, Carter BS, Patel J (1983) Prediction of adult height from height and bone age in childhood: A new system of equations (TW Mark II) based on a sample including very tall and very short children. Arch. Dis. Child. *58:*767–776.

Tanner JM, Whitehouse RH, Marshall WA, Carter BS (1975) Prediction of adult height from height, bone age and occurrence of menarche, at ages 4 to 16 with allowance for midparent height. Arch. Dis. Child. *50:*14–26.

Tukey JW (1972) Exploratory Data Analysis, 2nd preliminary edition. Reading, MA: Addison-Wesley.

Tukey JW (1977) Exploratory Data Analysis. Reading, MA: Addison-Wesley.

Voss LD, Bailey BJR, Mulligan J, Wilkin TJ, Betts PR (1991) Short stature and school performance—the Wessex Growth Study. Acta Paediatr. Scand. *377:*29–31.

Wainer H, Thissen D (1975) Multivariate semi-metric smoothing in multiple prediction. J. Am. Stat. Assoc. *70:*568–573.

Wold S (1974) Spline functions in data analysis. Technometrics *16:*1–11.

Zachman M, Sobradillo B, Frank M, Frisch H, Prader A (1978) Bayley-Pinneau, Roche-Wainer-Thissen, and Tanner height predictions in normal children and in patients with various pathologic conditions. J. Pediatr. *93:*749–755.

APPENDIX A

Brief mathematical descriptions are given for the MCS^2(k) and MKRS techniques.

MCS^2(k)

Mathematically, the cubic spline model with continuous first derivative and k knots is

$$S(x) = \sum_{j=o}^{3} \beta_{oj}x^j + \sum_{i-1}^{k} \sum_{j=2}^{3} \beta_{ij}(x - t_i)^j_+ + e,$$

$$\text{where } (x - t_i)_+ = \begin{cases} x - t_i, & \text{if } x > t_i, \\ 0, & \text{if } x \leq t_i. \end{cases}$$

These so-called "plus" functions, $(x - t_i)_+$, involve a simple statistical approach, allowing the data to be fit by ordinary least squares estimation (see Smith, 1979).

MKRS(b)

Mathematically, if $f(t_i)$ represents the observed value at time t_i, then

$$f(t_i) = f^*(t_i) + e_i,$$

where $f^*(t_i)$ is the actual value and e_i is the random error component. Define the weight function, $w_i(t)$ as

$$W_i(t) = \frac{1}{b+1} \int_{S_{i-1}}^{S_i} P\left(\frac{t - x}{b}\right) dx,$$

here b is the bandwidth, $s_i = (t_i + t_{i+1})/2$, and $P(*)$ is the kernel function, defined on $(-1, 1)$. Let $f_e(t_j)$ be the kernel estimate at time t_j; then

$$f_e(t_j) = \sum_{i=1}^{n} f(t_i)\, w_i(t_j)$$

for n observations.