1a

$$\ell(w) = \sum_{i=1}^{n} \left(\log |w| + \sum_{j=1}^{d} \log g'(w_j^T x^{(i)}) \right)$$

$$= n \log |w| + \sum_{i=1}^{n} \sum_{j=1}^{d} \log \frac{1}{\sqrt{2\pi}} \exp\left(\left(-\frac{1}{2} (w_j^T x^{(i)})^2 \right) \right)$$

$$= \cdot + \sum_{i=1}^{n} \sum_{j=1}^{d} -\frac{1}{2} (w_j^T x^{(i)})^2 + \log \frac{1}{\sqrt{2\pi}}$$

$$\ell'(w) = n W^{-T} + \frac{-1}{2} \sum_{i=1}^{n} \begin{bmatrix} 2(w_1^T x^{(i)} x^{(:)T}) \\ 2(w_2^T x^{(i)} x^{(:)T}) \\ \vdots \\ 2(w_d^T x^{(i)} x^{(:)T}) \end{bmatrix}$$

$$= n W^{-T} - W \sum_{i=1}^{n} \begin{bmatrix} x^{(i)} x^{(:)T} \\ x^{(i)} x^{(:)T} \\ \vdots \\ x^{(i)} x^{(:)T} \end{bmatrix}$$

$$\ell'(w) = 0 = n w^{-T} - w x^T X$$

$$w^T w = \left(\frac{1}{n} x^T X \right)^{-1}$$

(1b) $\ell = \log|W| + \sum_{j=1}^{d} \log \exp\left(-|w_j^T x^{(i)}|\right)$

$= \quad \cdot \quad - \sum |w_j^T x^{(i)}|$

$$\ell' = W^{-T} - \begin{bmatrix} \text{sign}(w_1^T x^{(i)}) x^{(i)T} \\ \text{sign}(w_2^T x^{(i)}) x^{(i)T} \\ \vdots \\ \text{sign}(w_d^T x^{(i)}) x^{(i)T} \end{bmatrix}$$

$$= W^{-T} - \begin{bmatrix} \text{sign}(w_1^T x^{(i)}) \\ \text{sign}(w_2^T x^{(i)}) \\ \vdots \\ \text{sign}(w_d^T x^{(i)}) \end{bmatrix} x^{(i)T}$$

Update rule:

$$W := W + \alpha \left(W^{-T} - \begin{bmatrix} \text{sign}(w_1^T x^{(i)}) \\ \text{sign}(w_2^T x^{(i)}) \\ \vdots \\ \text{sign}(w_d^T x^{(i)}) \end{bmatrix} x^{(i)T} \right)$$